Title: PSI 2016/2017 Quantum Field Theory III - Lecture 9

Date: Feb 09, 2017 11:30 AM

URL: http://pirsa.org/17020082

Abstract:

Recap:

$$T(z) = \sum_n \frac{L_n}{z^{n+2}}$$

$$L_{-n_1}\cdots$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^2-1)\cdot n$$

$$|h\rangle = \lim_{z,\bar{z}\to 0} \Phi(z,\bar{z})|0\rangle \qquad L_0|h\rangle = h|h\rangle$$

$n>0$ $\quad L_n|h\rangle = 0$ highest weight state

$$L_0(L_{-n}|h\rangle) = (h+n)(L_{-n}|h\rangle)$$

$$L_{-n_1} L_{-n_2} \cdots L_{-n_k} | h \rangle \equiv | \chi \rangle = | \{n_i\}, h \rangle$$

$$\mathcal{L}_{-n_1} \cdots \mathcal{L}_{-n_i} \cdots \mathcal{L}_{-n_k} \,|\, h\rangle \equiv |\chi\rangle = |\{n_i\}, h\rangle$$

$$T(z)\,\Phi(w) = \sum_{n \geq 0} (z-w)^{n-2}\, \hat{\mathcal{L}}_{-n}\Phi(w)$$

$$\langle \Phi_1 \Phi_2 \cdots \Phi_{n-1} \hat{\mathcal{L}}_{-k} \Phi_n \rangle = \mathcal{L}_{-k} \langle \Phi_1 \cdots \Phi_n \rangle$$

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How to build your own unitary solvable D=2 CFTs?

1) unitarity: $\| |\psi\rangle \|^2 \geq 0$ $\quad \langle h|h\rangle > 0$

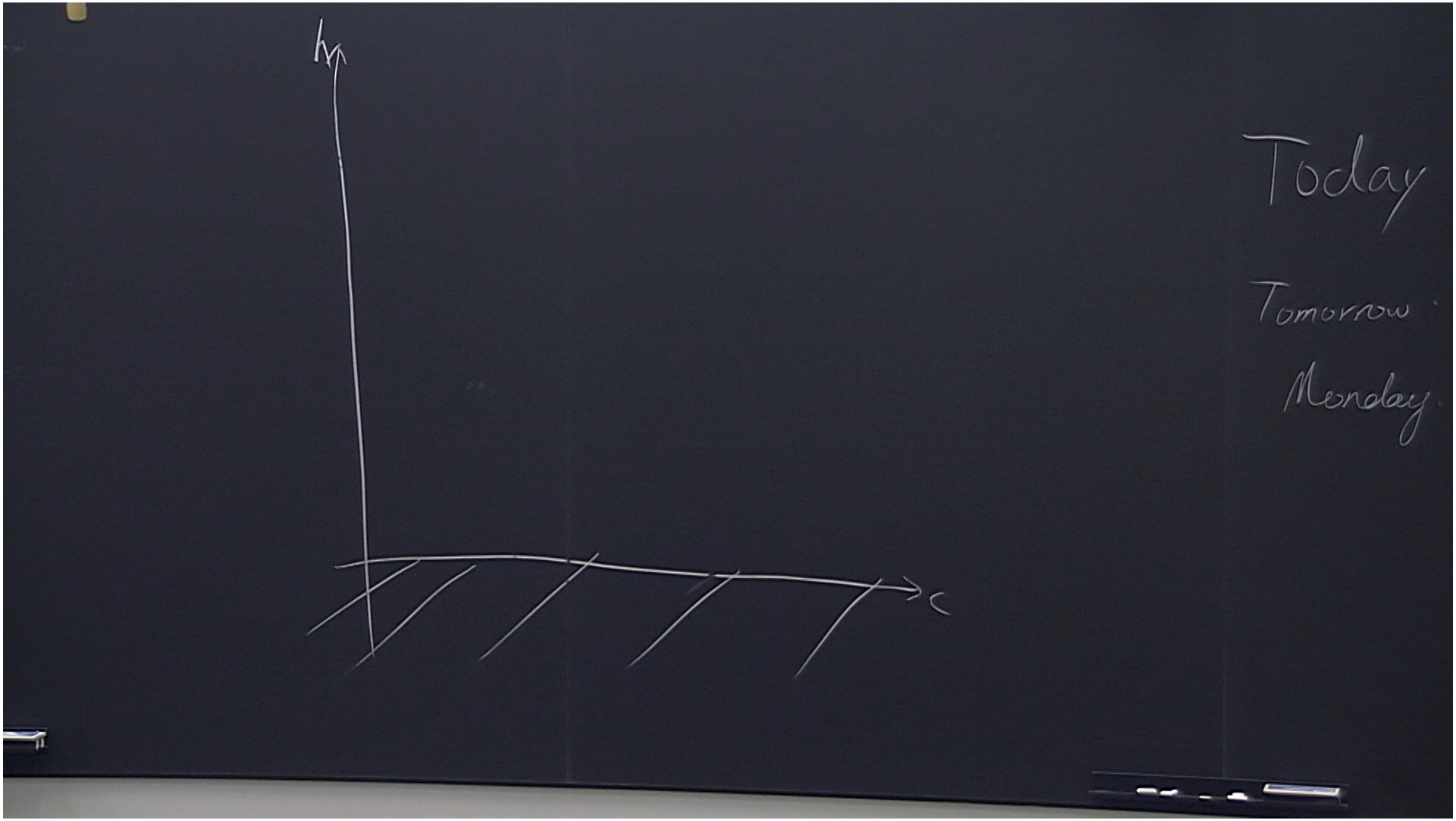
$$L_{-n}^\dagger = L_n \qquad \langle h| L_{-1}^\dagger L_{-1} |h\rangle = \langle h| L_1 L_{-1} |h\rangle$$

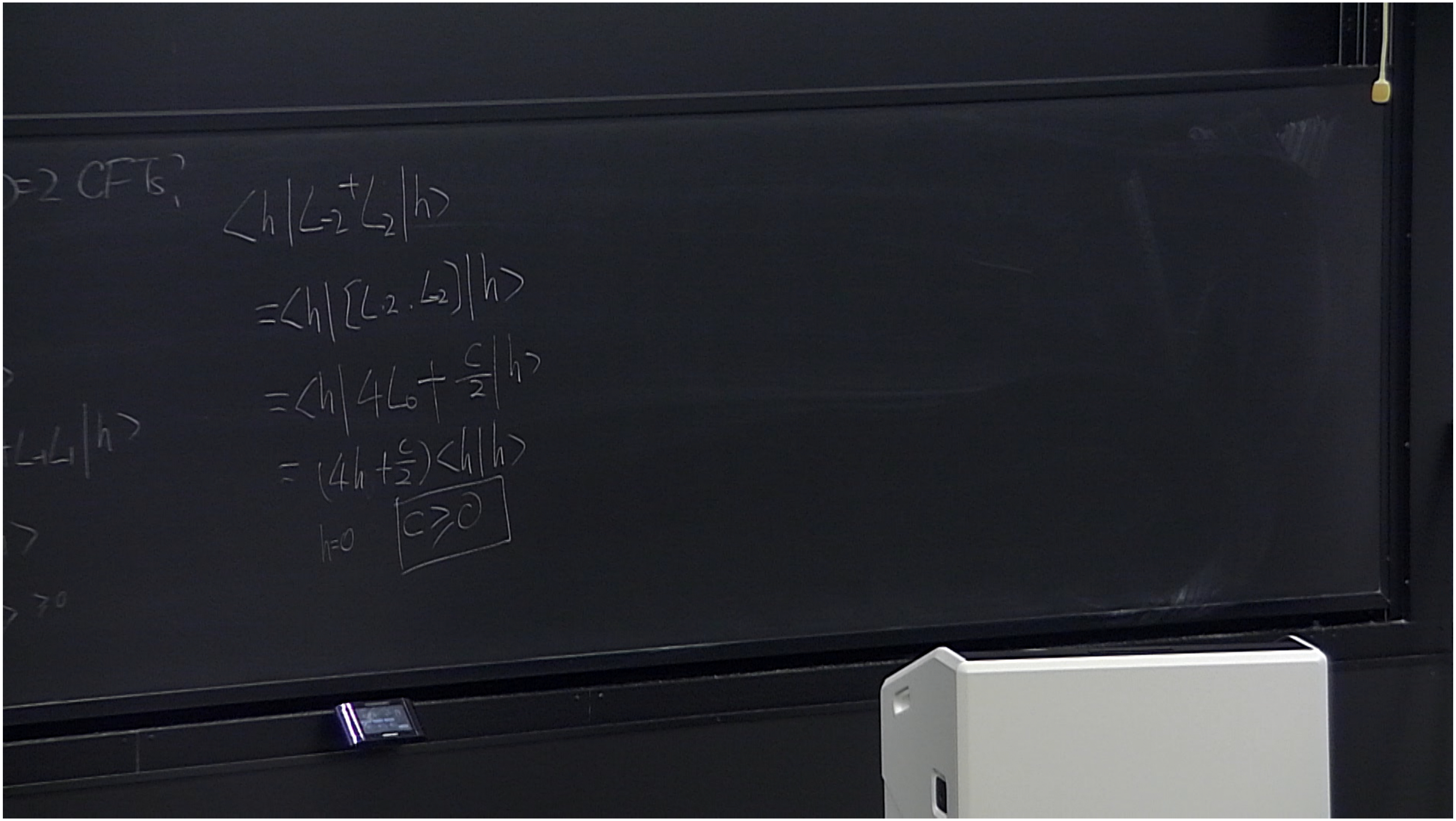
$$= \langle h| [L_1, L_{-1}] + L_{-1} L_1 |h\rangle$$

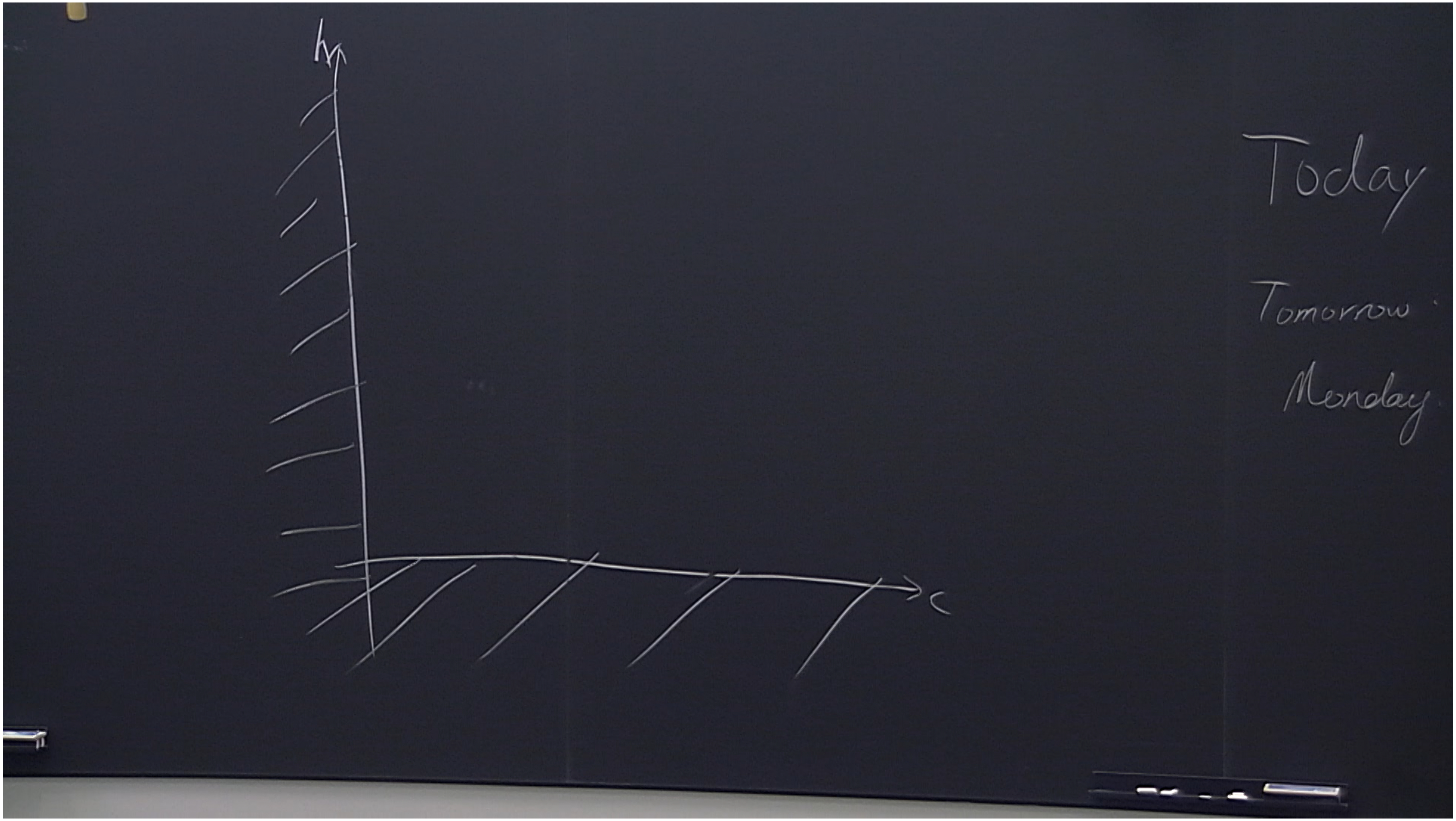
$$= \langle h| 2L_0 |h\rangle$$

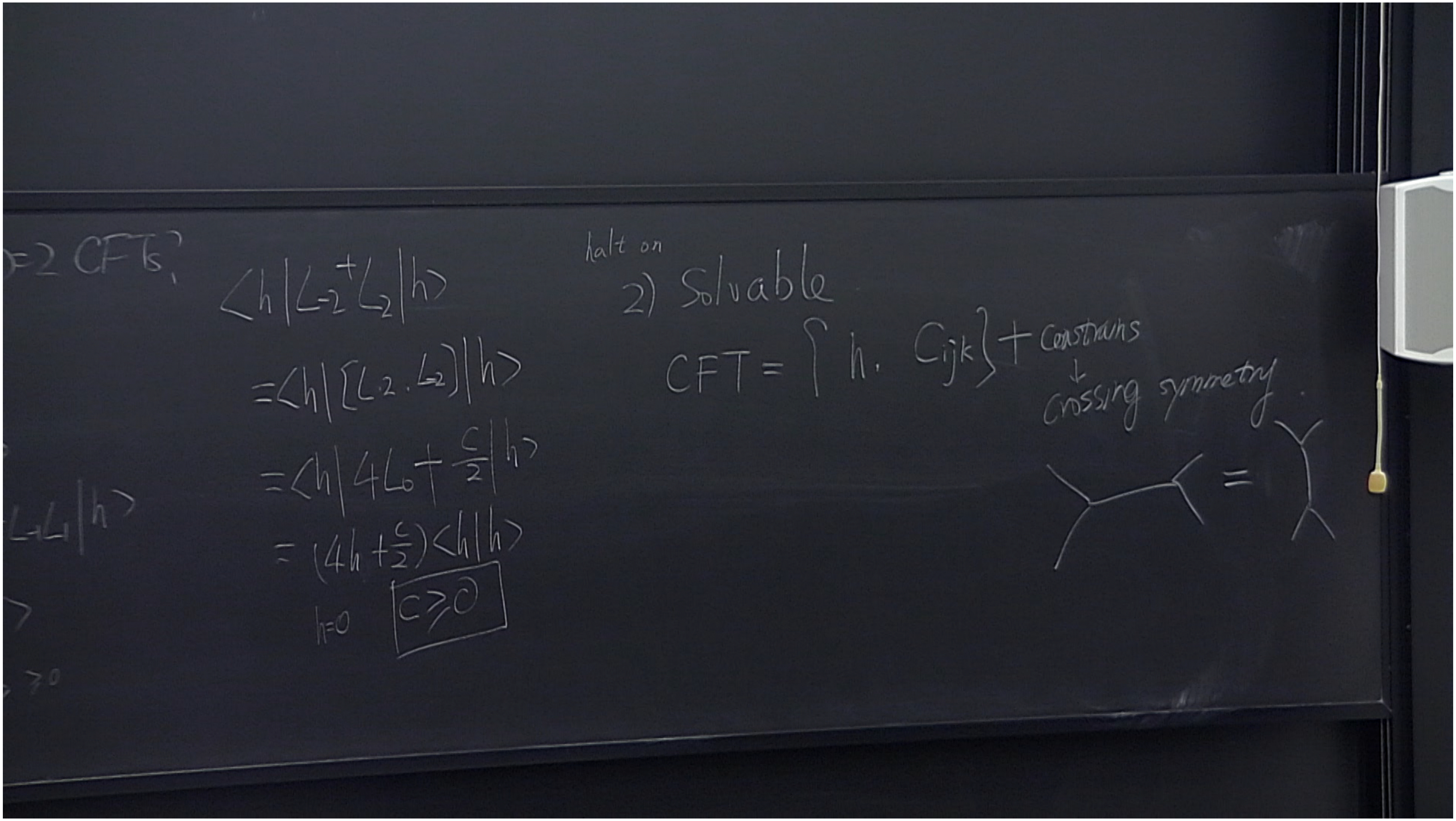
$$= 2h \langle h|h\rangle \geq 0$$

$$\boxed{h \geq 0}$$


h
c
Today
Tomorrow:
Monday:


=2 CFTs?
$\langle h | L_{-2}^{\dagger} L_{-2} | h \rangle$
$= \langle h | [L_{2}, L_{-2}] | h \rangle$
$= \langle h | 4L_0 + \frac{c}{2} | h \rangle$
$= (4h + \frac{c}{2}) \langle h | h \rangle$
$h=0$
$\boxed{c \geq 0}$
$L_{-1}L_1 | h \rangle$
≥ 0


Today
Tomorrow
Monday


=2 CFTs?
$\langle h|L_{-2}^{+}L_{2}|h\rangle$
$=\langle h|[L_{2}, L_{-2}]|h\rangle$
$=\langle h|4L_{0}+\frac{c}{2}|h\rangle$
$=(4h+\frac{c}{2})\langle h|h\rangle$
$h=0$ $\boxed{c\geq 0}$
halt on
2) Solvable
$\mathrm{CFT}=\{h, C_{ijk}\}$ + constraints
crossing symmetry
=

=2 CFTs

$L_1 L_1|h\rangle$

≥ 0

$$\langle h|L_{-2}^{+}L_{2}|h\rangle$$

$$=\langle h|[L_{2}, L_{-2}]|h\rangle$$

$$=\langle h|4L_0+\frac{c}{2}|h\rangle$$

$$=(4h+\frac{c}{2})\langle h|h\rangle$$

$h=0$ $\boxed{c\geq 0}$

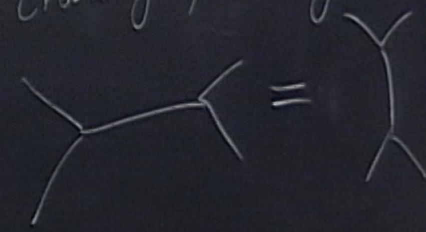
halt on

2) Solvable

$$\mathrm{CFT} = \{h,\ C_{ijk}\} + \text{constrains}$$

$\downarrow$

crossing symmetry



find $\langle \Phi_1 \Phi_2 \Phi_3 \Phi_4\rangle$

$z, \bar{z} \to 0$

$n > 0 \quad L_n | h \rangle = 0$ highest weight state

$L_0 (L_{-n} | h \rangle) = (h+n) (L_{-n} | h \rangle)$

make a wish $\langle \Phi_1 \Phi_2 \Phi_3 \chi \rangle = 0$

Claim: if: $| \chi \rangle$ also a highest weight state

the wish is granted

$n > 0 \quad L_n | \chi \rangle = 0$

observation $\langle \{n_i\}, h | \{n_i'\}, h' \rangle \propto \delta_{h h'} \delta_{\Sigma n_i = \Sigma n_i'}$

same level

$\langle \chi' | \chi \rangle$

$= (\langle h | L_{n_i} \cdots L_{n_k}) | \chi \rangle$

$= \langle h | L_{n_k} \cdots L_{n_1} | \chi \rangle$

$= 0$

$z, \bar{z} \to 0$

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observation $\langle \{n_i\}, h | \{n_i'\}, h' \rangle \propto \delta_{h h'} \delta_{\Sigma n_i = \Sigma n_i'}$

same level $\langle \chi' | \chi \rangle$

$= (\langle h | L_{n_1} \cdots L_{n_k}) |\chi\rangle$

$= \langle h | L_{n_k} \cdots L_{n_1} |\chi\rangle$

$= 0$

$(\langle h | \chi \rangle = 0)$

Null state

same level

$$\langle \chi' | \chi \rangle$$

$$= (\langle h | L_{n_1} \cdots L_{n_k}) | \chi \rangle$$

$$= \langle h | L_{n_k} \cdots L_{n_1} | \chi \rangle$$

$$= 0$$

$$(\langle h | \chi \rangle = 0)$$

Null state

Find Null states

level 1 $\quad L_{-1}|h\rangle$

$$L_1 L_{-1}|h\rangle = [L_1, L_{-1}]|h\rangle = 2L_0|h\rangle = 2h|h\rangle = 0$$

$$h=0$$

level 2 $\quad L_{-2}|h\rangle + \alpha L_{-1}^2 |h\rangle$

$$= 2h\langle h|h\rangle \geq 0$$

$$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^2|h\rangle$$

$$= [L_1, L_{-2}]|h\rangle + \alpha [L_1, L_{-1}^2]|h\rangle$$

$$= 3L_{-1}|h\rangle + \alpha\left(L_{-1}[L_1, L_{-1}] + [L_1, L_{-1}]L_{-1}\right)|h\rangle$$

$$= 3L_{-1}|h\rangle + \alpha\left(2L_{-1}L_0 + 2L_0 L_{-1}\right)|h\rangle$$

$$= 3L_{-1}|h\rangle + 2\alpha\left(hL_{-1} + (h+1)L_{-1}\right)|h\rangle$$

$$= \left(3 + 2\alpha(2h+1)\right)L_{-1}|h\rangle$$

$$3 + 2\alpha(2h+1) = 0$$

$$\alpha = -\frac{3}{2(2h+1)}$$

$$[L_2, L_{-2}] = 4L_0 + \frac{c}{2}$$

$$L_{-1}[L_2, L_{-1}] + [L_2, L_{-1}]L_{-1}$$

$$= 3L_1 L_{-1}$$

$$= 6L_0$$

$$L_2|\chi\rangle$$

$$3+2\alpha(2h+1)=0$$

$$\alpha=-\frac{3}{2(2h+1)}$$

$$[L_2, L_{-2}] = 4L_0+\frac{c}{2}$$

$$L_{-1}[L_2, L_{-1}] + [L_2, L_{-1}]L_{-1}$$

$$= 3L_1 L_{-1}$$

$$= 6L_0$$

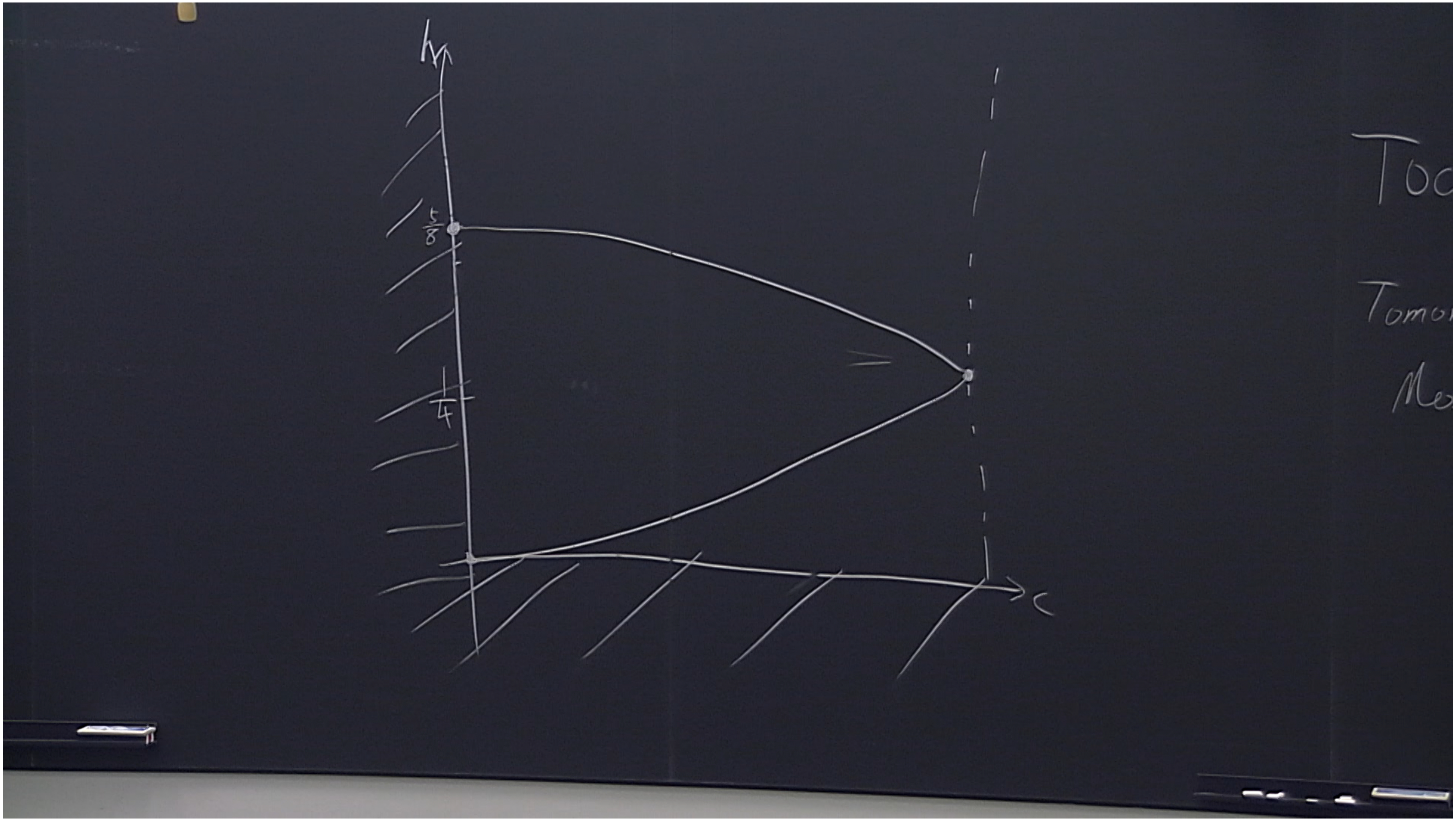
$$L_2|\chi\rangle$$

$$= \left(4h+\frac{c}{2}+6\alpha h\right)|h\rangle$$

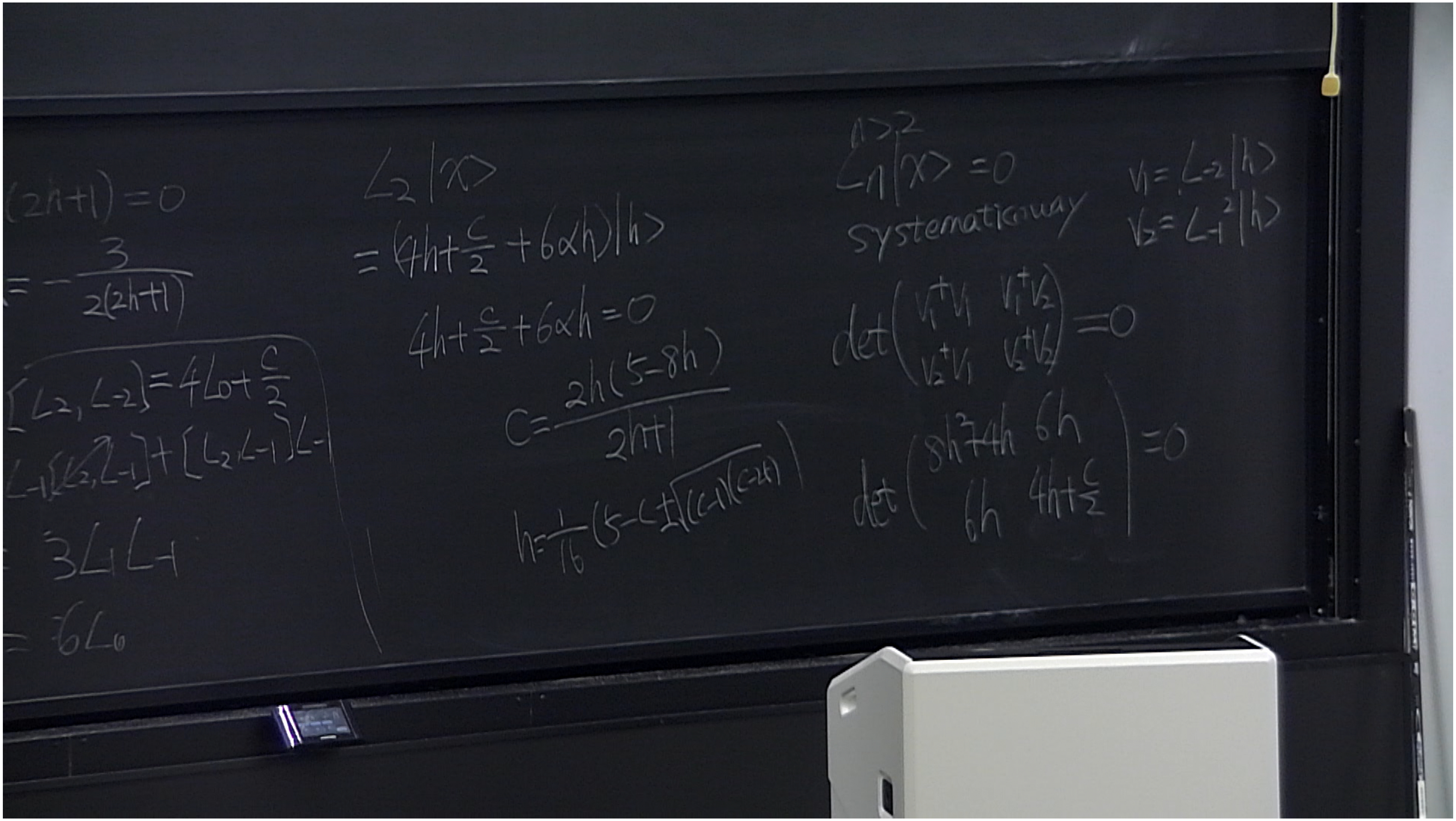
$$4h+\frac{c}{2}+6\alpha h = 0$$

$$c=\frac{2h(5-8h)}{2h+1}$$

$$h=\frac{1}{16}\left(5-c\pm\sqrt{(c-1)(c-25)}\right)$$



h
5/8
1/4
c
Tod
Tomo
Me

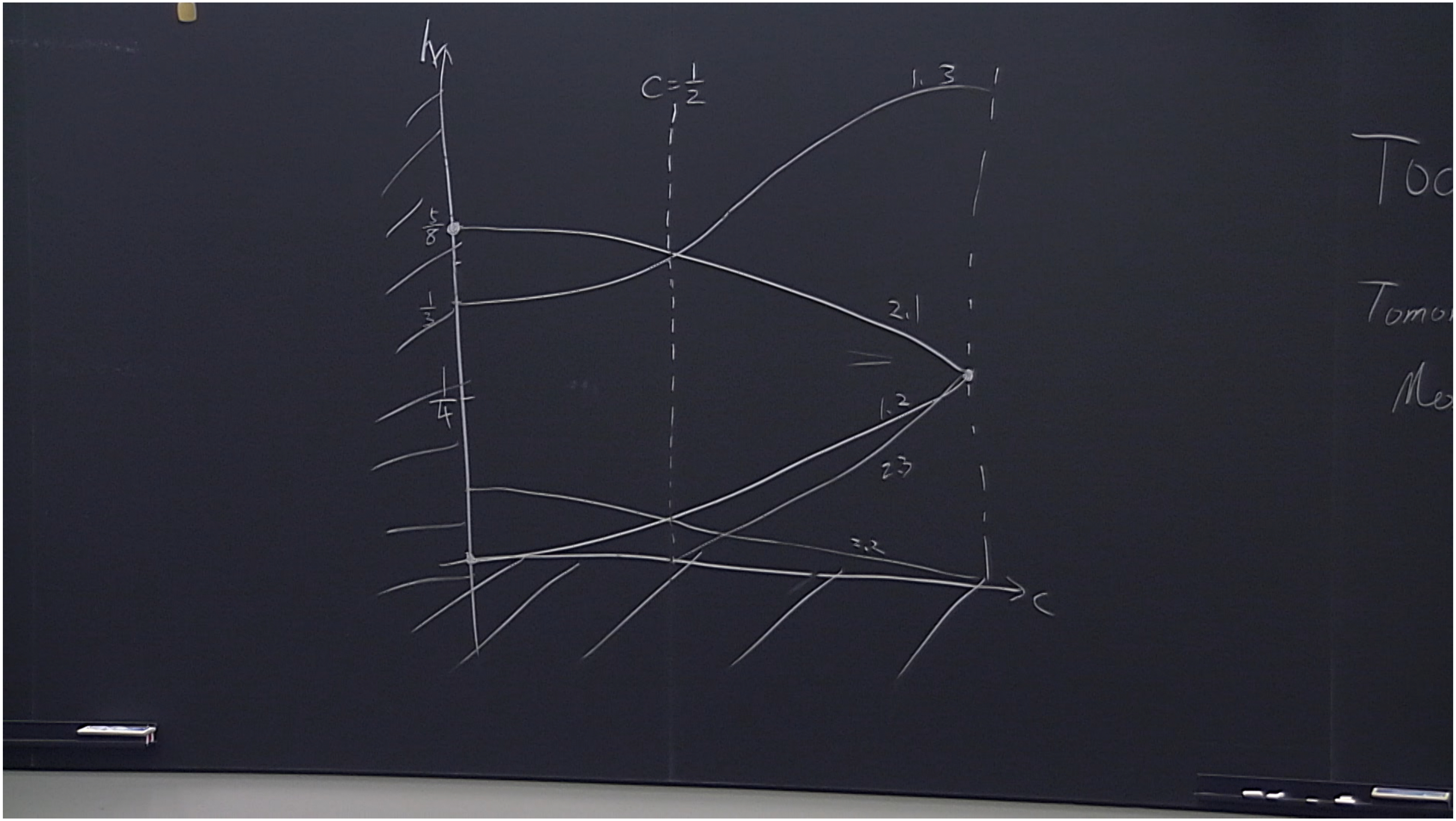

(2h+1) = 0
$= -\frac{3}{2(2h+1)}$
$[L_2, L_{-2}] = 4L_0 + \frac{c}{2}$
$3L_1L_{-1}$
$6L_0$
$L_2|\chi\rangle$
$= (4h + \frac{c}{2} + 6\alpha h)|h\rangle$
$4h + \frac{c}{2} + 6\alpha h = 0$
$c = \frac{2h(5-8h)}{2h+1}$
$L_1|\chi\rangle = 0$
systematic way
$v_1 = L_{-2}|h\rangle$
$v_2 = L_{-1}^2|h\rangle$
$\det\begin{pmatrix} v_1^\dagger v_1 & v_1^\dagger v_2 \\ v_2^\dagger v_1 & v_2^\dagger v_2 \end{pmatrix} = 0$
$\det\begin{pmatrix} 8h^2+4h & 6h \\ 6h & 4h+\frac{c}{2} \end{pmatrix} = 0$

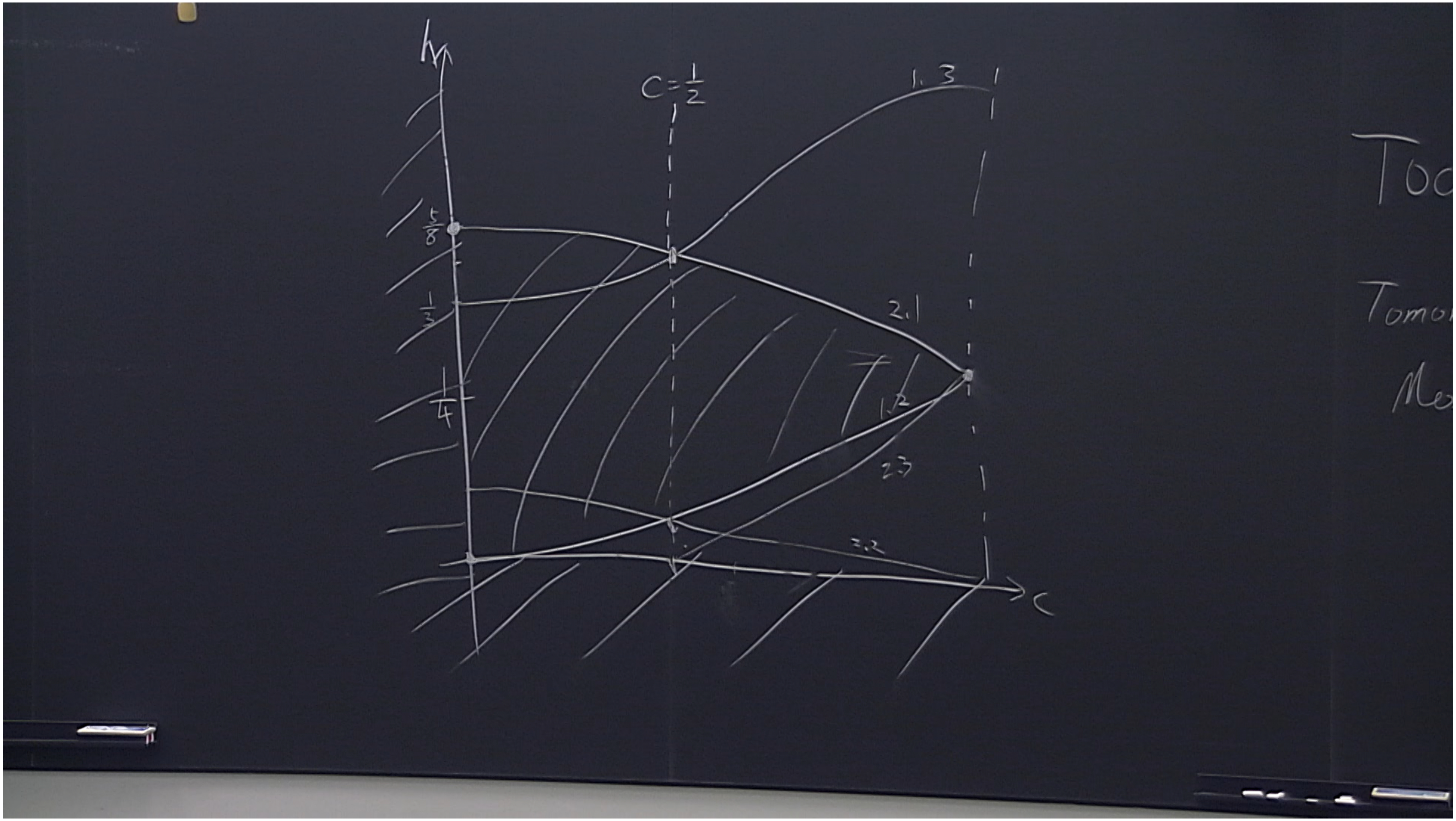
Kac

$$\det M_N \propto \prod_{pq \leq N} \left(h - h_{p,q}(c)\right)^{P(N-pq)}$$

$$h_{(p,q)}(c) = \frac{1-c}{96}\left(\left((p+q) \pm (p-q)\sqrt{\frac{25-c}{1-c}}\right)^2 - 4\right)$$

$p=1 \quad q=2$


h
c = 1/2
1.3
5/8
1/3
2.1
1/4
1.2
2.3
2.2
c
Tod
Tomo
Me


h
c
$c=\frac{1}{2}$
1.3
$\frac{5}{8}$
$\frac{1}{3}$
$\frac{1}{4}$
2.1
1.2
2.3
2.2

Unitarity. $\det \geq 0$

Unitary Minimal Models

$$c = 1 - \frac{6}{m(m+1)} \qquad m = 3, 4, 5, \ldots$$

$$h_{p,q}(m) = \frac{((m+1)p - mq)^2 - 1}{4m(m+1)} \qquad \begin{array}{l} p \to m-p \\ q \to m+1-q \end{array}$$

$$(p,q) \longleftrightarrow (m-p,\ m+1-q)$$

$t \geq 0$

Unitary Minimal Models

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$$(p, q) \longleftrightarrow (m-p, m+1-q)$$

$m = 3 \quad c = \frac{1}{2}$

$p, q \leq 3$

$\mathbb{1}$

$\langle \Phi_1 \Phi_2 \Phi_3 \rangle$

$\downarrow$ 2d Ising model