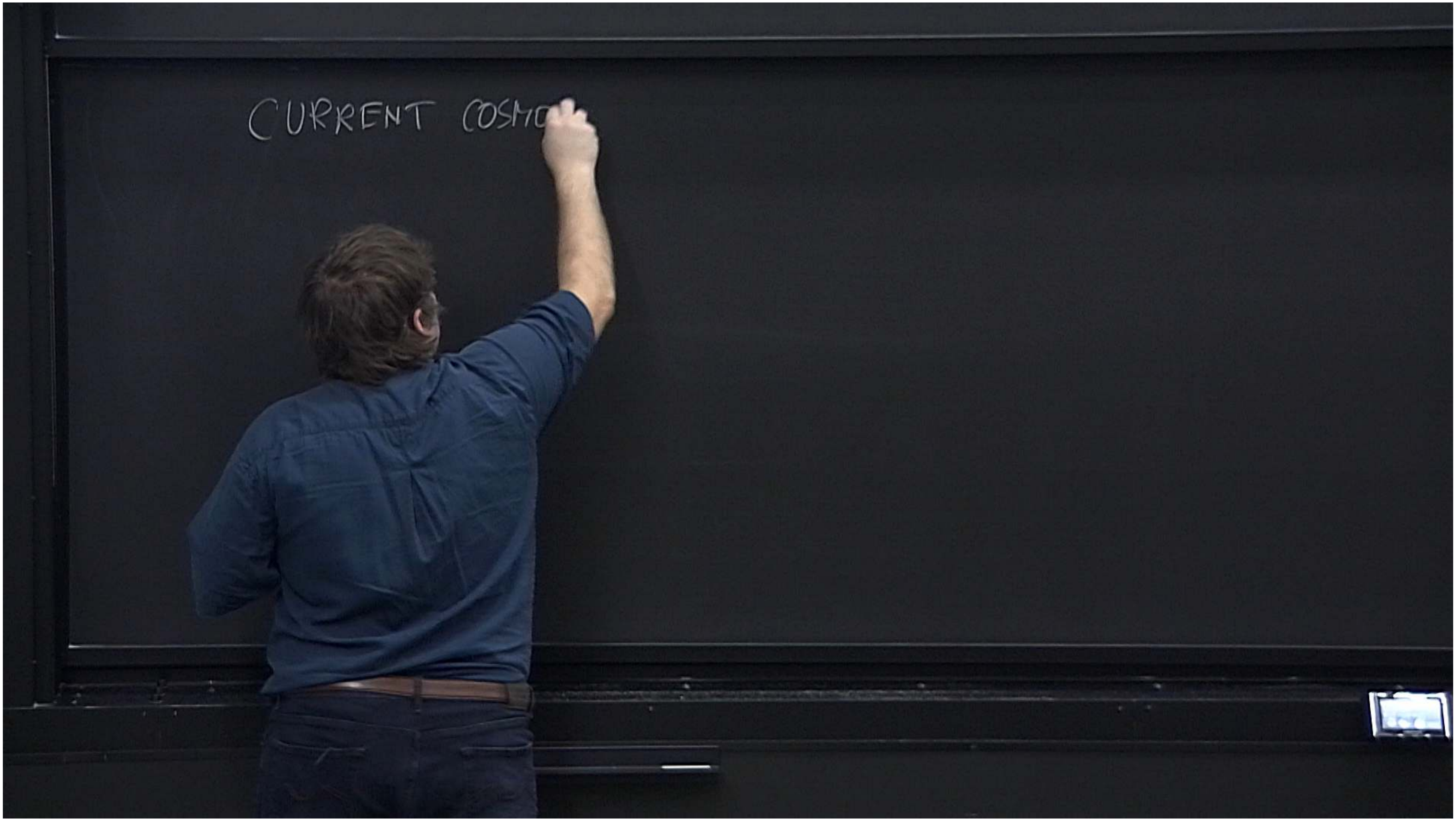
Title: PSI 2016/2017 Cosmology (Review) - Lecture 4

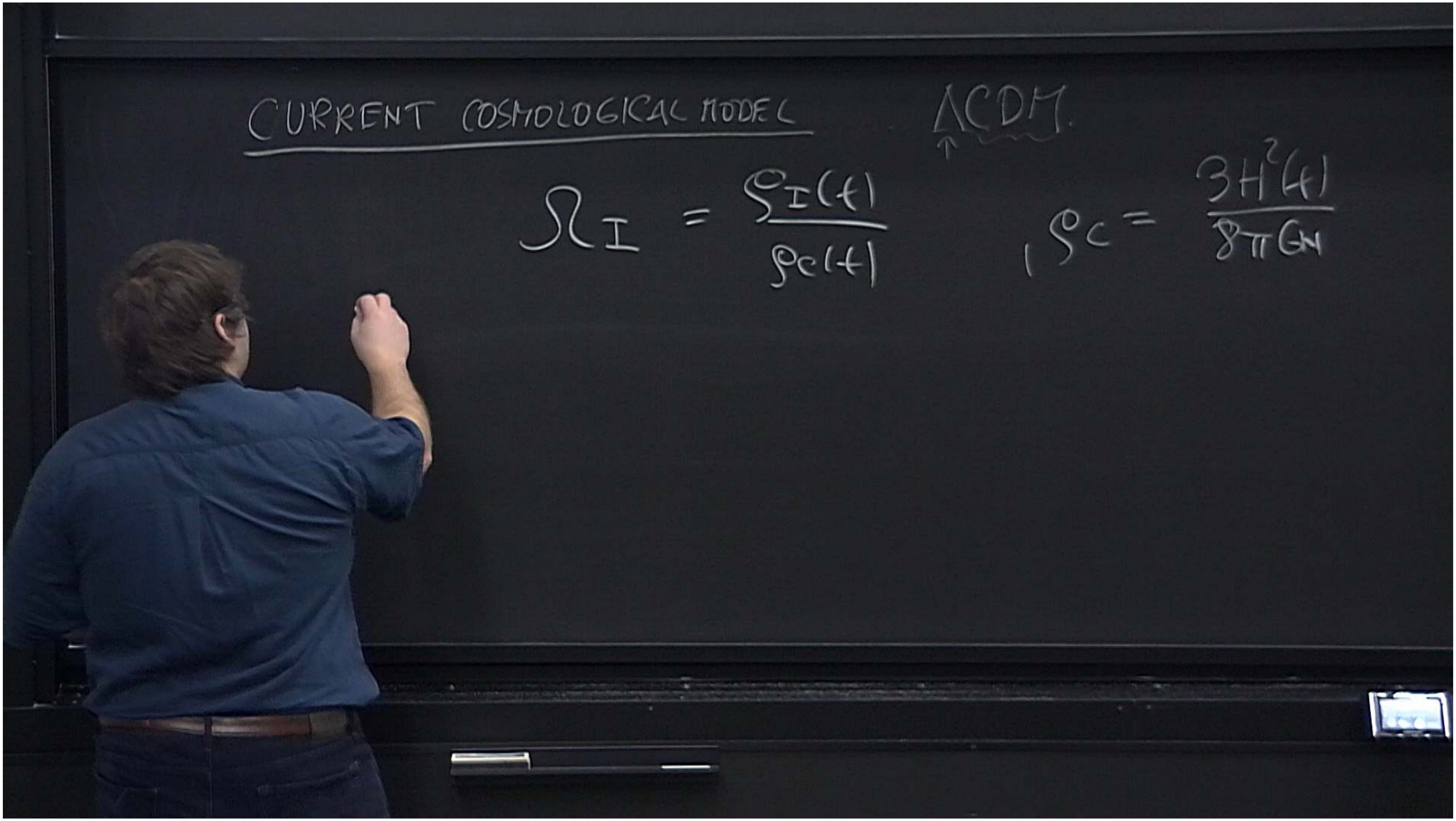
Date: Feb 03, 2017 10:15 AM

URL: http://pirsa.org/17020052

Abstract:

DOMINATING COMP	w	DENSITY	$a(\eta)$	$a(t)$
MATTER	0	$\rho \sim 1/a^3$	η^2	$t^{3/2}$
RADIATION	$1/3$	$\rho \sim 1/a^4$		


CURRENT COSM


CURRENT COSMOLOGICAL MODEL
ΛCDM.
$\Omega_I = \frac{\rho_I(t)}{\rho_c(t)}$
$, \rho_c = \frac{3H^2(t)}{8\pi G_N}$

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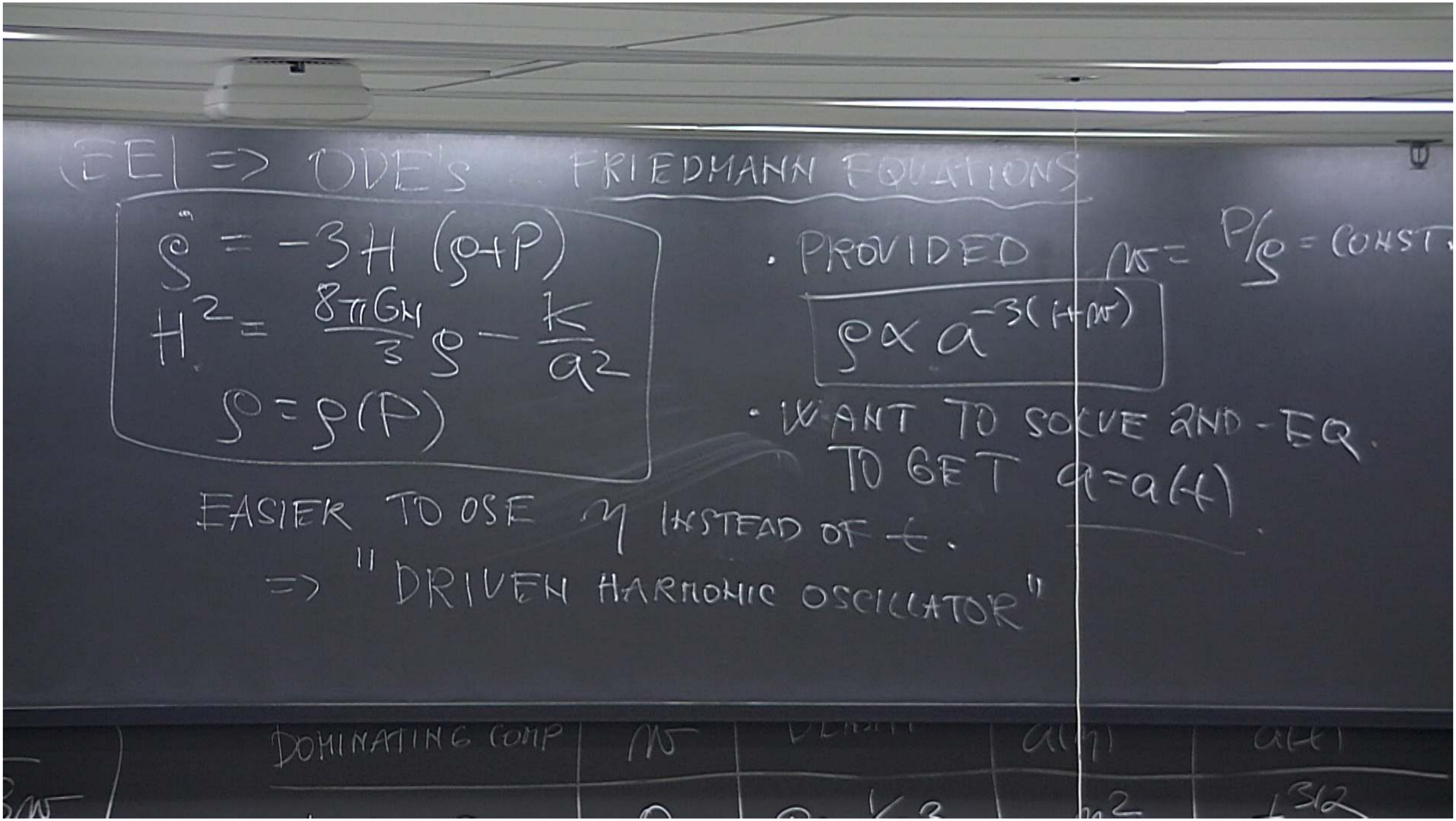
$$\Omega_I = \frac{\rho_I(t)}{\rho_c(t)} \quad , \quad \rho_c = \frac{3H^2(t)}{8\pi G_N}$$

CURREN MATTER CONTENT:

$$\Omega_m = 0.32 \begin{cases} \Omega_{DM} \simeq 0.27 \\ \Omega_B \simeq 0.05 \end{cases}$$


EEI => ODE's
FRIEDMANN EQUATIONS
$\dot{\rho} = -3H(\rho+P)$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$
$\rho = \rho(P)$
· PROVIDED
$w = P/\rho$ = CONST
$\rho \propto a^{-3(1+w)}$
· WANT TO SOLVE 2ND-EQ.
TO GET $a = a(t)$
EASIER TO USE η INSTEAD OF t.
=> "DRIVEN HARMONIC OSCILLATOR"
DOMINATING COMP
w
$a(\eta)$
$a(t)$

DYNAMICS OF FRW

HOM & ISOTROPY.

$$\boxed{\begin{aligned} ds^2 &= -dt^2 + a^2\, g^{(3,0)k}_{ij}\, dx^i dx^j \\ T_{\mu\nu} &= P g_{\mu\nu} + (\rho + P) u_\mu u_\nu \end{aligned}}$$

$$\Rightarrow \boxed{a(t),\ P(t),\ \rho(t)}$$

$$\Rightarrow \boxed{a(\eta) = C\, S_K^{\alpha}(\eta/\alpha),\quad \alpha = \frac{2}{1+3w}}$$

(EE) $\Rightarrow$ D

$\dot{\rho} = -3$

$H^2 = \frac{8}{}$

$\rho = \rho$

EASIER

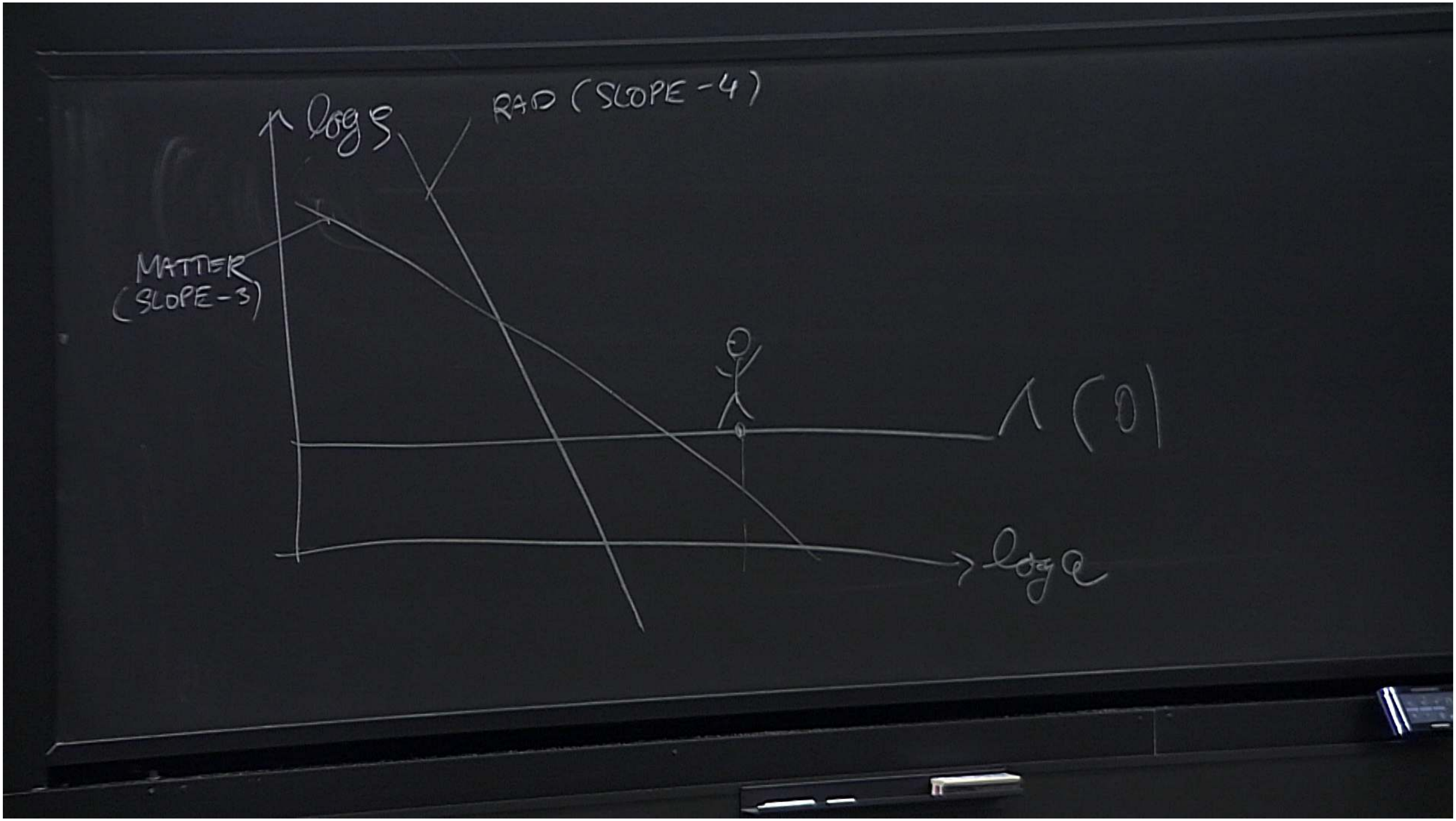
$\Rightarrow$

DOM

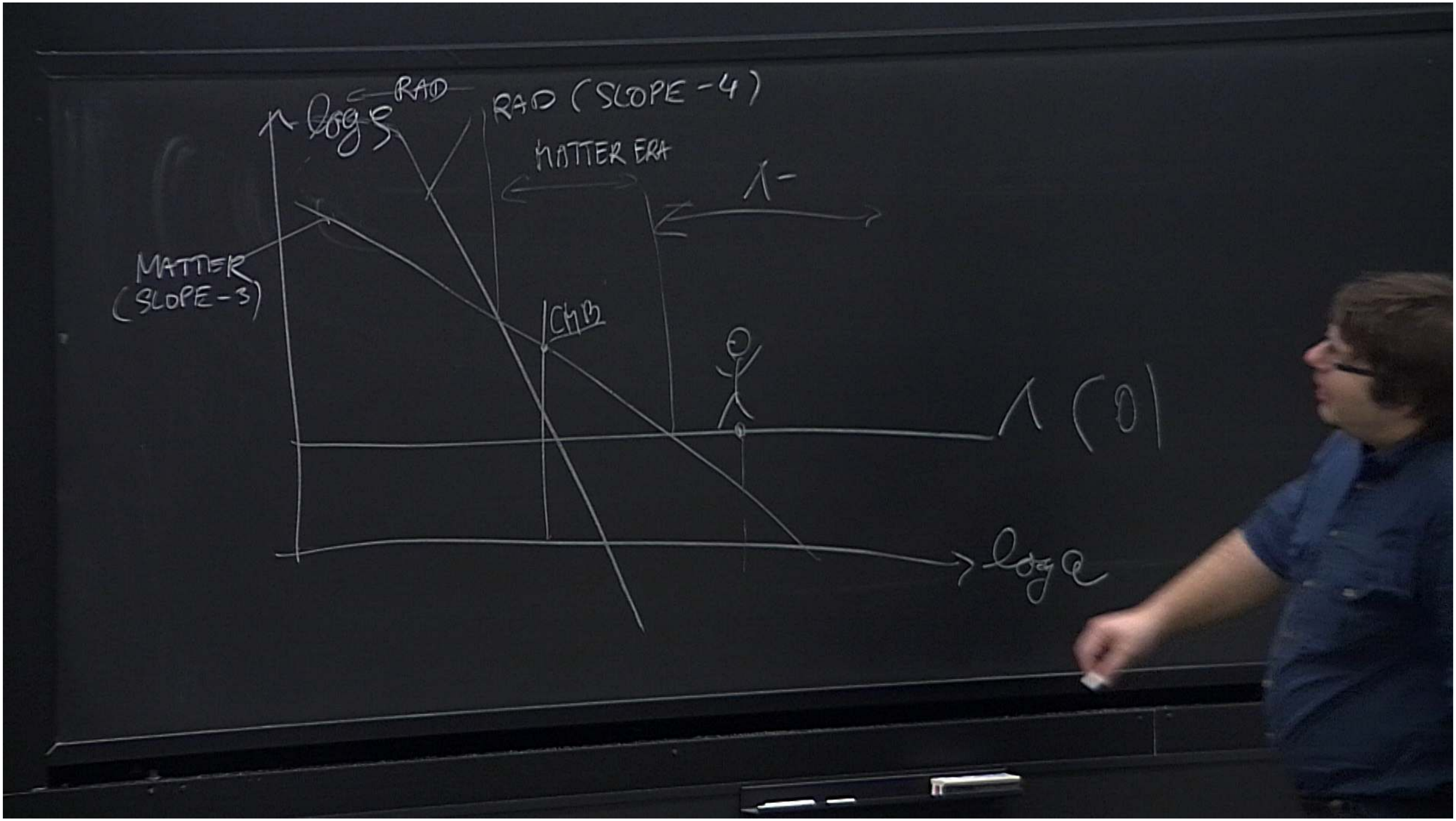
$\Omega_r \simeq 9.4 \times 10^{-5}$

$\Omega_\Lambda \simeq 0.68$

$\Omega_K \leq 0.01$



log ρ
RAD (SLOPE -4)
MATTER
(SLOPE -3)
Λ (0)
log a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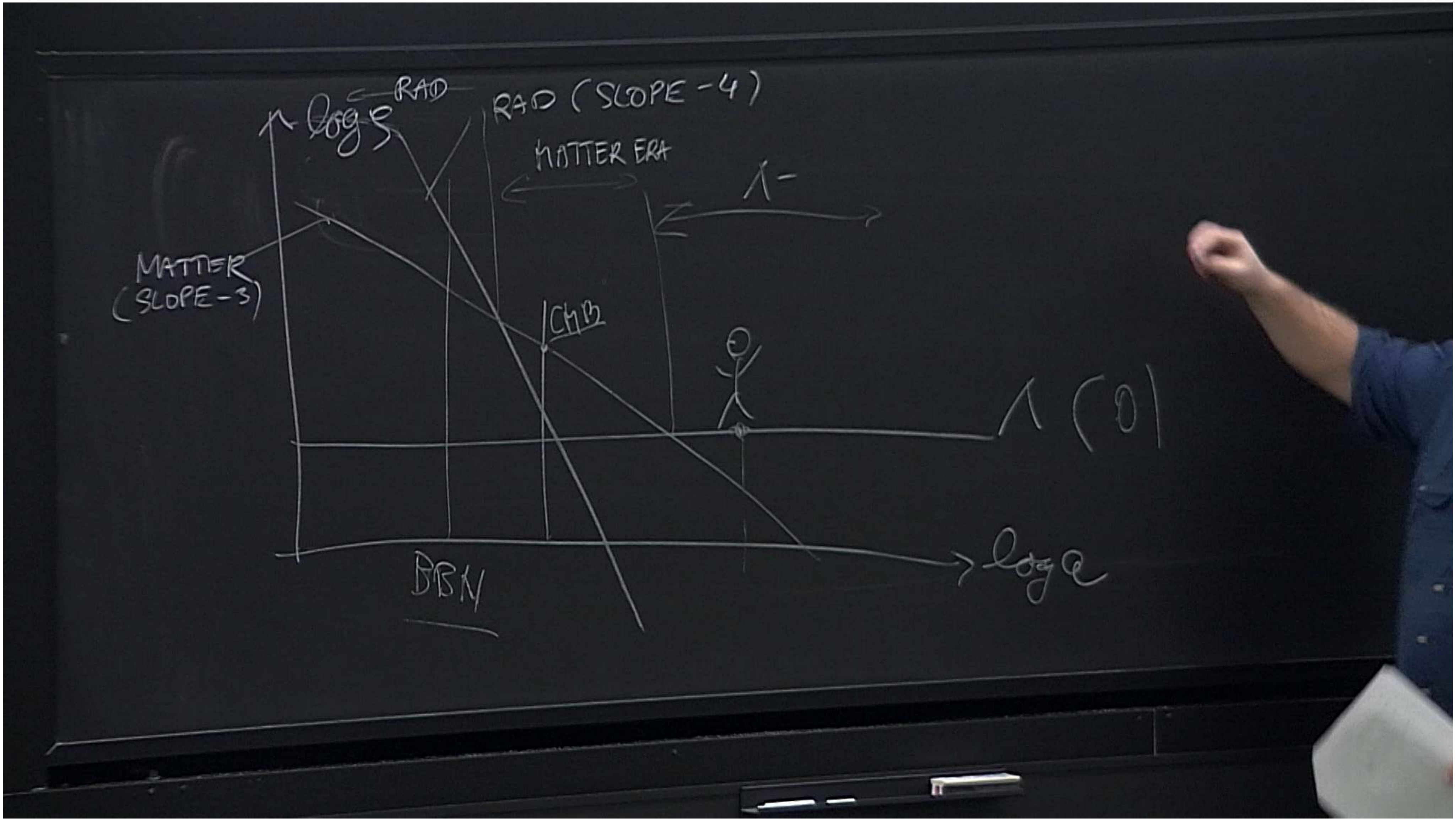
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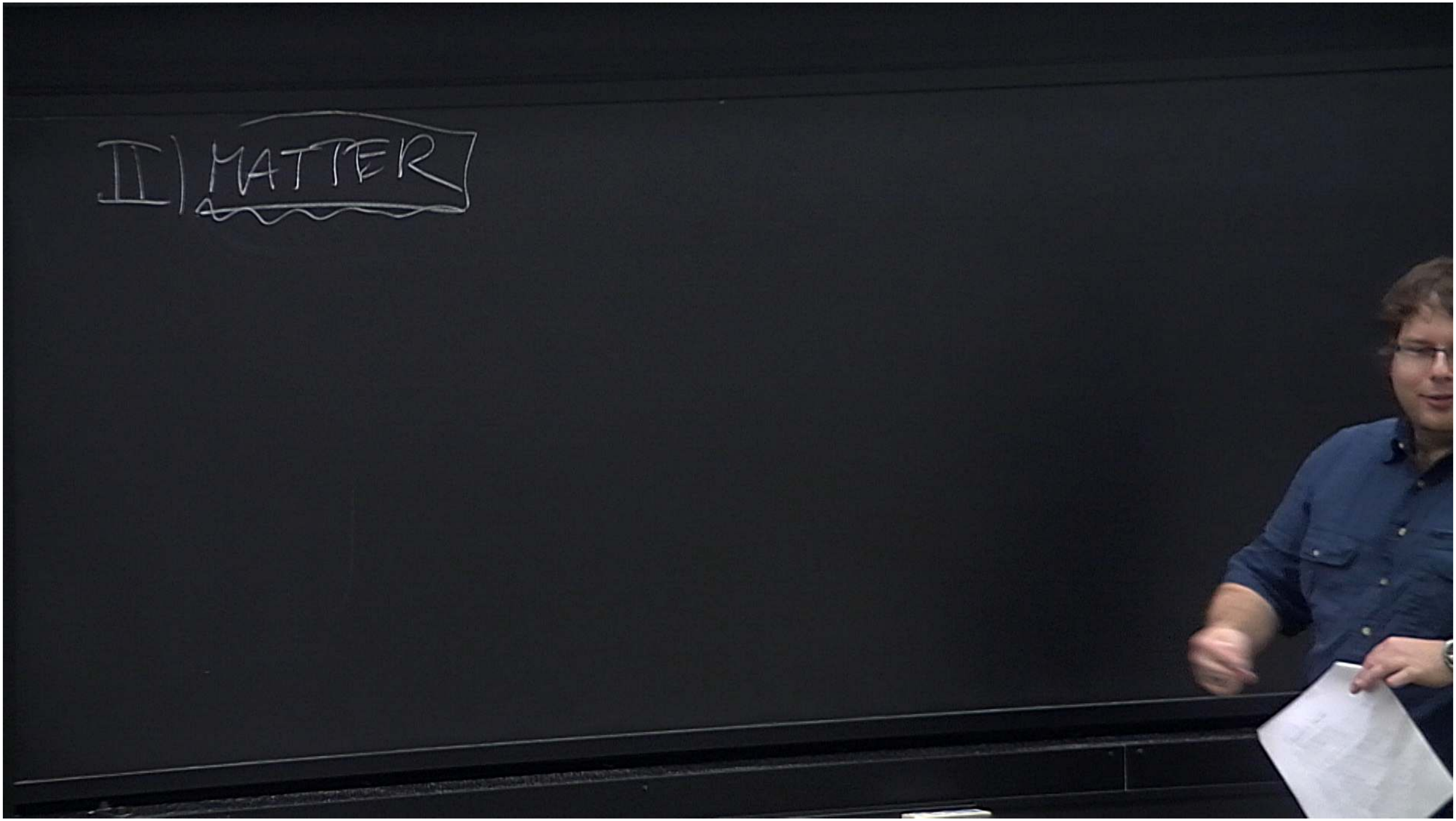
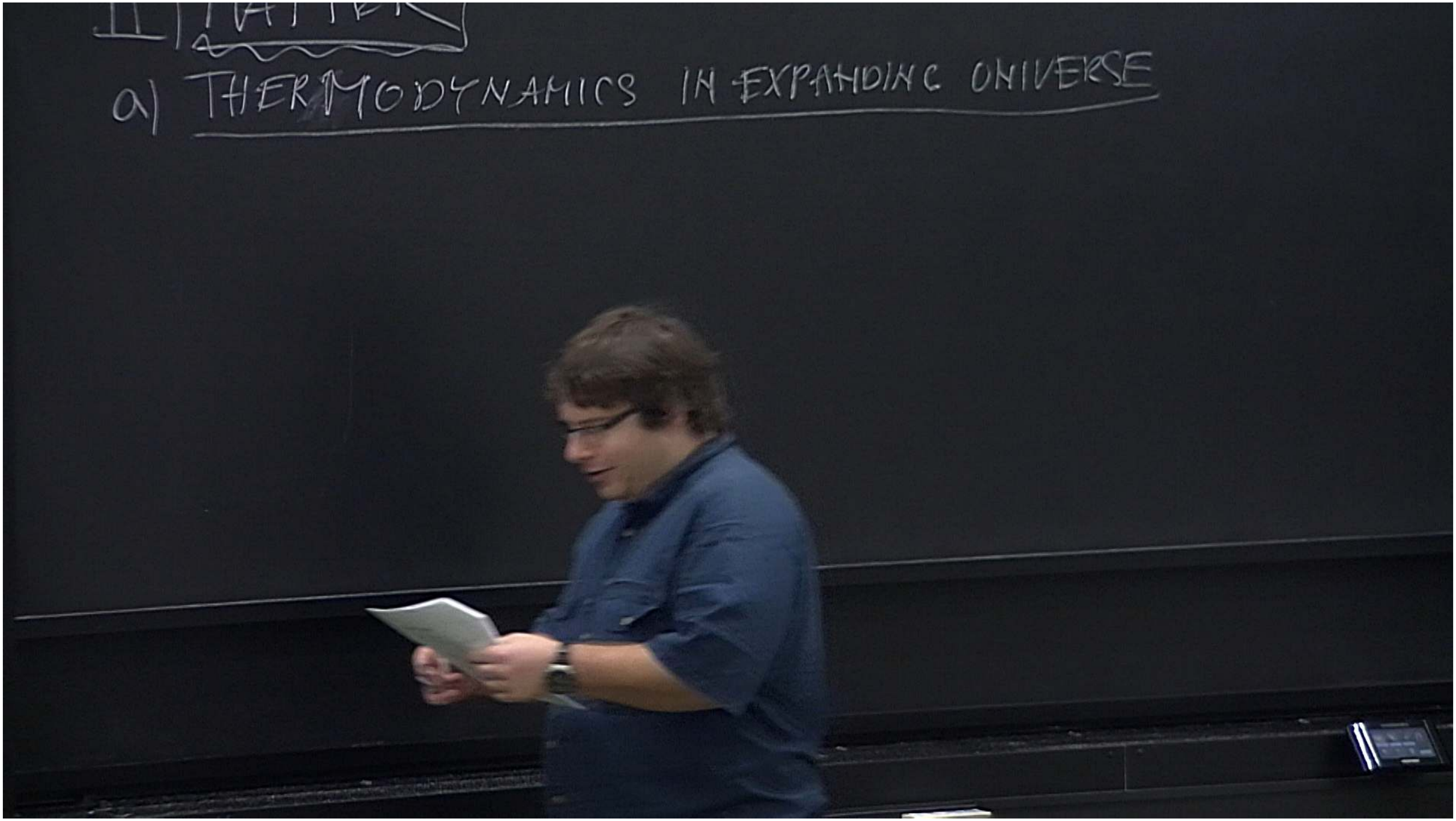
BBN

ANTHROPIC PRINCIPLE: THE UNIVERSE LOOK LIKE OURS BECAUSE WE ARE HERE TO OBSERVE IT.


II) MATTER


a) THERMODYNAMICS IN EXPANDING UNIVERSE

II) MATTER

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- DISTRIBUTION FUNCTION $f(\vec{x},\vec{p})$

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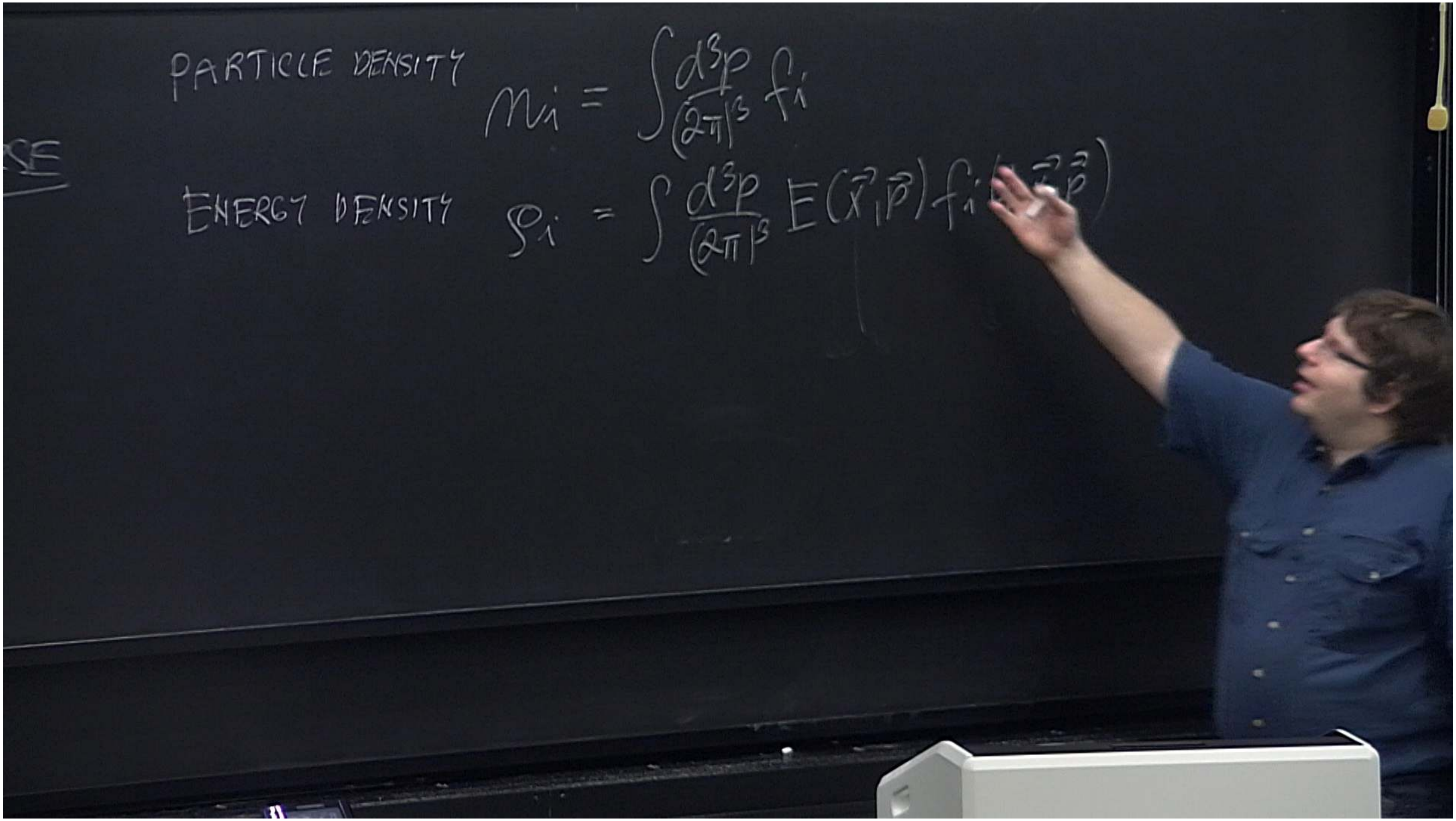
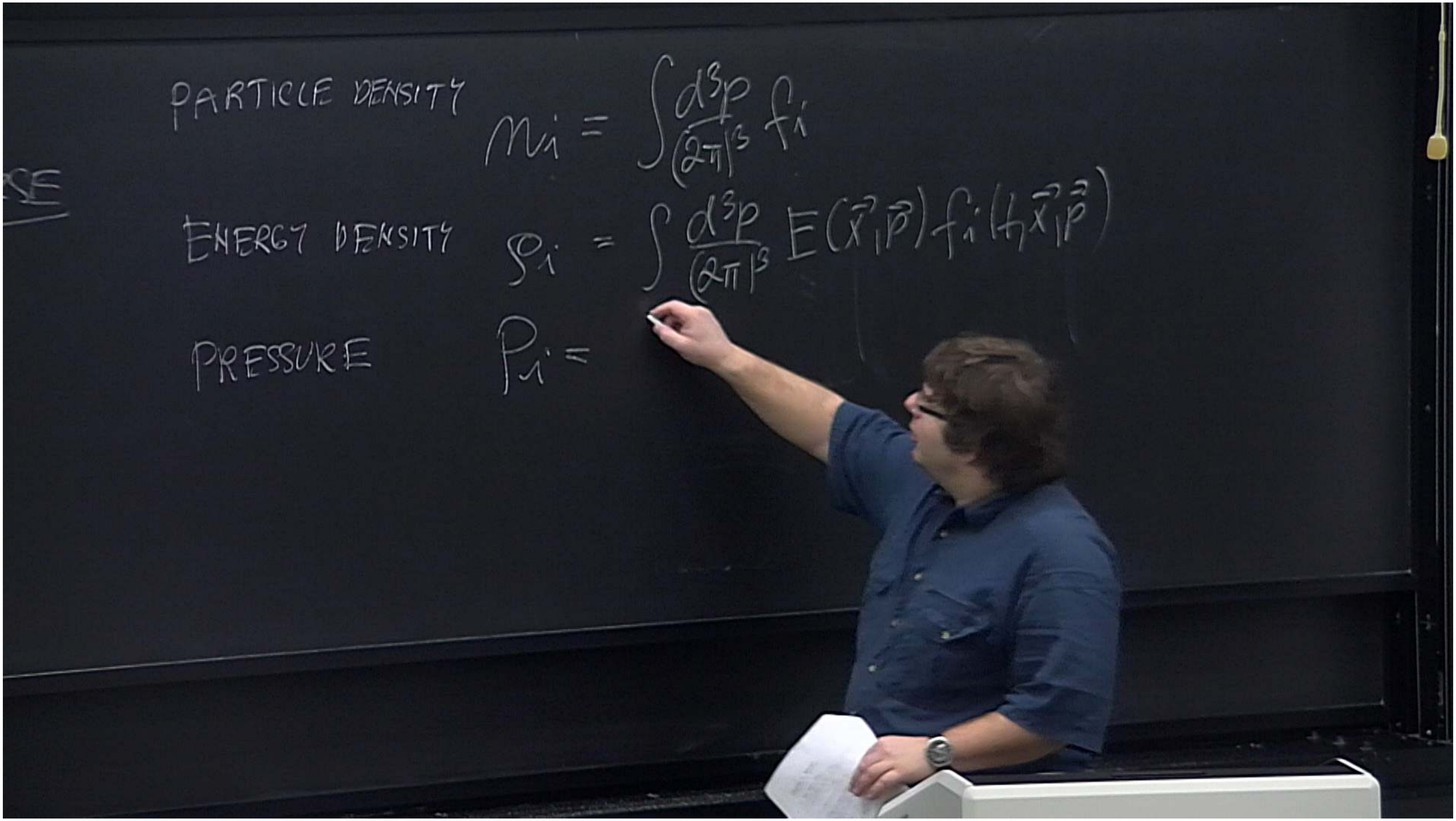
#PARTICLES = $dN_1 =$


PARTICLE DENSITY $n_i = \int \frac{d^3p}{(2\pi)^3} f_i$
ENERGY DENSITY $\rho_i = \int \frac{d^3p}{(2\pi)^3} E(\vec{x},\vec{p}) f_i(\vec{x},\vec{p})$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PRESSURE $P_i =$

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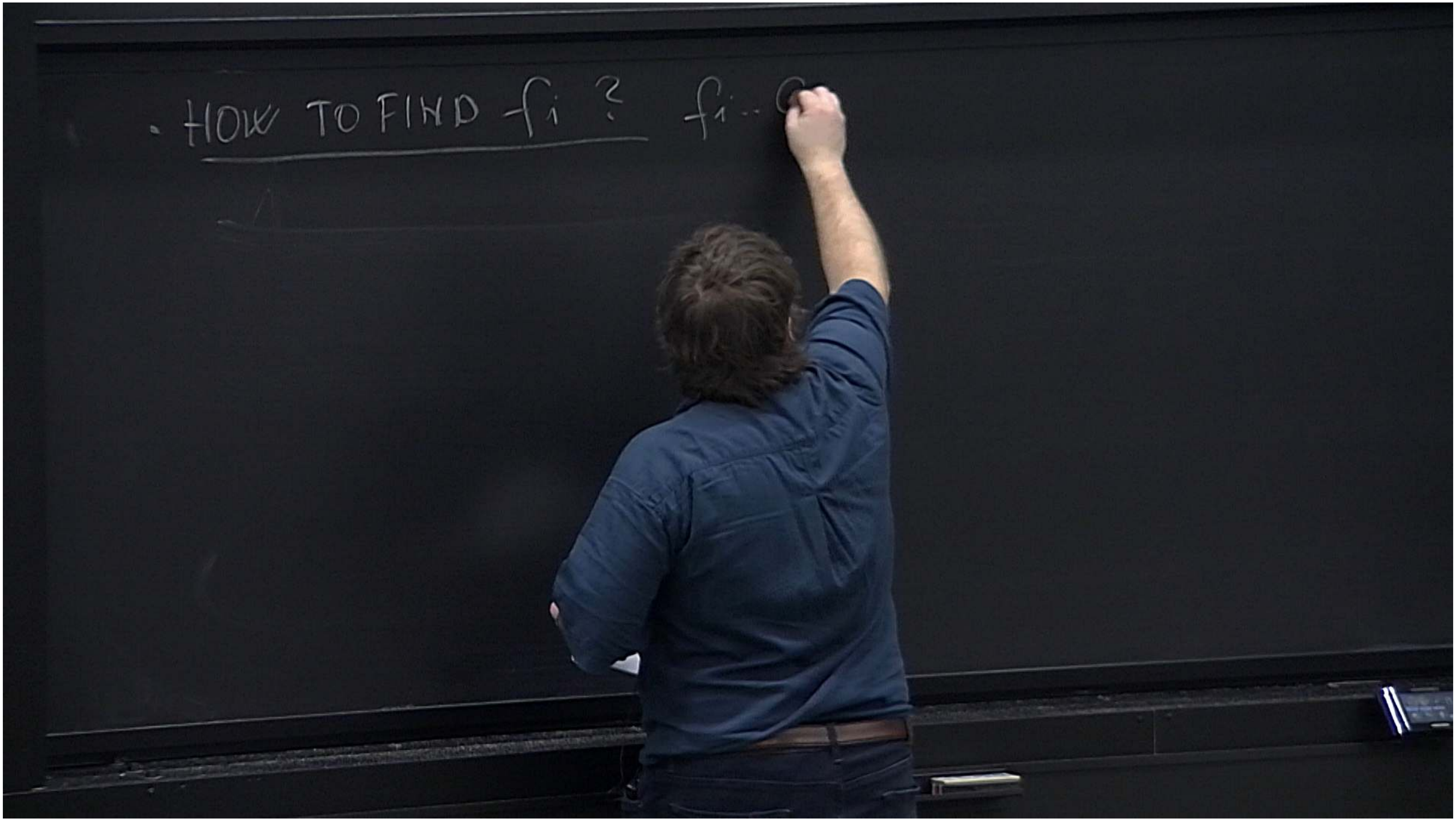
ENERGY DENSITY $\rho_i = \int \frac{d^3p}{(2\pi)^3} E(\vec{x},\vec{p}) f_i(t,\vec{x},\vec{p})$

PRESSURE $P_i = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{3E} f_i$


HOW TO FIND fi ?
fi ..

HOW TO FIND f_i ? f_i .. GOVERNED BY

BOLTZMANN EQUATION :

$$\boxed{\frac{df_i}{dt} = \frac{\partial f_i}{\partial t} + \frac{\partial f_i}{\partial x_j}\dot{x}_j + \frac{\partial f_i}{\partial p_j}\dot{p}_j = \mathrm{Col}_i(f_j)}$$

HOW TO FIND f_i ? f_i.. GOVERNED BY

HIDING DIRTY PHYSICS

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INSTEAD WE CONSIDER "LOCAL THERMAL EQUILIBRIUM"

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BOLTZMANN EQUATION :

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INSTEAD WE CONSIDER "LOCAL THERMAL EQUILIBRIUM"

$$\frac{1}{\Gamma} = \text{COLLISION TIME} \ll \text{EXPANSION TIME} \simeq t_H = \frac{1}{H}$$

REACTION RATE

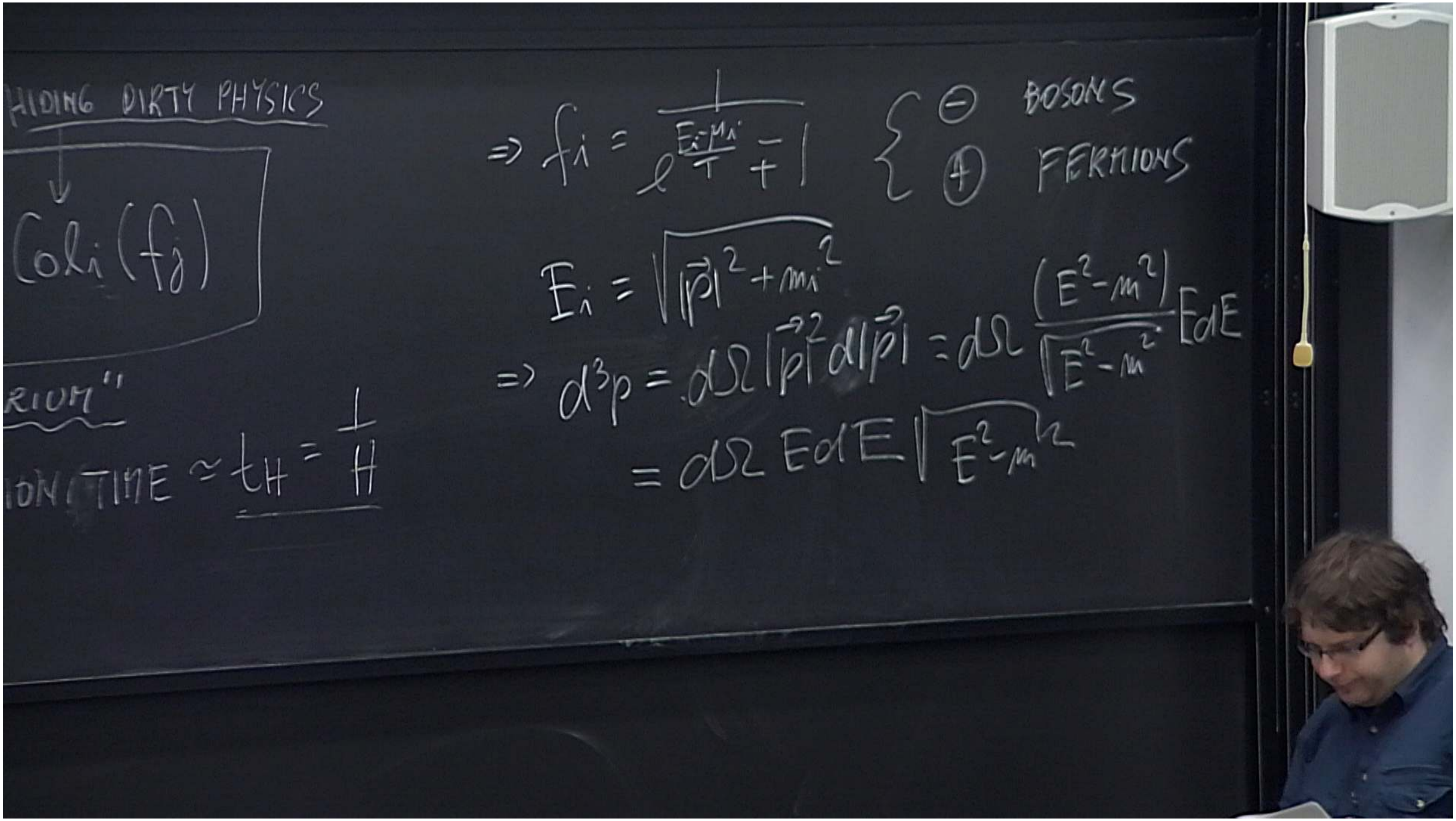
HIDING DIRTY PHYSICS

$$\boxed{\mathrm{Coll}_i(f_j)}$$

RIUM"

ION TIME $\simeq t_H = \frac{1}{H}$

$$\Rightarrow f_i = \frac{1}{e^{\frac{E-\mu_i}{T}} \mp 1} \quad \begin{cases} \ominus & \text{BOSONS} \\ \oplus & \text{FERMIONS} \end{cases}$$


HIDING DIRTY PHYSICS
$Col_i(f_j)$
RIUM"
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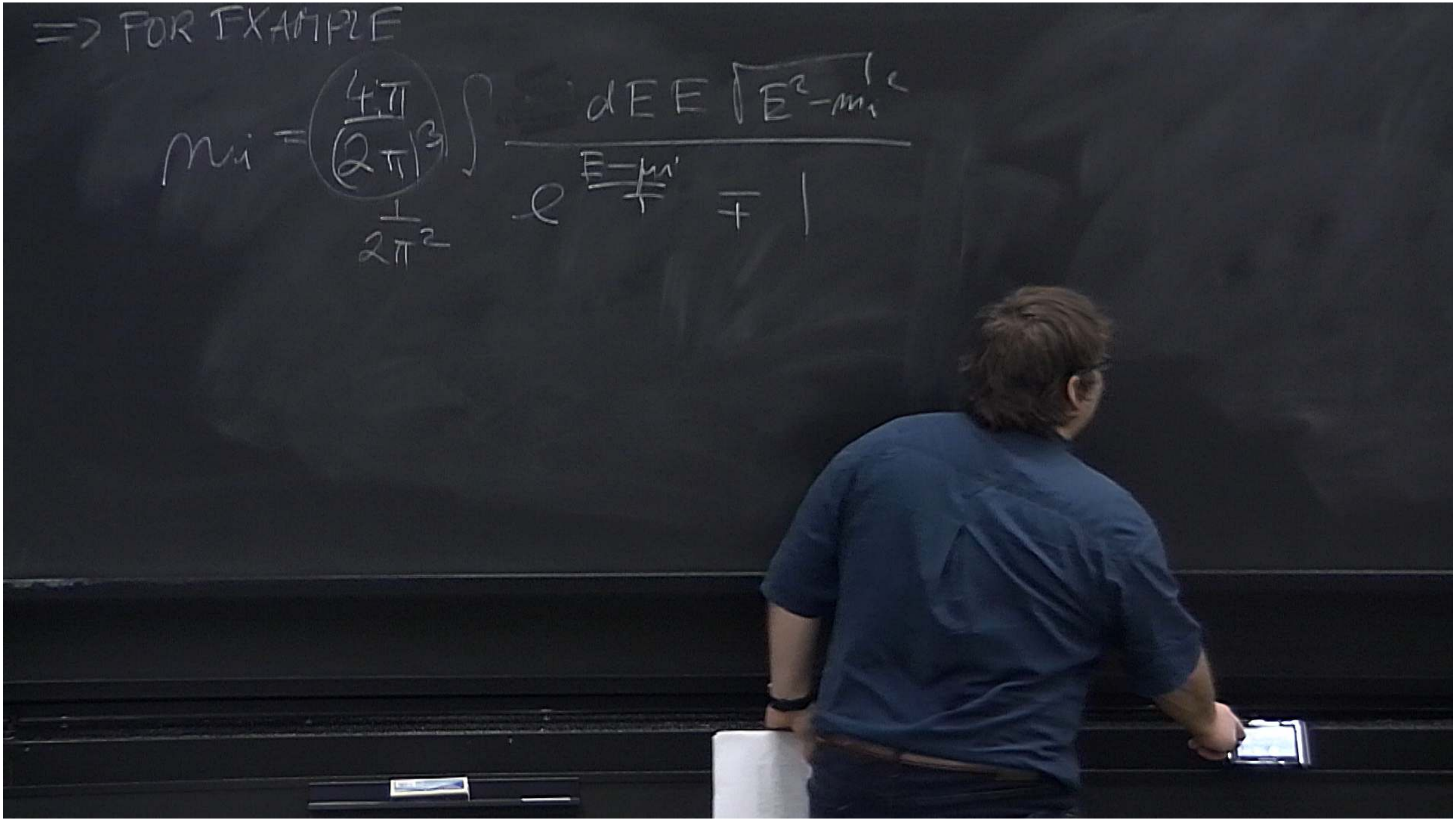
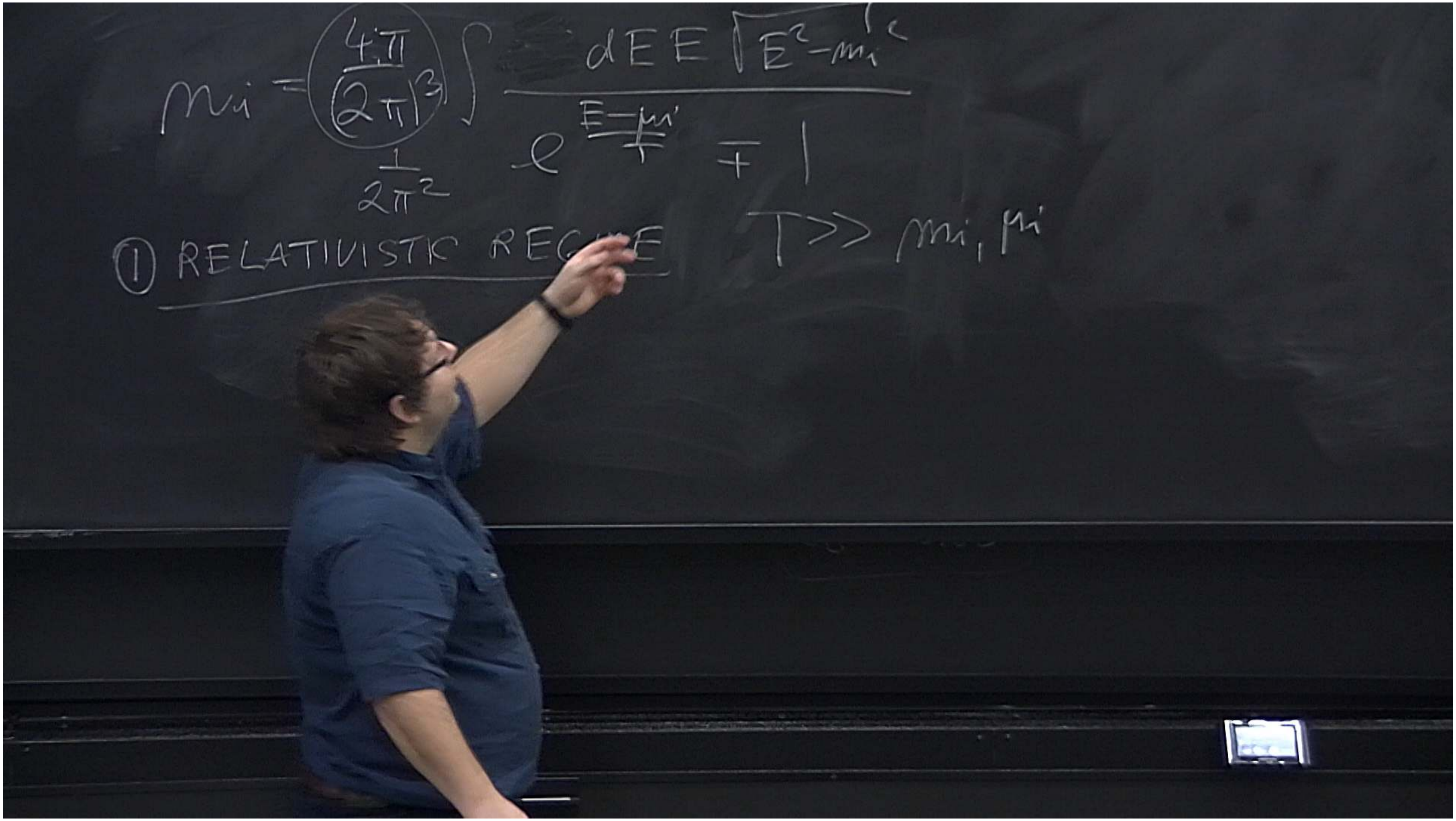
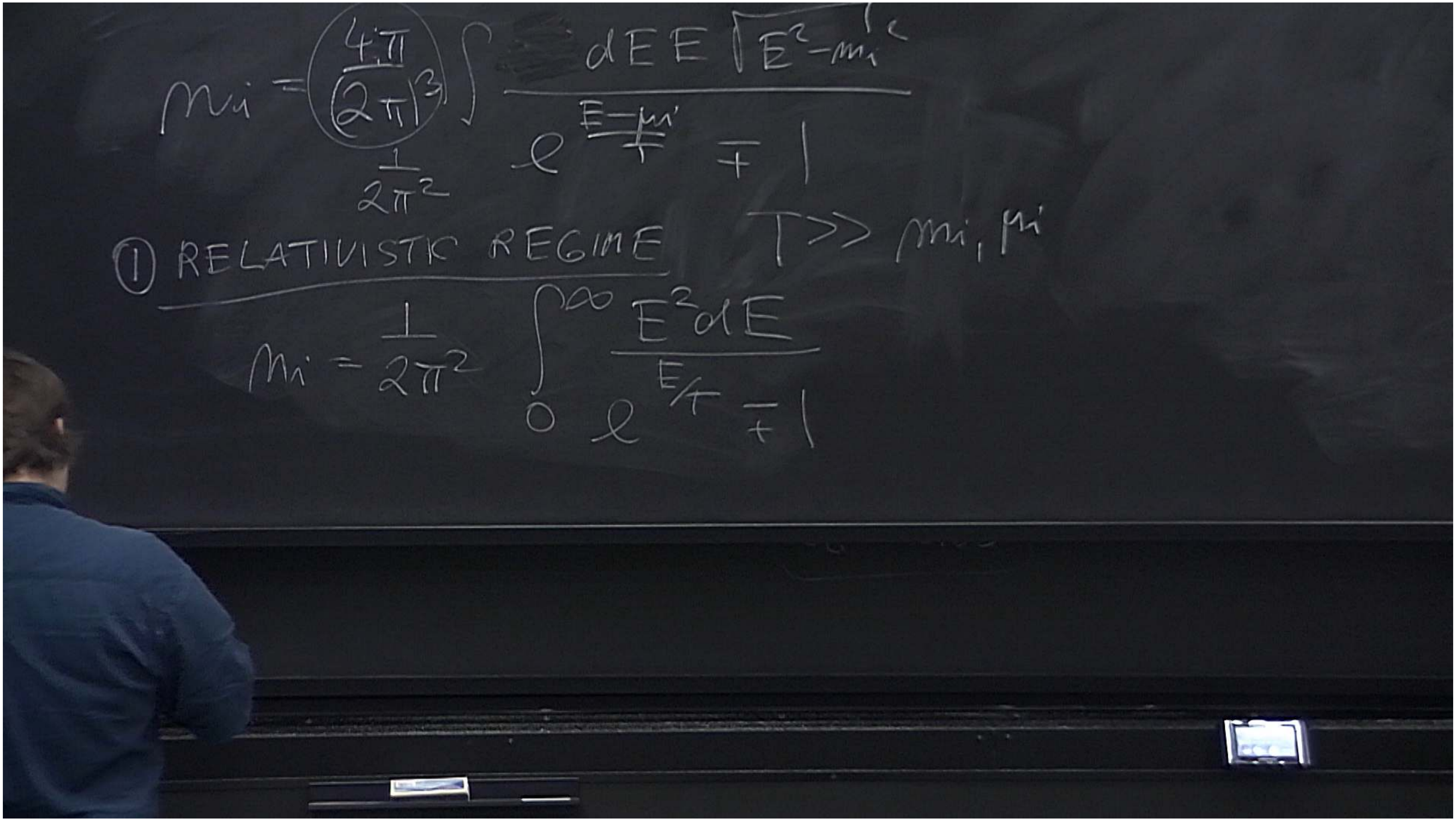
$\Rightarrow f_i = \frac{1}{e^{\frac{E_i - \mu_i}{T}} \mp 1}$
$\ominus$ BOSONS
$\oplus$ FERMIONS
$E_i = \sqrt{|\vec{p}|^2 + m_i^2}$
$\Rightarrow d^3p = d\Omega\, |\vec{p}|^2\, d|\vec{p}| = d\Omega \frac{(E^2 - m^2)}{\sqrt{E^2 - m^2}} E\, dE$
$= d\Omega\, E\, dE \sqrt{E^2 - m^2}$


⇒ FOR EXAMPLE
$n_i = \frac{4\pi}{(2\pi)^3} \int \frac{dE\, E \sqrt{E^2 - m_i^2}}{e^{\frac{E-\mu_i}{T}} \mp 1}$
$\frac{1}{2\pi^2}$


n_i = 4π/(2π)^3 ∫ dE E √(E^2 − m_i^2) / (e^((E − μ_i)/T) ∓ 1)
1/(2π^2)
① RELATIVISTIC REGIME
T >> m_i, μ_i



$n_i = \frac{4\pi}{(2\pi)^3} \int \frac{dE\, E \sqrt{E^2 - m_i^2}}{e^{\frac{E-\mu_i}{T}} \mp 1}$
$\frac{1}{2\pi^2}$
① RELATIVISTIC REGIME $T \gg m_i, \mu_i$
$n_i = \frac{1}{2\pi^2} \int_0^\infty \frac{E^2\, dE}{e^{E/T} \mp 1}$

⇒ FOR EXAMPLE

$$n_i = \frac{4\pi}{(2\pi)^3} \int \frac{dE\, E \sqrt{E^2 - m_i^2}}{e^{\frac{E-\mu_i}{T}} \mp 1}$$

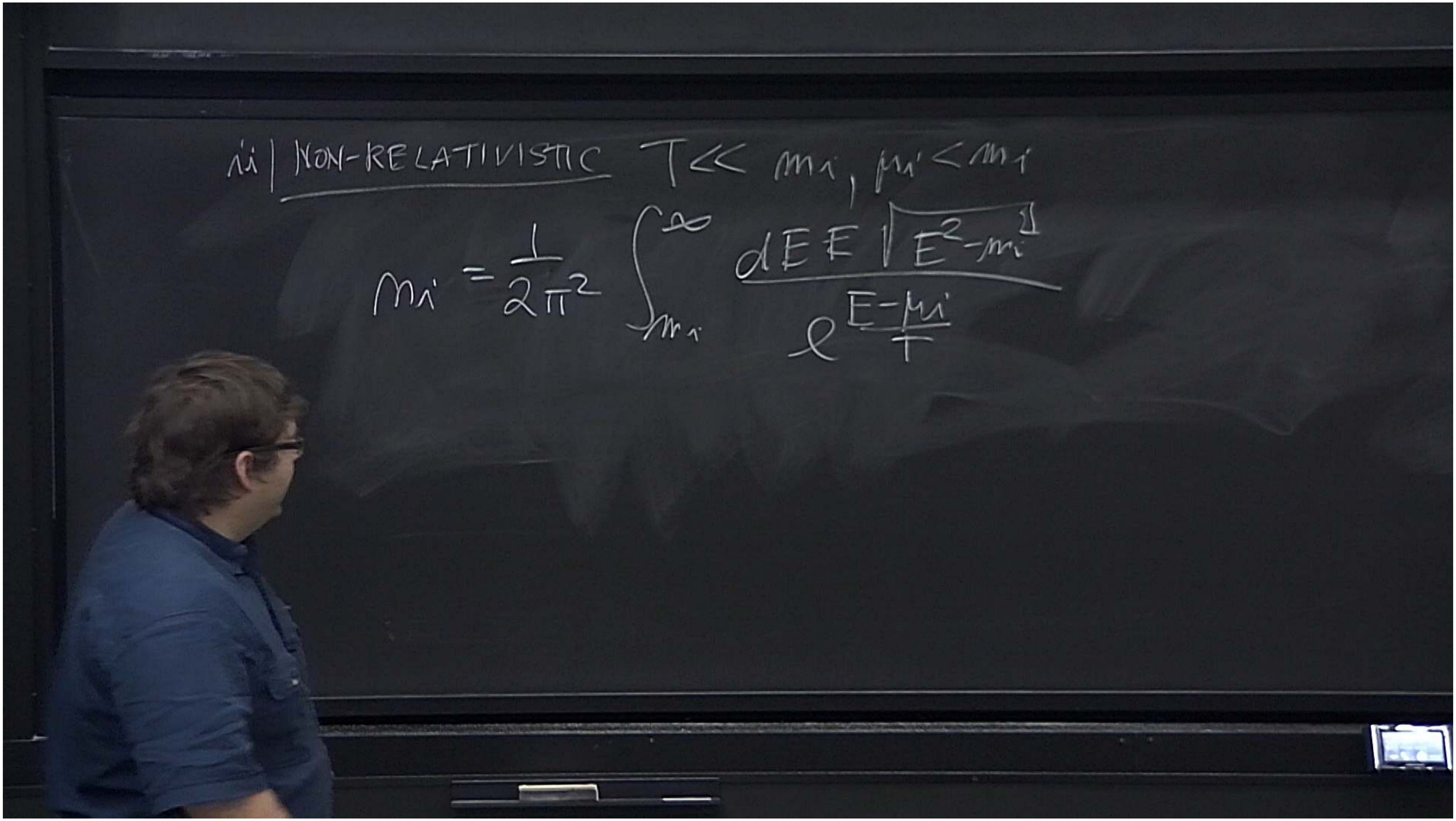
$$\frac{1}{2\pi^2}$$

① RELATIVISTIC REGIME $\quad T \gg m_i, \mu_i$

$$n_i = \frac{1}{2\pi^2} \int_0^\infty \frac{E^2\, dE}{e^{E/T} \mp 1} = \left| \begin{array}{l} x = E/T \\ dE = T\,dx \end{array} \right| = \frac{T^3}{2\pi^2}$$

ii) <u>NON-RELATIVISTIC</u> $T \ll m_i$, $\mu_i < m_i$

$$n_i = \frac{1}{2\pi^2} \int$$


ii) NON-RELATIVISTIC $T \ll m_i$, $\mu_i < m_i$
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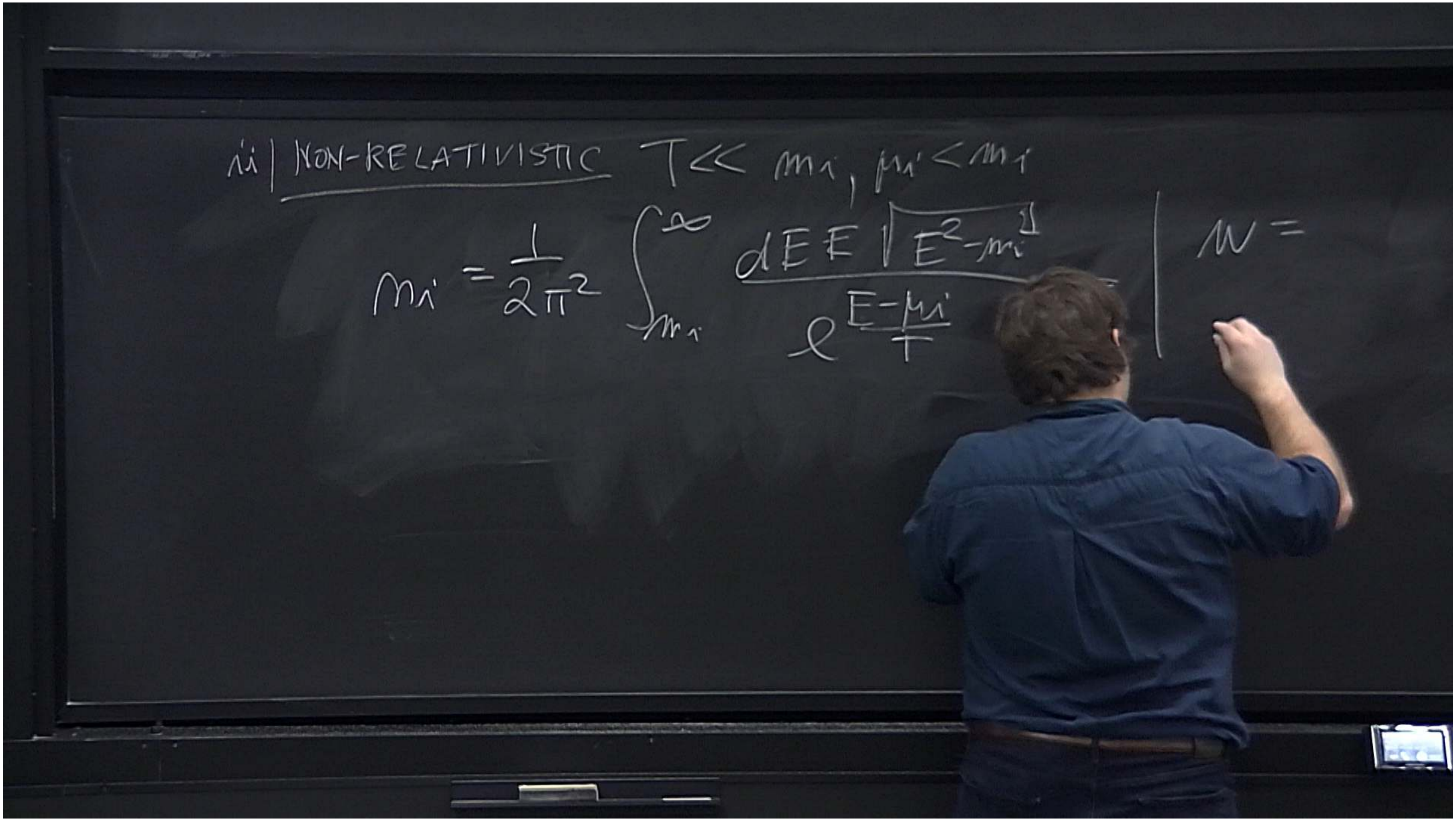
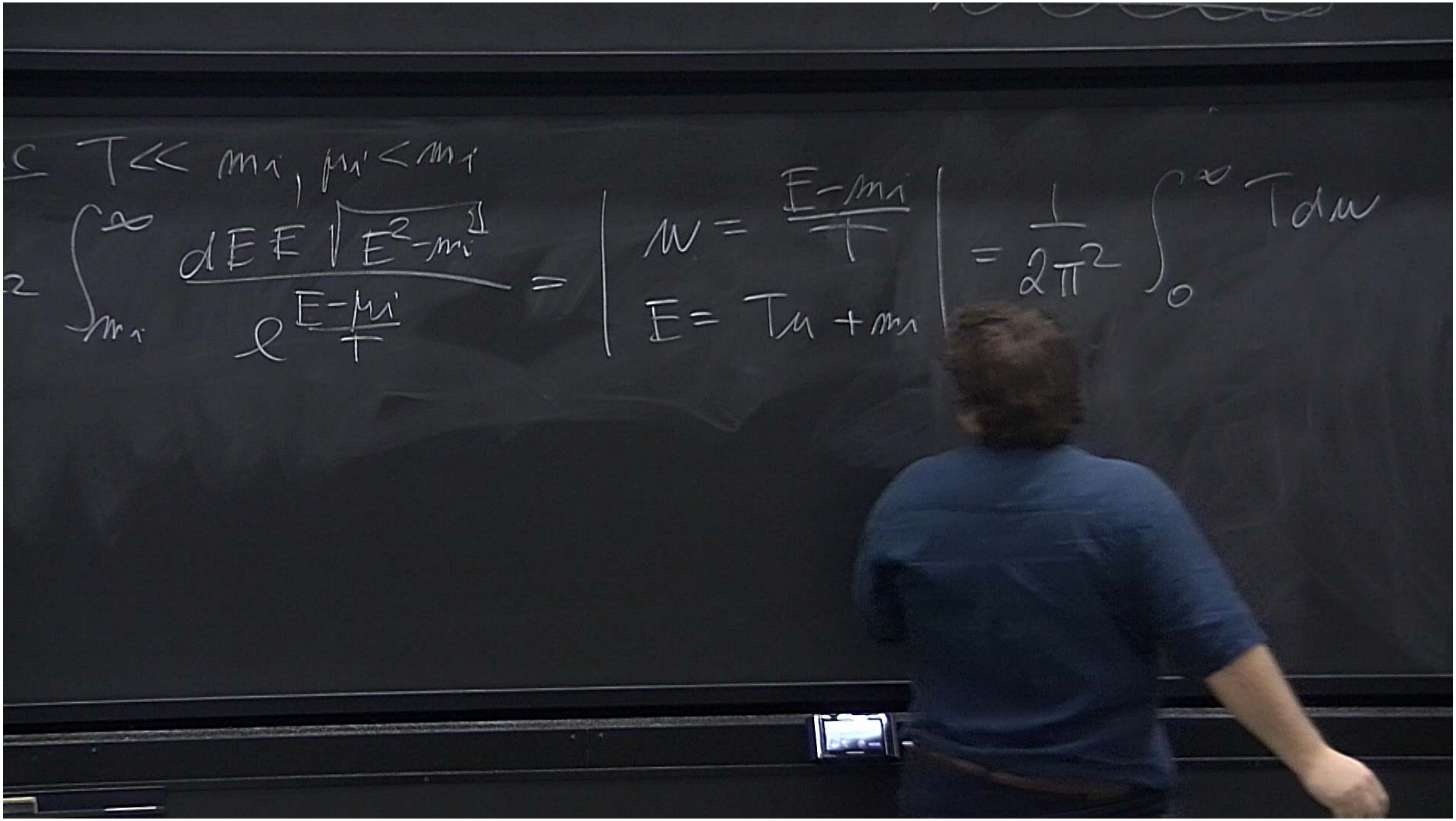
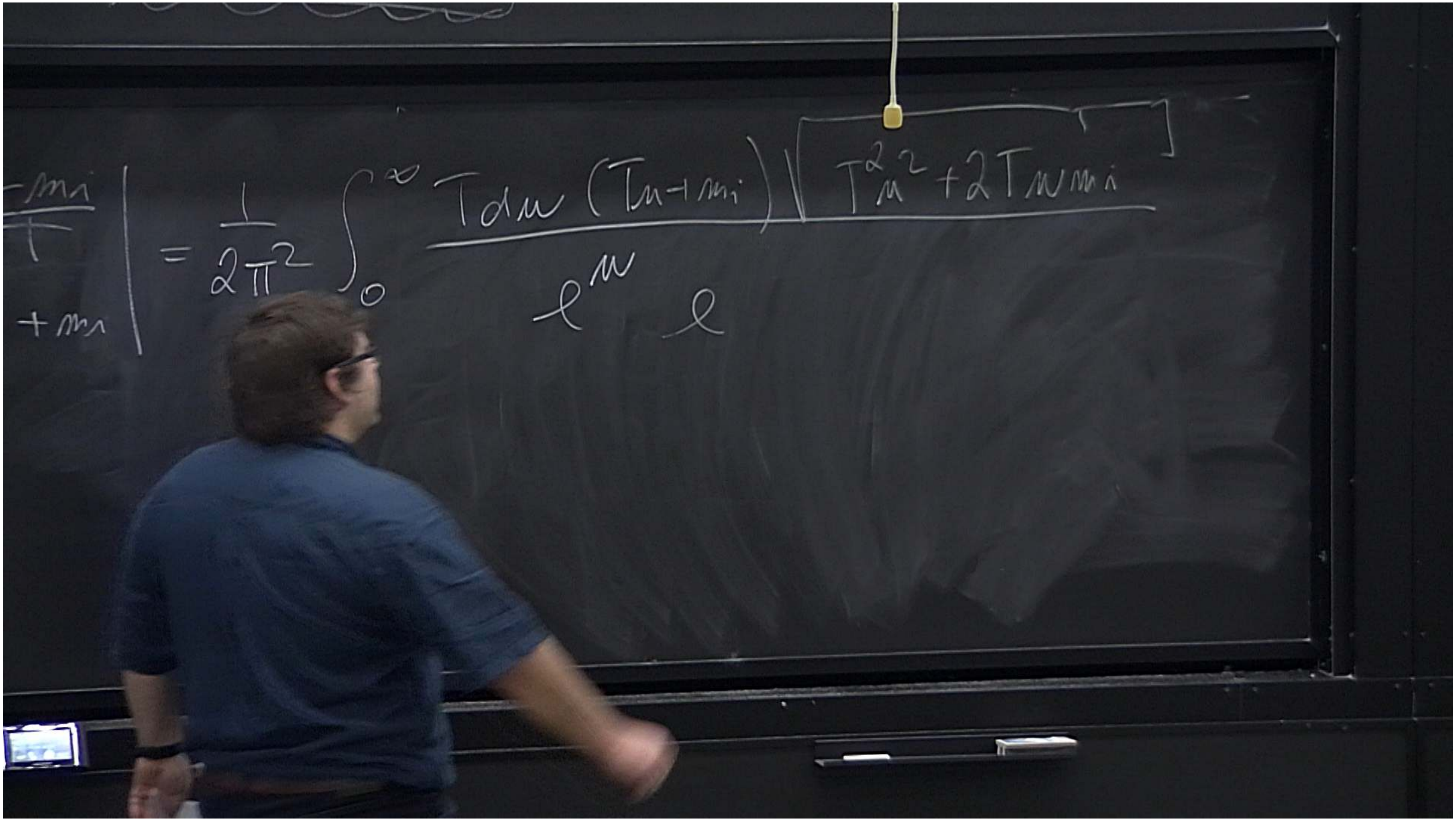

ii) NON-RELATIVISTIC T << m_i , μ_i < m_i
n_i = 1/(2π²) ∫_{m_i}^∞ dE E √(E² − m_i²) / e^{(E − μ_i)/T}
n =

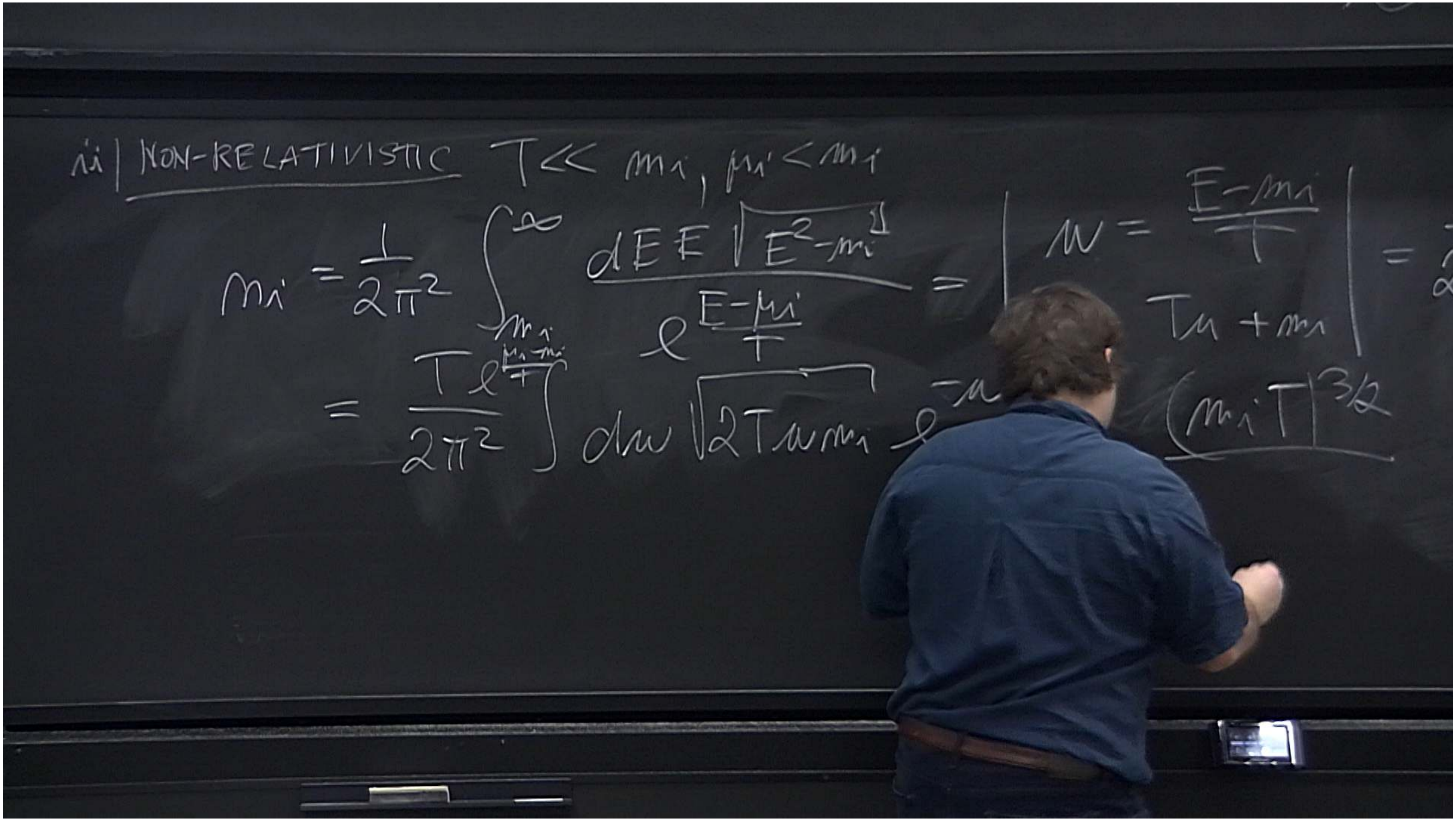

$T \ll m_i,\ \mu_i < m_i$
$\int_{m_i}^{\infty} \frac{dE\, E \sqrt{E^2 - m_i^2}}{e^{\frac{E-\mu_i}{T}}} =$
$u = \frac{E - m_i}{T}$
$E = Tu + m_i$
$= \frac{1}{2\pi^2} \int_0^{\infty} T du$

$T \ll m_i, \ \mu_i < m_i$

$$\int_{m_i}^{\infty} \frac{dE\, E \sqrt{E^2 - m_i^2}}{e^{\frac{E-\mu_i}{T}}} = \left| \begin{array}{l} u = \frac{E - m_i}{T} \\ E = Tu + m_i \end{array} \right| = \frac{1}{2\pi^2} \int_0^{\infty} T\, du\, (T$$


$= \frac{1}{2\pi^2} \int_0^\infty \frac{T\,du\,(Tu + m_i)\sqrt{T^2u^2 + 2Tu\,m_i}}{e^u - 1}$

$$\frac{\;-m_i}{T} \qquad + m_i \Bigg| = \frac{1}{2\pi^2}\int_0^\infty \frac{T\,du\,(Tu + m_i)\sqrt{T^2u^2 + 2Tu\,m_i}}{e^{u}\;e^{\frac{m_i-\mu_i}{T}}}$$


ii) NON-RELATIVISTIC T << m_i, μ_i < m_i
n_i = 1/2π² ∫_{m_i}^∞ dE E √(E² − m_i²) / e^{(E−μ_i)/T} =
u = (E − m_i)/T
Tu + m_i
= T e^{(μ_i − m_i)/T} / 2π² ∫ du √(2Tu m_i) e^{−u}
(m_i T)^{3/2}

$$u = \frac{E - m_i}{T} \qquad E = Tu + m_i$$

$$= \frac{1}{2\pi^2}\int_0^\infty \frac{T\,du\,(\cancel{T}u + m_i)\sqrt{\cancel{T^2u^2} + 2Tum_i}}{e^{u}\, e^{\frac{m_i - \mu_i}{T}}}$$

$$= \frac{(m_i T)^{3/2}}{2\pi^2}\, e^{\frac{\mu_i - m_i}{T}} \int \sqrt{2u}\, e^{-u}\, du$$

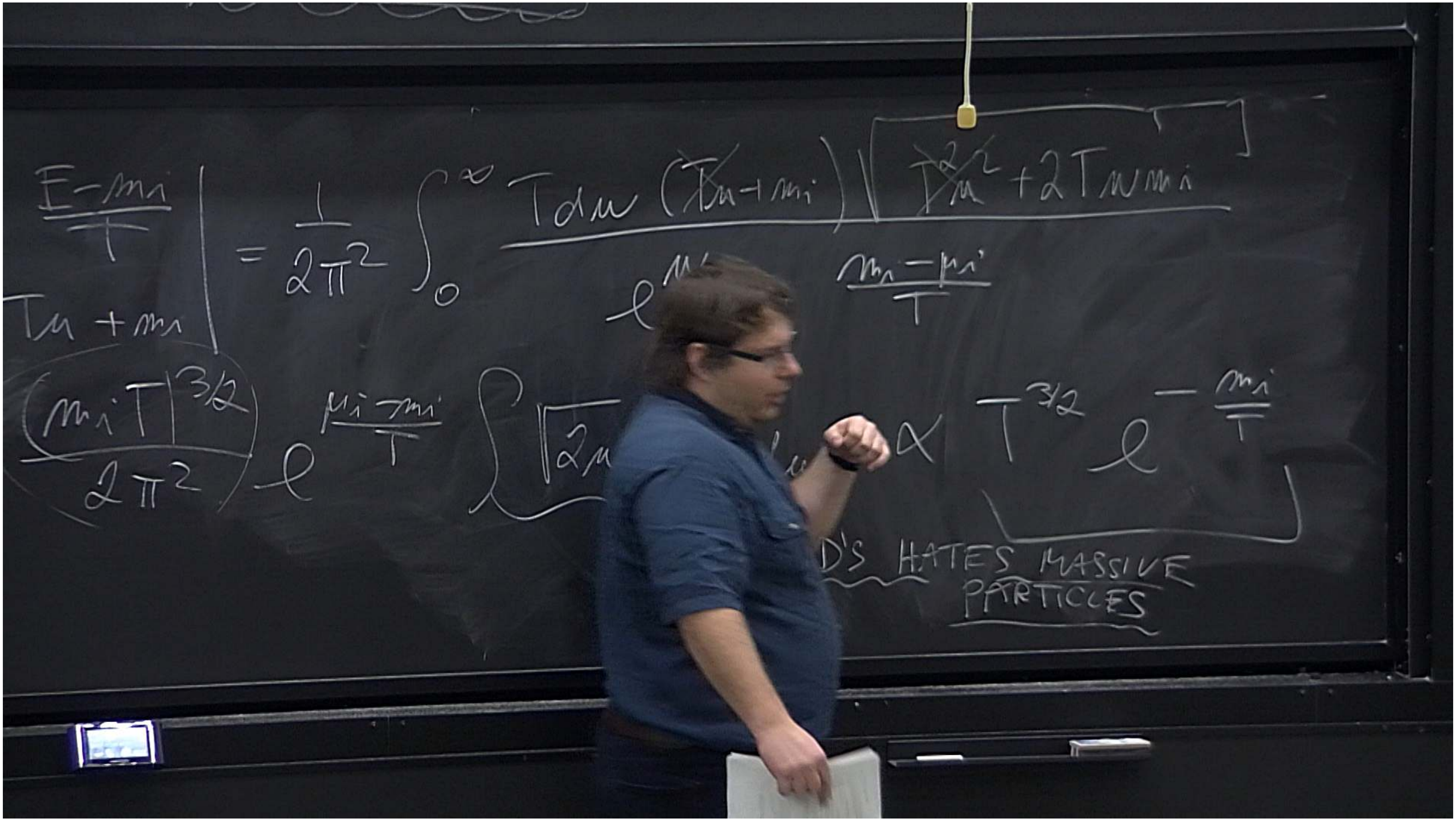
$$\frac{E-m_i}{T}\Bigg|_{Tu+m_i} = \frac{1}{2\pi^2}\int_0^\infty \frac{T\,du\,(\cancel{T}u+m_i)\sqrt{\cancel{T^2}u^2+2Tum_i}}{e^{u}\, e^{\frac{m_i-\mu_i}{T}}}$$

$$\frac{(m_i T)^{3/2}}{2\pi^2}\, e^{\frac{\mu_i-m_i}{T}} \int \sqrt{2u}\, e^{-u}\, du$$

$\mu_i < m_i$

$$\frac{\sqrt{E^2 - m_i^2}}{E - \mu_i \over T} = \left| \begin{array}{l} u = \frac{E - m_i}{T} \\ E = Tu + m_i \end{array} \right| = \frac{1}{2\pi^2} \int_0^\infty \frac{T\,du\,(\cancel{T}u + m_i)\sqrt{\cancel{T^2}u^2 \ldots}}{e^{u}\, e^{\frac{m_i - \mu_i}{T}}}$$

$$\sqrt{2Tu\,m_i}\; e^{-u} = \frac{(m_i T)^{3/2}}{2\pi^2}\, e^{\frac{\mu_i - m_i}{T}} \underbrace{\int \sqrt{2u}\; e^{-u}\, du}_{\#}$$


D'S HATES MASSIVE
PARTICLES

$$\frac{E - m_i}{T} \Big|_{Tu + m_i} = \frac{1}{2\pi^2} \int_0^\infty \frac{T\,du\,(\cancel{T}u + m_i)\sqrt{\cancel{T^2 u^2} + 2Tu\,m_i}}{e^{u}\, e^{\frac{m_i - \mu_i}{T}}}$$

$$\left(\frac{m_i T}{2\pi^2}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}} \underbrace{\int \sqrt{2u}\, e^{-u}\, du}_{\#} \propto \boxed{T^{3/2}\, e^{-\frac{m_i}{T}}}$$

TD'S HATES MASSIVE PARTICLES