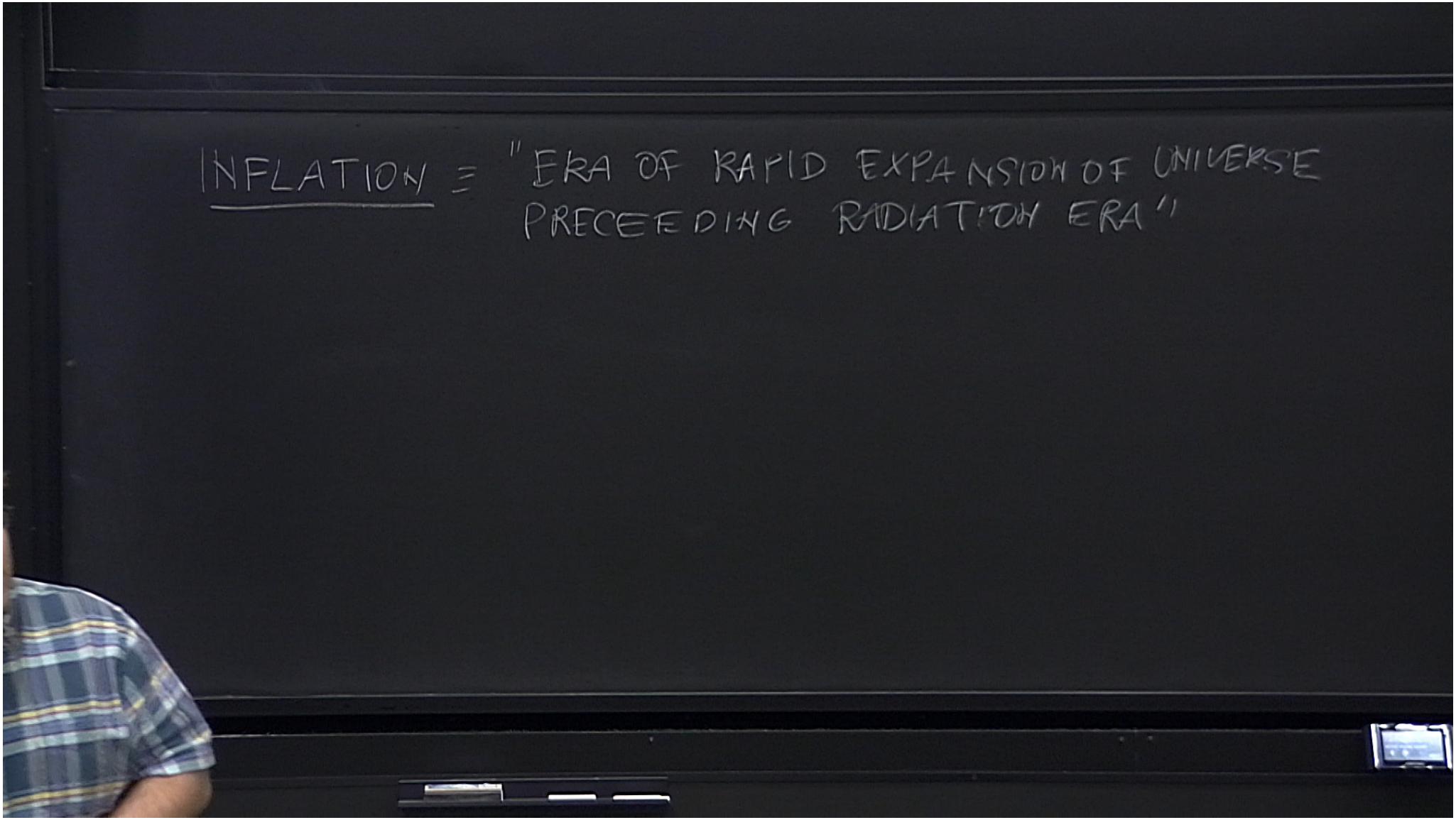
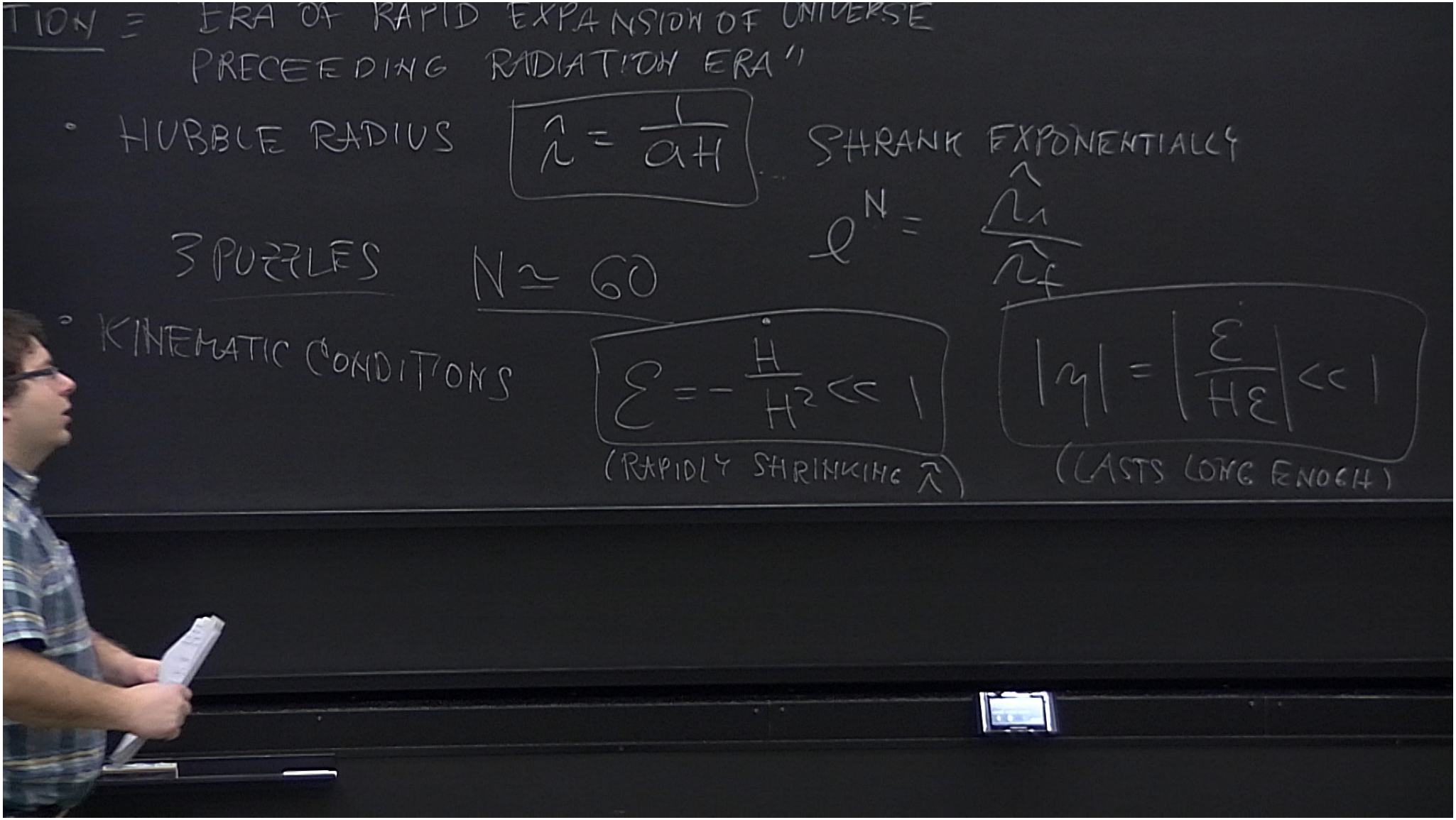
Title: PSI 2016/2017 Cosmology (Review) - Lecture 10

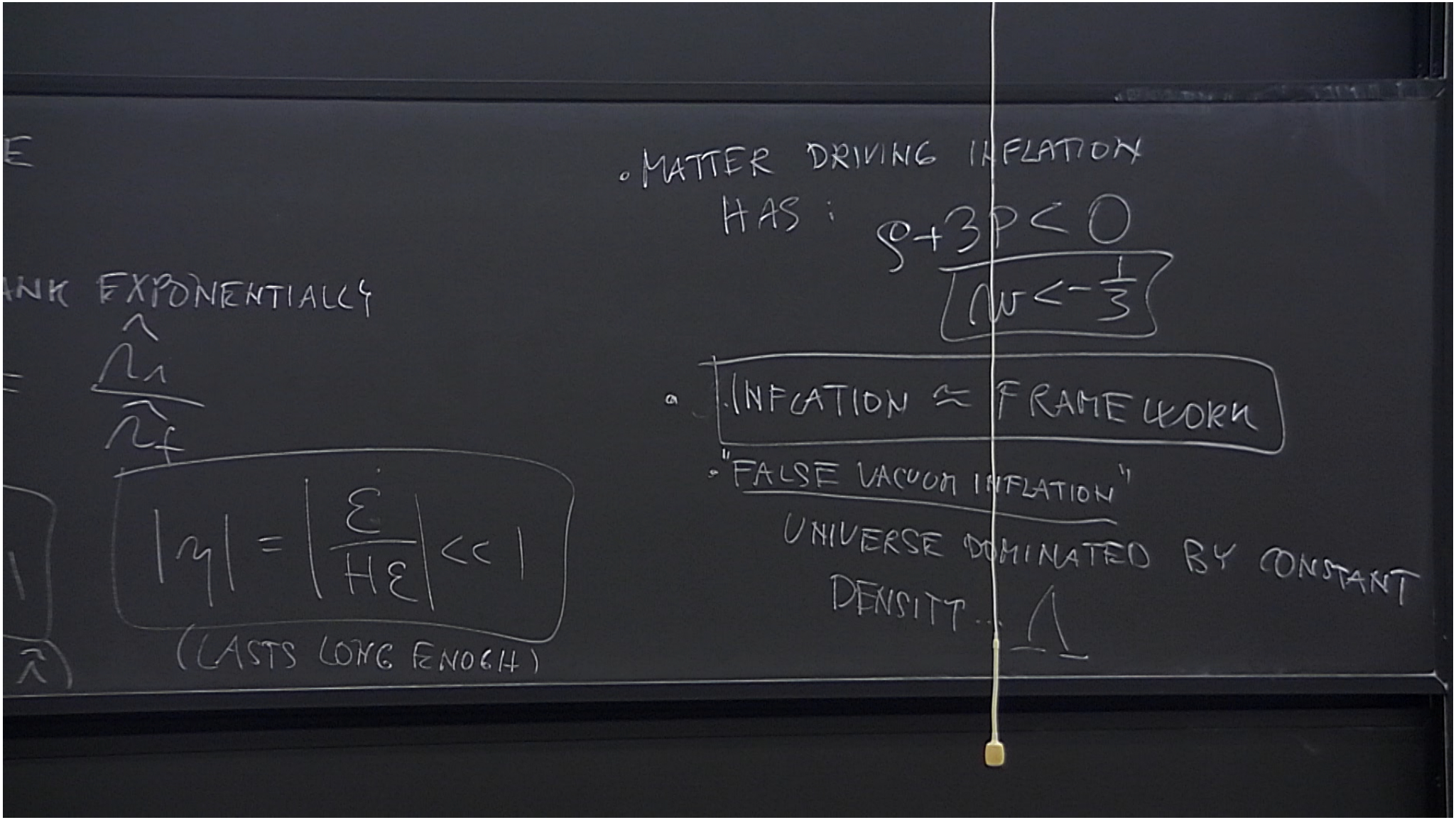
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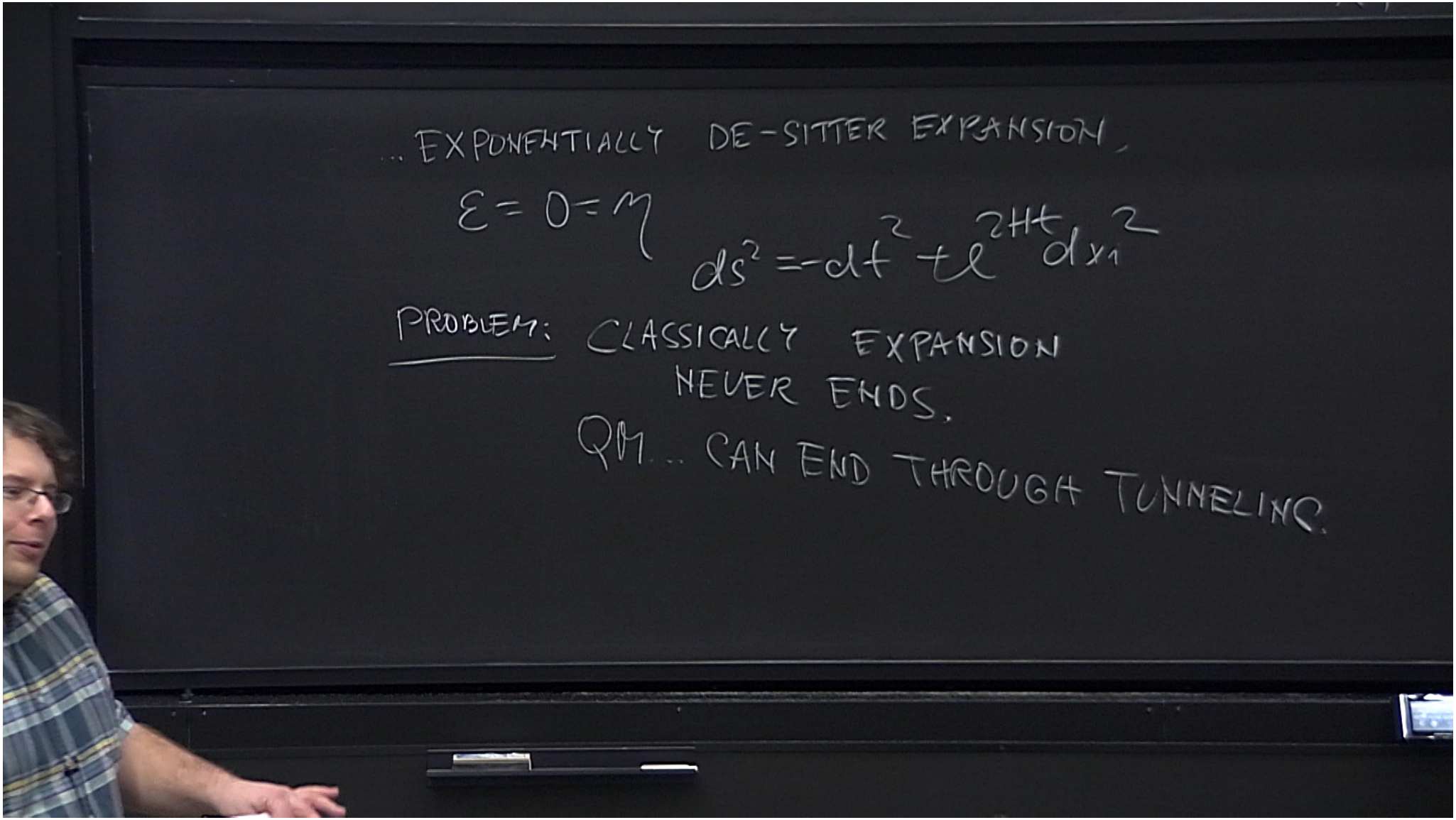
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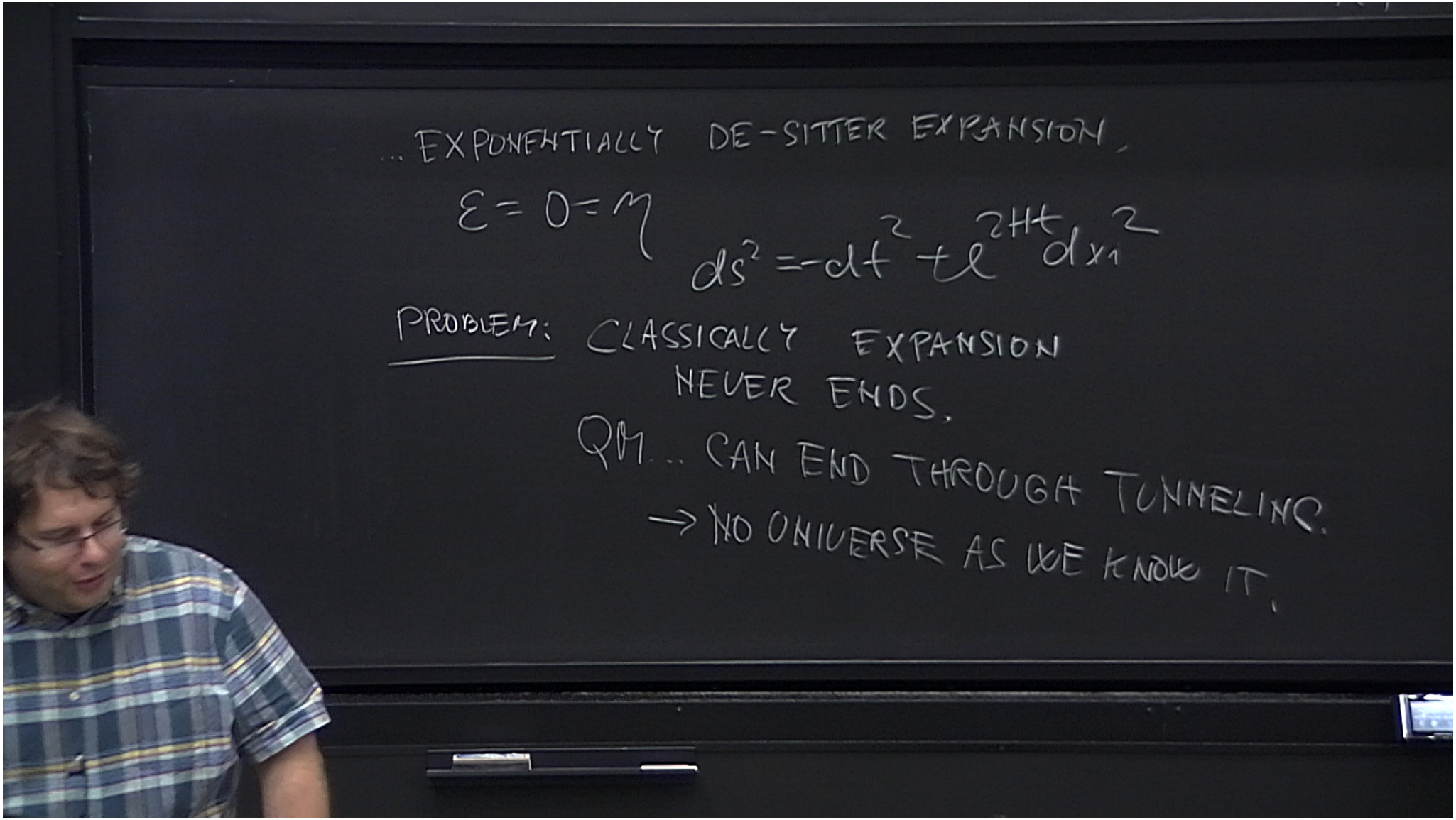
Abstract:

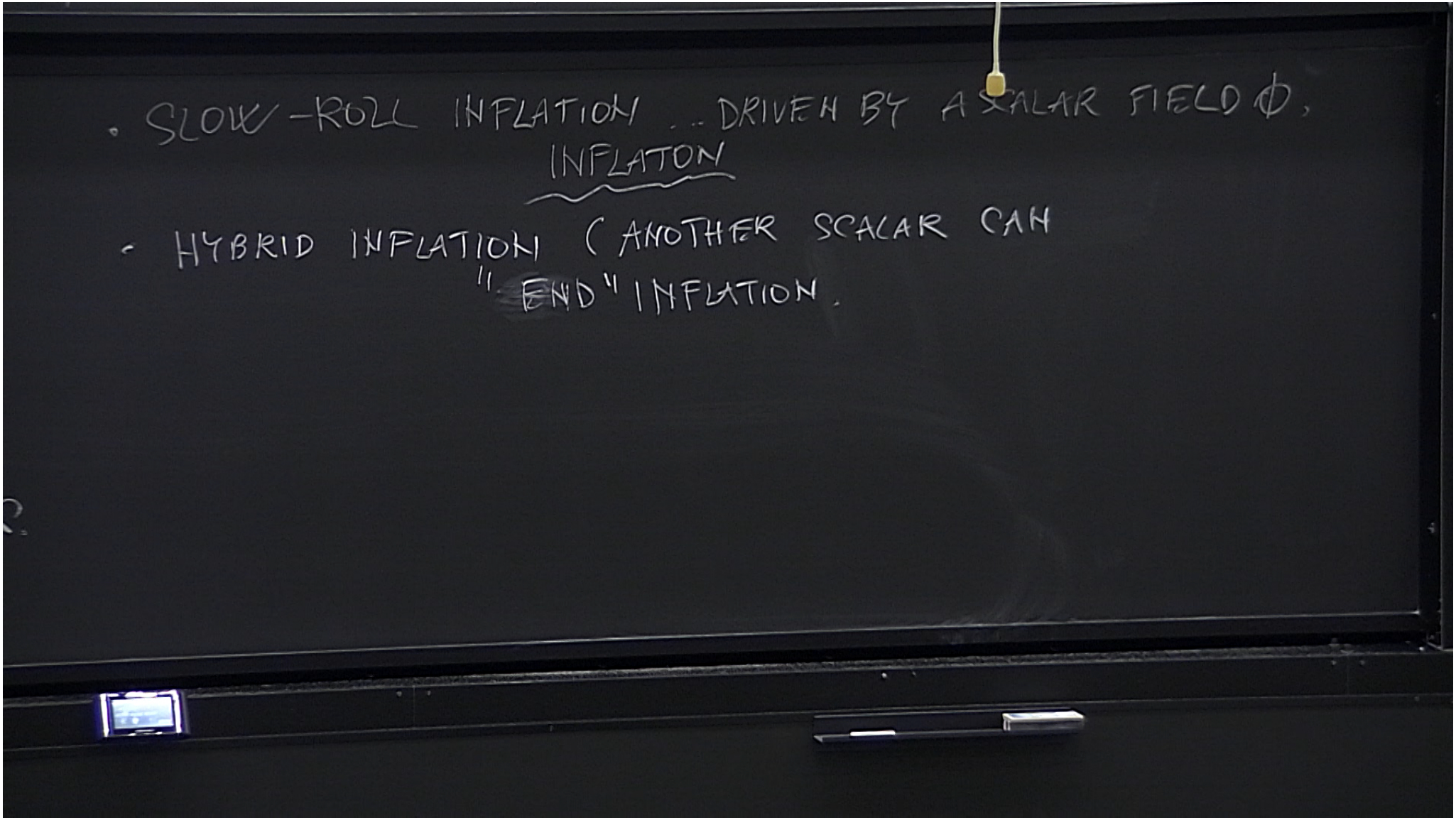

INFLATION = "ERA OF RAPID EXPANSION OF UNIVERSE
PRECEEDING RADIATION ERA"

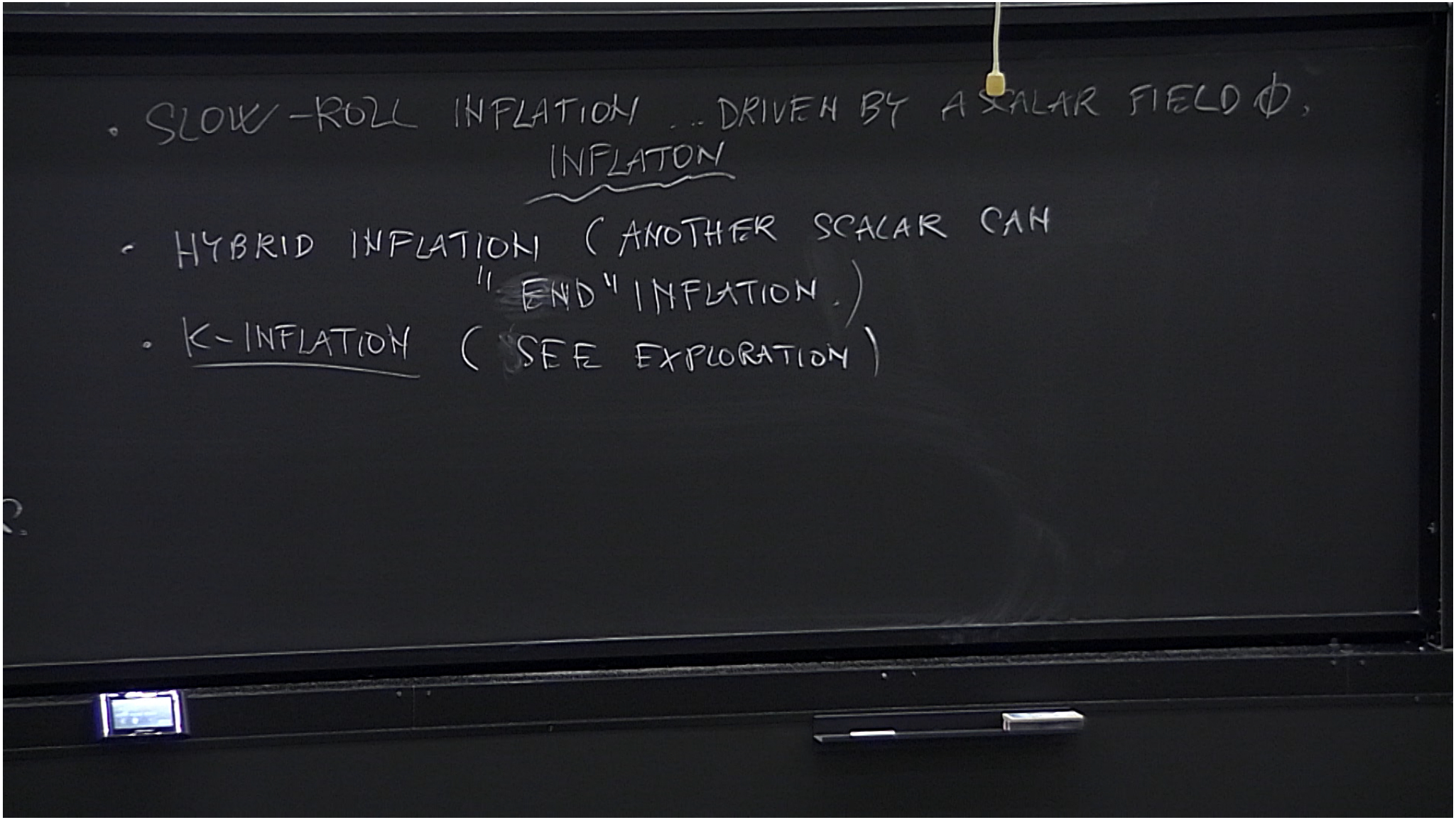

TION = ERA OF RAPID EXPANSION OF UNIVERSE
PRECEEDING RADIATION ERA"
• HUBBLE RADIUS $\hat{\lambda} = \frac{1}{aH}$ SHRANK EXPONENTIALLY
$e^N = \frac{\hat{\lambda}_i}{\hat{\lambda}_f}$
3 PUZZLES
$N \simeq 60$
• KINEMATIC CONDITIONS
$\varepsilon = -\frac{\dot{H}}{H^2} \ll 1$
(RAPIDLY SHRINKING $\hat{\lambda}$)
$|\eta| = \left|\frac{\varepsilon}{H\varepsilon}\right| \ll 1$
(LASTS LONG ENOUGH)

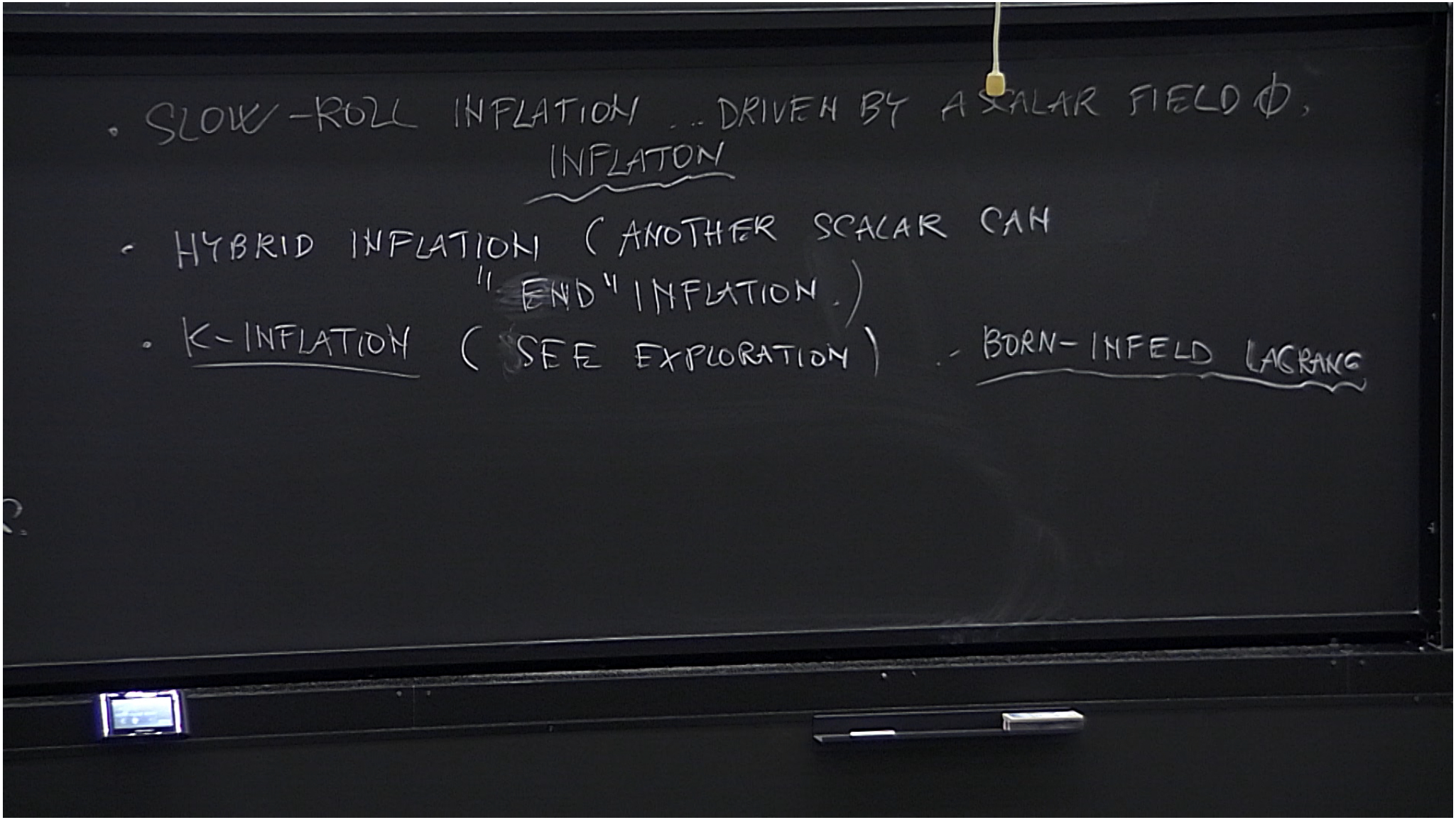

E
ANK EXPONENTIALLY
$\frac{\Lambda_i}{\Lambda_f}$
$|\eta| = \left|\frac{\dot{\varepsilon}}{H\varepsilon}\right| \ll 1$
(LASTS LONG ENOUGH)
• MATTER DRIVING INFLATION HAS: $\rho + 3P < 0$
$w < -\frac{1}{3}$
• INFLATION ≃ FRAMEWORK
• "FALSE VACUUM INFLATION"
UNIVERSE DOMINATED BY CONSTANT DENSITY
Λ


.. EXPONENTIALLY DE-SITTER EXPANSION
$\mathcal{E} = 0 = \mathcal{M}$
$ds^2 = -dt^2 + e^{2Ht} dx_1^2$
PROBLEM: CLASSICALLY EXPANSION NEVER ENDS.
QM .. CAN END THROUGH TUNNELING


.. EXPONENTIALLY DE-SITTER EXPANSION.
$\mathcal{E} = 0 = \mathcal{M}$
$ds^2 = -dt^2 + e^{2Ht} dx_1^2$
PROBLEM: CLASSICALLY EXPANSION NEVER ENDS.
QM .. CAN END THROUGH TUNNELING.
→ NO UNIVERSE AS WE KNOW IT.


• SLOW-ROLL INFLATION ... DRIVEN BY A SCALAR FIELD ϕ,
INFLATON
- HYBRID INFLATION (ANOTHER SCALAR CAN
"END" INFLATION.

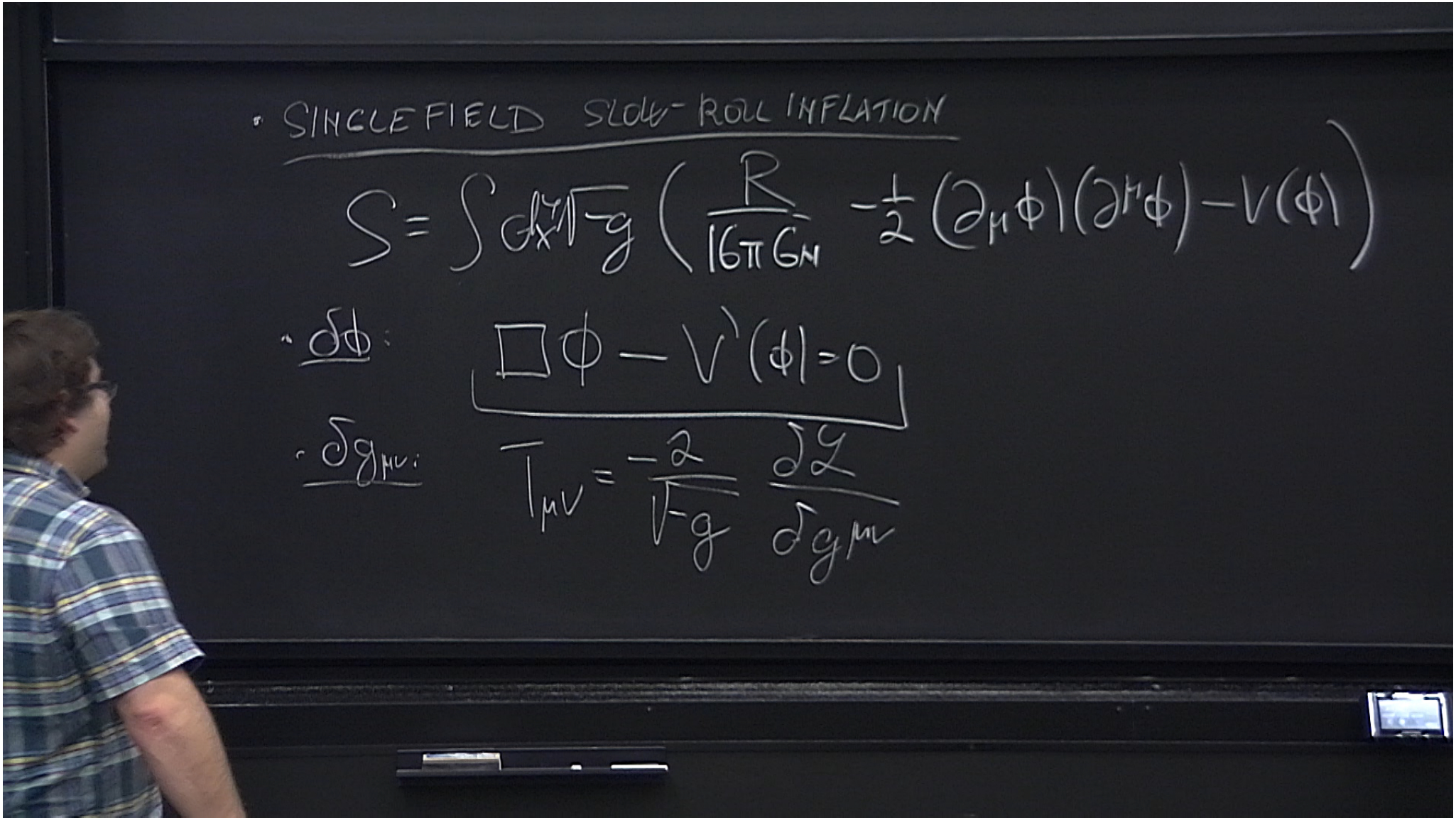

• SLOW-ROLL INFLATION ... DRIVEN BY A SCALAR FIELD φ,
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• HYBRID INFLATION (ANOTHER SCALAR CAN "END" INFLATION.)
• K-INFLATION (SEE EXPLORATION)


• SLOW-ROLL INFLATION ... DRIVEN BY A SCALAR FIELD ϕ,
INFLATON
• HYBRID INFLATION (ANOTHER SCALAR CAN "END" INFLATION.)
• K-INFLATION (SEE EXPLORATION) - BORN-INFELD LAGRANG

- SINGLE FIELD SLOW-ROLL INFLATION

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \right)$$

- $\delta\phi$: $\boxed{\Box\phi - V'(\phi) = 0}$

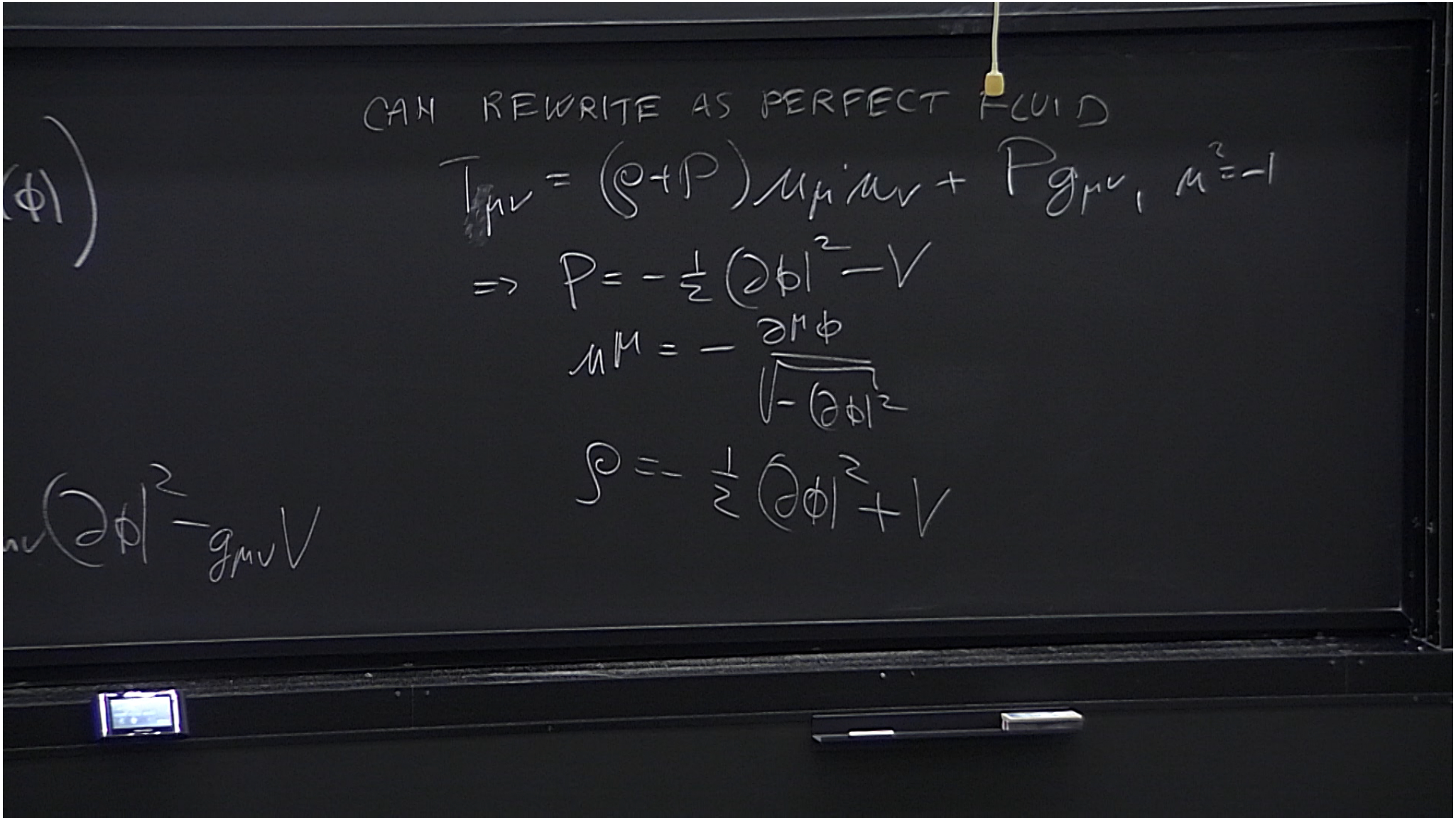

• SINGLE FIELD SLOW-ROLL INFLATION
$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \right)$
• $\delta\phi$: $\Box\phi - V'(\phi) = 0$
• $\delta g_{\mu\nu}$: $T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}}$

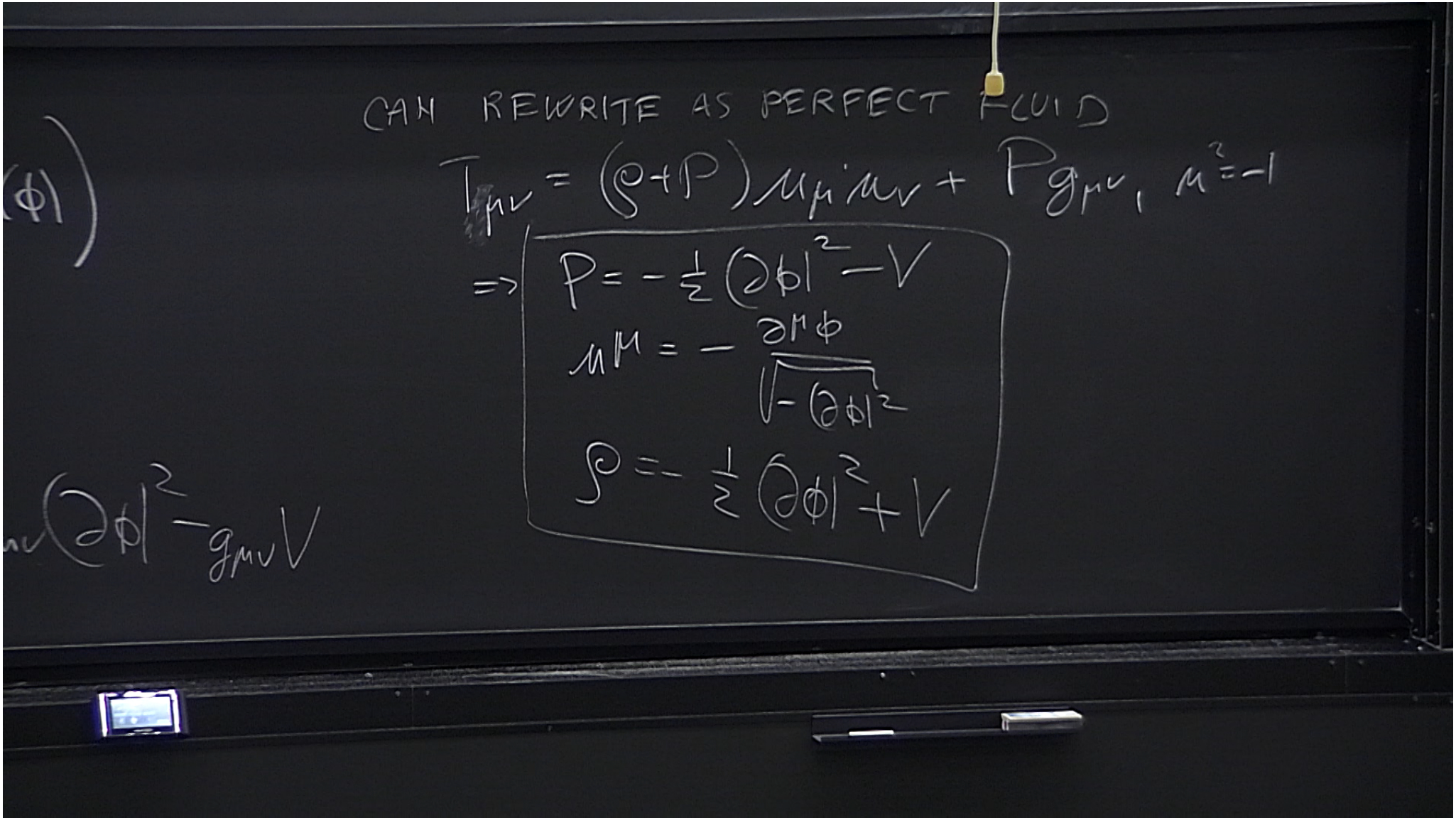
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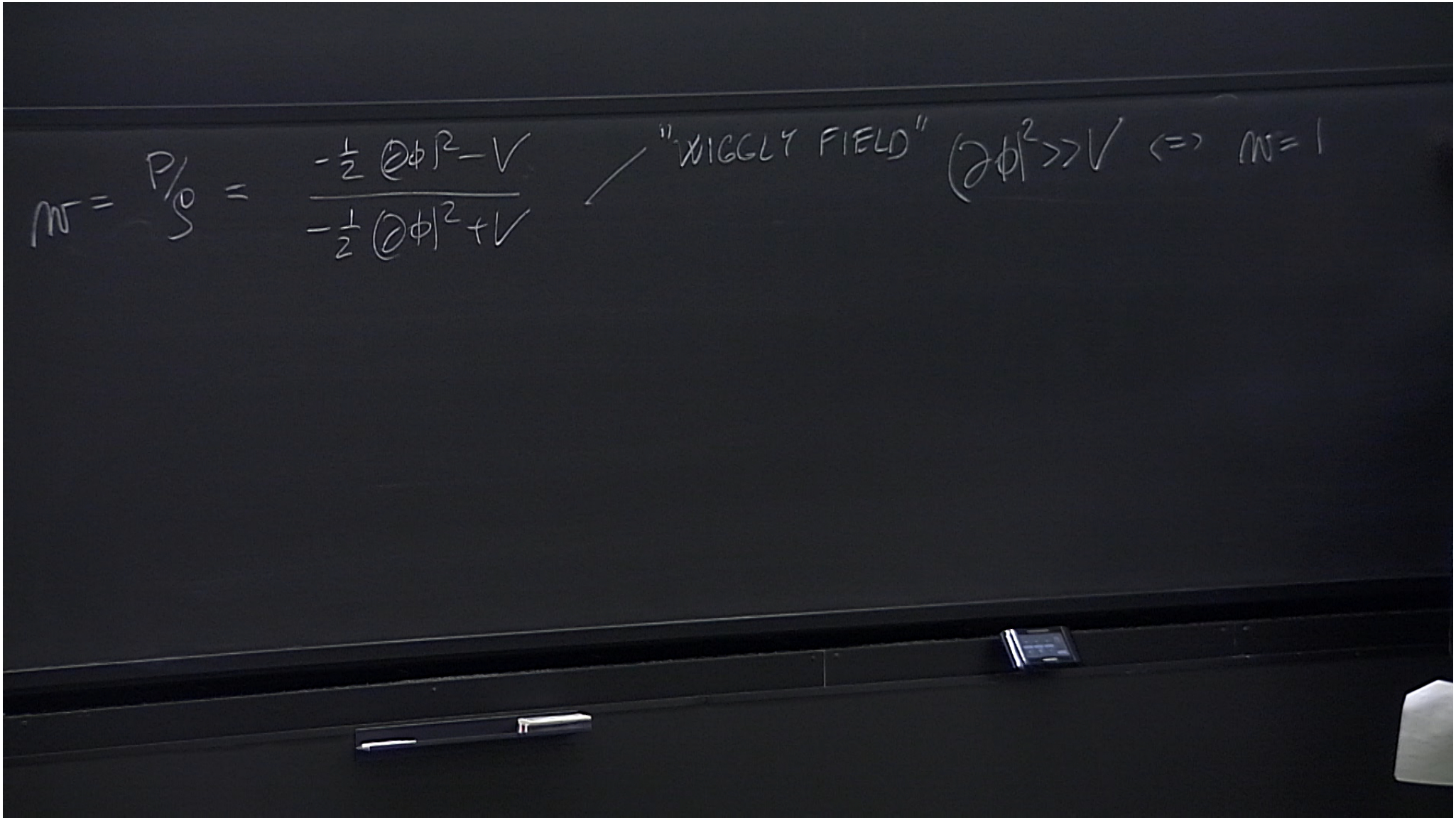
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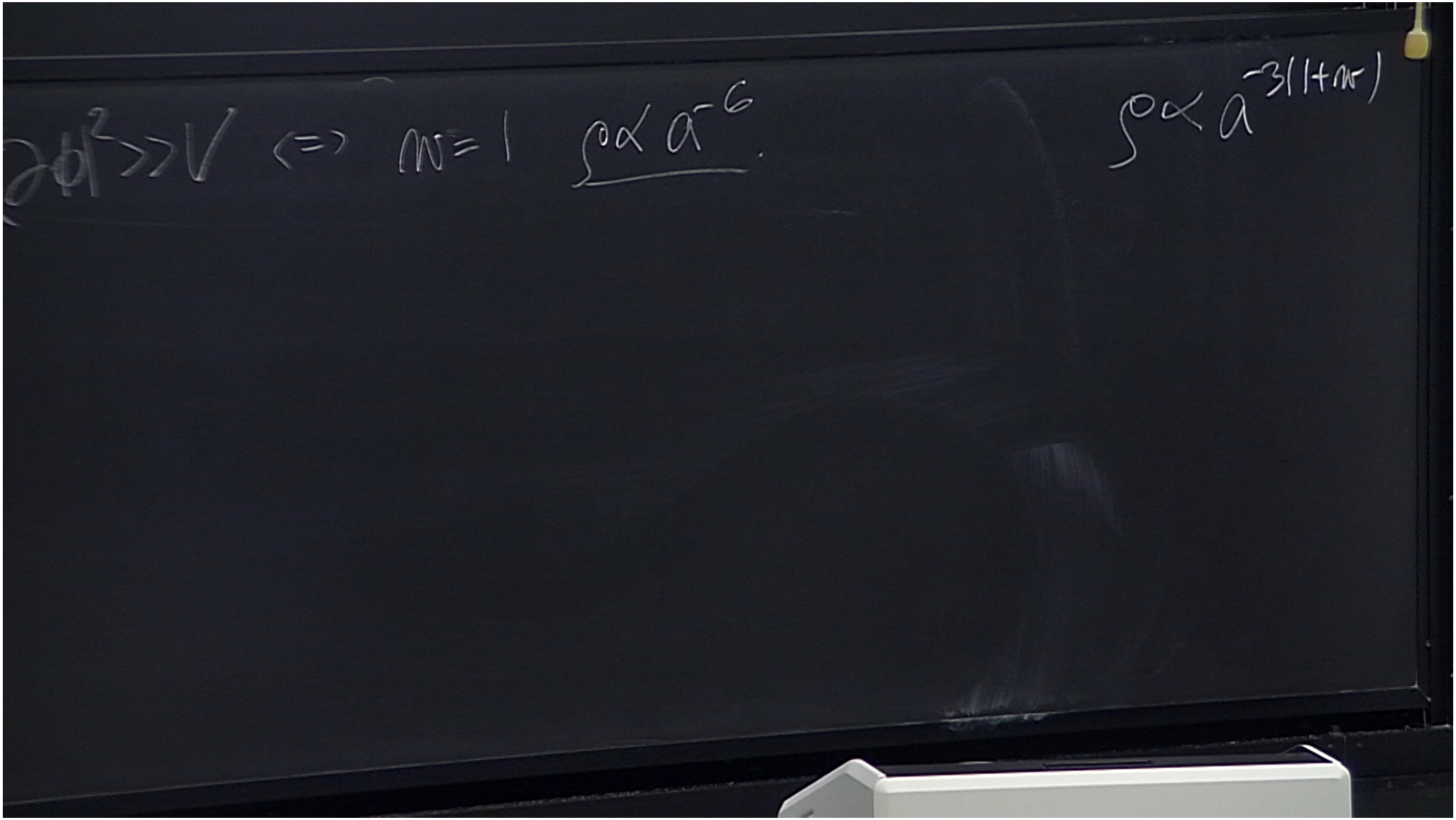
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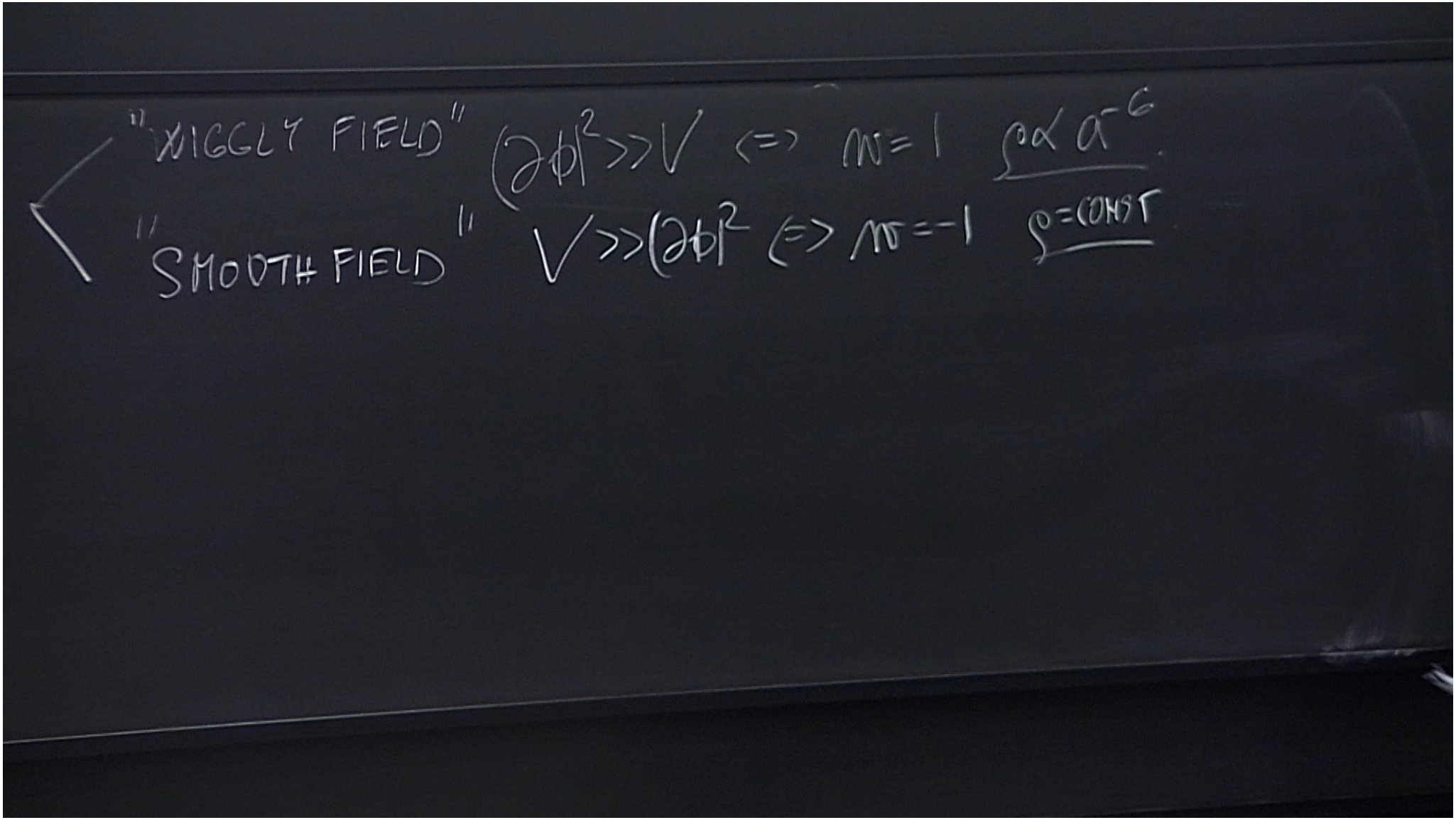
- $\delta g_{\mu\nu}$: $T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V$


CAN REWRITE AS PERFECT FLUID
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}, \quad u^2 = -1$
$\Rightarrow P = -\frac{1}{2}(\partial\phi)^2 - V$
$u^\mu = -\frac{\partial^\mu \phi}{\sqrt{-(\partial\phi)^2}}$
$\rho = -\frac{1}{2}(\partial\phi)^2 + V$
(ϕ)
$(\partial\phi)^2 - g_{\mu\nu} V$


CAN REWRITE AS PERFECT FLUID
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}, \quad u^2 = -1$
$\Rightarrow$
$P = -\frac{1}{2}(\partial\phi)^2 - V$
$u^\mu = -\frac{\partial^\mu \phi}{\sqrt{-(\partial\phi)^2}}$
$\rho = -\frac{1}{2}(\partial\phi)^2 + V$


$w = P/\rho = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$
"WIGGLY FIELD"
$(\partial\phi)^2 >> V \iff w = 1$


$\dot{\phi}^2 \gg V \Leftrightarrow w = 1 \quad \rho \propto a^{-6}$.
$\rho \propto a^{-3(1+w)}$


"WIGGLY FIELD" $(\partial\phi)^2 \gg V \Leftrightarrow w = 1$ $\rho \propto a^{-6}$
"SMOOTH FIELD" $V \gg (\partial\phi)^2 \Leftrightarrow w = -1$ $\rho = \text{CONST}$

$$w = \frac{P}{\rho} = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$$

"WIGGLY FIELD" $(\partial\phi)^2 >> V$

"SMOOTH FIELD" $V >> (\partial\phi)^2$

FIELD EQS: $\phi = \phi(t)$

$$3 m_p^2 H^2 = -\frac{1}{2}(\partial\phi)^2 + V = \frac{1}{2}\dot{\phi}^2 + V$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$w = P/\rho = \dfrac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$

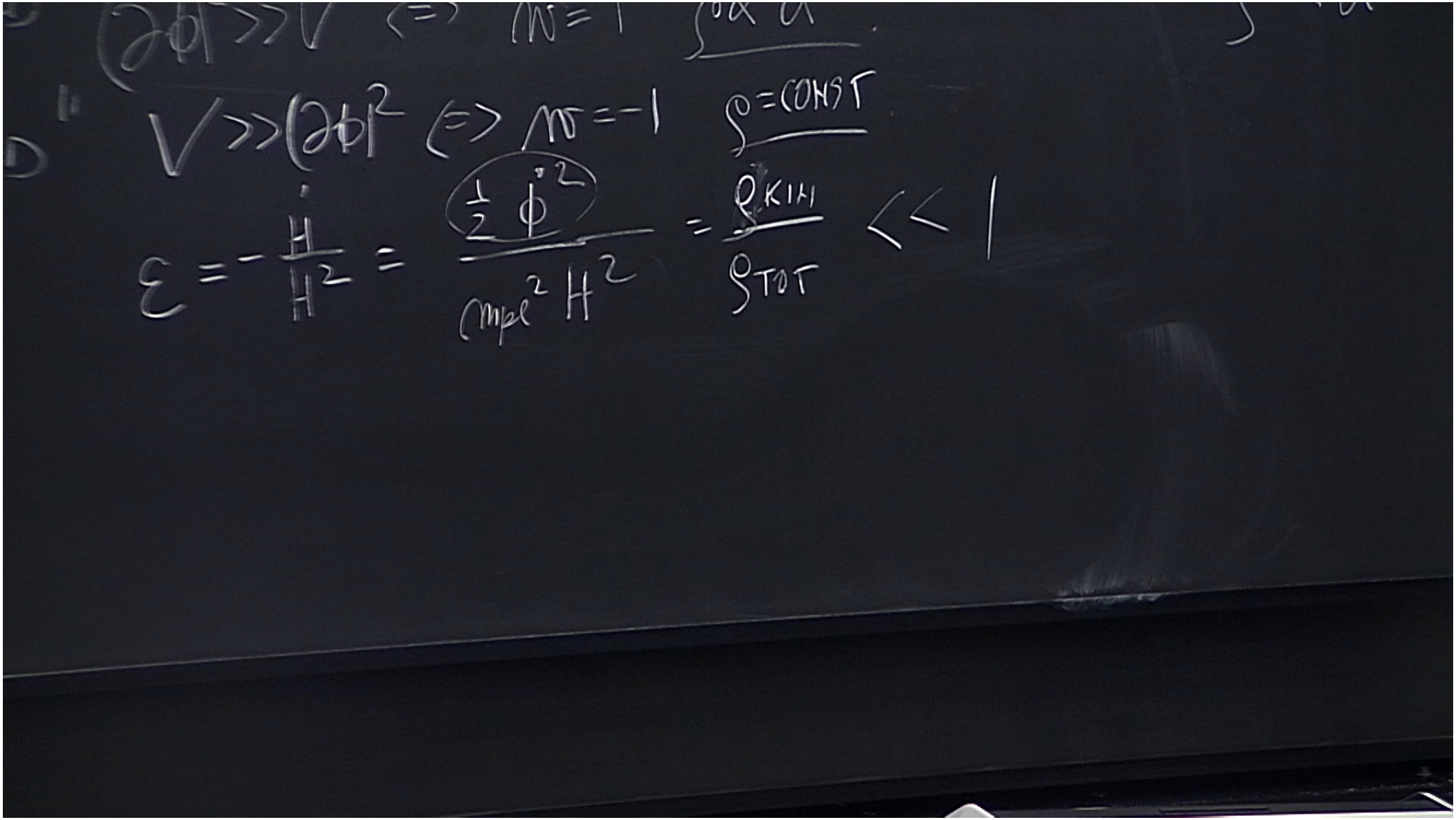
"WIGGLY FIELD" $(\partial\phi)^2 >> V$

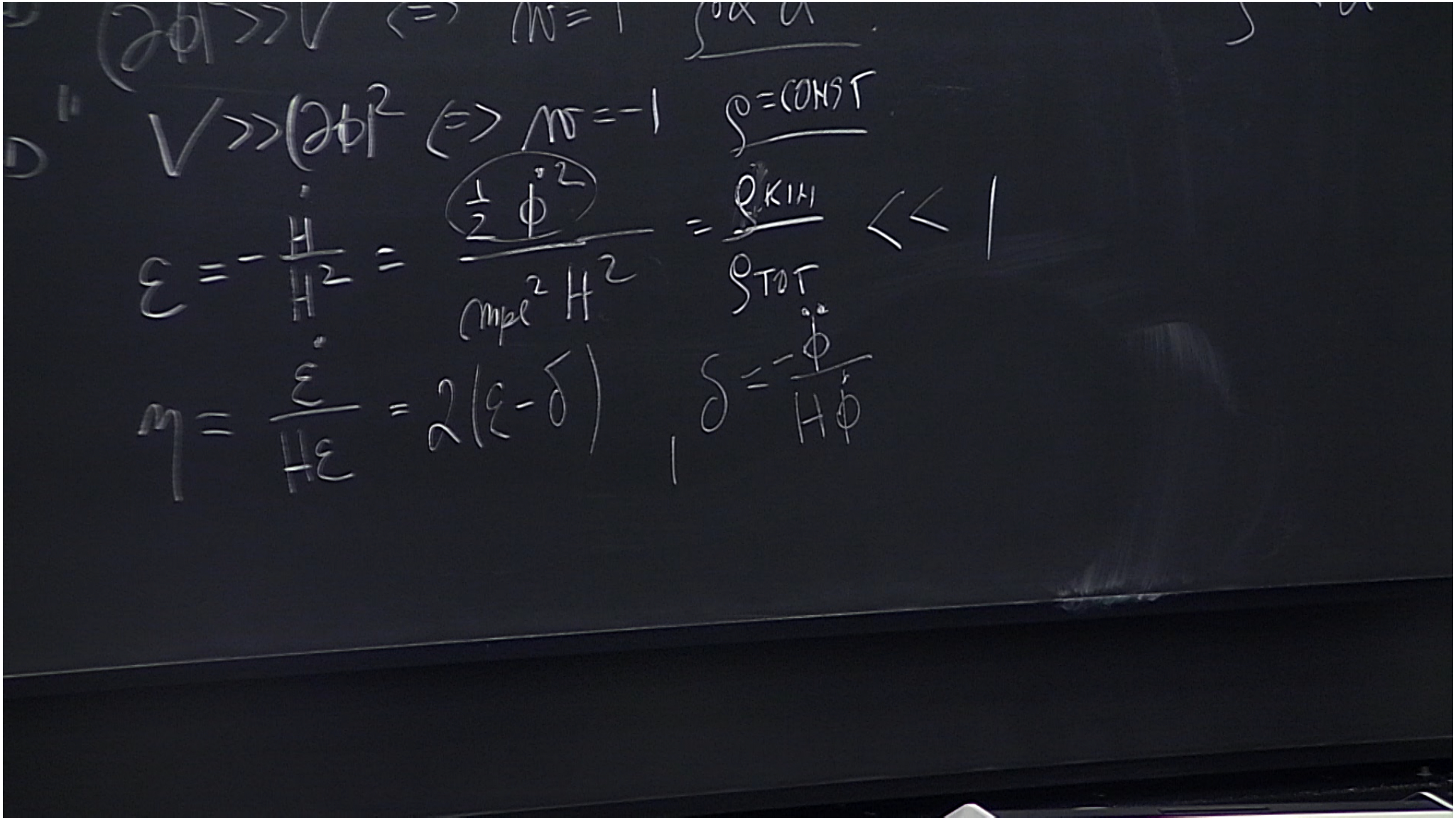
"SMOOTH FIELD" $V >> (\partial\phi)^2$

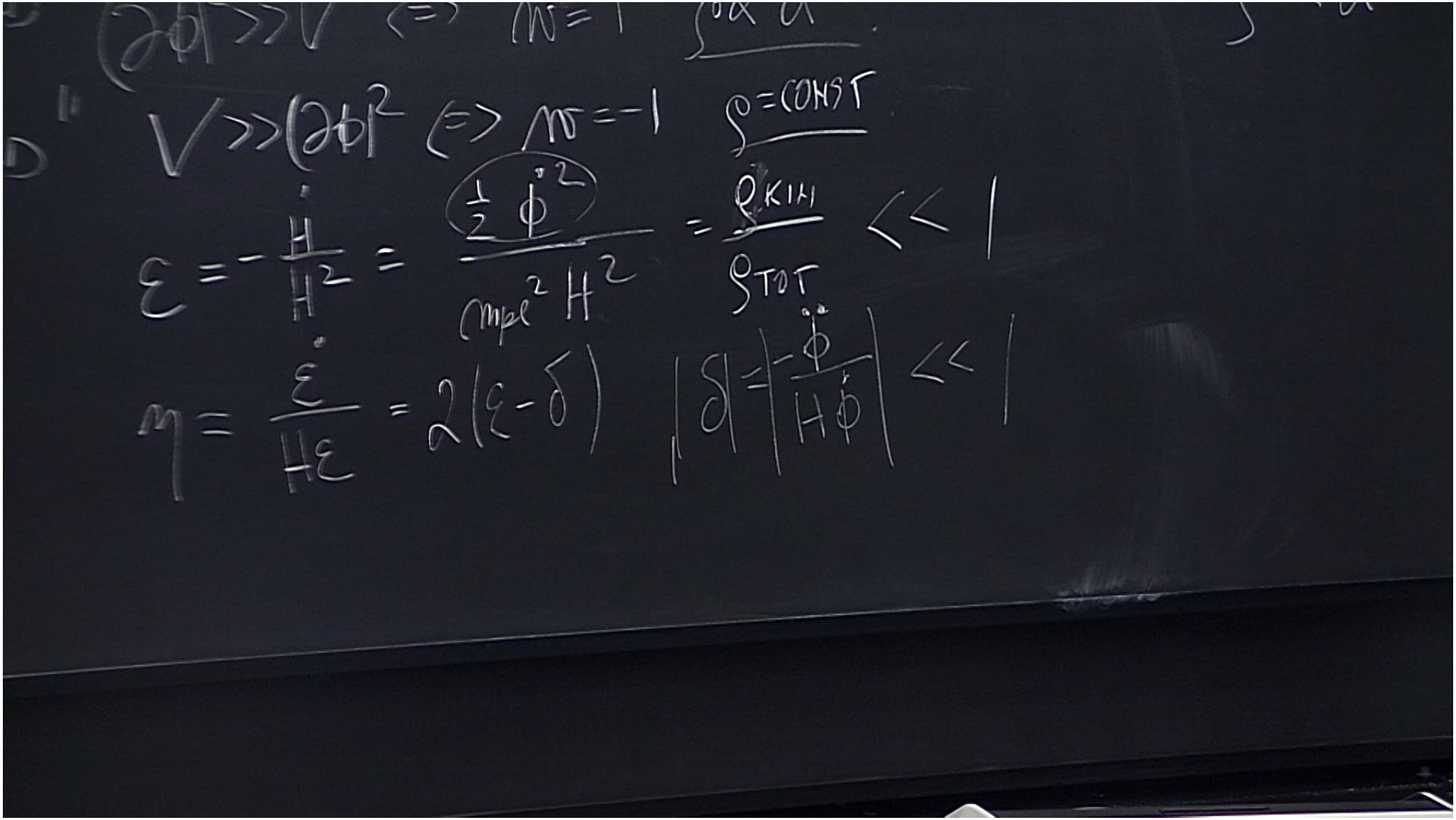
FIELD EQS: $\phi = \phi(t)$

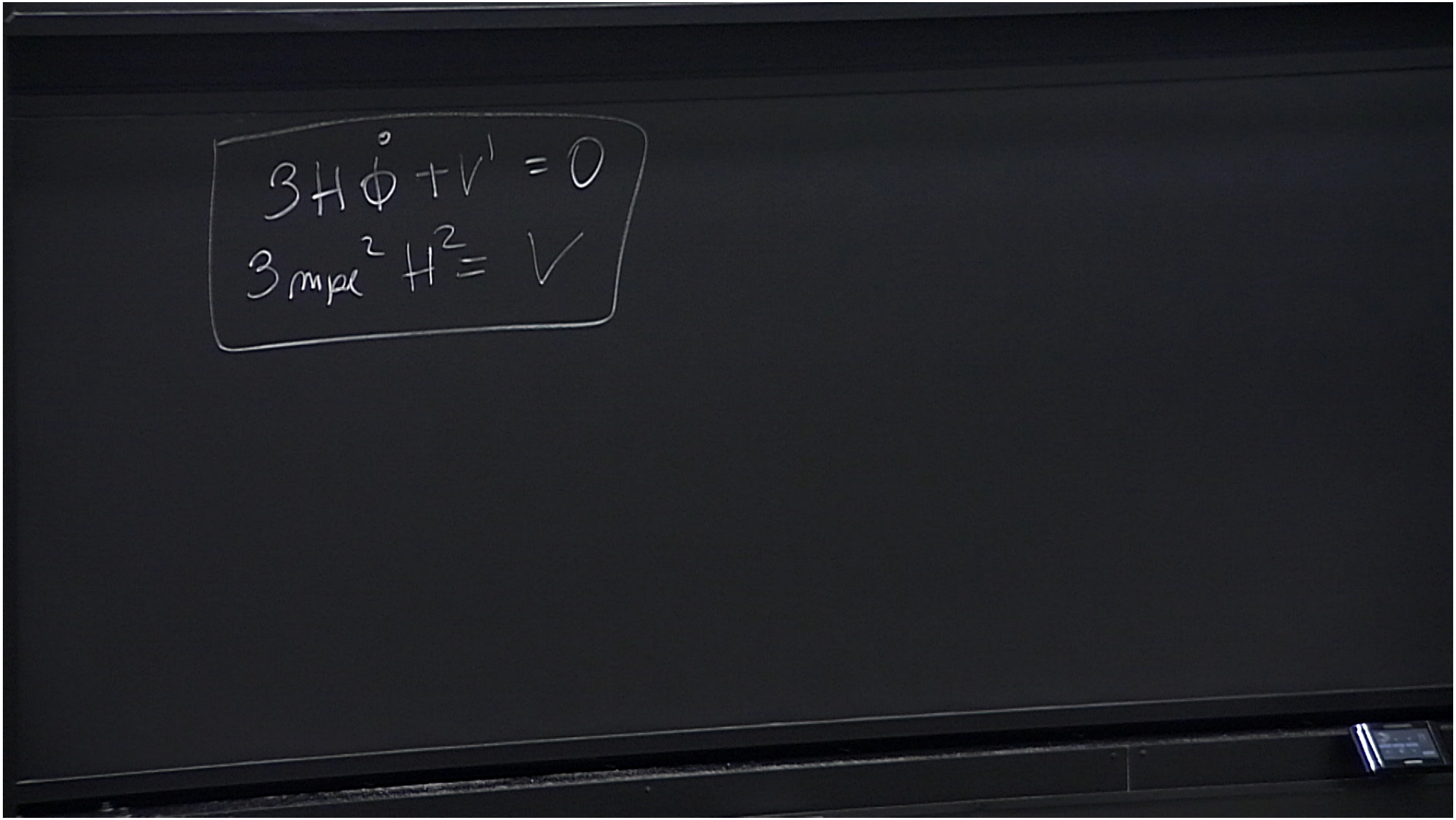
$$3 m_p^2 H^2 = -\frac{1}{2}(\partial\phi)^2 + V = \frac{1}{2}\dot{\phi}^2 + V$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$


$V \gg \dot{\phi}^2 \Leftrightarrow w = -1$
$\rho = \text{CONST}$
$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{m_{pl}^2 H^2} = \frac{\rho_{KIN}}{\rho_{TOT}} \ll 1$


$|\dot{\phi}| \gg V \Leftrightarrow w = 1$
$V \gg |\dot{\phi}|^2 \Leftrightarrow w = -1$
$\rho = \text{CONST}$
$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{m_{pl}^2 H^2} = \frac{\rho_{KIN}}{\rho_{TOT}} \ll 1$
$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2(\varepsilon - \delta)$
$\delta = -\frac{\ddot{\phi}}{H\dot{\phi}}$

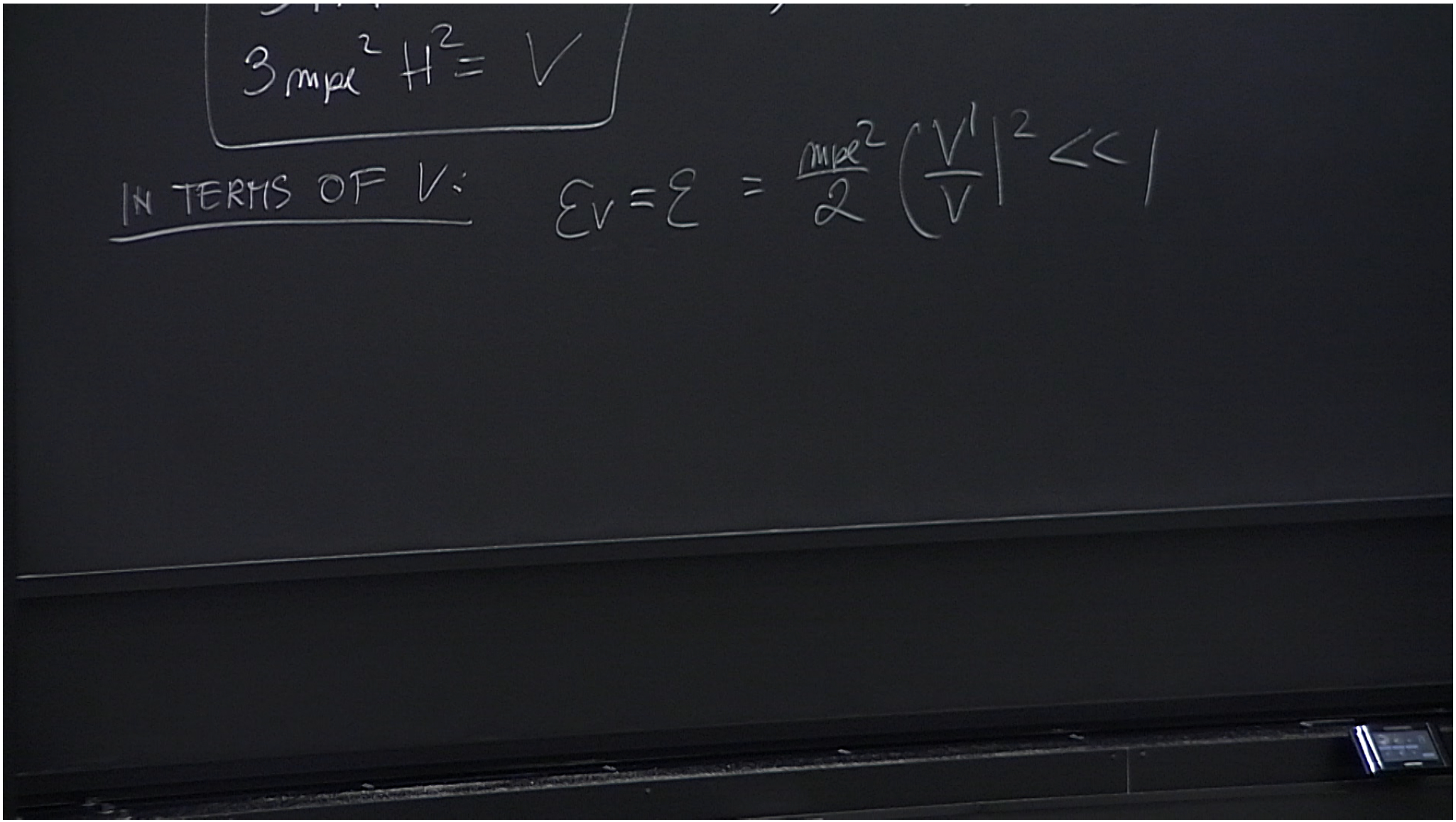

$\dot{\phi}^2 \gg V \Leftrightarrow w = 1$
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$3H\dot{\phi} + V' = 0$
$3 m_{pl}^2 H^2 = V$

$3H\dot{\phi} + V' = 0$

$3 m_{pl}^2 H^2 = V$

$\rho_{KIN} \ll \rho_{TOT}$ ... SLOW-ROLL

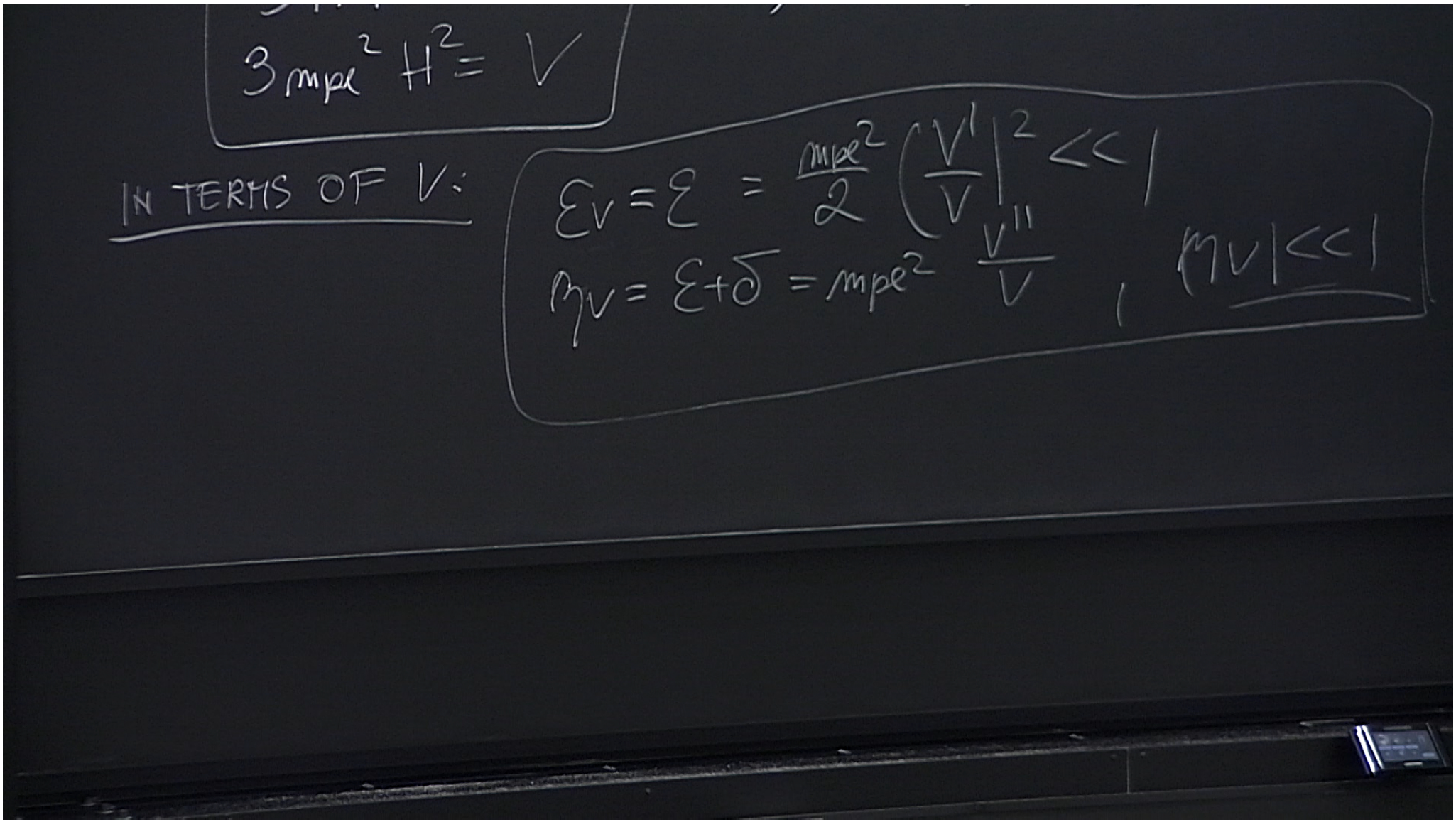

$3 m_{pl}^2 H^2 = V$
IN TERMS OF V:
$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 << 1$

$3 m_{pl}^2 H^2 = V$

IN TERMS OF V:

$$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V}, \quad |\eta_V| \ll 1$$


$3 m_{pl}^2 H^2 = V$
IN TERMS OF V:
$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V}$, $|\eta_V| \ll 1$

$$3H\dot{\phi} + V' = 0 \quad (1)$$

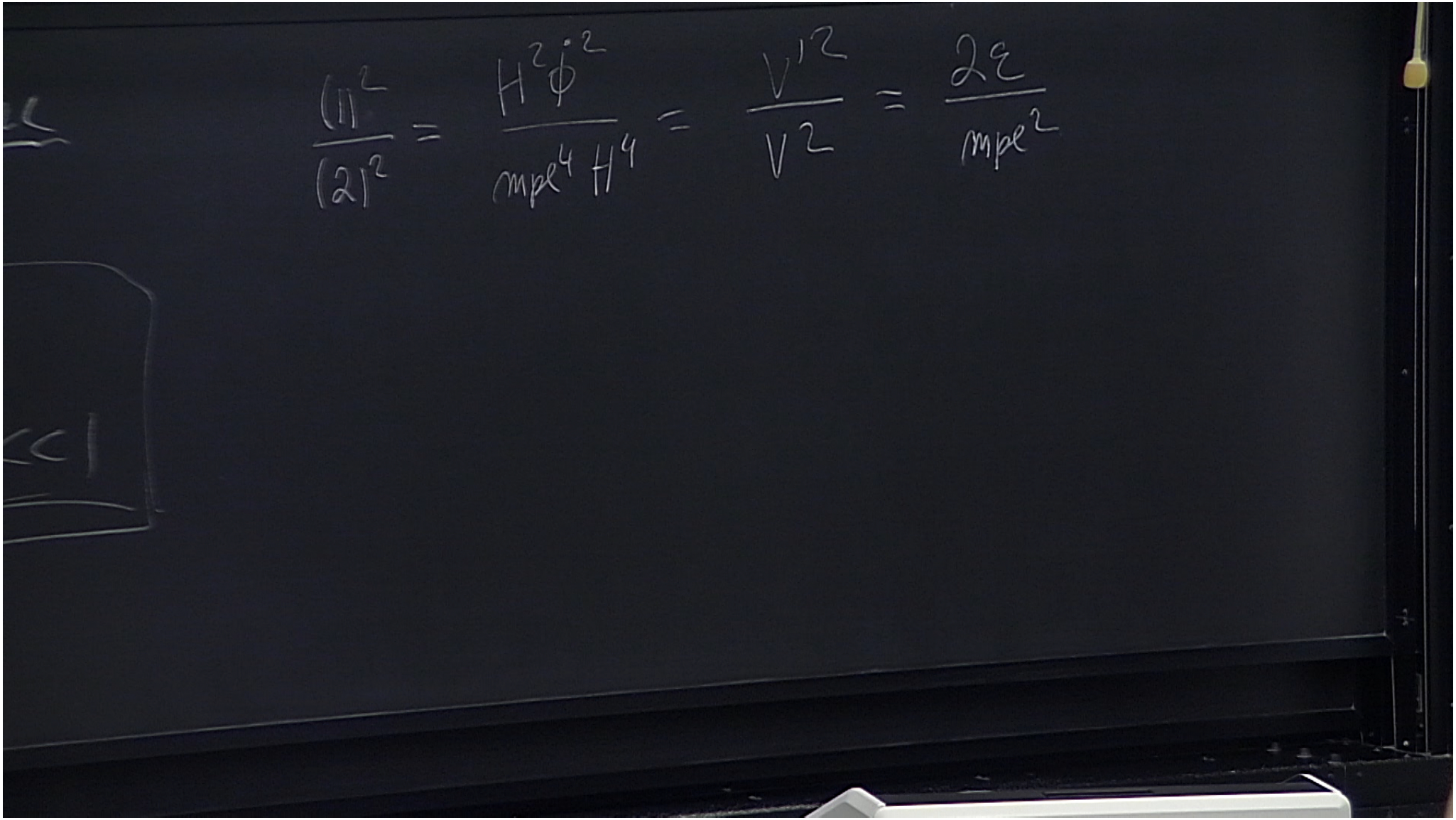
$$3 m_{pl}^2 H^2 = V \quad (2)$$

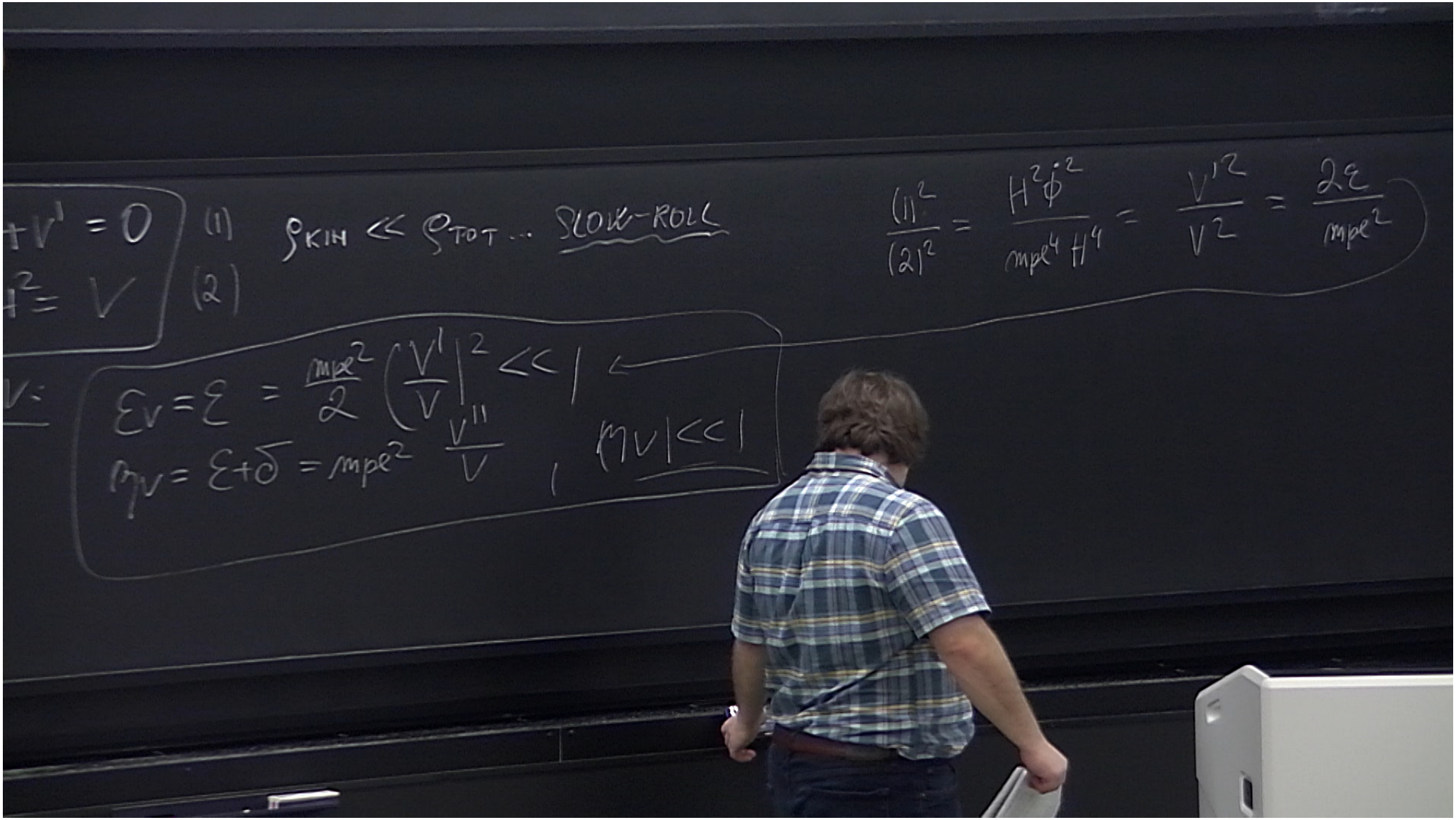
$\rho_{KIN} \ll \rho_{TOT}$ ... SLOW-ROLL

IN TERMS OF V:

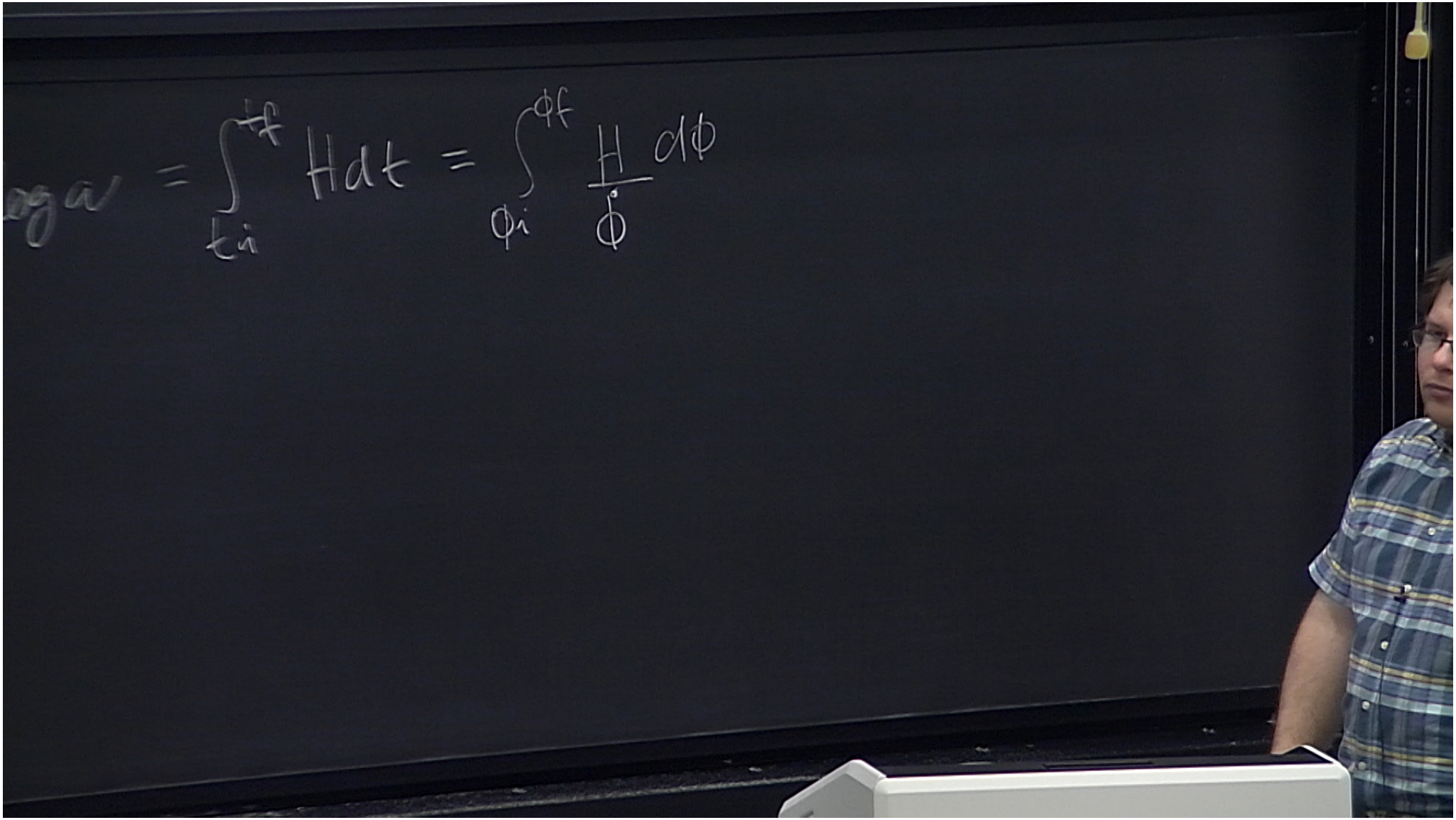
$$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V}, \quad |\eta_V| \ll 1$$


$\frac{\Pi^2}{(2)^2} = \frac{H^2\dot{\phi}^2}{m_{pl}^4 H^4} = \frac{V'^2}{V^2} = \frac{2\varepsilon}{m_{pl}^2}$


+V' = 0 (1)
H² = V (2)
ρKIN ≪ ρTOT ... SLOW-ROLL
(1)²/(2)² = H²φ̇²/mpl⁴H⁴ = V'²/V² = 2ε/mpl²
εV = ε = (mpl²/2)(V'/V)² ≪ 1
ηV = ε+δ = mpl² V''/V , |ηV| ≪ 1

$$N = \log \frac{\hat{a}_i}{\hat{a}_f} = \log \frac{a_f H_f}{a_i H_i} \approx \log \frac{a_f}{a_i} = \int_{a_i}^{a_f} d\log a = \int_{t_i}^{t_f} H dt$$

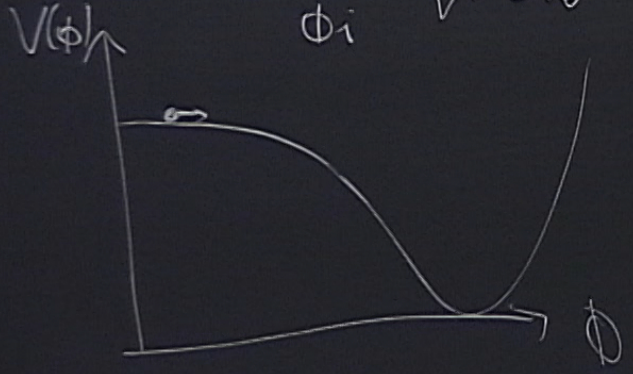

$\log a = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi$

$$N = \log \frac{\hat{a}_i}{\hat{a}_f} = \log \frac{a_f H_f}{a_i H_i} \approx \log \frac{a_f}{a_i} = \int_{a_i}^{a_f} d\log a = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi$$

$$\overset{(1)}{=} -3 \int_{\phi_i}^{\phi_f} \frac{H^2}{V'} d\phi \overset{(2)}{=} \frac{1}{m_{pl}^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$$

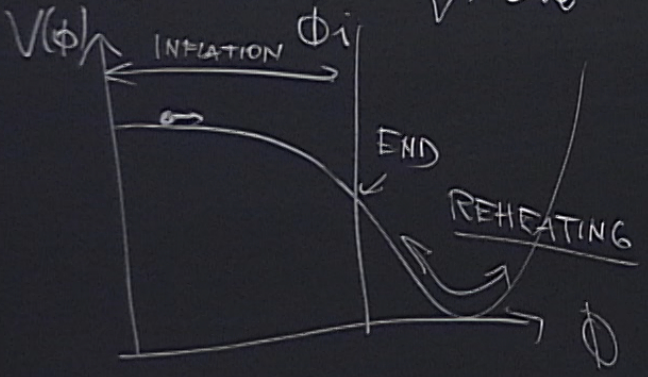
$$N = \log \frac{\hat{a}_i}{\hat{a}_f} = \log \frac{a_f H_f}{a_i H_i} \approx \log \frac{a_f}{a_i} = \int_{a_i}^{a_f} d\log a = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi$$

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$$N = \log \frac{\hat{a}_i}{\hat{a}_f} = \log \frac{a_f H_f}{a_i H_i} \simeq \log \frac{a_f}{a_i} = \int_{a_i}^{a_f} d\log a = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi$$

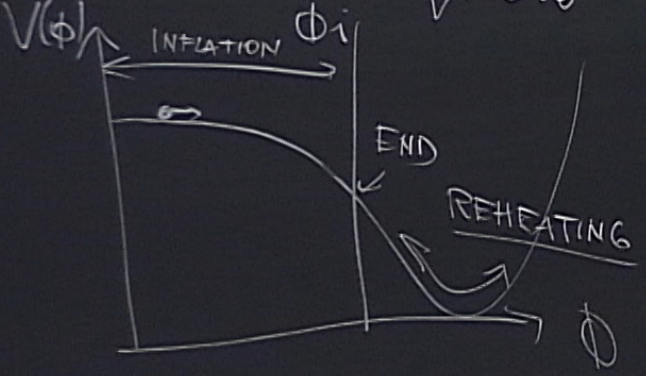
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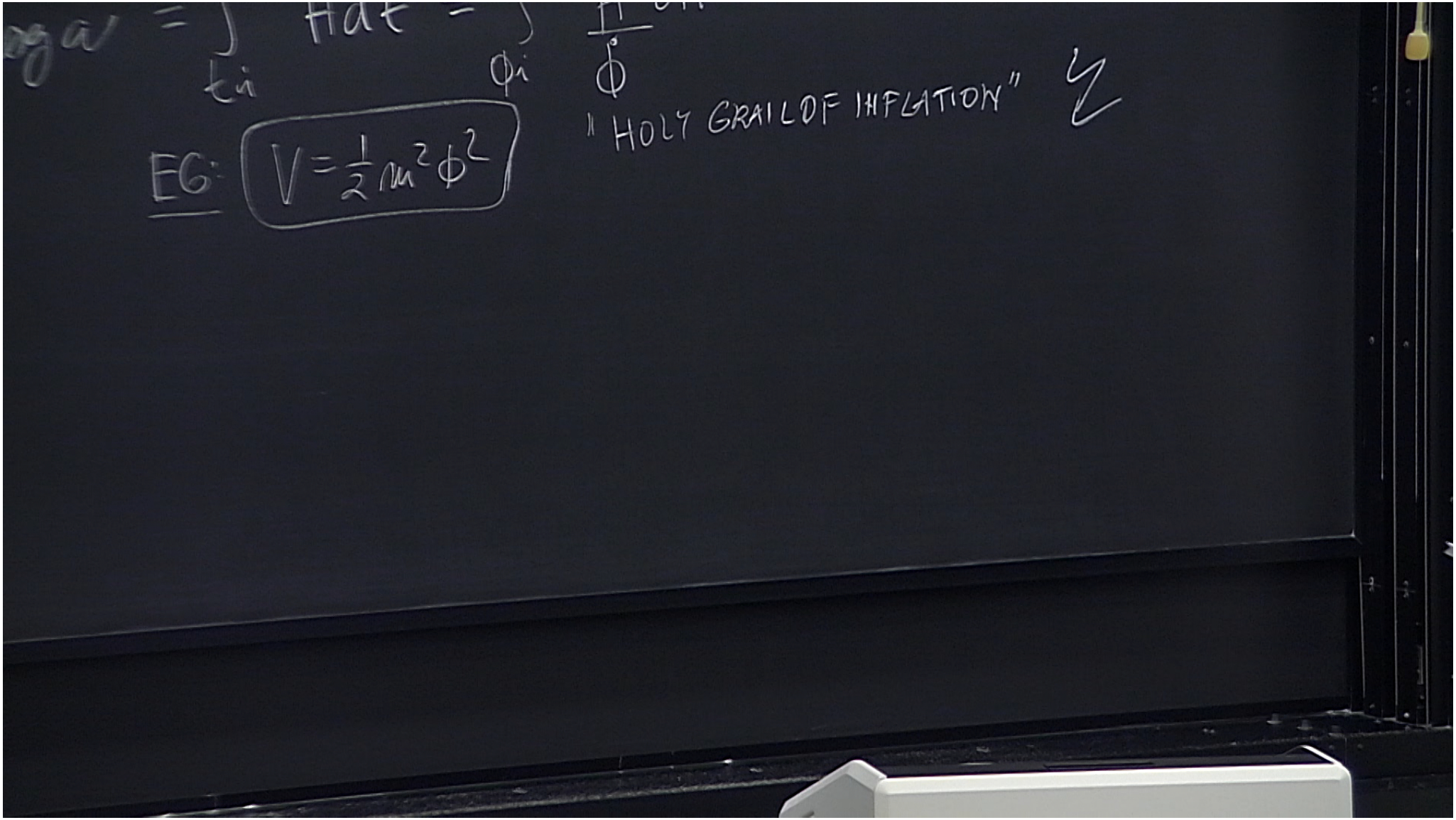


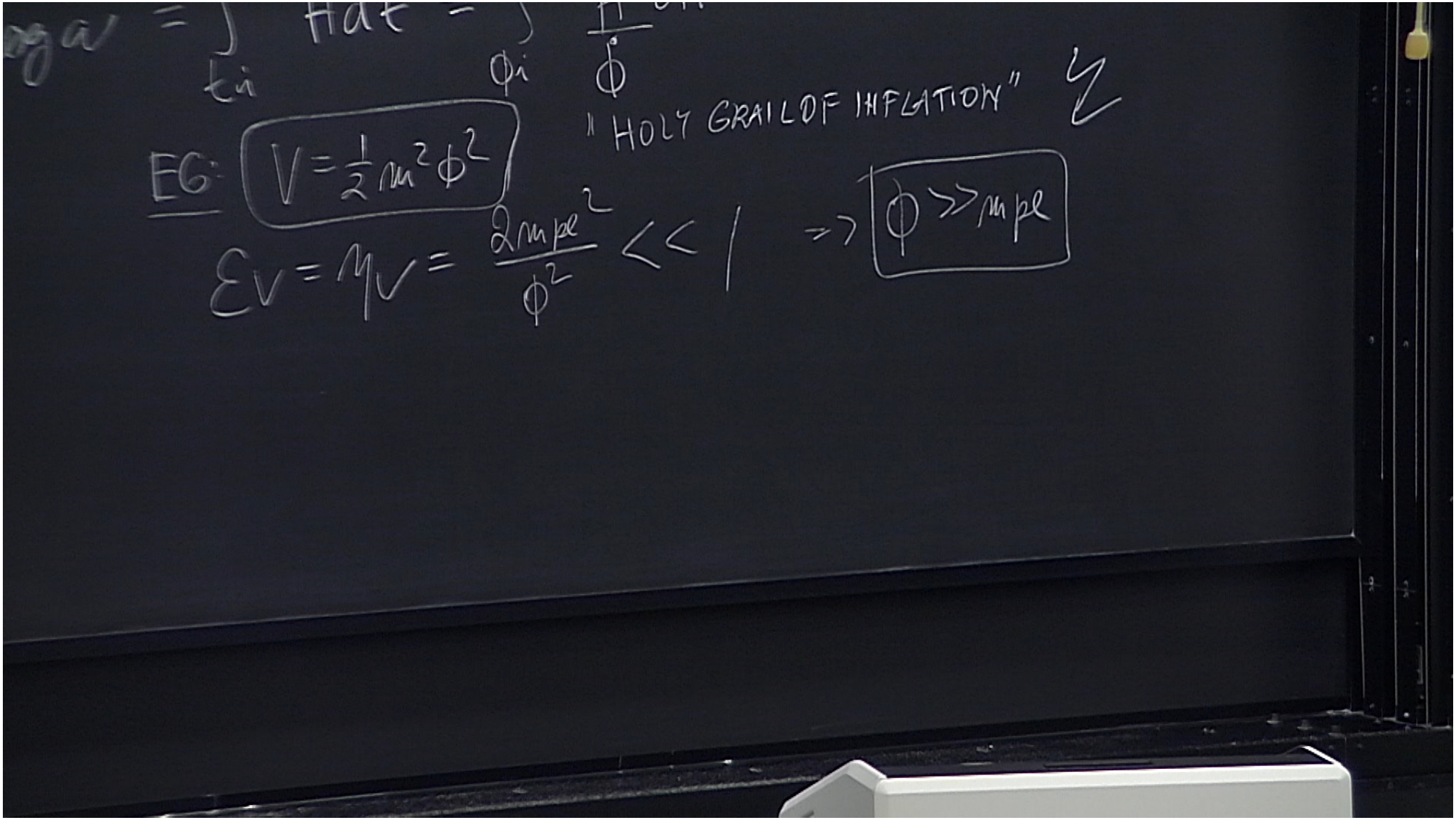
$$N = \log \frac{a_i}{a_f} = \log \frac{a_f H_f}{a_i H_i} \simeq \log \frac{a_f}{a_i} = \int_{a_i}^{a_f} d\log a = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi$$

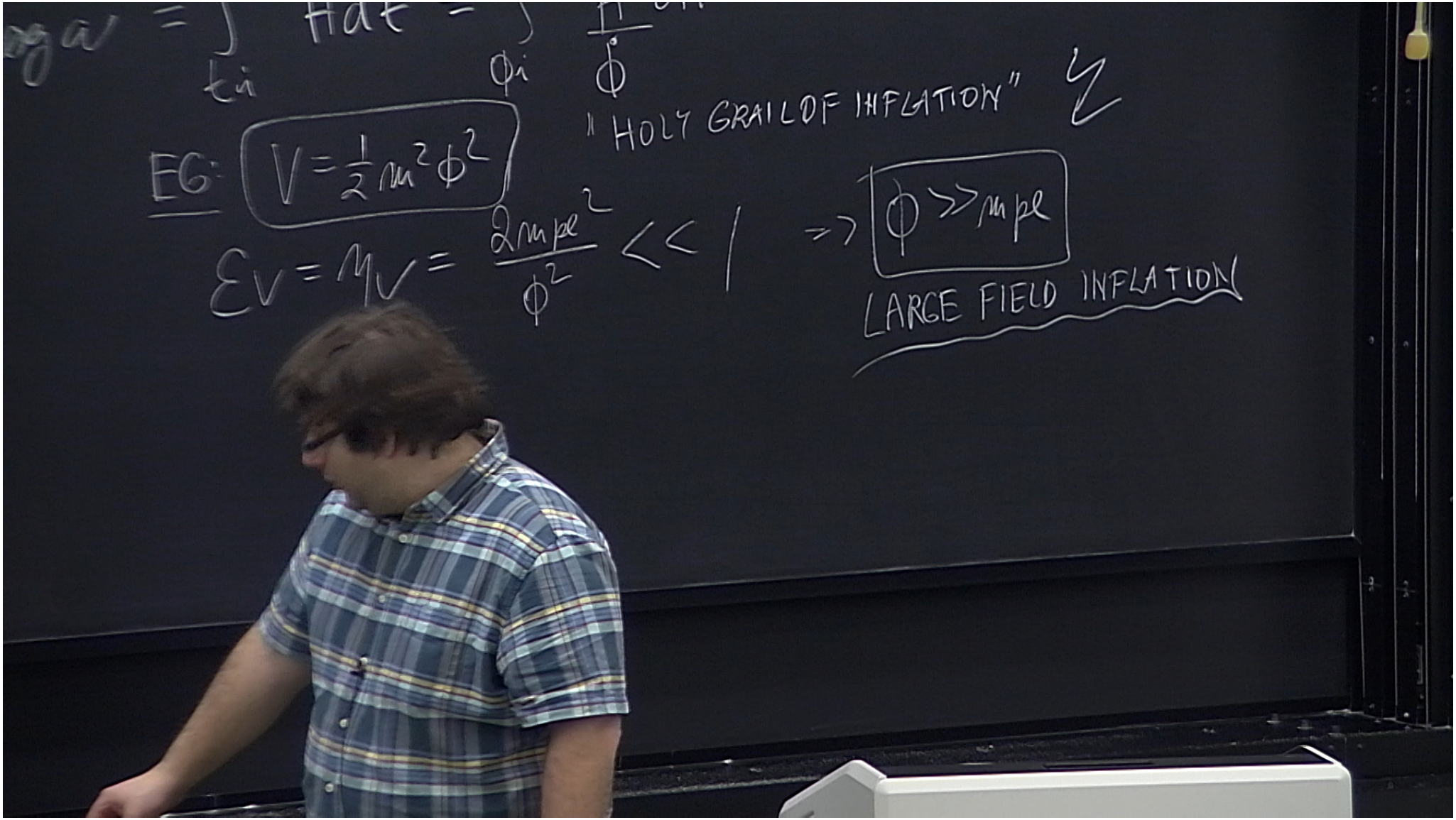
$$\overset{(1)}{=} -3 \int_{\phi_i}^{\phi_f} \frac{H^2}{V'} d\phi \qquad \overset{(2)}{=} \frac{1}{m_{pl}^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$$

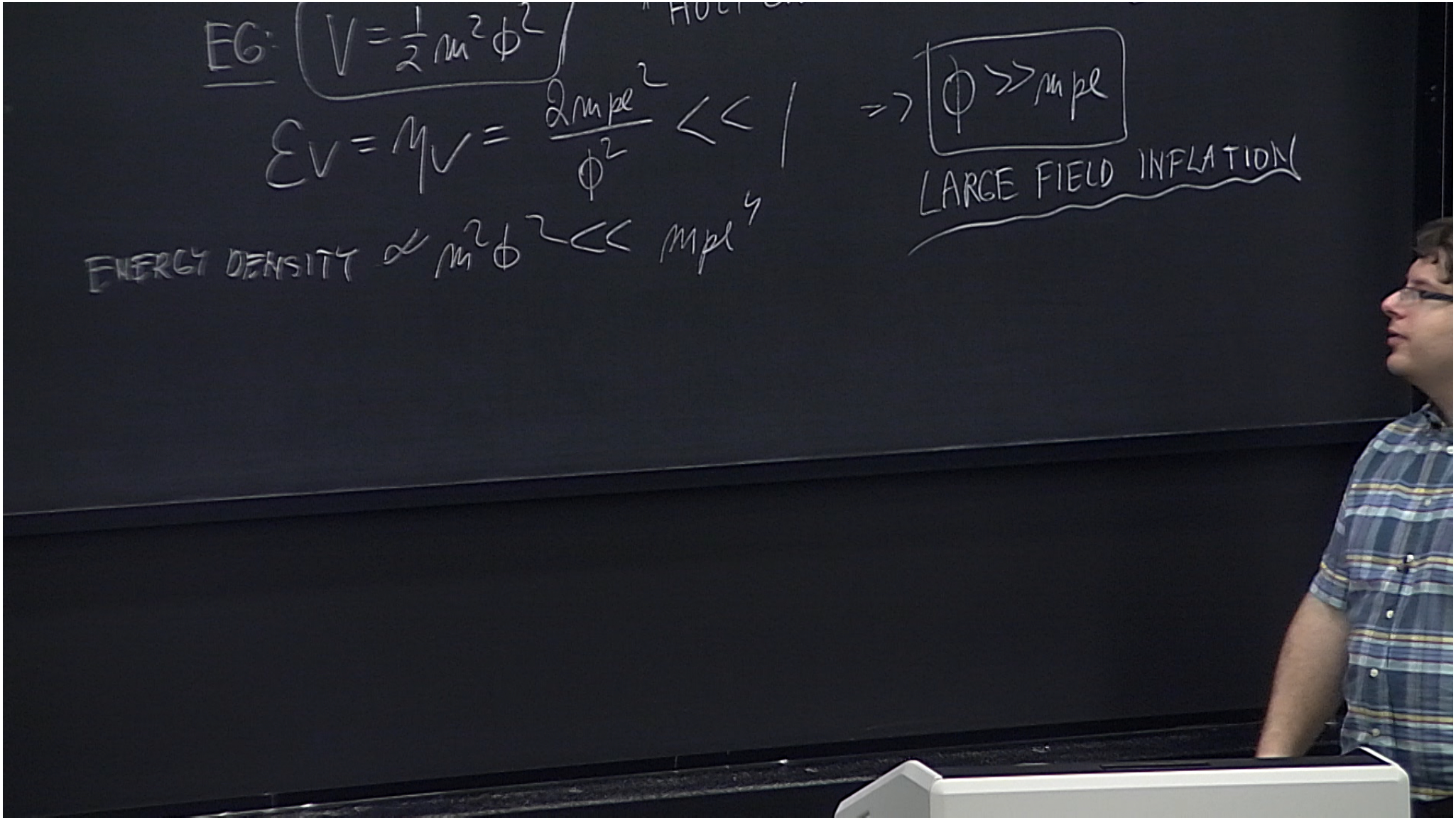
EG: $V = \frac{1}{2} m^2 \phi^2$

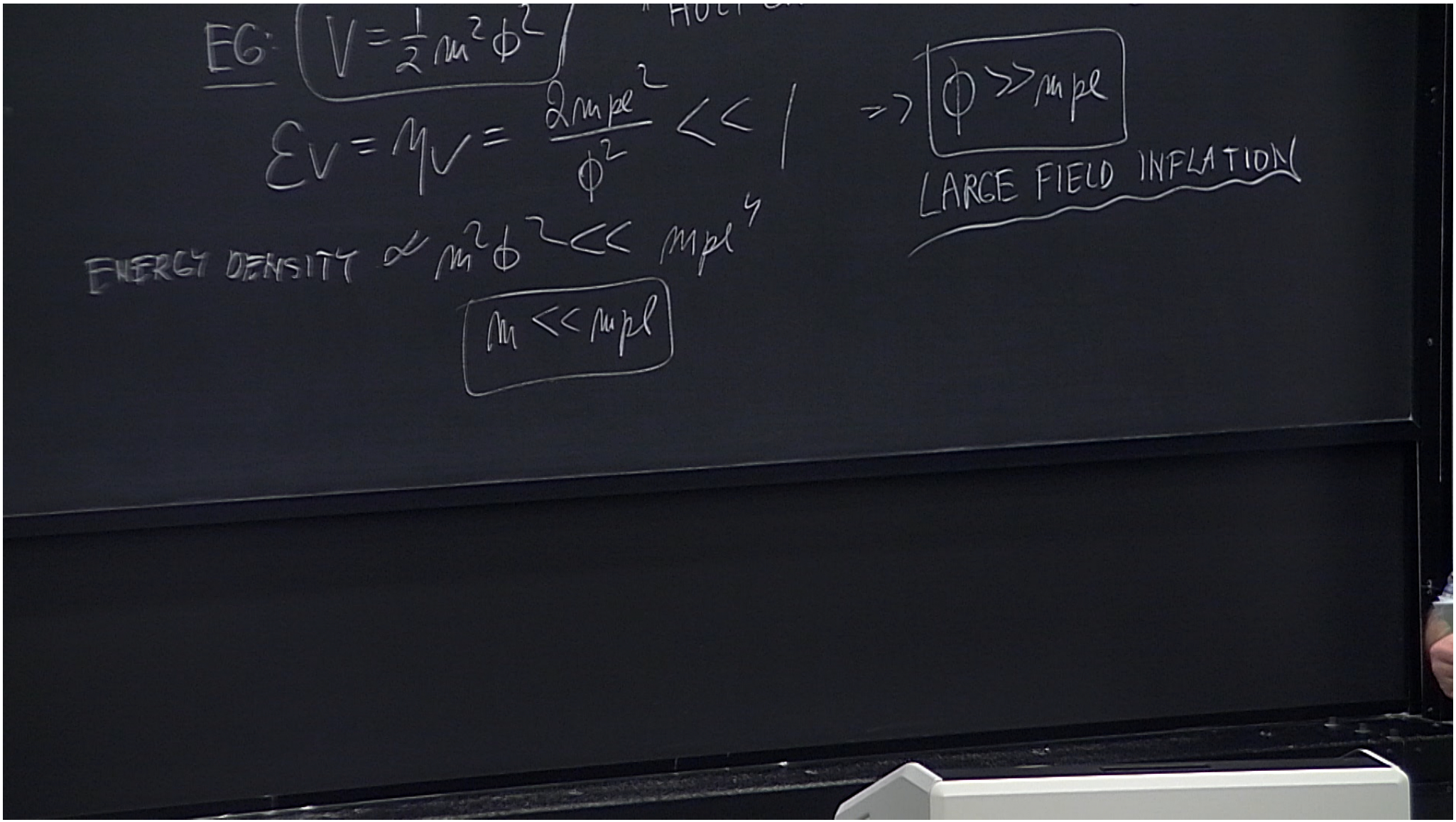


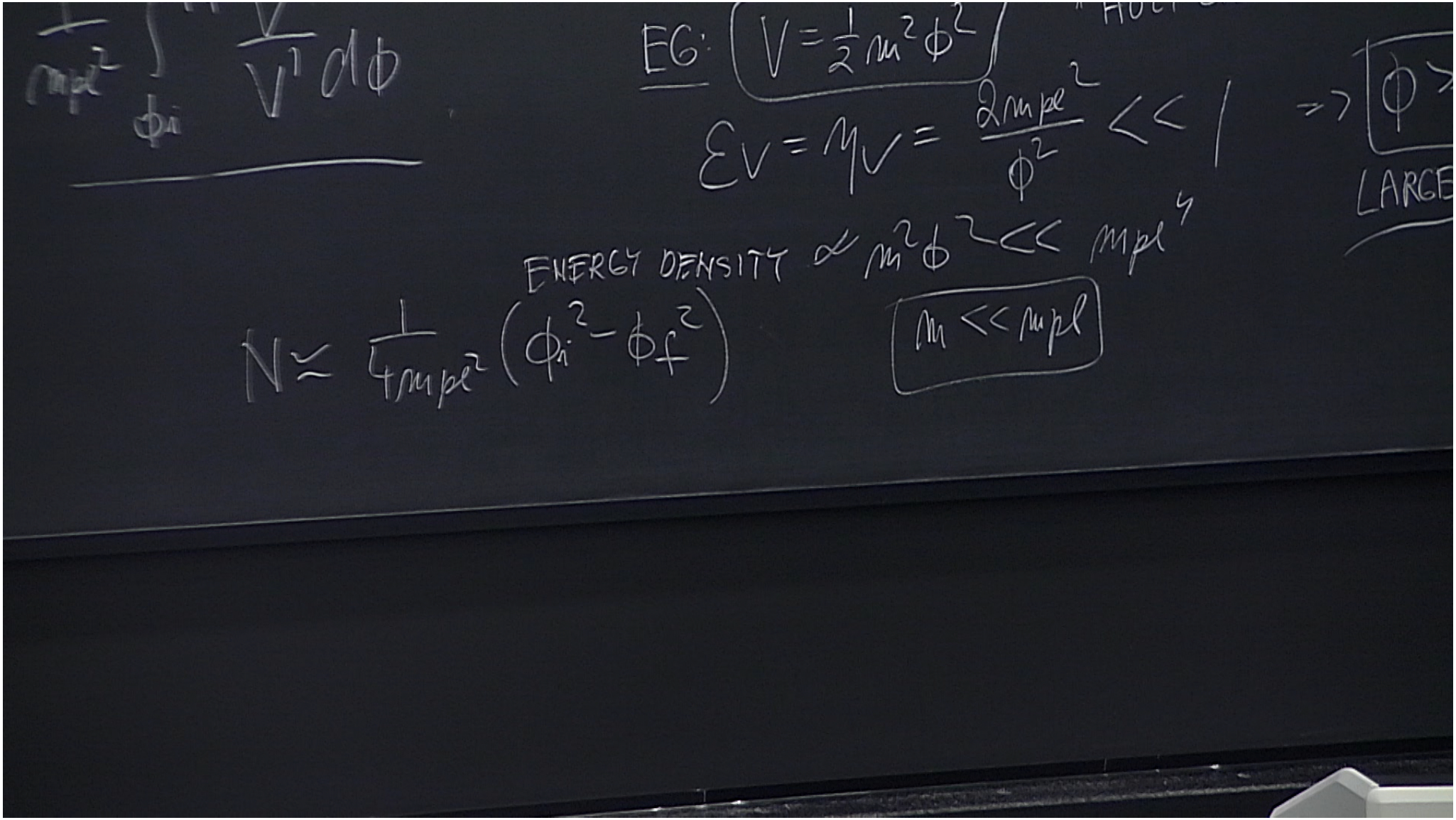

$\ln a = \int_{t_i}^{t} H dt = \int_{\phi_i}^{\phi} \frac{H}{\dot{\phi}}$
EG: $V = \frac{1}{2} m^2 \phi^2$
"HOLY GRAIL OF INFLATION"

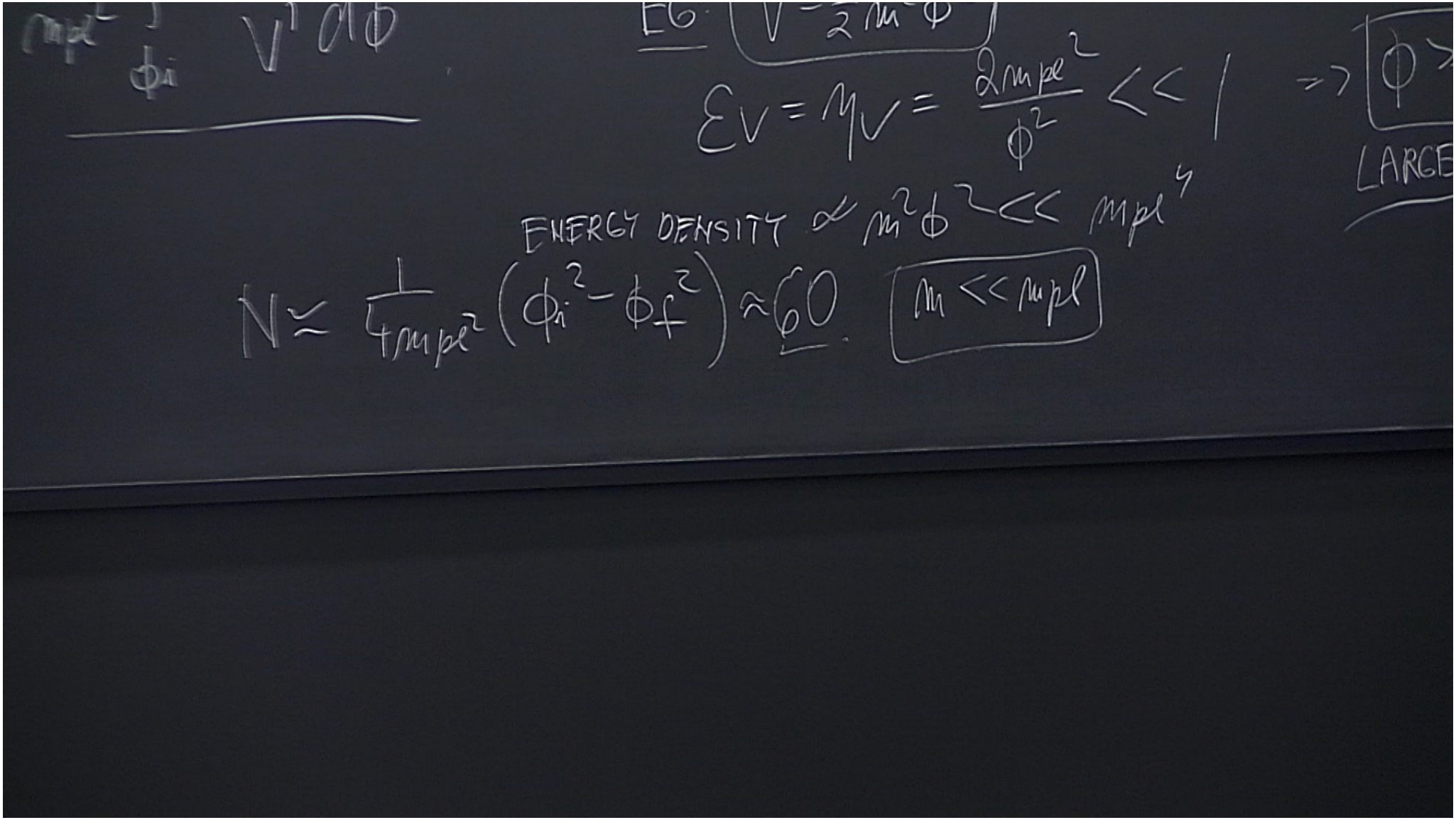

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$\varepsilon_V = \eta_V = \frac{2 m_{pl}^2}{\phi^2} << 1 \Rightarrow \phi >> m_{pl}$

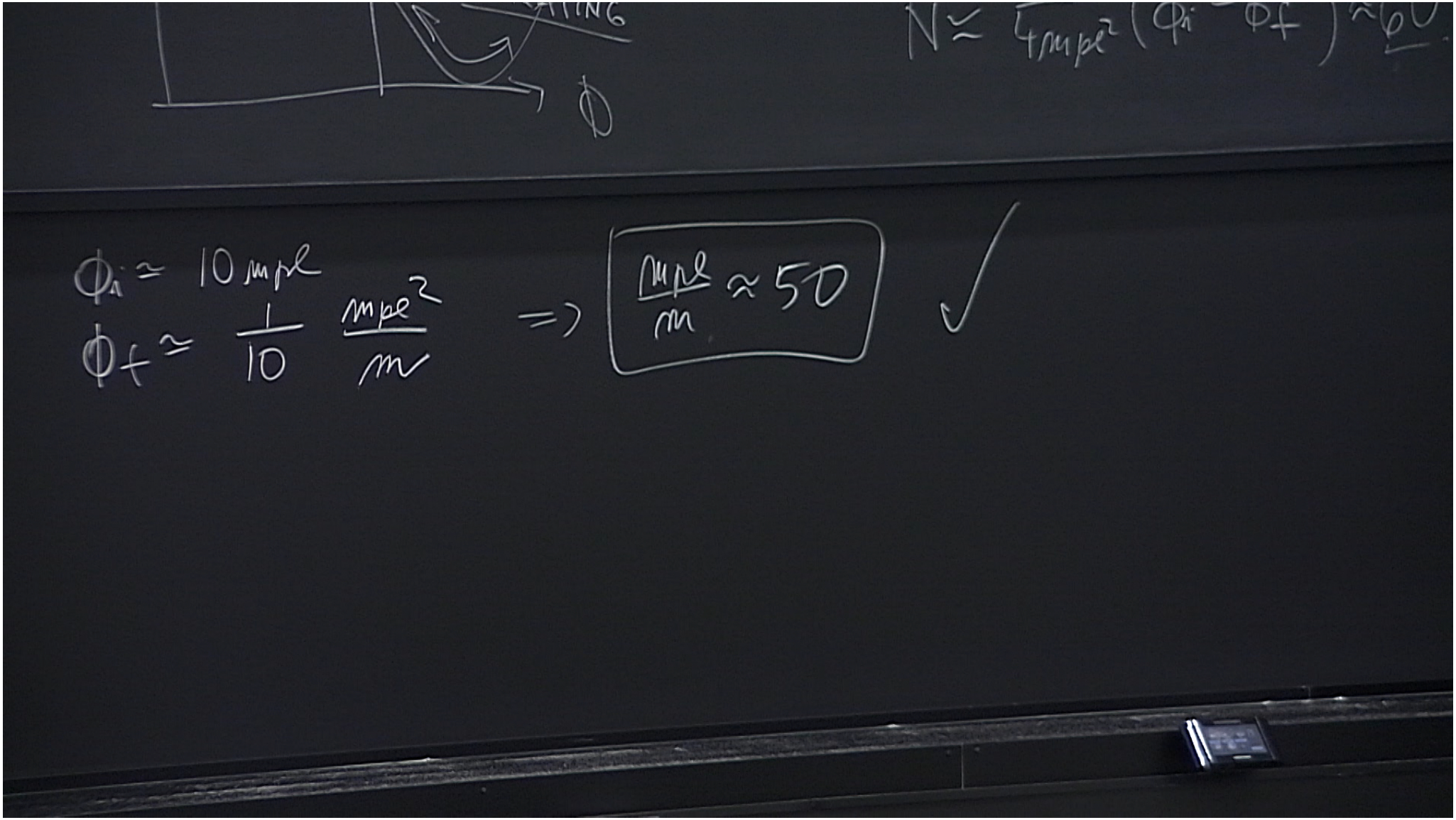

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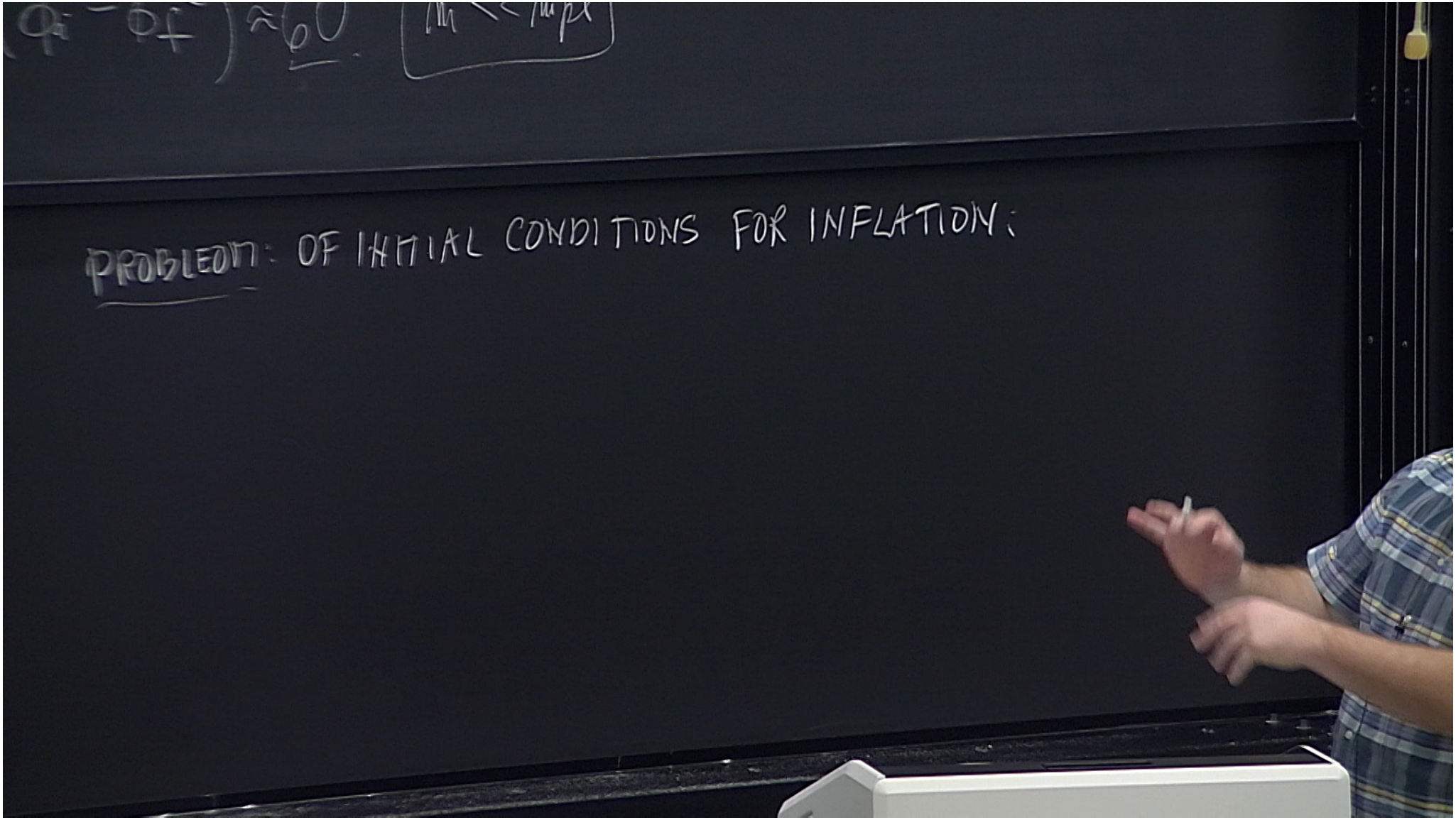

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LARGE FIELD INFLATION
ENERGY DENSITY $\propto m^2 \phi^2 << m_{pl}^4$

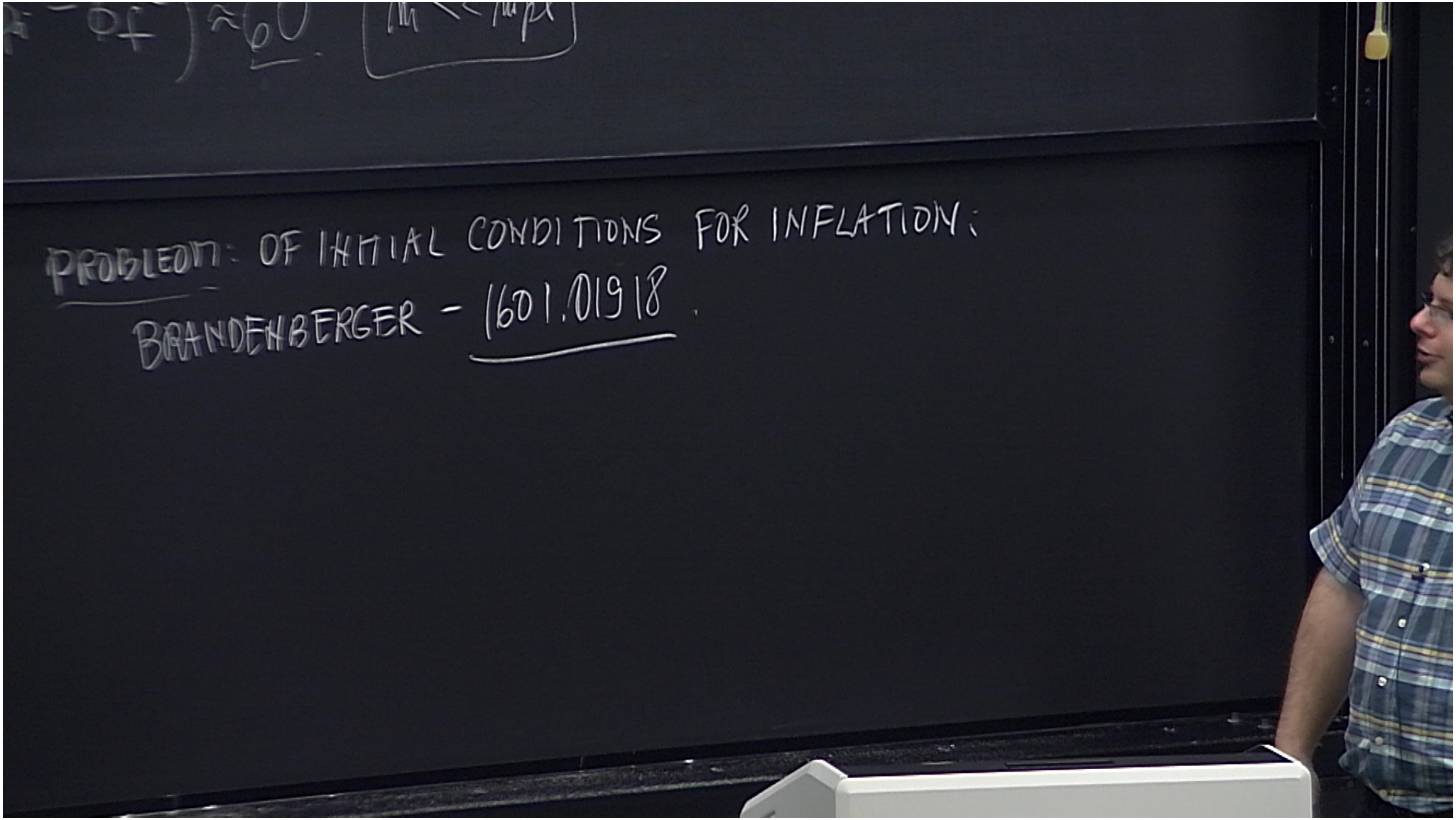

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LARGE FIELD INFLATION
ENERGY DENSITY $\propto m^2 \phi^2 << m_{pl}^4$
$m << m_{pl}$


$\frac{1}{m_{pl}^2}\int_{\phi_i}^{\phi_f}\frac{V}{V'}d\phi$
EG: $V=\frac{1}{2}m^2\phi^2$
$\varepsilon_V=\eta_V=\frac{2m_{pl}^2}{\phi^2}<<1$
$\Rightarrow$
LARGE
ENERGY DENSITY $\propto m^2\phi^2<<m_{pl}^4$
$m<<m_{pl}$
$N\simeq\frac{1}{4m_{pl}^2}(\phi_i^2-\phi_f^2)$


$\int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$
EG: $V = \frac{1}{2} m^2 \phi^2$
$\varepsilon_V = \eta_V = \frac{2 m_{pl}^2}{\phi^2} << 1$
$\Rightarrow \phi >$
LARGE
ENERGY DENSITY $\propto m^2 \phi^2 << m_{pl}^4$
$N \simeq \frac{1}{4 m_{pl}^2} (\phi_i^2 - \phi_f^2) \approx 60$
$m << m_{pl}$


ϕ
$N \simeq \frac{1}{4\pi m_{pl}^2}(\phi_i - \phi_f) \sim 60$
$\phi_i \simeq 10\, m_{pl}$
$\phi_f \simeq \frac{1}{10}\frac{m_{pl}^2}{m}$
$\Rightarrow$
$\frac{m_{pl}}{m} \simeq 50$
✓


PROBLEM: OF INITIAL CONDITIONS FOR INFLATION;


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