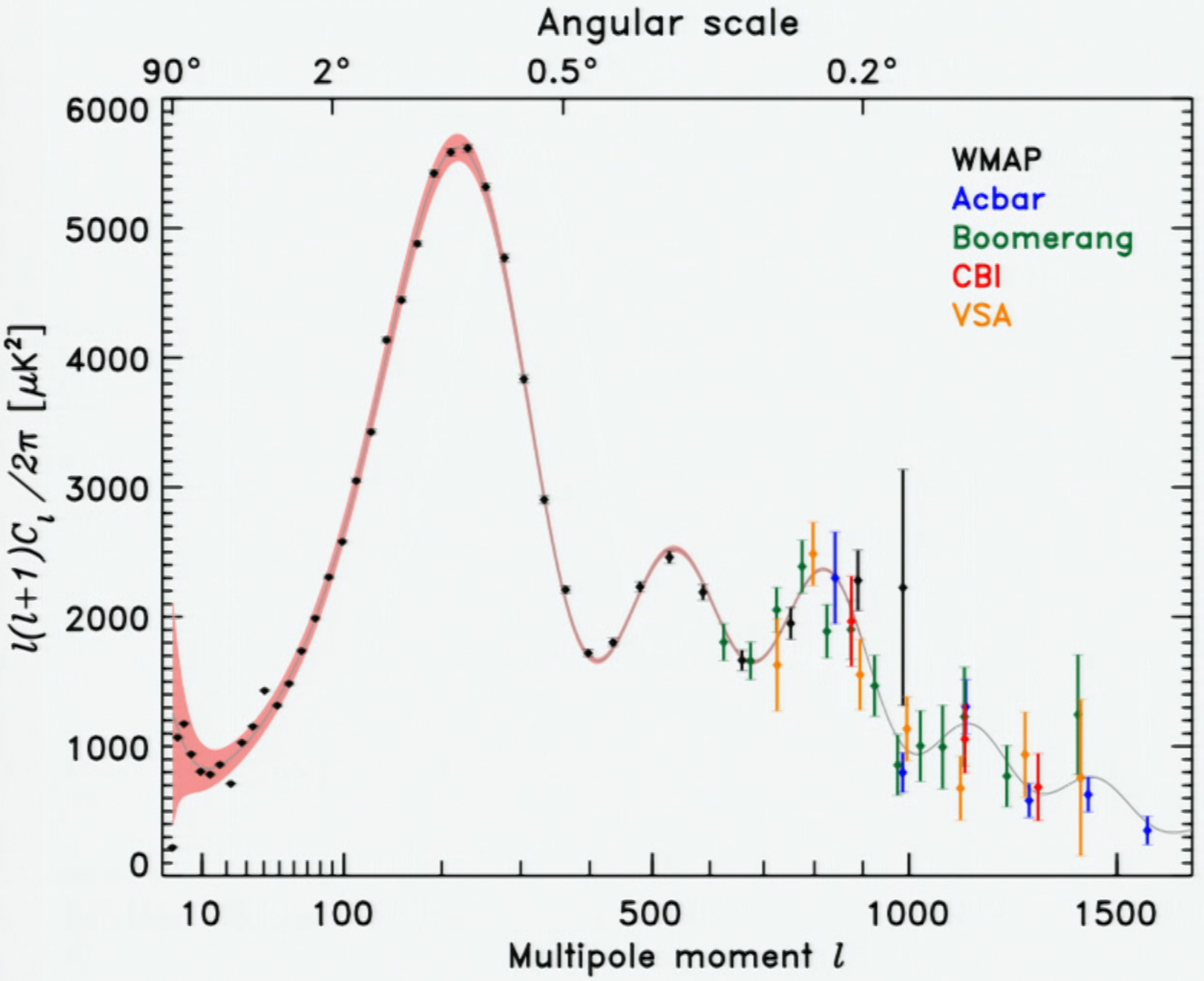
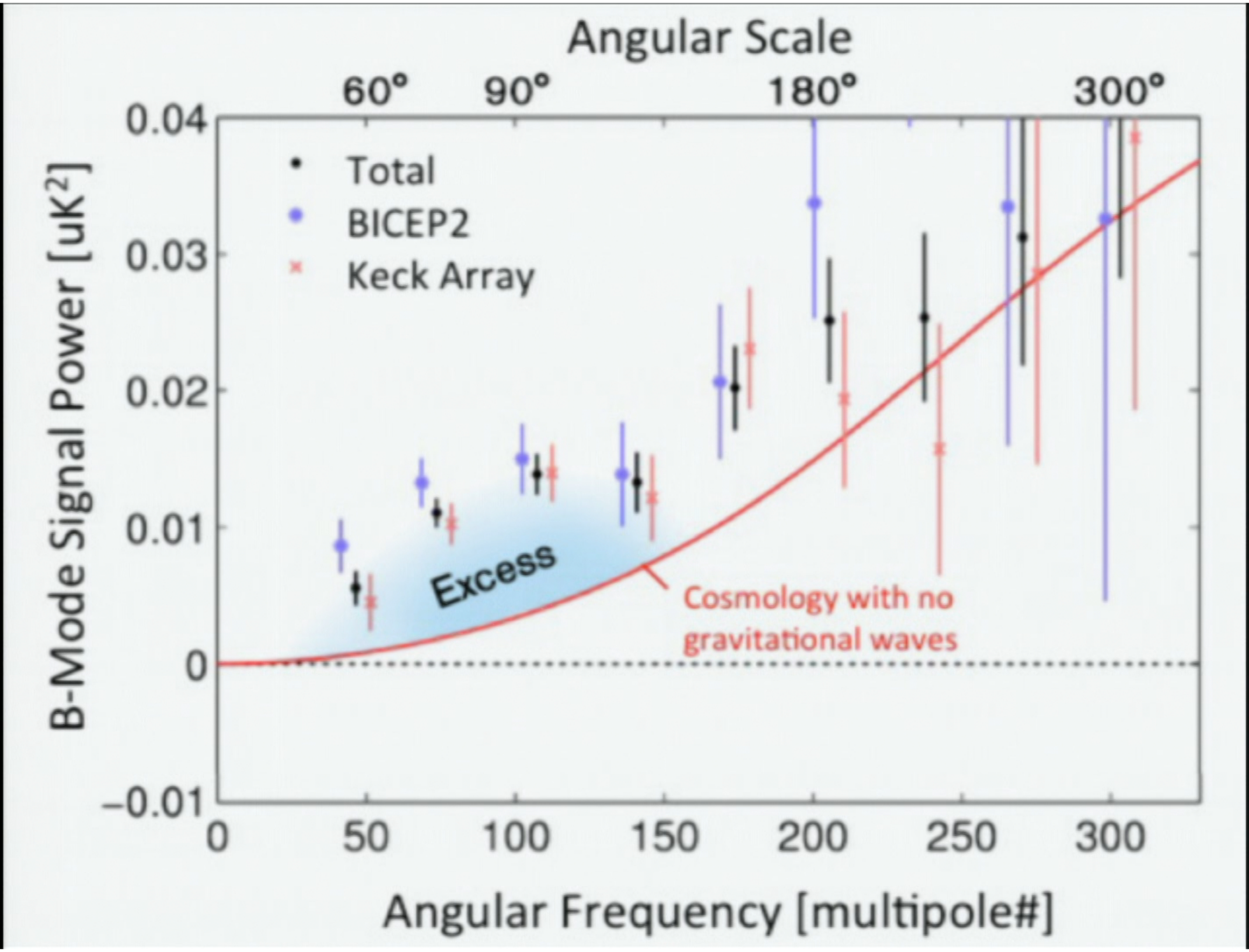
Title: PSI 2016/2017 Cosmology (Review) - Lecture 12

Date: Feb 15, 2017 10:15 AM

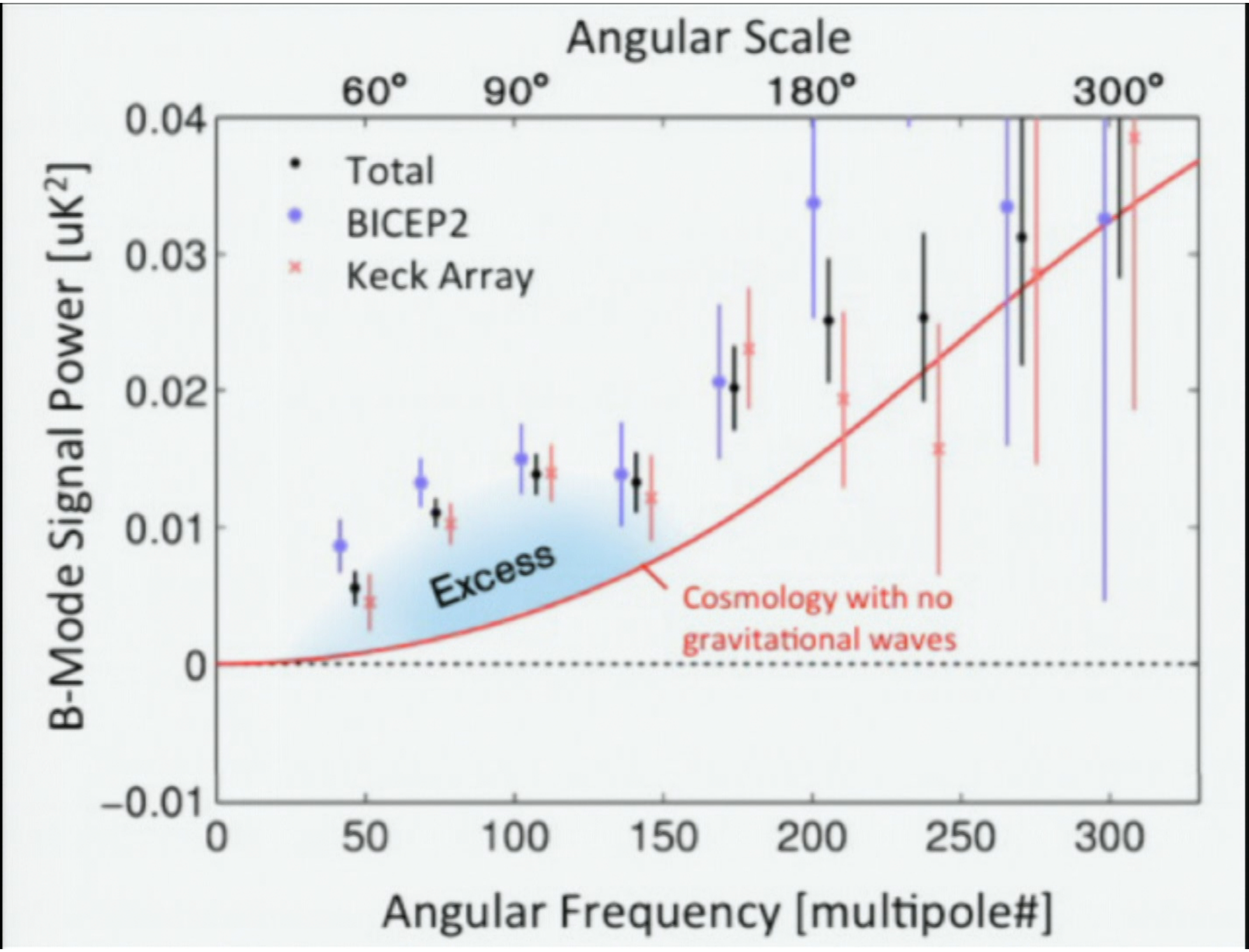
URL: http://pirsa.org/17020060

Abstract:


Angular scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l\ /2\pi\ [\mu K^2]$
WMAP
Acbar
Boomerang
CBI
VSA
10
100
500
1000
1500
Multipole moment $l$



Angular Scale
60°
90°
180°
300°
B-Mode Signal Power [uK²]
0.04
0.03
0.02
0.01
0
−0.01
● Total
● BICEP2
× Keck Array
Excess
Cosmology with no gravitational waves
0
50
100
150
200
250
300
Angular Frequency [multipole#]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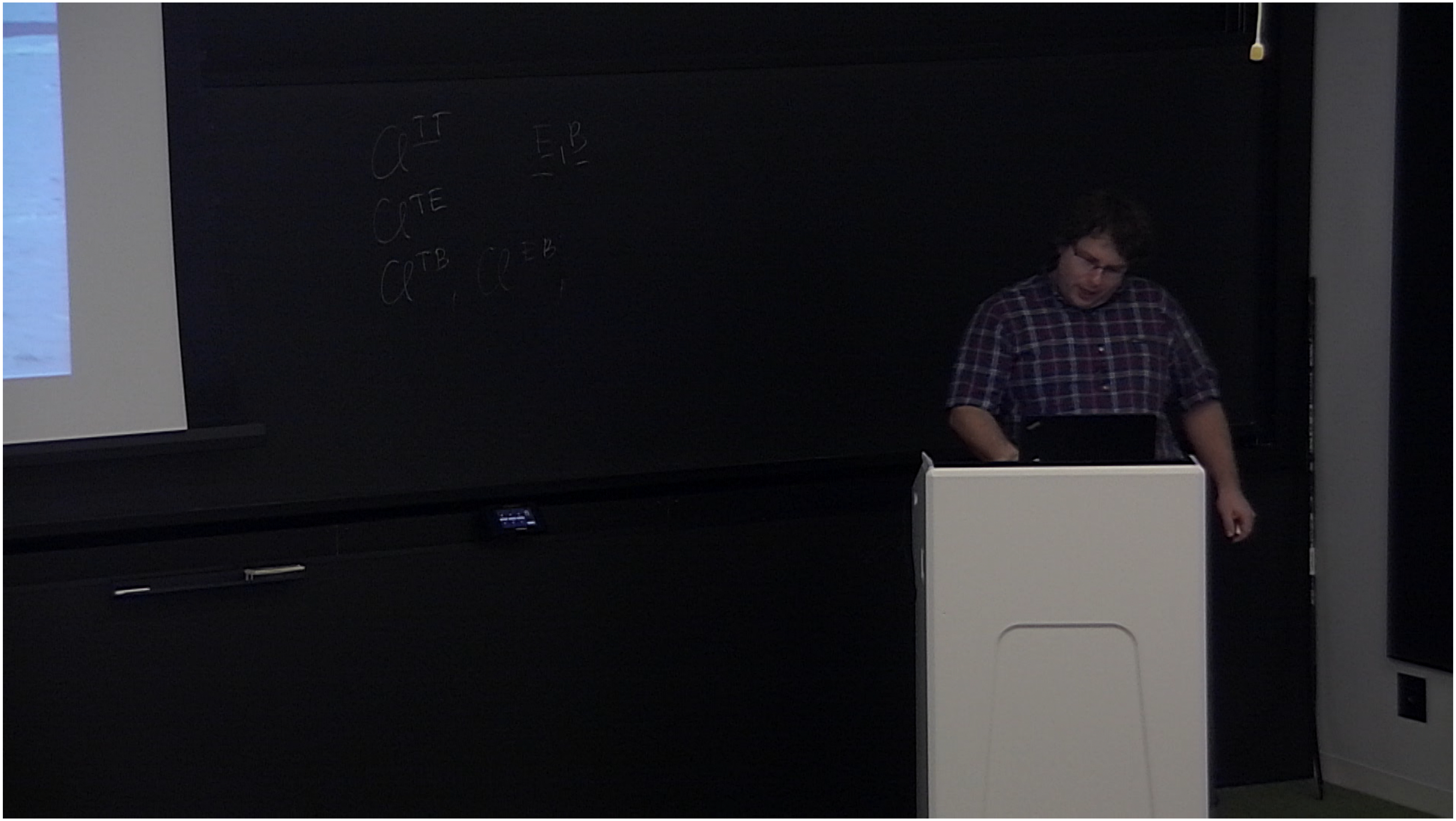
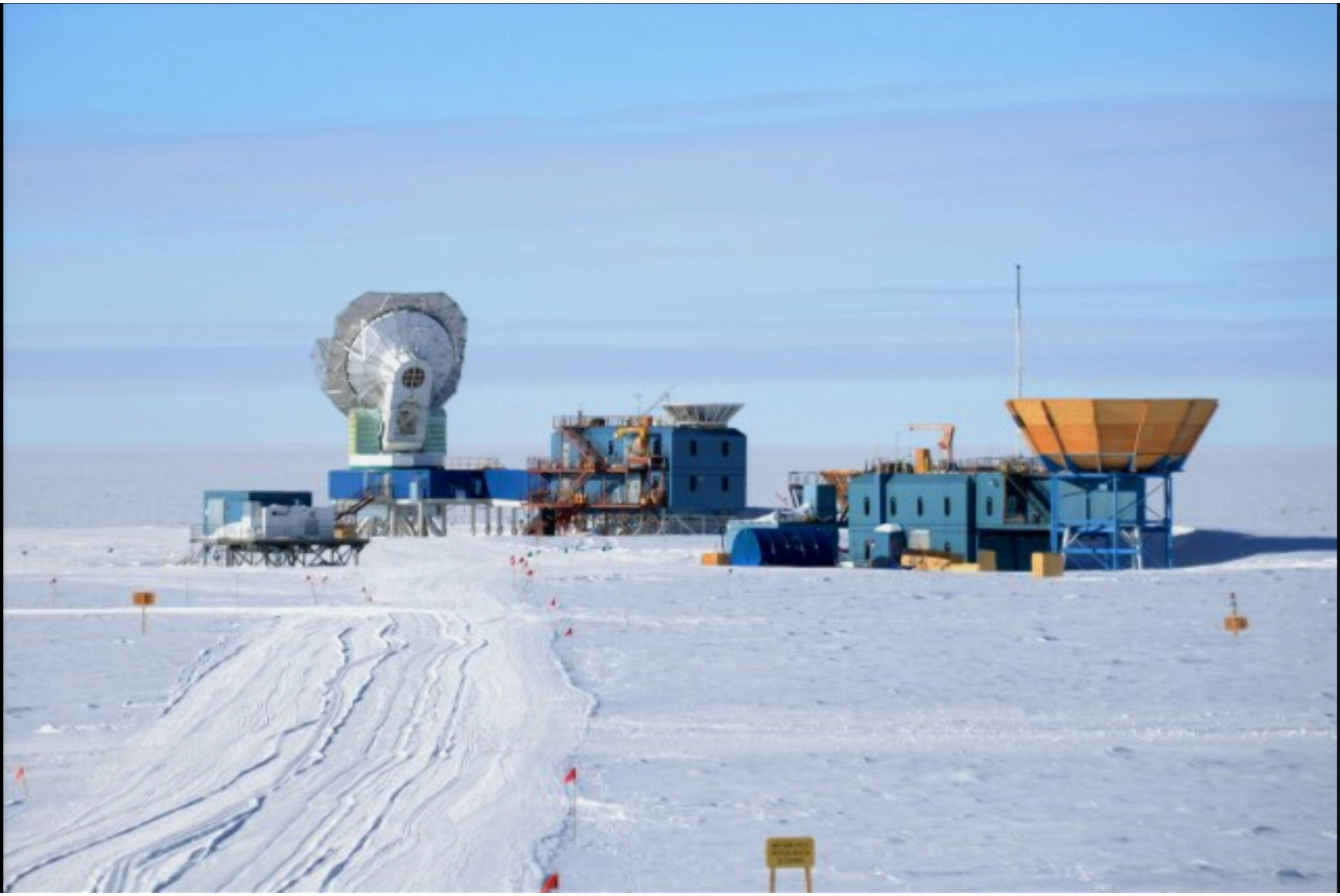
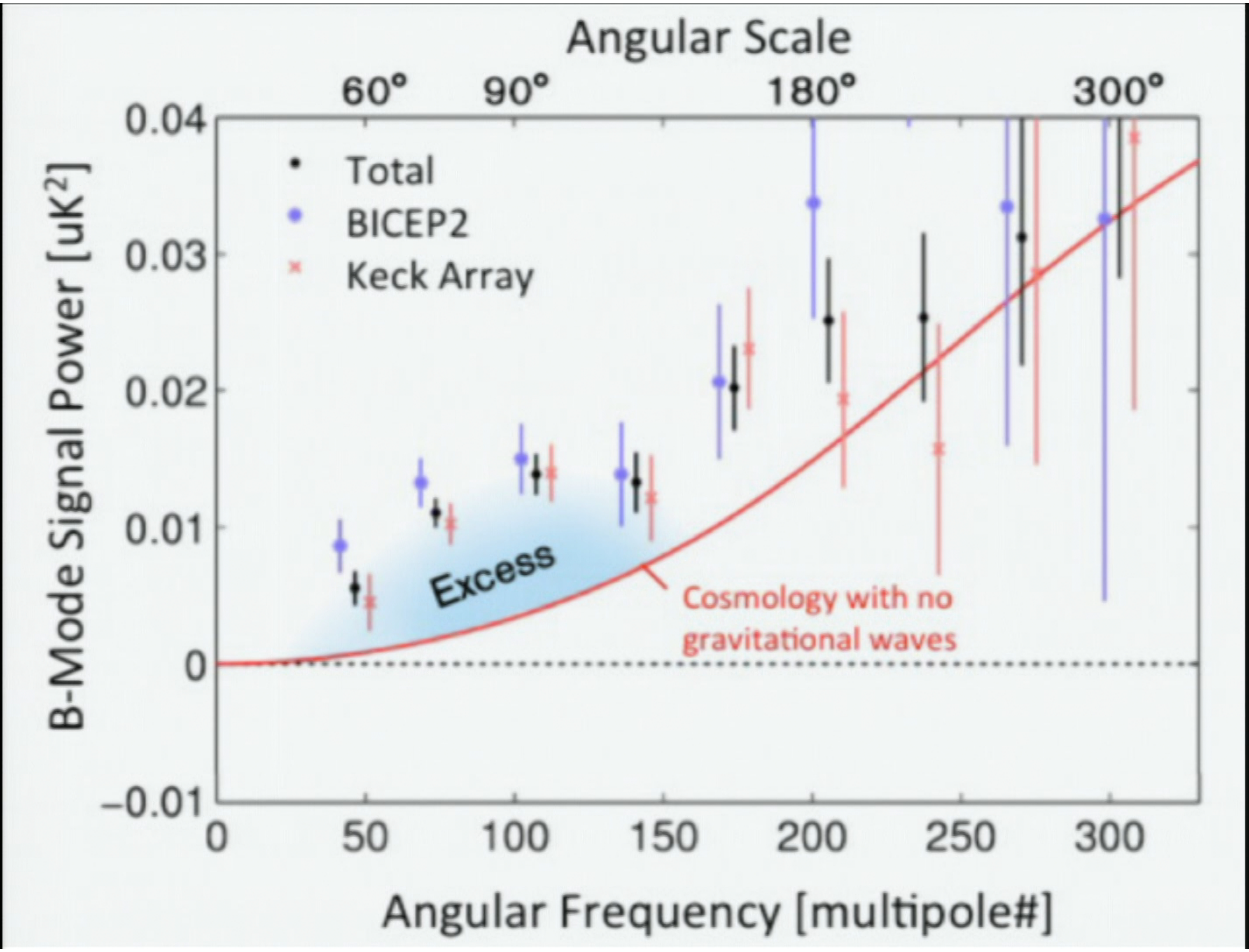

$C_\ell^{TT}$
$\underline{E}, \underline{B}$
$C_\ell^{TE}$
$C_\ell^{TB}, C_\ell^{EB},$

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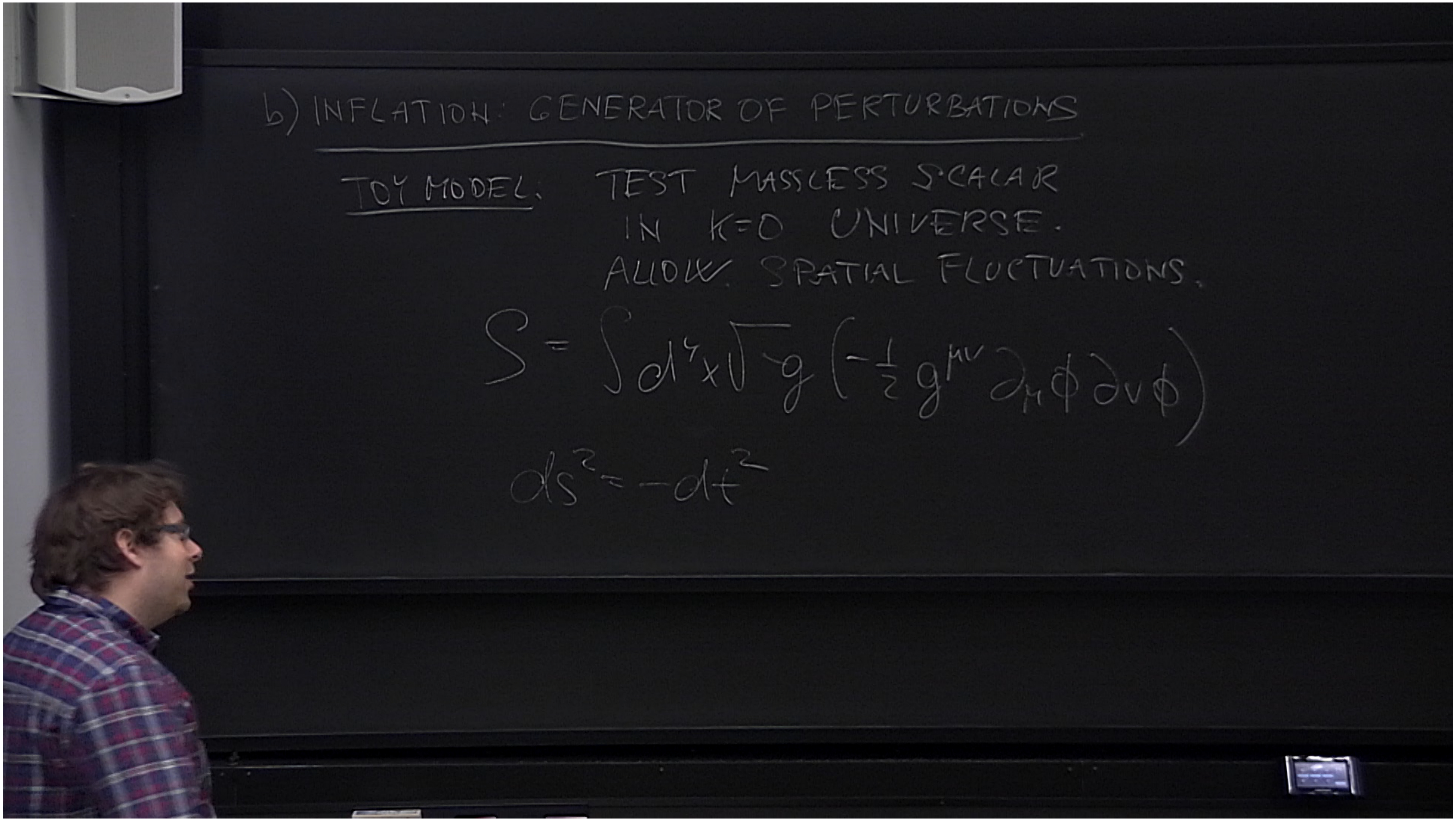
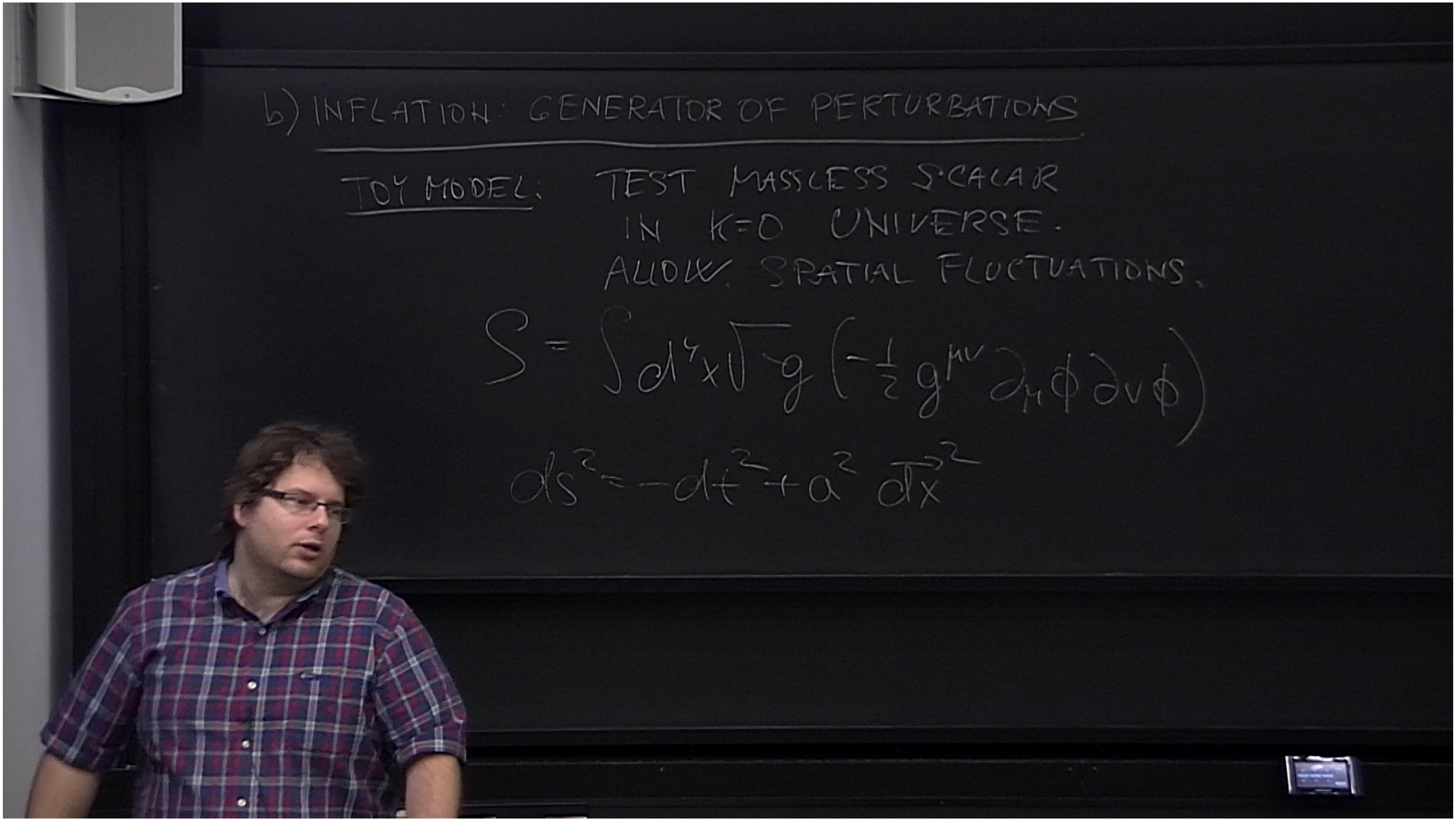


b) INFLATION: GENERATOR OF PERTURBATIONS

TOY MODEL: TEST MASSLESS SCALAR
IN K=0 UNIVERSE.
ALLOW SPATIAL FLUCTUATIONS.

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right)$$

$$ds^2 = -dt^2$$



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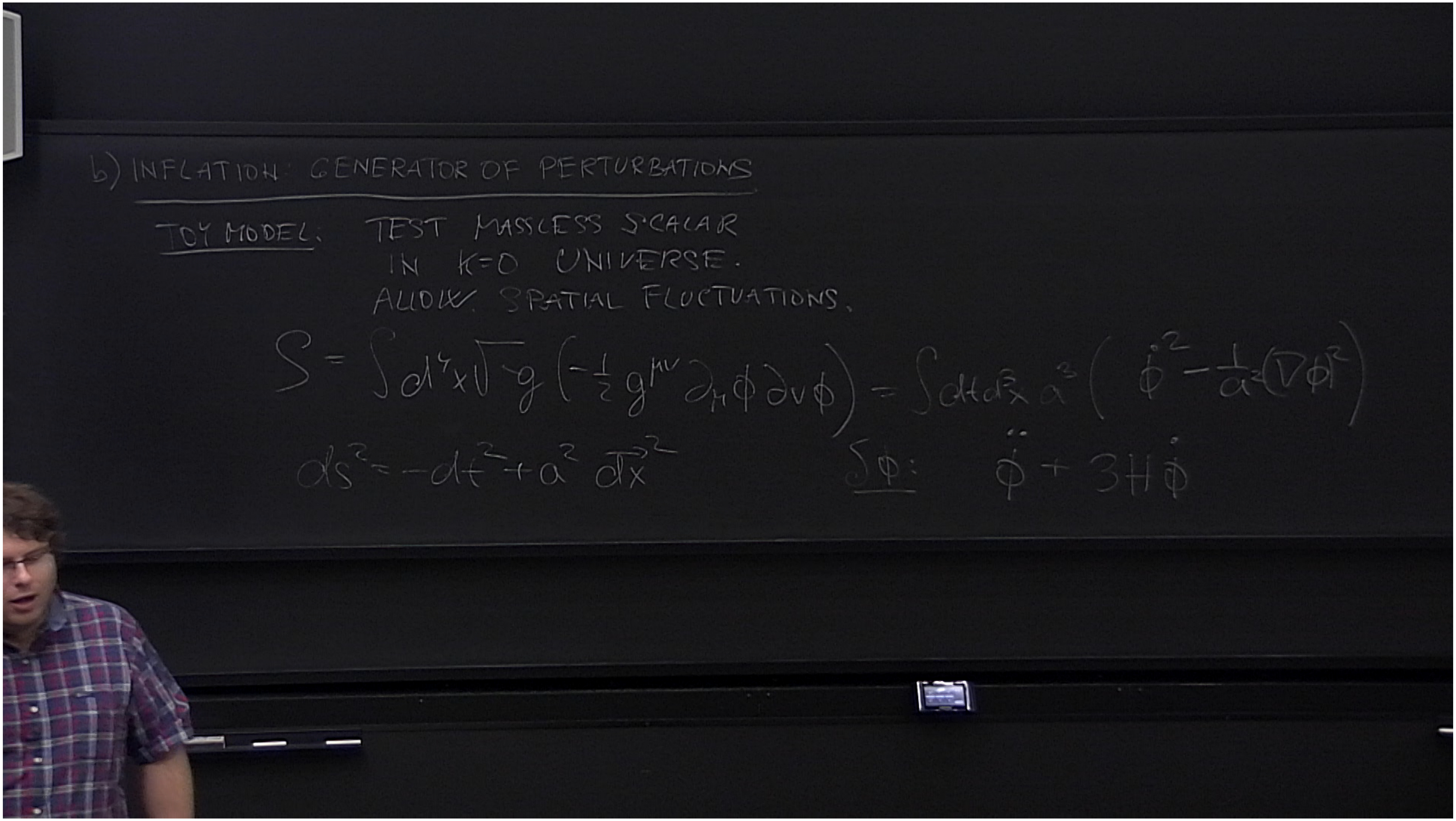
$$ds^2 = -dt^2 + a^2 \, d\vec{x}^2$$



b) INFLATION: GENERATOR OF PERTURBATIONS

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IN K=0 UNIVERSE.
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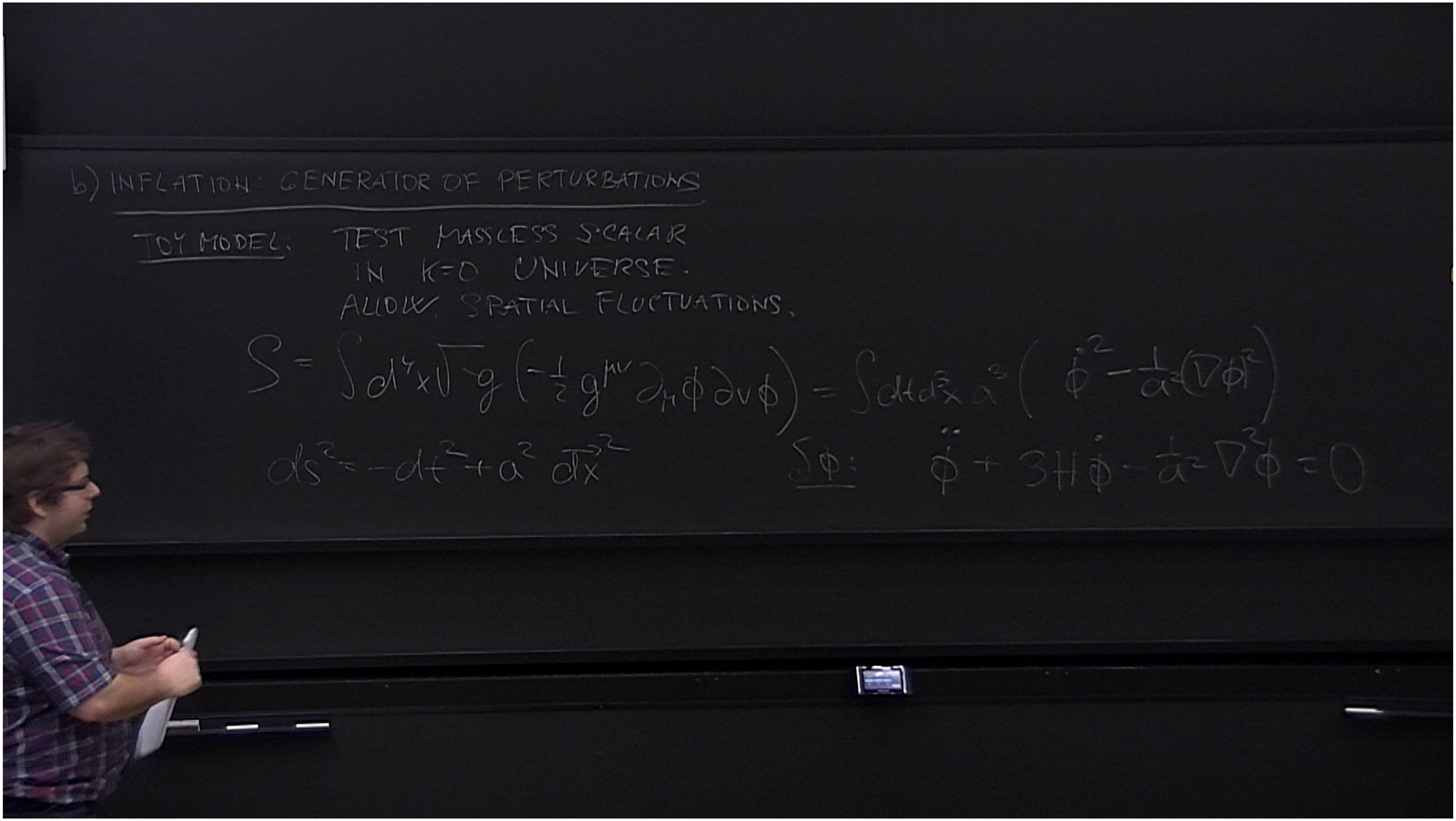
$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) = \int dt\, d^3x\, a^3 \left( \dot{\phi}^2 - \frac{1}{a^2} (\nabla \phi)^2 \right)$$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

$\delta\phi$: $\ddot{\phi} + 3H\dot{\phi}$



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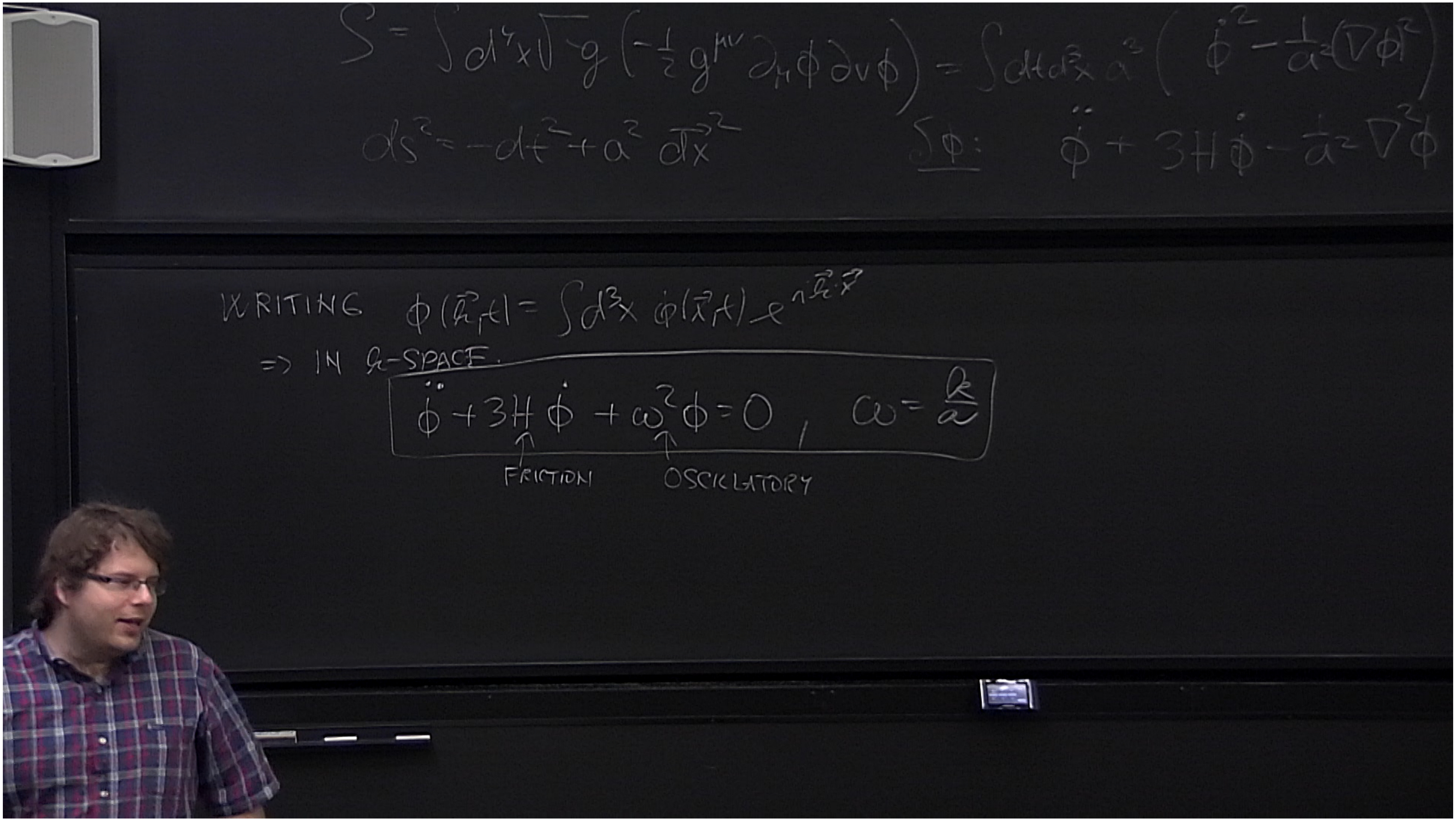
$$\underline{\delta\phi:}\quad \ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi = 0$$



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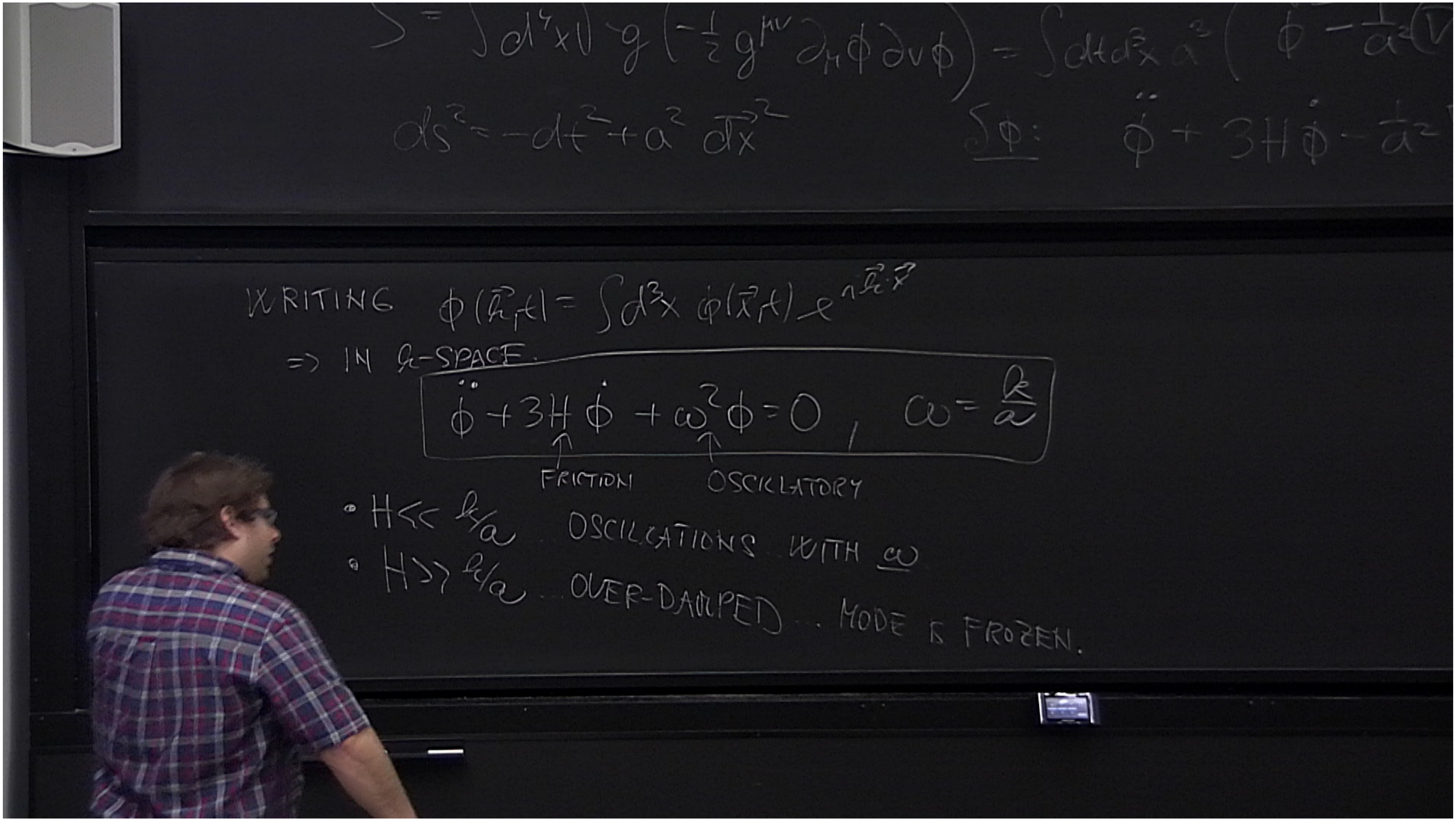
$$\underline{\delta\phi:}\quad \ddot\phi + 3H\dot\phi - \frac{1}{a^2}\nabla^2\phi$$

WRITING $\phi(\vec{k},t) = \int d^3x\, \phi(\vec{x},t)\, e^{i\vec{k}\cdot\vec{x}}$

=> IN k-SPACE.

$$\boxed{\ddot\phi + 3\underset{\overset{\uparrow}{\text{FRICTION}}}{H}\dot\phi + \underset{\overset{\uparrow}{\text{OSCILLATORY}}}{\omega^2}\phi = 0, \quad \omega = \frac{k}{a}}$$



$$S = \int d^4x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right) = \int dt\, d^3x\, a^3 \left( \phi - \frac{1}{a^2}(\nabla\right.$$

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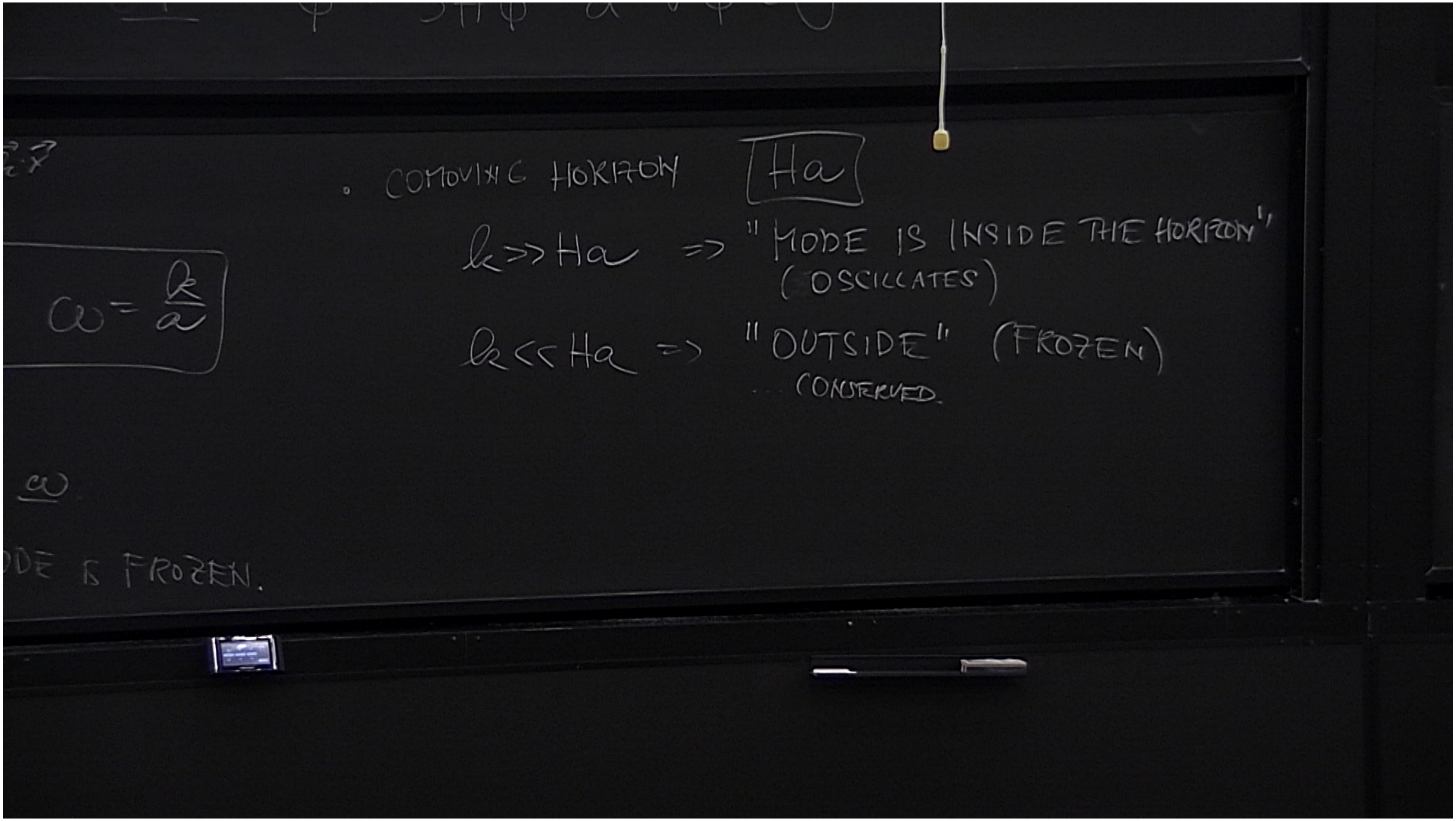
- $H \ll k/a$ ... OSCILLATIONS WITH $\underline{\omega}$
- $H \gg k/a$ ... OVER-DAMPED ... MODE IS FROZEN.



$$\boxed{\omega = \frac{k}{a}}$$

- COMOVING HORIZON $\boxed{Ha}$

$k \gg Ha \Rightarrow$ "MODE IS INSIDE THE HORIZON" (OSCILLATES)

$k \ll Ha \Rightarrow$ "OUTSIDE" (FROZEN)

... CONSERVED.

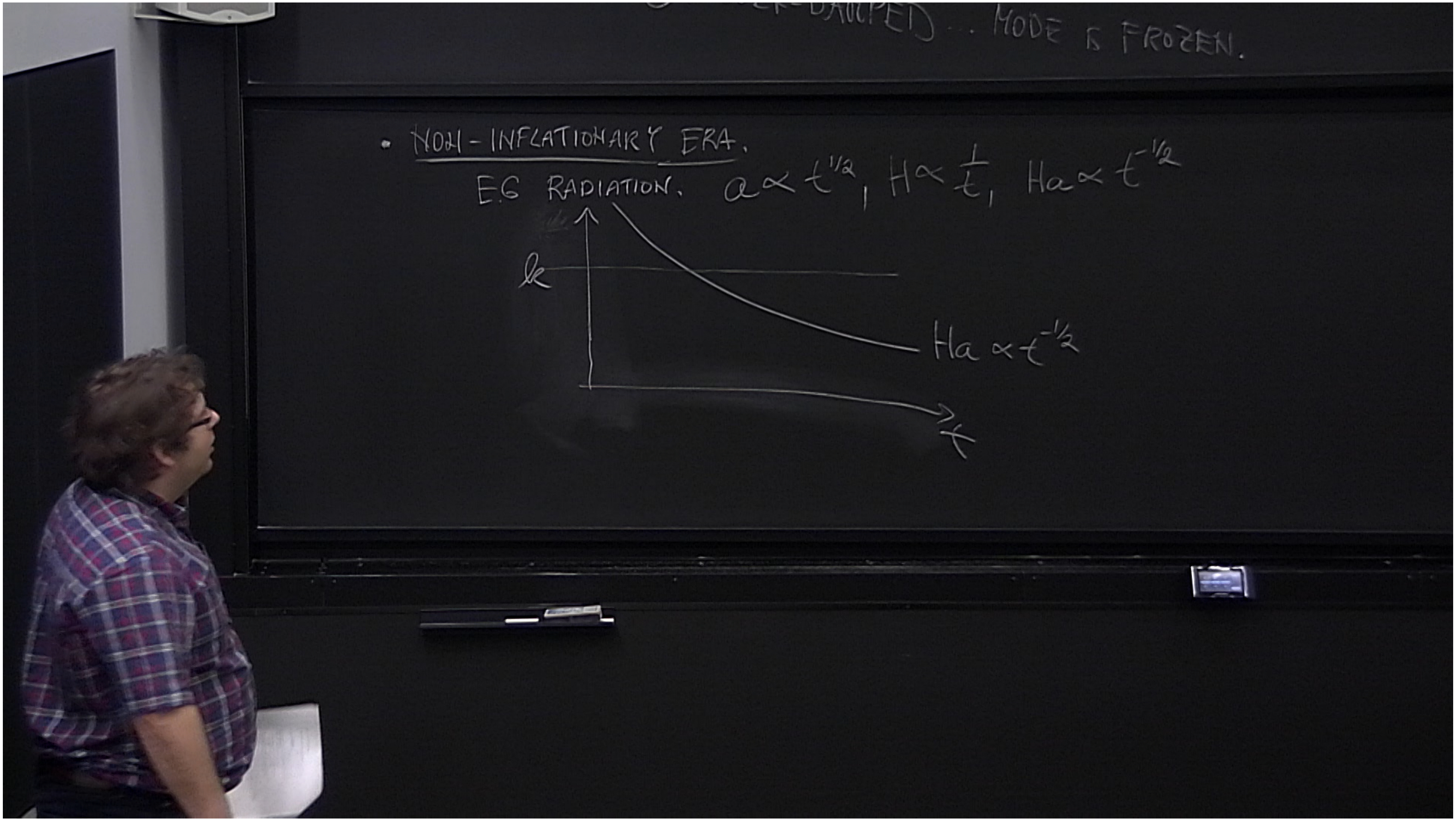
$\underline{\omega}$

DE IS FROZEN.

DAMPED) ... MODE IS FROZEN.

- NON-INFLATIONARY ERA:

E.G RADIATION. $a \propto t^{1/2}$, $H \propto \frac{1}{t}$, $Ha \propto t^{-1/2}$



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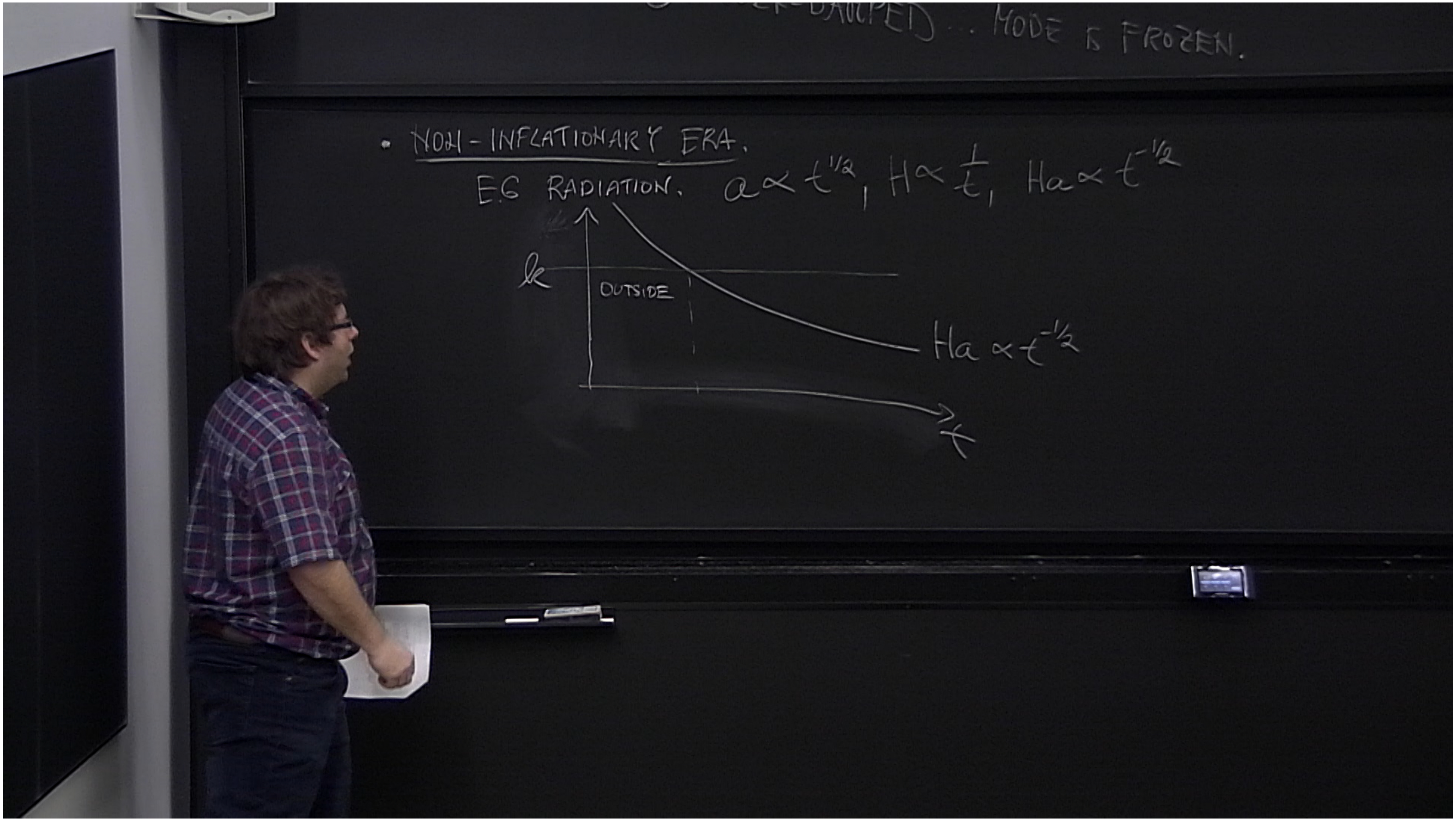

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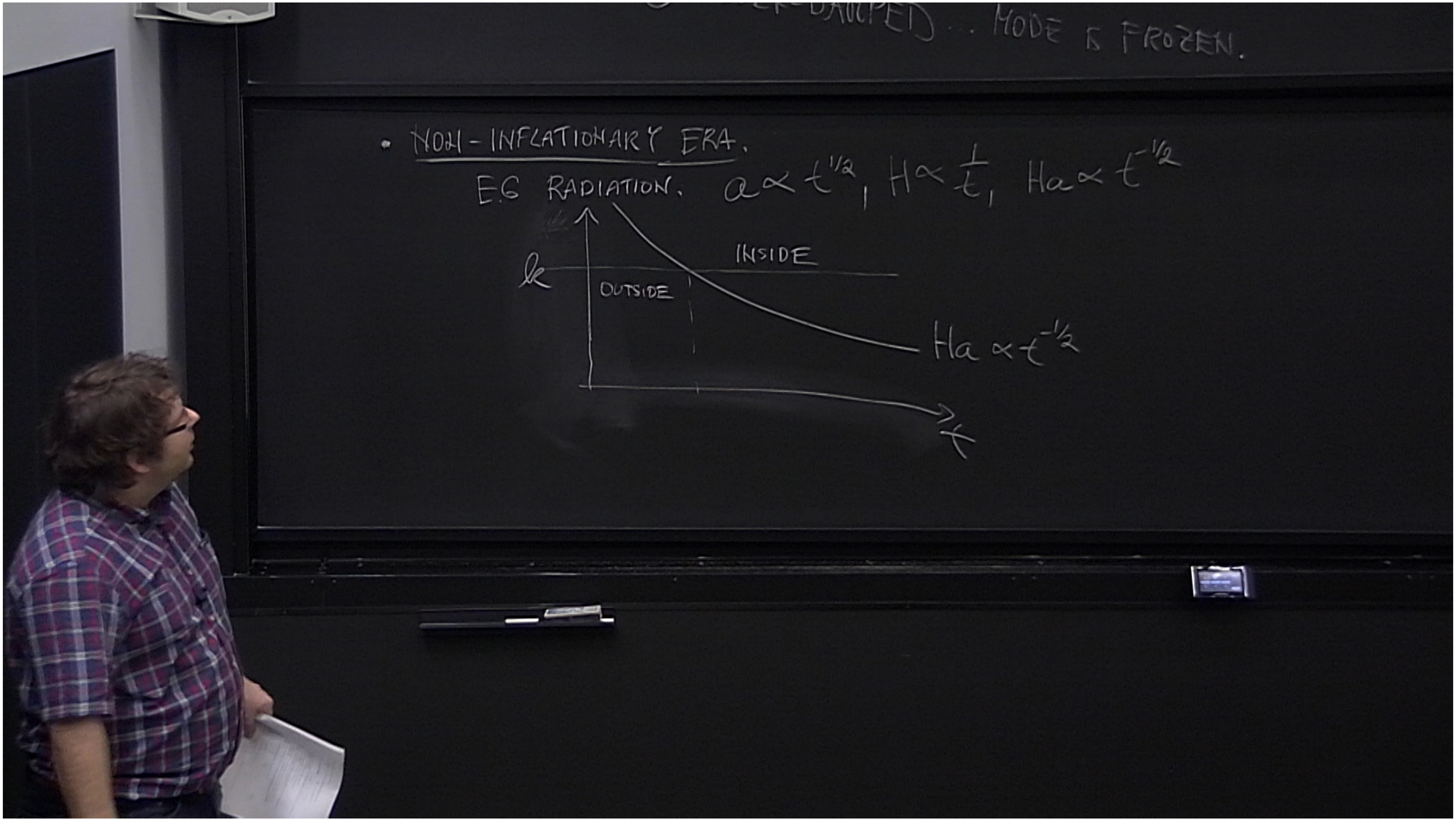

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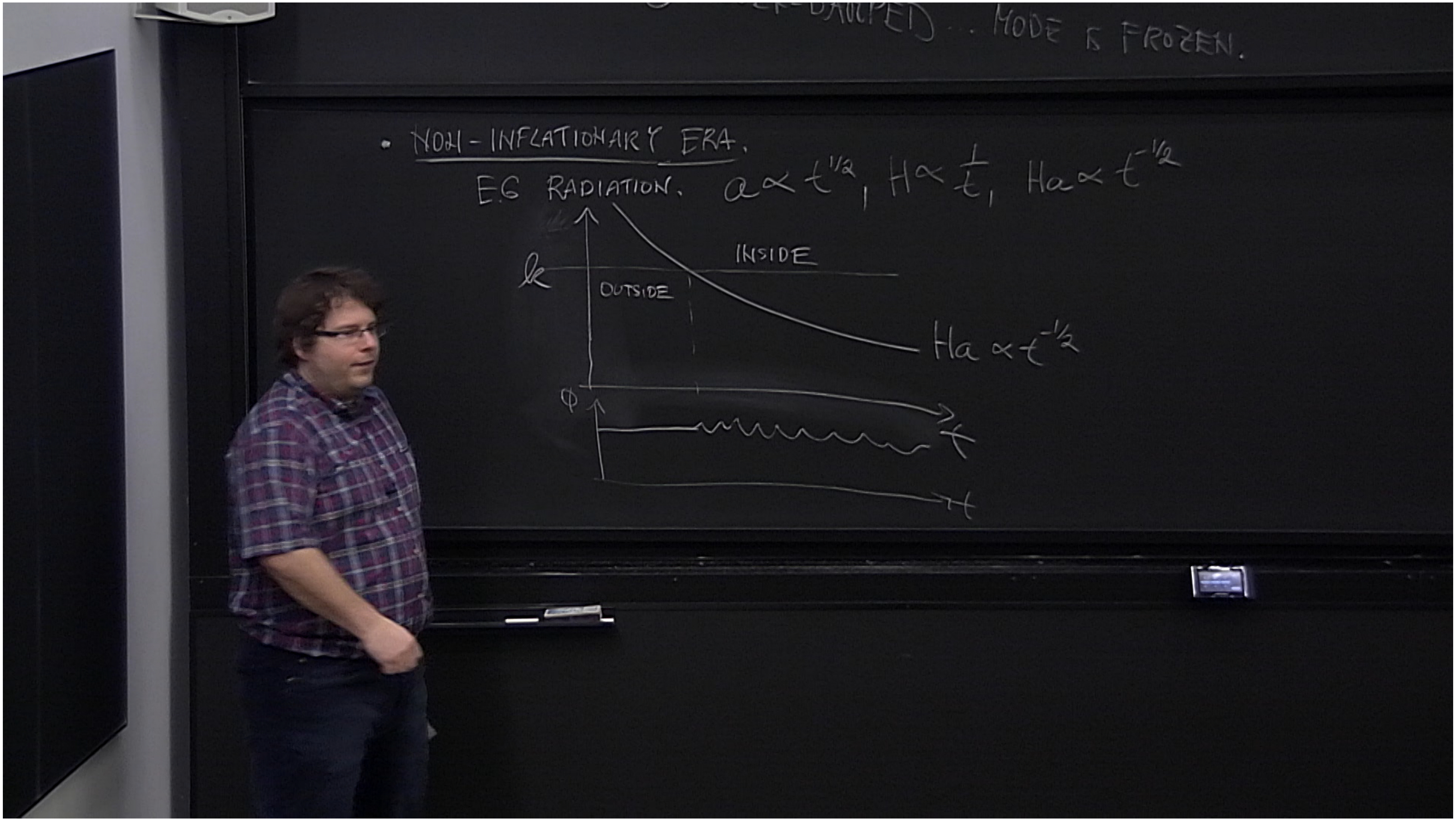

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- NON-INFLATIONARY ERA.

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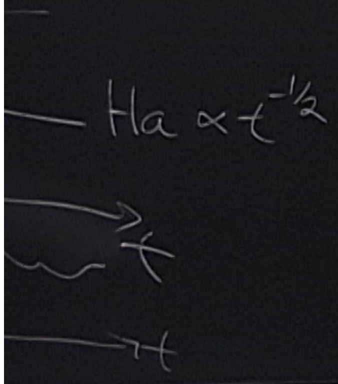
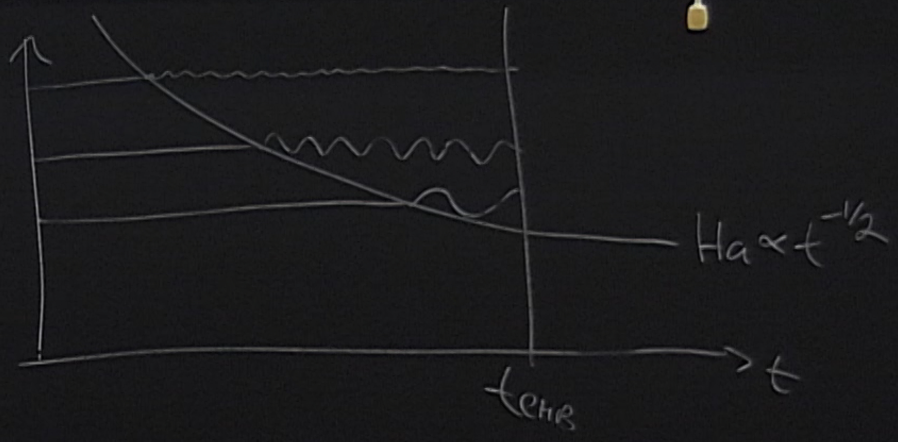
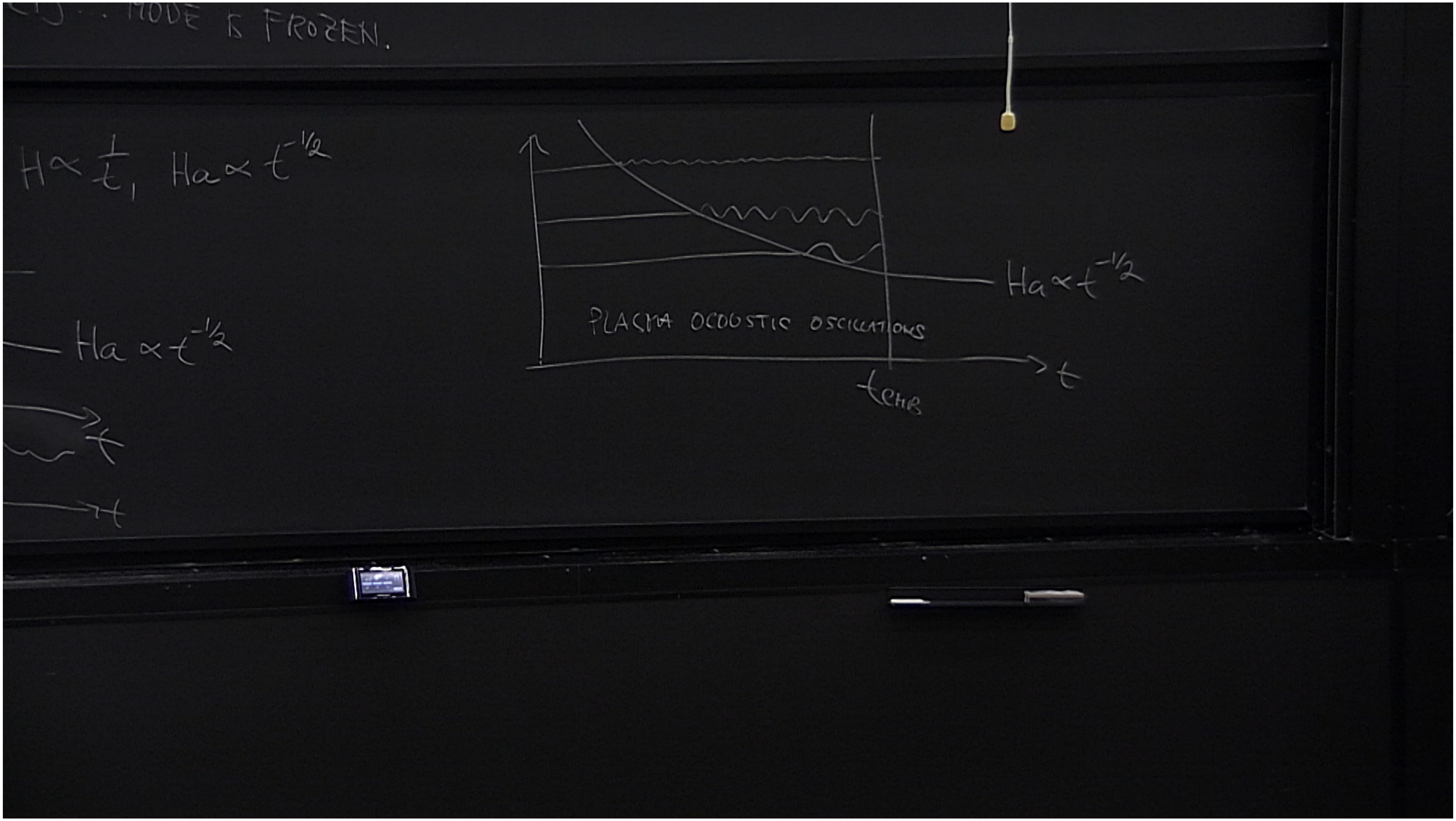
... MODE IS FROZEN.

$H \propto \frac{1}{t}, \quad Ha \propto t^{-1/2}$







... MODE IS FROZEN.

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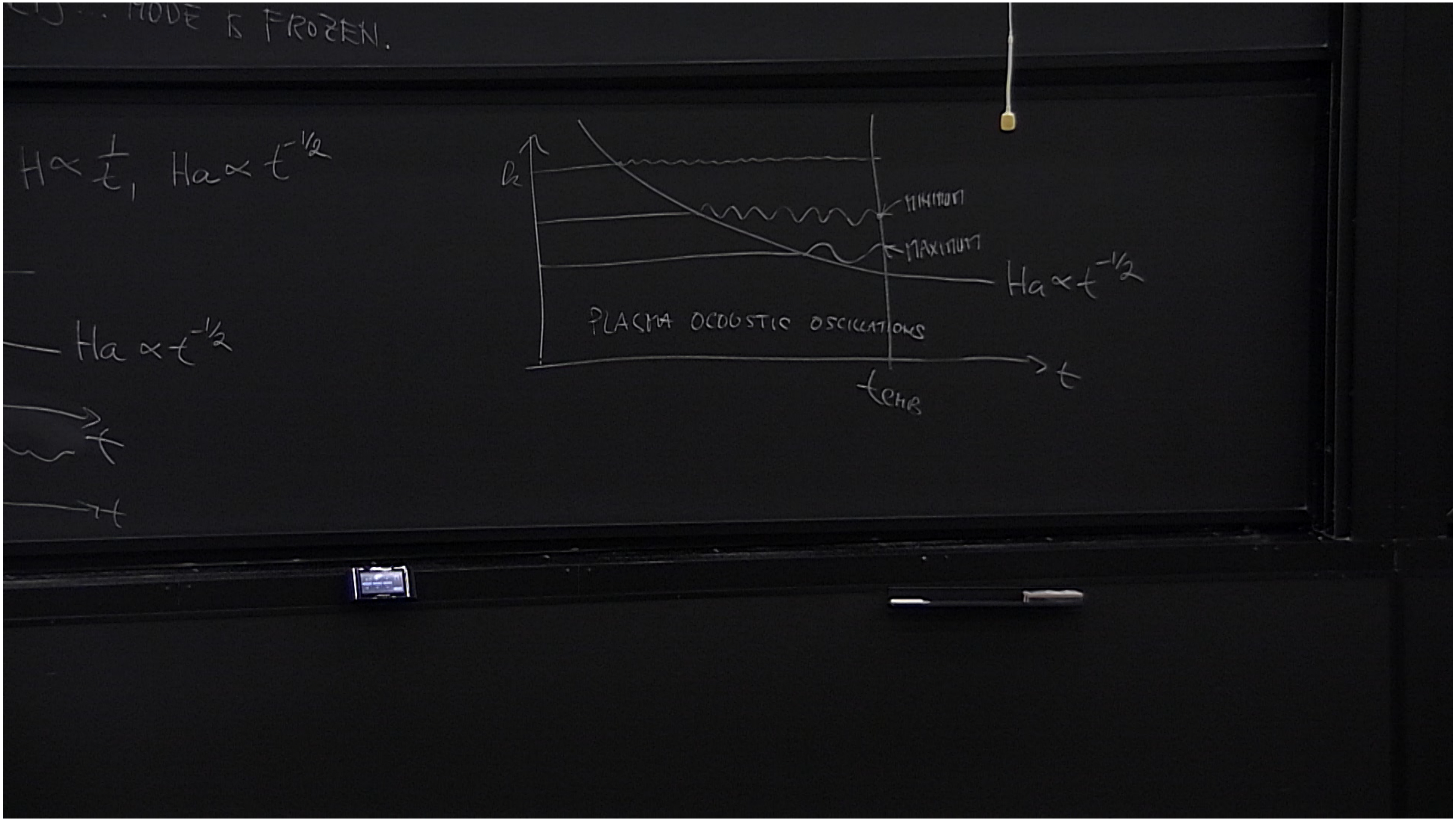

MODE IS FROZEN.

$H \propto \frac{1}{t}$, $Ha \propto t^{-1/2}$

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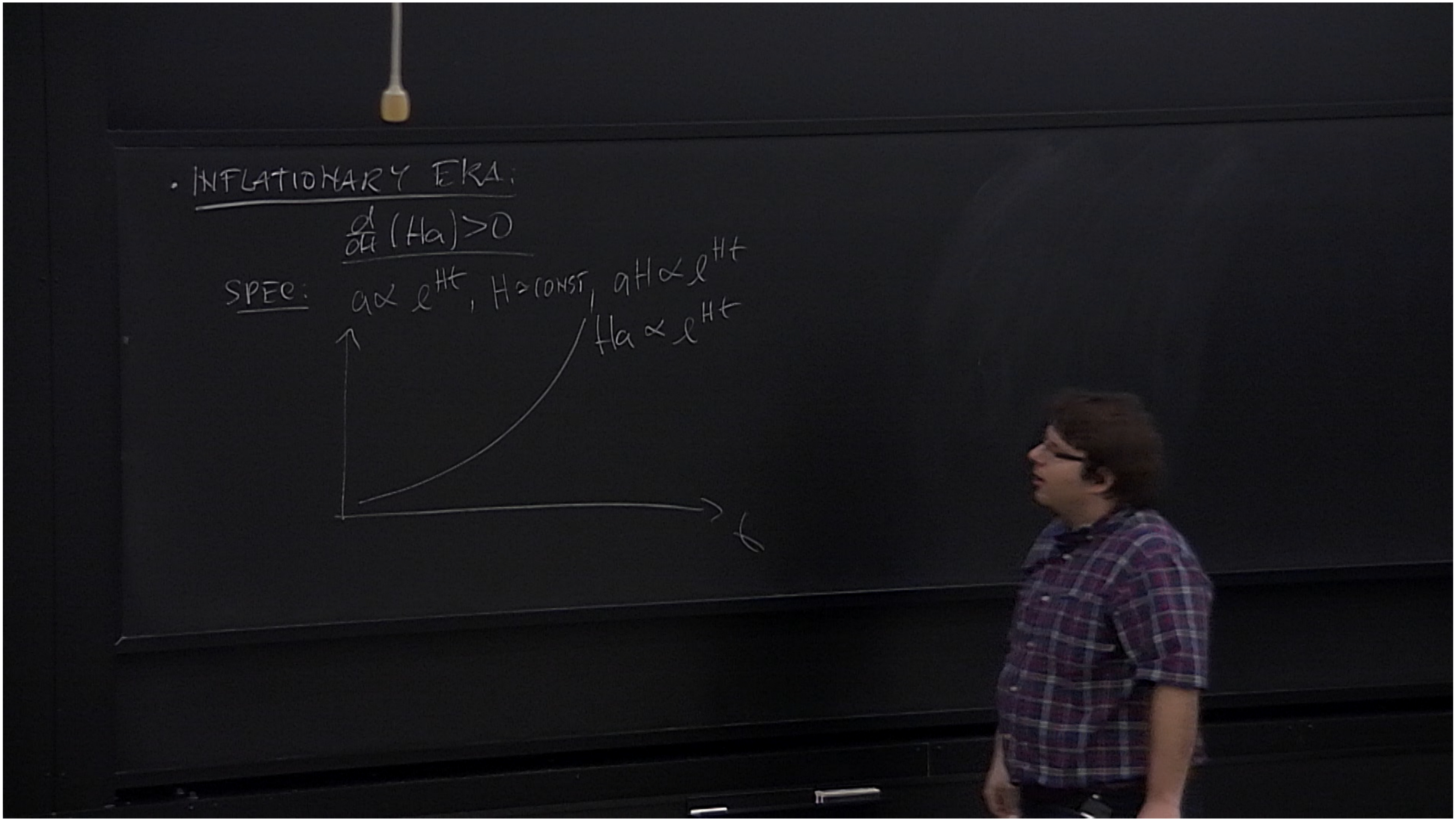
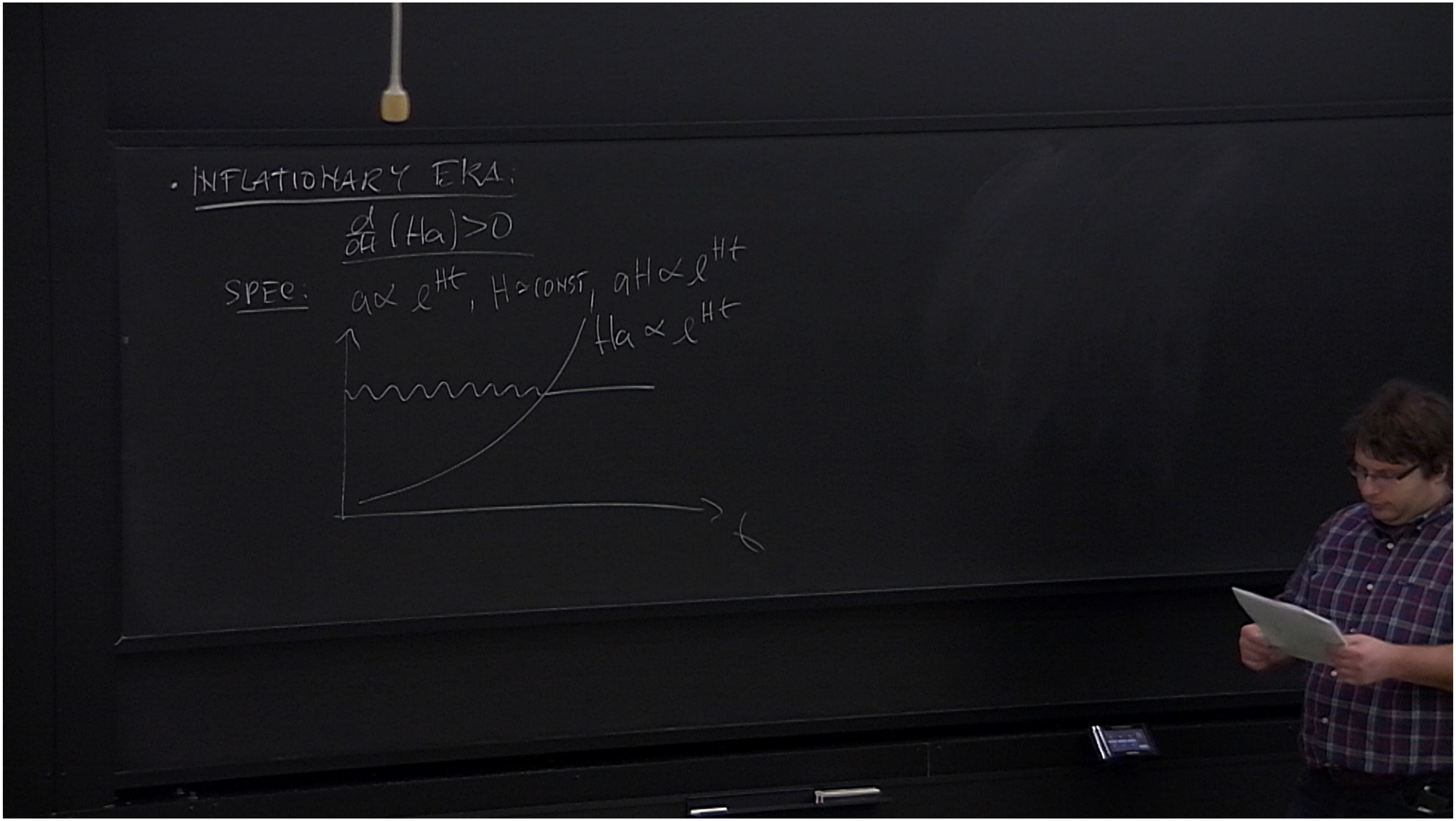


- INFLATIONARY ERA:

$$\frac{d}{dt}(Ha)>0$$

SPEC: $a \propto e^{Ht}$, $H \simeq \text{CONST}$, $aH \propto e^{Ht}$

$Ha \propto e^{Ht}$



- INFLATIONARY ERA:

$$\frac{d}{dt}(Ha) > 0$$

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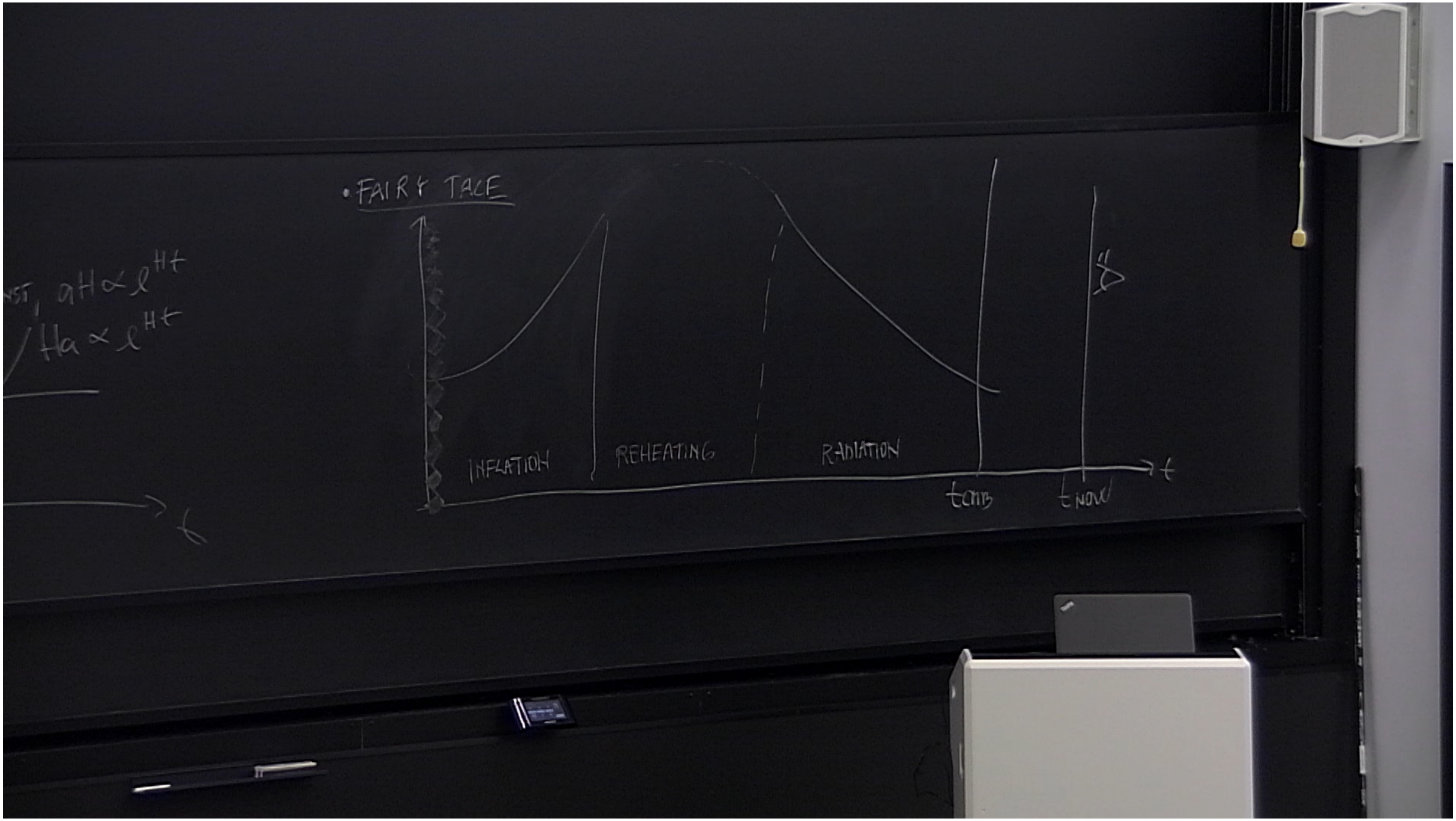
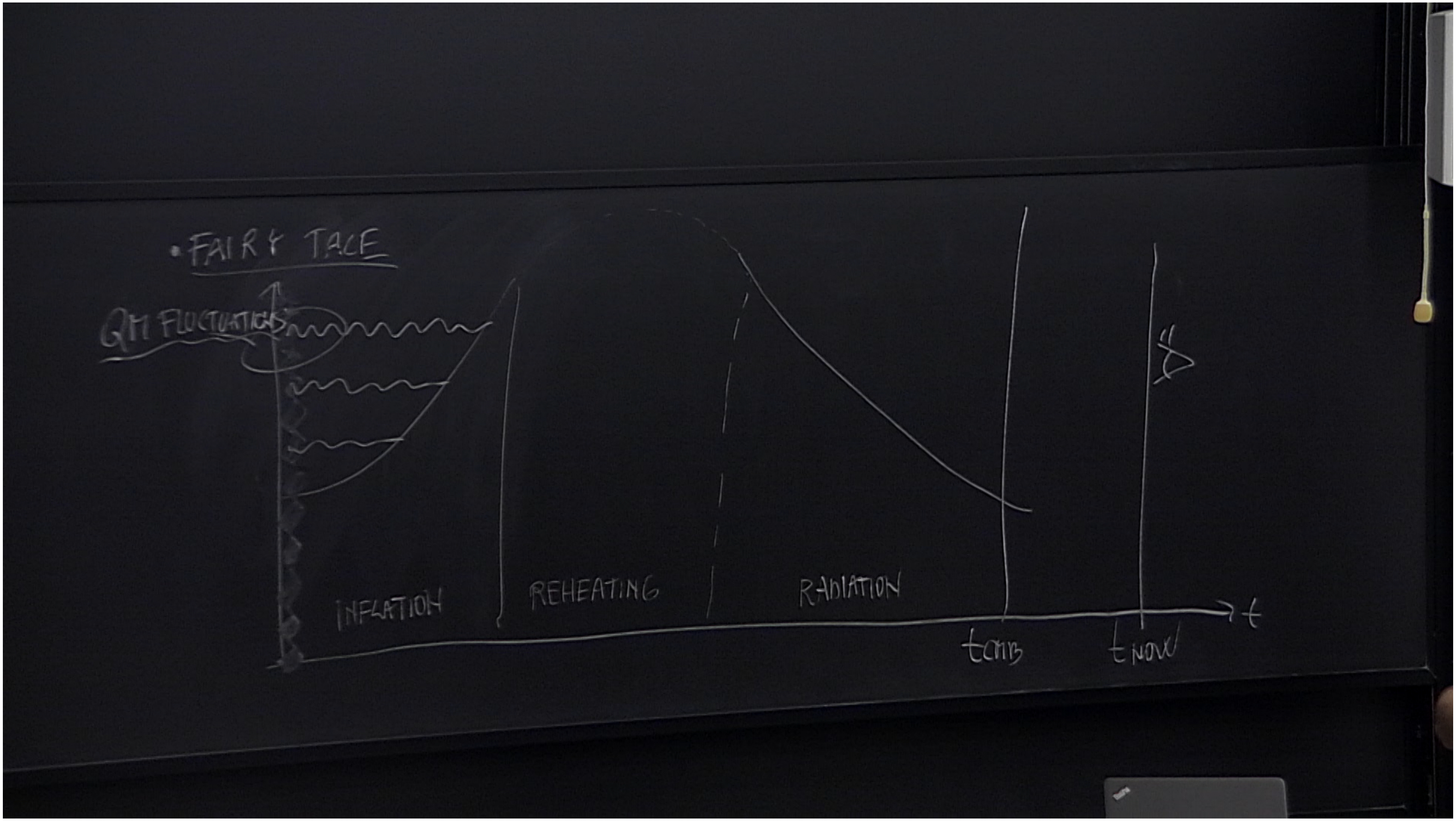
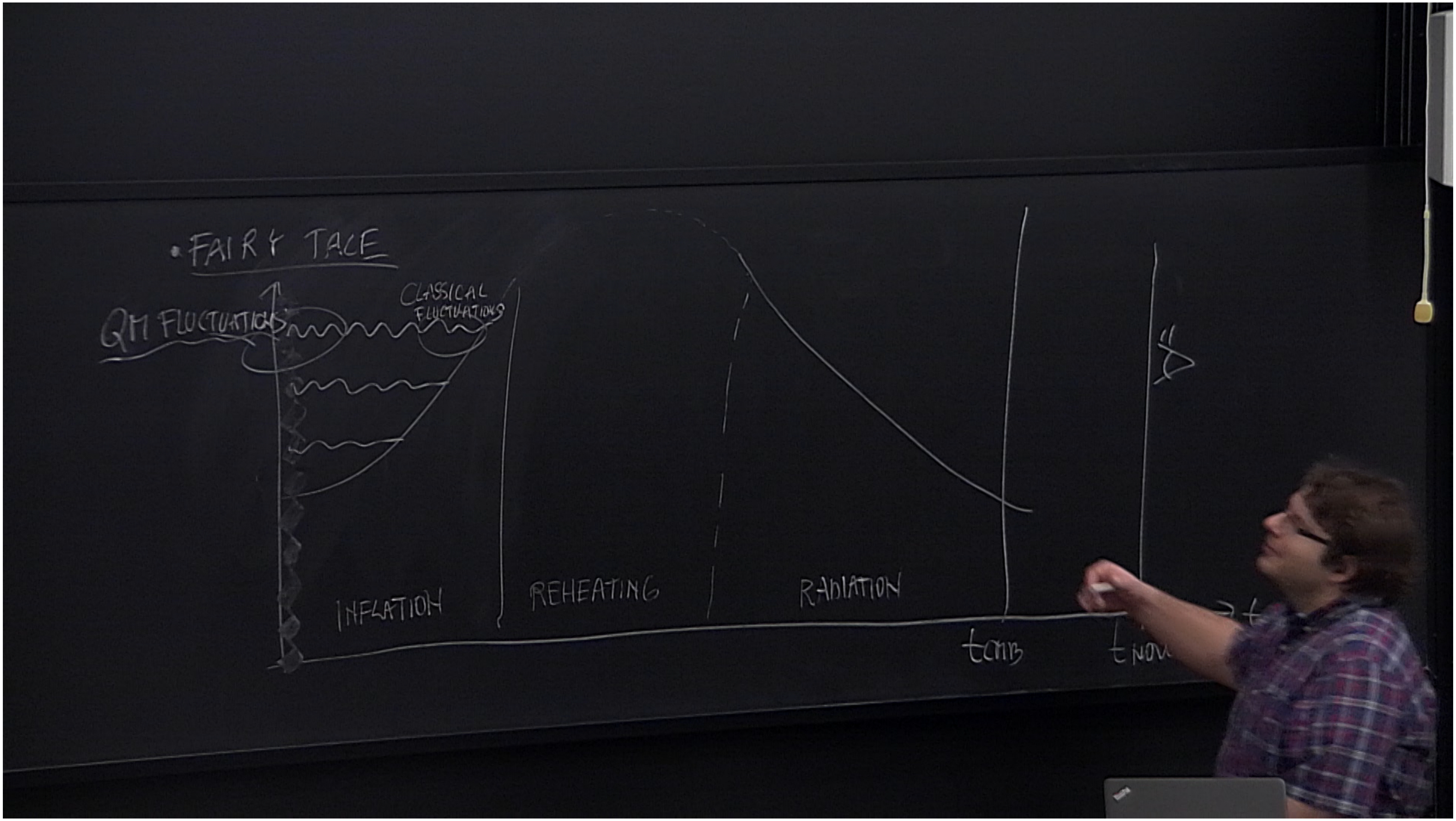
$Ha \propto e^{Ht}$


• FAIRY TALE
aH ∝ e^{Ht}
Ha ∝ e^{Ht}
INFLATION
REHEATING
RADIATION
t
t_CMB
t_NOW
t


• FAIRY TALE
QM FLUCTUATIONS
INFLATION
REHEATING
RADIATION
t
t_CMB
t_NOW
δ


• FAIRY TALE
QM FLUCTUATIONS
CLASSICAL FLUCTUATIONS
INFLATION
REHEATING
RADIATION
tCMB
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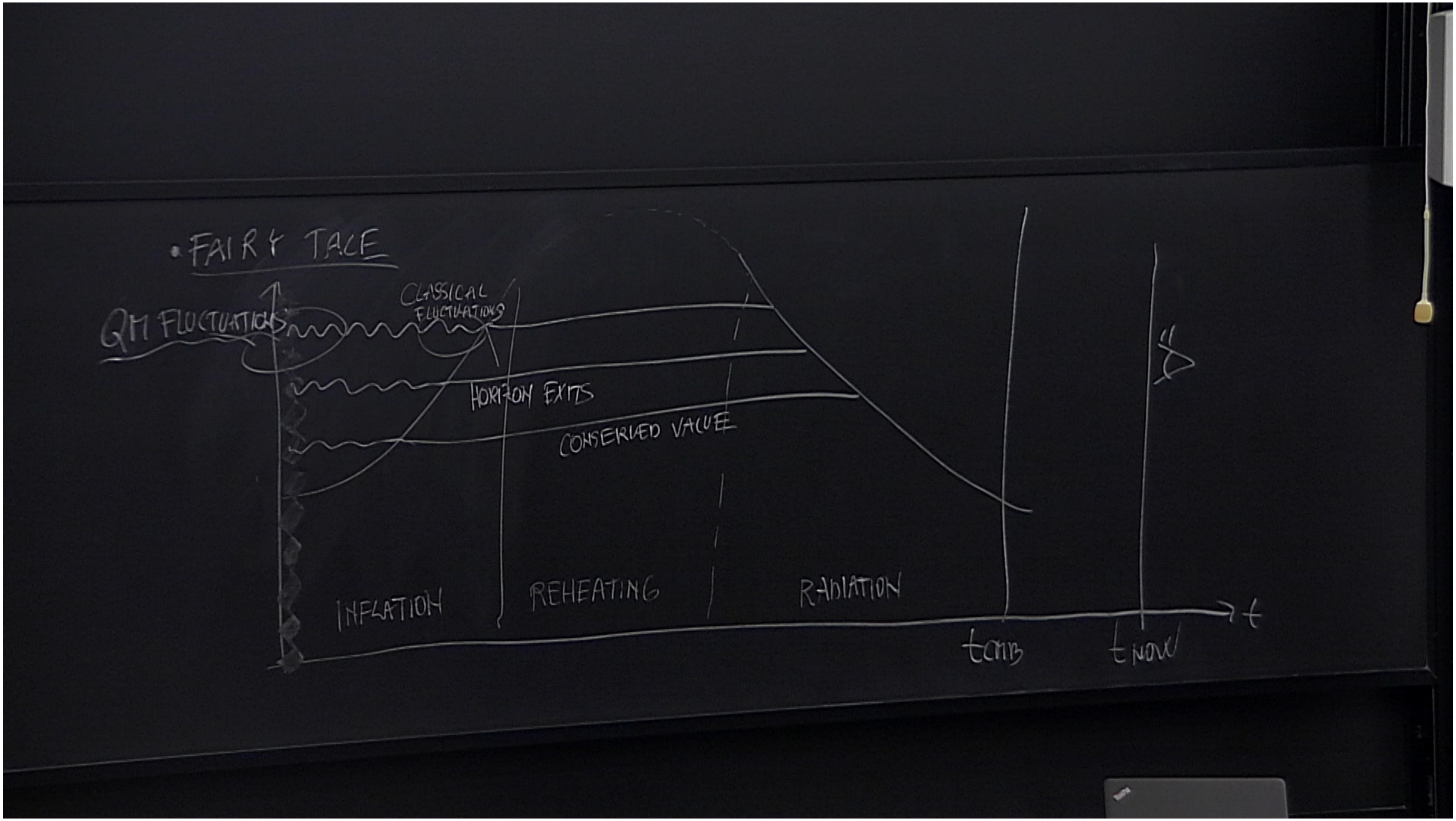
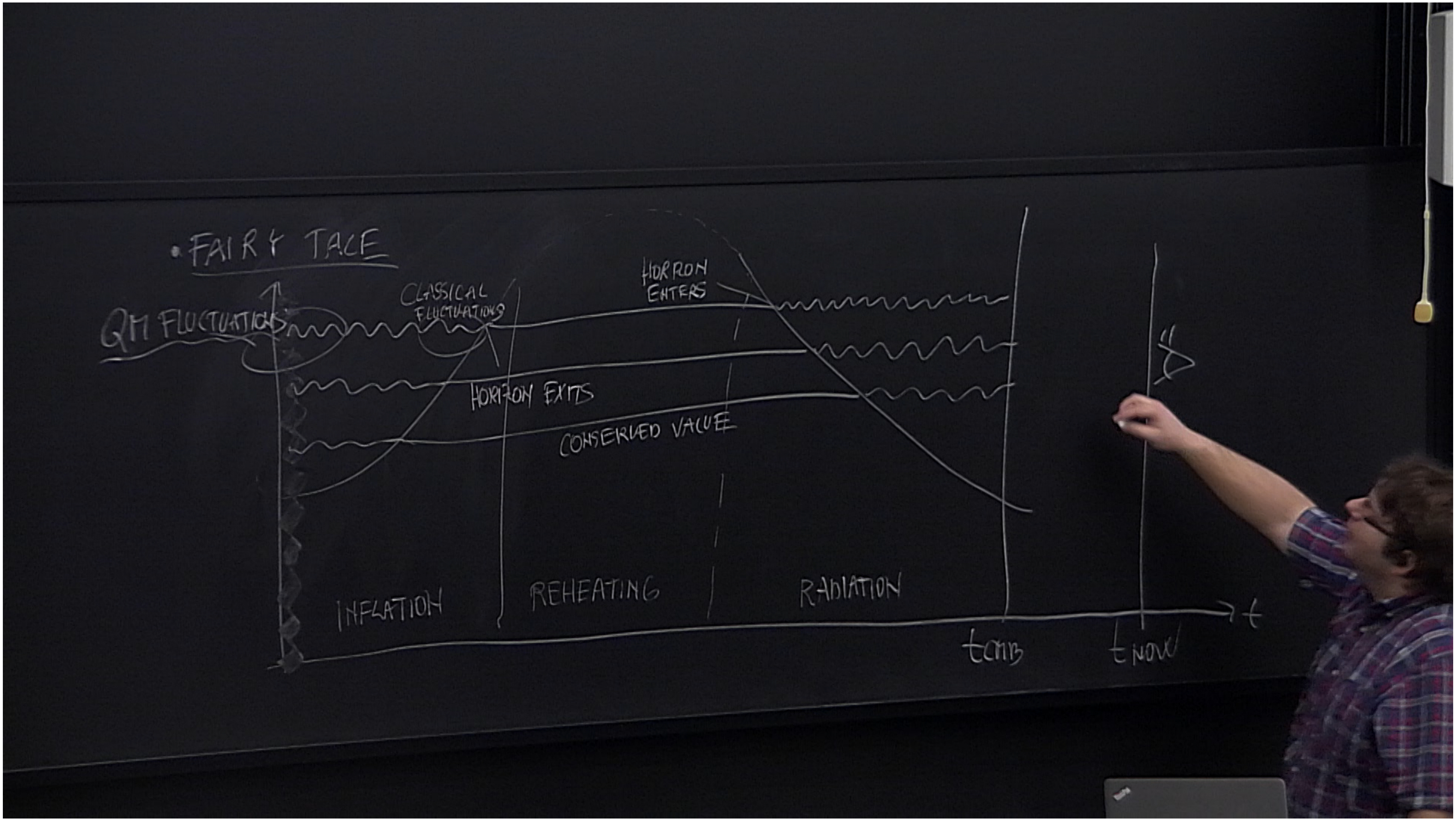
t


• FAIRY TALE
QM FLUCTUATIONS
CLASSICAL FLUCTUATIONS
HORIZON EXITS
CONSERVED VALUE
INFLATION
REHEATING
RADIATION
t
$t_{CMB}$
$t_{NOW}$


• FAIRY TALE
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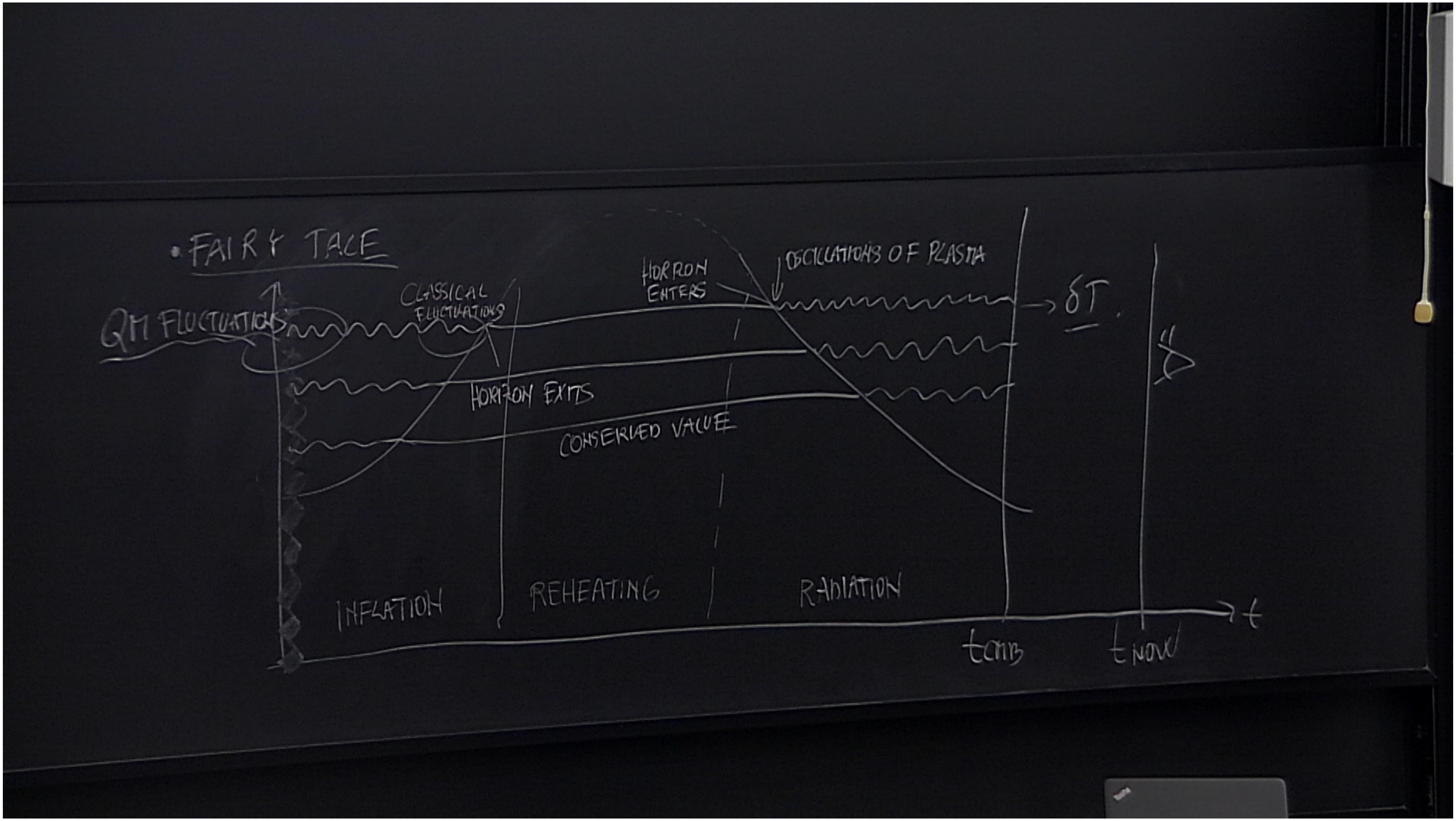
• FAIRY TALE
QM FLUCTUATIONS
CLASSICAL FLUCTUATIONS
HORIZON ENTERS
OSCILLATIONS OF PLASMA
→ δT
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CONSERVED VALUE
INFLATION
REHEATING
RADIATION
t
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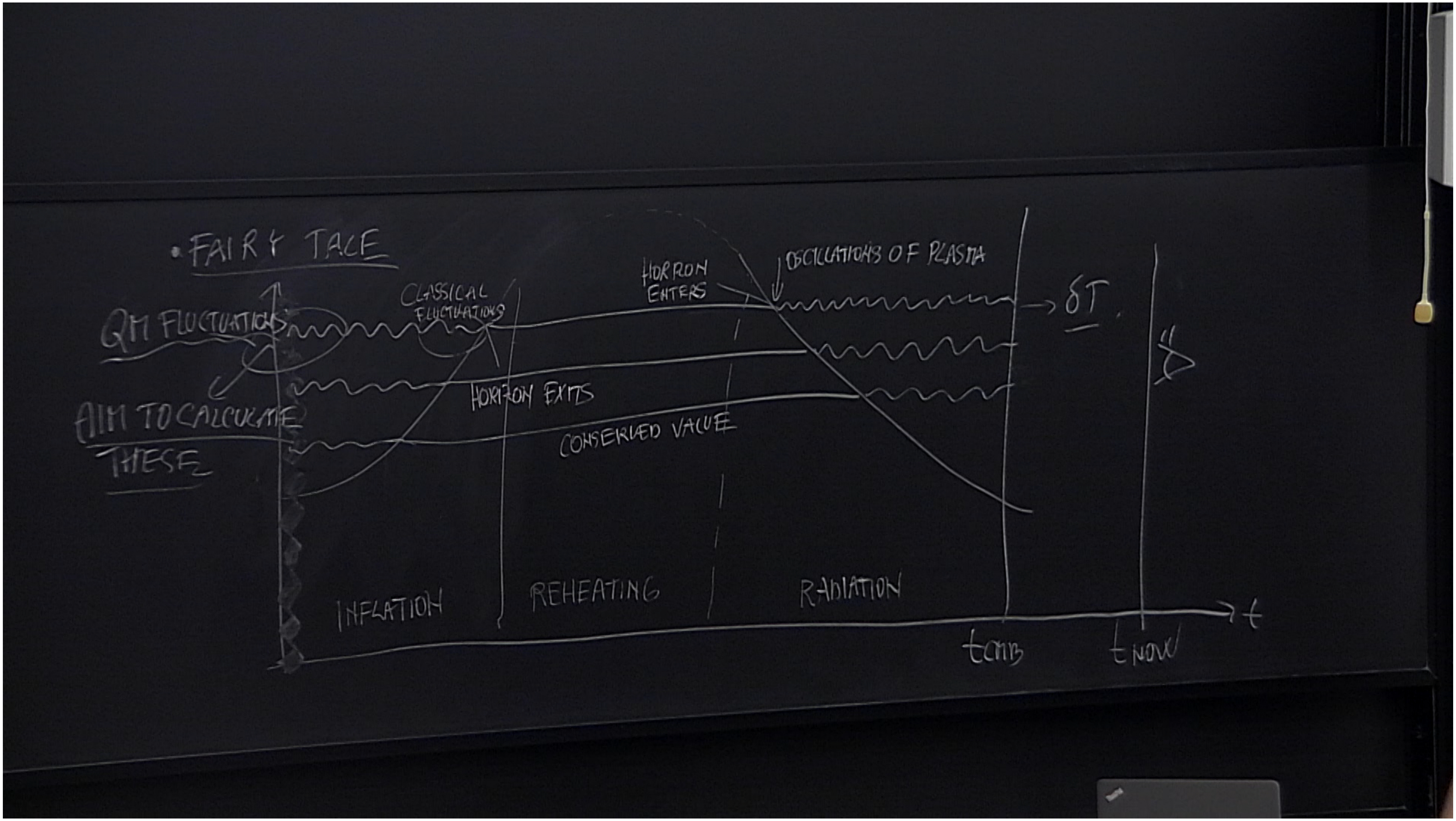
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HORIZON ENTERS
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AIM TO CALCULATE THESE
HORIZON EXITS
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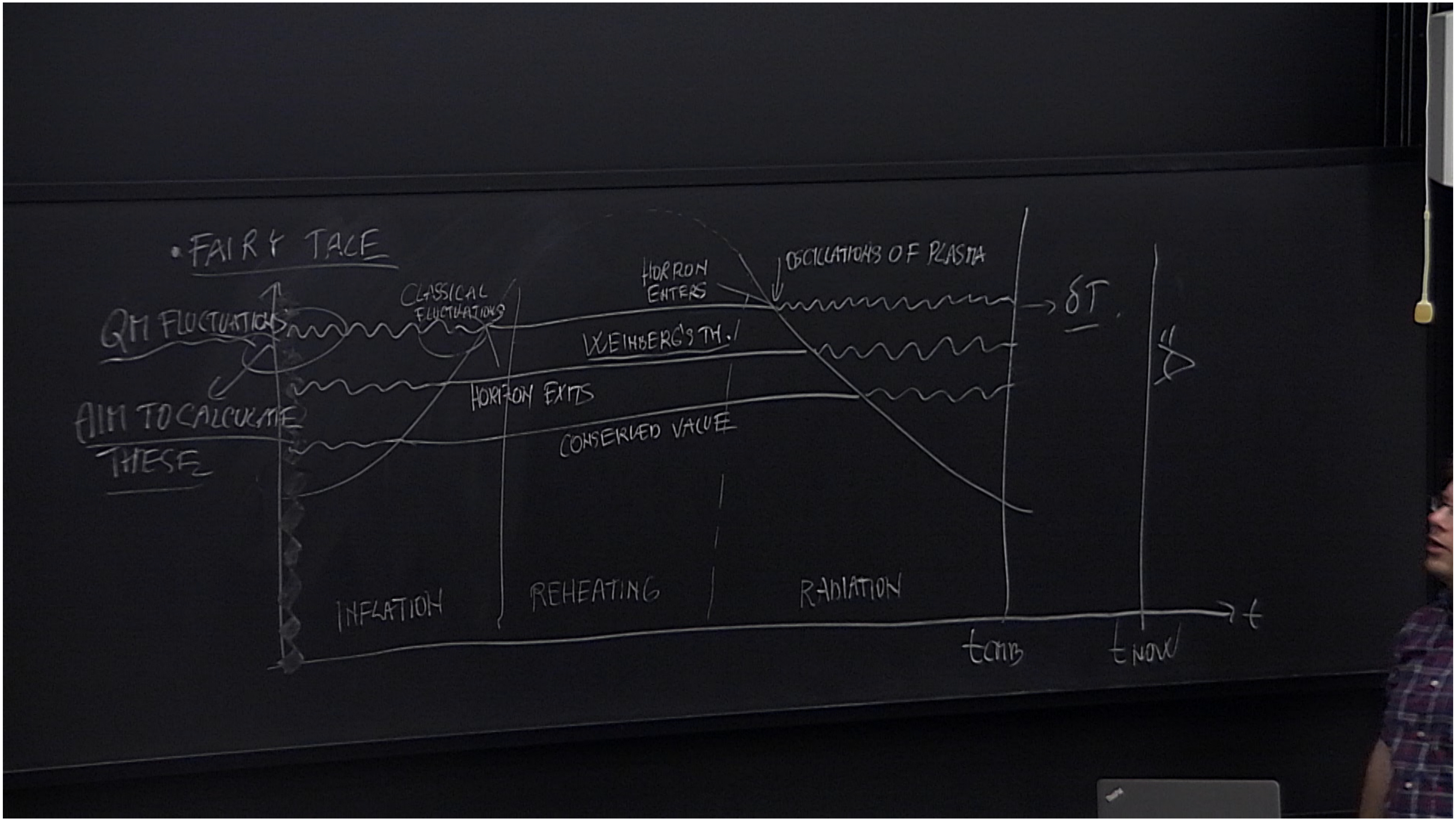
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→ δT.
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HORIZON EXITS
CONSERVED VALUE
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RADIATION
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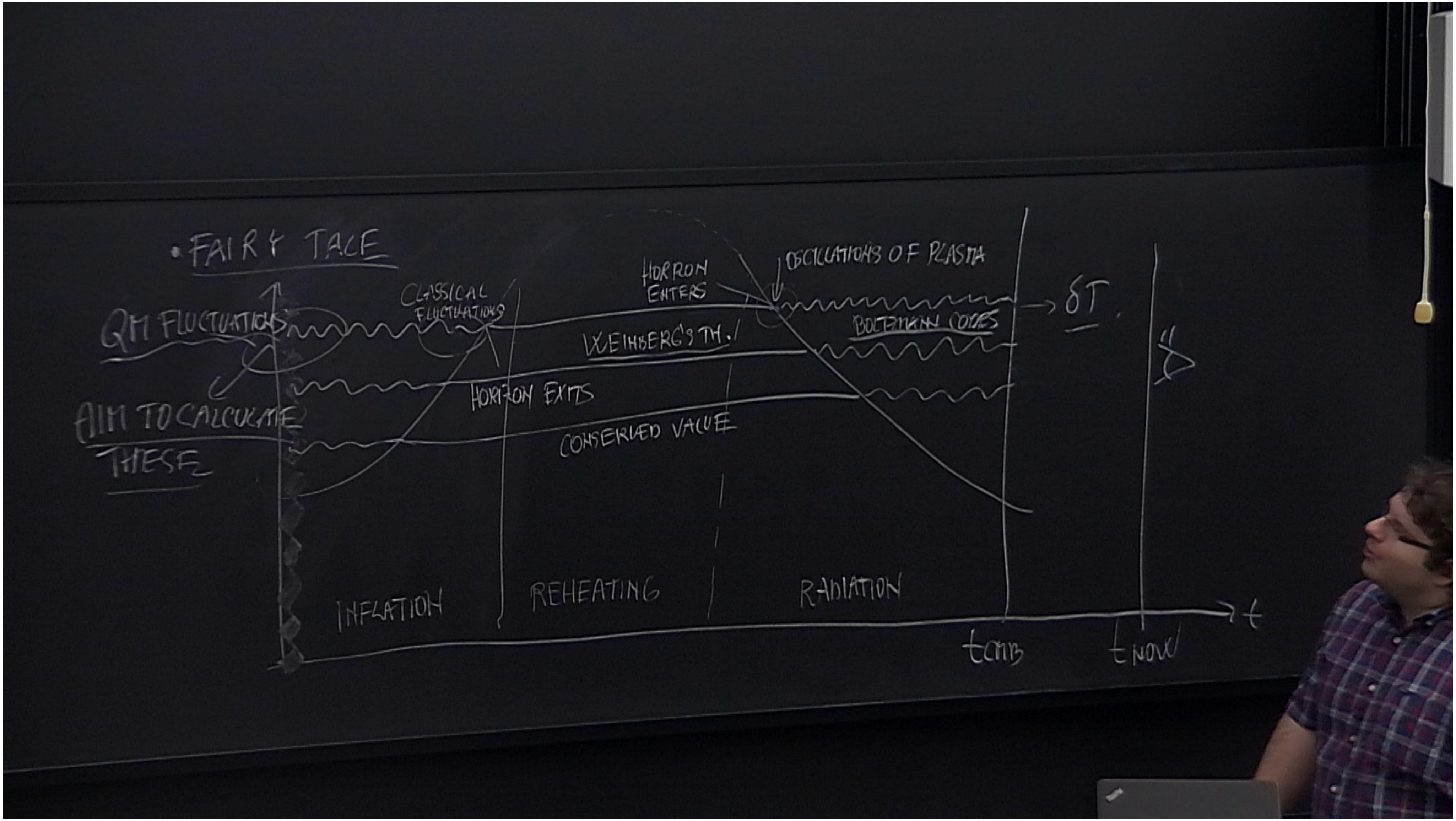
t_NOW
t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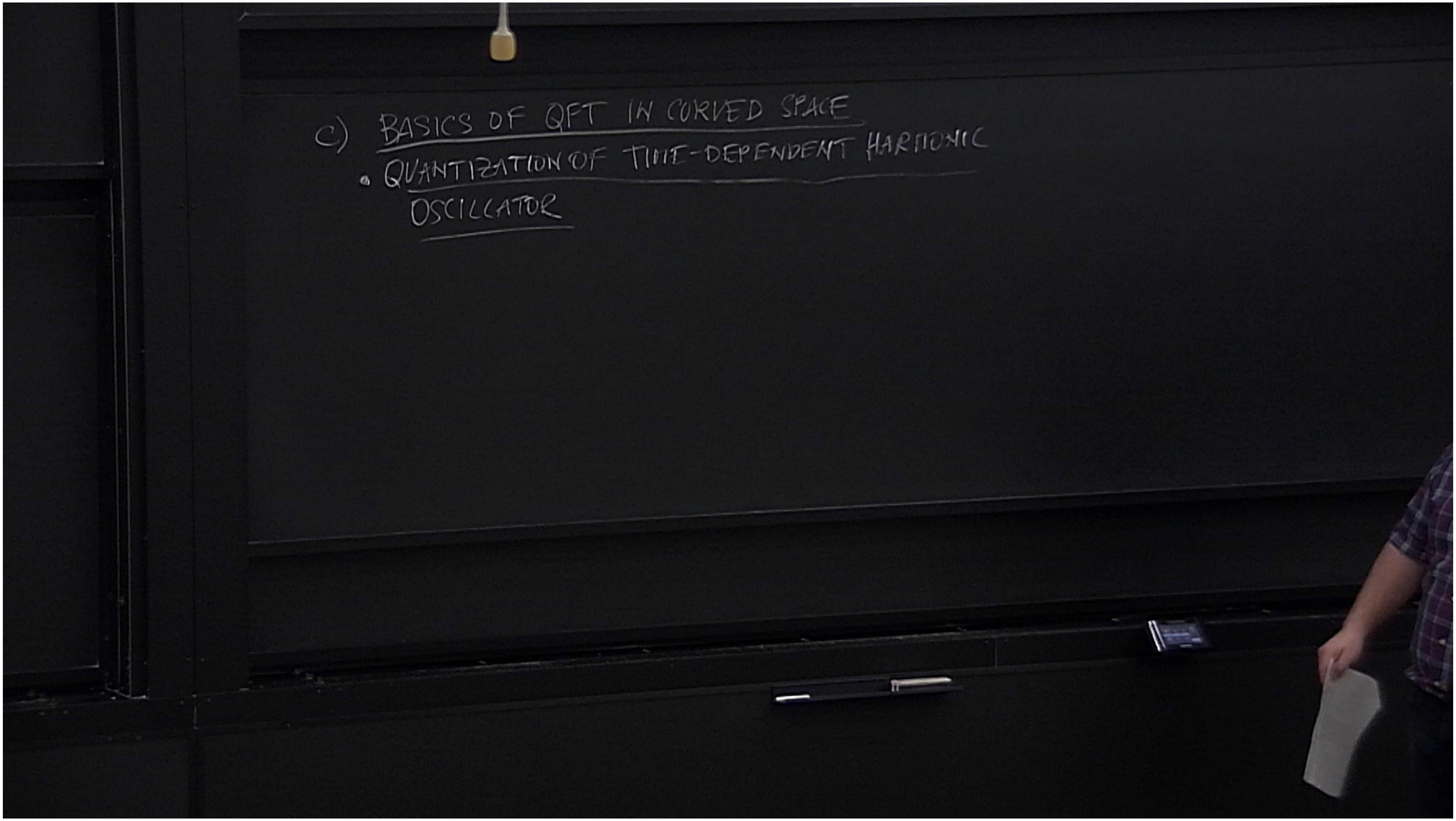
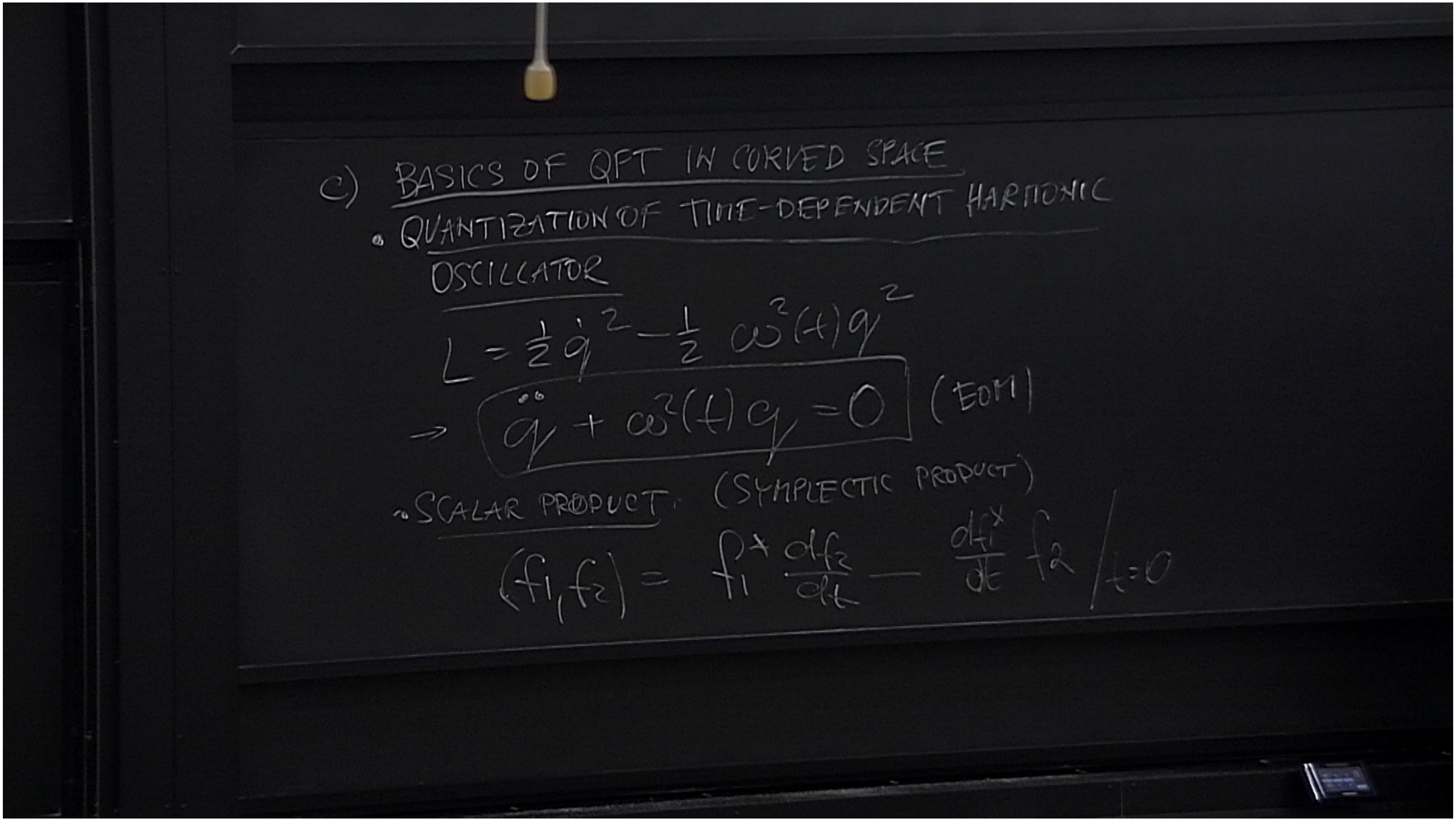


c) BASICS OF QFT IN CURVED SPACE

- QUANTIZATION OF TIME-DEPENDENT HARMONIC OSCILLATOR



c) BASICS OF QFT IN CURVED SPACE

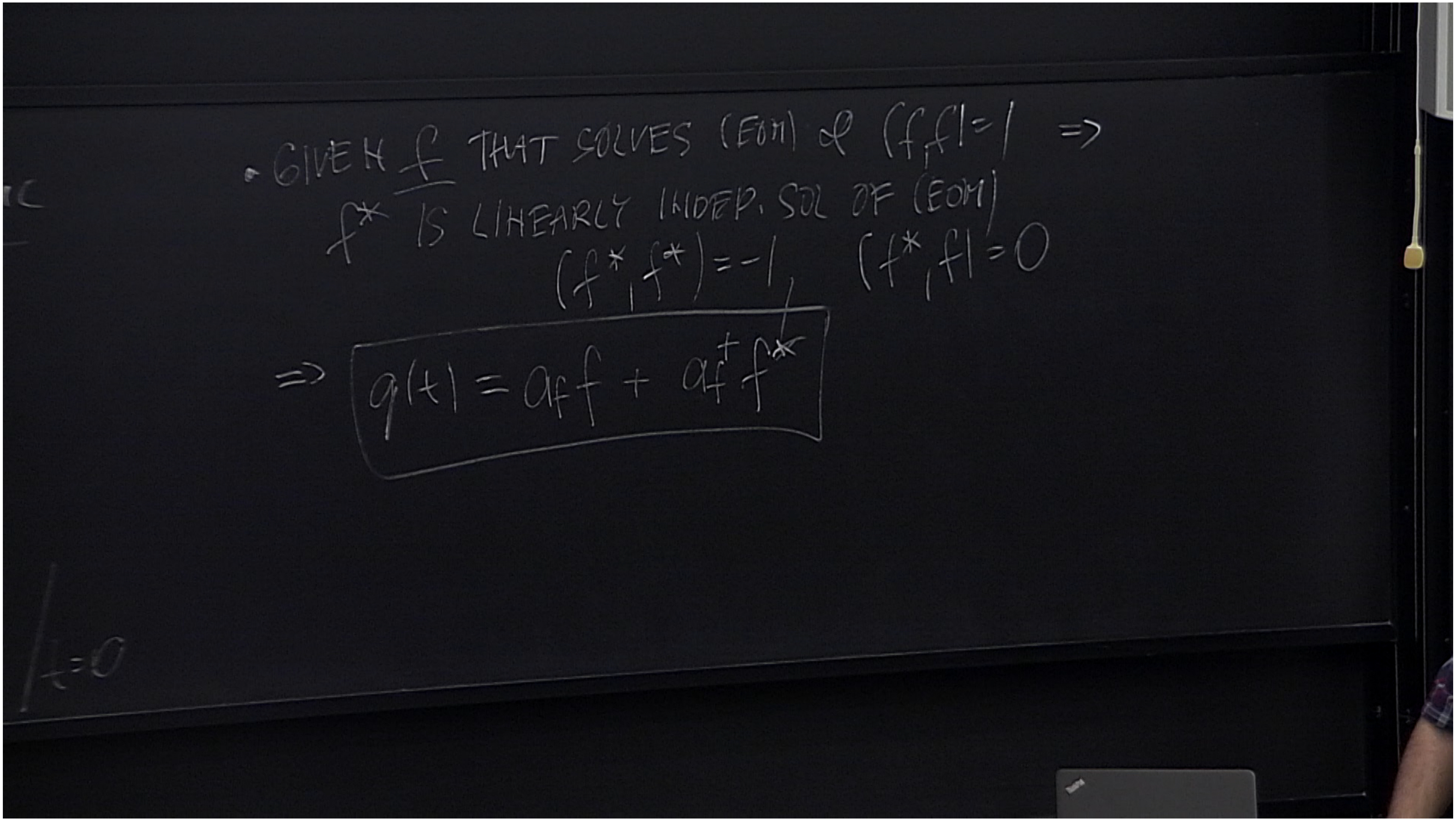
- QUANTIZATION OF TIME-DEPENDENT HARMONIC OSCILLATOR

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}\omega^2(t)q^2$$

$$\rightarrow \boxed{\ddot{q} + \omega^2(t)q = 0} \quad (\text{EOM})$$

- SCALAR PRODUCT. (SYMPLECTIC PRODUCT)

$$(f_1, f_2) = f_1^* \frac{df_2}{dt} - \frac{df_1^*}{dt} f_2 \Big|_{t=0}$$



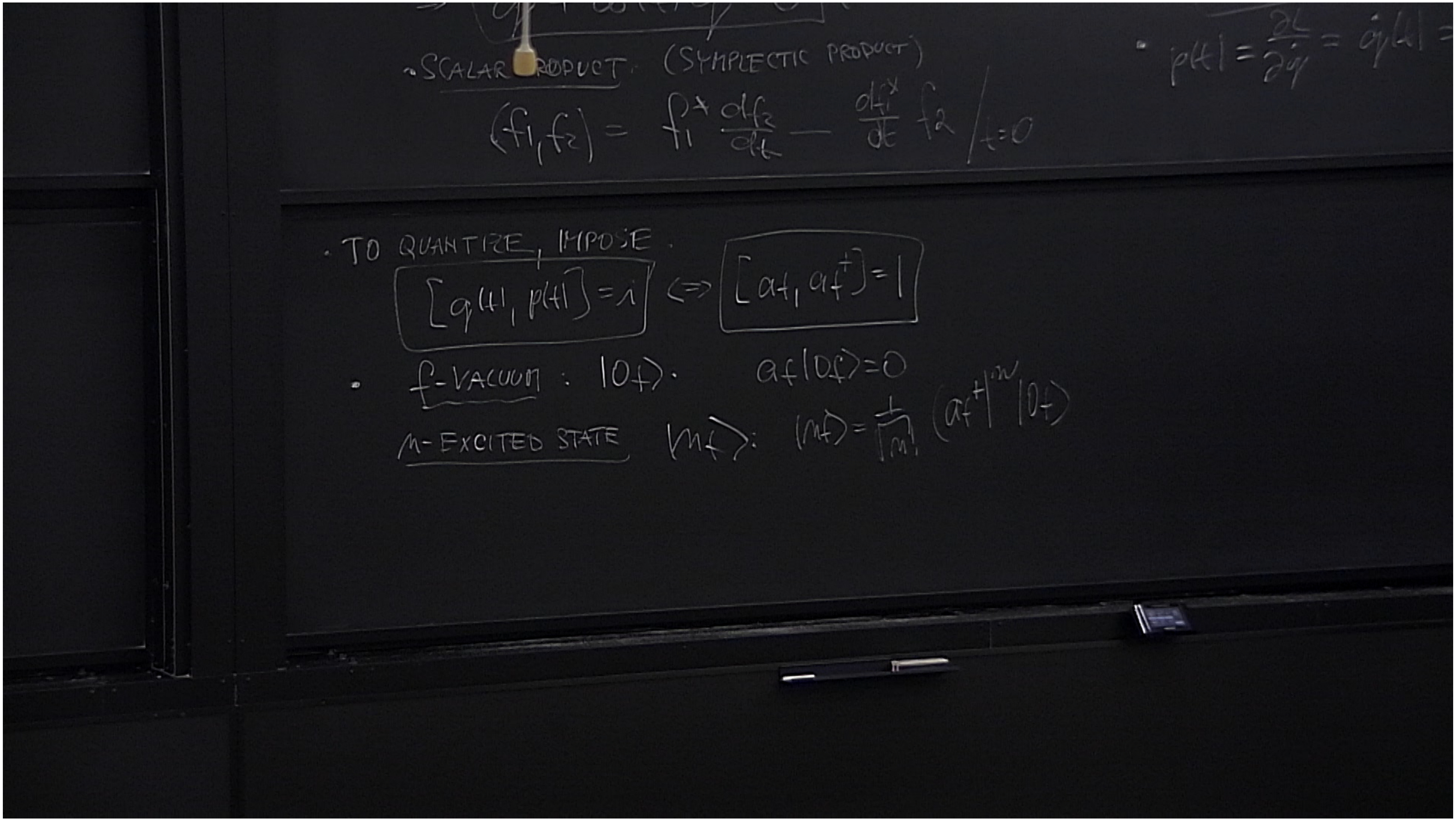
- GIVEN $\underline{f}$ THAT SOLVES (EOM) & $(f,f)=1$ $\Rightarrow$

$f^*$ IS LINEARLY INDEP. SOL OF (EOM)

$$(f^*,f^*)=-1, \quad (f^*,f)=0$$

$$\Rightarrow \boxed{q(t) = a_f f + a_f^\dagger f^*}$$

$t=0$



- SCALAR PRODUCT. (SYMPLECTIC PRODUCT)

$$(f_1, f_2) = f_1^* \frac{df_2}{dt} - \frac{df_1^*}{dt} f_2 \Big|_{t=0}$$

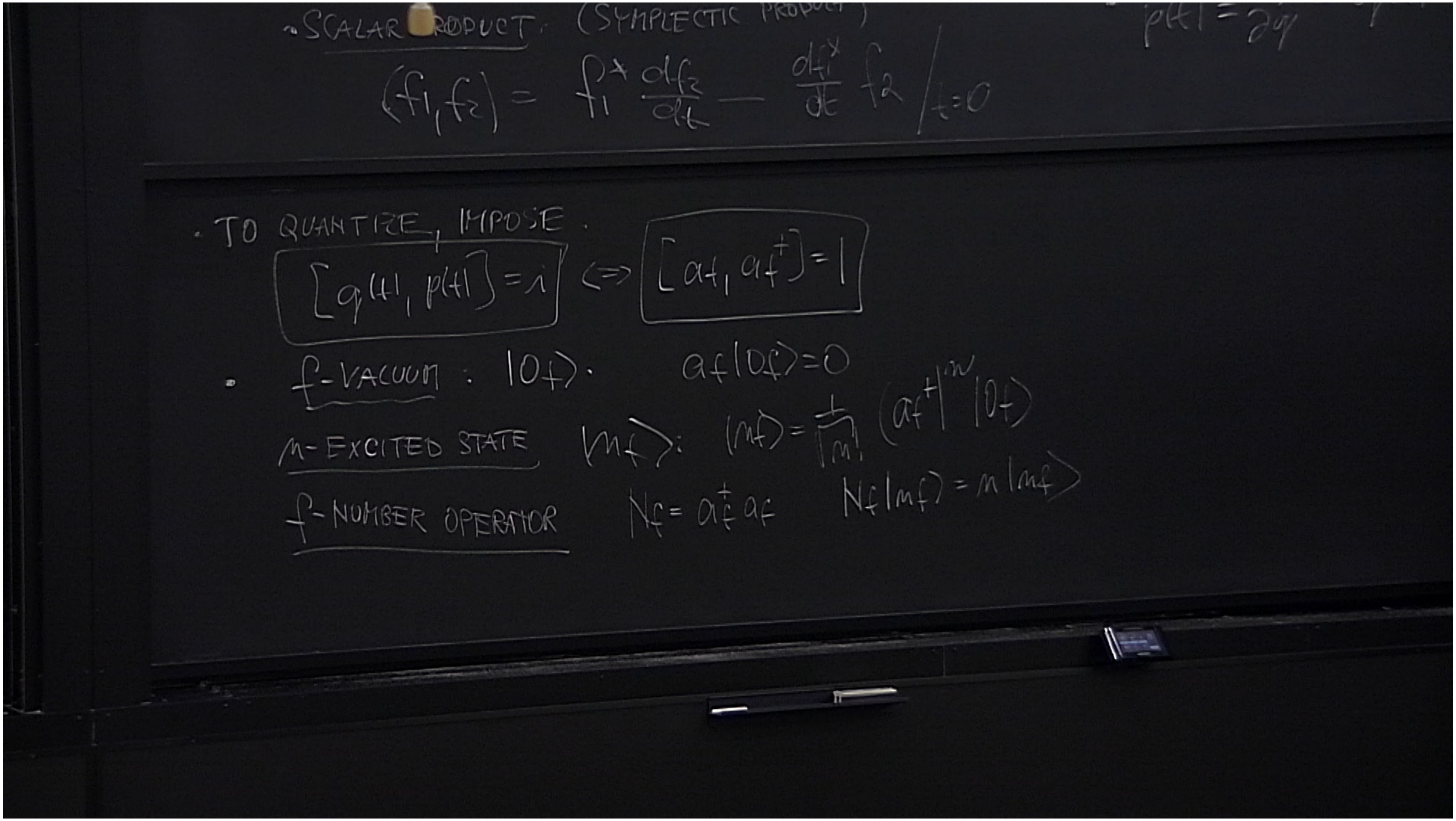
- $p(t) = \frac{\partial L}{\partial \dot{q}} = \dot{q}(t) =$

- TO QUANTIZE, IMPOSE.

$$\boxed{[q(t), p(t)] = i} \Leftrightarrow \boxed{[a_f, a_f^\dagger] = 1}$$

- f-VACUUM : $|0_f\rangle$. $\quad a_f|0_f\rangle = 0$

n-EXCITED STATE $|n_f\rangle$: $|n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^\dagger)^n |0_f\rangle$

<br>

- SCALAR RODUCT: (SYMPLECTIC PRODUCT)

$$(f_1, f_2) = f_1^* \frac{df_2}{dt} - \frac{df_1^*}{dt} f_2 \Big|_{t=0}$$

$p(t) = \partial q$

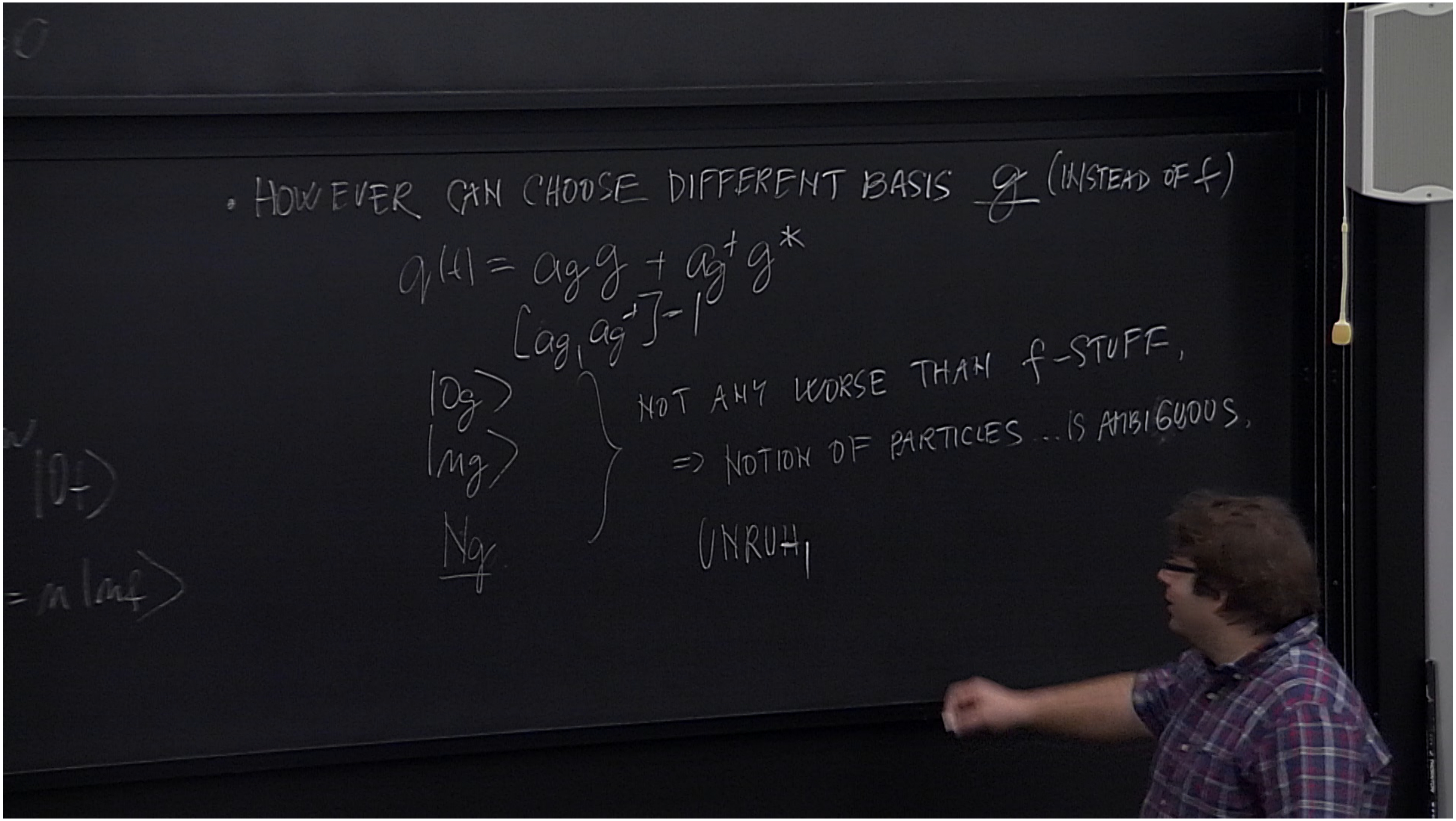
- TO QUANTIZE, IMPOSE.

$$\boxed{[q(t), p(t)] = i} \Leftrightarrow \boxed{[a_f, a_f^+] = 1}$$

- f-VACUUM : $|0_f\rangle$. $\quad a_f|0_f\rangle = 0$

n-EXCITED STATE $|n_f\rangle$: $\quad |n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^+)^n |0_f\rangle$

f-NUMBER OPERATOR $\quad N_f = a_f^+ a_f \quad N_f|n_f\rangle = n|n_f\rangle$



• HOWEVER CAN CHOOSE DIFFERENT BASIS $\underline{g}$ (INSTEAD OF f)

$$q(t) = a_g g + a_g^\dagger g^*$$

$$[a_g, a_g^\dagger] = 1$$

$$\left.\begin{array}{l} |0_g\rangle \\ |m_g\rangle \\ \underline{N_g} \end{array}\right\}$$

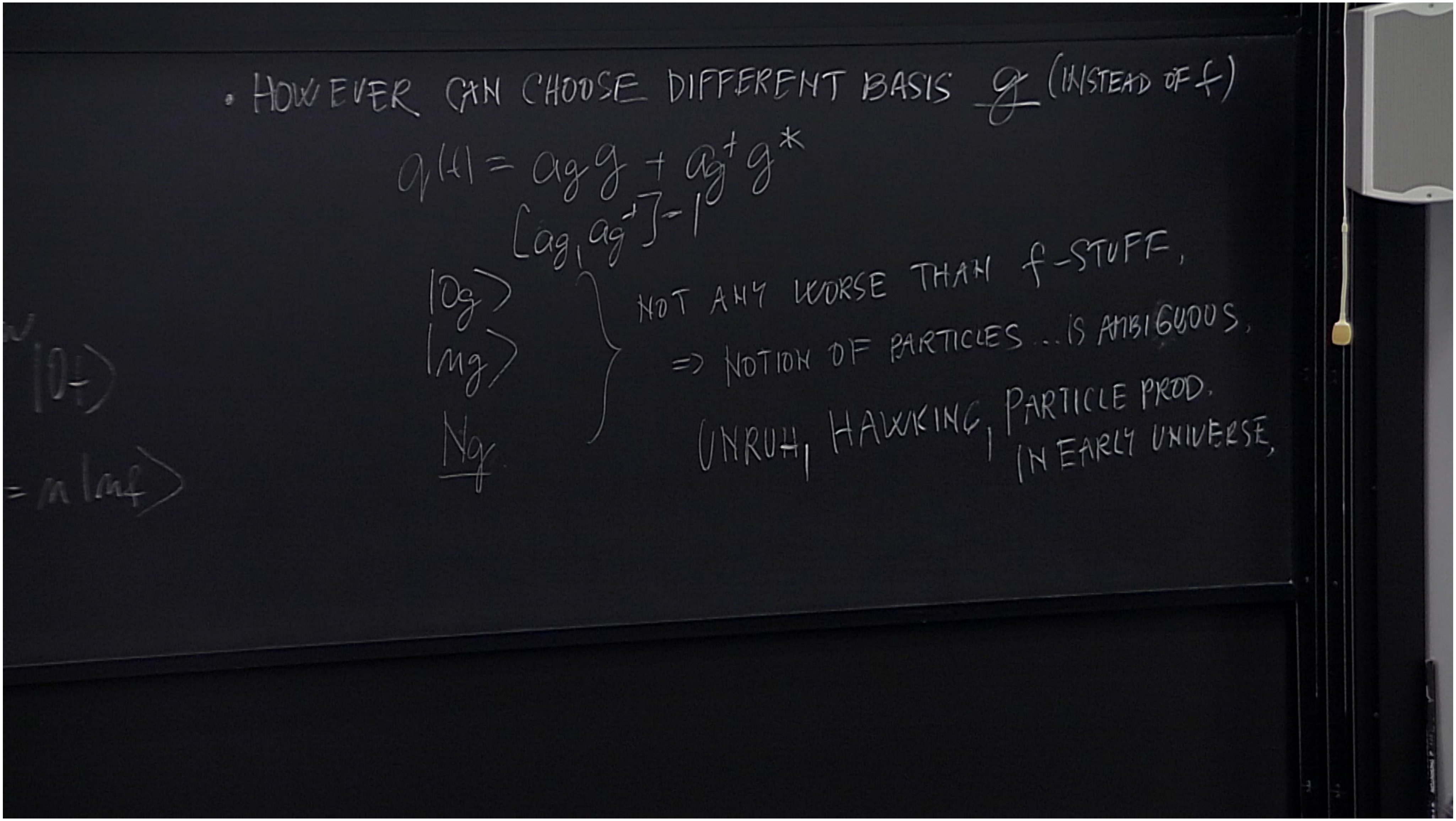
NOT ANY WORSE THAN f-STUFF,

⇒ NOTION OF PARTICLES ... IS AMBIGUOUS.

UNRUH,

$|0_f\rangle$

$= m|m_f\rangle$



• HOWEVER CAN CHOOSE DIFFERENT BASIS $\underline{g}$ (INSTEAD OF f)

$$q(t) = a_g g + a_g^\dagger g^*$$

$$[a_g, a_g^\dagger] = 1$$

$$\left.\begin{array}{l} |0_g\rangle \\ |n_g\rangle \\ \underline{N_g} \end{array}\right\}$$

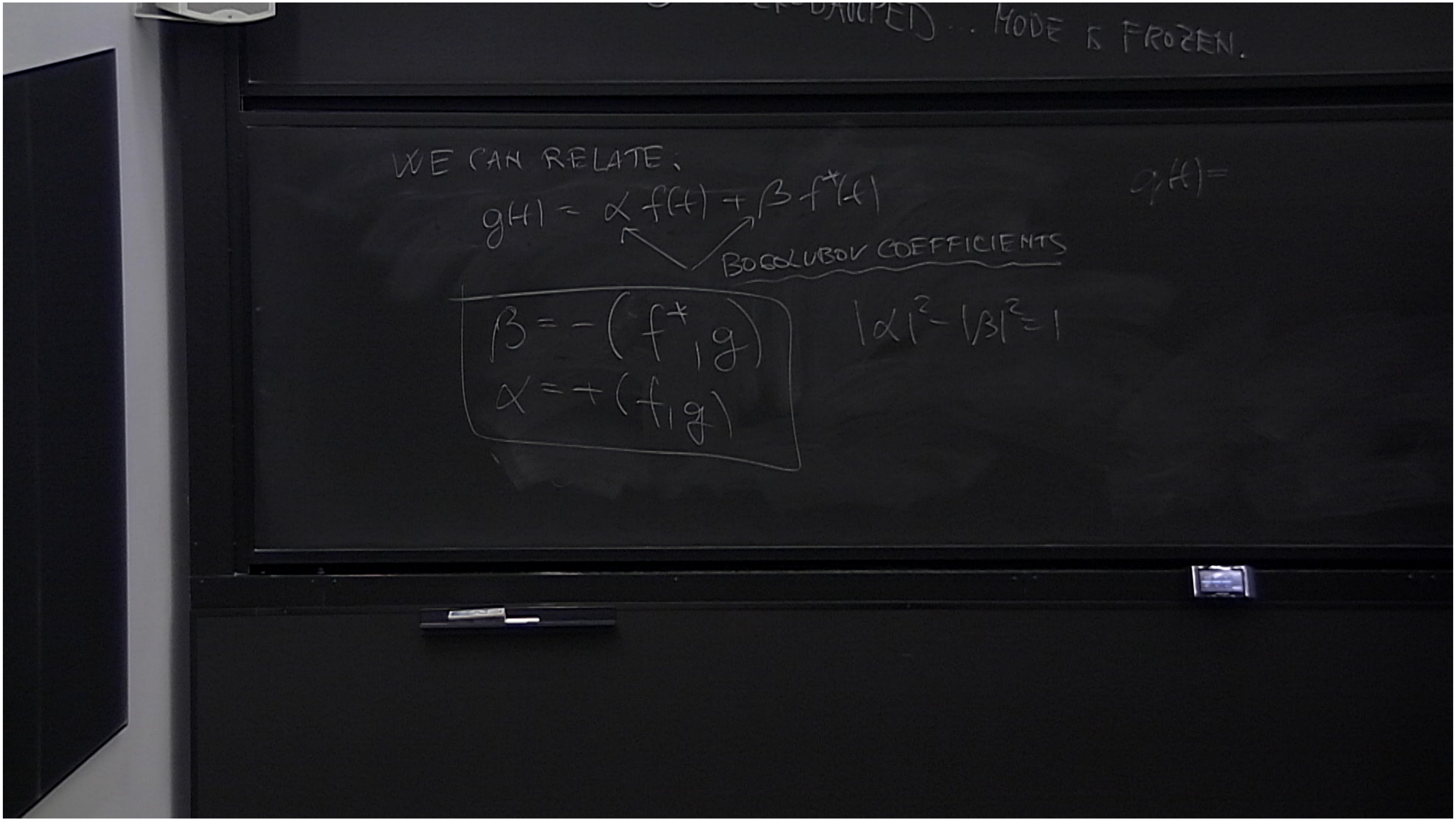
NOT ANY WORSE THAN f-STUFF.

⇒ NOTION OF PARTICLES ... IS AMBIGUOUS.

UNRUH, HAWKING, PARTICLE PROD. IN EARLY UNIVERSE.

$|0_f\rangle$

$= n|n_f\rangle$



DAMPED) ... MODE IS FROZEN.

WE CAN RELATE:

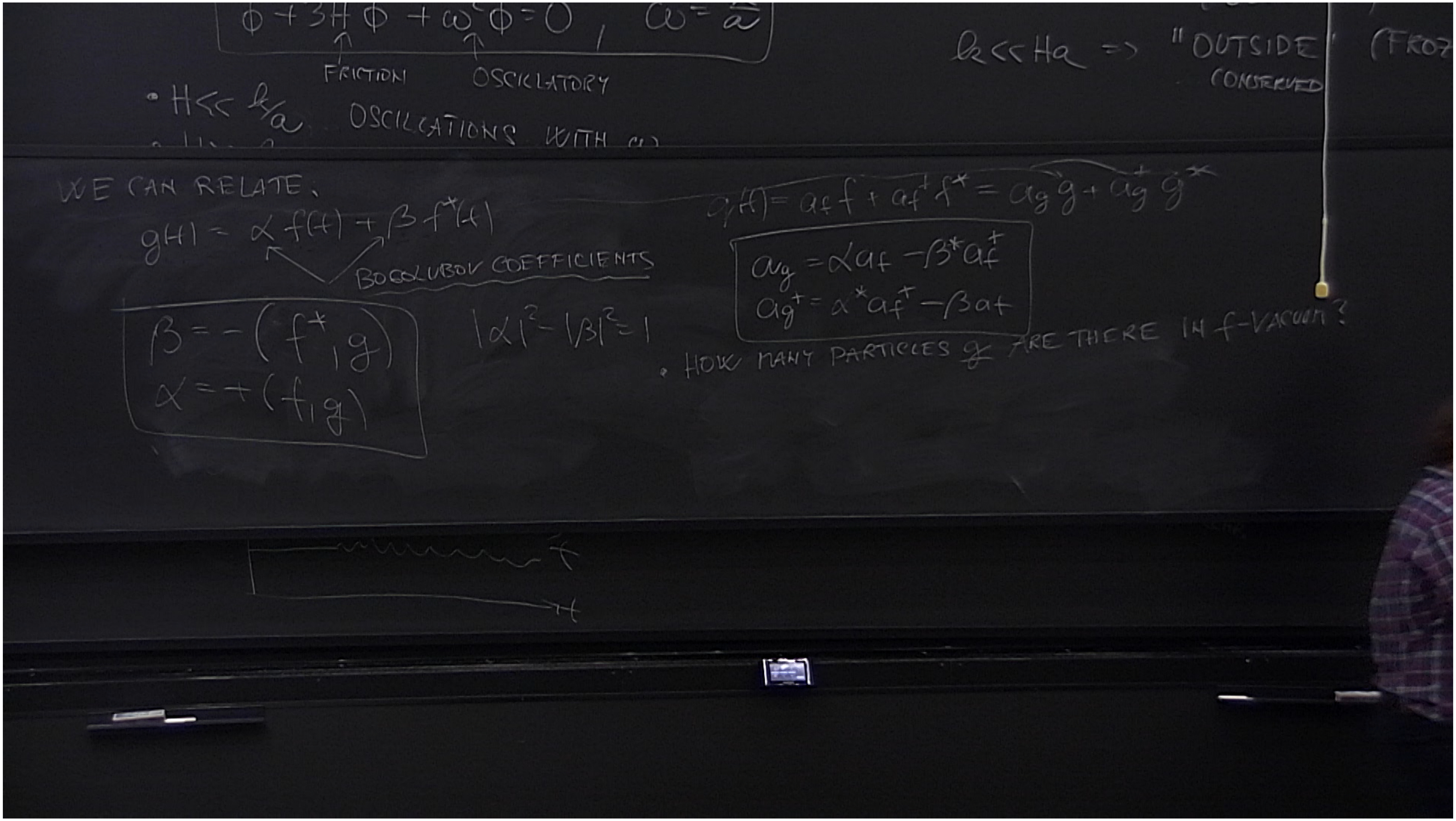
$$g(t) = \alpha f(t) + \beta f^*(t)$$

BOGOLUBOV COEFFICIENTS

$$\boxed{\begin{aligned}\beta &= -(f^*, g)\\ \alpha &= +(f, g)\end{aligned}}$$

$$|\alpha|^2 - |\beta|^2 = 1$$

$g(t) =$



$$\boxed{\phi + 3H\phi + \omega^2\phi = 0 \; , \quad \omega = \frac{}{a}}$$

FRICTION OSCILLATORY

- $H << k/a$ OSCILLATIONS WITH $\omega$

$k << Ha \Rightarrow$ "OUTSIDE" (FRO

CONSERVED

WE CAN RELATE.

$$g(t) = \alpha f(t) + \beta f^*(t)$$

BOGOLUBOV COEFFICIENTS

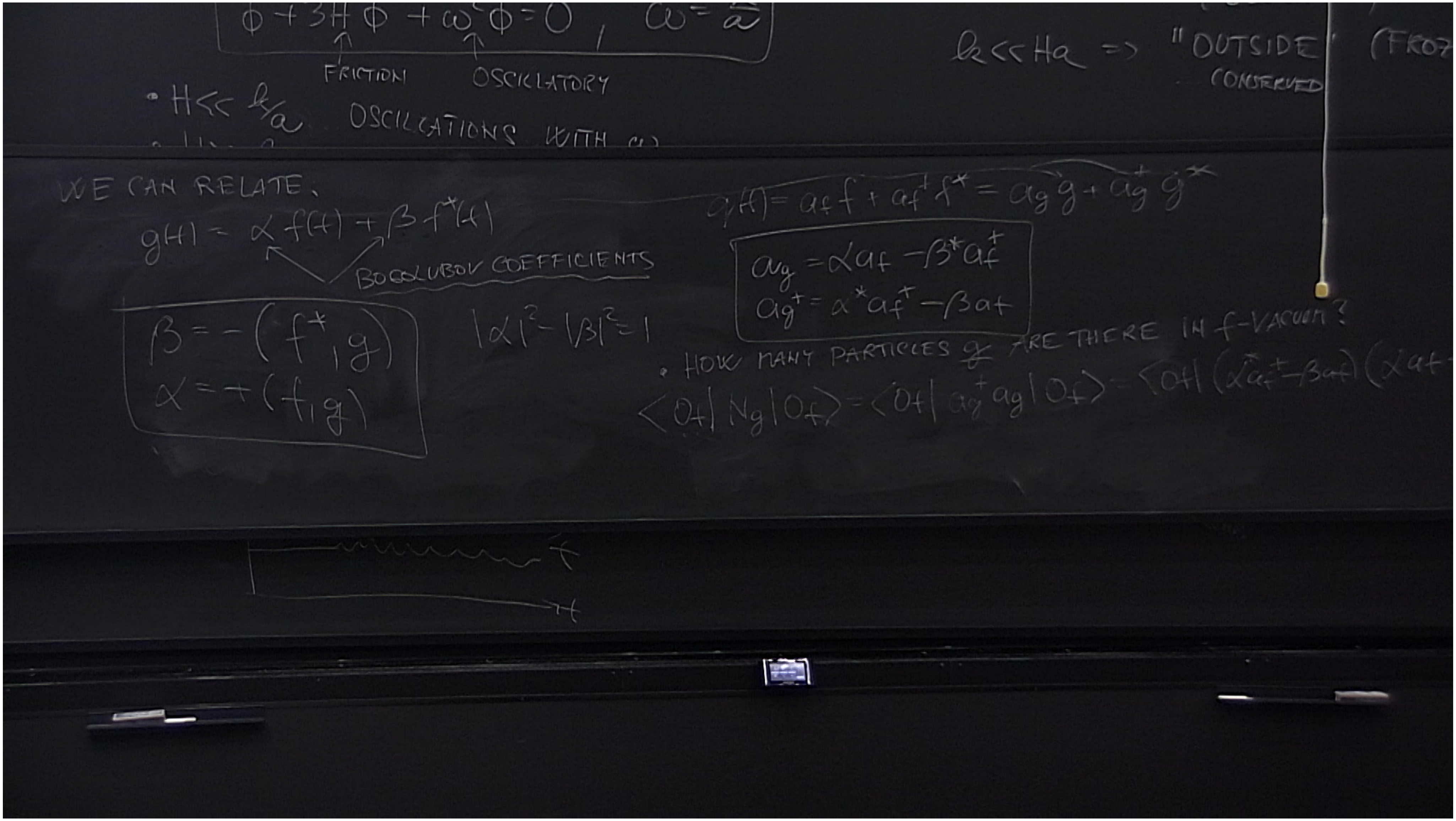
$$\boxed{\begin{aligned} \beta &= -(f^*, g) \\ \alpha &= +(f, g) \end{aligned}}$$

$$|\alpha|^2 - |\beta|^2 = 1$$

$$\phi(t) = a_f f + a_f^+ f^* = a_g g + a_g^+ g^*$$

$$\boxed{\begin{aligned} a_g &= \alpha a_f - \beta^* a_f^+ \\ a_g^+ &= \alpha^* a_f^+ - \beta a_f \end{aligned}}$$

- HOW MANY PARTICLES $g$ ARE THERE IN f-VACUUM?



$$\boxed{\phi + 3H\phi + \omega^2\phi = 0 \quad , \quad \omega = \frac{}{a}}$$

FRICTION OSCILLATORY

- $H \ll k/a$ OSCILLATIONS WITH $\omega$

$k \ll Ha \Rightarrow$ "OUTSIDE" (FRO

CONSERVED

WE CAN RELATE:

$$g(t) = \alpha f(t) + \beta f^*(t)$$

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$$|\alpha|^2 - |\beta|^2 = 1$$

$$q(t) = a_f f + a_f^\dagger f^* = a_g g + a_g^\dagger g^*$$

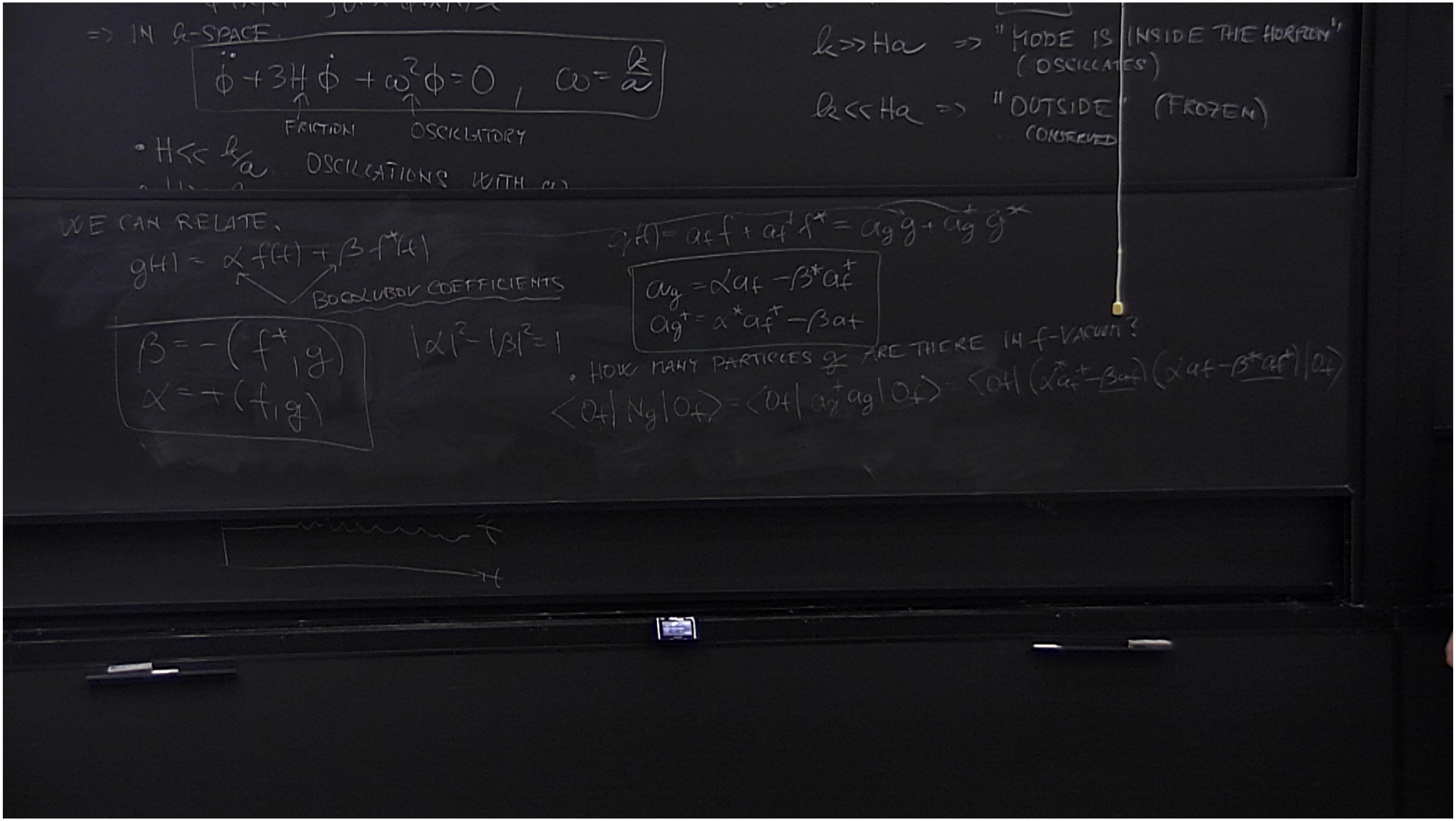
$$\boxed{\begin{aligned} a_g &= \alpha a_f - \beta^* a_f^\dagger \\ a_g^\dagger &= \alpha^* a_f^\dagger - \beta a_f \end{aligned}}$$

- HOW MANY PARTICLES $g$ ARE THERE IN $f$-VACUUM?

$$\langle 0_f | N_g | 0_f \rangle = \langle 0_f | a_g^\dagger a_g | 0_f \rangle = \langle 0_f | (\alpha^* a_f^\dagger - \beta a_f)(\alpha a_f$$

$t$

$t$



=> IN k-SPACE.

$$\boxed{\ddot{\phi}+3\underset{\uparrow}{H}\dot{\phi}+\underset{\uparrow}{\omega^2}\phi=0\ ,\quad \omega=\frac{k}{a}}$$

FRICTION OSCILATORY

- $H \ll k/a$ OSCILLATIONS WITH $\omega$

$k \gg Ha \Rightarrow$ "MODE IS INSIDE THE HORIZON" (OSCILLATES)

$k \ll Ha \Rightarrow$ "OUTSIDE" (FROZEN) (CONSERVED

WE CAN RELATE.

$$g(t)=\alpha f(t)+\beta f^*(t)$$

BOGOLUBOV COEFFICIENTS

$$\boxed{\begin{aligned}\beta&=-(f^*,g)\\ \alpha&=+(f,g)\end{aligned}}$$

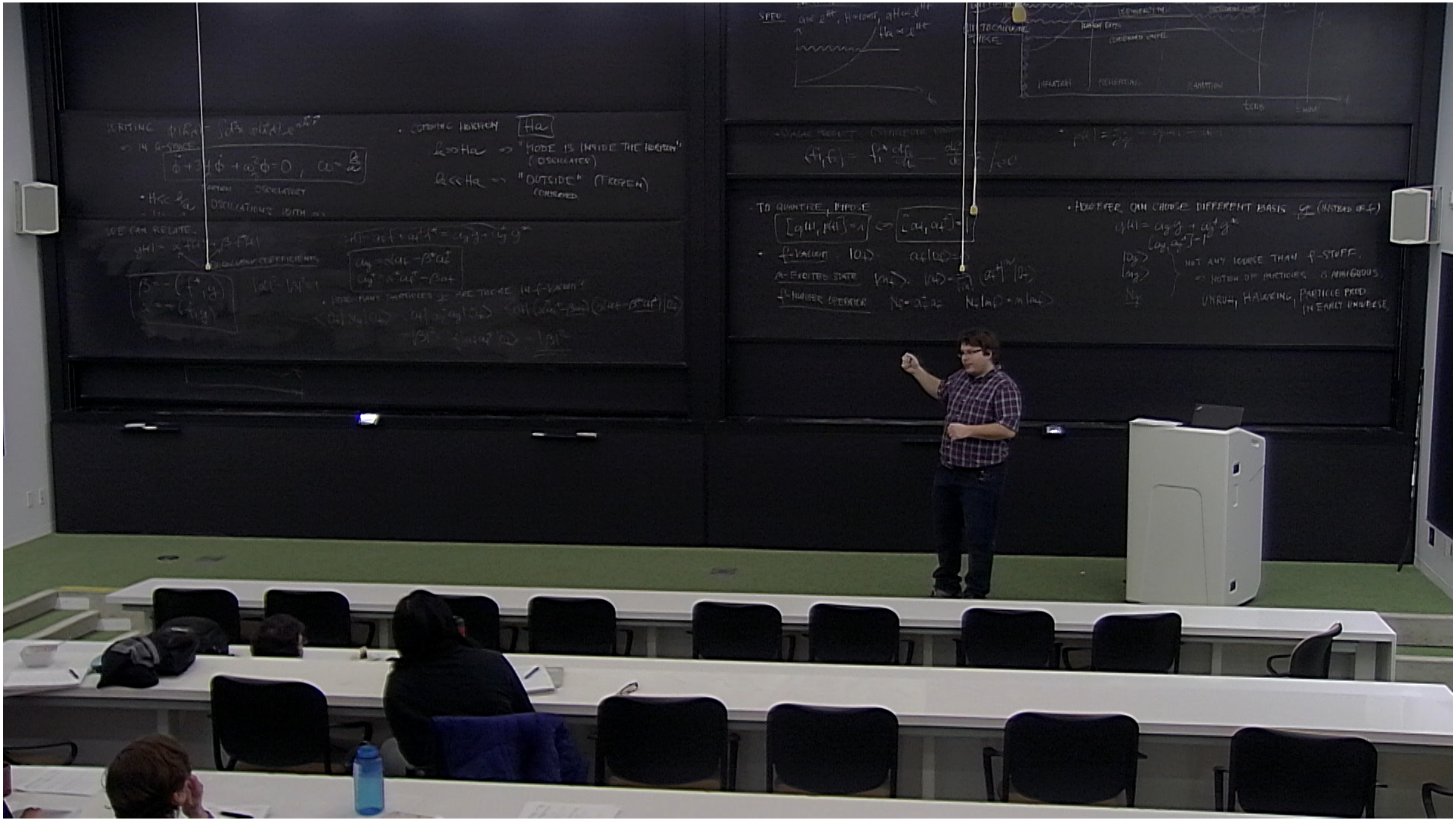
$$|\alpha|^2-|\beta|^2=1$$

$$\phi(t)=a_f f+a_f^\dagger f^*=a_g g+a_g^\dagger g^*$$

$$\boxed{\begin{aligned}a_g&=\alpha a_f-\beta^* a_f^\dagger\\ a_g^\dagger&=\alpha^* a_f^\dagger-\beta a_f\end{aligned}}$$

- HOW MANY PARTICLES g ARE THERE IN f-VACUUM?

$$\langle 0_f|N_g|0_f\rangle=\langle 0_f|a_g^\dagger a_g|0_f\rangle=\langle 0_f|(\alpha^* a_f^\dagger-\underline{\beta a_f})(\alpha a_f-\underline{\beta^* a_f^\dagger})|0_f\rangle$$


COMOVING HORIZON
"MODE IS INSIDE THE HORIZON"
(OSCILLATES)
"OUTSIDE" (FROZEN)
WE CAN RELATE
BOGOLIUBOV COEFFICIENTS
TO QUANTIZE, IMPOSE
f-VACUUM
n-EXCITED STATE
f-NUMBER OPERATOR
HOWEVER CAN CHOOSE DIFFERENT BASIS (INSTEAD OF f)
NOT ANY WORSE THAN f-STUFF.
NOTION OF PARTICLES IS AMBIGUOUS.
UNRUH, HAWKING, PARTICLE PROD. IN EARLY UNIVERSE
INFLATION
REHEATING
RADIATION