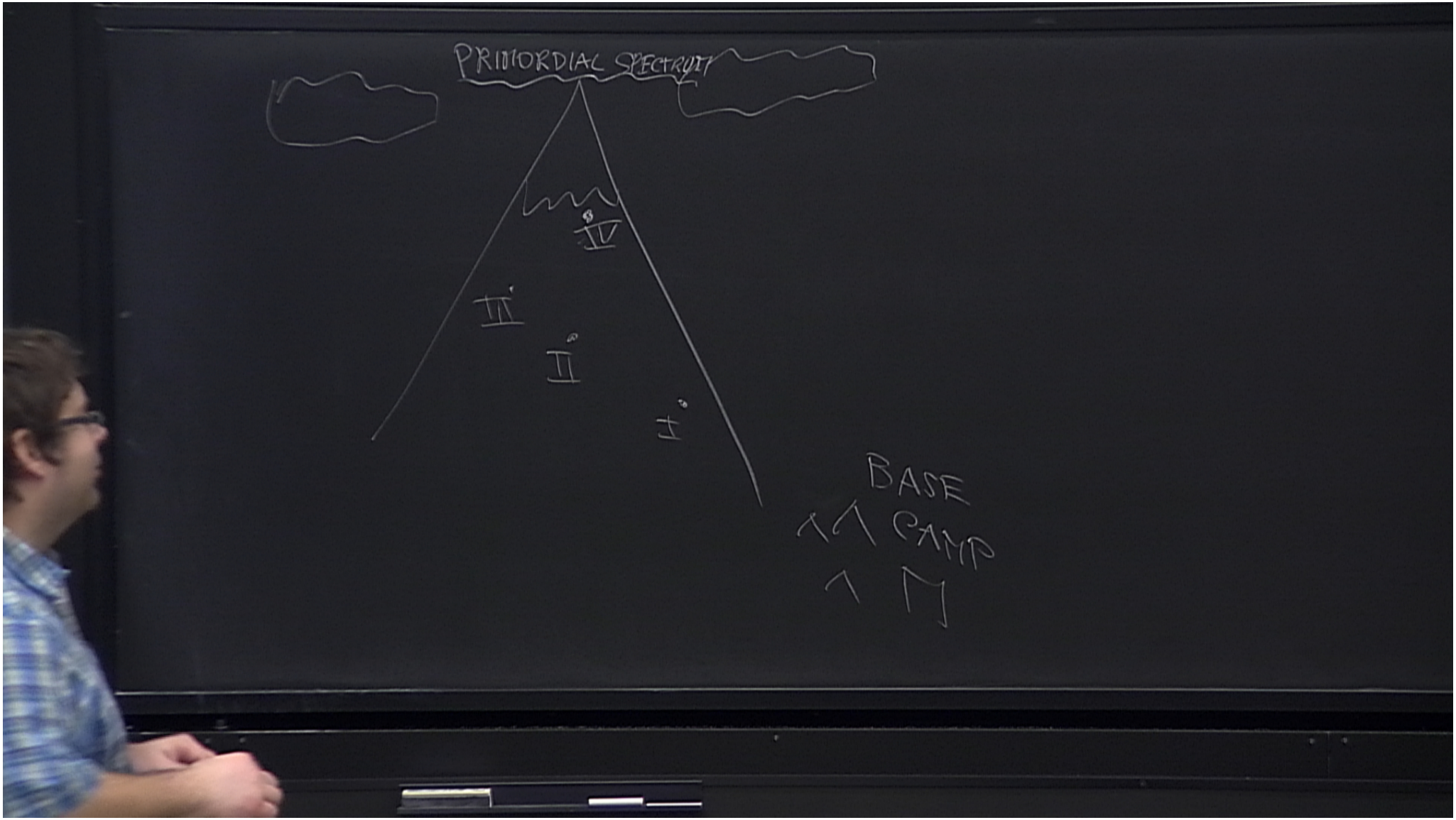
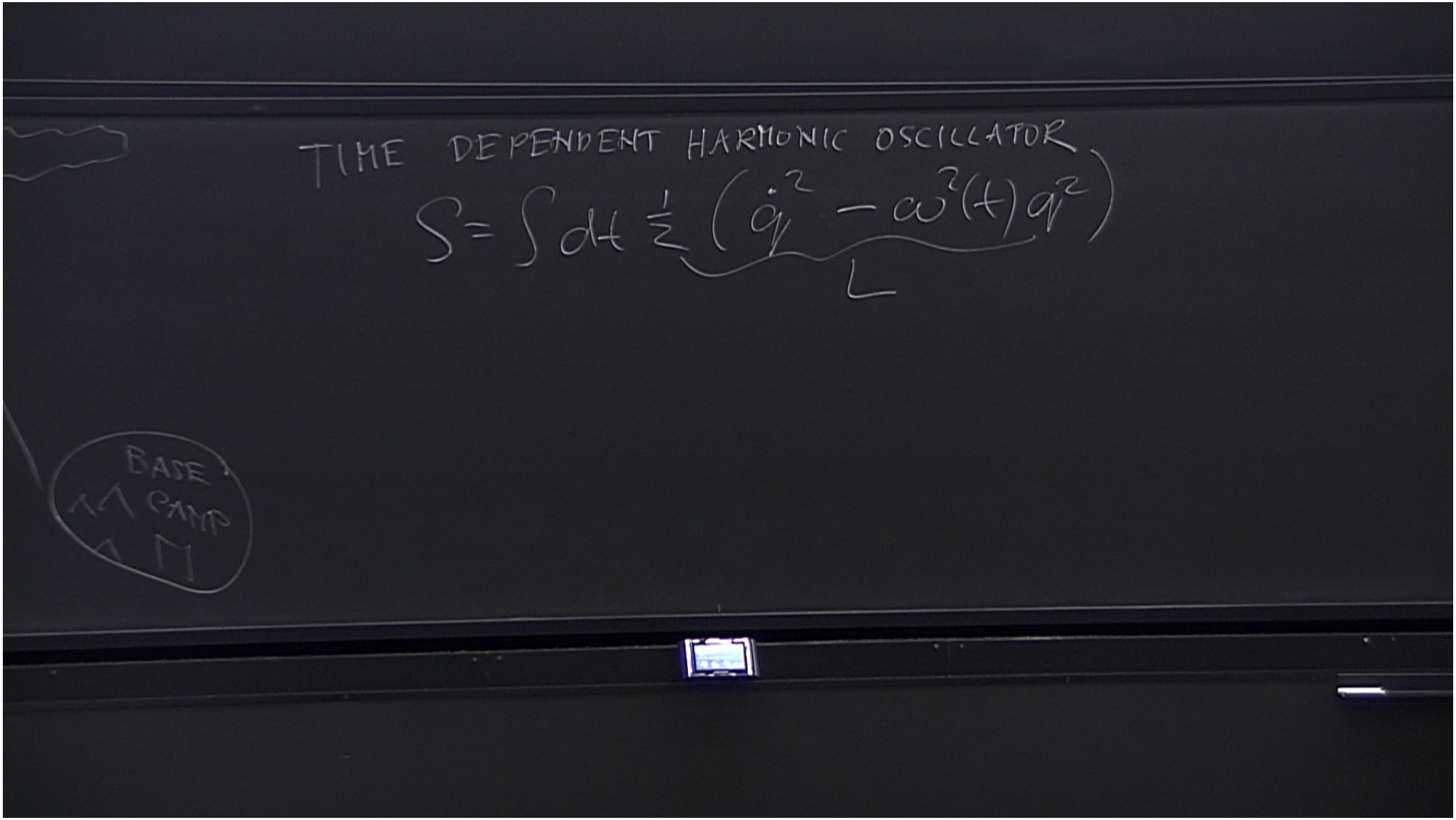
Title: PSI 2016/2017 Cosmology (Review) - Lecture 13

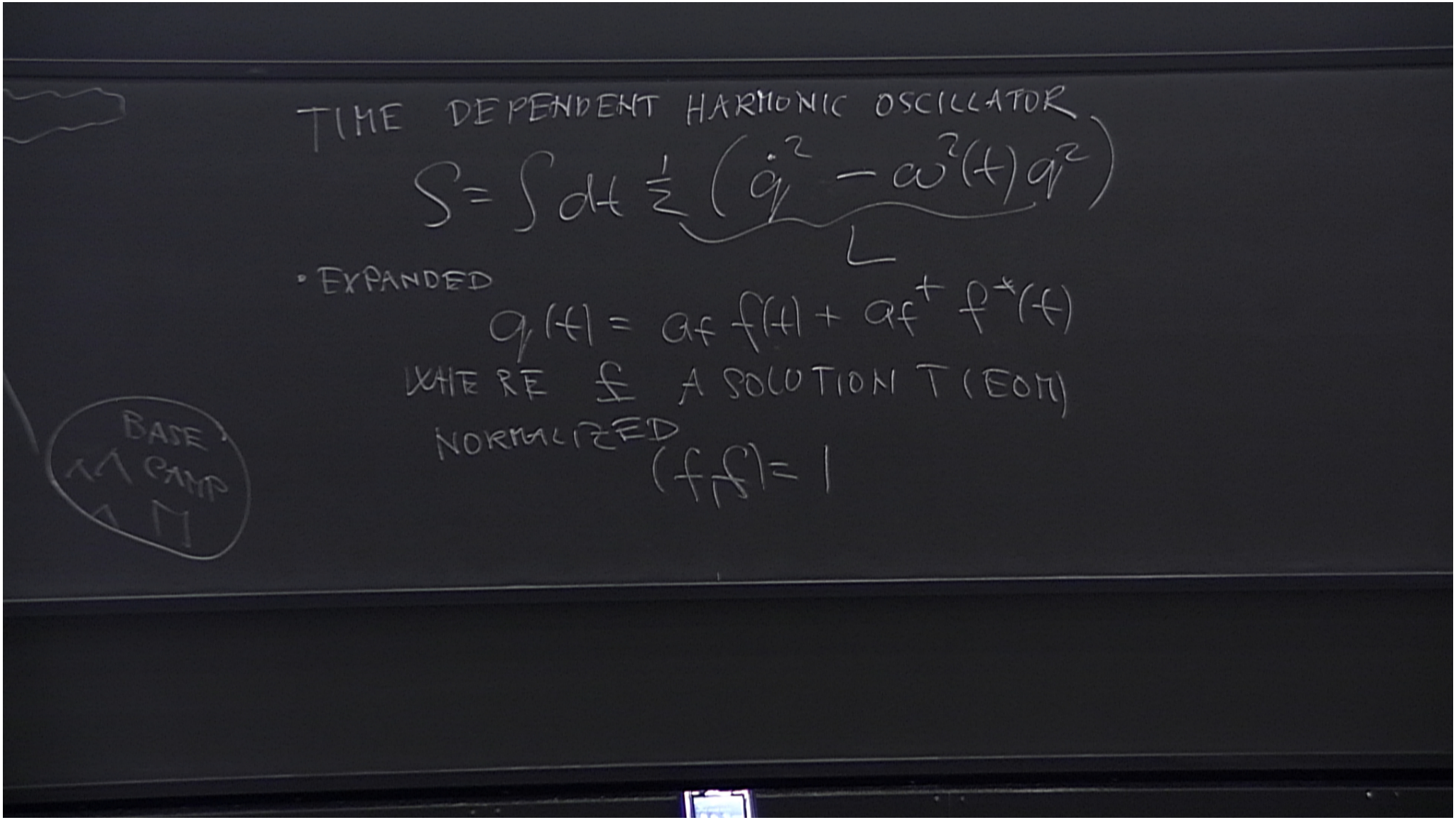
Date: Feb 16, 2017 10:15 AM

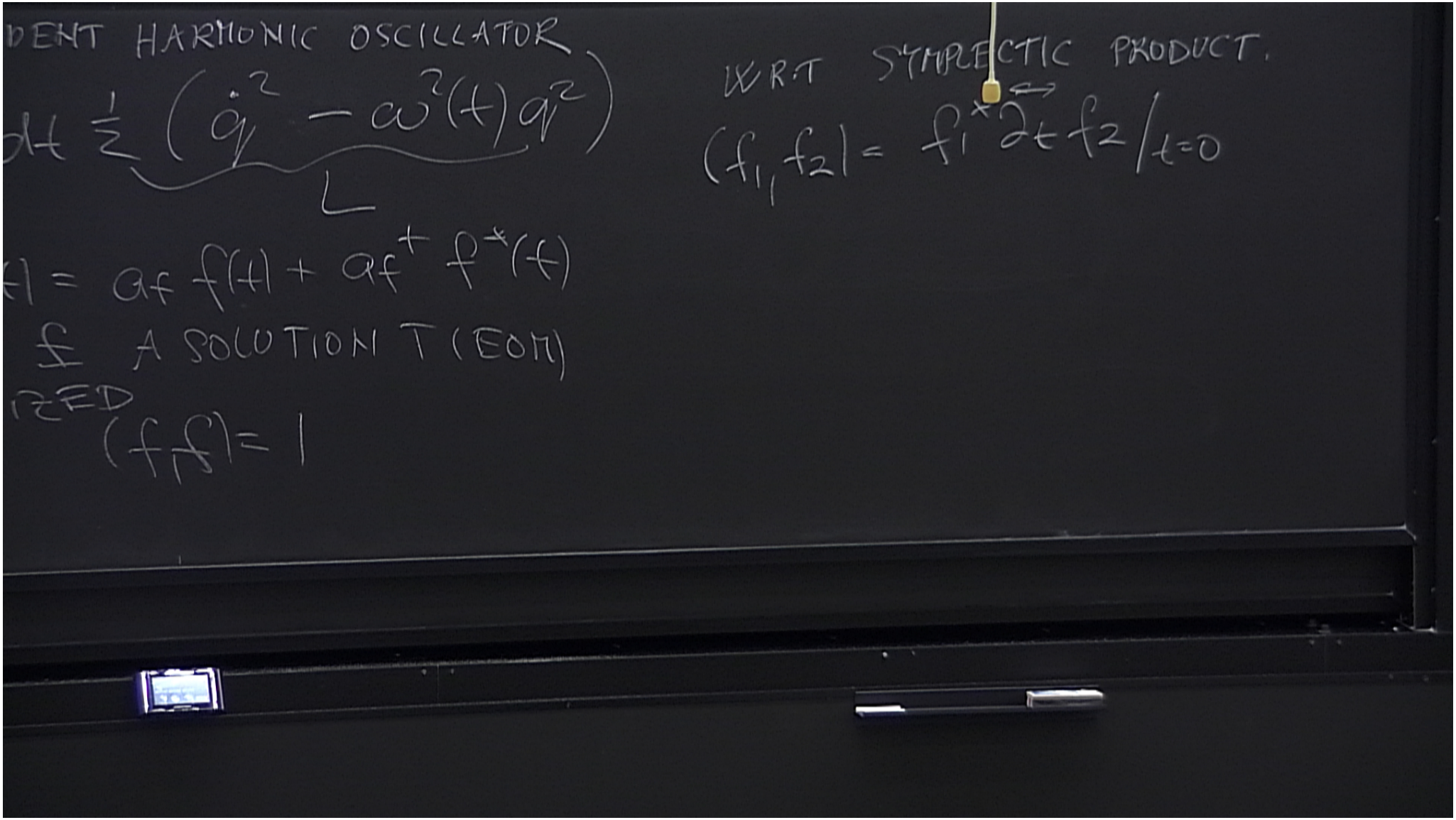
URL: http://pirsa.org/17020061

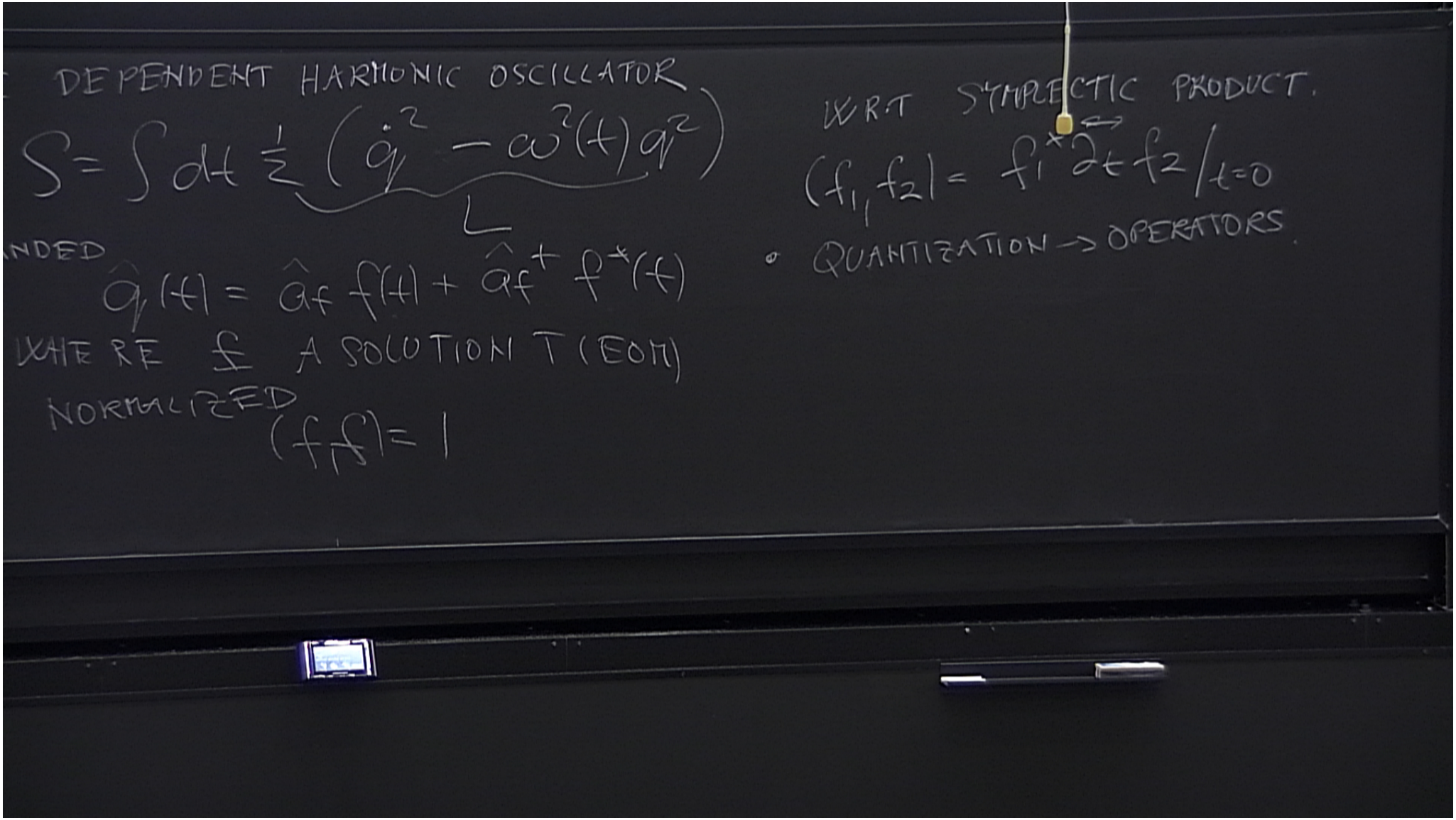
Abstract:

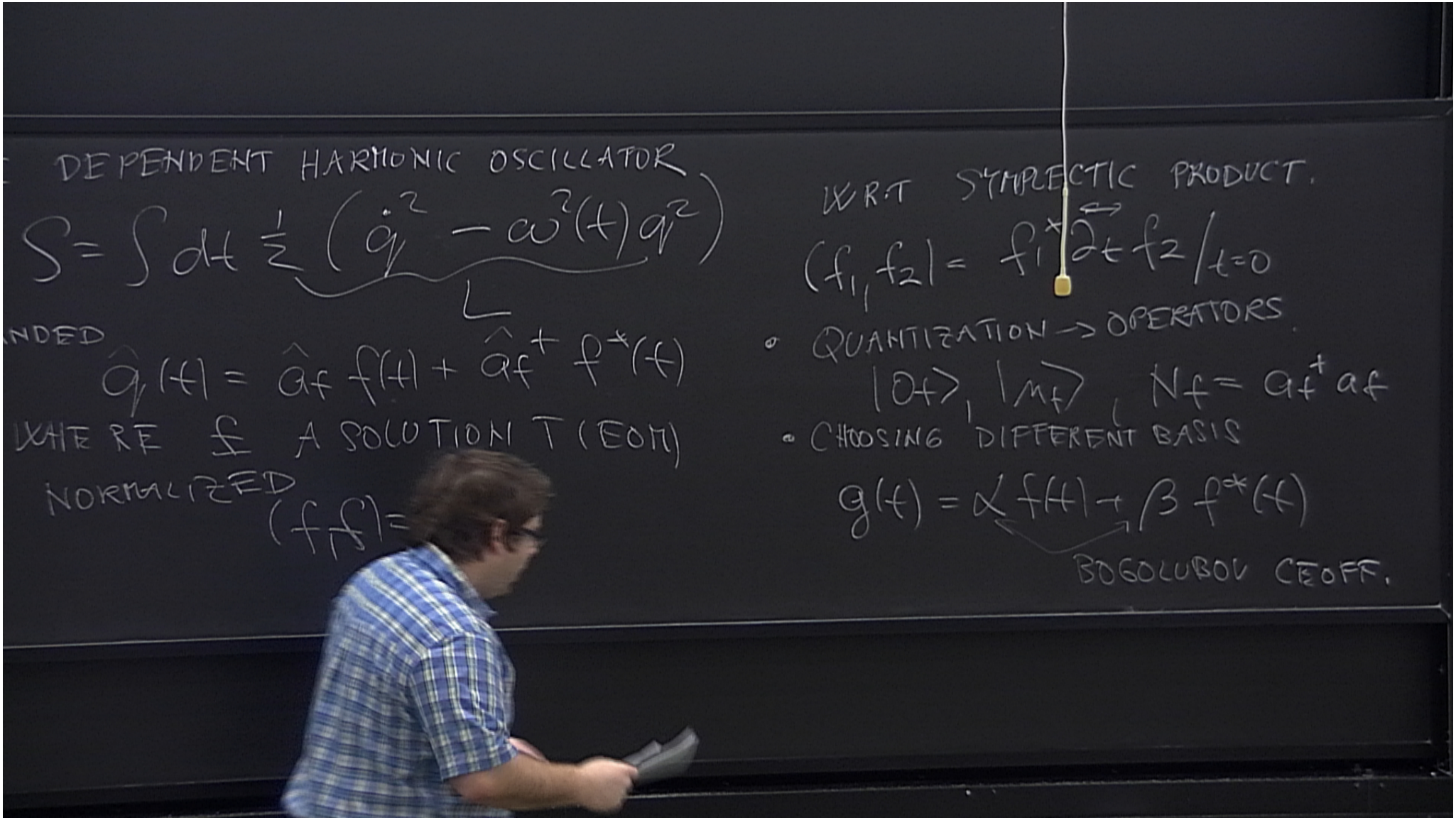

PRIMORDIAL SPECTRUM
IV
III
II
I
BASE
CAMP

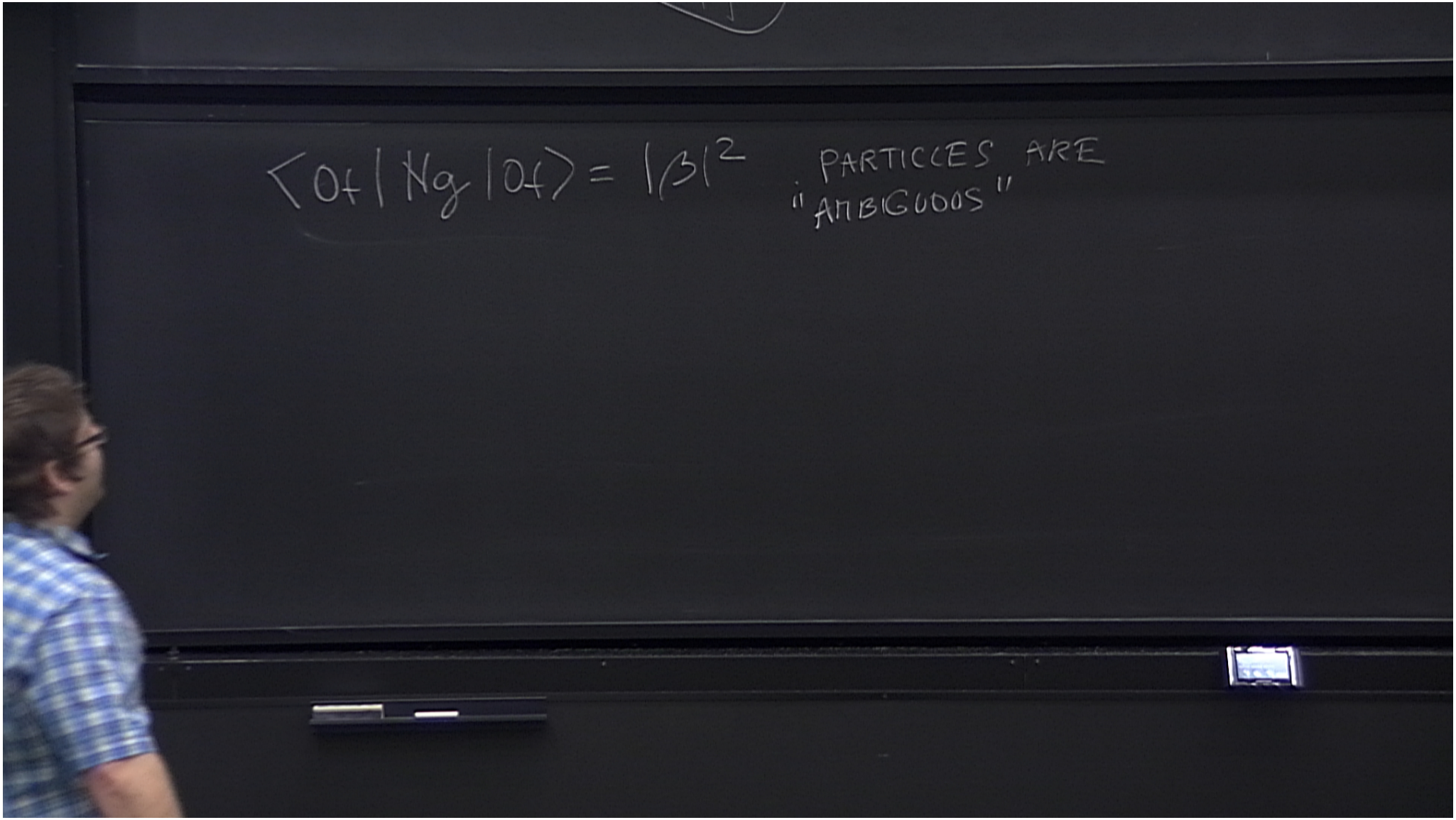

TIME DEPENDENT HARMONIC OSCILLATOR
$S = \int dt \, \underbrace{\frac{1}{2}\left(\dot{q}^2 - \omega^2(t) q^2\right)}_{L}$
BASE CAMP


TIME DEPENDENT HARMONIC OSCILLATOR
$S = \int dt \, \frac{1}{2} \underbrace{(\dot{q}^2 - \omega^2(t) q^2)}_{L}$
• EXPANDED
$q(t) = a f f(t) + a f^+ f^*(t)$
WHERE f A SOLUTION T (EOM)
NORMALIZED
$(f, f) = 1$
BASE CAMP


DENT HARMONIC OSCILLATOR
$dt \frac{1}{2}(\dot{q}^2 - \omega^2(t) q^2)$
L
$= a_f f(t) + a_f^+ f^*(t)$
A SOLUTION T (EOM)
$(f,f) = 1$
W.R.T SYMPLECTIC PRODUCT.
$(f_1, f_2) = f_1^* \overleftrightarrow{\partial_t} f_2 \big|_{t=0}$


DEPENDENT HARMONIC OSCILLATOR
$S = \int dt \, \frac{1}{2} \underbrace{(\dot{q}^2 - \omega^2(t) q^2)}_{L}$
NDED
$\hat{q}(t) = \hat{a}_f f(t) + \hat{a}_f^\dagger f^*(t)$
WHERE f A SOLUTION T (EOM)
NORMALIZED
$(f,f) = 1$
W.R.T SYMPLECTIC PRODUCT.
$(f_1, f_2) = f_1^* \overleftrightarrow{\partial_t} f_2 \big|_{t=0}$
• QUANTIZATION -> OPERATORS.


DEPENDENT HARMONIC OSCILLATOR
$S = \int dt \, \frac{1}{2} (\dot{q}^2 - \omega^2(t) q^2)$
L
NDED
$\hat{q}(t) = \hat{a}_f f(t) + \hat{a}_f^+ f^*(t)$
WHERE f A SOLUTION T (EOM)
NORMALIZED
$(f, f) =$
W.R.T SYMPLECTIC PRODUCT.
$(f_1, f_2) = f_1^* \overleftrightarrow{\partial_t} f_2 \big|_{t=0}$
• QUANTIZATION -> OPERATORS
$|0_f\rangle, |n_f\rangle, N_f = a_f^+ a_f$
• CHOOSING DIFFERENT BASIS
$g(t) = \alpha f(t) + \beta f^*(t)$
BOGOLUBOV COEFF.

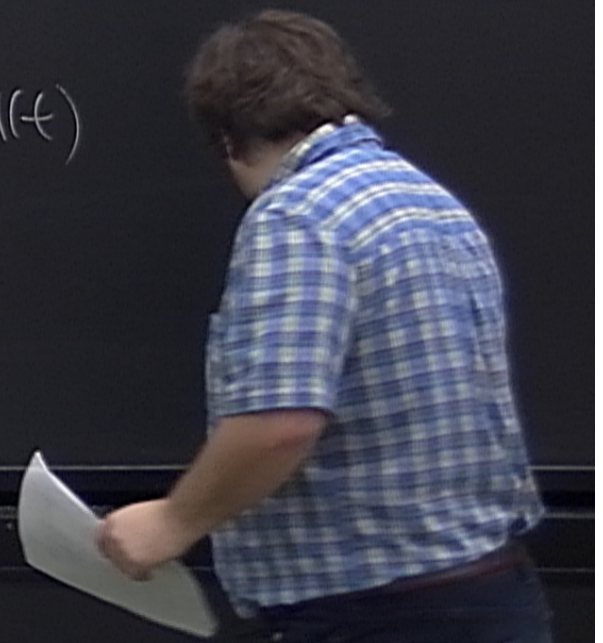

$\langle 0_f | N_g | 0_f \rangle = |\beta|^2$
PARTICLES ARE "AMBIGUOUS"

$\langle 0_f | N_g | 0_f \rangle = |\beta|^2$ PARTICLES ARE "AMBIGUOUS"

(3) IS THE VACUUM LOWEST ENERGY STATE?

EIGENSTATES OF HAMILTONIAN?

$$H = p\dot{q} - L = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$$



$\langle 0_f | N_g | 0_f \rangle = |\beta|^2$ PARTICLES ARE "AMBIGUOUS"

- (?) IS THE VACUUM LOWEST ENERGY STATE? EIGENSTATES OF HAMILTONIAN?

$$H = p\dot{q} - L = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$$

$$= \frac{1}{2}\left(a_f \dot{f} + a_f^\dagger \dot{f}^*\right)^2 + \frac{1}{2}\left(a_f f + a_f^\dagger f^*\right)^2 \omega^2$$

$$= \frac{1}{2}\Big(\left(\dot{f}^2 + \omega^2 f^2\right) a_f a_f + \left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger + \left(|\dot{f}|^2 + \omega^2 |f|^2\right)\left(a_f a_f^\dagger + a_f^\dagger a_f\right)\Big)$$

$\langle 0_f | N_g | 0_f \rangle = |\beta|^2$ PARTICLES ARE "AMBIGUOUS"

- (?) IS THE VACUUM LOWEST ENERGY STATE?
  EIGEN STATES OF HAMILTONIAN?

$$H = p\dot{q} - L = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$$

$$= \frac{1}{2}\left(a\dot{f} + a^\dagger \dot{f}^*\right)^2 + \frac{1}{2}\left(af + a^\dagger f^*\right)^2\omega^2$$

$$= \frac{1}{2}\Big(\left(\dot{f}^2 + \omega^2 f^2\right) a_f a_f + \left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger$$
$$\left(|\dot{f}|^2 + \omega^2 |f|^2\right)\left(a_f a_f^\dagger + a_f^\dagger a_f\right)\Big)$$

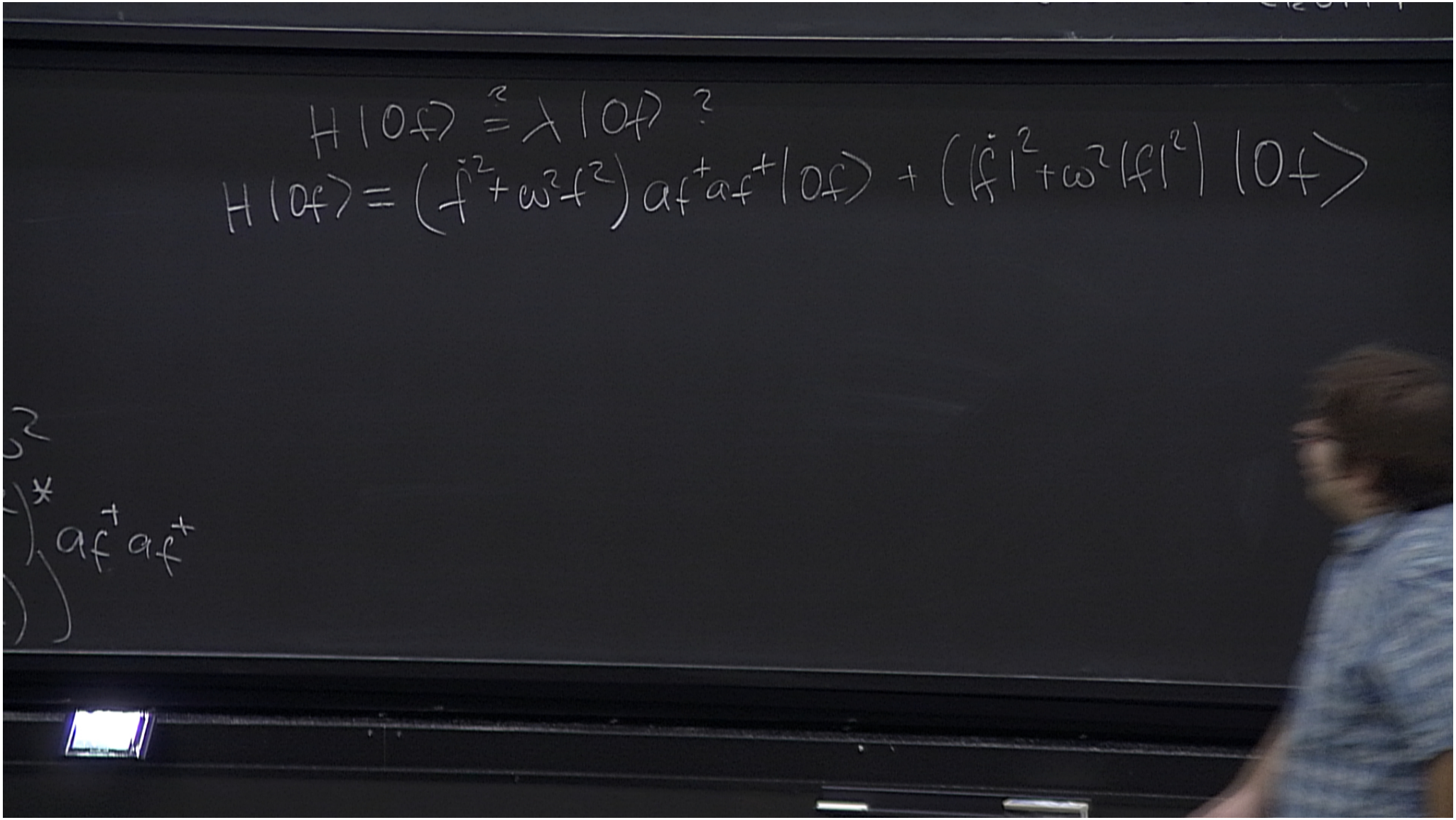
H

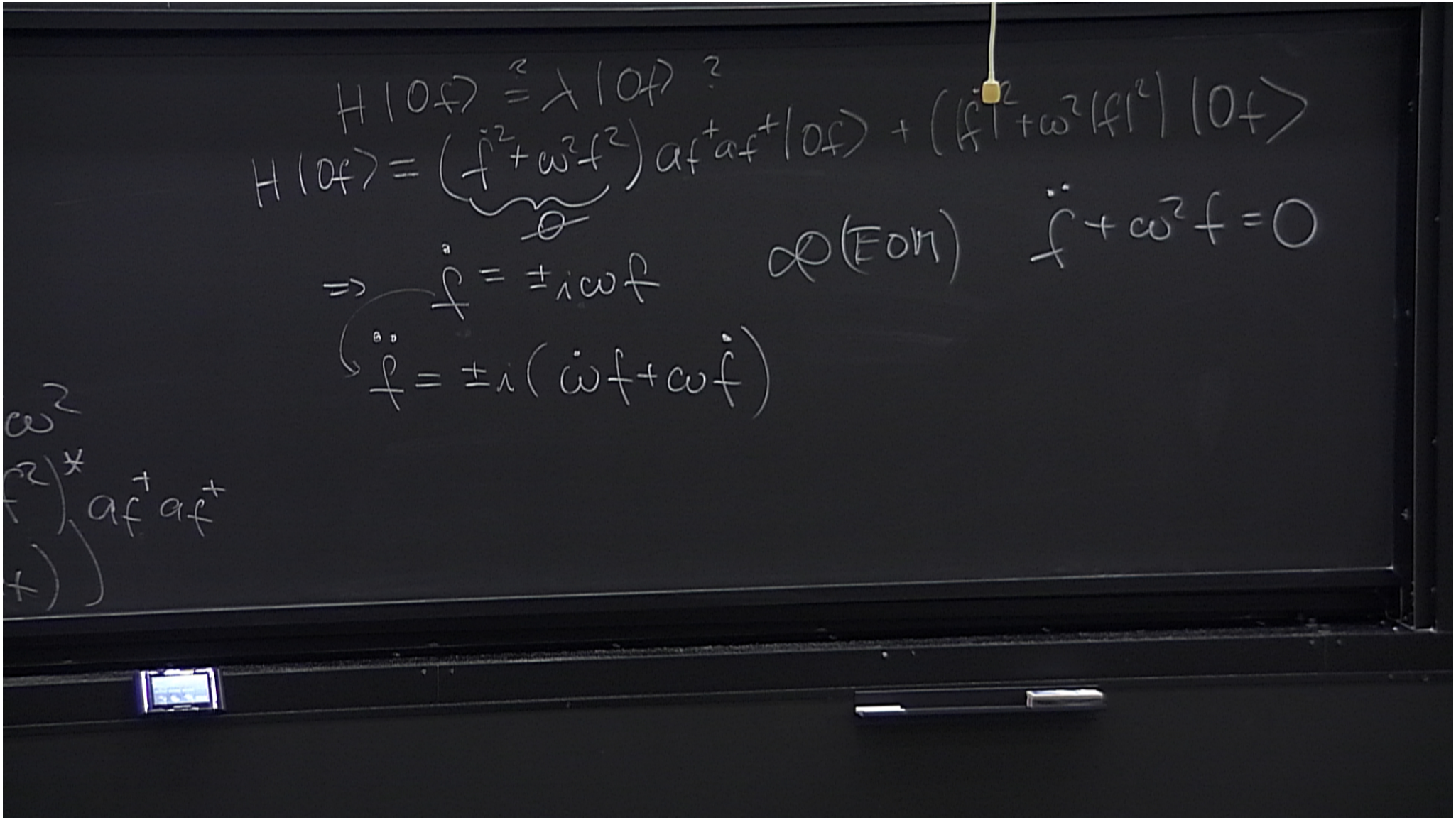
$H|0_f\rangle$

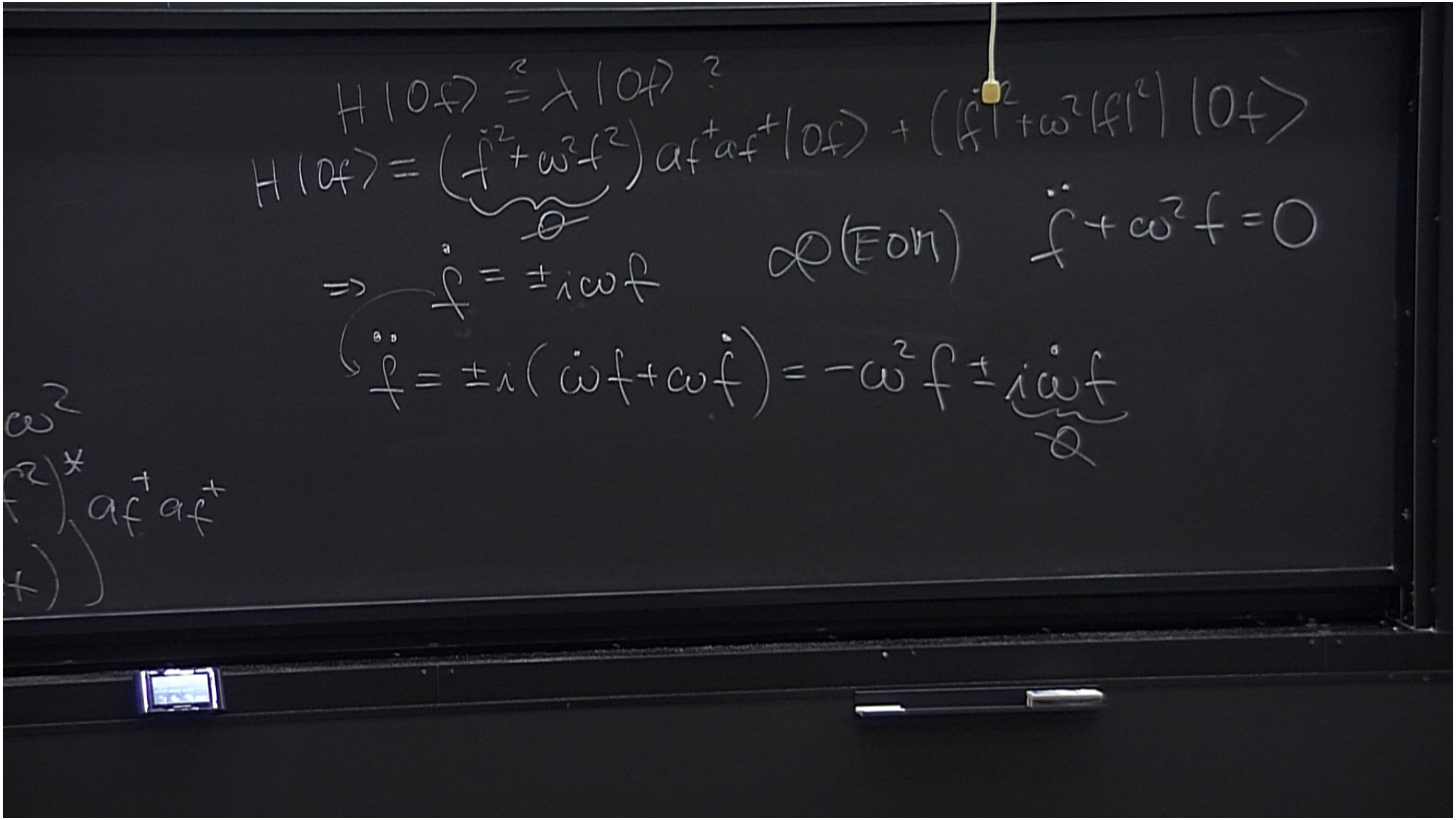
$$H|0_f\rangle = \lambda|0_f\rangle$$

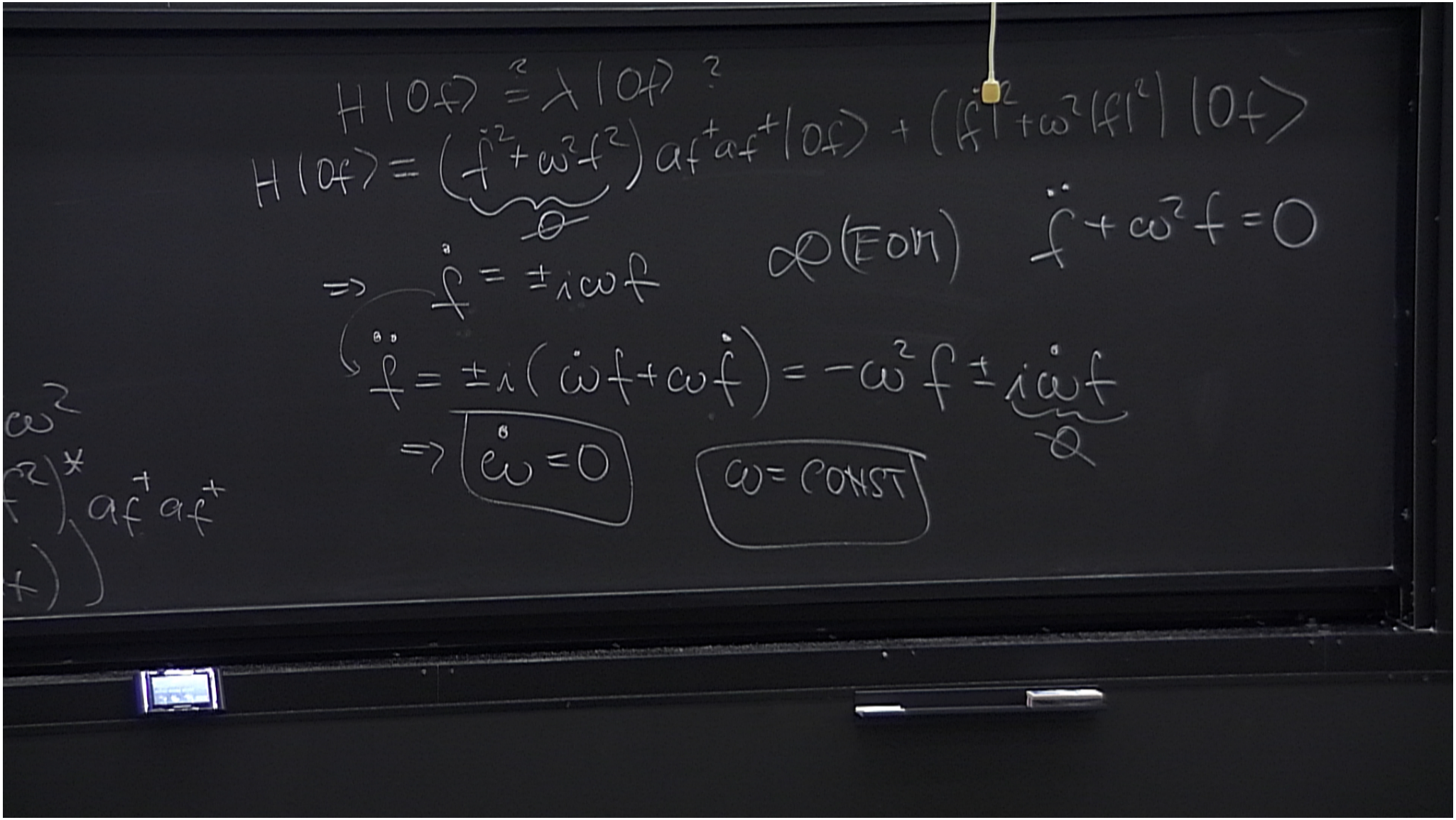
$$H|0_f\rangle = \left(\dot{f}^2 + \omega^2 f^2\right) a_f^\dagger a_f^\dagger |0_f\rangle + \left(|\dot{f}|^2 + \omega^2 |f|^2\right)|0_f\rangle$$

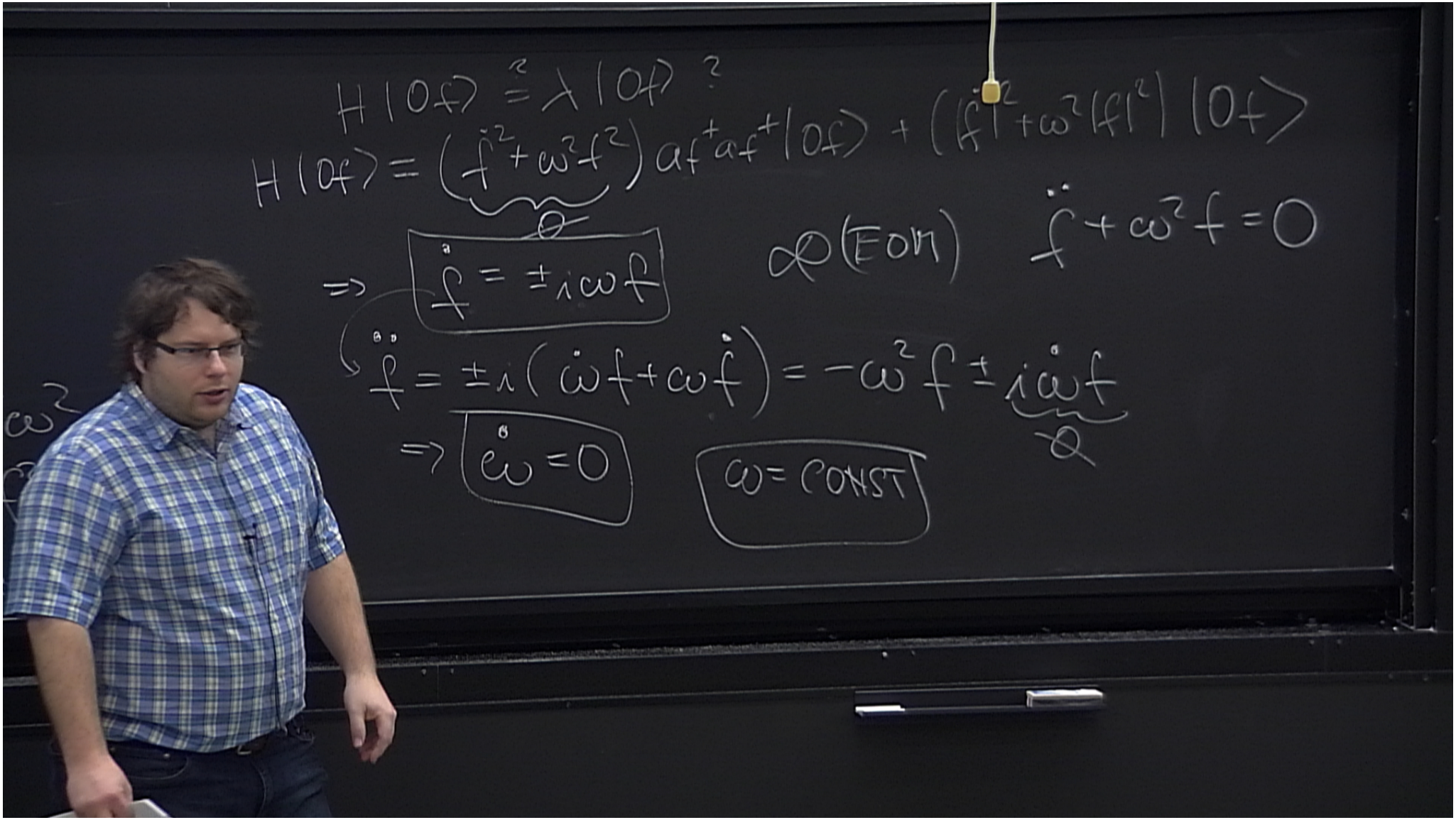
$$a_f^\dagger a_f^\dagger$$



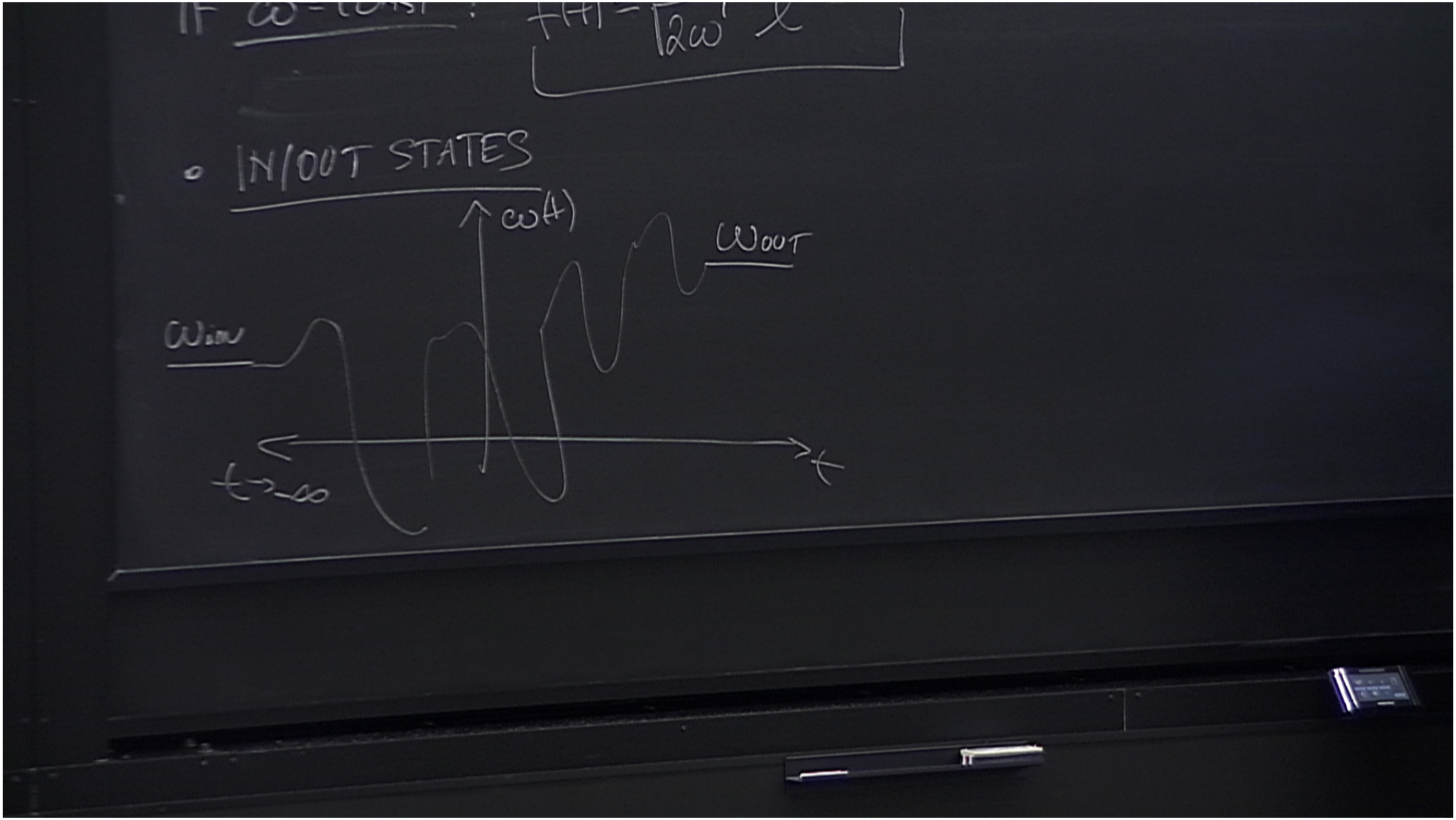

$H|0_f\rangle = \lambda|0_f\rangle$?
$H|0_f\rangle = (\dot{f}^2 + \omega^2 f^2)\, a_f^\dagger a_f^\dagger |0_f\rangle + (\dot{f}^2 + \omega^2|f|^2)|0_f\rangle$
$\Rightarrow \dot{f} = \pm i\omega f$
$\propto$ (EOM) $\ddot{f} + \omega^2 f = 0$
$\ddot{f} = \pm i(\dot{\omega} f + \omega \dot{f})$

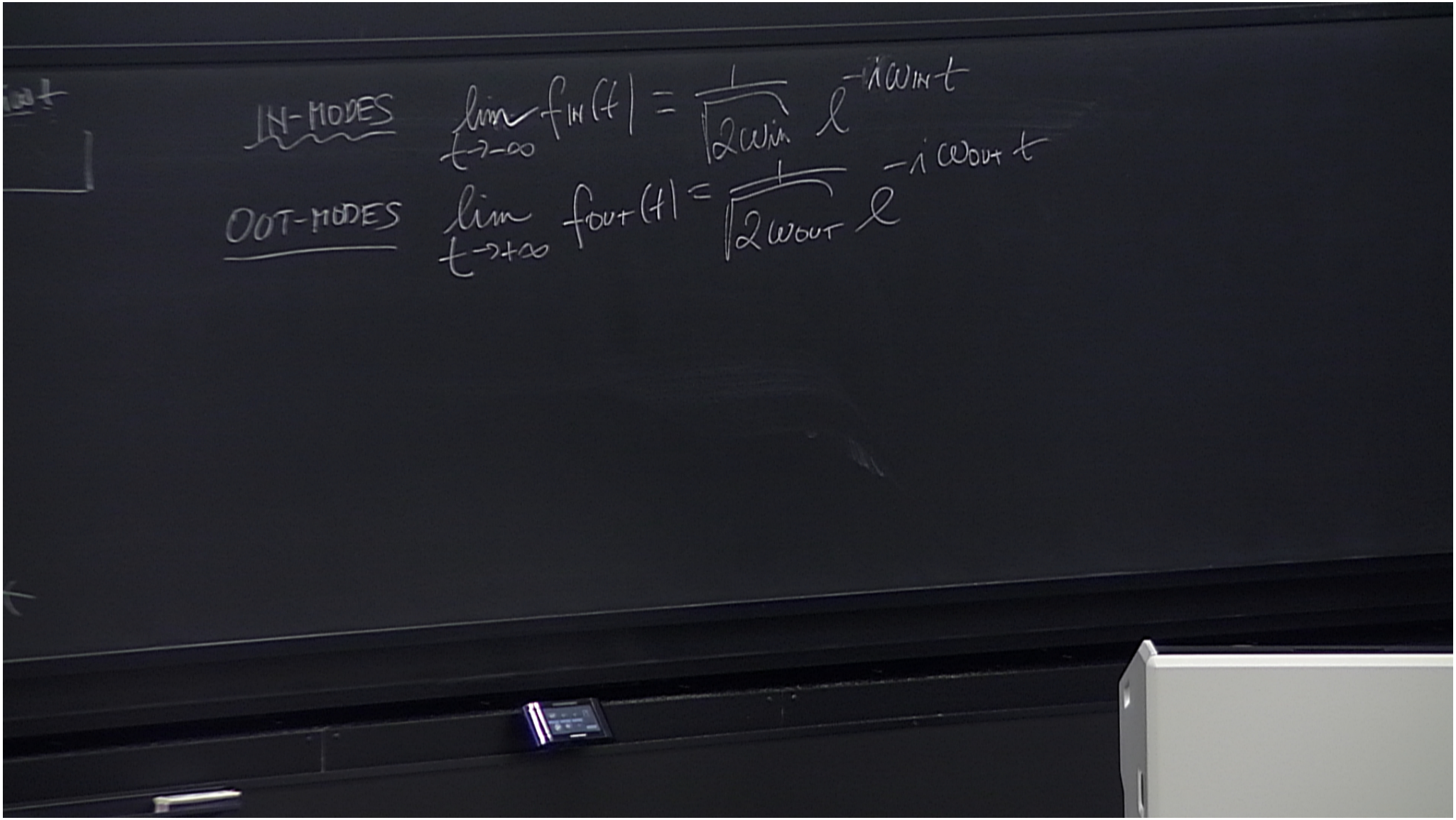

$H|0_f\rangle = \lambda|0_f\rangle$?
$H|0_f\rangle = (\dot{f}^2 + \omega^2 f^2)\, a_f^\dagger a_f^\dagger |0_f\rangle + (\dot{f}^2 + \omega^2 |f|^2)\,|0_f\rangle$
$\Rightarrow \dot{f} = \pm i\omega f$
$\propto$ (EOM) $\ddot{f} + \omega^2 f = 0$
$\ddot{f} = \pm i(\dot{\omega} f + \omega \dot{f}) = -\omega^2 f \pm i\dot{\omega} f$
$a_f^\dagger a_f^\dagger$

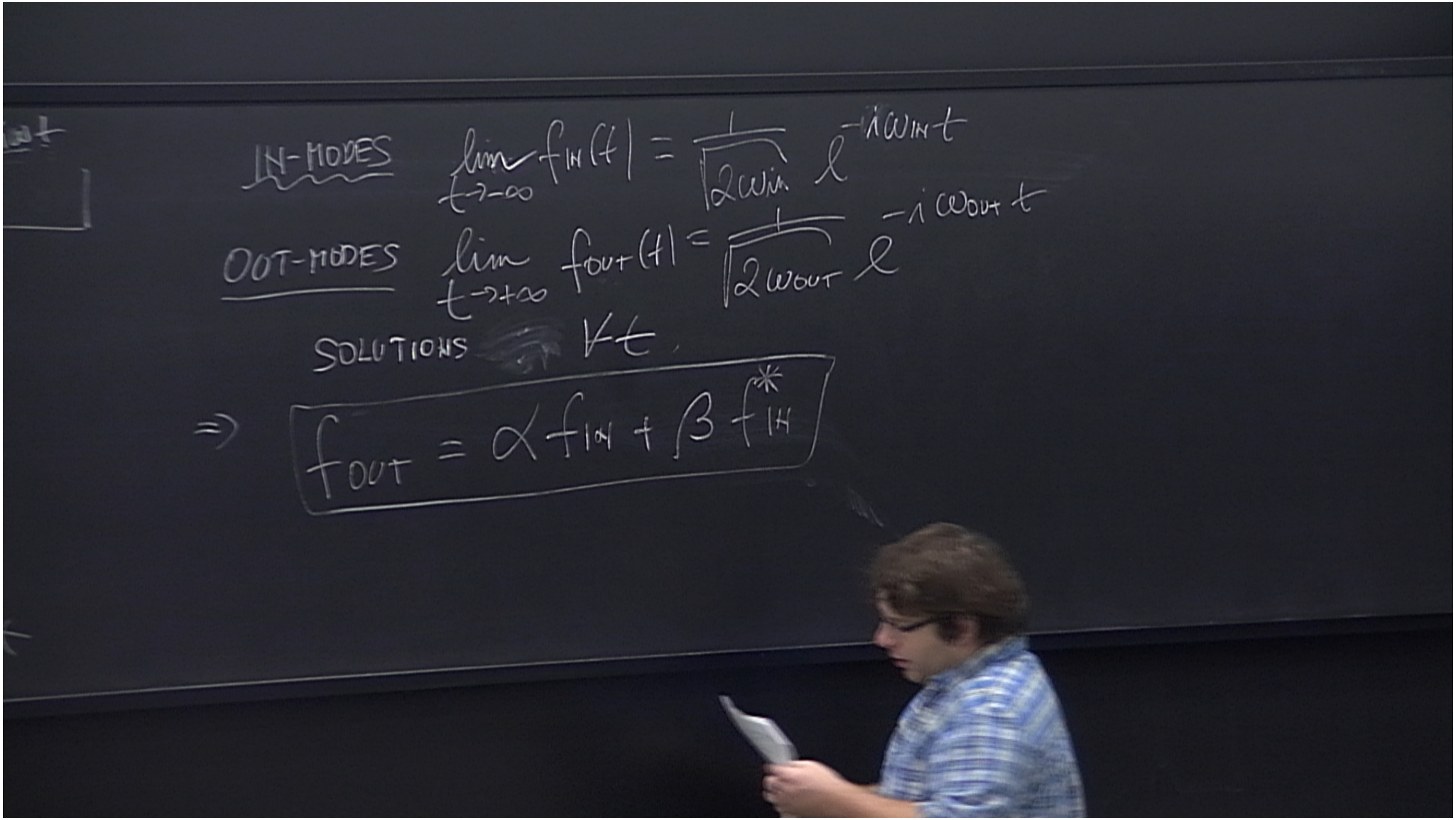

$H|0_f\rangle \stackrel{?}{=} \lambda|0_f\rangle$?
$H|0_f\rangle = (\dot{f}^2 + \omega^2 f^2)\, a_f^\dagger a_f^\dagger |0_f\rangle + ((\dot{f}^2 + \omega^2 f^2)|0_f\rangle$
$\Rightarrow \dot{f} = \pm i\omega f$
$\propto$ (EOM) $\ddot{f} + \omega^2 f = 0$
$\ddot{f} = \pm i(\dot{\omega} f + \omega \dot{f}) = -\omega^2 f \pm i\dot{\omega} f$
$\Rightarrow \dot{\omega} = 0$
$\omega = \text{CONST}$
$a_f^\dagger a_f^\dagger$

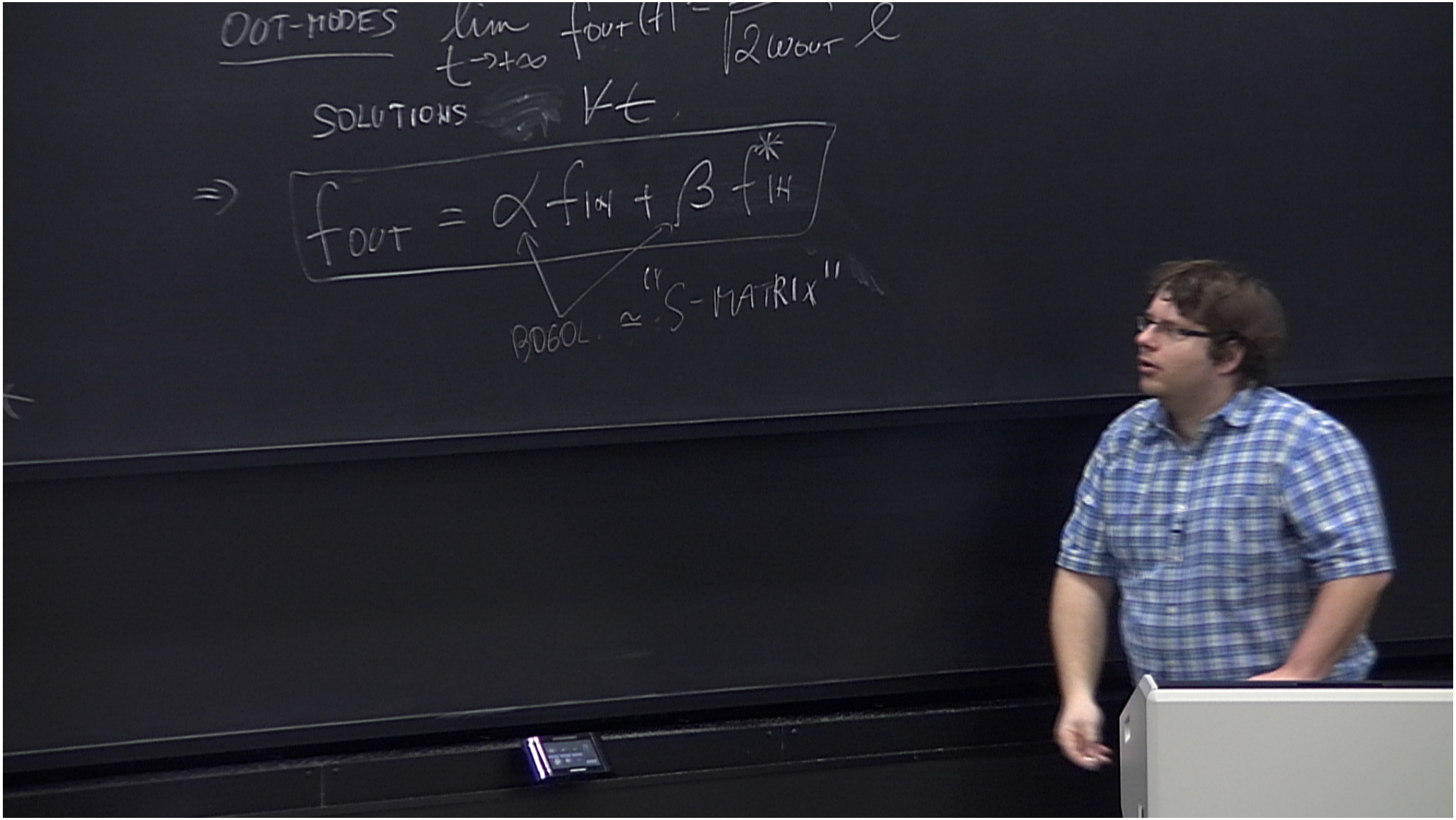

H|0f⟩ = λ|0f⟩ ?
H|0f⟩ = (ḟ² + ω²f²) a_f† a_f† |0f⟩ + (ḟ² + ω²|f|²)|0f⟩
0
⇒ ḟ = ±iωf
∝ (EOM)
f̈ + ω²f = 0
f̈ = ±i(ω̇f + ωḟ) = −ω²f ± iω̇f
0
⇒ ω̇ = 0
ω = CONST

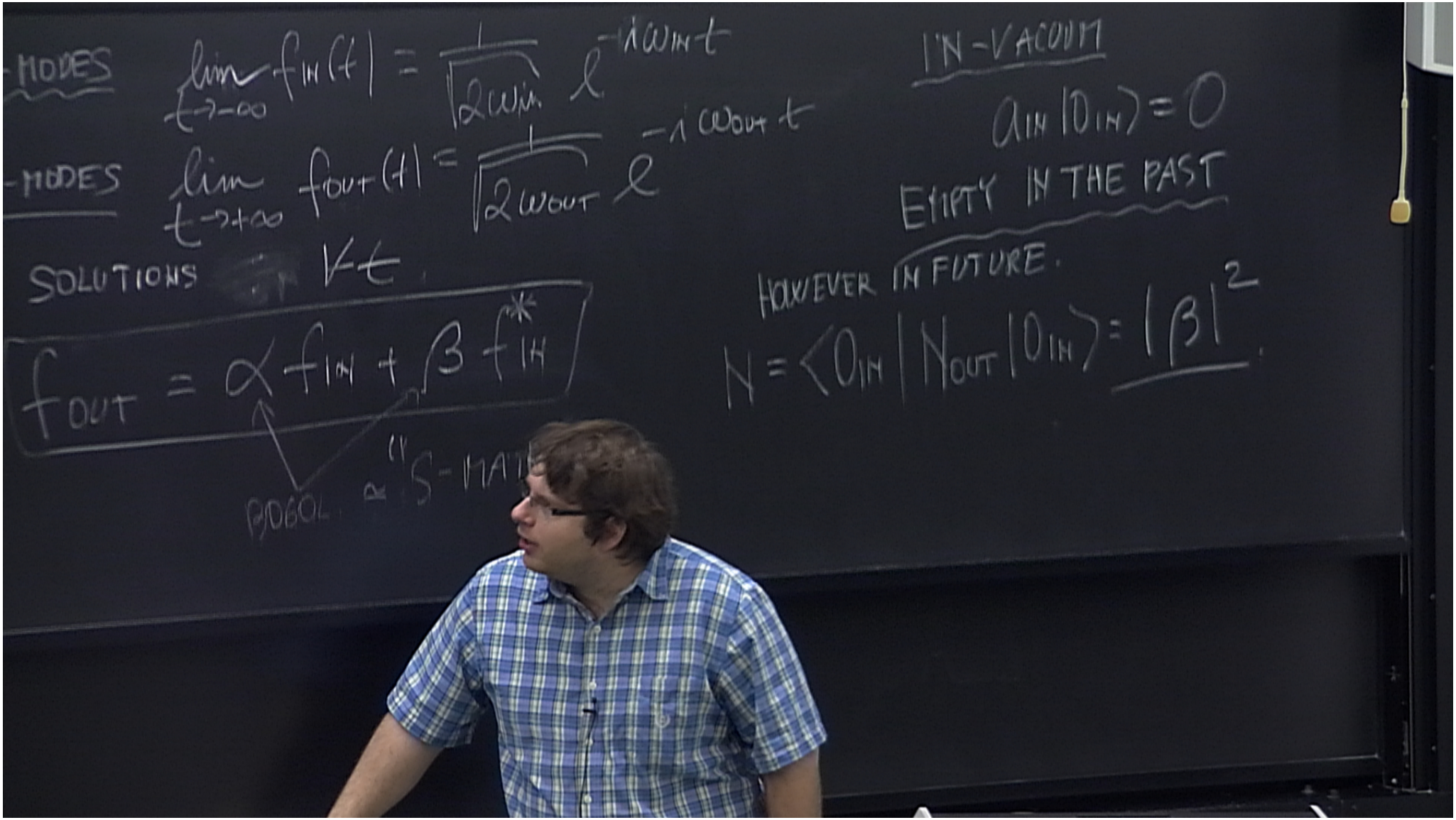
IF $\omega = \text{CONST}$: $f(t) = \frac{1}{\sqrt{2\omega}} e^{-i\omega t}$

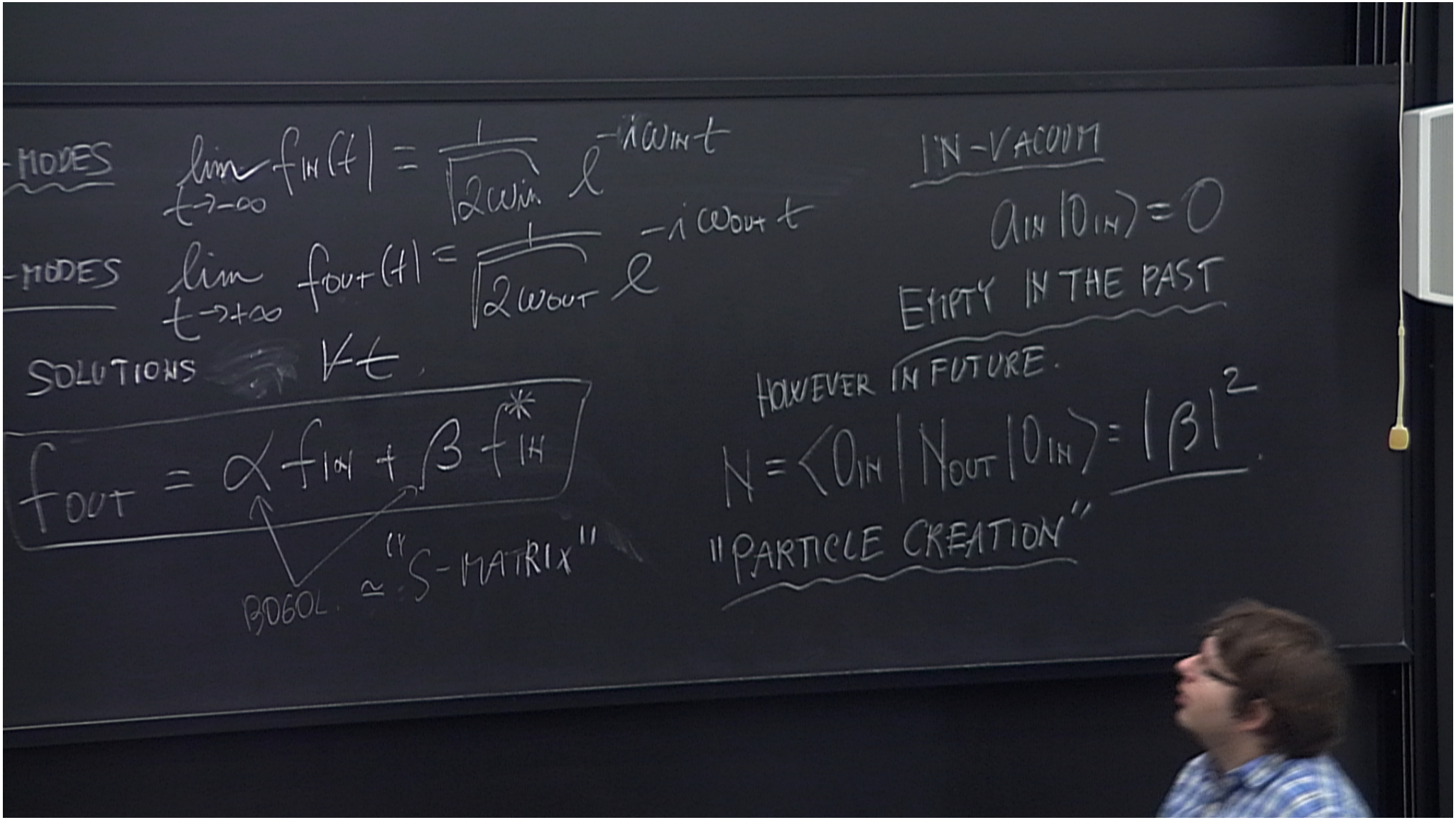

IF ω = const
• IN/OUT STATES
ω(t)
ω_OUT
ω_IN
t → −∞
t

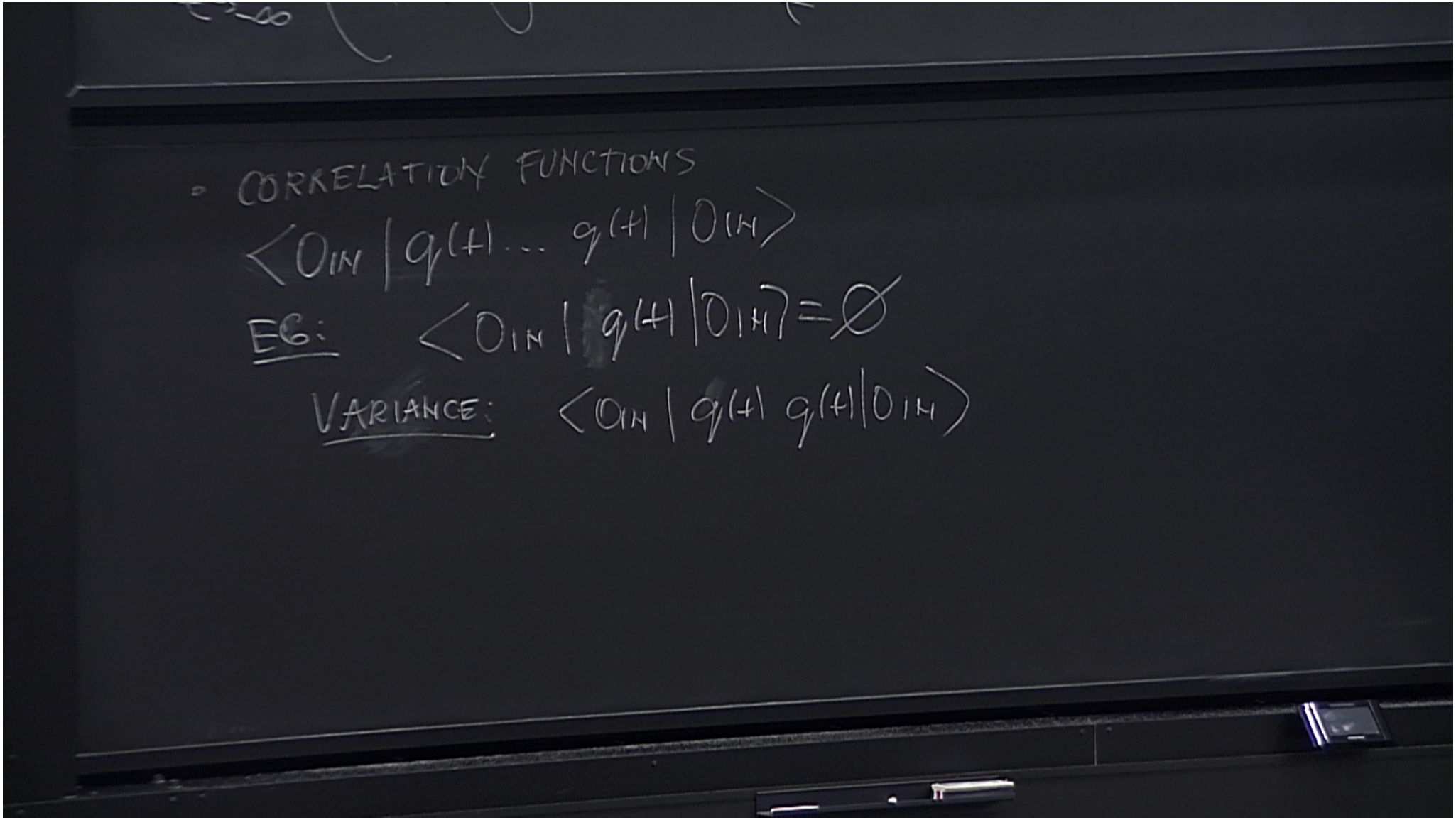

IN-MODES
$\lim_{t\to-\infty} f_{IN}(t) = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}$
OUT-MODES
$\lim_{t\to+\infty} f_{OUT}(t) = \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT}t}$


IN-MODES
$\lim_{t\to-\infty} f_{IN}(t) = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}$
OUT-MODES
$\lim_{t\to+\infty} f_{OUT}(t) = \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT}t}$
SOLUTIONS
$\Rightarrow$
$f_{OUT} = \alpha f_{IN} + \beta f^*_{IN}$


OOT-MODES
SOLUTIONS
$f_{OUT} = \alpha f_{IN} + \beta f_{IN}^*$
BOGOL. ≃ "S-MATRIX"


MODES
$\lim_{t\to-\infty} f_{IN}(t) = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}$
MODES
$\lim_{t\to+\infty} f_{OUT}(t) = \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT}t}$
SOLUTIONS
$f_{OUT} = \alpha f_{IN} + \beta f_{IN}^*$
IN-VACUUM
$a_{IN}|0_{IN}\rangle = 0$
EMPTY IN THE PAST
HOWEVER IN FUTURE.
$N = \langle 0_{IN}|N_{OUT}|0_{IN}\rangle = |\beta|^2$


IN-MODES
$\lim_{t\to-\infty} f_{IN}(t) = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}$
OUT-MODES
$\lim_{t\to+\infty} f_{OUT}(t) = \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT}t}$
SOLUTIONS $\forall t$.
$f_{OUT} = \alpha f_{IN} + \beta f^*_{IN}$
BOGOL. $\simeq$ "S-MATRIX"
IN-VACUUM
$a_{IN}|0_{IN}\rangle = 0$
EMPTY IN THE PAST
HOWEVER IN FUTURE.
$N = \langle 0_{IN}|N_{OUT}|0_{IN}\rangle = |\beta|^2$
"PARTICLE CREATION"


• CORRELATION FUNCTIONS
$\langle 0_{IN} | q(t) \dots q(t) | 0_{IN} \rangle$
EG: $\langle 0_{IN} | q(t) | 0_{IN} \rangle = \emptyset$
VARIANCE: $\langle 0_{IN} | q(t)\, q(t) | 0_{IN} \rangle$

- CORRELATION FUNCTIONS

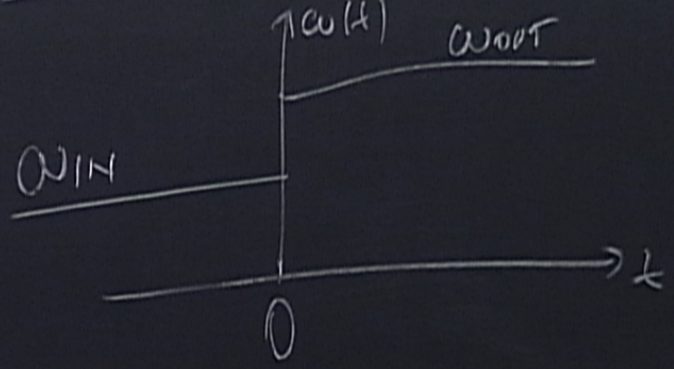
$$\langle 0_{IM} | q(t) \dots q(t) | 0_{IM} \rangle$$

EG: $\langle 0_{IM} | q(t) | 0_{IM} \rangle = \emptyset$

VARIANCE: $\langle 0_{IM} | q(t)\, q(t) | 0_{IM} \rangle = |f_{IM}(t)|^2$.

EXAMPLE OF PARTICLE CREATION

$\omega(t) = \omega_{IN}\,\theta(-t) + \omega_{OUT}\,\theta(t)$

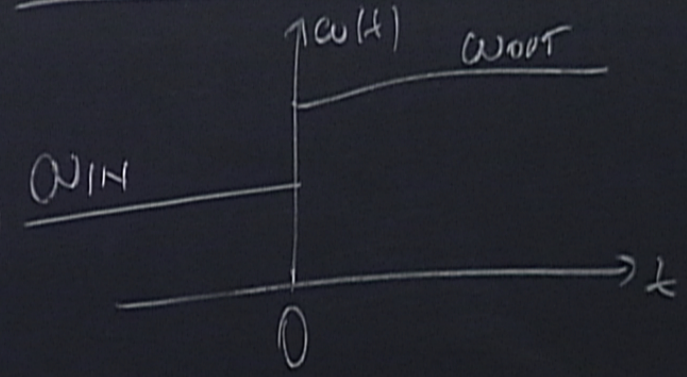


$|\ldots(t)|^2$

EXAMPLE OF PARTICLE CREATION

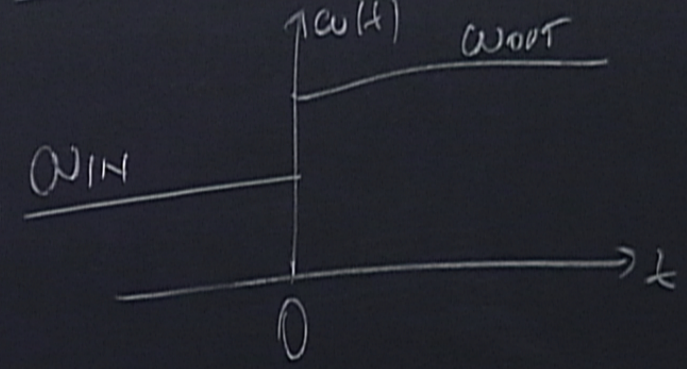
$$\omega(t) = \omega_{IN}\,\theta(-t) + \omega_{OUT}\,\theta(t)$$

$$f_{IN} = \frac{1}{\sqrt{2\omega_{IN}}}\, e^{-i\omega_{IN} t}\,\theta(-t)$$



$|f_{IN}(t)|^2$ 0

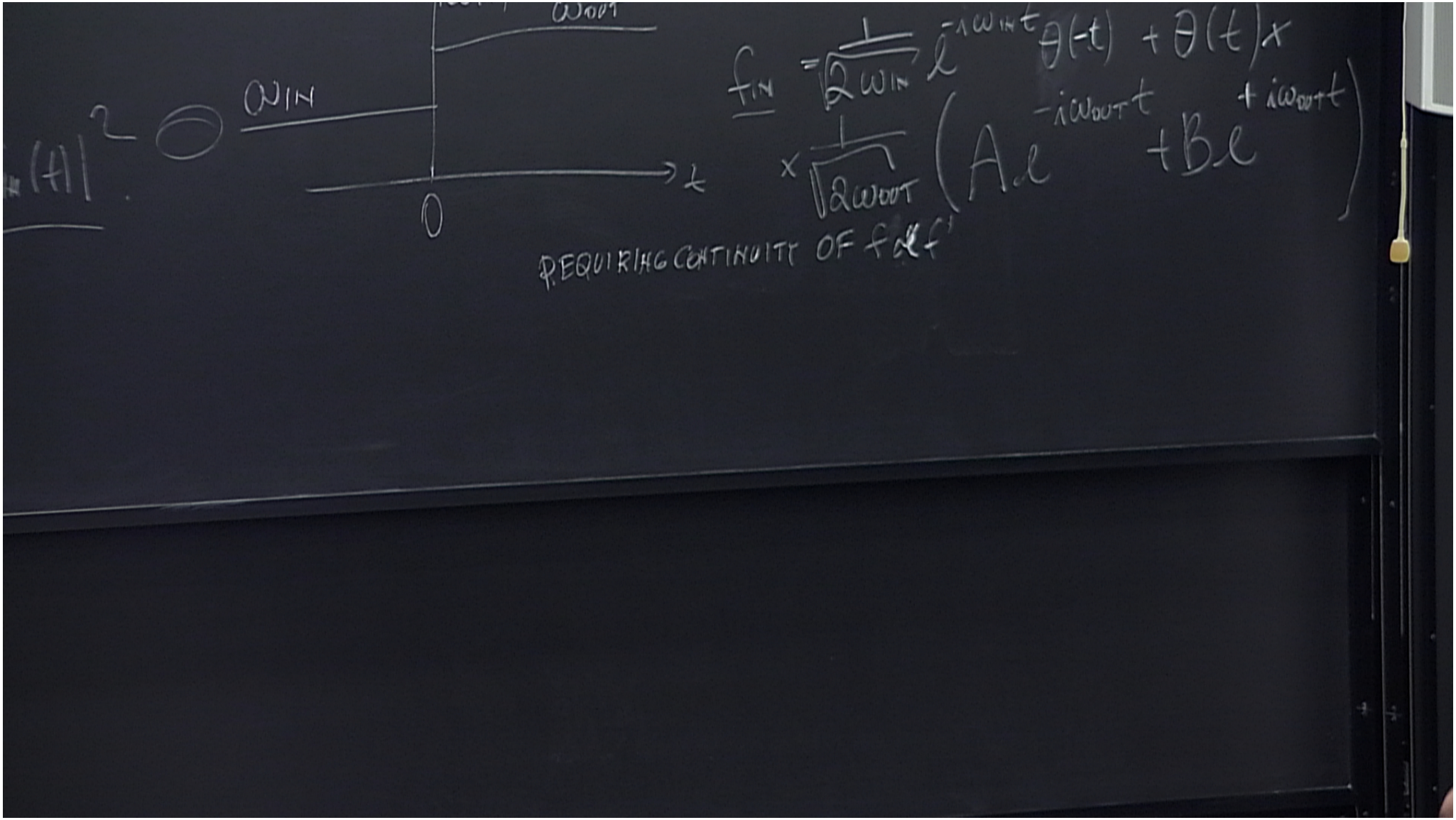
EXAMPLE OF PARTICLE CREATION

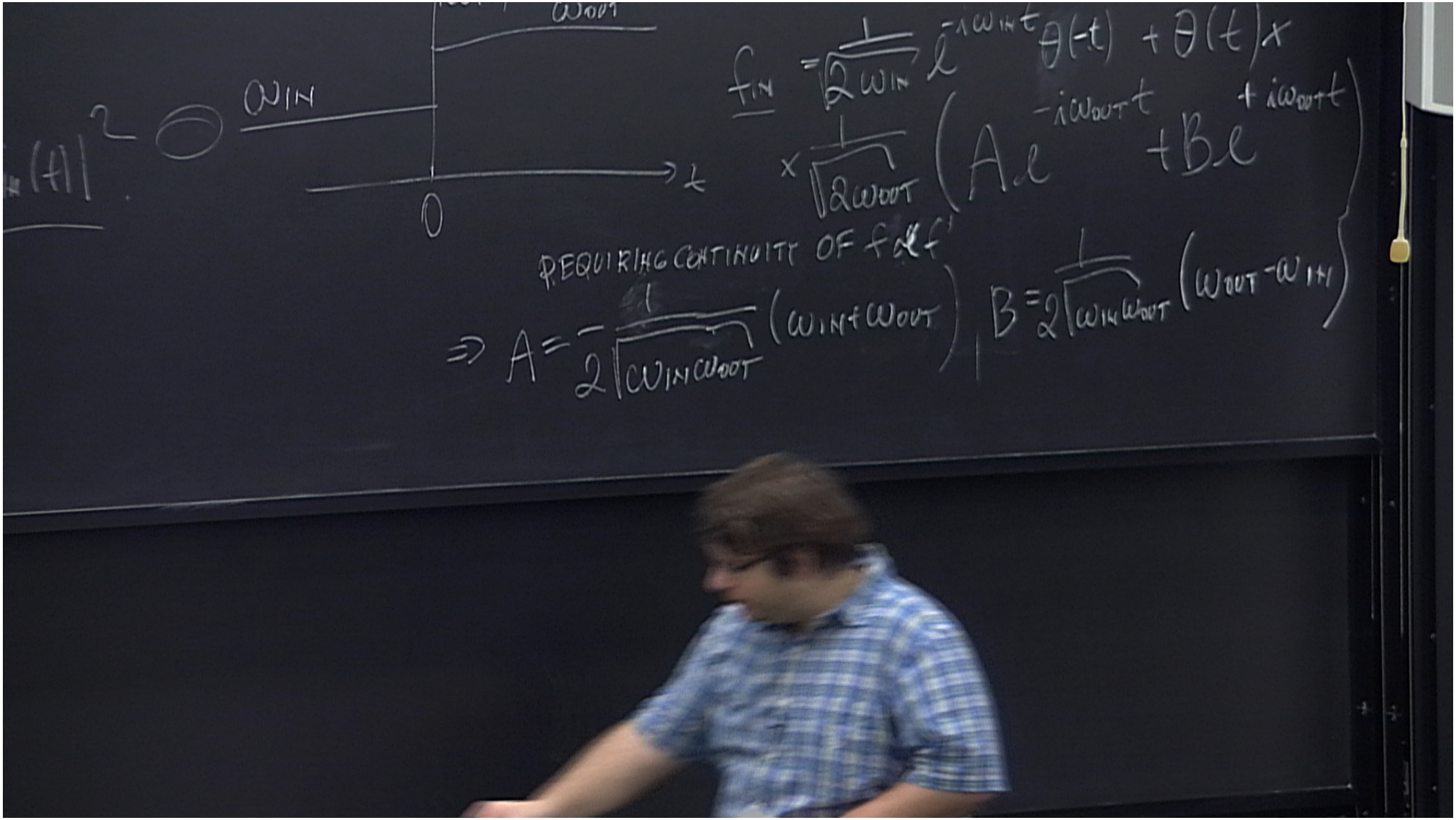


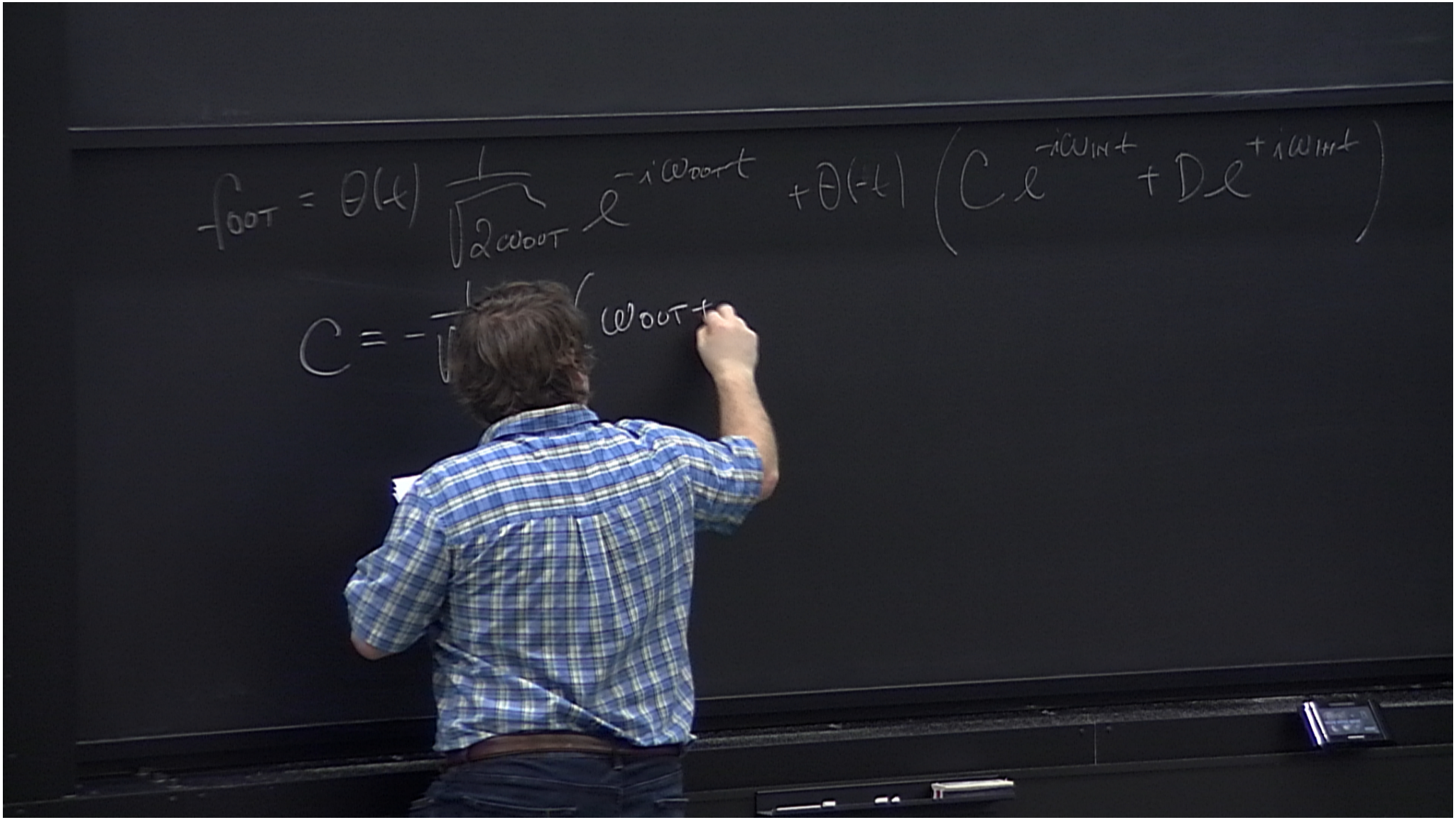
$$\omega(t) = \omega_{IN}\,\theta(-t) + \omega_{OUT}\,\theta(t)$$

$$f_{IN} = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}\,\theta(-t) + \theta(t)\,x$$

$$x \frac{1}{\sqrt{2\omega_{OUT}}} \left(A e^{-i\omega_{OUT}t} + B e^{+i\omega_{OUT}t} \right)$$


ω_{OUT}
ω_{IN}
0
t
$f_{IN} = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}\theta(-t) + \theta(t)x$
$\times \frac{1}{\sqrt{2\omega_{OUT}}}\left(A e^{-i\omega_{OUT}t} + B e^{+i\omega_{OUT}t}\right)$
REQUIRING CONTINUITY OF f & f'


ω_{IN}
0
t
$f_{IN} = \frac{1}{\sqrt{2\omega_{IN}}} e^{-i\omega_{IN}t}\theta(-t) + \theta(t)\times$
$\times \frac{1}{\sqrt{2\omega_{OUT}}}\left(Ae^{-i\omega_{OUT}t} + Be^{+i\omega_{OUT}t}\right)$
REQUIRING CONTINUITY OF f & f'
$\Rightarrow A = -\frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}(\omega_{IN}+\omega_{OUT})$, $B = \frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}(\omega_{OUT}-\omega_{IN})$


$f_{OUT} = \theta(t) \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT}t} + \theta(-t)\left(C e^{-i\omega_{IN}t} + D e^{+i\omega_{IN}t}\right)$
$C = -\frac{1}{\sqrt{}}\left(\omega_{OUT}+\right.$

$$f_{OUT} = \theta(t)\frac{1}{\sqrt{2\omega_{OUT}}}e^{-i\omega_{OUT}t} + \theta(-t)\left(Ce^{-i\omega_{IN}t} + De^{+i\omega_{IN}t}\right)$$

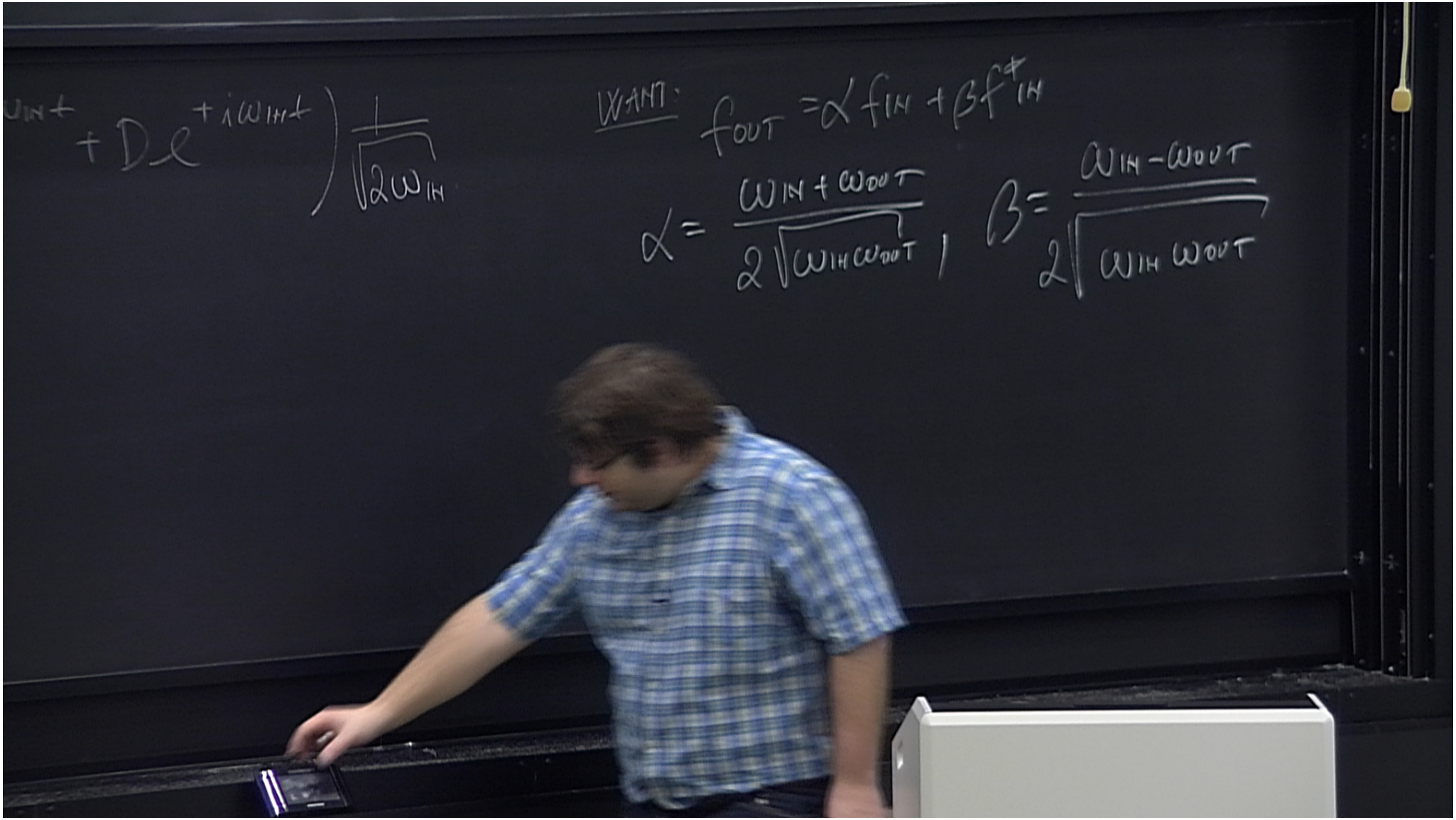
$$C = -\frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}\left(\omega_{OUT} + \omega_{IN}\right)$$

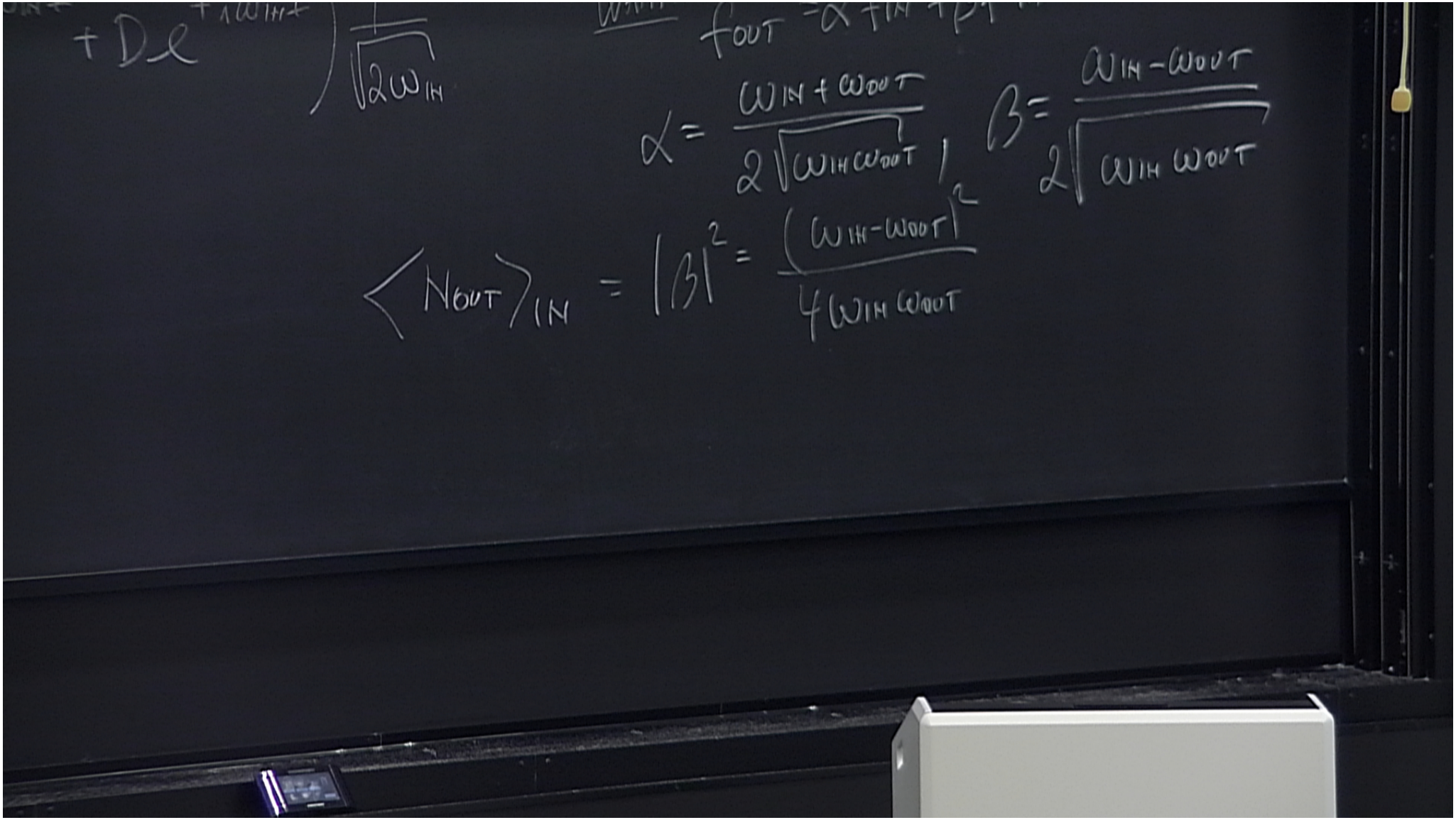
$$D = \oplus\!\!-\!\!|\!|\!-\left(\omega_{IN} - \omega_{OUT}\right)$$

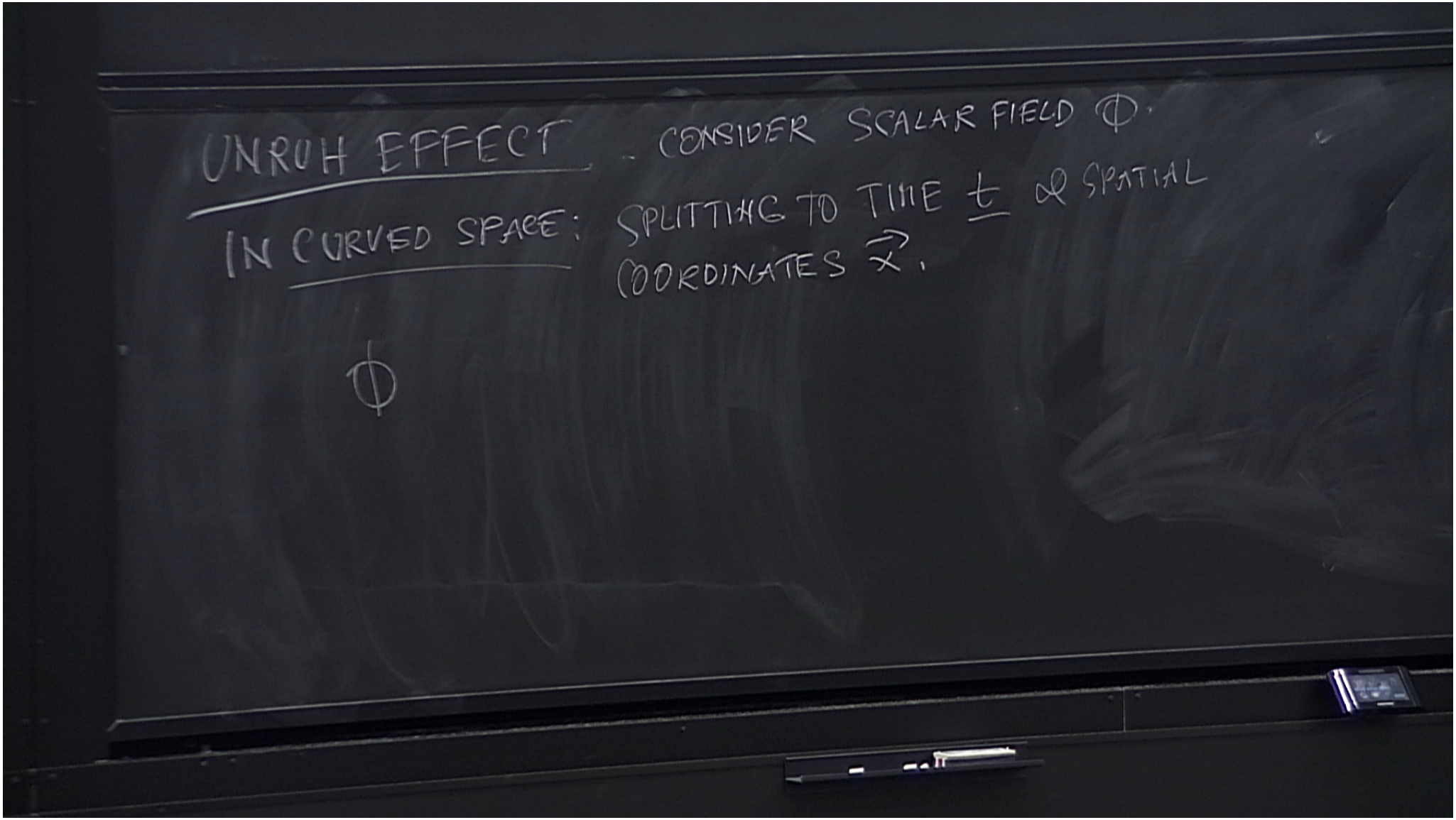
$$f_{OUT} = \theta(t)\frac{1}{\sqrt{2\omega_{OUT}}}e^{-i\omega_{OUT}t} + \theta(-t)\left(Ce^{-i\omega_{IN}t} + De^{+i\omega_{IN}t}\right)\frac{1}{\sqrt{2\omega_{IN}}}$$

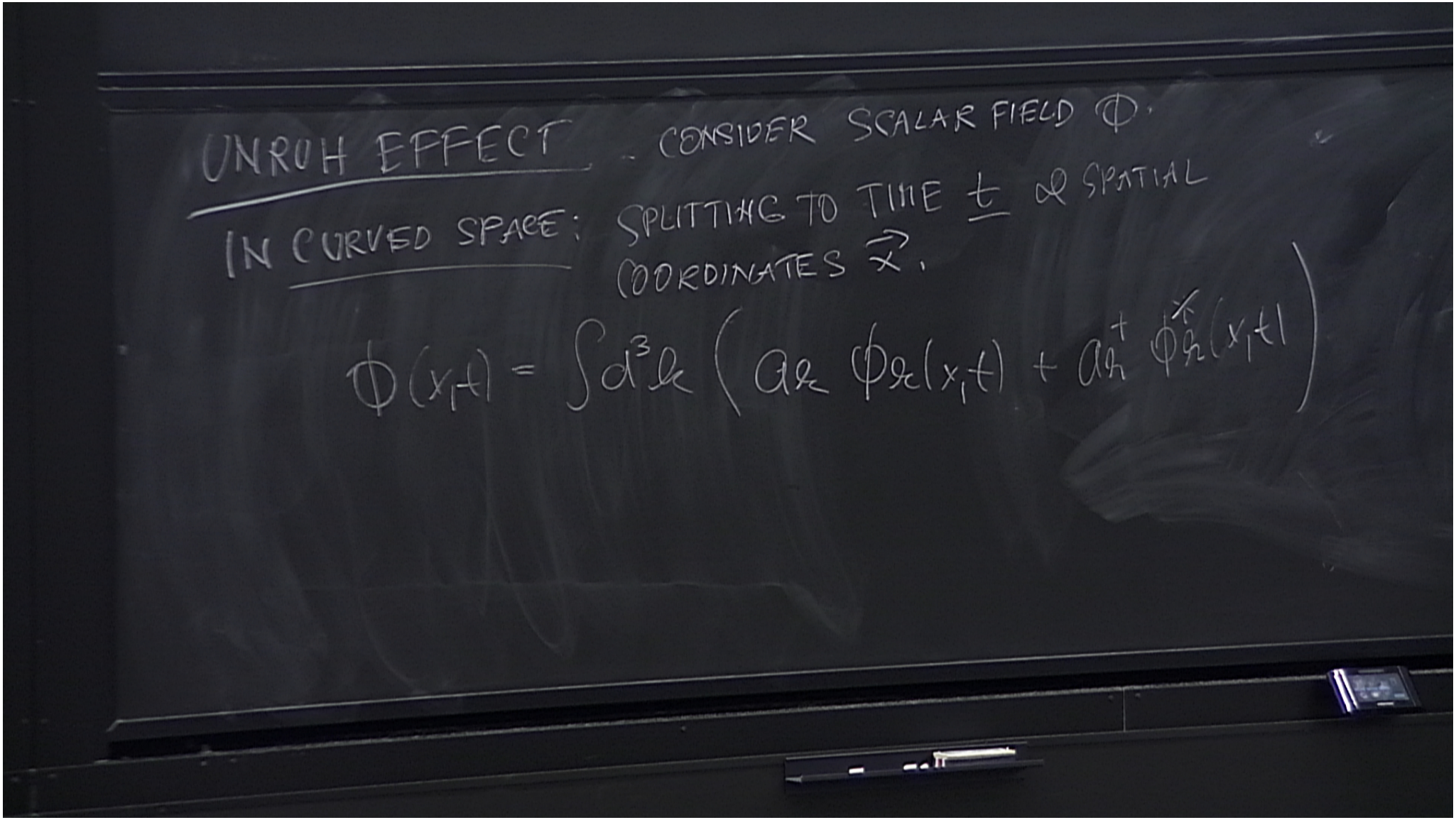
$$C = -\frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}\left(\omega_{OUT} + \omega_{IN}\right)$$

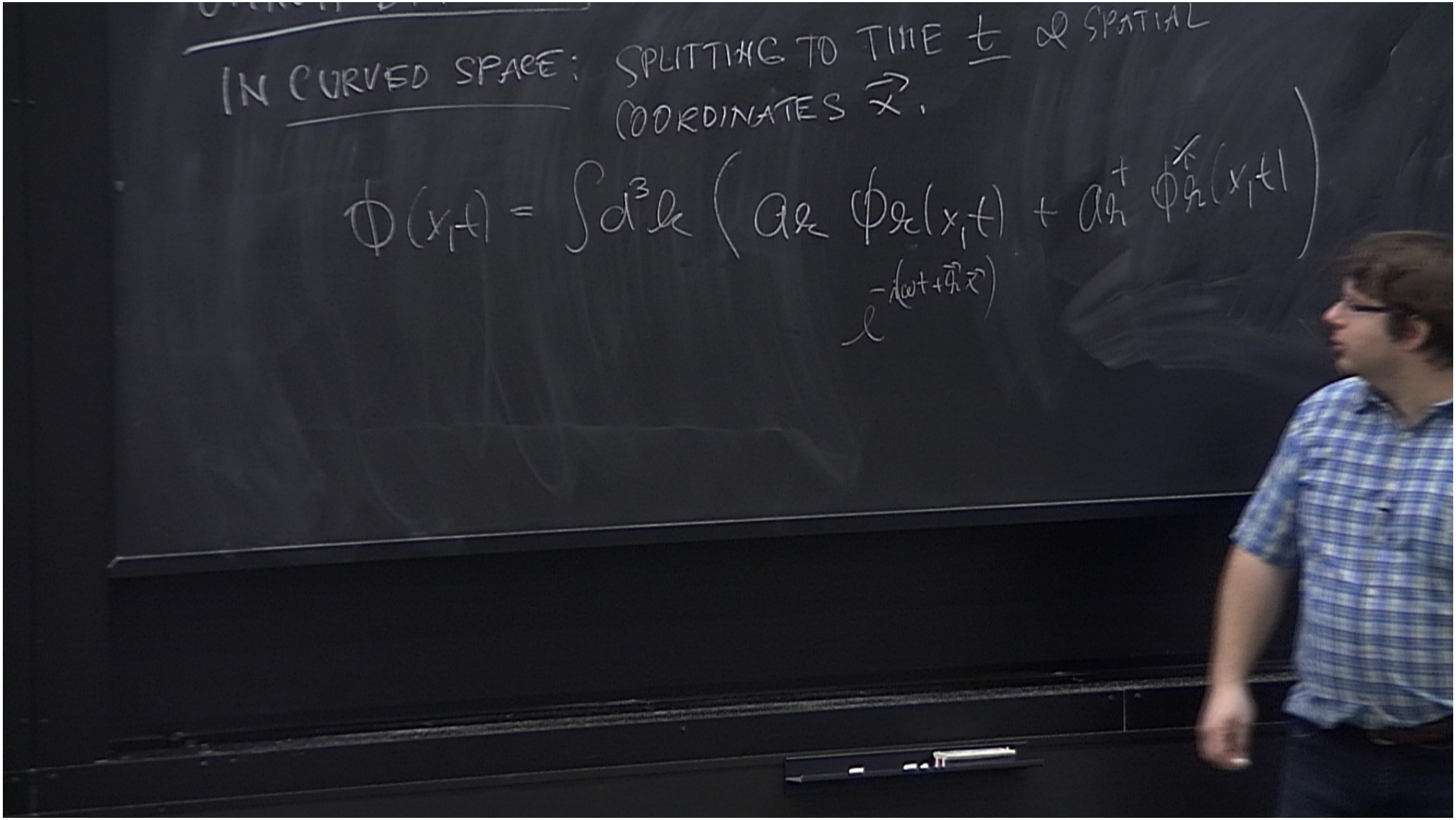
$$D = \oplus\!\!\!\!-\,11\,-\left(\omega_{IN} - \omega_{OUT}\right)$$


$+De^{+i\omega_{IN}t}\Big)\frac{1}{\sqrt{2\omega_{IN}}}$
WANT:
$f_{OUT} = \alpha f_{IN} + \beta f^{\dagger}_{IN}$
$\alpha = \frac{\omega_{IN} + \omega_{OUT}}{2\sqrt{\omega_{IN}\omega_{OUT}}}$, $\beta = \frac{\omega_{IN} - \omega_{OUT}}{2\sqrt{\omega_{IN}\omega_{OUT}}}$


$+De^{+i\omega_{IN}t}\Big) \frac{1}{\sqrt{2\omega_{IN}}}$
$f_{OUT} = \alpha f_{IN} + \beta$
$\alpha = \frac{\omega_{IN}+\omega_{OUT}}{2\sqrt{\omega_{IN}\omega_{OUT}}}$, $\beta = \frac{\omega_{IN}-\omega_{OUT}}{2\sqrt{\omega_{IN}\omega_{OUT}}}$
$\langle N_{OUT}\rangle_{IN} = |\beta|^2 = \frac{(\omega_{IN}-\omega_{OUT})^2}{4\omega_{IN}\omega_{OUT}}$


UNRUH EFFECT
. CONSIDER SCALAR FIELD Φ.
IN CURVED SPACE: SPLITTING TO TIME t & SPATIAL
COORDINATES x⃗.
Φ

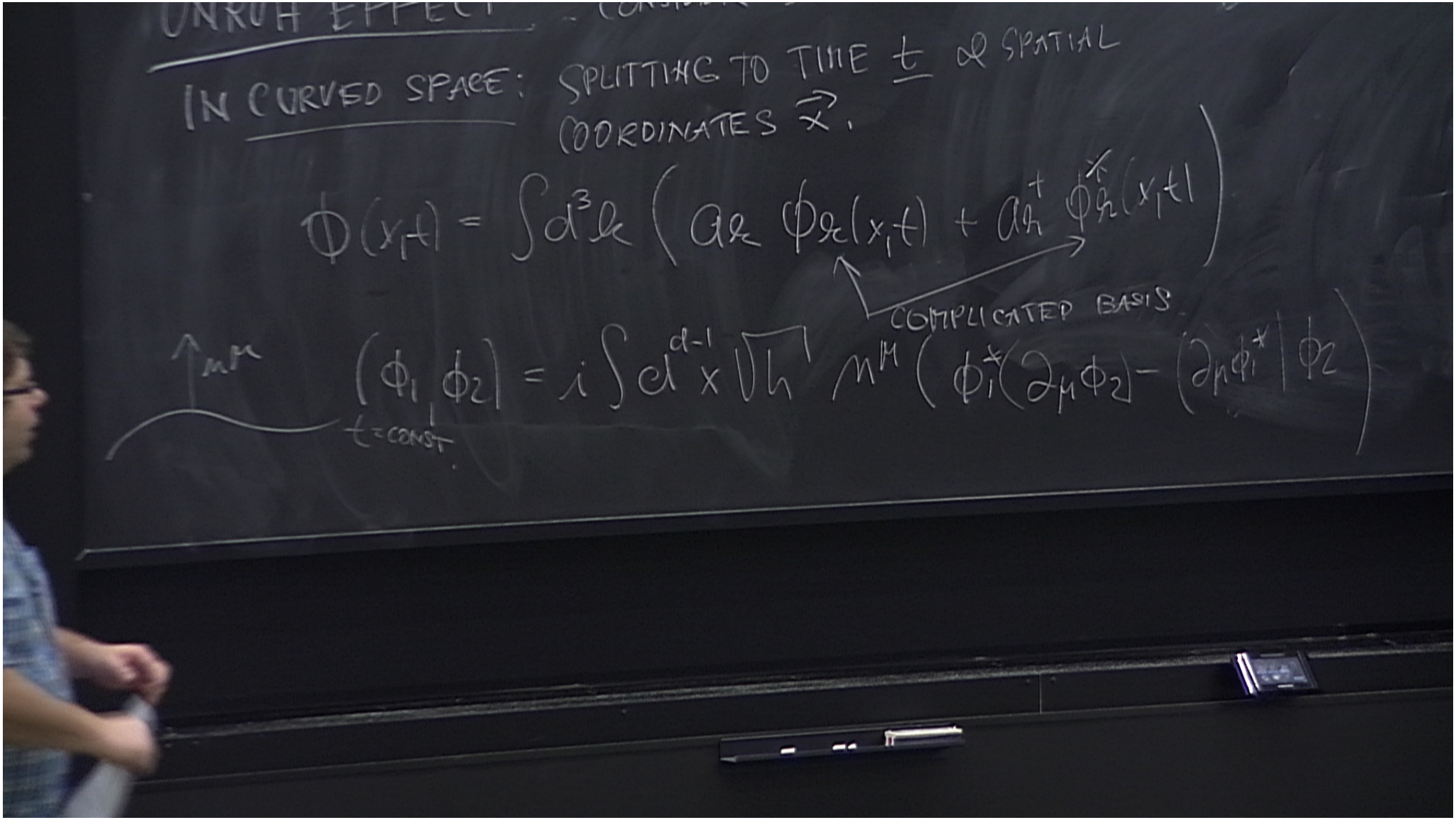

UNRUH EFFECT - CONSIDER SCALAR FIELD Φ.
IN CURVED SPACE: SPLITTING TO TIME t & SPATIAL COORDINATES x⃗.
$\Phi(x,t) = \int d^3k \left(a_k \phi_k(x,t) + a_k^\dagger \phi_k^*(x,t) \right)$


IN CURVED SPACE: SPLITTING TO TIME t & SPATIAL COORDINATES $\vec{x}$.
$\phi(x,t) = \int d^3k \left(a_k \phi_{\vec{k}}(x,t) + a_k^\dagger \phi^*_{\vec{k}}(x,t) \right)$
$e^{-i(\omega t + \vec{k}\cdot\vec{x})}$

IN CURVED SPACE: SPLITTING TO TIME t & SPATIAL COORDINATES $\vec{x}$.

$$\phi(x,t) = \int d^3k \left(a_k \phi_k(x,t) + a_k^\dagger \phi_k^*(x,t) \right)$$

COMPLICATED BASIS


UNRUH EFFECT
IN CURVED SPACE: SPLITTING TO TIME t & SPATIAL COORDINATES x⃗.
$\phi(x,t) = \int d^3k \left(a_k \phi_{\vec{k}}(x,t) + a_k^\dagger \phi^*_{\vec{k}}(x,t) \right)$
COMPLICATED BASIS
n^μ
$(\phi_1, \phi_2) = i \int_{t=\text{CONST}} d^{d-1}x \sqrt{h}\, n^\mu \left(\phi_1^* (\partial_\mu \phi_2) - (\partial_\mu \phi_1^*)\, \phi_2 \right)$

UNRUH EFFECT

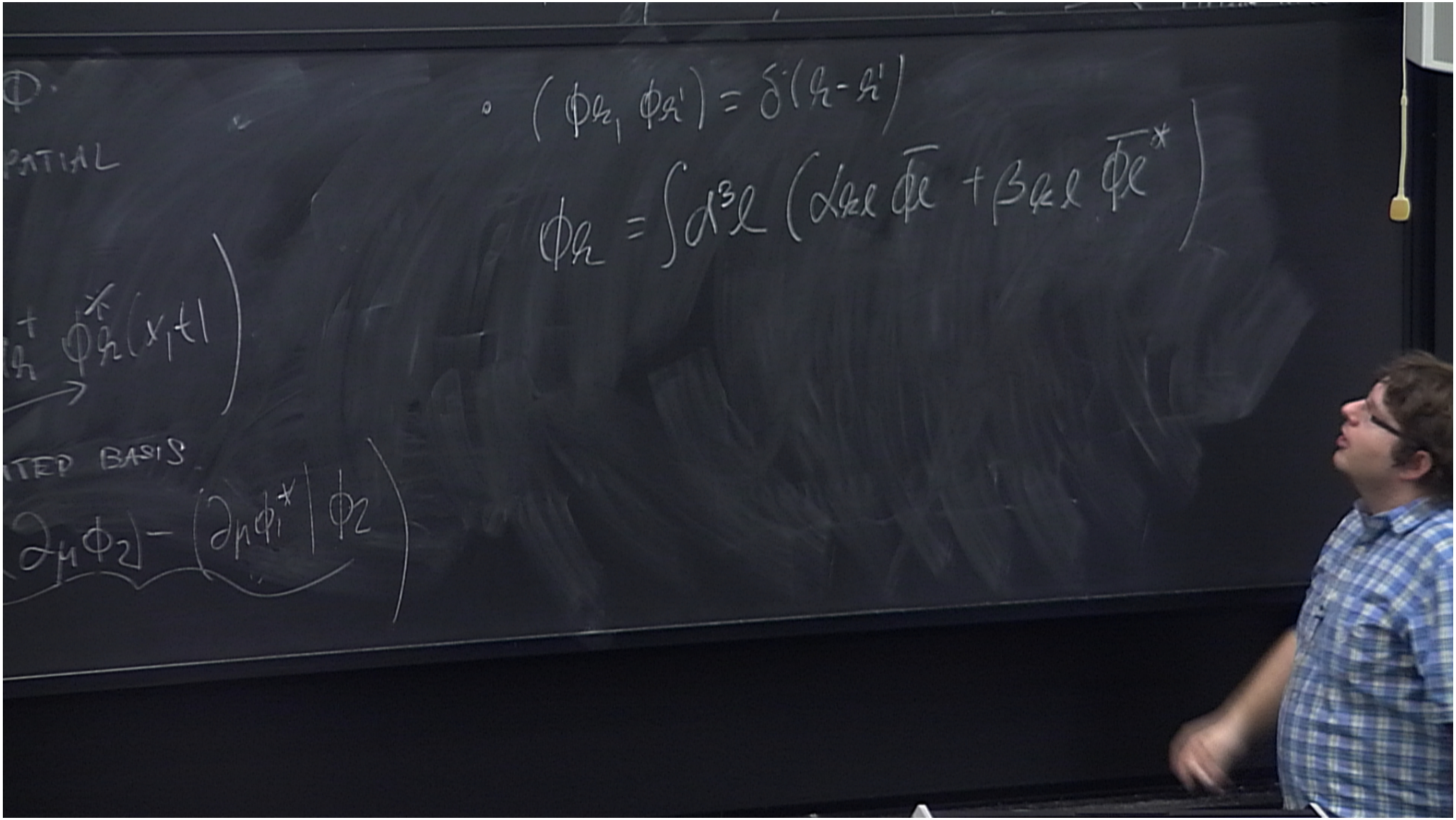
IN CURVED SPACE: SPLITTING TO TIME t & SPATIAL COORDINATES $\vec{x}$.

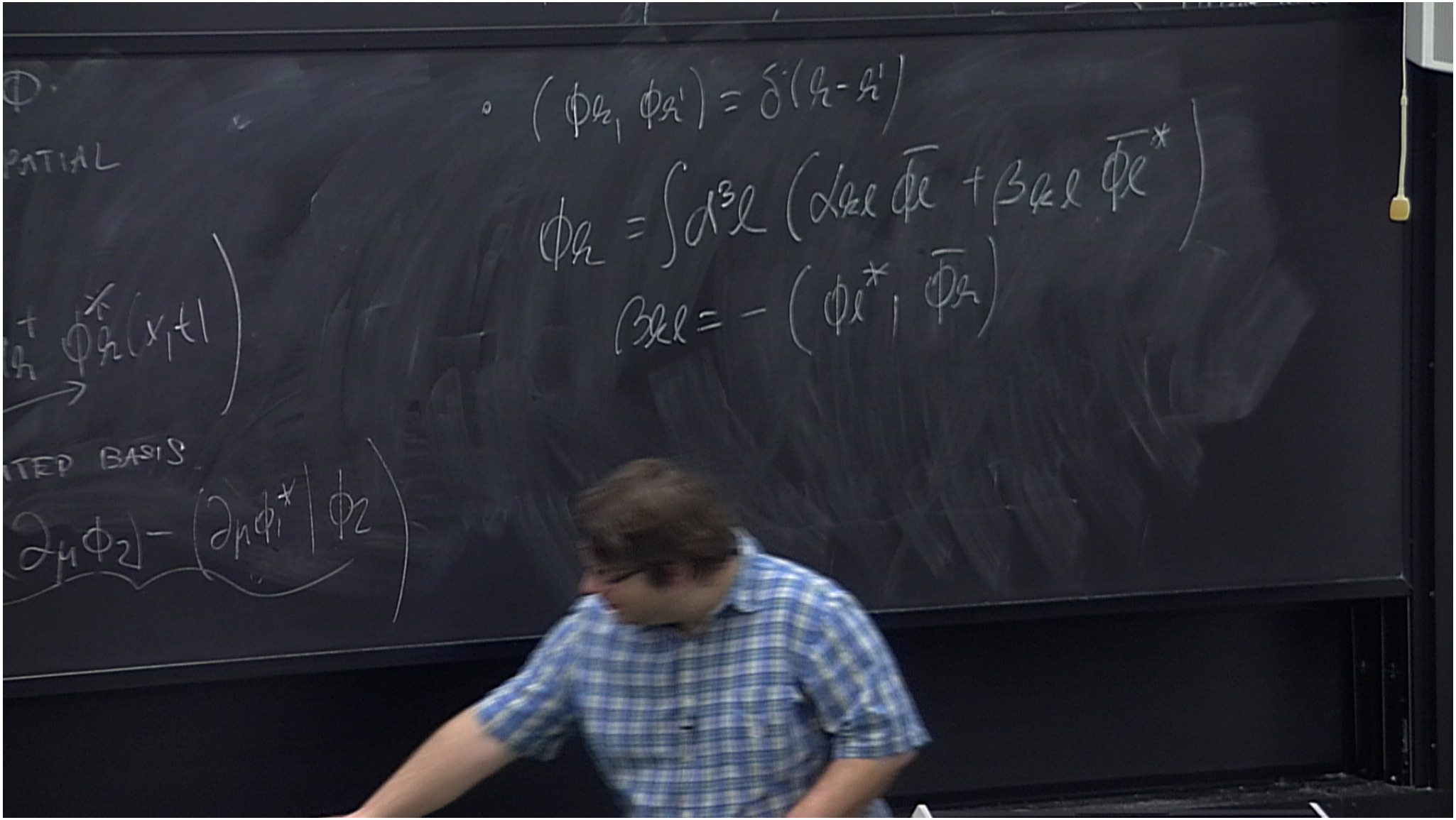
$$\phi(x,t) = \int d^3k \left(a_k \phi_{k}(x,t) + a_k^\dagger \phi_k^*(x,t) \right)$$

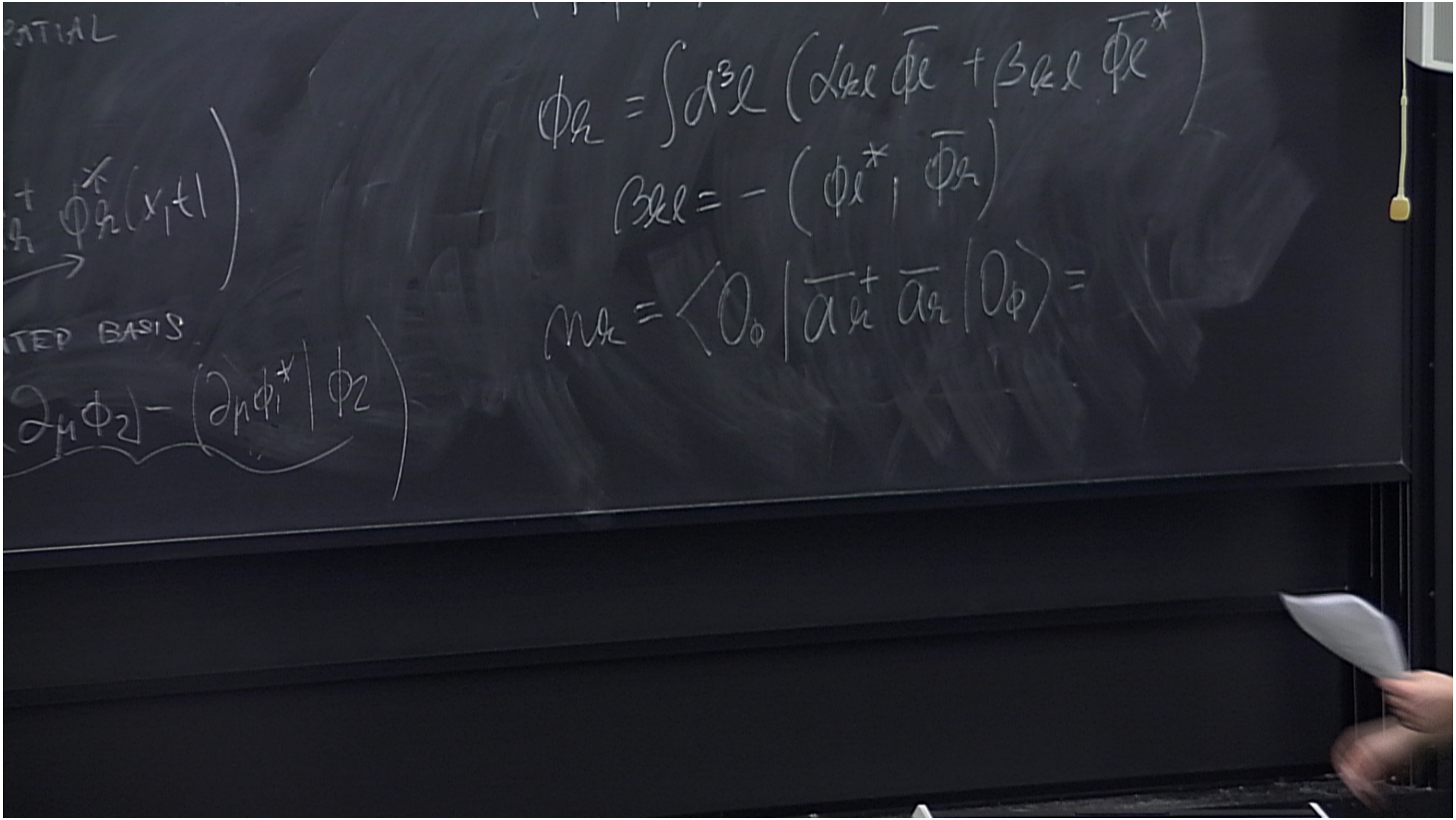
COMPLICATED BASIS

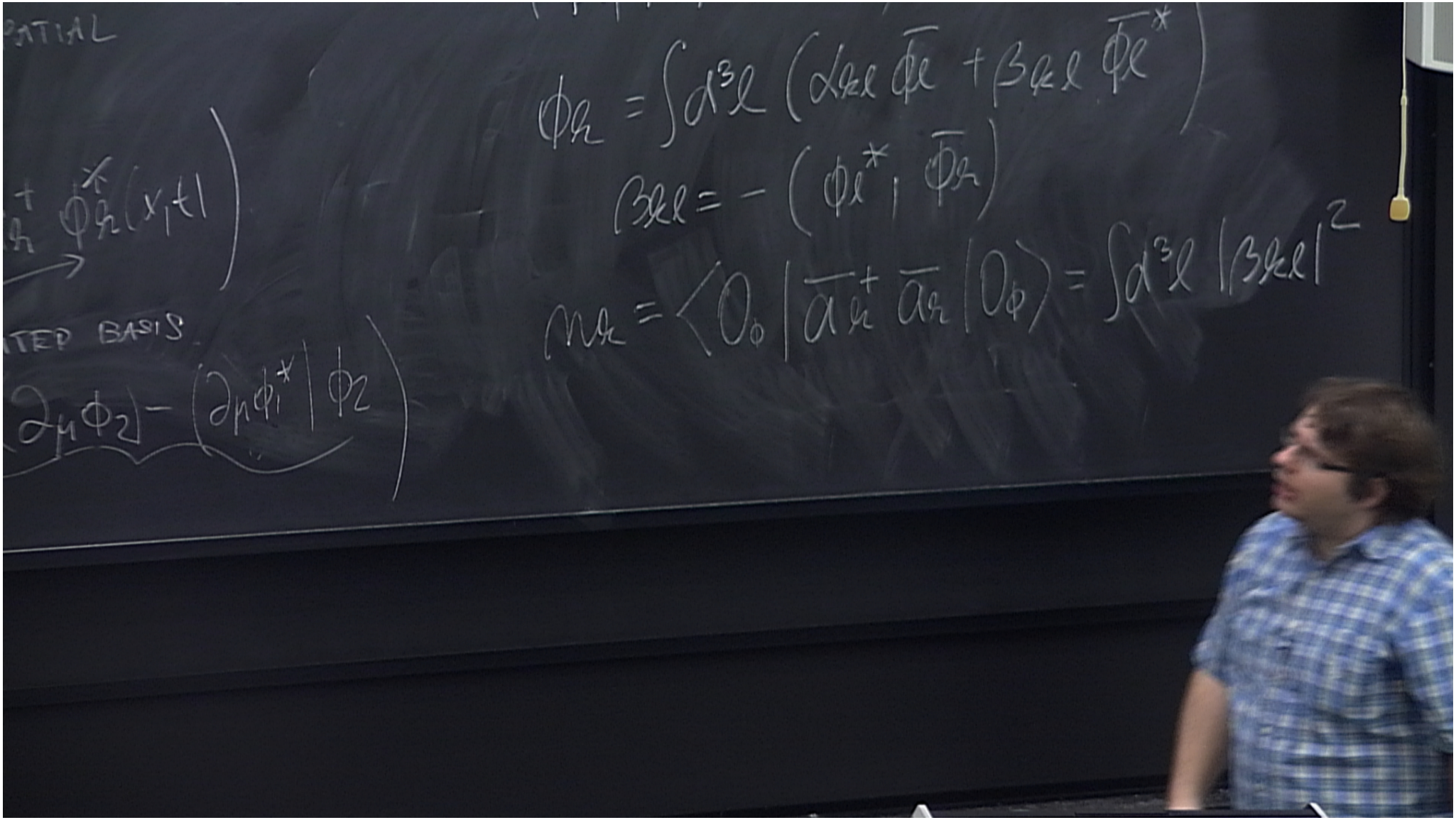
$$(\phi_1, \phi_2) = i \int_{t=\text{CONST}} d^{d-1}x \sqrt{h}\, n^\mu \left(\phi_1^* (\partial_\mu \phi_2) - (\partial_\mu \phi_1^*)\, \phi_2 \right)$$

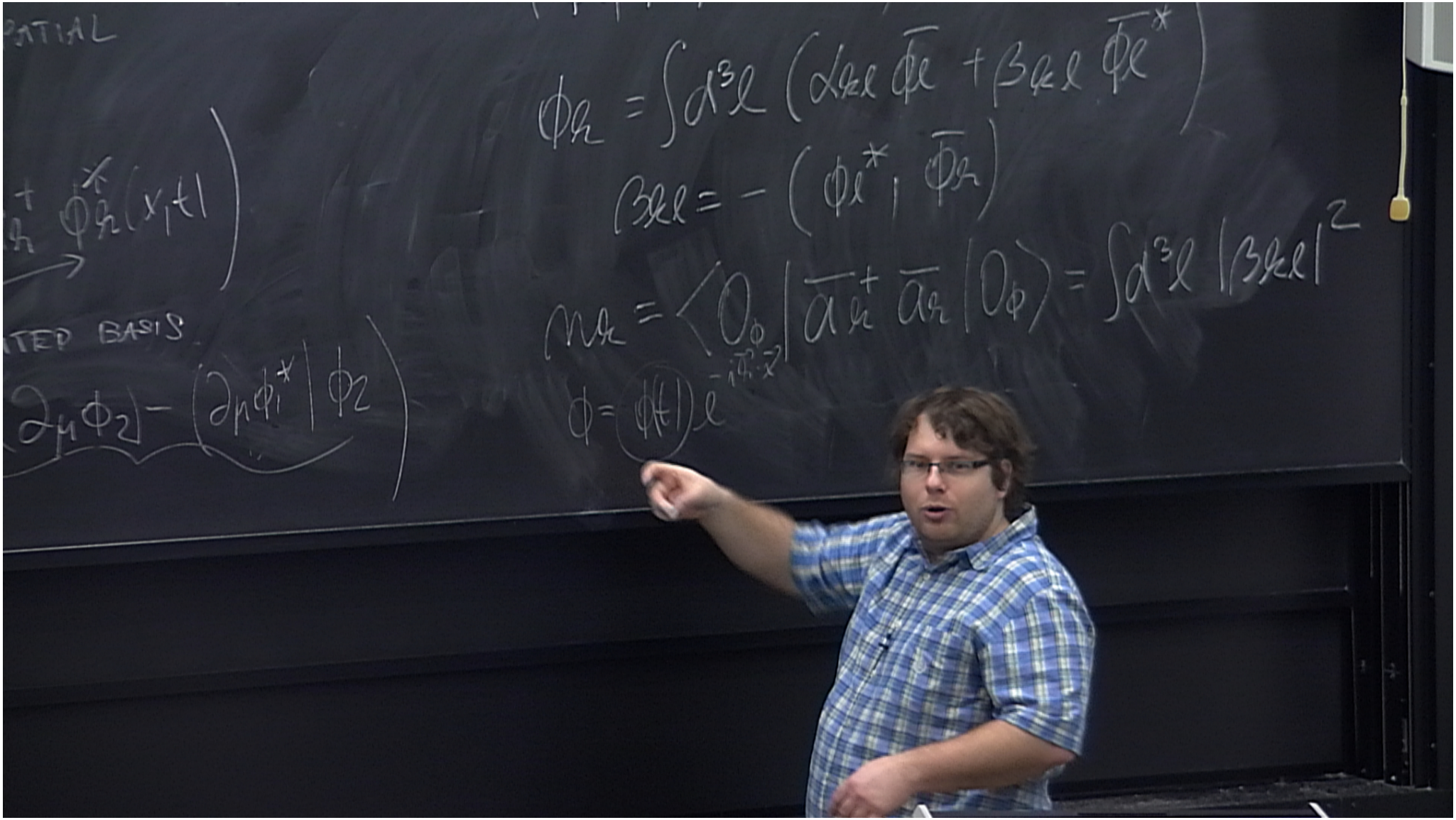
n^μ

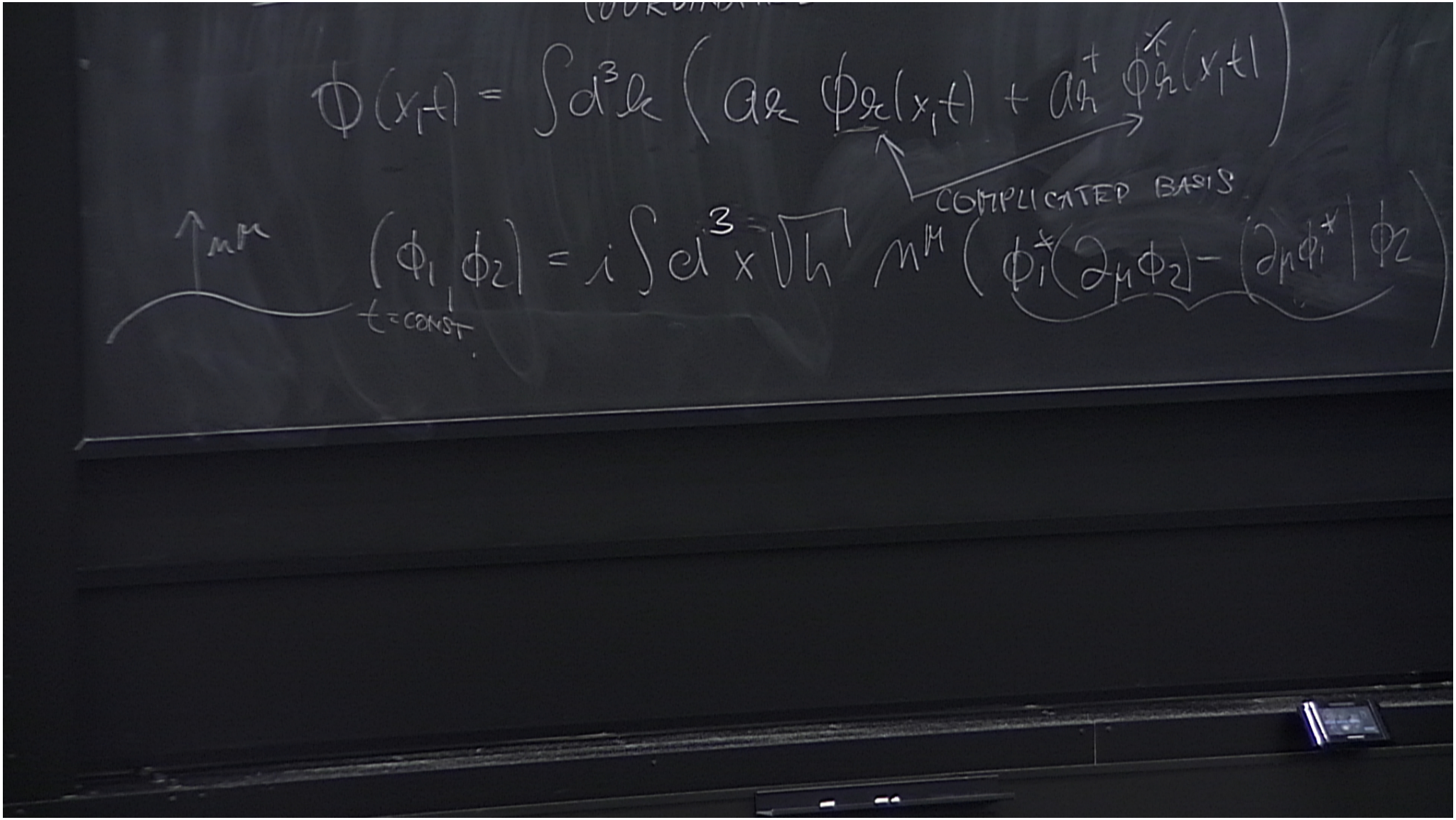

$(\phi_k, \phi_{k'}) = \delta(k-k')$
$\phi_k = \int d^3x \left(\alpha_{kx}\phi_x + \beta_{kx}\phi_x^*\right)$
SPATIAL
$\phi_k^*(x,t)$
INTER BASIS
$(\partial_\mu\phi_2) - (\partial_\mu\phi_1^* | \phi_2)$

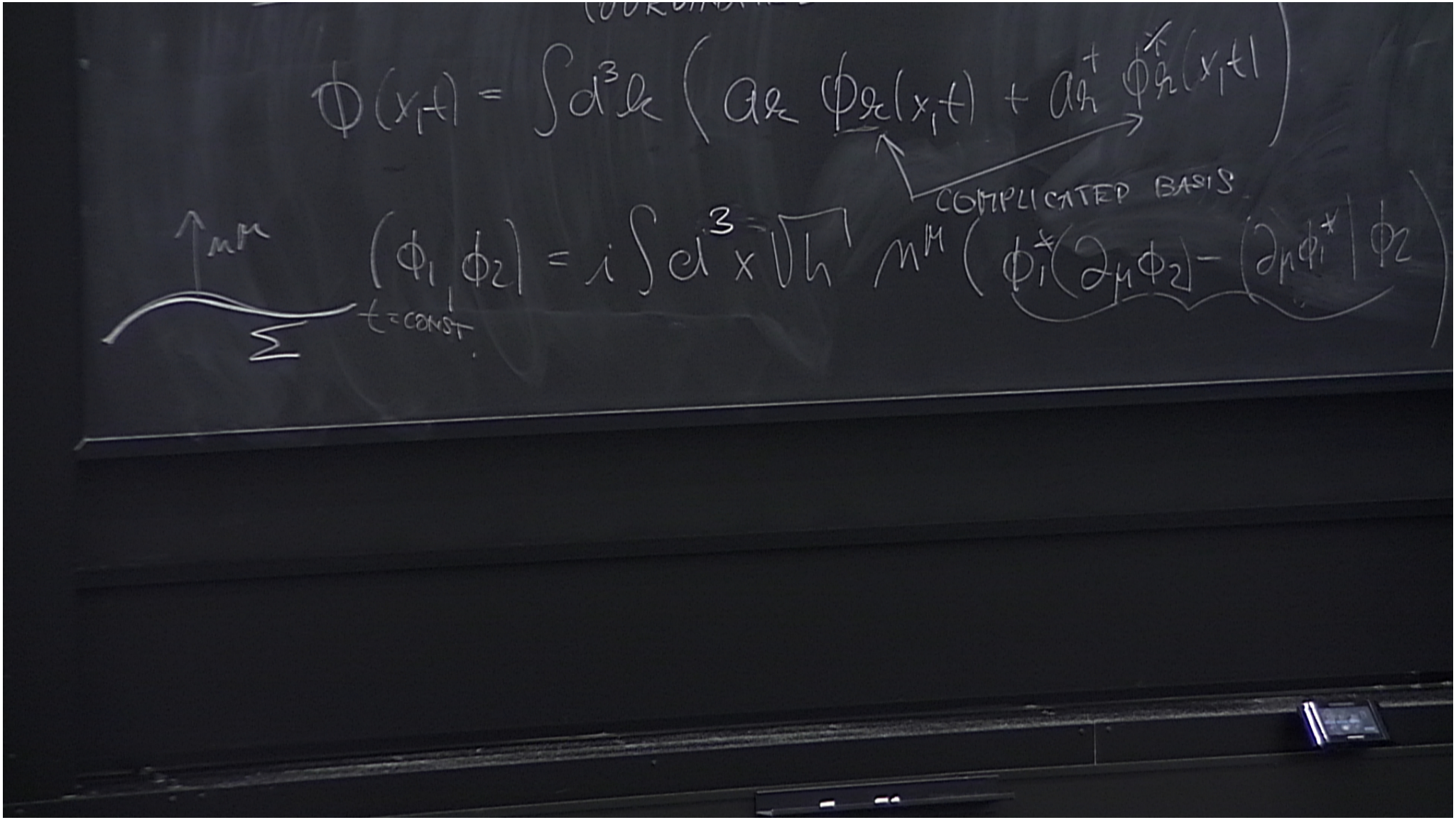

$(\phi_k, \phi_{k'}) = \delta(k-k')$
$\phi_k = \int d^3l \left(\alpha_{kl}\,\phi_l + \beta_{kl}\,\phi_l^*\right)$
$\beta_{kl} = -(\phi_l^*, \phi_k)$
SPATIAL
$\phi_k^*(x,t)$
INTERP BASIS
$(\partial_\mu \phi_2) - (\partial_\mu \phi_1^* | \phi_2)$

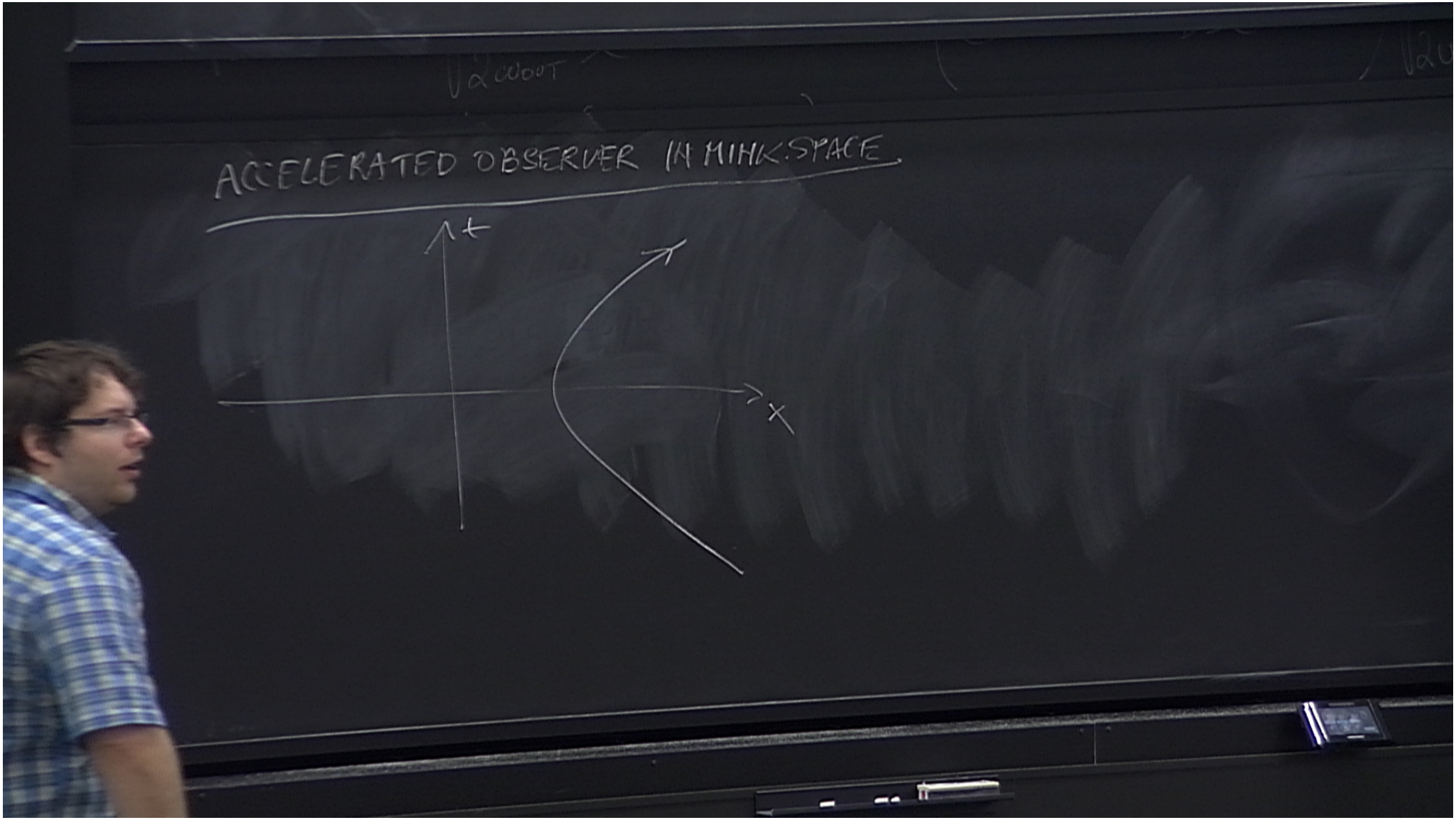

SPATIAL
$\phi_k = \int d^3l \left(\alpha_{kl} \bar{\phi}_l + \beta_{kl} \bar{\phi}_l^* \right)$
$\beta_{kl} = -\left(\phi_l^* ; \bar{\phi}_k \right)$
$n_k = \langle 0_\phi | \bar{a}_k^\dagger \bar{a}_k | 0_\phi \rangle =$
$\phi_k^*(x,t)$
ATED BASIS
$(\partial_\mu \phi_2) - (\partial_\mu \phi_1^* | \phi_2)$

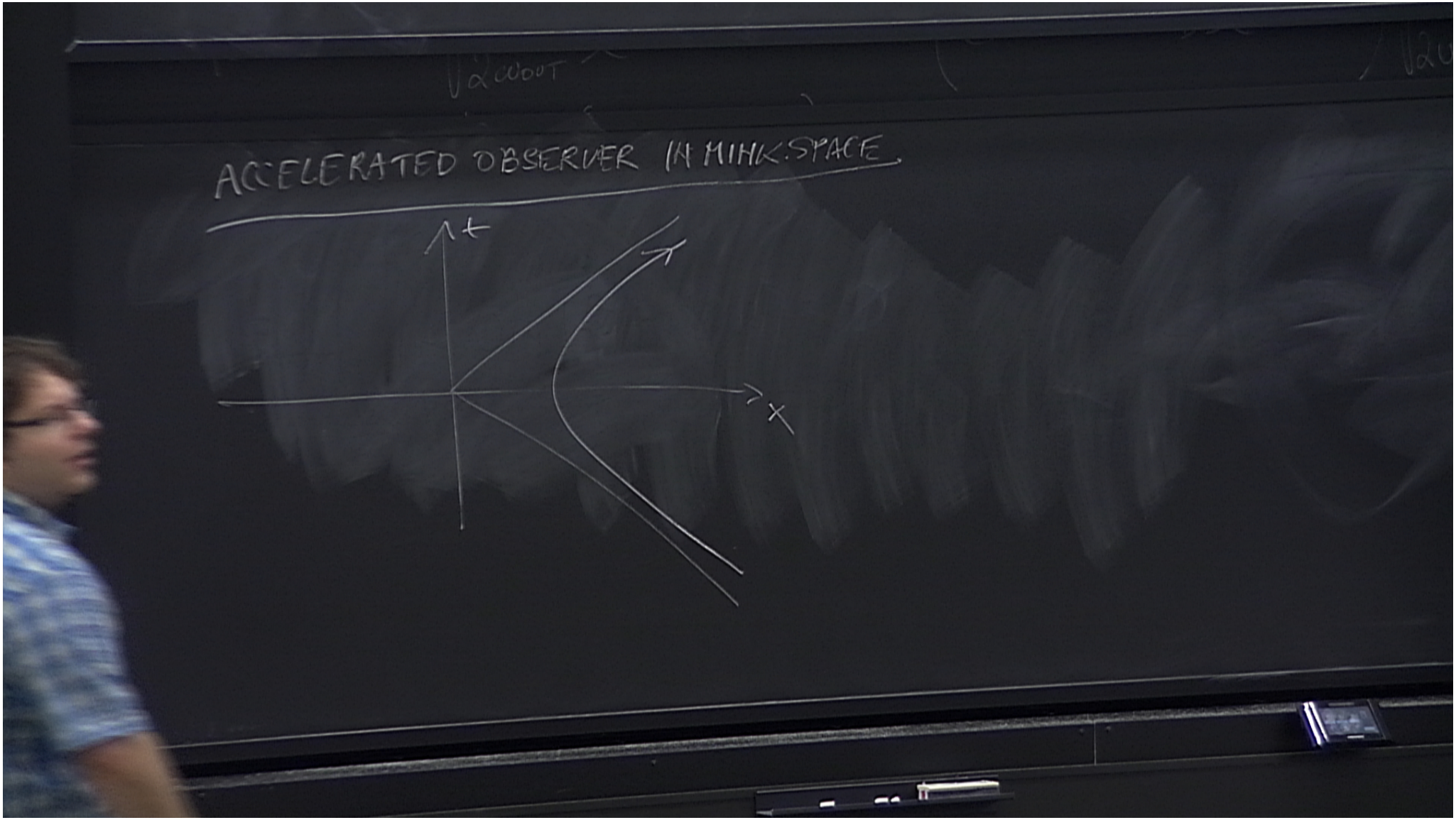

PATIAL
$\phi_k = \int d^3\ell \left(\alpha_{k\ell}\, \phi_\ell + \beta_{k\ell}\, \phi_\ell^* \right)$
$\beta_{k\ell} = -\left(\phi_\ell^* ; \phi_k \right)$
$n_k = \langle 0_\phi | a_k^\dagger a_k | 0_\phi \rangle = \int d^3\ell\, |\beta_{k\ell}|^2$
$\phi_k^*(x,t)$
ATED BASIS
$(\partial_\mu \phi_2) - (\partial_\mu \phi_1^* | \phi_2)$

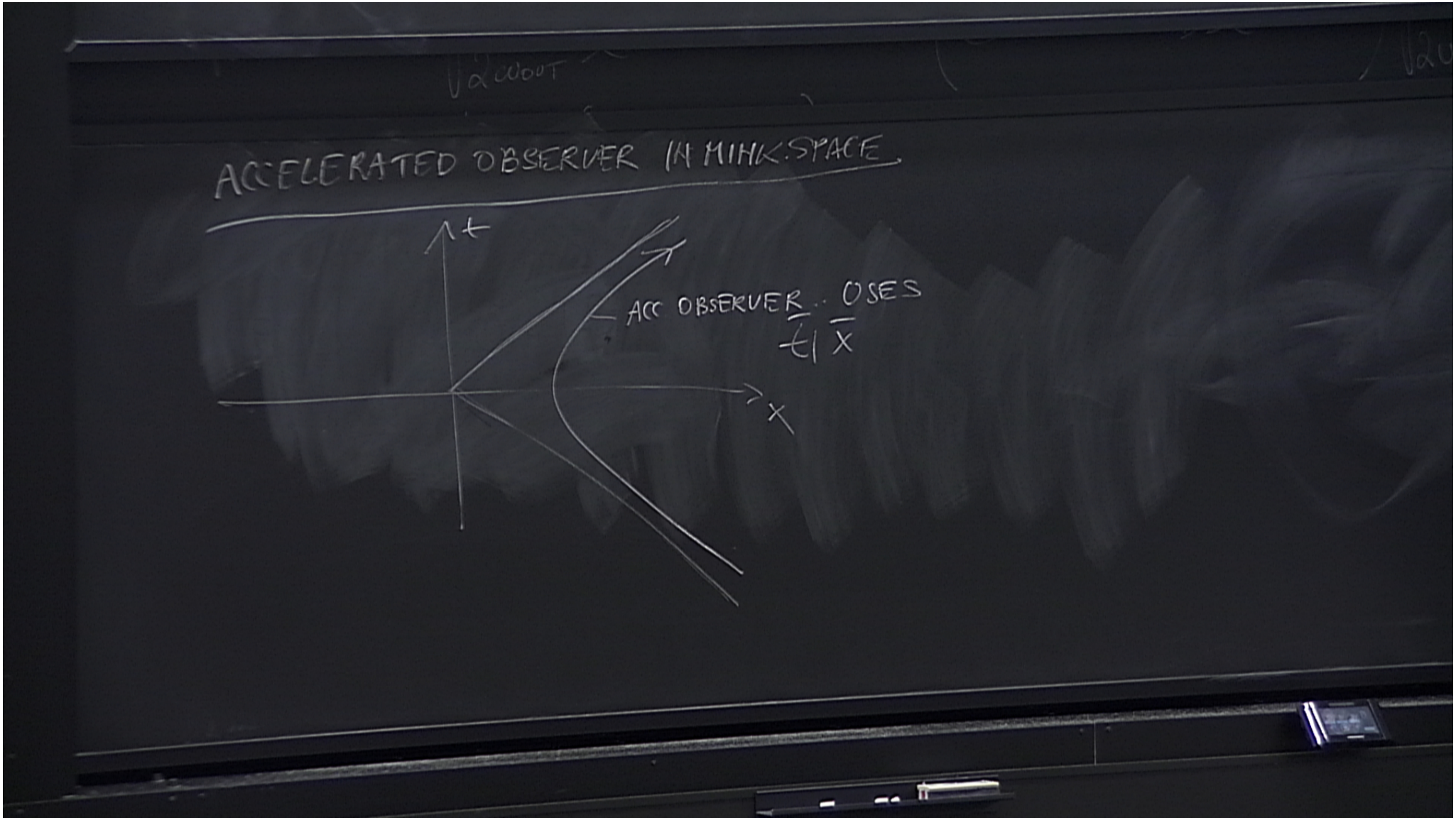

PATIAL
ATED BASIS


$\phi(x,t) = \int d^3k \left(a_k \phi_{k}(x,t) + a_k^\dagger \phi^*_{k}(x,t) \right)$
COMPLICATED BASIS
$(\phi_1, \phi_2) = i \int_{t=\text{CONST}} d^3x \sqrt{h}\, n^\mu \left(\phi_1^* (\partial_\mu \phi_2) - (\partial_\mu \phi_1^*)\, \phi_2 \right)$
n^μ

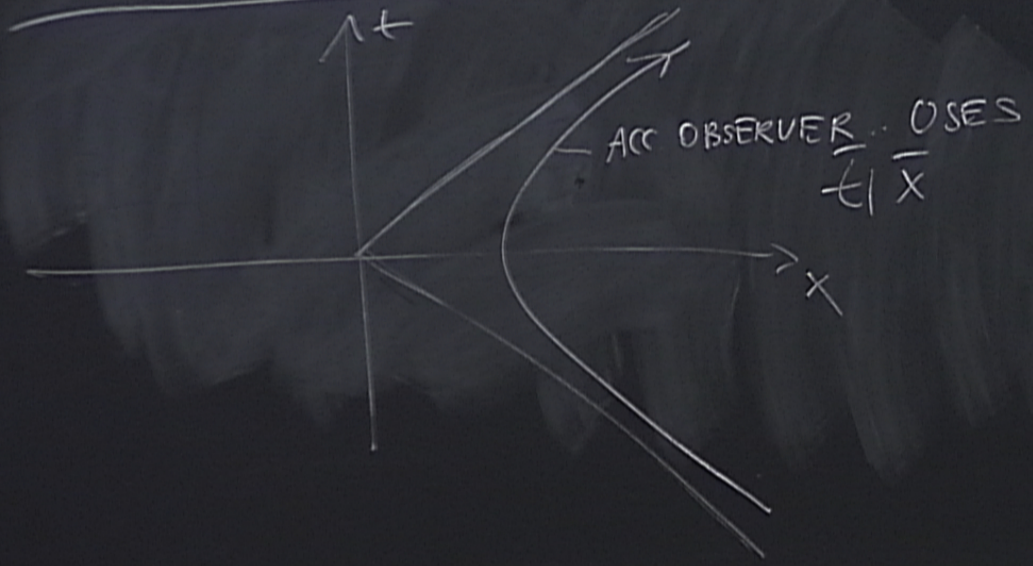

$\phi(x,t) = \int d^3k \left(a_k \phi_{k}(x,t) + a_k^\dagger \phi^*_{k}(x,t) \right)$
COMPLICATED BASIS
$(\phi_1, \phi_2) = i \int_{t=\text{CONST}} d^3x \sqrt{h}\, n^\mu \left(\phi_1^* (\partial_\mu \phi_2) - (\partial_\mu \phi_1^*)\, \phi_2 \right)$
n^μ
Σ
$t = \text{CONST}$


ACCELERATED OBSERVER IN MINK. SPACE
t
x


ACCELERATED OBSERVER IN MINK. SPACE
t
x


ACCELERATED OBSERVER IN MINK. SPACE
t
x
ACC OBSERVER SEES
t
x

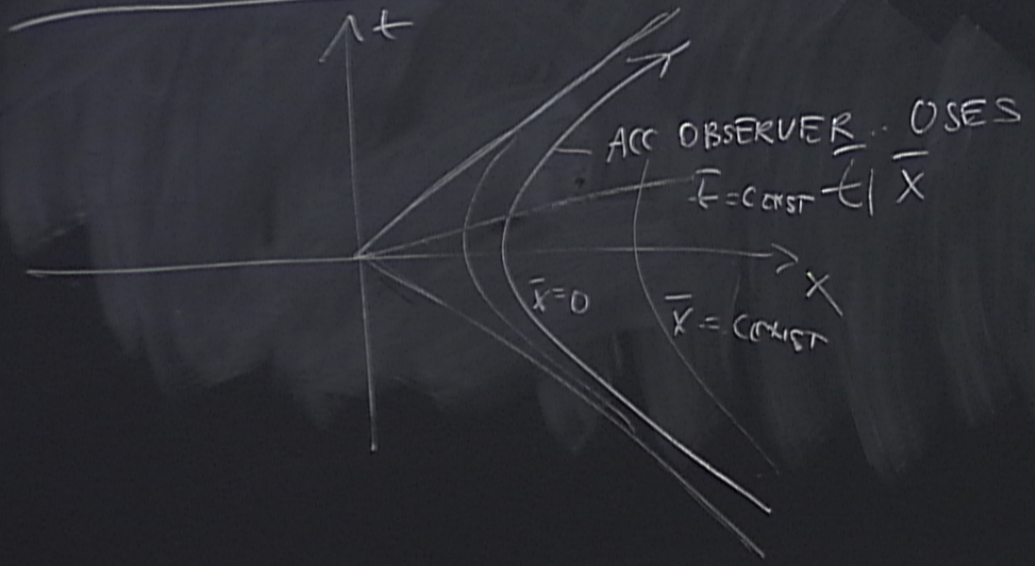
ACCELERATED OBSERVER IN MINK. SPACE



$$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$$

$$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$$

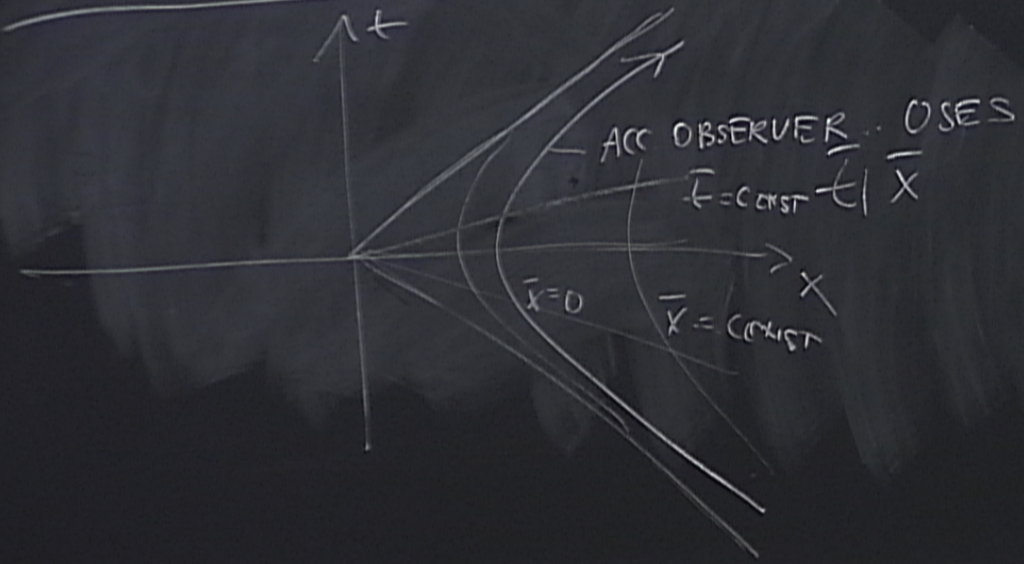
ACCELERATED OBSERVER IN MINK. SPACE



$$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$$

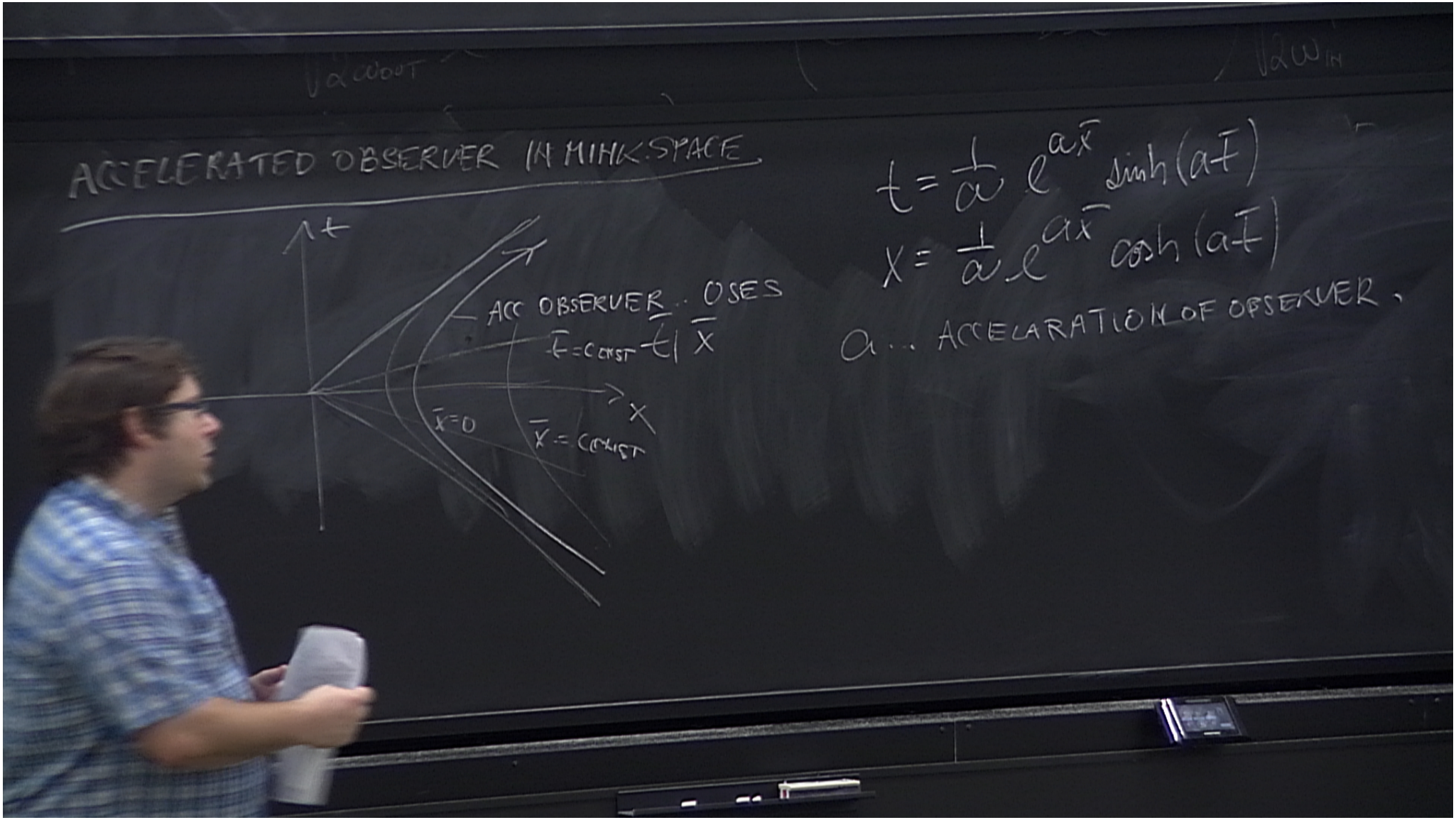
$$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$$

ACCELERATED OBSERVER IN MINK. SPACE

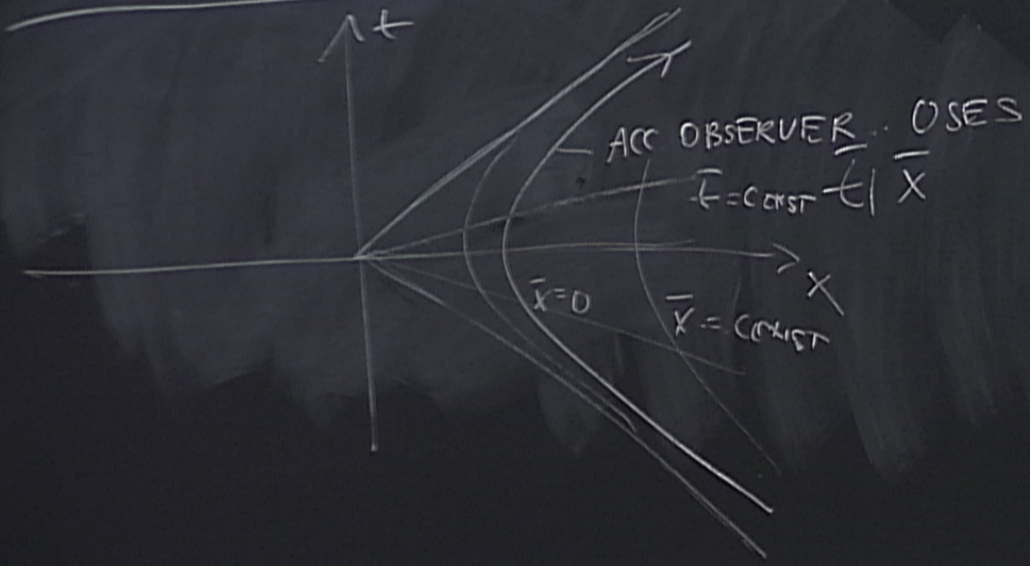


$$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$$

$$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$$


ACCELERATED OBSERVER IN MINK. SPACE
t
x
ACC OBSERVER USES
$\bar{t}$ = CONST
$\bar{x}$ = 0
$\bar{x}$ = CONST
$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$
$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$
a ... ACCELARATION OF OBSERVER.

ACCELERATED OBSERVER IN MINK. SPACE



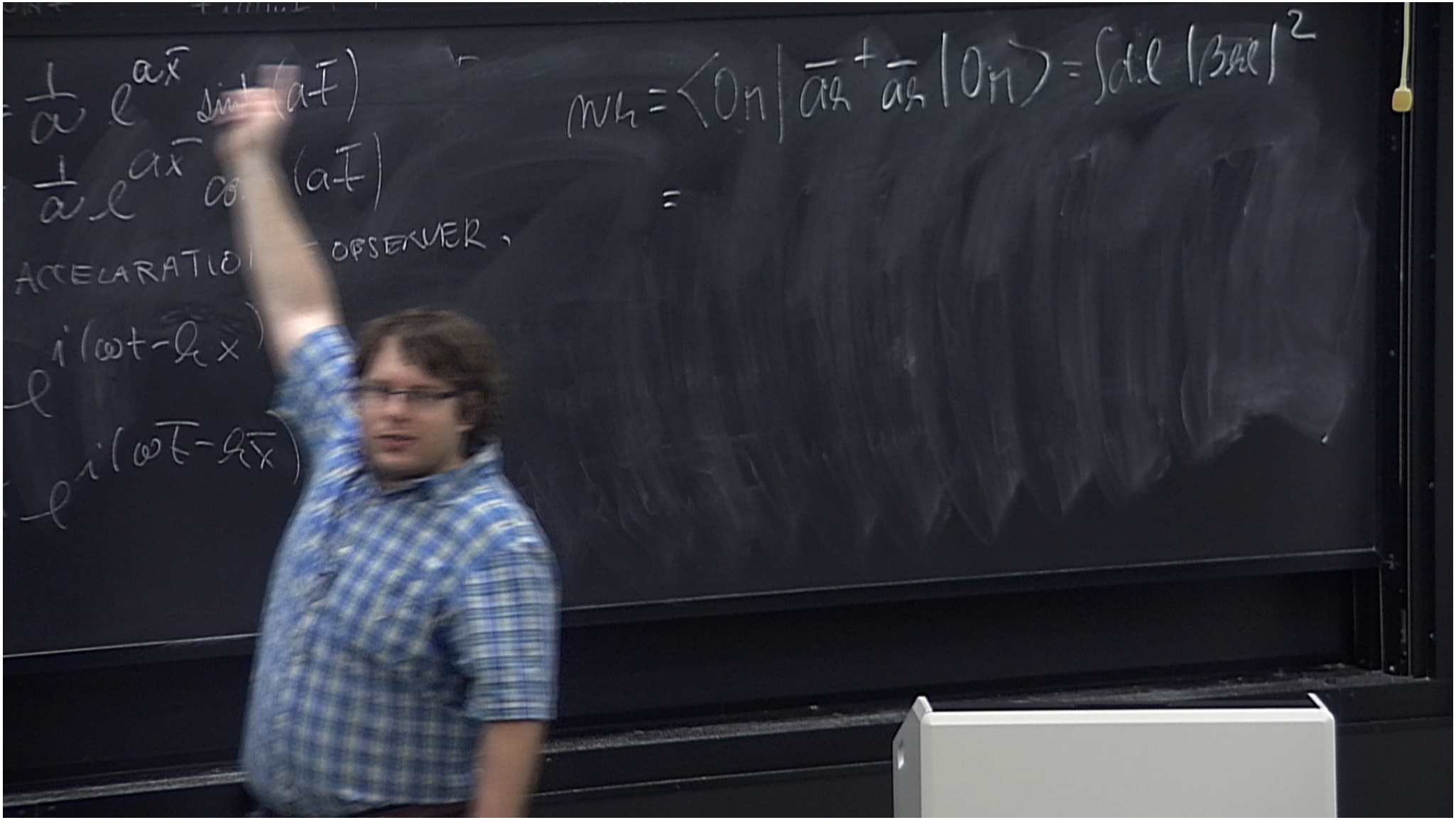
$$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$$

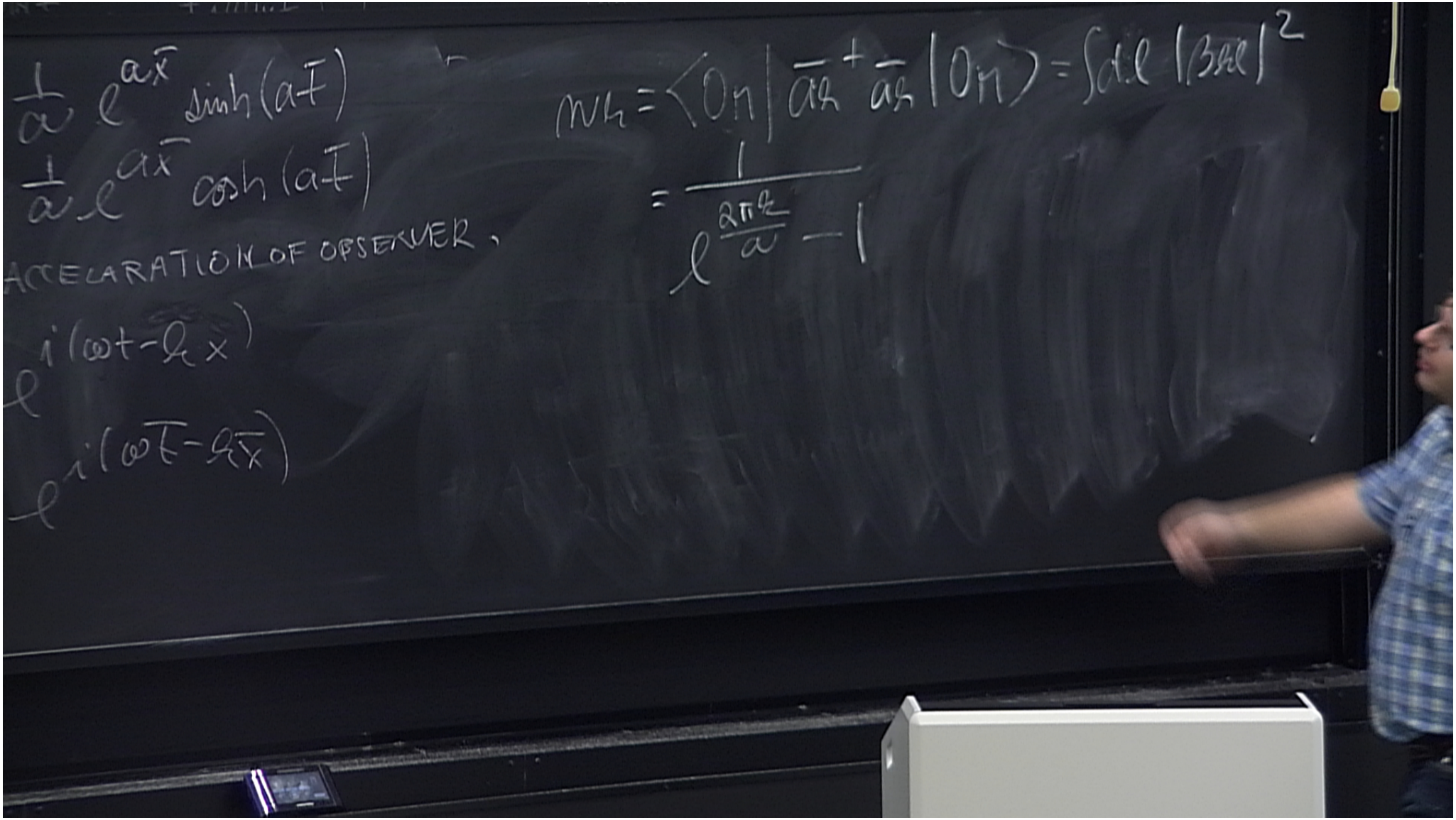
$$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$$

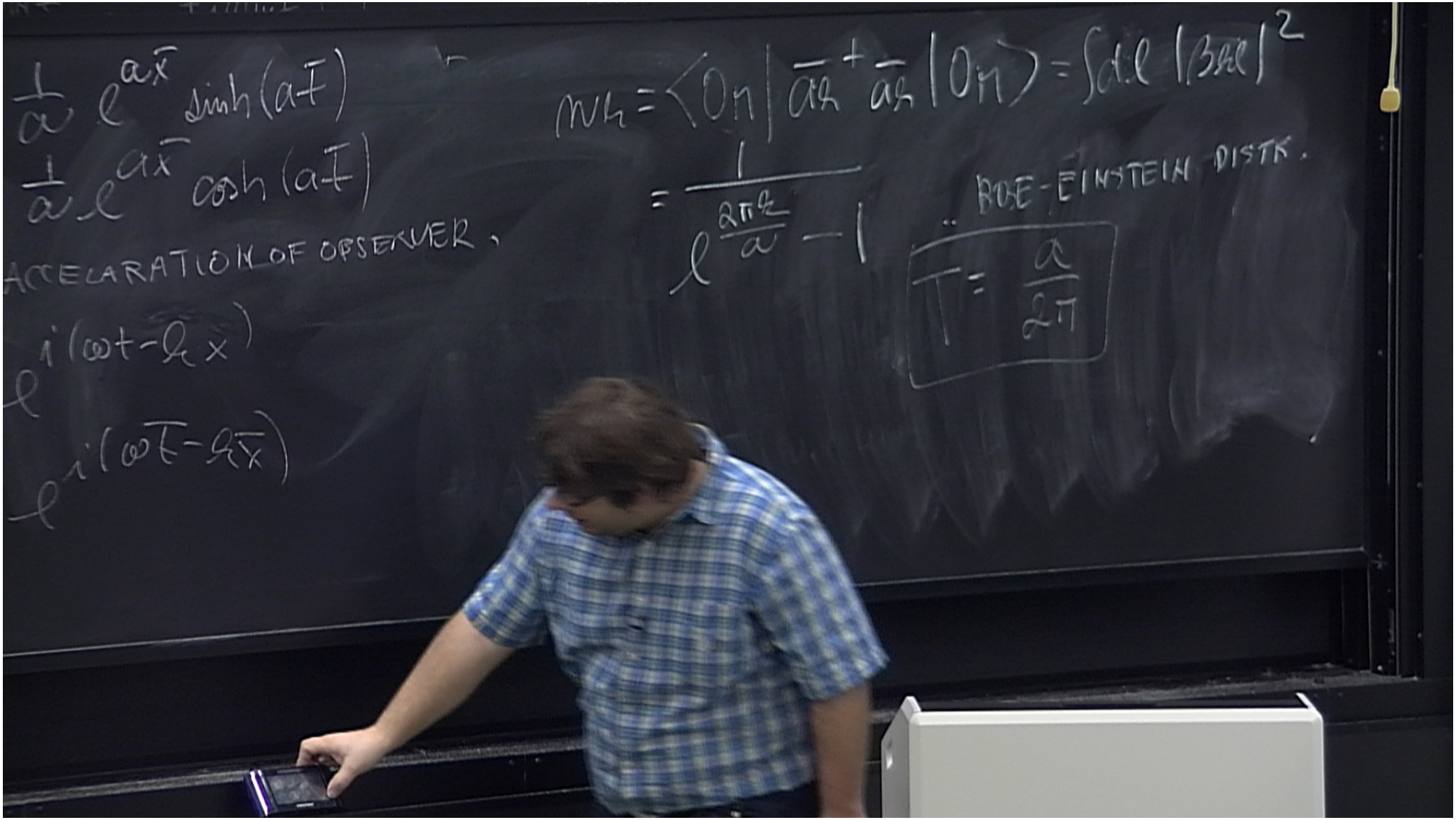
a ... ACCELARATION OF OBSERVER.

$$\phi_\omega \propto e^{i(\omega t - \omega x)}$$

$$\bar{\phi}_\omega \propto e^{i(\omega \bar{t} - \omega \bar{x})}$$


ACCELARATIO
OBSERVER,


$\frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$
$\frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$
ACCELARATION OF OBSERVER.
$e^{i(\omega t - k x)}$
$e^{-i(\omega \bar{t} - k \bar{x})}$
$n_k = \langle 0_M | \bar{a}_k^+ \bar{a}_k | 0_M \rangle = \int d\ell \, |\beta_{k\ell}|^2$
$= \frac{1}{e^{\frac{2\pi k}{a}} - 1}$


ACCELARATION OF OBSERVER.
BOE-EINSTEIN DISTR.
$T = \frac{a}{2\pi}$