

Title: PSI 2016/2017 Quantum Field Theory III - Lecture 7

Date: Feb 07, 2017 11:30 AM

URL: <http://pirsa.org/17020080>

Abstract:

Recap

$D=2$  CT.  $z \rightarrow f(z)$

$$\text{local } [l_n, l_m] = (1-m)l_n + m$$

$$\text{Global } f(z) = \frac{az+b}{cz+d} \quad ad-bc=1$$

$$\frac{SL(2, \mathbb{C})}{\mathbb{Z}_2} \simeq SO(3, 1)$$



Recap

$D=2$  CT.  $z \rightarrow f(z)$

$$\text{local } [l_n, l_m] = (n-m)l_{n+m}$$

$$\text{Global } f(z) = \frac{az+b}{cz+d} \quad ad-bc=1$$

$$\text{VIP} \stackrel{\text{chiral}}{=} \text{primary } \tilde{\Phi}(f(z)) = (\partial f)^{-1} \tilde{\Phi}(z) \quad \frac{SL(2, \mathbb{C})}{\mathbb{Z}_2} \simeq SO(3,1)$$



Recap

$D=2$  CT.  $z \rightarrow f(z)$

local  $[l_n, l_m] = (n-m)l_{n+m}$

Global  $f(z) = \frac{az+b}{cz+d}$   $ad-bc=1$   
 $SL(2, \mathbb{C})$

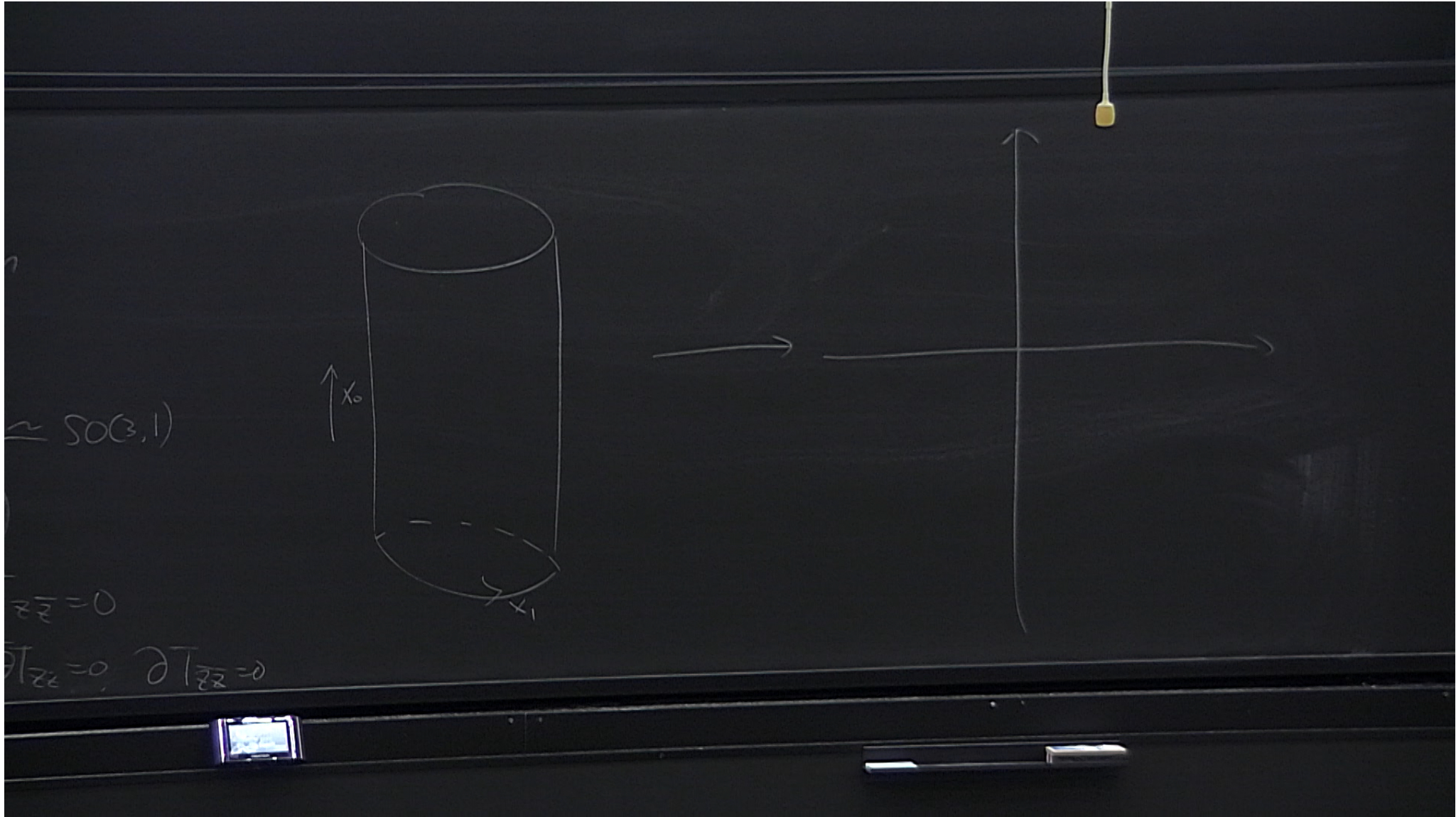
chiral  
VIP: primary  $\tilde{\Phi}(-f(z)) = (\partial f)^{-h} \tilde{\Phi}(z)$   
 $SO(3,1)$

$T(z) = -2\pi T_{zz}$

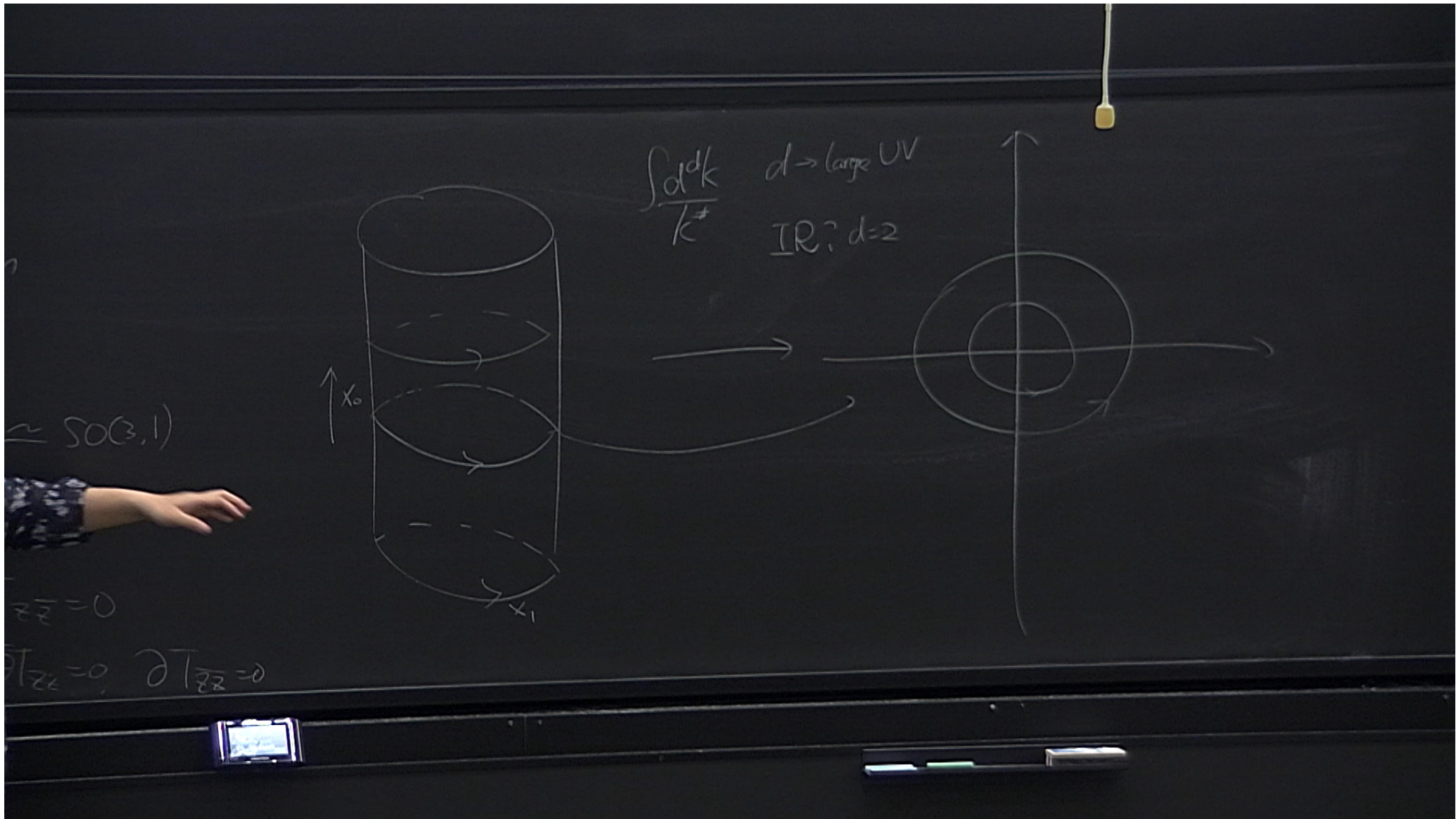
$T_{\mu\nu} = \begin{pmatrix} + & \\ & - \end{pmatrix}$

$T_{\mu}^{\mu} = 0 \Rightarrow T_{z\bar{z}} = 0$   
 $\partial_{\mu} T^{\mu\nu} = 0 \Rightarrow \bar{\partial}_{\bar{z}} T_{z\bar{z}} = 0$

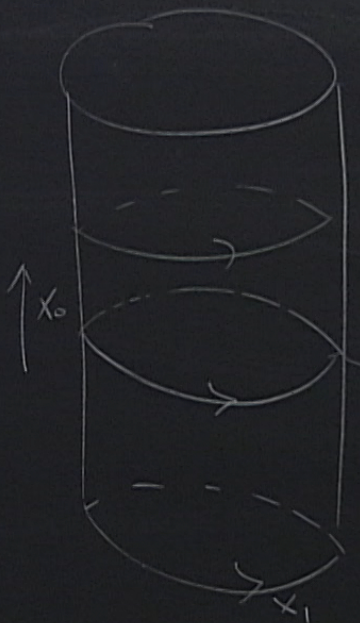




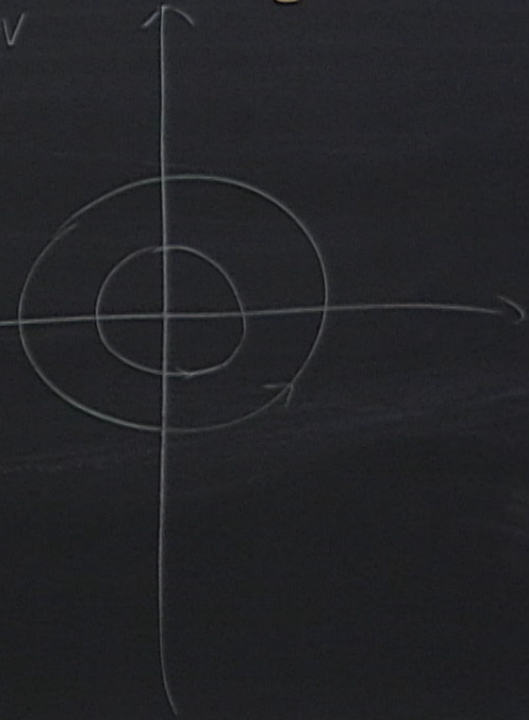




$\sim SO(3,1)$

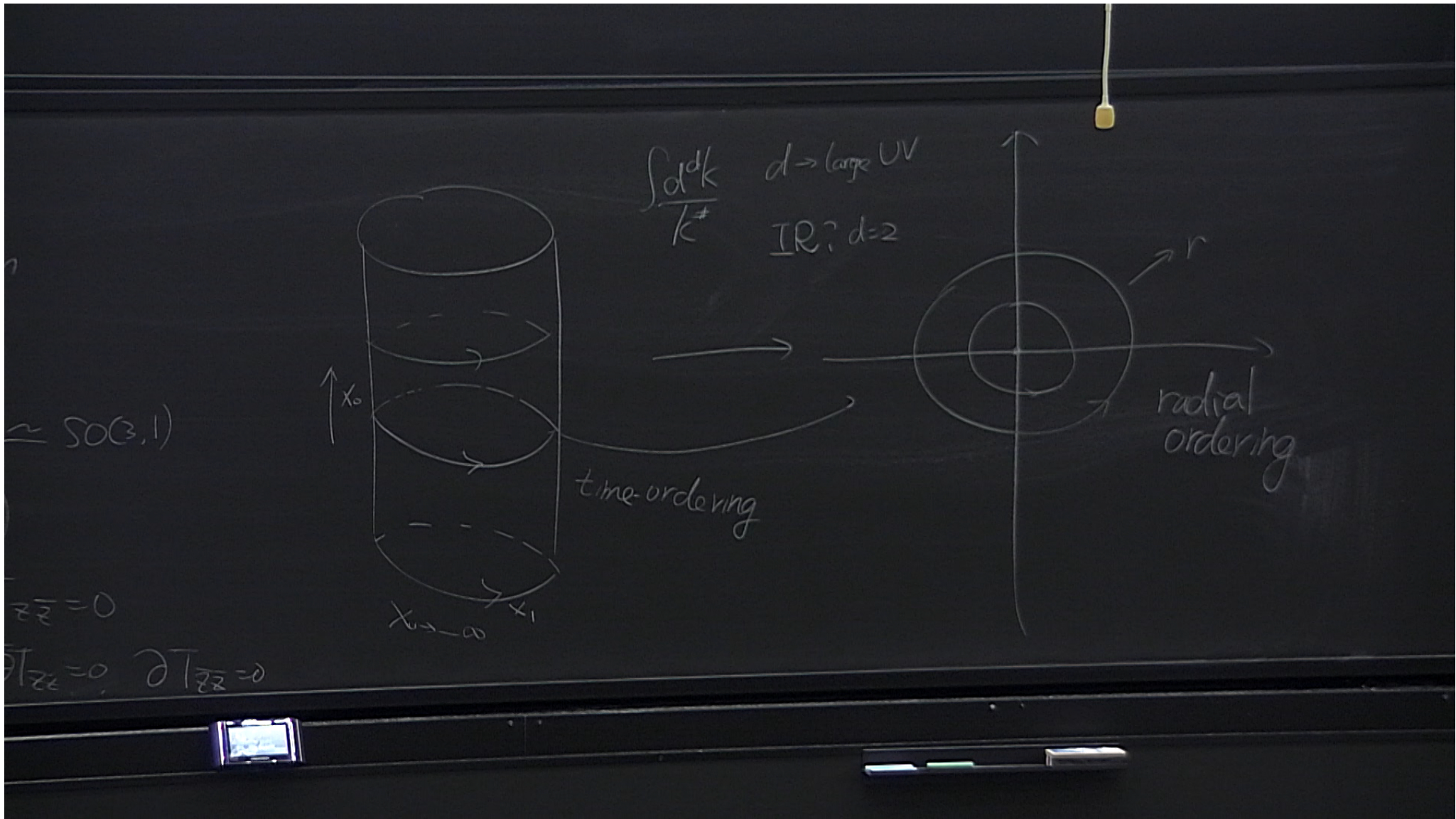


$\int \frac{d^d k}{k^4}$   $d \rightarrow \text{large UV}$   
IR?  $d=2$

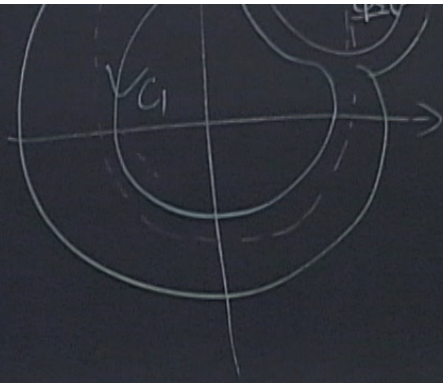


$z_2 = 0$   
 $\partial T / \partial z_2 = 0$





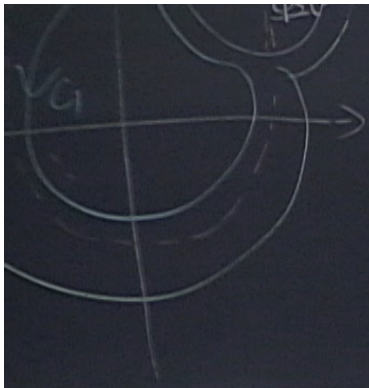




$$\begin{aligned}
 \oint_{|z-w|=\varepsilon} \frac{dZ \Phi_1(z) \Phi_2(w)}{C} &= \oint_{C_2} \bar{\Phi}_1(z) \bar{\Phi}_2(w) - \oint_{C_1} \bar{\Phi}_2(w) \bar{\Phi}_1(z) \\
 &= \oint_{C_2} \bar{\Phi}_1(z) \bar{\Phi}_2(w) - \oint_{C_1} \bar{\Phi}_2(w) \bar{\Phi}_1(z)
 \end{aligned}$$

$$= \lim_{\substack{\varepsilon \rightarrow 0 \\ C_2 \rightarrow -C_1}}$$





$$\oint_{|z-w|=\varepsilon} \bar{d}z \Phi_1(z) \Phi_2(w)$$

$$= \oint_{C_2} \bar{d}z \Phi_1(z) \Phi_2(w) - \oint_{C_1} \bar{d}z \Phi_1(z) \Phi_2(w)$$

$$\lim_{\varepsilon \rightarrow 0} \oint_{C_2 \rightarrow -C_1} \bar{d}z [\Phi_1(z), \Phi_2(w)]$$



2) Generators

$$\delta\Phi = a_n G_n \Phi$$

$$\delta\Phi = [Q, \Phi]$$

$$Q = \int dx^i j^0$$

$$\partial_\mu j^\mu = 0$$

$$\epsilon_\mu T^{\mu\nu}$$

$$Q = -\oint dz \epsilon(z) T(z)$$



2) Generators

$$\delta \Phi = a_n G_n \Phi$$

$$Q = \int dx j^0$$

$$\delta \Phi = [Q, \Phi]$$

$$\partial_\mu j^\mu = 0$$

$$j^\nu = \epsilon_{\mu\nu} T^{\mu\nu}$$

$$Q = - \oint dz \epsilon(z) T(z) + \dots$$

$$= - \oint_c dz \epsilon(z) [T(z), \Phi(w)]$$

$$= - \oint_{c'} dz \epsilon(z) T(z) \Phi(w)$$



$z \bar{z} = 0$   $z \bar{z} = 0$

$x^j$

0

$w$

$w$

$$= - \oint_{c'} dz a_n z^{n+1} T(z) \bar{\Phi}(w)$$

want  $- a_n L_n \bar{\Phi}(w)$

$$T(z) = \sum_m \frac{L_m}{z^{m+2}}$$

$$L_m = \oint T(z) z^m$$

$$Q = - \oint dz \epsilon(z) T(z) + \dots$$



$$= - \oint_{c'} dz a_n z^{n+1} T(z) \bar{\Phi}(w)$$

$T(z) + \dots$

want  $-a_n L_n \bar{\Phi}(w)$

$$T(z) = \sum_m \frac{L_m}{z^{m+2}}$$

$$L_m = \oint dz T(z) z^{m+1}$$

$$[L_m, L_n]$$



OPE

$$\langle O_1(z_1) O_2(z_2) \dots \rangle = \langle \sum_k C_k O_k(z) \dots \rangle$$



OPE

$$\langle O_1(z_1) O_2(z_2) \dots \rangle = \langle \sum_k c_k O_k(z) \dots \rangle$$

$$T(z) \Phi(w) \sim ?$$

$$\delta \bar{\Phi} = \delta \Phi$$

$$\hat{\Phi}(f(z)) = (df)^{-h} \bar{\Phi}(z)$$

$$\delta \Phi = \hat{\Phi}(z) - \Phi(z)$$



$$\delta\Phi = \tilde{\Phi}(z) - \Phi(z)$$

$$\tilde{\Phi}(z + \epsilon(z)) = (1 + \partial\epsilon)^{-h} \Phi(z)$$

$$\tilde{\Phi}(z) + \epsilon(z)\partial\Phi(z) = (1 - h\partial\epsilon)\Phi(z)$$

$$\delta\Phi = -\epsilon\partial\Phi - h\partial\epsilon\Phi$$

$$\lim_{\epsilon \rightarrow 0} \int_{C_2} \rightarrow -C_1 \rightarrow \bigcirc$$

$$= \oint_0 dz [\Phi_1(z); \Phi_2(w)]$$



$$\delta \bar{\Phi}(w) = -\epsilon(w) \partial \bar{\Phi}(w) - h \partial \epsilon(w) \bar{\Phi}(w)$$

$$\epsilon(w) \partial \bar{\Phi}(w) = \oint dz \frac{\epsilon(z)}{z-w} \partial \bar{\Phi}(w)$$

$$h \partial \epsilon(w) \bar{\Phi}(w) = \oint dz \frac{h \epsilon(z)}{(z-w)^2} \bar{\Phi}(w)$$

$$T(z) \bar{\Phi}(w) \sim \frac{\partial \bar{\Phi}(w)}{z-w} + \frac{h \bar{\Phi}(w)}{(z-w)^2} + \dots$$



$$T(z)T(w) \sim \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} + \dots$$



$$(z)T(w) \sim \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} + \dots$$

$$[L_n, L_m] = \left[ \oint dz z^{n+1} T(z), \oint dw w^{m+1} T(w) \right]$$

$$= \oint dw w^{m+1} \oint dz z^{n+1} \left( \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \right)$$

$$= \oint dw w^{m+1} \left( \frac{c}{12} (n+1)n(n-1) w^{n-2} + 2(n+1)w^n T(w) + w^{n+1} \partial T(w) \right)$$



$$= \frac{c}{12} n(n+1)(n-1) \delta_{n+m=0} + 2(n+1) \int \phi \psi w^{m+n+1} T(w) - (n+m+2) \int \phi \psi$$

==



$$= \boxed{\frac{c}{12} n(n+1)(n-1) \delta_{n+m=0}} + 2(n+1) \int \phi \psi w^{m+n+1} T(w) - (n+m+2) \int \phi \psi$$

$$+ (n-m) L_{n+m}$$



$$\boxed{\delta_{n+m=0}} + 2(n+1) \int_{\text{ctw}} \underline{w^{m+n+1} T(w)} - (n+m+2) \int_{\text{ctw}} w^{n+m+1} T(w) \\ + (n-m) L_{n+m}$$



$$\delta_{n+m=0} \left[ +2(n+1) \int_0^1 w^{m+n+1} T(w) - (n+m+2) \int_0^1 w^{n+m+1} T(w) \right. \\ \left. + (n-m) L_{n+m} \right]$$