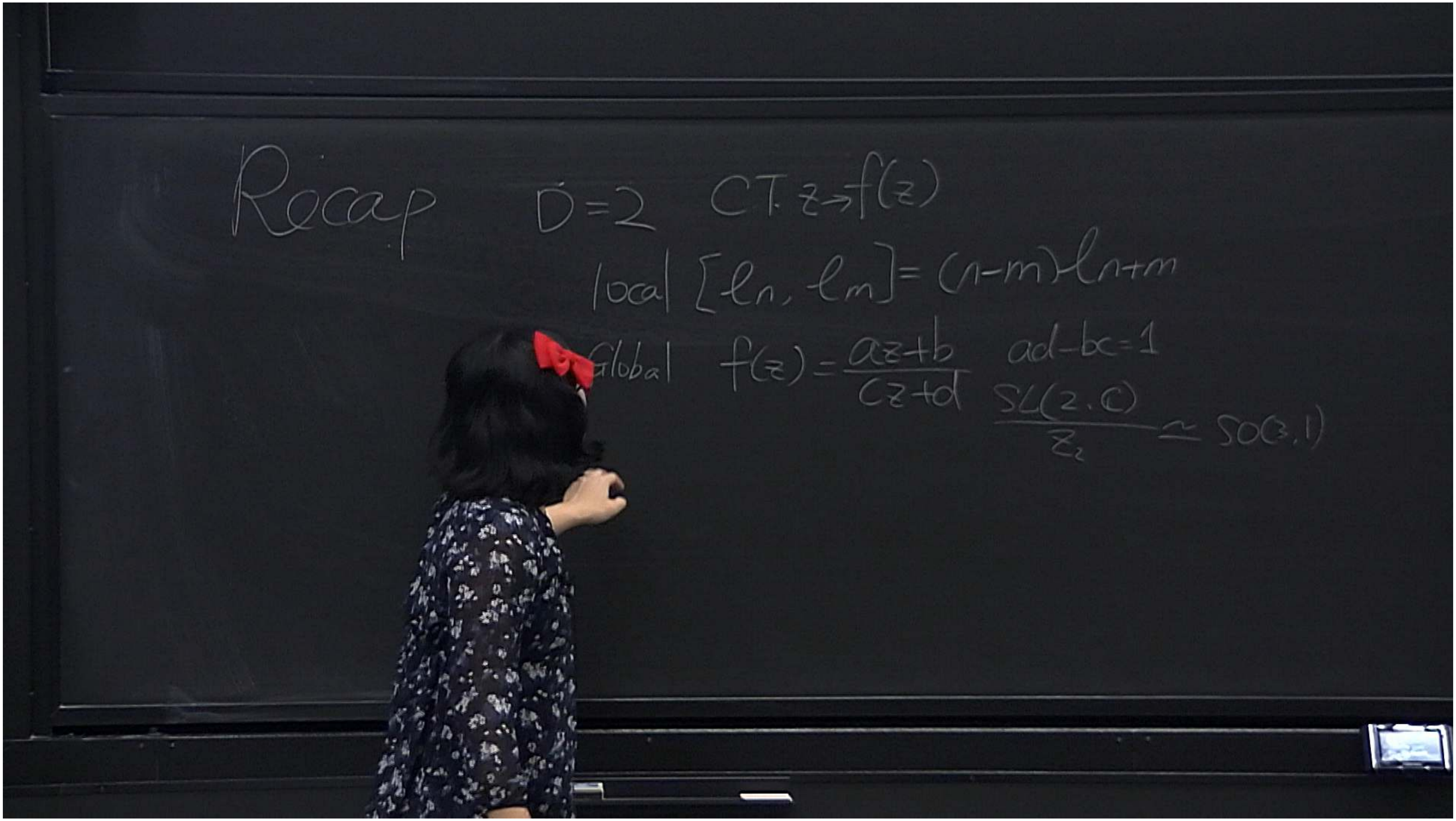
Title: PSI 2016/2017 Quantum Field Theory III - Lecture 7

Date: Feb 07, 2017 11:30 AM

URL: http://pirsa.org/17020080

Abstract:


Recap
D=2
C.T. z→f(z)
local [ℓn, ℓm] = (n−m)ℓn+m
Global f(z) = (az+b)/(cz+d)
ad−bc=1
SL(2,C)/Z2 ≃ SO(3,1)

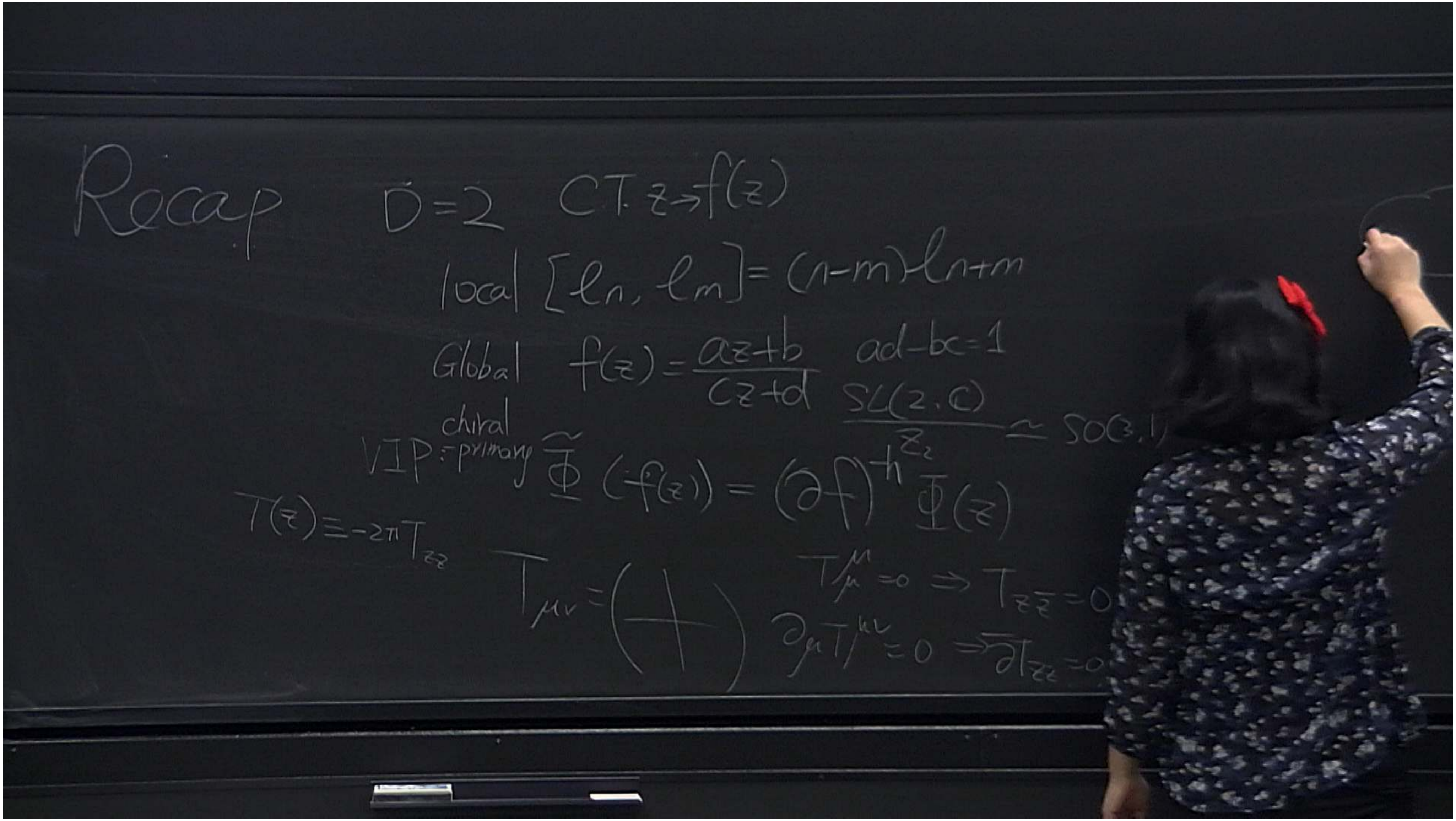
Recap

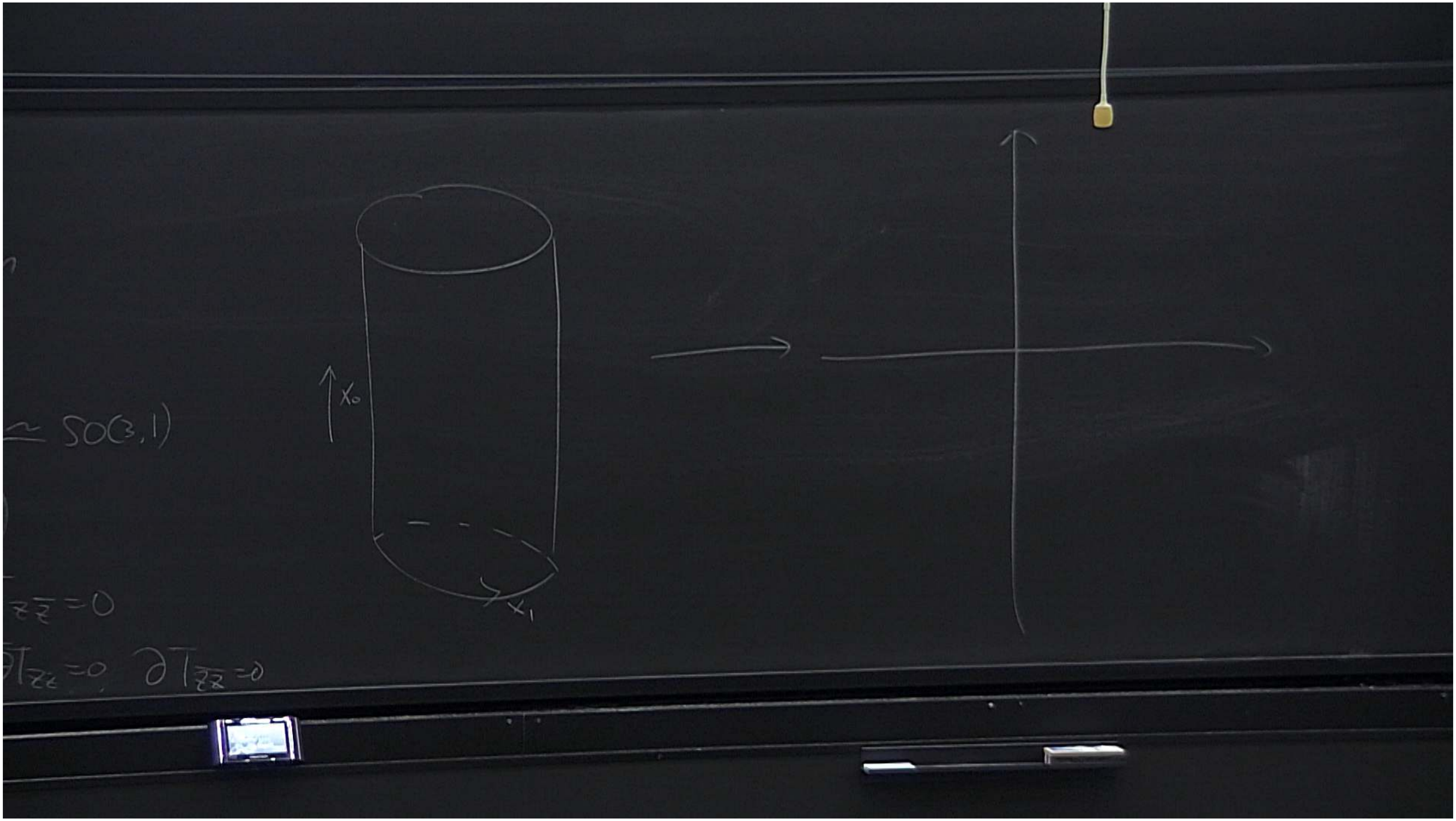
$D=2$ C.T. $z \to f(z)$

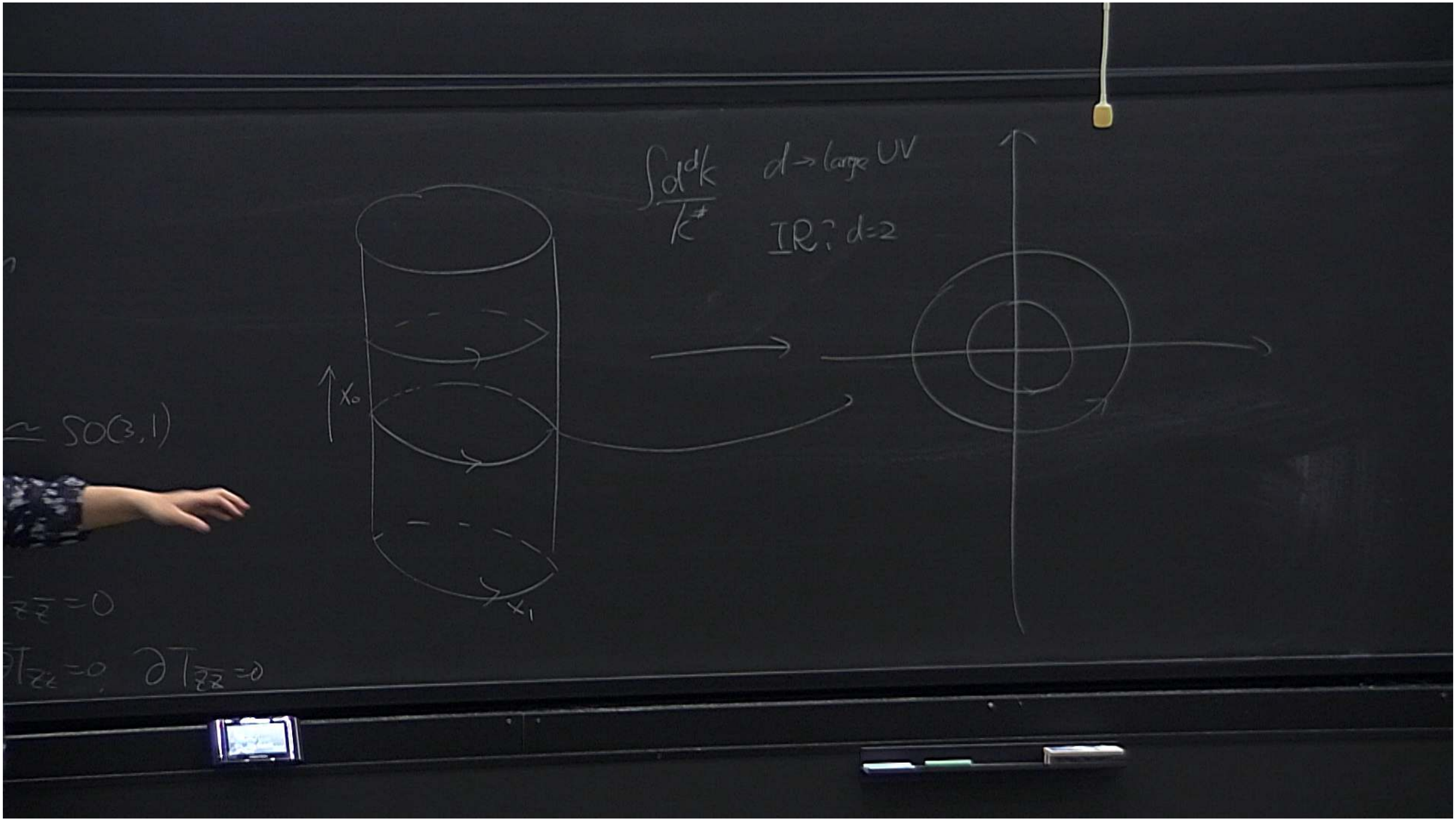
local $[\ell_n, \ell_m] = (n-m)\,\ell_{n+m}$

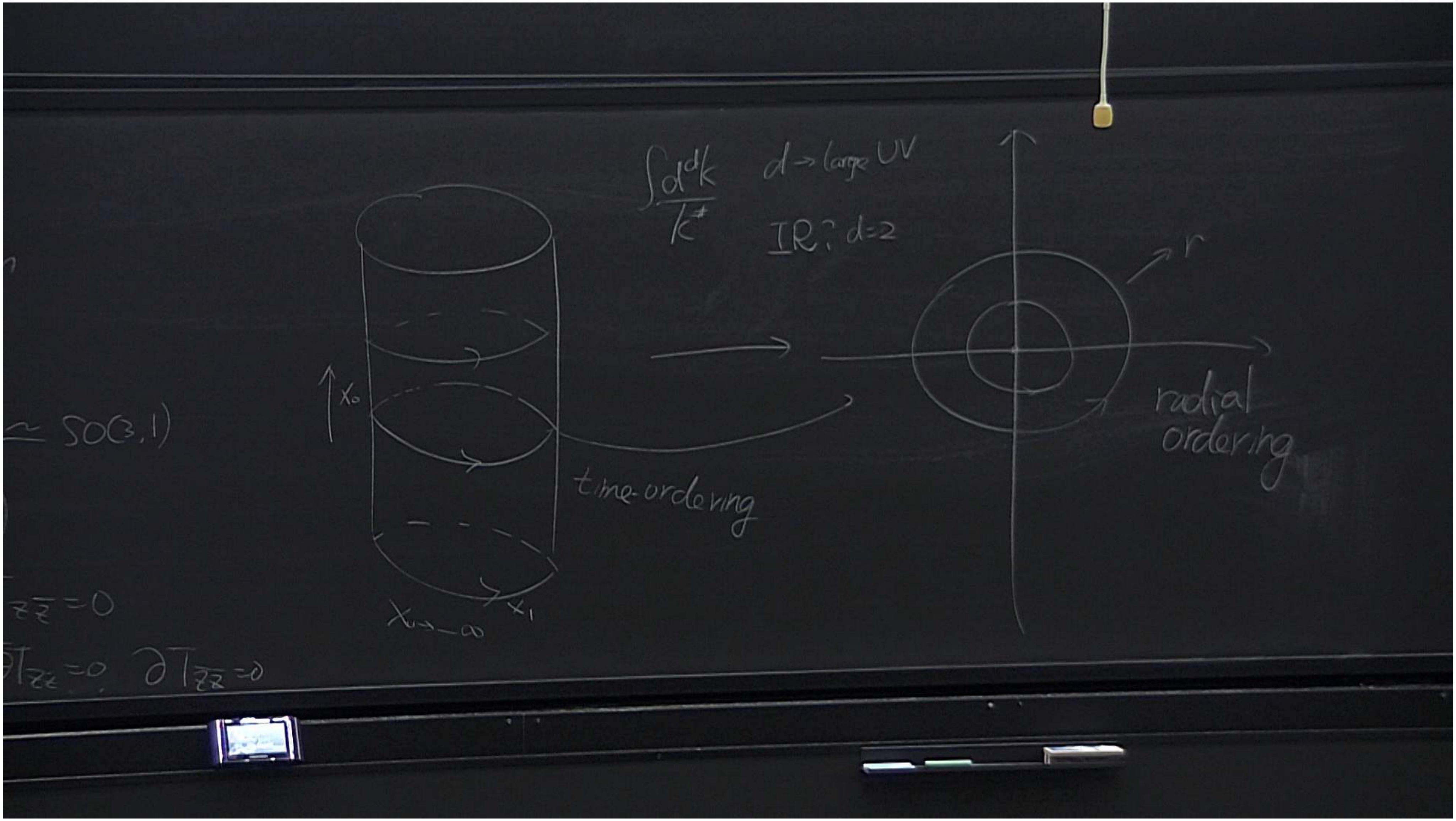
Global $f(z) = \dfrac{az+b}{cz+d}$ $\quad ad-bc=1$ $\quad \dfrac{SL(2,\mathbb{C})}{\mathbb{Z}_2} \simeq SO(3,1)$

VIP = chiral primary $\quad \tilde{\Phi}(f(z)) = (\partial f)^{-h}\,\Phi(z)$

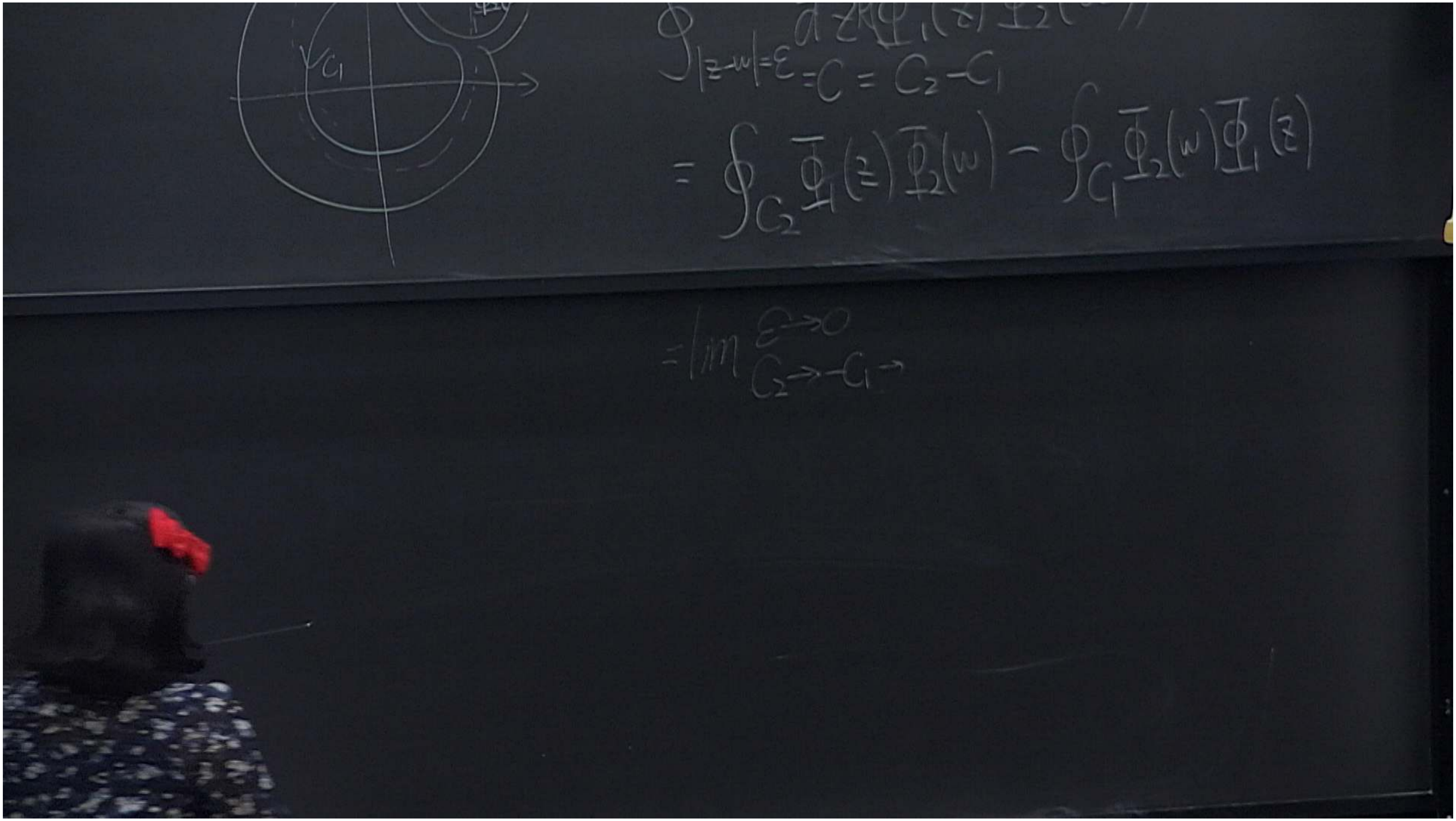

Recap
D=2
C.T. z→f(z)
local [ln, lm] = (n−m) ln+m
Global f(z) = (az+b)/(cz+d)
ad−bc=1
SL(2,C)/Z2 ≃ SO(3,1)
chiral
VIP = primary
Φ̃(f(z)) = (∂f)^−h Φ(z)
T(z) ≡ −2π Tzz
Tμν =
T^μ_μ = 0 ⇒ Tzz̄ = 0
∂μT^μν = 0 ⇒ ∂̄Tzz = 0

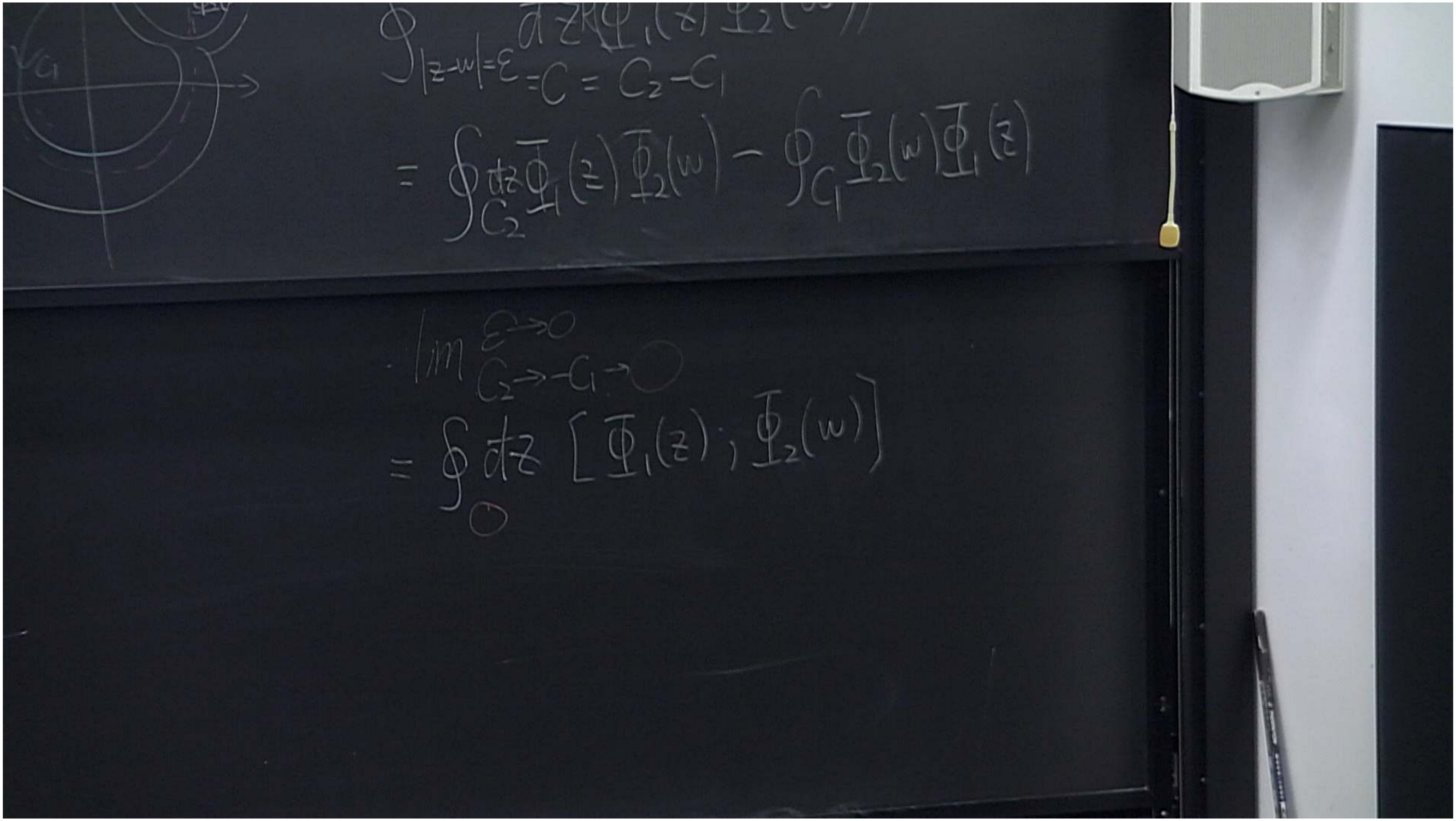

$\simeq SO(3,1)$
x_0
x_1
$\partial T_{\bar z \bar z} = 0$

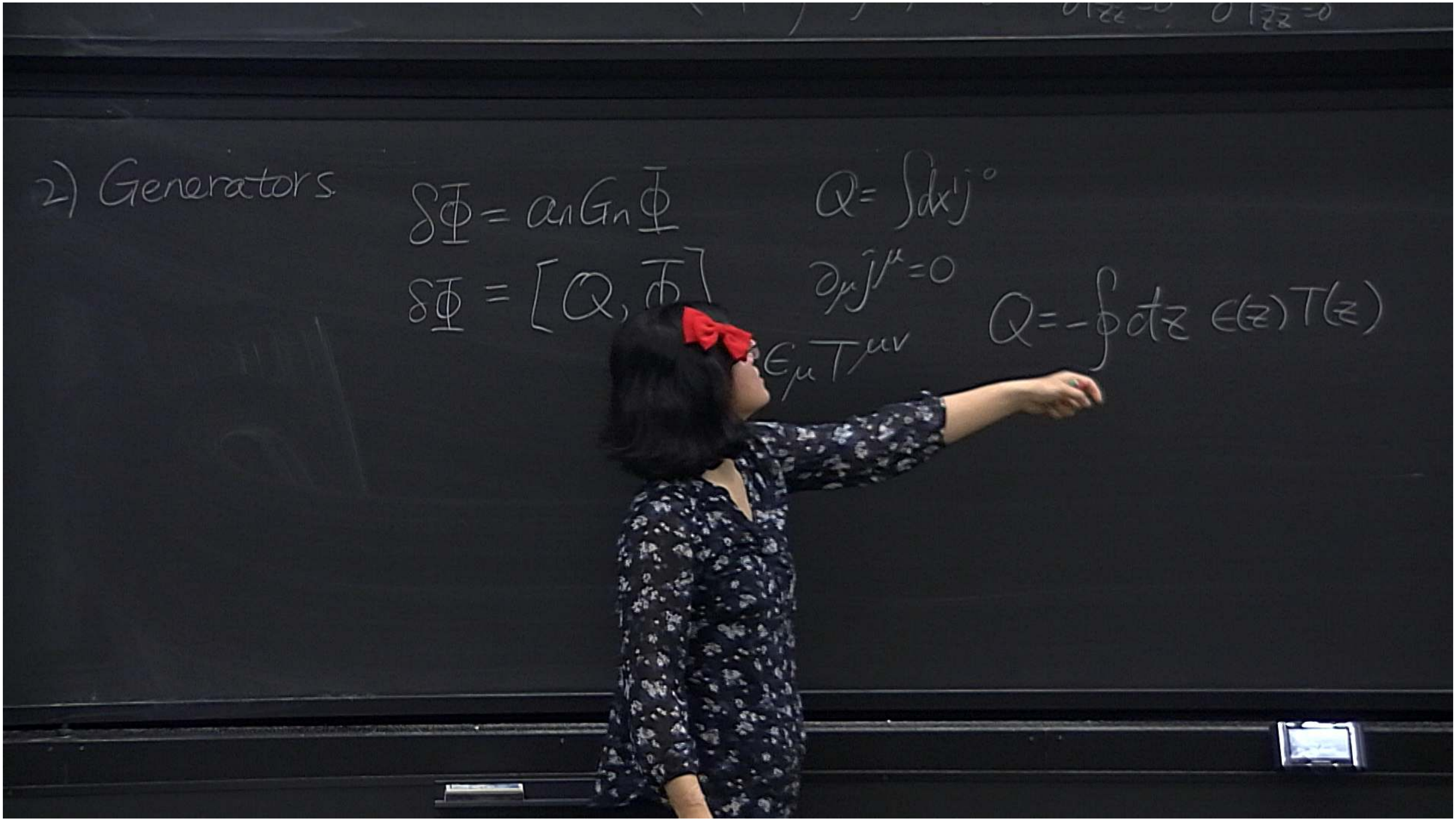

d → large UV
IR: d=2
SO(3,1)



d → large UV
IR ? d=2
r
radial ordering
time-ordering
x0
x1
x0 → −∞
≃ SO(3,1)


$\oint_{|z-w|=\varepsilon} dz\, R(\Phi_1(z)\Phi_2(w))$
$= C = C_2 - C_1$
$= \oint_{C_2} \Phi_1(z)\Phi_2(w) - \oint_{C_1} \Phi_2(w)\Phi_1(z)$
$= \lim_{\substack{\varepsilon \to 0 \\ C_2 \to -C_1}}$


C_1
$\oint_{|z-w|=\varepsilon} dz\, \Phi_1(z)\,\Phi_2(w)$
$= C = C_2 - C_1$
$= \oint_{C_2} dz\, \Phi_1(z)\,\Phi_2(w) - \oint_{C_1} \Phi_2(w)\,\Phi_1(z)$
$\lim_{\varepsilon \to 0,\; C_2 \to -C_1 \to \bigcirc}$
$= \oint_{\bigcirc} dz\, [\Phi_1(z), \Phi_2(w)]$


2) Generators.
$\delta\Phi = a_n G_n \Phi$
$\delta\Phi = [Q, \Phi]$
$Q = \int dx^1 j^0$
$\partial_\mu j^\mu = 0$
$\epsilon_\mu T^{\mu\nu}$
$Q = -\oint dz\, \epsilon(z) T(z)$

$\partial T_{zz}=0, \quad \partial T_{\bar z\bar z}=0$

2) Generators

$$\delta\Phi = a_n G_n \Phi \qquad Q = \int dx^1 j^0$$

$$\delta\Phi = [Q, \Phi] \qquad \partial_\mu j^\mu = 0 \qquad Q = -\oint dz\, \epsilon(z) T(z) + \ldots$$

$$j^\nu = \epsilon_\mu T^{\mu\nu}$$

$$= -\oint_C dz\, \epsilon(z)[T(z), \Phi(w)]$$

$$= -\oint_{C'} dz\, \epsilon(z) T(z) \Phi(w)$$

$\partial T_{\bar z z}=0$; $\partial T_{\bar z \bar z}=0$

$dx^i j^\circ$

$=0$

$$Q = -\oint dz\, \epsilon(z)\, T(z) + \dots$$

$$= -\oint_{C'} dz\, a_n z^{n+1}\, T(z)\, \Phi(w)$$

$\underline{\underline{\text{want}}}$ $-a_n L_n \Phi(w)$

$$T(z) = \sum_n \frac{L_m}{z^{m+2}} \qquad L_m = \oint T(z)\, z^{m}$$

$(w)]$

(w)

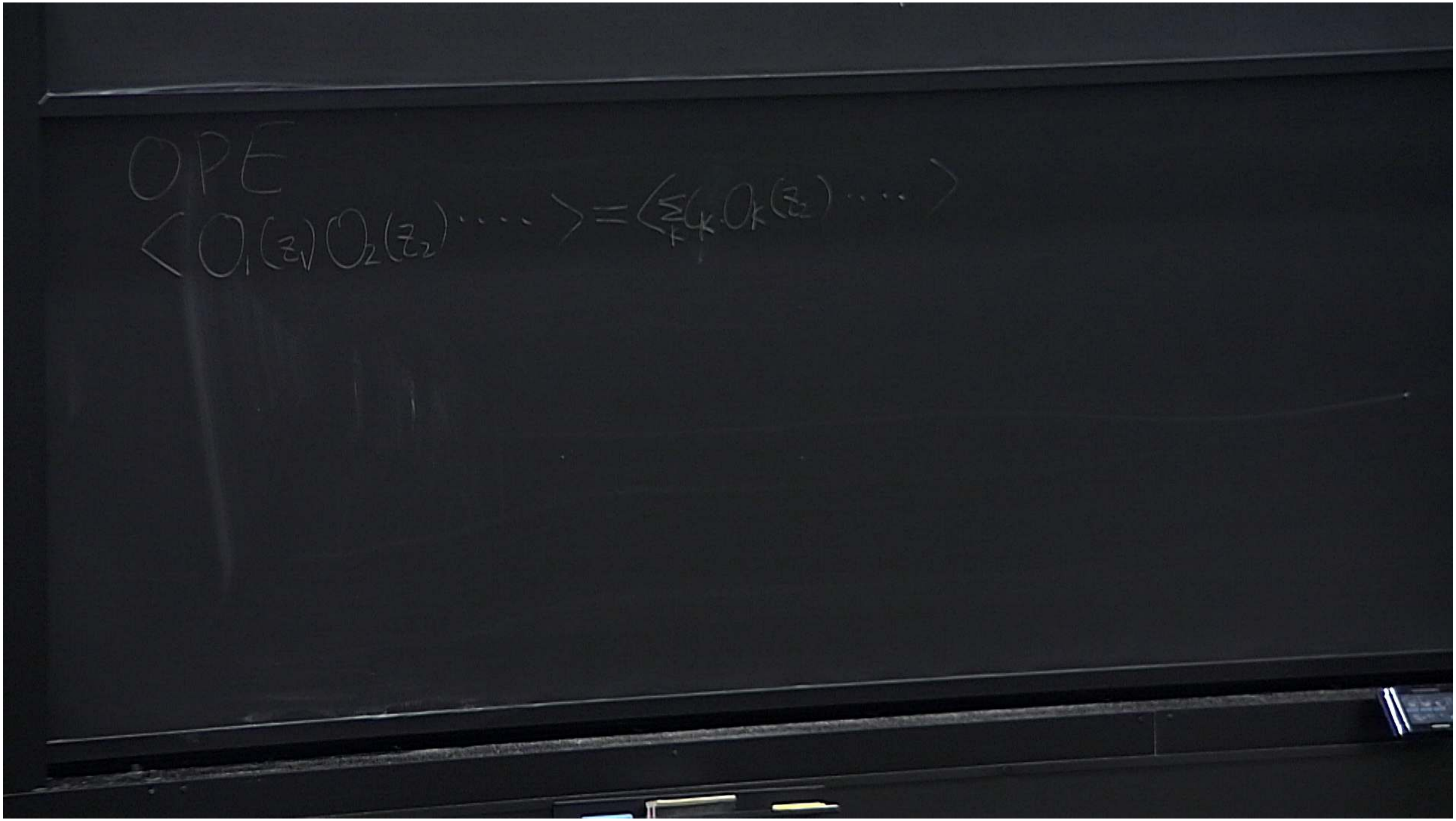
$) T(z) + \ldots$

$$= -\oint_{C'} dz\, a_n z^{n+1}\, T(z)\, \Phi(w)$$

$\underline{\underline{\text{want}}}$ $\; -a_n L_n \Phi(w)$

$$T(z) = \sum_n \frac{L_m}{z^{m+2}} \qquad L_m = \oint dz\, T(z)\, z^{m+1}$$

$$[L_m, L_n]$$


OPE
$\langle O_1(z_1) O_2(z_2) \cdots \rangle = \langle \sum_k C_k O_k(z_2) \cdots \rangle$

OPE

$$\langle O_1(z_1)\, O_2(z_2) \cdots \rangle = \langle \sum_k c_k\, O_k(z_2) \cdots \rangle$$

$$\delta\Phi = \tilde{\Phi}(z) - \Phi(z$$

$$T(z)\,\Phi_h(w) \sim \;?$$

$$\delta\Phi = \delta\Phi$$

$$\tilde{\Phi}(f(z)) = (\partial f)^{-h}\,\Phi(z)$$

$$\delta\Phi = \tilde{\Phi}(z) - \Phi(z)$$

$$\tilde{\Phi}(z+\epsilon(z)) = (1+\partial\epsilon)^{-h}\,\Phi(z)$$

$$\tilde{\Phi}(z) + \epsilon(z)\partial\Phi(z) = (1-h\partial\epsilon)\Phi(z)$$

$$\delta\Phi = -\epsilon\partial\Phi - h\partial\epsilon\,\Phi$$

$$\lim_{\substack{\epsilon\to 0 \\ C_2\to -C_1\to \bigcirc}} = \oint_{\bigcirc} dz\,\left[\Phi_1(z)\,;\,\Phi_2(w)\right]$$

$$\delta \Phi(w) = -\epsilon(w)\partial \Phi(w) - h\partial\epsilon(w)\Phi(w)$$

$$\epsilon(w)\partial\Phi(w) = \oint dz \frac{\epsilon(z)}{z-w}\partial\Phi(w)$$

$$h\partial\epsilon(w)\Phi(w) = \oint dz \frac{h\epsilon(z)}{(z-w)^2}\Phi(w)$$

$$T(z)\Phi(w) \sim \frac{\partial\Phi(w)}{z-w} + \frac{h\Phi(w)}{(z-w)^2} + \cdots$$

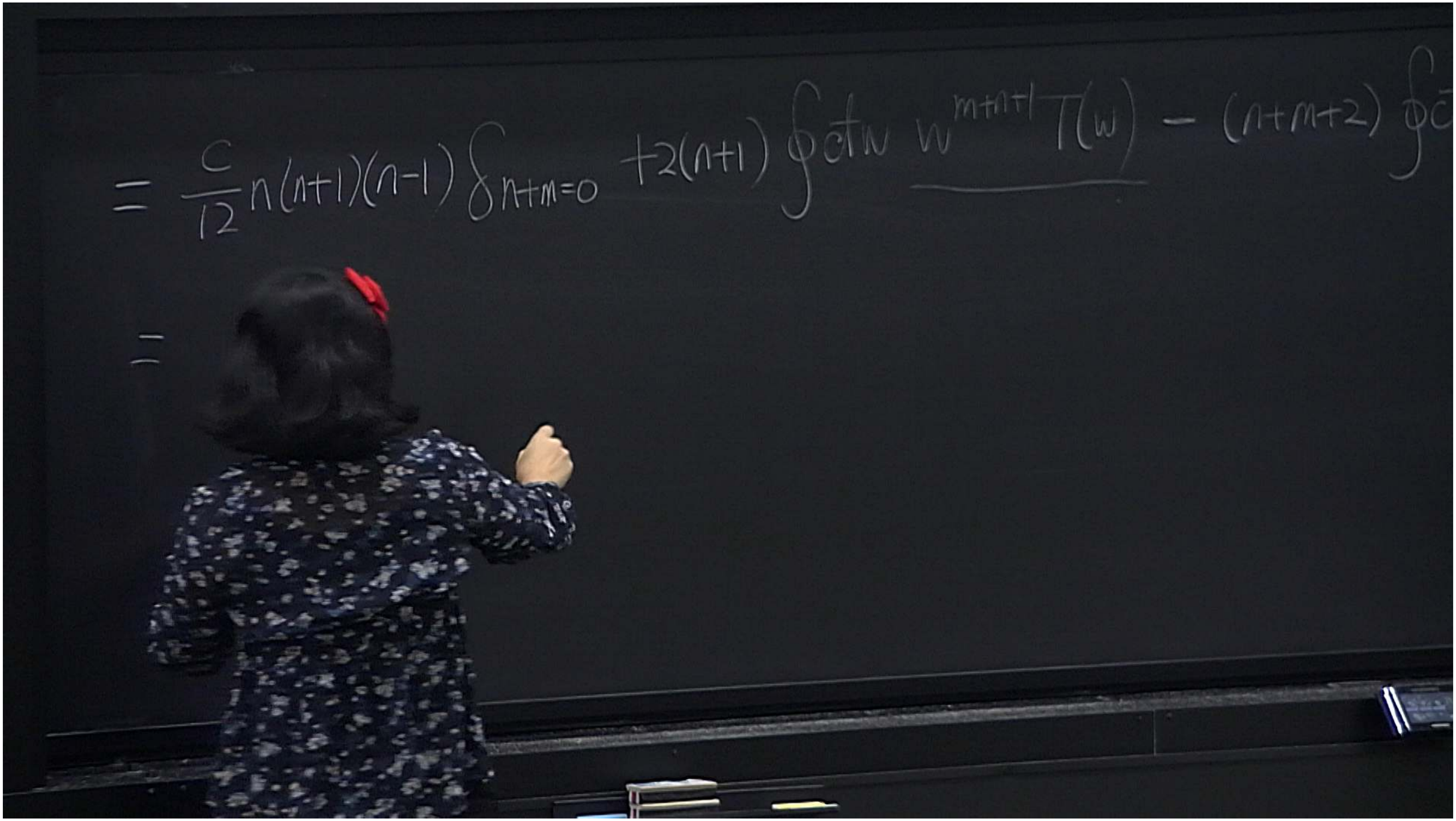
$$T(z)T(w) \sim \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} + \cdots$$

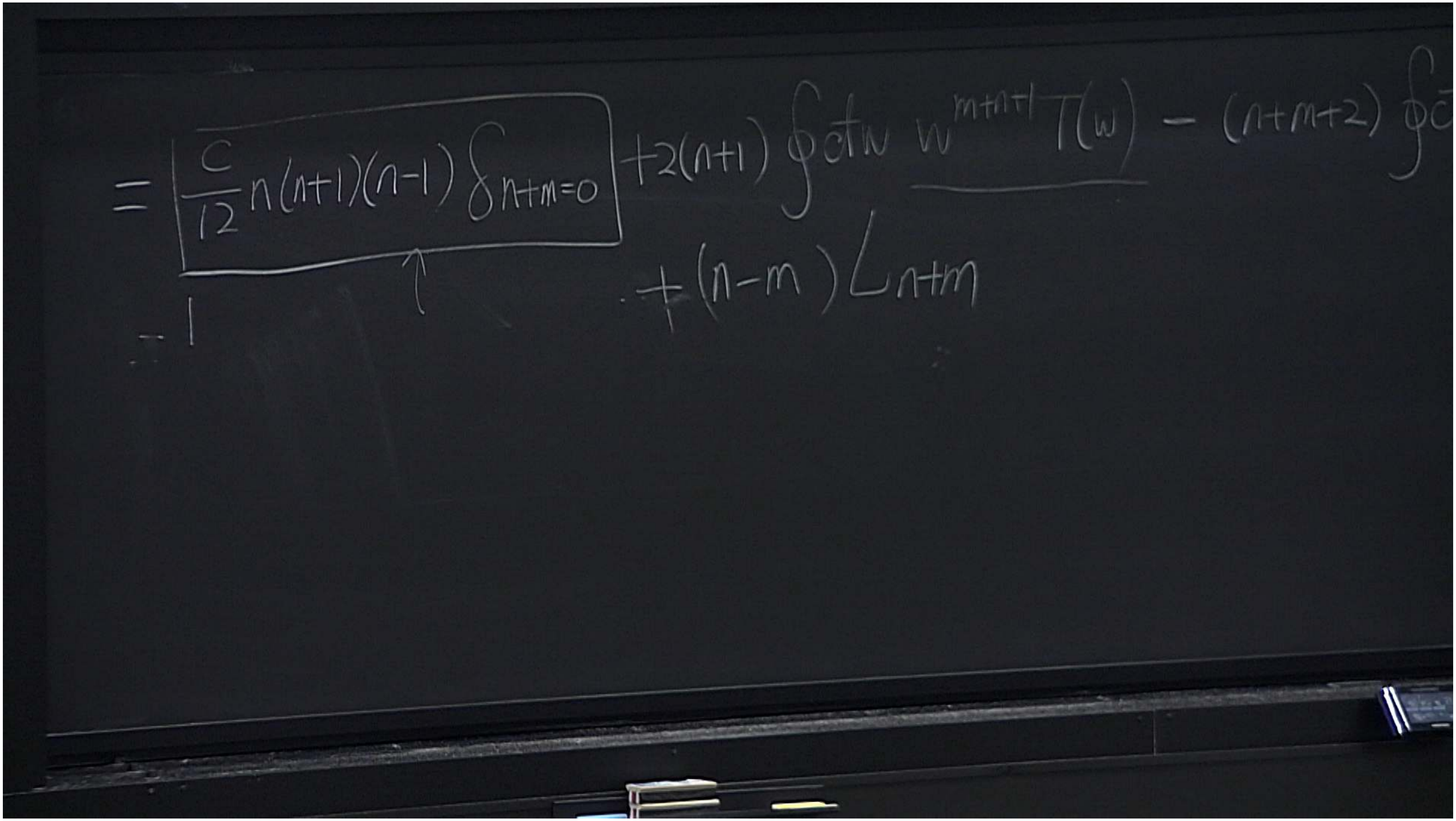
$$(z)T(w) \sim \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} + \cdots$$

$$L_n, L_m] = \left[\oint dz\, z^{n+1} T(z), \oint dw\, w^{m+1} T(w) \right]$$

$$= \oint dw\, w^{m+1} \oint dz\, z^{n+1} \left(\frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \right)$$

$$= \oint dw\, w^{m+1} \left(\frac{c}{12}(n+1)n(n-1)w^{n-2} + 2(n+1)w^{n} T(w) + w^{n+1} \partial T(w) \right)$$


$= \frac{c}{12} n(n+1)(n-1) \delta_{n+m=0} + 2(n+1) \oint dw\, w^{m+n+1} T(w) - (n+m+2) \oint d$
$=$


$= \boxed{\frac{c}{12} n(n+1)(n-1)\,\delta_{n+m=0}} + 2(n+1) \oint dw\, w^{m+n+1} T(w) - (n+m+2) \oint d$
$+ (n-m) L_{n+m}$

$$\boxed{\delta_{n+m=0}} + 2(n+1)\oint dw\, \underline{w^{m+n+1}\, T(w)} - (n+m+2)\oint dw\, w^{n+m+1}\, T(w)$$

$$+ (n-m) L_{n+m}$$

$$\boxed{\delta_{n+m=0}} + 2(n+1)\oint dw\, \underline{w^{m+n+1}\, T(w)} - (n+m+2)\oint dw\, w^{n+m+1}\, T(w)$$

$$+ (n-m) L_{n+m}$$