

Title: PSI 2016/2017 Quantum Field Theory III - Lecture 3

Date: Feb 01, 2017 11:30 AM

URL: <http://pirsa.org/17020076>

Abstract:

wavefunction renormalization of operators.

$$\phi_0 = Z \phi$$

$$\gamma =$$

anomalous dimensions.

anomalous dimension

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma(\lambda) \right) G_2(p, \lambda, \mu) = 0$$

- At a fixed of RG $\beta(\lambda^*) = 0$

$$\phi_0 = z \phi \quad \gamma = \frac{1}{z} \mu \frac{dz}{d\mu}$$

$$\left(\mu \frac{\partial}{\partial \mu} + 2\gamma(\lambda_*) \right) G_2(p, \lambda, \mu) = 0$$

$$G_2(p, \lambda) = 0$$

$$G_2 = \frac{1}{p^2} \left(\frac{p^2}{\mu^2} \right)^{\gamma(\lambda_*)}$$

$$\langle \phi(x) \phi(0) \rangle = G_2(x) \sim \int d^D p e^{ip \cdot x} \frac{1}{p^2 - 2\gamma(\mu)}$$

Gaussian / Free

$$\phi_0 = Z \phi \quad \gamma = \frac{1}{2} \mu \frac{dZ}{d\mu}$$

$$\left(\mu \frac{\partial}{\partial \mu} + 2\gamma(\lambda^*) \right) G_2(p, \lambda, \mu) = 0$$

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$$G_2 = \frac{1}{p^2} \left(\frac{p^2}{\mu^2} \right)^{\gamma(\lambda^*)}$$

$$\langle \phi(x) \phi(0) \rangle = G_2(x) \sim \int d^D p e^{ip \cdot x} \frac{1}{p^2 - 2\gamma(\mu)} \sim \frac{1}{x^{D-2-2\gamma(\lambda^*)}}$$

$$D-2-2\gamma$$

Gaussian / Free theory. $\frac{1}{\sqrt{D-2}} \rightarrow \frac{1}{\sqrt{D-2-2\gamma(\lambda^*)}}$
 ? anomalous dimension

$$X^M \rightarrow \lambda X^M$$

$$\hat{\phi}(x) = \lambda^{-\frac{D-2-2\gamma(\lambda^*)}{2}} \phi(\lambda^{-1}x)$$

$$\omega = 0$$

$$D-2-2\gamma(\lambda^*)$$

conformal transformation = angle preserving coordinate transformations: $x^M \rightarrow \tilde{x}^M(x)$

Transformations connected to identity (i.e. $\tilde{x}^M = x^M + \xi^M(x)$) obey CKV equation.

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D} (\partial \cdot \xi) \eta_{\mu\nu} \quad (\partial^3 \xi = 0)$$

General solution ($D \geq 2$):

$$\xi^M = \underbrace{a^M}_{P^M} + \underbrace{\omega^M{}_\nu}_{M^{\mu\nu}} x^\nu + \underbrace{\lambda}_{D} x^M + \underbrace{b^M}_{K^M} x^2 - 2 x^M b \cdot x$$

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \implies \text{conformal algebra} \simeq \begin{matrix} SO(2, D) \\ \mathbb{R}^{1|D-1} \end{matrix} \quad \text{or} \quad \begin{matrix} SO(1, D-1) \\ \mathbb{R}^D \end{matrix}$$

- A Poincaré invariant QFT has a conserved + symmetric energy-momentum tensor

$$\partial^\mu T_{\mu\nu} = 0 \quad T_{\mu\nu} = T_{\nu\mu}$$

- A CFT has a traceless energy-momentum tensor

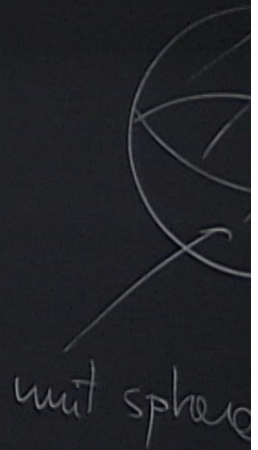
$$T^\mu{}_\mu = 0$$

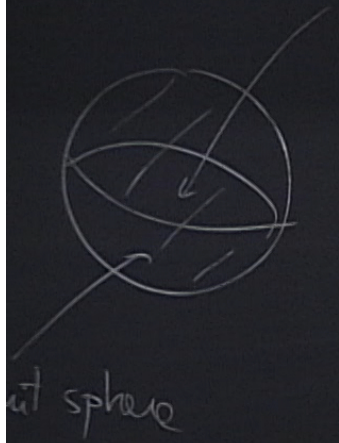
Inversion transformation I

$$\hat{x}^M = \frac{x^M}{x^2} \quad \text{exchanges} \quad 0 \leftrightarrow \infty$$

$$I^2 = 1 \quad \mathbb{Z}_2 \text{ transformation.}$$

Conformal group generated by Poincaré + I .





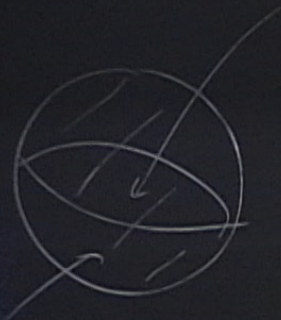
$$I P_\mu I \quad x^\mu \rightarrow \frac{x^\mu + b^\mu x^2}{1 + 2b \cdot x + b^2 x^2} \simeq x^\mu + \underbrace{b^\mu x^2 - 2b \cdot x x^\mu}_{\propto K_\mu \text{ transformation}}$$

$P_\mu, M_{\mu\nu}, K_\mu$

$$[P_\mu, K_\nu] \supset D \eta_{\mu\nu}$$

Conformal group has 4 disconnected components
 $SO(2,4)_+$: Λ connected to identity (all CFTs)

$$\left\{ \begin{array}{l} I \cdot \Lambda \\ T \cdot \Lambda \\ IT \cdot \Lambda \end{array} \right. \quad T \text{ time reversal}$$



sphere

$$I P_{\mu} I \quad x^{\mu} \rightarrow \frac{x^{\mu} + b^{\mu} x^2}{1 + 2b \cdot x + b^2 x^2} \simeq x^{\mu} + \underbrace{b^{\mu} x^2 - 2b \cdot x}_{\propto K_{\mu} \text{ transformation}} x^{\mu}$$

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Conformal group has 4 disconnected components
 $SO(2,4)_+$: Λ connected to identity (all CFTs)
 Parity $\cdot P$

- $\left\{ \begin{array}{l} I \cdot \Lambda \\ T \cdot \Lambda \\ IT \cdot \Lambda \end{array} \right.$ $\xrightarrow{\text{Parity } P}$ $T \cdot \text{time reversal}$

Inversion transformation I

$$d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2(x) dy_\mu dy^\mu \quad \tilde{x}^\mu = \frac{x^\mu}{x^2} \quad \text{exchanges } 0 \leftrightarrow \infty$$

$$I^2 = 1 \quad \mathbb{Z}_2 \text{ transformation.}$$

Conformal group generated by Poincaré + I .



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$$T^\mu{}_\mu = 0$$

massive scalar

$$T^\mu{}_\mu \neq 0 \propto m^2 \phi^2$$

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Question. what is the conserved currents corresponding to conformal generators?

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Question: what is the conserved currents corresponding to conformal generators?

$$j_\mu^{(\xi)} = T_{\mu\nu} \xi^\nu$$

$$\partial^\mu j_\mu = 0 + T_{\mu\nu} (\partial^\mu \xi^\nu)$$

$$= \frac{1}{2} T_{\mu\nu} (\partial^\mu \xi^\nu + \partial^\nu \xi^\mu)$$

T time reversal

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$$j_\mu^{(\xi)} = T_{\mu\nu} \xi^\nu$$

$$j_\mu^D = T_{\mu\nu} X^\nu$$

$$\partial^\mu j_\mu = 0 + T_{\mu\nu} (\partial^\mu \xi^\nu)$$

$$= \frac{1}{2} T_{\mu\nu} (\partial^\mu \xi^\nu + \partial^\nu \xi^\mu) \propto \eta_{\mu\nu} T^{\mu\nu} (\partial \cdot \xi) = 0$$

$$\partial_\mu j^\mu = 0 \implies Q = \int d^{D-1}x j^0$$

$$j_\mu^D = T_{\mu\nu} X^\nu$$

$$D \equiv \int d^{D-1}x T_{0\nu} X^\nu$$

$$P_\mu = \int d^{D-1}x T_{0\mu}$$

(3) \implies conformal algebra $\simeq SO(2, D)$ or $SO(1, D-1)$
 $\mathbb{R}^{1, D-1}$ \mathbb{R}^D

$$J_{\mu\nu} = T_{\mu\nu} \\ J_{\mu}^{\nu} = T_{\mu\alpha} X^{\alpha\nu}$$

What constraints does conf. inv. give on the correlation functions of operators in CFT?

Classify operators into representations of conformal group.

CFT: operators are labeled by Lorentz quantum # + Dilatation eigenvalue. (scaling dimension)

P_{μ} : raising operator

\mathcal{O}_{Δ} has dim Δ

$P_{\mu} \mathcal{O}_{\Delta}$ has dimension $\Delta+1$

K_{μ} : lowering operator

"

$K_{\mu} \mathcal{O}_{\Delta}$ " " $\Delta-1$

$s = a + w^{\mu} x^{\mu} + \lambda x^{\mu} + b^{\mu} x^{\mu} - 2x^{\mu} b_{\mu} x$
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $P^{\mu} \quad M^{\mu\nu} \quad D \quad K^{\mu}$
 $[\xi_{\mu}, \xi_{\nu}] = \xi_{\mu} \xi_{\nu} \implies$ conformal algebra $\simeq SO(2, D)$ or $SO(1, D-1)$
 $\mathbb{R}^{1, D-1} \quad \mathbb{R}^D$

- A CFT has a traceless energy-momentum tensor
 $T_{\mu}^{\mu} = 0$

massive scalar
 $T_{\mu}^{\mu} \neq 0 \propto m^2 \phi^2$

Question: what are the conserved currents corresponding to conformal generators?

$$j_{\mu}^{(s)} = T_{\mu\nu} x^{\nu}$$

$$j_{\mu}^{\nu} = T_{\mu\nu} x^{\nu}$$

$$\partial^{\mu} j_{\mu} = 0 + T_{\mu\nu} (\partial^{\mu} x^{\nu})$$

$$= \frac{1}{2} T_{\mu\nu} (\partial^{\mu} x^{\nu} + \partial^{\nu} x^{\mu}) \propto \eta_{\mu\nu} T^{\mu\nu} (\partial \cdot \xi) = 0$$

What constraints does conformal inv. give on the correlation functions of operators in CFT?

Classify operators into representations of conformal group

CFT operators are labeled by Lorentz quantum # + Dilatation eigenvalue (scaling dimension)

P_{μ} raising operator
 K_{μ} lowering operator
 \mathcal{O}_{Δ} has dim Δ
 $P_{\mu} \mathcal{O}_{\Delta}$ has dimension $\Delta + 1$
 $K_{\mu} \mathcal{O}_{\Delta}$ " " $\Delta - 1$

\Rightarrow Primary operator $K_{\mu} \mathcal{O}_{\Delta} = 0$
 \mathcal{O}_{Δ} $\frac{\text{Dimension}}{\Delta}$
 $P_{\mu}, J_{\mu\nu}$ \mathcal{O} $\Delta + n$

\Rightarrow Primary operator O_Δ k_μ $\frac{O_\Delta(\delta) = 0}{\Delta}$

$P_{\mu_1} \dots P_{\mu_n} \circ$ } descendant
 $\Delta + n.$

sion)

$$x \rightarrow \tilde{x} = g \cdot x$$

Primaner: $\theta(x) \rightarrow \tilde{\theta}_A(x) = \left| \frac{\partial x}{\partial \tilde{x}} \right| L_A^B(\mathcal{R}(x)) \theta_B(\tilde{x})$

- Dimension Δ

- spin

$$\frac{\partial \tilde{x}^\mu}{\partial x^\rho} \frac{\partial \tilde{x}^\nu}{\partial x^\sigma} \eta_{\mu\nu} = \Omega^2 \eta_{\rho\sigma}$$

$$R^\mu_\rho(x) = \frac{\partial \tilde{x}^\mu}{\partial x^\rho} \Omega^{-1}(x)$$

$$R^T \eta R = \eta$$

$$L(g_1) L(g_2) = L(g_1 g_2)$$

$$\tilde{x}^M = x^M + \xi^M$$

$$\delta O_A(x) = \tilde{O}_A(x) - O_A(x) = -\xi^M \partial_M O_A + \frac{i}{2} \underbrace{S_{\mu\nu}(x) (M_{\mu\nu}^A)^B}_{\text{local Lorentz}} \phi_B(x) - \underbrace{[\Delta \omega(x)]}_{\text{scale transf.}} O_A(x)$$

$$S_{\mu\nu}(x) = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu)$$

$$\omega(x) = \lambda - 2x \cdot b$$

r : representation of Lorentz group

Spinors $M_{\mu\nu} \propto [\Gamma_\mu, \Gamma_\nu]$

$$[\xi^{(1)}, \xi^{(2)}] = \xi^{(3)}$$

$$[\delta_{\xi^{(1)}}, \delta_{\xi^{(2)}}] \theta_A = \delta_{\xi^{(3)}} \theta_A$$

$\langle \theta \rangle$

Constraint correlators of primary operators.

$$\langle \theta_1(x_1) \theta_2(x_2) \rangle = F(x_1, x_2)$$

$$\delta_{\xi} \langle \theta_1(x_1) \theta_2(x_2) \rangle = 0$$

$$\langle \delta_{\xi} \theta_1(x_1) \cdot \theta_2(x_2) \rangle + \langle \theta_1(x_1) \delta_{\xi} \theta_2(x_2) \rangle = 0$$

\Rightarrow Differential equation for $F(x_1, x_2)$ PomCase! : $F(x_1, x_2) = f(|x_1 - x_2|)$

Show 1) $\langle \theta_{\Delta} \rangle = 0$ whess \perp

2) $\langle O_{\Delta_1}(x_1) \rangle$

$\langle \theta(x) \rangle = 0$

$F(x_1, x_2) = f(|x_1 - x_2|)$

Show 1) $\langle \sigma_{\Delta} \rangle = 0$ unless \parallel

σ_{Δ} : scalars under Lorentz

2) $\langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) \rangle = \frac{S_{\Delta_1, \Delta_2}}{|x_1 - x_2|^{2\Delta_1}} \Rightarrow$ spectrum of dimension 1

3) $\langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) O_{\Delta_3}(x_3) \rangle = \frac{C}{(x_{12})^{\frac{2\Delta_3 - \Delta_1 - \Delta_2}{2}} (x_{13})^{\frac{\Delta_2 - \Delta_1 - \Delta_2}{2}} (x_{23})^{\frac{\Delta_1 - \Delta_2 - \Delta_3}{2}}}$

$P_{\mu}, M_{\mu\nu}, D$ only

$x_{ij} = x_i - x_j$

$f(|x_1 - x_2|)$

add k_{μ}