Title: PSI 2016/2017 Quantum Field Theory III - Lecture 3

Date: Feb 01, 2017 11:30 AM

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Abstract:

wavefunction renormalization of operators. $\phi_0 = Z\,\phi$ $\gamma =$

anomalous dimensions.

anomalous dimension

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + 2\,\gamma(\lambda) \right) G_2(p, \lambda, \mu) = 0$$

$(\mu$

- At a fixed of RG $\beta(\lambda^*) = 0$

$\phi_0 = Z\,\phi \qquad \gamma = \frac{1}{Z}\mu\frac{dZ}{d\mu}$

$$\left(\mu\frac{\partial}{\partial\mu} + 2\gamma(\lambda_*)\right)G_2(p,\lambda,\mu) = 0$$

$\lambda,\mu) = 0$

$$G_2 = \frac{1}{p^2}\left(\frac{p^2}{\mu^2}\right)^{\gamma(\lambda_*)}$$

$$\langle\phi(x)\phi(0)\rangle = G_2(x) \sim \int d^D p\, e^{ip\cdot x}\frac{1}{p^{2-2\gamma(\lambda_*)}}$$

Gaussian / Free

$$\phi_0 = Z\,\phi \qquad \gamma = \frac{1}{Z}\mu\frac{dZ}{d\mu}$$

$$\left(\mu\frac{\partial}{\partial\mu} + 2\gamma(\lambda_*)\right) G_2(p, \lambda, \mu) = 0$$

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$$\langle\phi(x)\phi(0)\rangle = G_2(x) \sim \int d^D p\, e^{ip\cdot x}\frac{1}{p^{2-2\gamma(\lambda_*)}} \sim \frac{1}{x^{D-2-2\gamma(\lambda^*)}}$$

$D-2-2\gamma$

Gaussian / Free theory. $\dfrac{1}{x^{D-2}} \longrightarrow \dfrac{1}{x^{D-2-2\gamma(\lambda^*)}}$

anomalous dimension

$x^\mu \longrightarrow \lambda x^\mu$

$\hat{\phi}(x) = \lambda^{-\frac{D-2}{2}-\gamma(\lambda^*)}\,\phi(\lambda^{-1} x)$

$\omega) = 0$

$D-2-2\gamma(\lambda^*)$

conformal transformation = angle preserving coordinate transformations : $x^\mu \longrightarrow \tilde{x}^\mu(x)$

* Transformations connected to identity (i.e. $\tilde{x}^\mu = x^\mu + \xi^\mu(x)$) obey CKV equation.

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D}(\partial\cdot\xi)\,\eta_{\mu\nu} \qquad \left(\partial^3 \xi = 0\right)$$

General solution $(D>2)$: $\xi^\mu = \underbrace{a^\mu}_{P^\mu} + \underbrace{\omega^\mu{}_\nu x^\nu}_{M^{\mu\nu}} + \underbrace{\lambda x^\mu}_{D} + \underbrace{b^\mu x^2 - 2x^\mu b\cdot x}_{K^\mu}$

$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \Longrightarrow$ conformal algebra $\simeq \underbrace{SO(2,D)}_{\mathbb{R}^{1,D-1}}$ or $\underbrace{SO(1,D-1)}_{\mathbb{R}^{D}}$

- A Poincaré invariant QFT has a conserved + symmetric energy-momentum tensor

$$\partial^\mu T_{\mu\nu} = 0 \qquad T_{\mu\nu} = T_{\nu\mu}.$$

- A CFT has a traceless energy-momentum tensor

$$T^\mu_\mu = 0$$

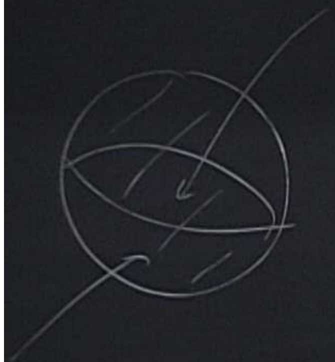
Inversion transformation I

$$\tilde{x}^\mu = \frac{x^\mu}{x^2}$$ exchanges $0 \longleftrightarrow \infty$

$I^2 = 1$ $\quad \mathbb{Z}_2$ transformation.

Conformal group generated by Poincare + I.

unit sphe



it sphere

$$I\,P_\mu\,I \quad x^\mu \longrightarrow \frac{x^\mu + b^\mu x^2}{1 + 2b\cdot x + b^2 x^2} \simeq x^\mu + \underbrace{b^\mu x^2 - 2b\cdot x\; x^\mu}_{\infty' \; K_\mu \text{ transformation}}$$

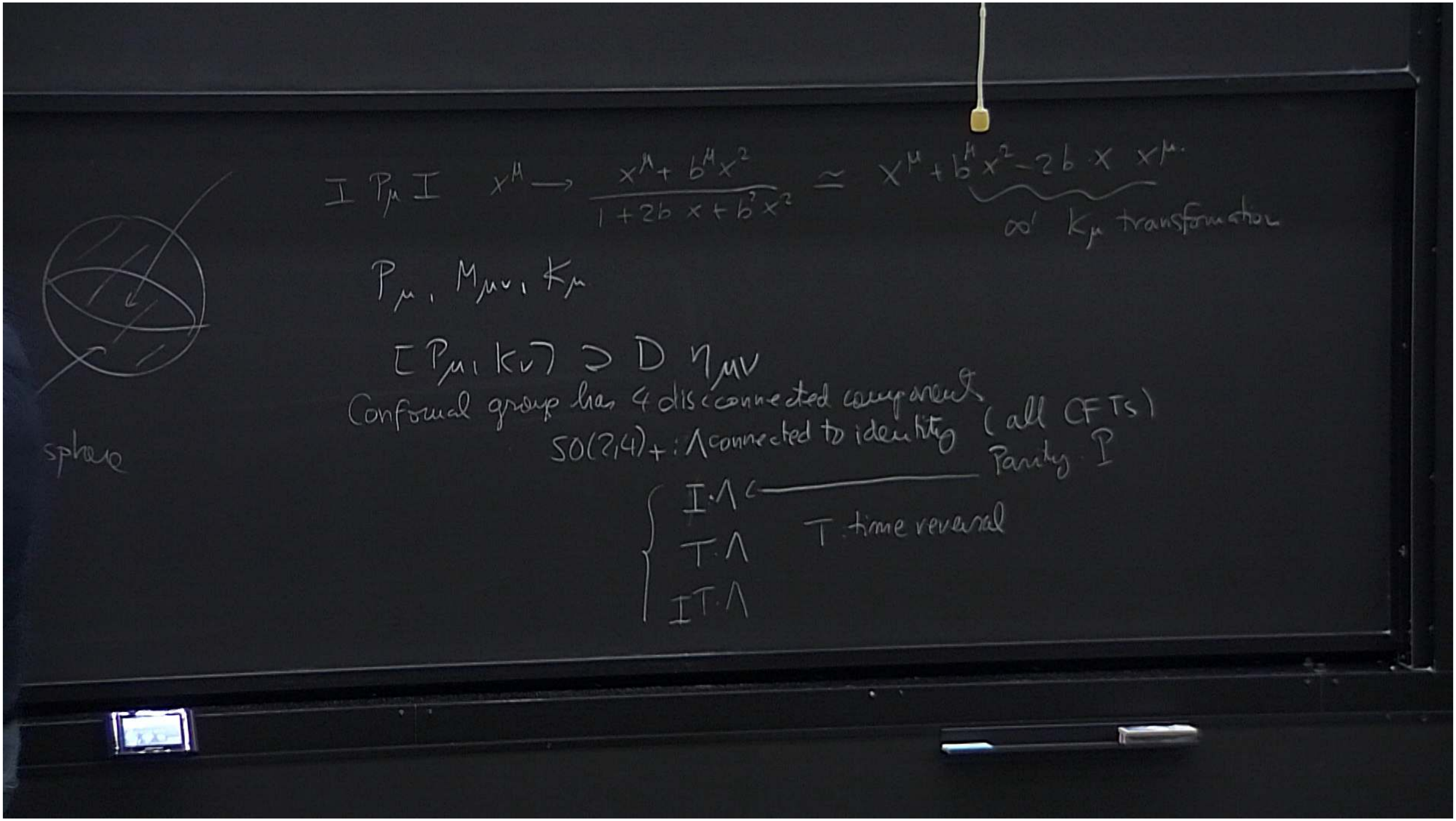
$P_\mu,\ M_{\mu\nu},\ K_\mu$

$[P_\mu, K_\nu] \supset D\,\eta_{\mu\nu}$

Conformal group has 4 disconnected components (all CFTs)

$SO(2,4)_+$: Λ connected to identity

$$\begin{cases} I\cdot\Lambda \\ T\cdot\Lambda \\ IT\cdot\Lambda \end{cases} \qquad T\text{: time reversal}$$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∞' K_μ transformation
$P_\mu, M_{\mu\nu}, K_\mu$
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$I\cdot\Lambda$
$T\cdot\Lambda$
T: time reversal
$IT\cdot\Lambda$
sphere

$\mathbb{R}^{1,D-1}$ $\mathbb{R}$

Inversion transformation I

$d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2(x)\, dx_\mu dx^\mu$ $\quad \tilde{x}^\mu = \frac{x^\mu}{x^2}$ $\quad$ exchanges $0 \longleftrightarrow \infty$

$I^2 = 1$ $\quad \mathbb{Z}_2$ transformation.

Conformal group generated by Poincare + I.

unit sphere

$\to \tilde{x}^\mu(x)$

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massive scalar

$$T^\mu_\mu \neq\,: \alpha\, m^2 \phi^2$$

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Question. what is the conserved currents corresponding to conformal generators?

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Question: what is the conserved currents corresponding to conformal generators?

$$j^{(\xi)}_\mu = T_{\mu\nu}\,\xi^\nu \qquad \partial^\mu j_\mu = 0 + T_{\mu\nu}(\partial^\mu \xi^\nu)$$

$$= \frac{1}{2} T_{\mu\nu}(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu)$$

T time reversal

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$$= \tfrac{1}{2} T_{\mu\nu}(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu) \propto \eta_{\mu\nu} T^{\mu\nu} (\partial\cdot\xi) = 0$$

$$j^D_\mu = T_{\mu\nu} x^\nu$$

$$\partial_\mu j^\mu = 0 \implies Q = \int d^{D-1}x \; j^0$$

$$j^D_\mu = T_{\mu\nu} x^\nu \qquad D \equiv \int d^{D-1}x \; T_{0\nu} x^\nu$$

$$P_\mu = \int d^{D-1}x \; T_{0\mu}$$

$\Rightarrow$ Conformal algebra $\simeq$ $SO(2,D)$ or $SO(1,D-1)$

$\mathbb{R}^{1,D-1}$ $\qquad$ $\mathbb{R}^{D}$

$j^{(\xi)}_\mu = T_{\mu\nu}\xi^\nu$

$j^D_\mu = T_{\mu\nu}x^\nu$

What constraints does conf inv. give on the correlation functions of operators in CFT?

Classify operators into representations of conformal group.

CFT: operators are labeled by Lorentz quantum # + Dilatation eigenvalue. (scaling dimension)

P_μ. raising operator $\qquad$ $\mathcal{O}_\Delta$ has dim Δ $\qquad$ $P_\mu\mathcal{O}_\Delta$ has dimension $\Delta+1$

K_μ. lowering operator $\qquad$ " $\qquad$ $K_\mu\mathcal{O}_\Delta$ " " $\Delta-1$

general solution $(D>2)$ $\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu b\cdot x$

$P^\mu \quad M^{\mu\nu} \quad D \quad K^\mu$

$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \Longrightarrow$ conformal algebra $\cong SO(2,D)$ or $SO(1,D-1)$

$\mathbb{R}^{1,D-1} \qquad \mathbb{R}^D$

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Question. what is the conserved currents corresponding to conformal generators?

$j^{(\xi)}_\mu = T_{\mu\nu}\xi^\nu \qquad \partial^\mu j_\mu = 0 + T_{\mu\nu}(\partial^\mu \xi^\nu)$

$j^D_\mu = T_{\mu\nu} x^\nu \qquad = \frac{1}{2} T_{\mu\nu}(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu) \propto \eta_{\mu\nu} T^{\mu\nu}(\partial\cdot\xi) = 0$

What constraints does conf inv. give on the correlation functions of operators in CFT?

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P_μ raising operator $\quad \mathcal{O}_\Delta$ has dim $\Delta \quad P_\mu \mathcal{O}_\Delta$ has dimension $\Delta+1$

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$\Rightarrow$ Primary operator $\quad K_\mu \mathcal{O}_\Delta = 0$

	Dimension
$\mathcal{O}_\Delta$	Δ
$P_{\mu_1} \cdots P_{\mu_n} \mathcal{O}$	$\Delta+n$

$\Rightarrow$ Primary operator $\quad K_\mu \, \mathcal{O}_\Delta(0) = 0$

$\mathcal{O}_\Delta \qquad \dfrac{\text{Dimension}}{\Delta}$

sion)

$$\left.\begin{array}{c} P_{\mu_1} \cdots P_{\mu_n} \mathcal{O} \\ \vdots \end{array}\right\} \text{descendants} \qquad \Delta + n.$$

Δ / D.

$x \to \tilde{x} = g \cdot x$

Primaries: $\mathcal{O}(x) \longrightarrow \tilde{\mathcal{O}}_A(x) = \left| \frac{\partial x}{\partial \tilde{x}} \right| L_A{}^B(R(x))\, \mathcal{O}_B(g^{-1}x)$

- Dimension Δ
- spin

$$\frac{\partial \tilde{x}^\mu}{\partial x^\rho} \frac{\partial \tilde{x}^\nu}{\partial x^\sigma} \eta_{\mu\nu} = \Omega^2 \eta_{\rho\sigma}$$

$$R^\mu{}_\rho(x) = \frac{\partial \tilde{x}^\mu}{\partial x^\rho} \Omega^{-1}(x) \qquad R^T \eta R = \eta$$

$$L(g_1)\,L(g_2) = L(g_1 g_2)$$

$$\tilde{x}^\mu = x^\mu + \xi^\mu$$

$$\delta O_A(x) = \hat{O}_A(x) - O_A(x) = -\xi^\mu \partial_\mu O_A + \underbrace{\tfrac{i}{2}\,\Omega_{\mu\nu}(x)\,(M^{\mu\nu}_{r})_A{}^B\, O_B(x)}_{\text{local Lorentz}} - \underbrace{\boxed{\Delta}\,\omega(x)\, O_A(x)}_{\text{scale transf}}$$

$$\Omega_{\mu\nu}(x) = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu)$$

$$\omega(x) = \lambda - 2x\cdot b$$

r: representation of Lorentz group

Spinor $M_{\mu\nu} \propto [\Gamma_\mu, \Gamma_\nu]$

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)}$$

$$[\delta_{\xi_{(1)}}, \delta_{\xi_{(2)}}]\, \theta_A = \delta_{\xi_3}\, \theta_A$$

Constraint correlators of primary operators. $\langle O \rangle$

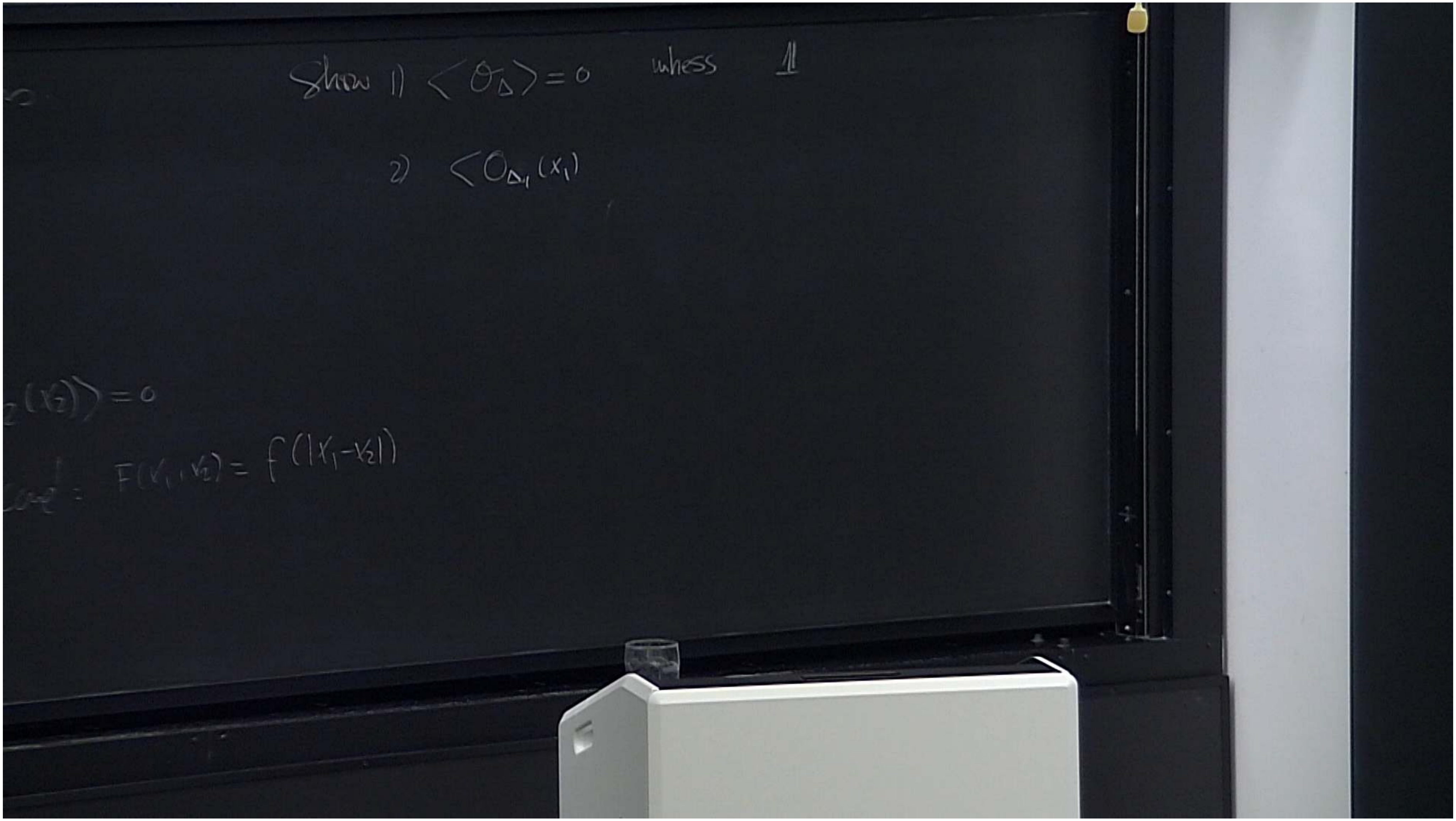
$$\langle O_1(x_1)\, O_2(x_2)\rangle = F(x_1, x_2)$$

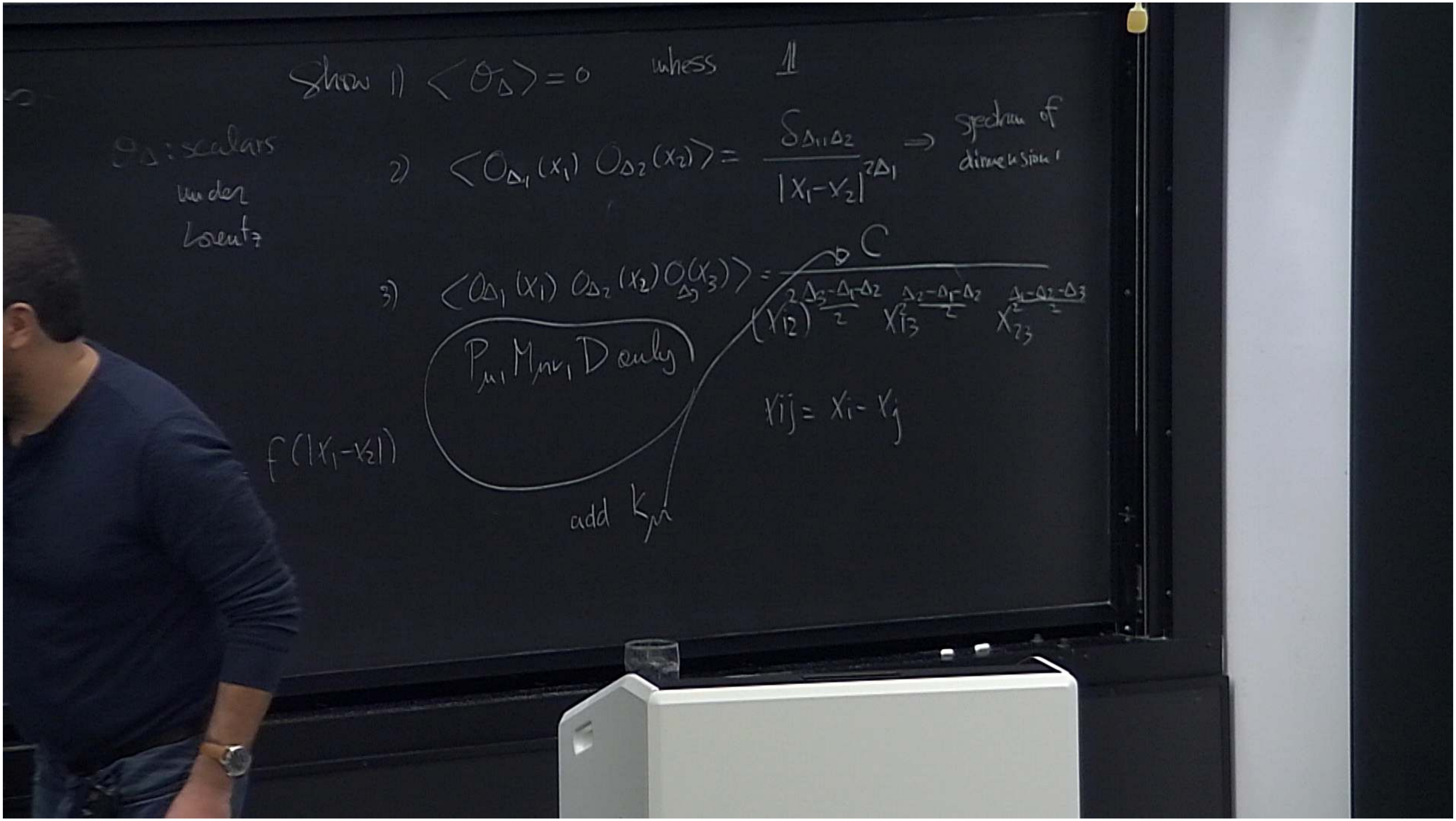
$$\delta_\xi \langle O_1(x_1)\, O_2(x_2)\rangle = 0$$

$$\langle \delta_\xi O_1(x_1) \cdot O_2(x_2)\rangle + \langle O_1(x_1)\, \delta_\xi O_2(x_2)\rangle = 0$$

Poincaré: $F(x_1, x_2) = f(|x_1 - x_2|)$

$\Rightarrow$ Differential equation for $F(x_1, x_2)$


Show 1) $\langle O_\Delta \rangle = 0$ unless $\mathbb{1}$
2) $\langle O_{\Delta_1}(x_1)$
$(x_2)\rangle = 0$
$F(x_1, x_2) = f(|x_1 - x_2|)$


Show 1) < O_Δ > = 0 unless 1
O_Δ : scalars under Lorentz
2) < O_Δ1 (x1) O_Δ2 (x2) > = δ_Δ1,Δ2 / |x1-x2|^2Δ1 ⇒ spectrum of dimension!
3) < O_Δ1 (x1) O_Δ2 (x2) O_Δ3 (x3) > = C / (x12^2)^((Δ3-Δ1-Δ2)/2) x13^2 ^((Δ2-Δ1-Δ2)/2) x23^2 ^((Δ1-Δ2-Δ3)/2)
P_μ, M_μν, D only
add K_μ
f(|x1-x2|)
xij = xi - xj