Title: PSI 2016/2017 Cosmology (Review) - Lecture 3

Date: Feb 02, 2017 10:15 AM

URL: http://pirsa.org/17020051

Abstract:

LAST TIME: HOMOGENEITY + ISOTROPY CONSTRAIN GEOMETRY OF OUR UNIVERSE TO FRW FORM:

$$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)k}_{ij}\, dx^i dx^j$$

LAST TIME: HOMOGENEITY + ISOTROPY CONSTRAIN GEOMETRY OF OUR UNIVERSE TO FRW FORM:

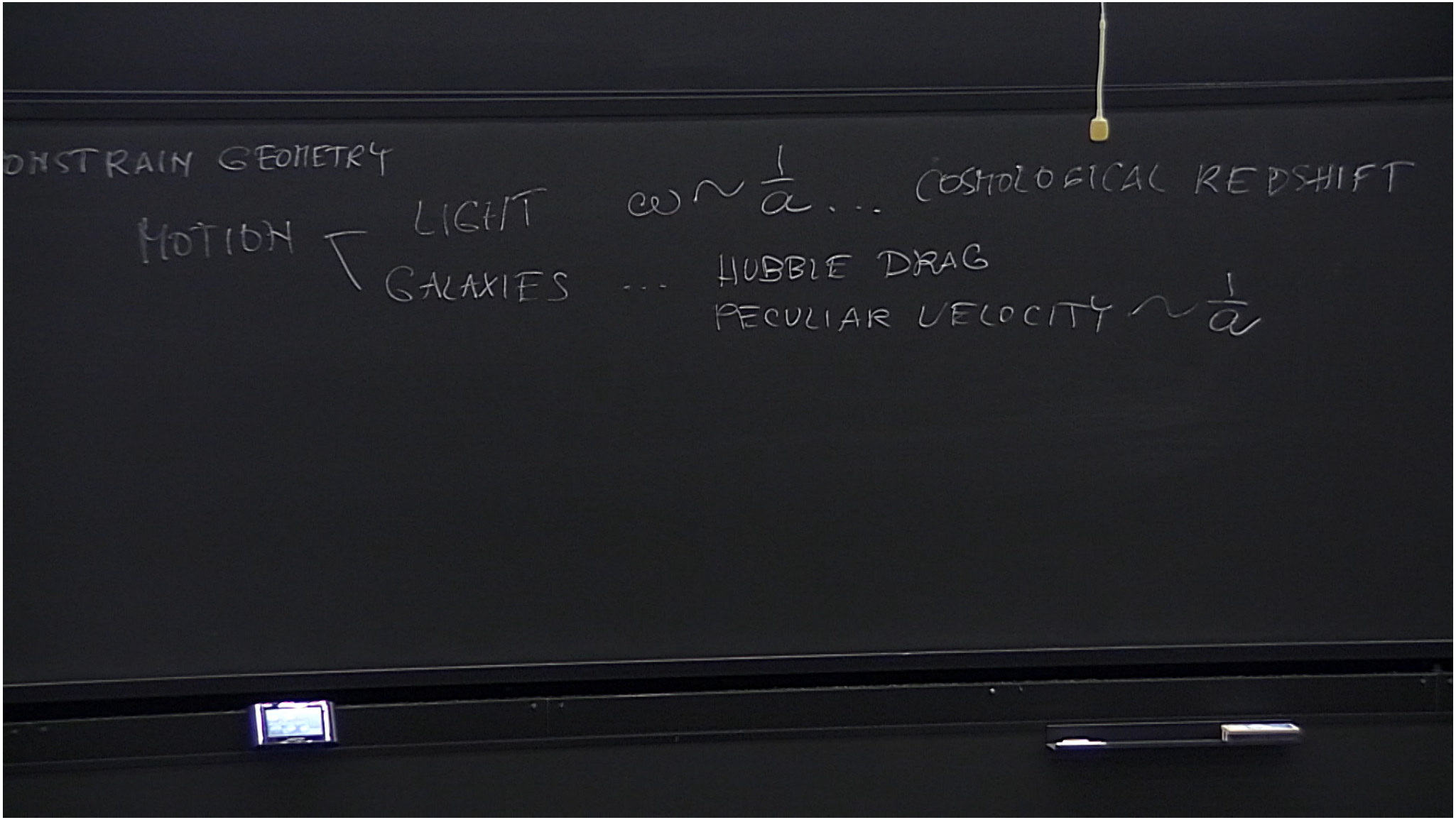
$$ds^2 = -dt^2 + a^2(t)\, g^{(3,0|k)}_{ij}\, dx^i dx^j$$

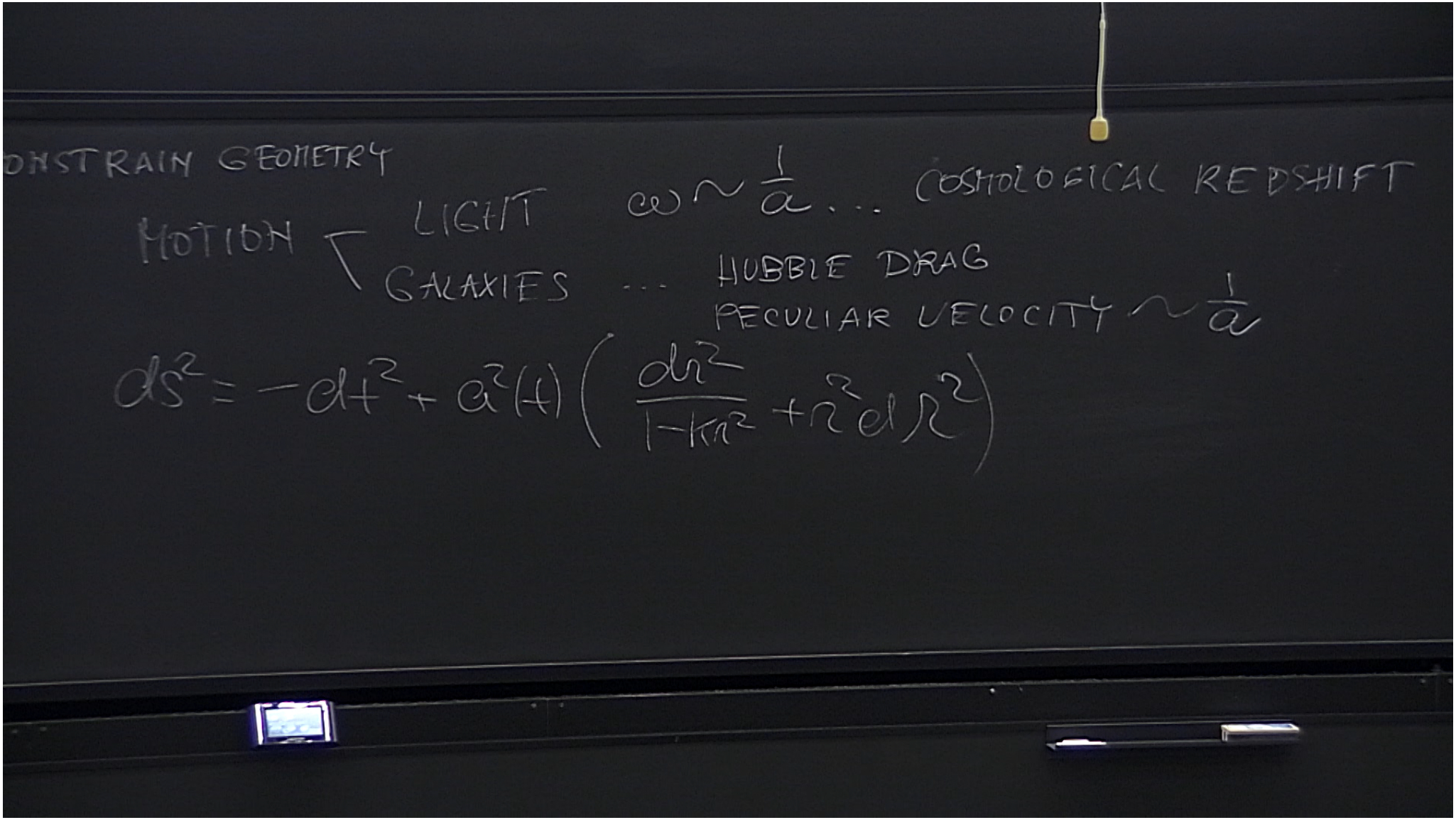
"FRAME IS ADAPTED TO COMOVING OBSERVERS"

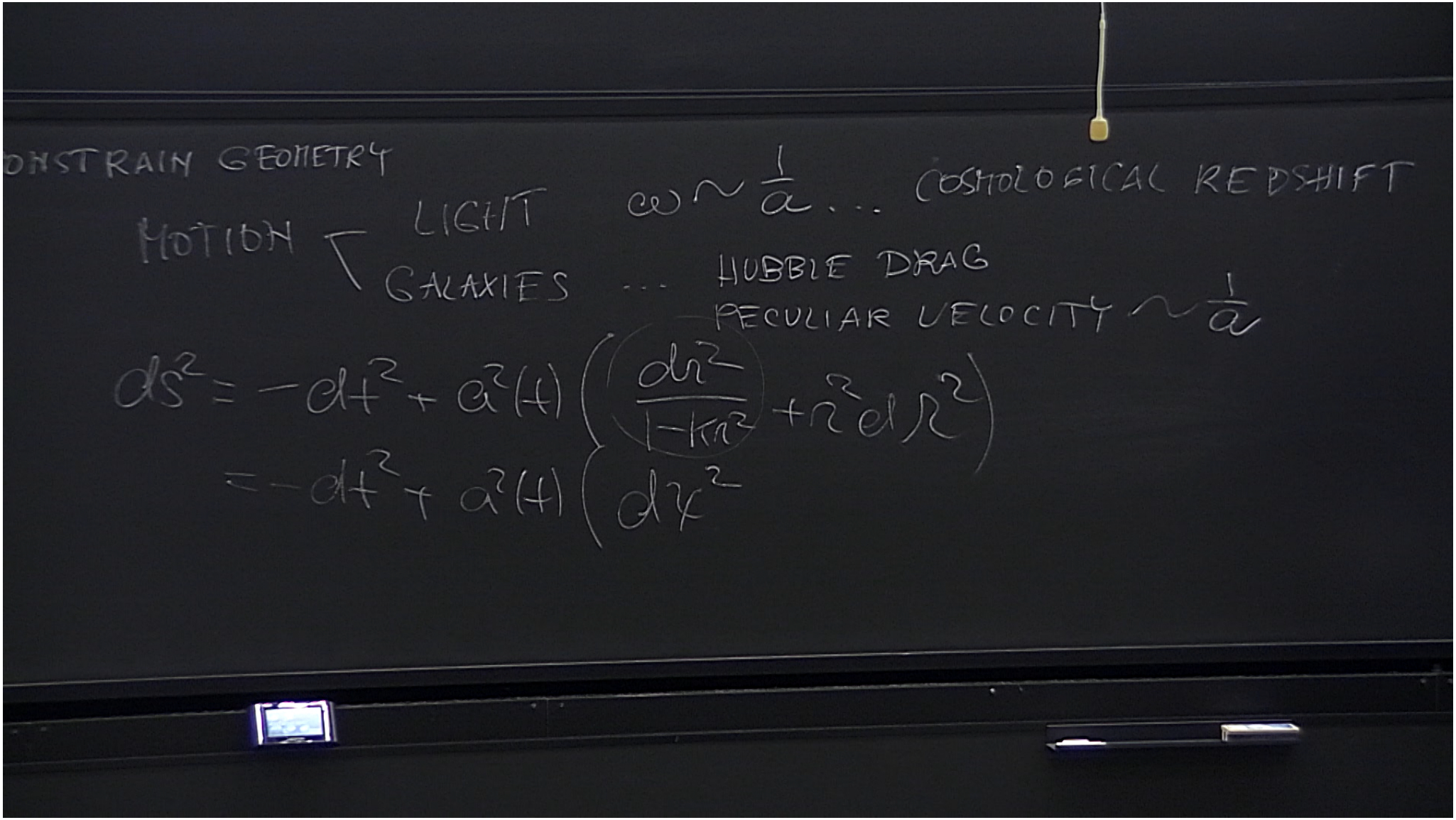
LAST TIME: HOMOGENEITY + ISOTROPY CONSTRAIN GEOMETRY OF OUR UNIVERSE TO FRW FORM:

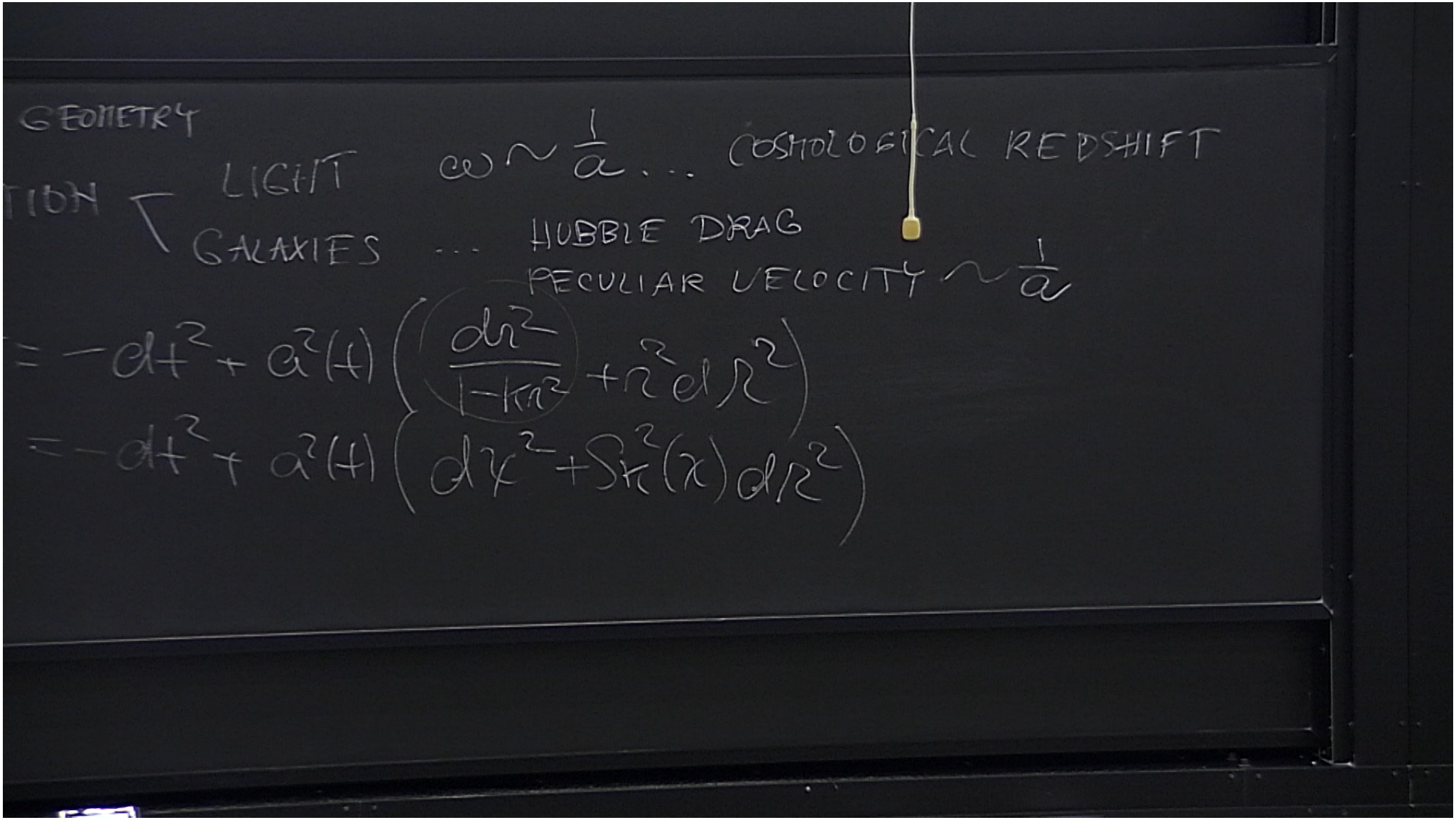
$$ds^2 = -dt^2 + a^2(t)\, g^{(3,0|k}_{ij}\, dx^i dx^j$$

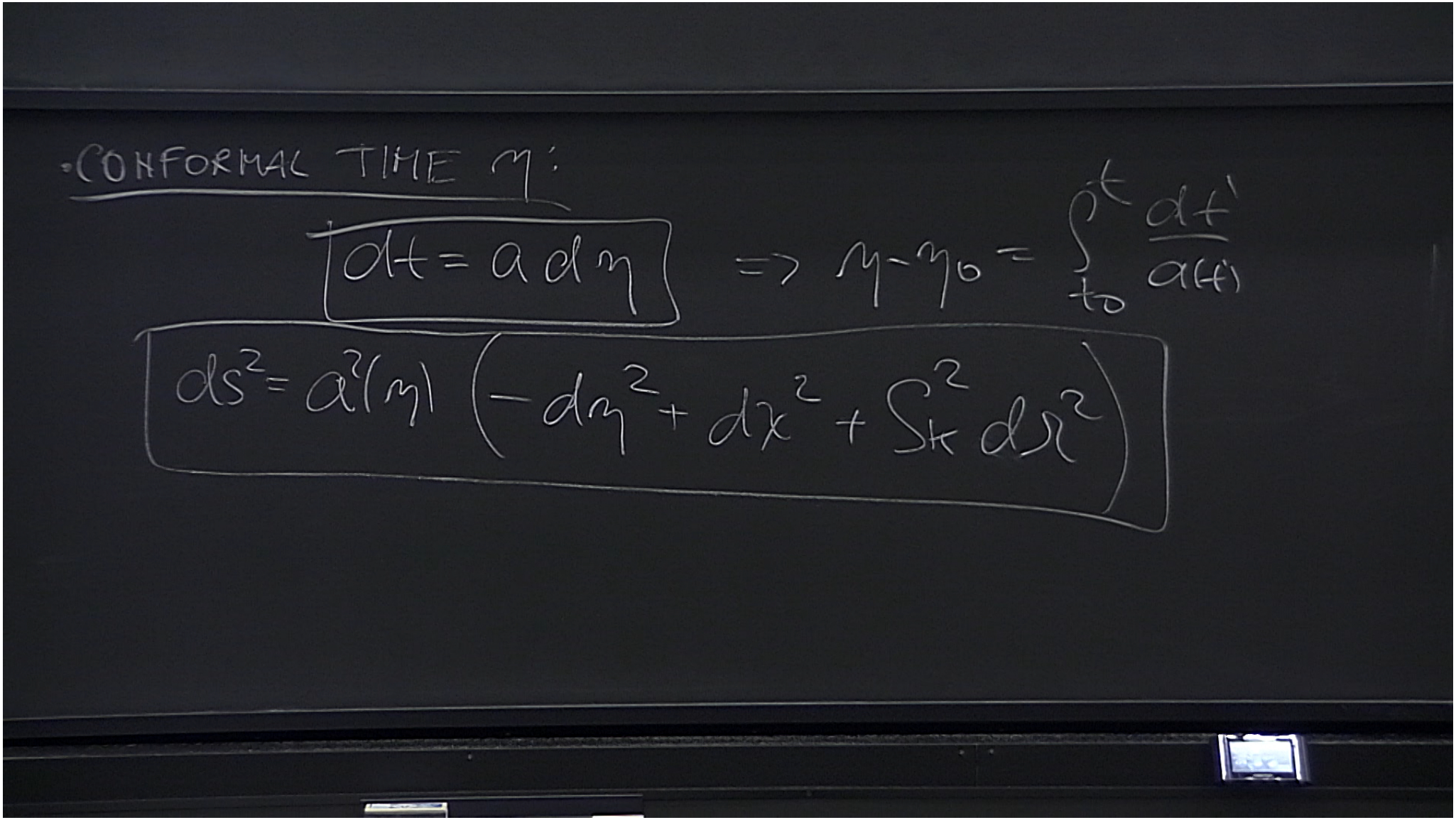
"FRAME IS ADAPTED TO COMOVING OBSERVERS"

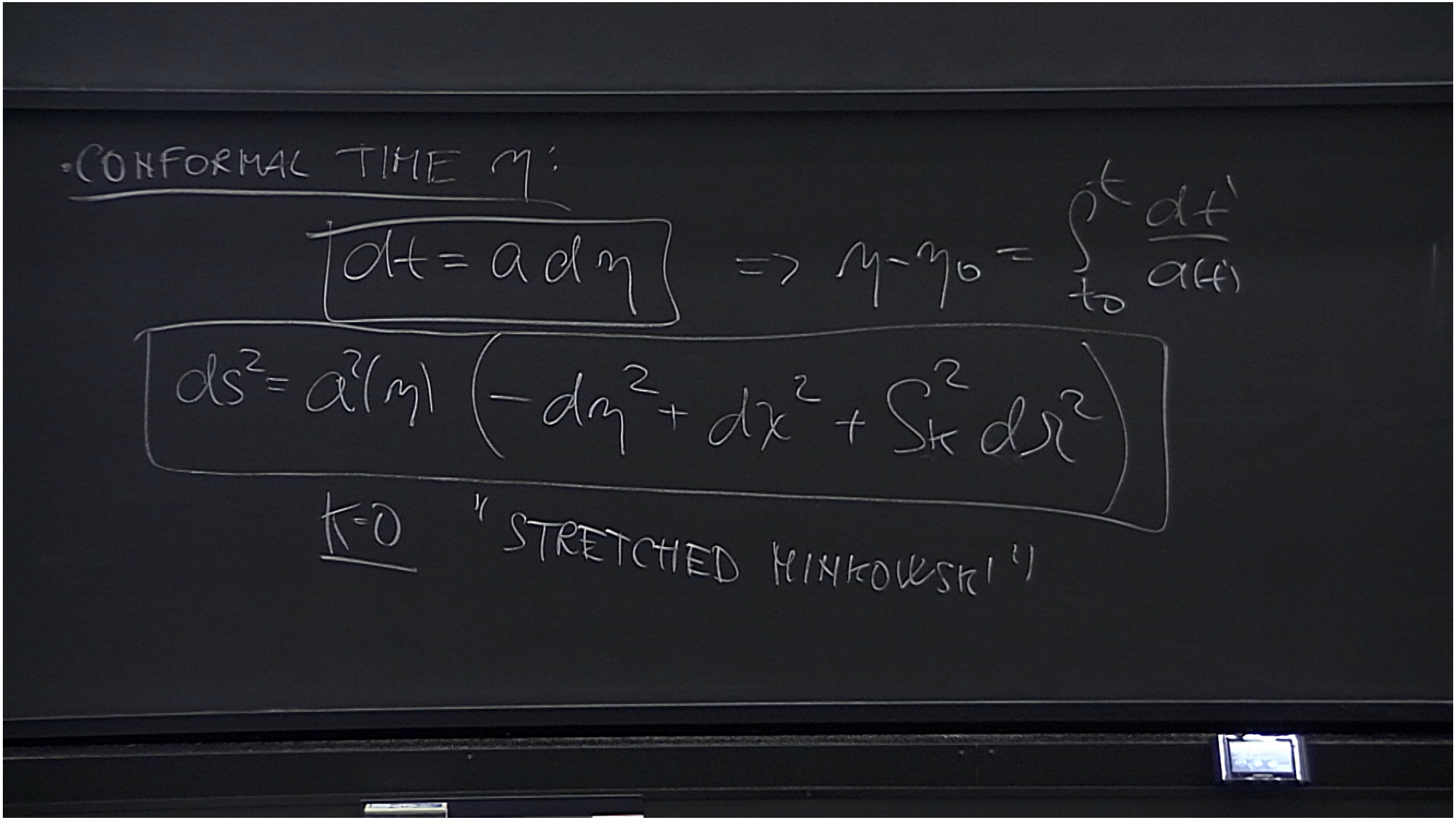

ONSTRAIN GEOMETRY
MOTION
LIGHT
GALAXIES
$\omega \sim \frac{1}{a}$ ... COSMOLOGICAL REDSHIFT
... HUBBLE DRAG
PECULIAR VELOCITY $\sim \frac{1}{a}$


ONSTRAIN GEOMETRY
MOTION
LIGHT
GALAXIES
$\omega \sim \frac{1}{a}$ ... COSMOLOGICAL REDSHIFT
... HUBBLE DRAG
PECULIAR VELOCITY $\sim \frac{1}{a}$
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$


ONSTRAIN GEOMETRY
MOTION
LIGHT
GALAXIES
$\omega \sim \frac{1}{a}$ ... COSMOLOGICAL REDSHIFT
... HUBBLE DRAG
PECULIAR VELOCITY $\sim \frac{1}{a}$
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$= -dt^2 + a^2(t)\left(d\chi^2\right.$


GEOMETRY
TION
LIGHT
GALAXIES
$\omega \sim \frac{1}{a}$ ... COSMOLOGICAL REDSHIFT
... HUBBLE DRAG
PECULIAR VELOCITY $\sim \frac{1}{a}$
$= -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$= -dt^2 + a^2(t)\left(d\chi^2 + S_k^2(\chi) d\Omega^2\right)$


• CONFORMAL TIME η:
$dt = a\,d\eta \Rightarrow \eta - \eta_0 = \int_{t_0}^{t} \frac{dt'}{a(t')}$
$ds^2 = a^2(\eta)\left(-d\eta^2 + dx^2 + S_k^2\, d\Omega^2\right)$


• CONFORMAL TIME η:
$dt = a\,d\eta \Rightarrow \eta - \eta_0 = \int_{t_0}^{t} \frac{dt'}{a(t')}$
$ds^2 = a^2(\eta)\left(-d\eta^2 + dx^2 + S_k^2\, d\Omega^2\right)$
k=0 "STRETCHED MINKOWSKI"

$\frac{dt'}{a(t)}$

· HORIZONS

CONSIDER RADIAL PHOTONS

$$d\eta = \pm dx = \frac{dt}{a}$$

· OBS. FACT ... $\dot{a} > 0$, $t: 0 \to \infty$

MODEL:

$$a = a_0 \left(\frac{t}{t_0}\right)^p$$

$\frac{dt'}{a(t)}$

- HORIZONS CONSIDER RADIAL PHOTONS

$$d\eta = \pm dx = \frac{dt}{a}$$

- OBS. FACT ... $\dot{a} > 0$, $t: 0 \to \infty$

MODEL:

$$a = a_0 \left(\frac{t}{t_0}\right)^p \propto t^p$$

$\frac{dt'}{a(t)}$

HORIZONS

CONSIDER RADIAL PHOTONS

$$d\eta = \pm dx = \frac{dt}{a}$$

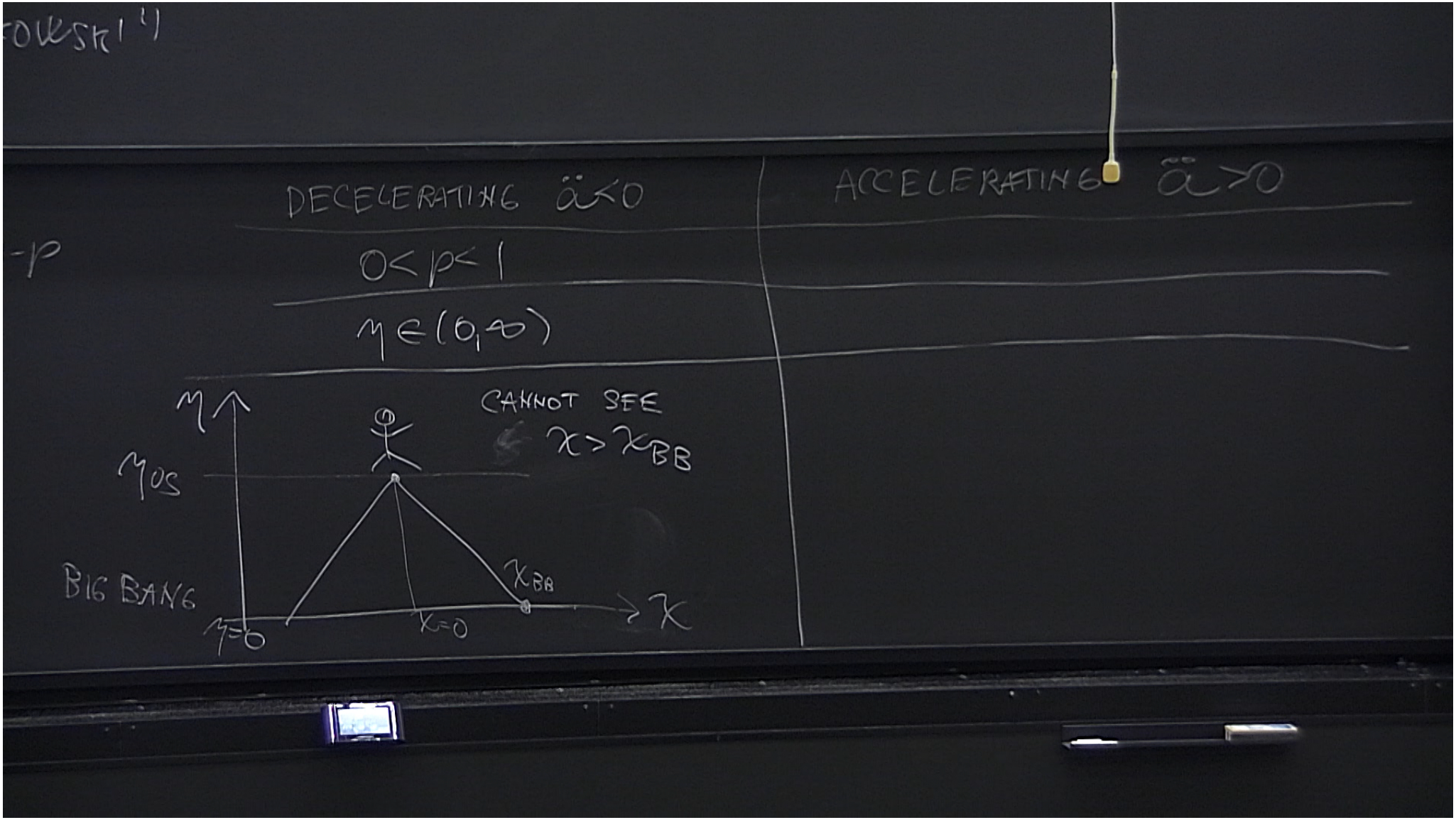
- OBS. FACT ... $\dot{a} > 0$, $t: 0 \to \infty$

MODEL:

$$a = a_0 \left(\frac{t}{t_0}\right)^p \propto t^p \quad , p > 0$$

$$\ddot{a} \propto p(p-1)\, t^{p-2}$$

$$\eta = \int \frac{dt}{a} \propto \int t^{-p}\, dt \propto t^{1-p}$$

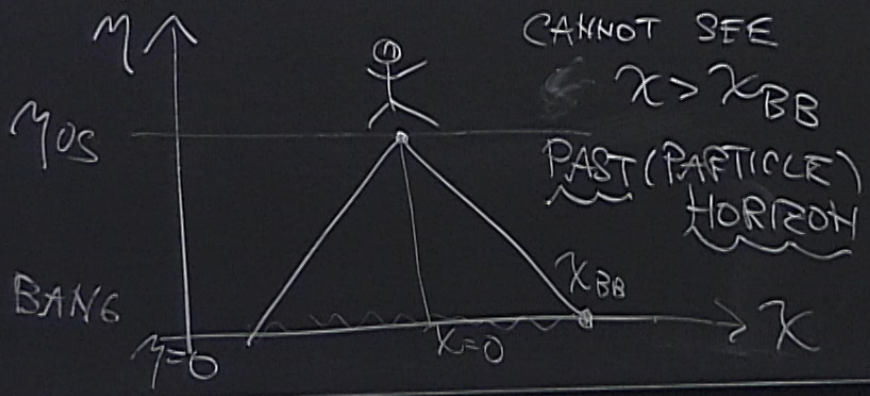

DECELERATING $\ddot{a}<0$
ACCELERATING $\ddot{a}>0$
$0<p<1$
$\eta \in (0,\infty)$
CANNOT SEE
$\chi > \chi_{BB}$
η_{OS}
η
BIG BANG
$\eta=0$
$\chi=0$
χ_{BB}
χ

DECELERATING $\ddot{a} < 0$

$0 < p < 1$

$\eta \in (0, \infty)$

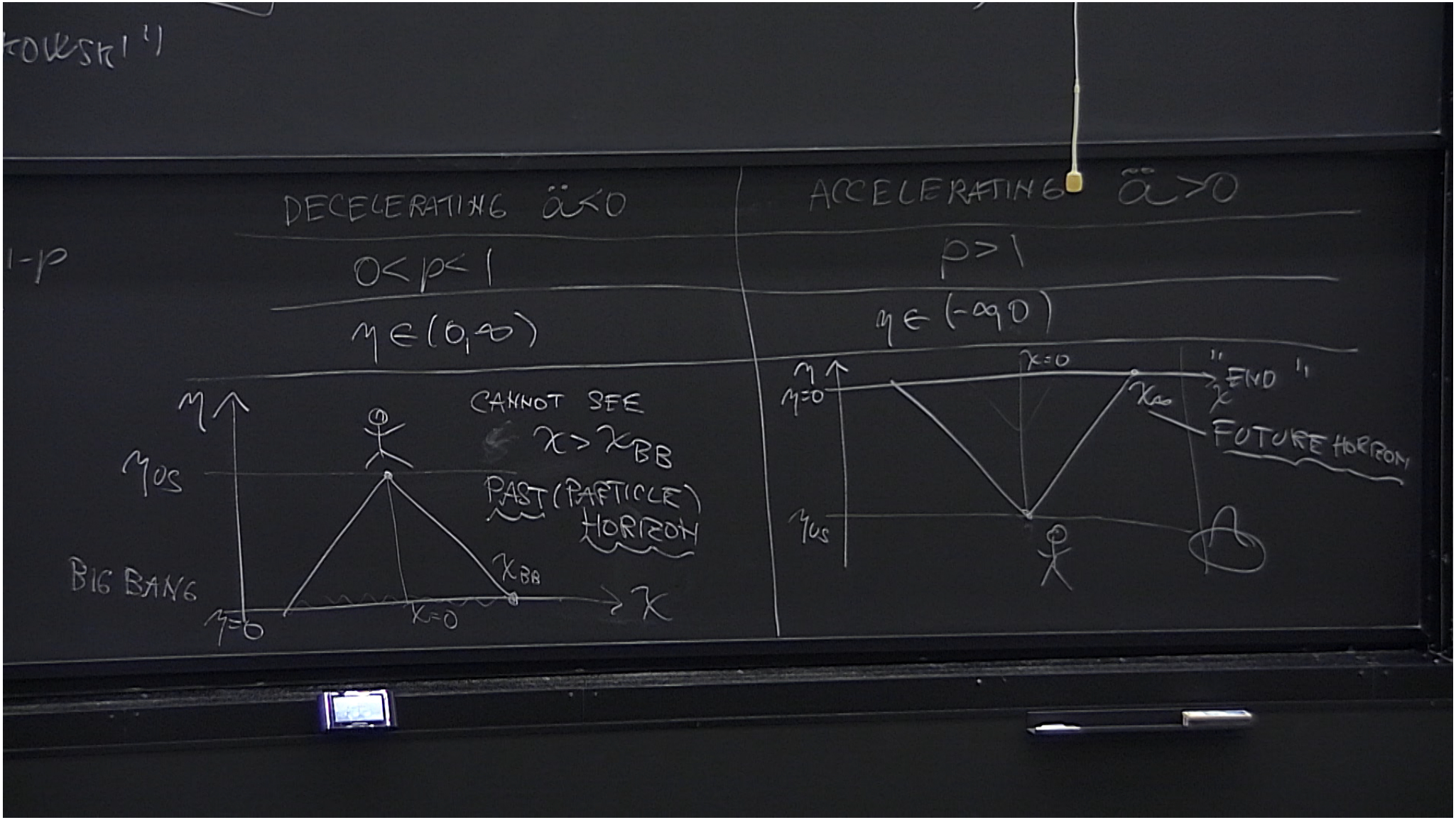
ACCELERATING $\ddot{a} > 0$



CANNOT SEE

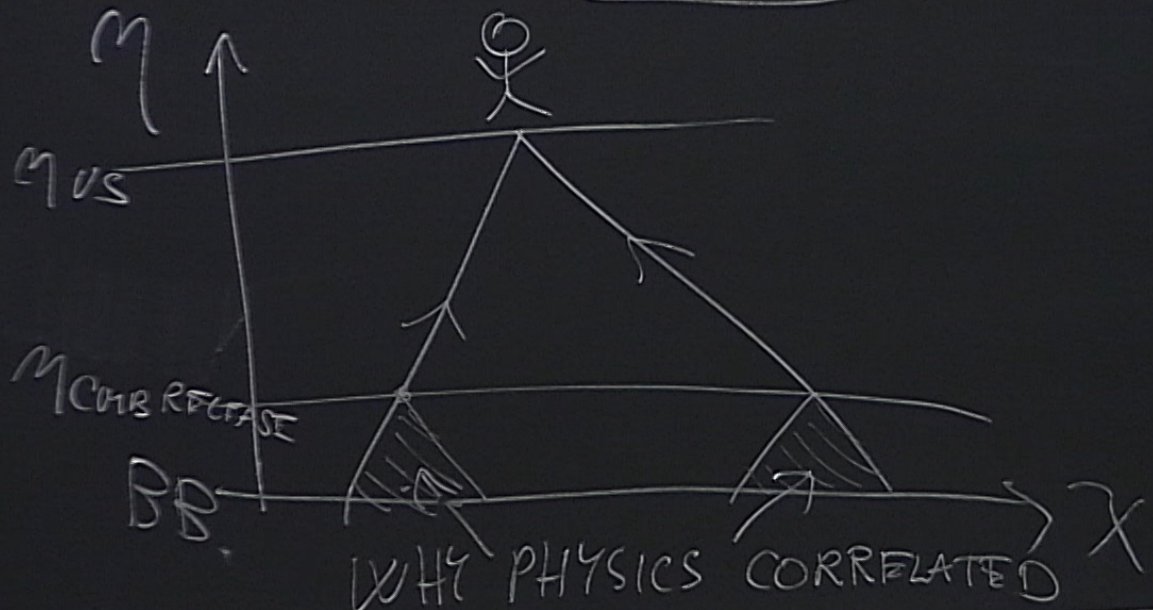
$x > x_{BB}$

PAST (PARTICLE) HORIZON


DECELERATING $\ddot{a}<0$
$0<p<1$
$\eta \in (0,\infty)$
CANNOT SEE
$\chi > \chi_{BB}$
PAST (PARTICLE) HORIZON
BIG BANG
η_{US}
$\eta=0$
χ_{BB}
$\chi=0$
χ
ACCELERATING $\ddot{a}>0$
$p>1$
$\eta \in (-\infty, 0)$
$\chi=0$
"END"
χ_∞
FUTURE HORIZON
$\eta=0$
η_{US}

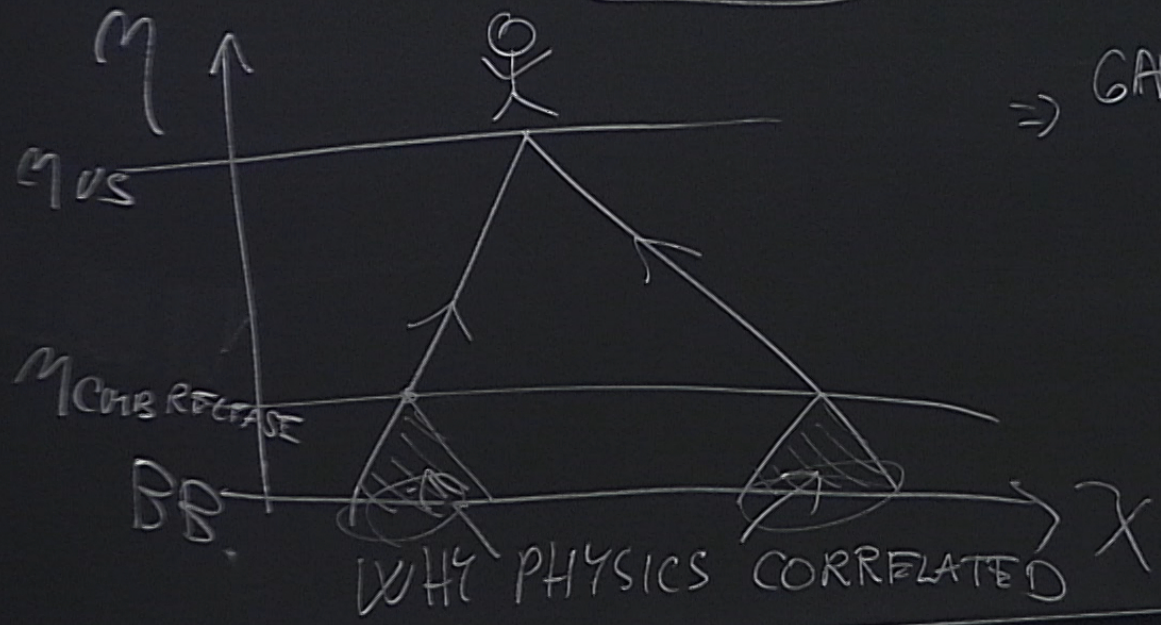
- HORIZON PUZZLE BACK IN 70's $\delta_{rms}, \delta_{\Omega}$... MEASURED.

$\Rightarrow \ddot{a} < 0$ .. ALL TIME.

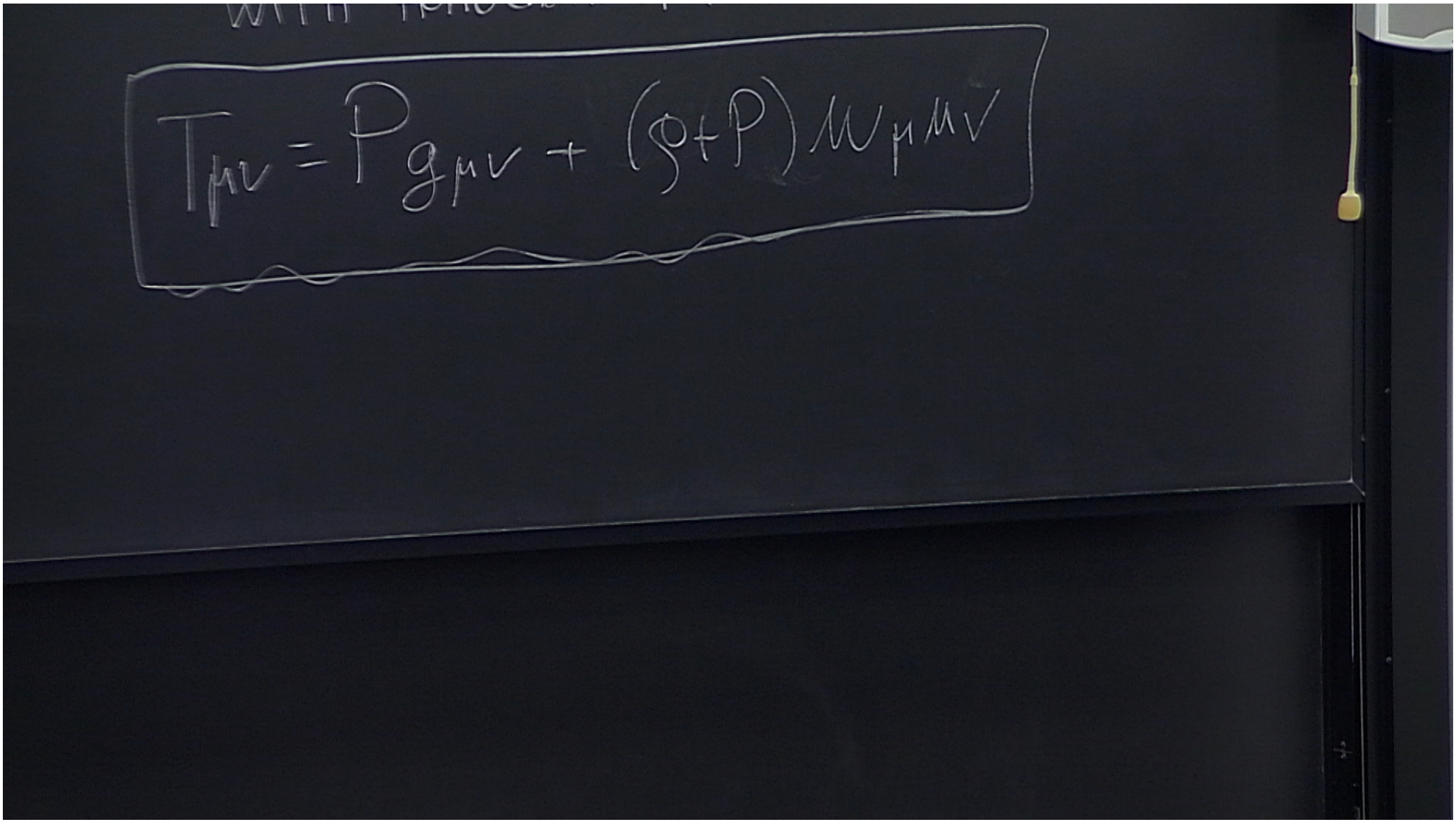


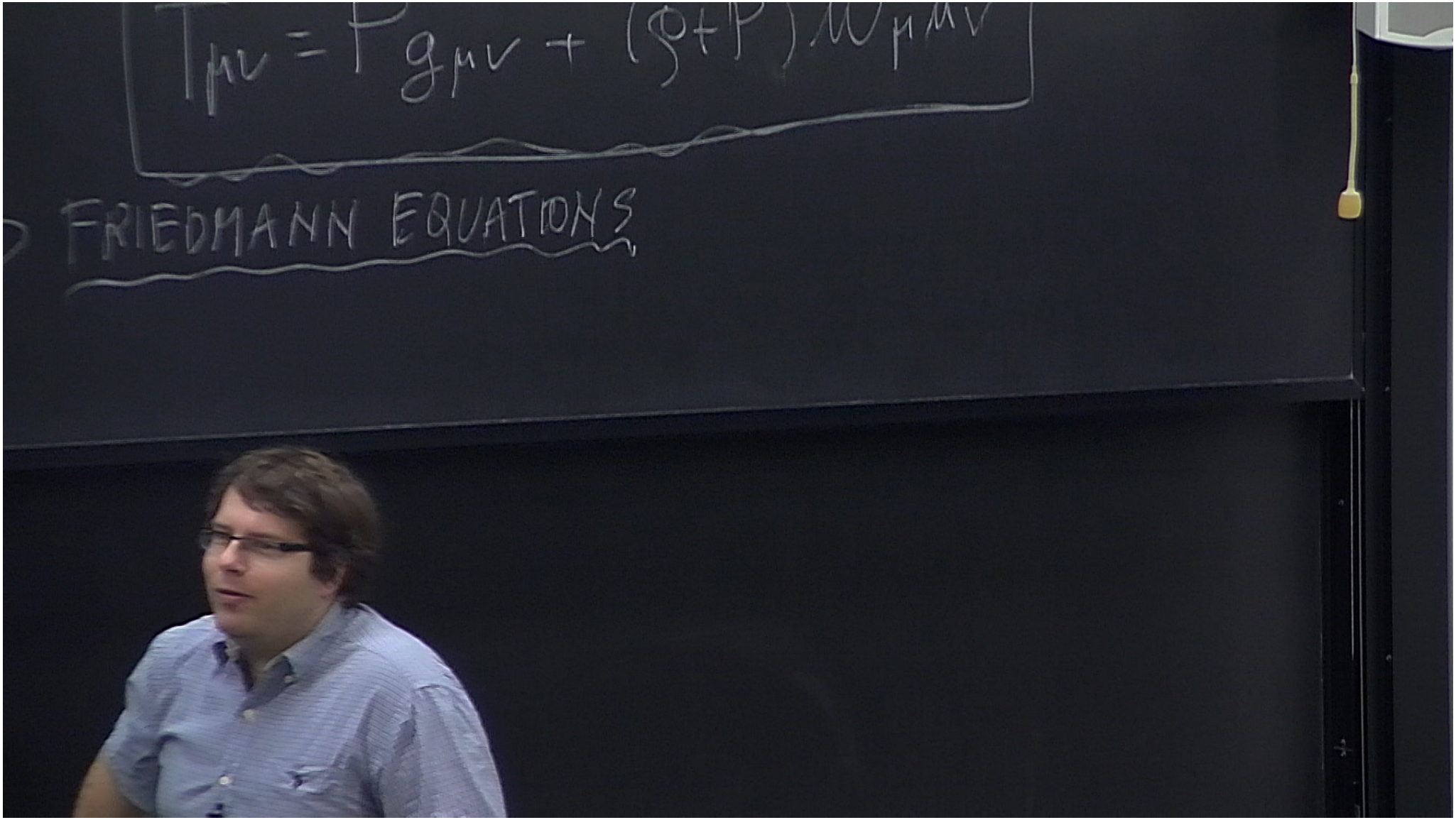
- HORIZON PUZZLE BACK IN 70's Ω_m, Ω_Λ... MEASURED.

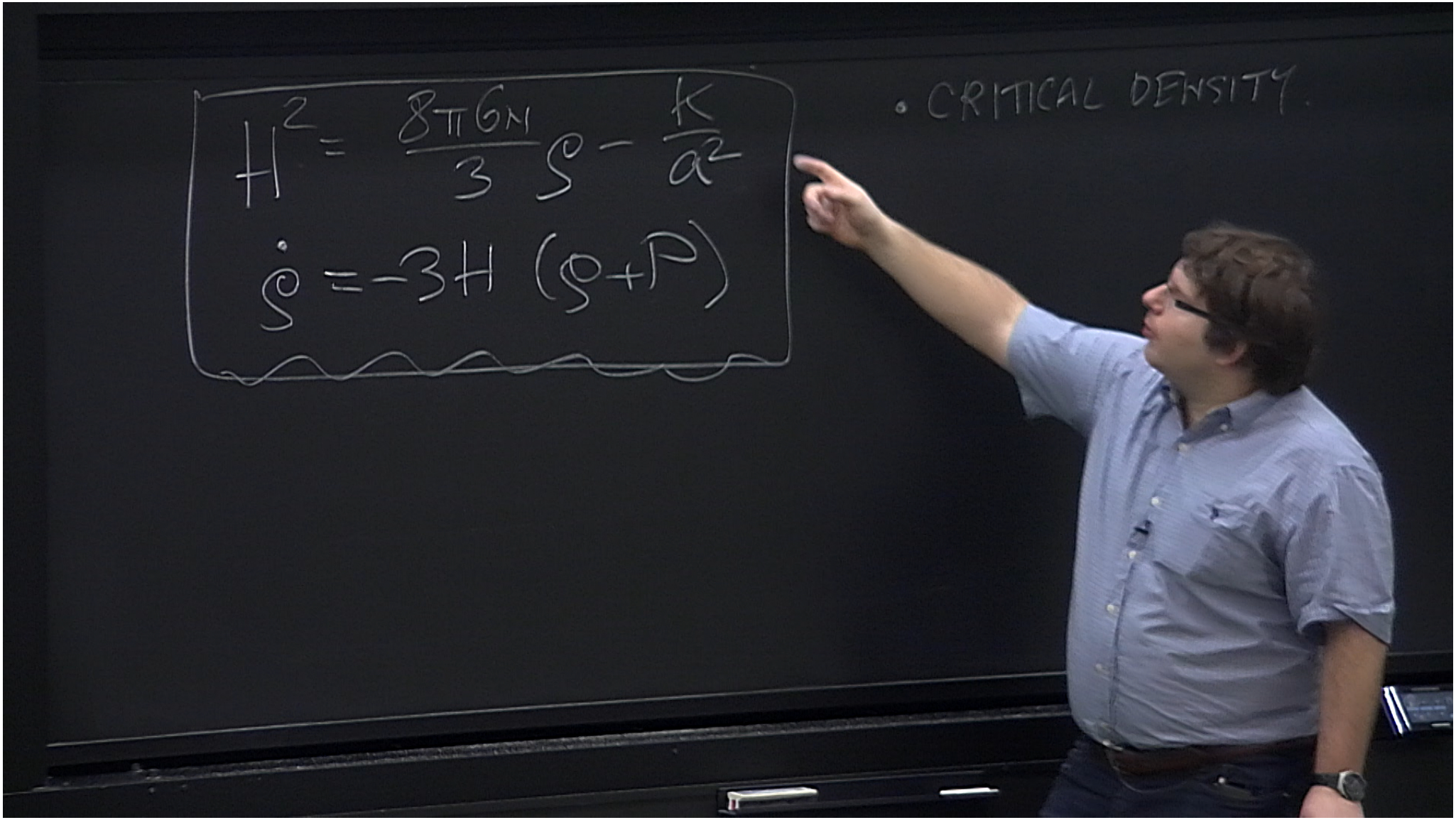
$\Rightarrow \ddot{a} < 0$ ... ALL TIME.

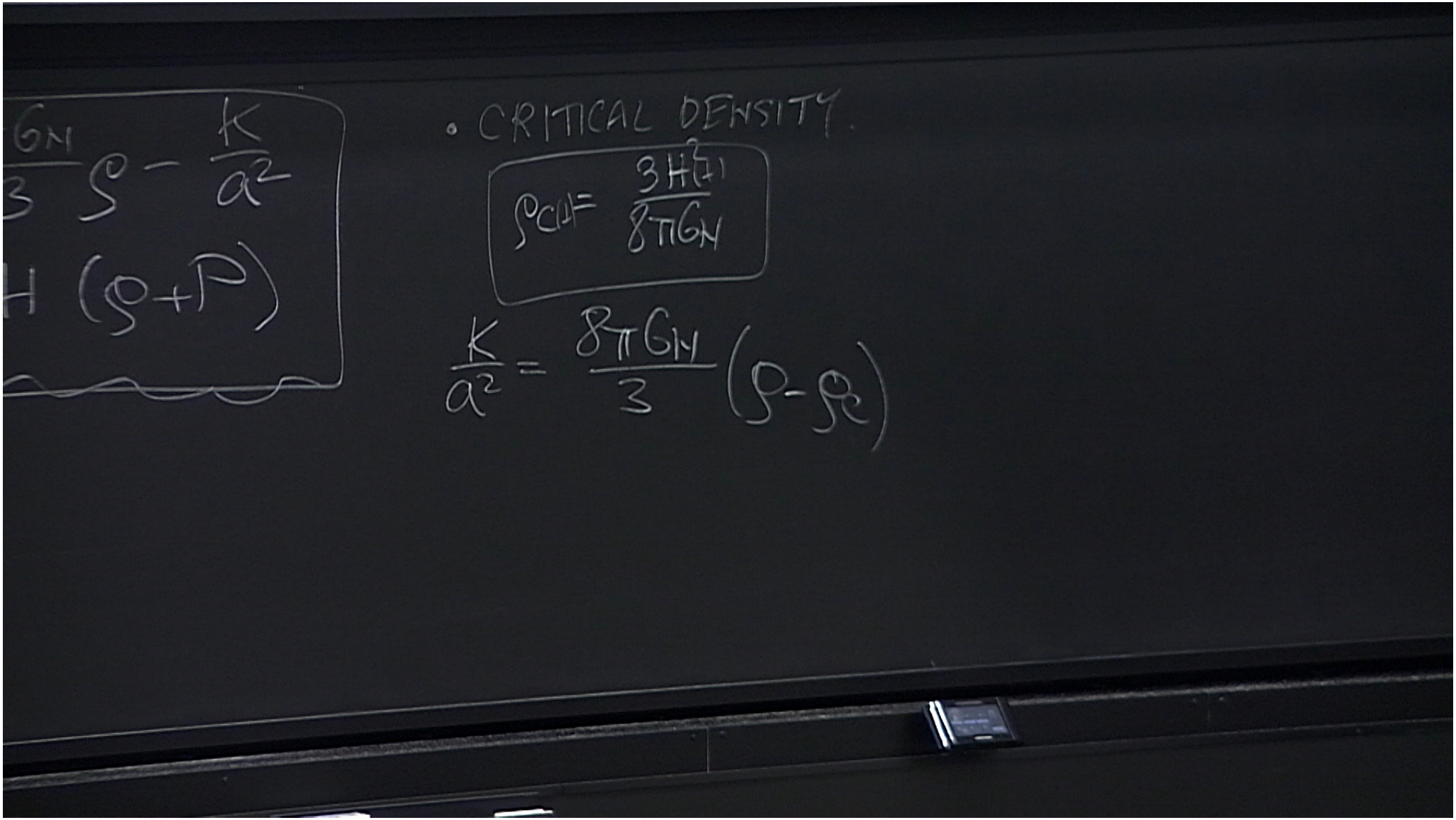


$\Rightarrow$ GAVE RISE TO INFLATION THEORY


$T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu$


$T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu$
FRIEDMANN EQUATIONS


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$
$\dot{\rho} = -3H(\rho + P)$
• CRITICAL DENSITY.


$\frac{G_N}{3}\rho - \frac{k}{a^2}$
$H(\rho+P)$
• CRITICAL DENSITY.
$\rho_{CH} = \frac{3H^2}{8\pi G_N}$
$\frac{k}{a^2} = \frac{8\pi G_N}{3}(\rho - \rho_c)$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + P)$$

• CRITICAL DENSITY.

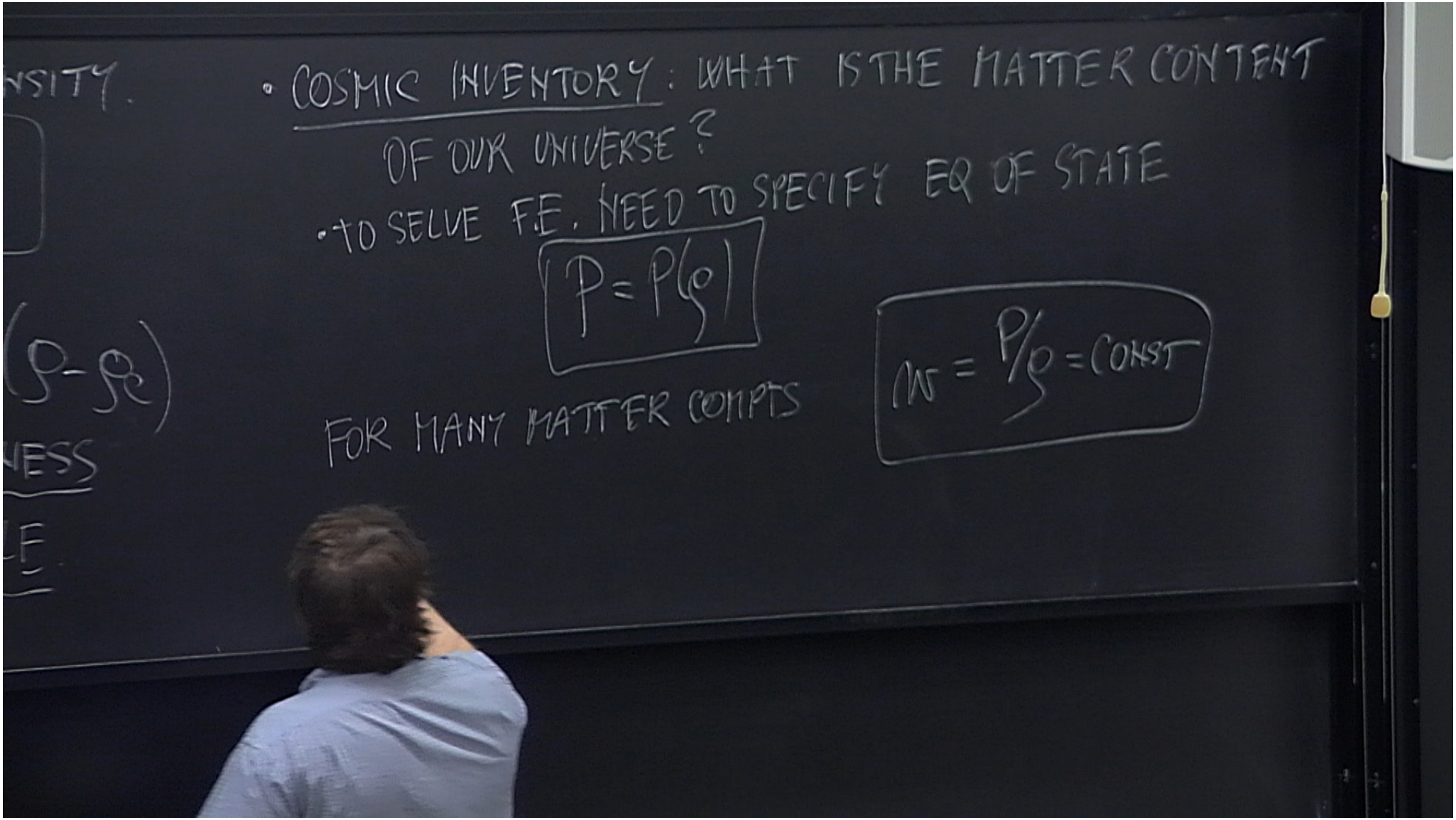
$$\rho_{cr} = \frac{3H^2}{8\pi G_N}$$

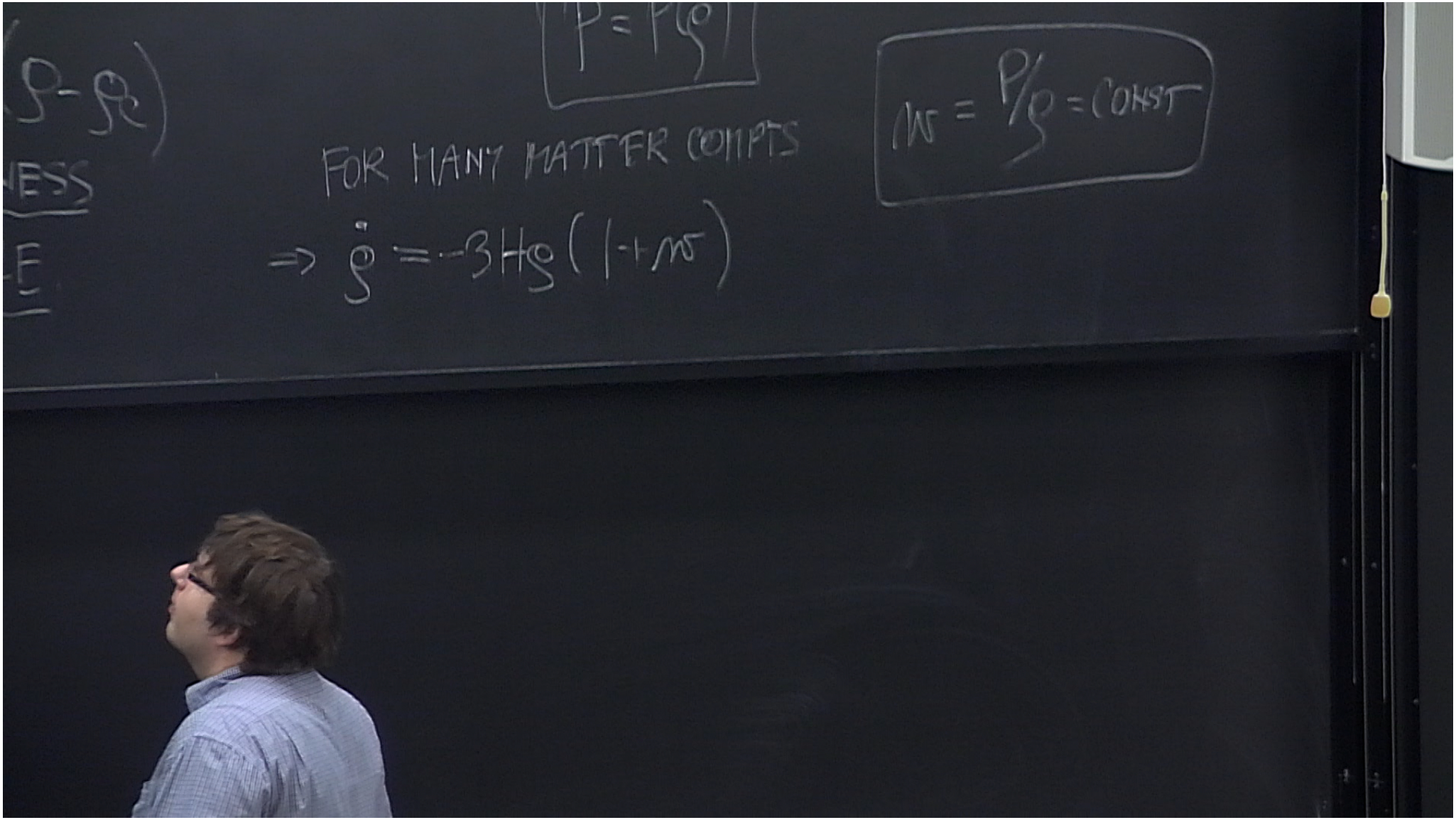
$$\frac{k}{a^2} = \frac{8\pi G_N}{3}(\rho - \rho_c)$$

WE OBSERVE

$$\rho - \rho_c (1 \pm 0.01)$$

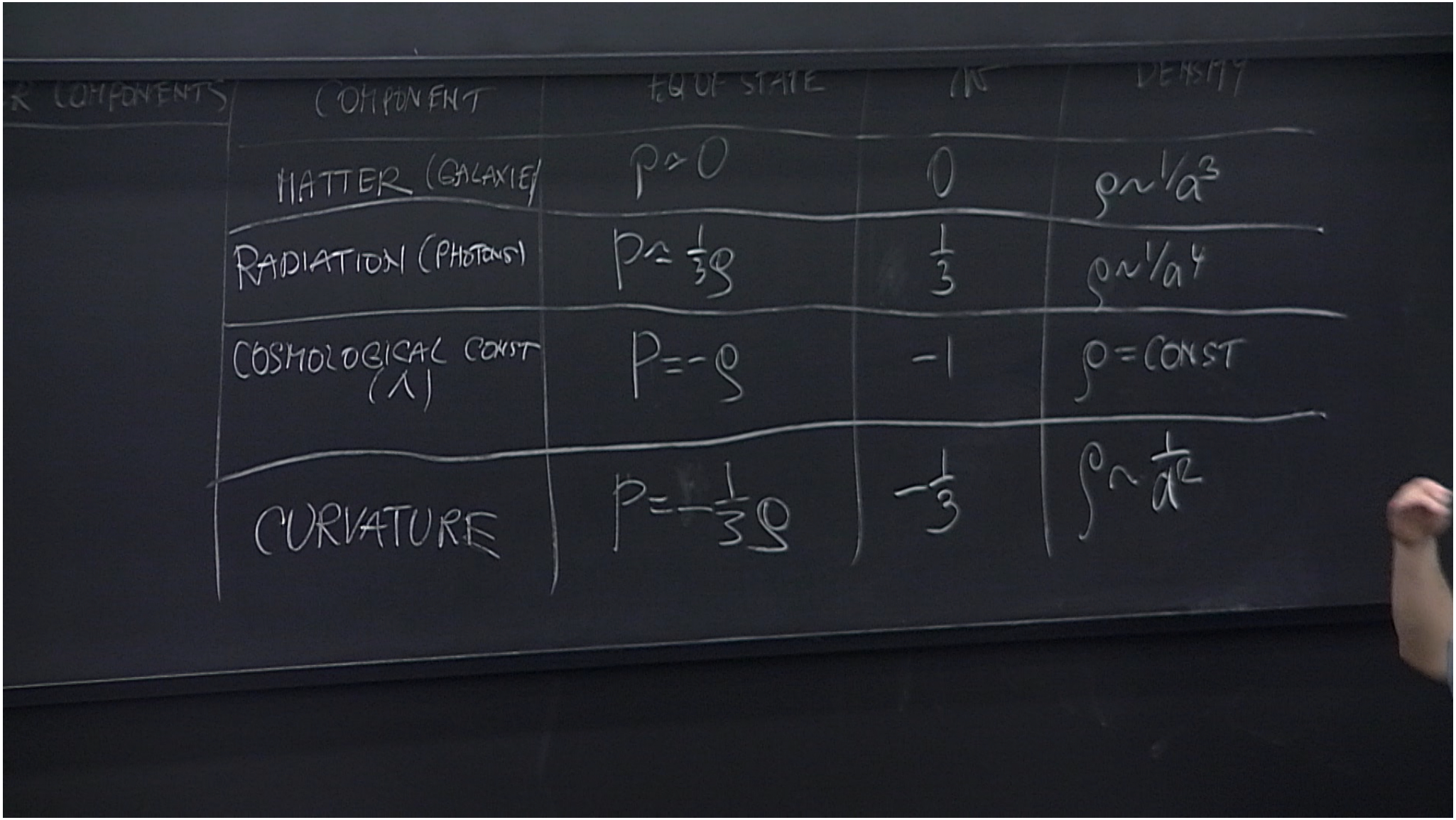
FLATNESS PUZZLE

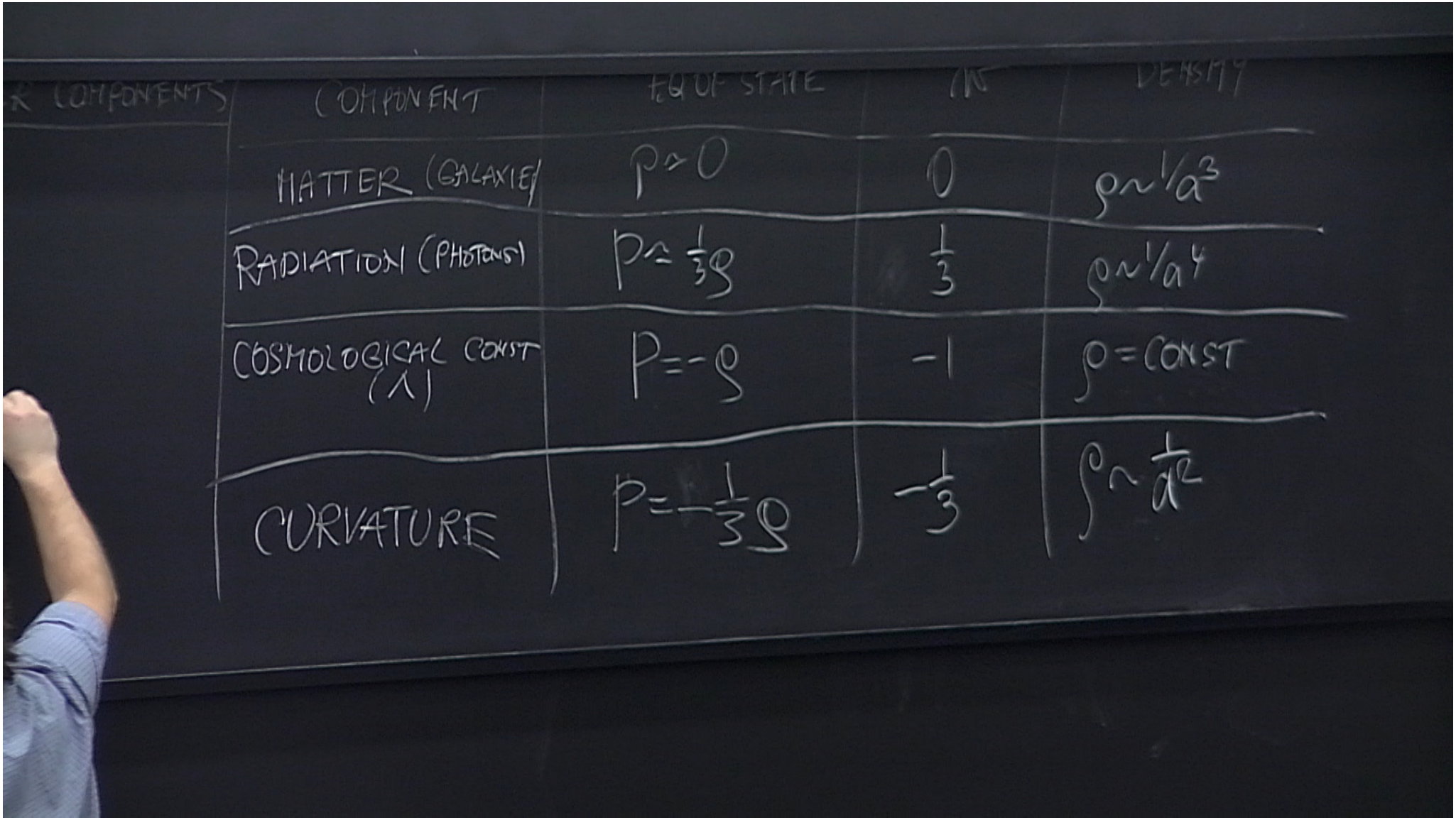

NSITY.
• COSMIC INVENTORY: WHAT IS THE MATTER CONTENT
OF OUR UNIVERSE?
• TO SOLVE F.E. NEED TO SPECIFY EQ OF STATE
P = P(ρ)
FOR MANY MATTER COMPTS
w = P/ρ = CONST
(ρ-ρc)
NESS
LE

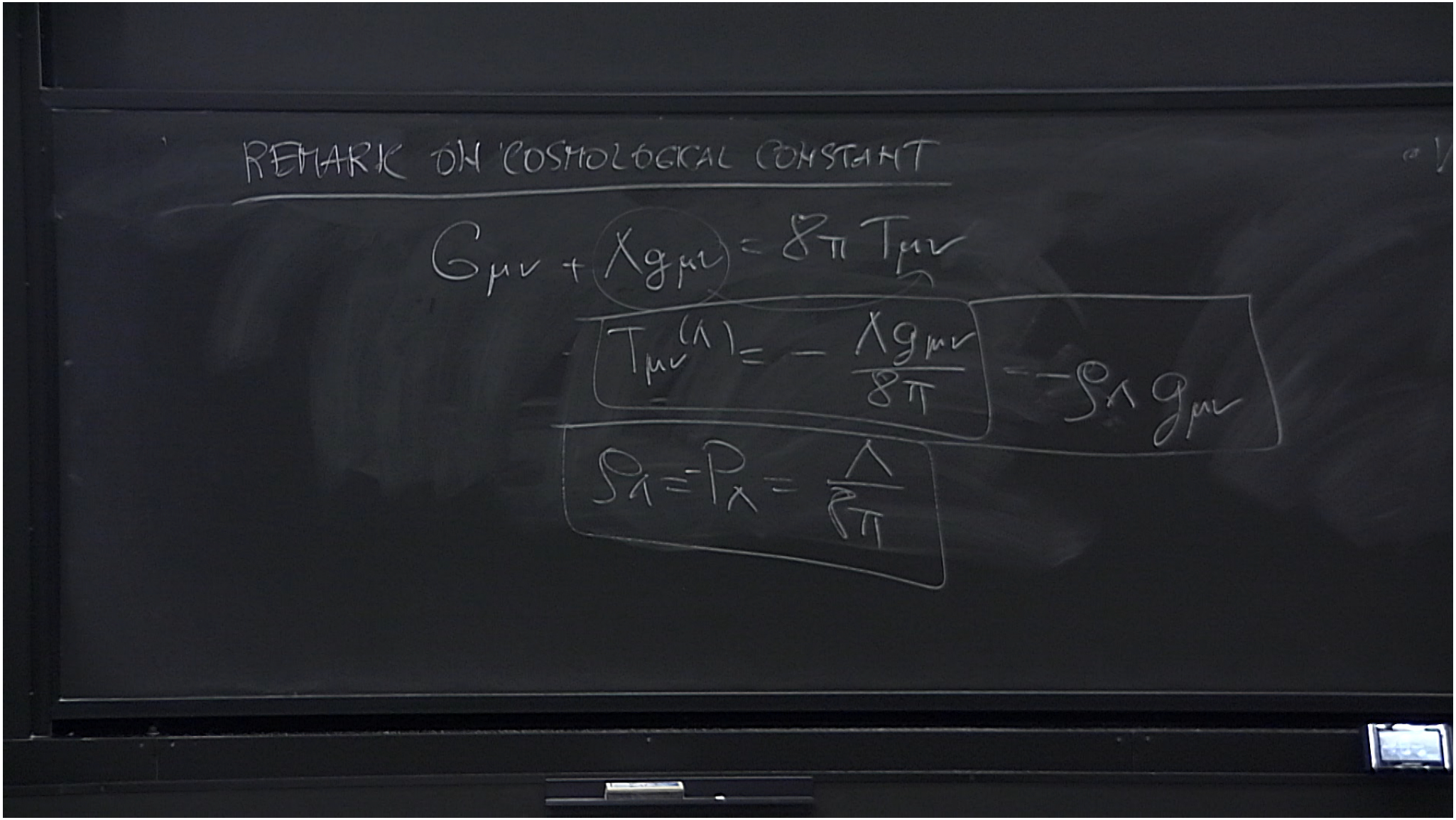

P = P(ρ)
FOR MANY MATTER COMPTS
w = P/ρ = CONST
→ ρ̇ = -3Hρ(1+w)

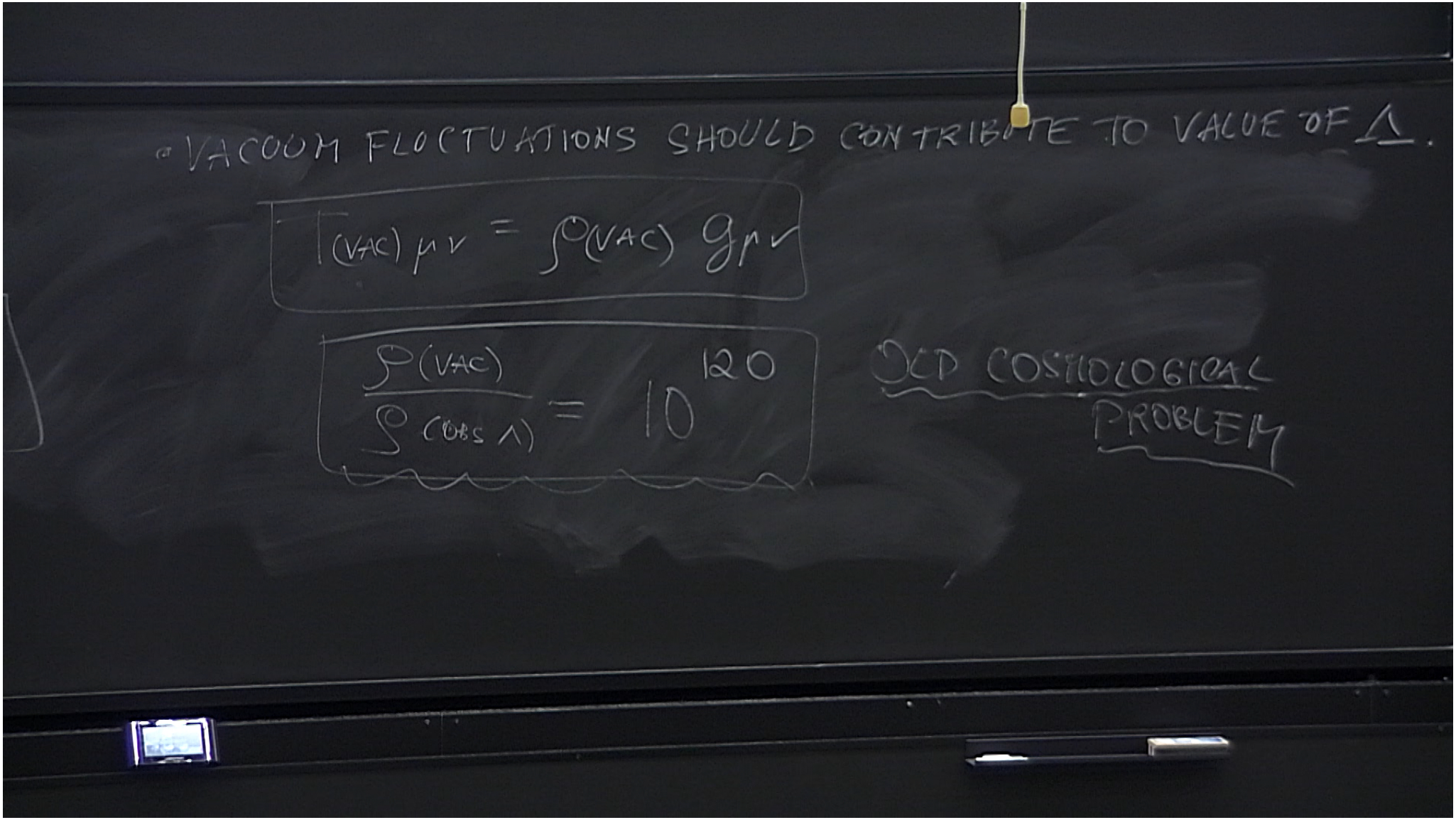
$$\Rightarrow \frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

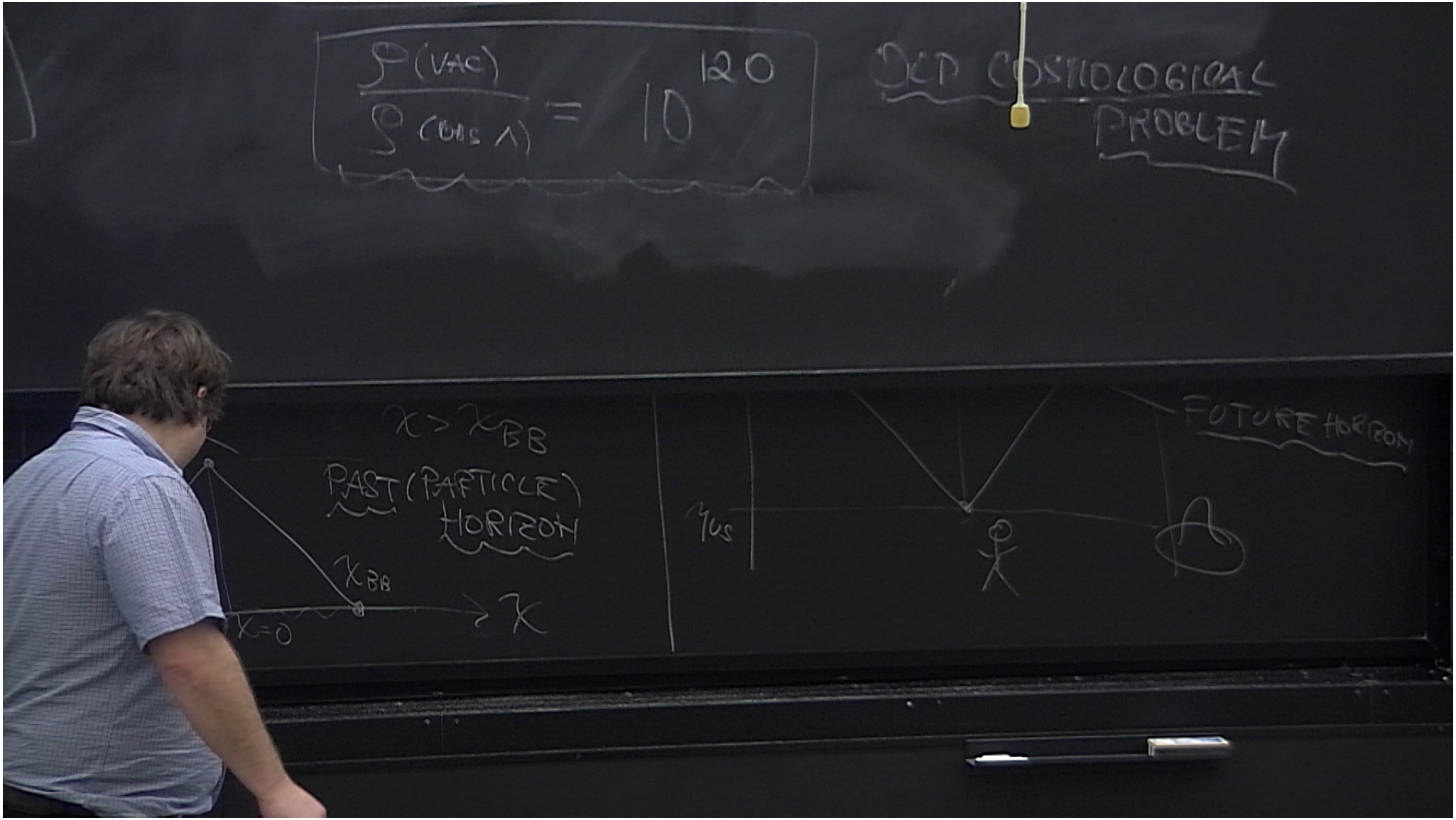
$$\Rightarrow \boxed{\rho \propto a^{-3(1+w)}}$$


R COMPONENTS
COMPONENT
EQ OF STATE
DENSITY
MATTER (GALAXIES)
$p=0$
0
$\rho \sim 1/a^3$
RADIATION (PHOTONS)
$p=\frac{1}{3}\rho$
$\frac{1}{3}$
$\rho \sim 1/a^4$
COSMOLOGICAL CONST (Λ)
$p=-\rho$
-1
$\rho = \text{CONST}$
CURVATURE
$p=-\frac{1}{3}\rho$
$-\frac{1}{3}$
$\rho \sim \frac{1}{a^2}$

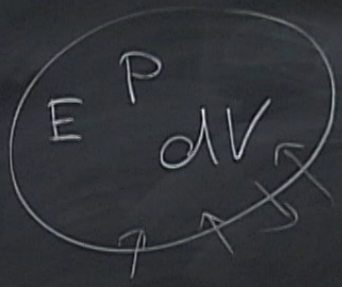

K COMPONENTS
COMPONENT
EQ OF STATE
DENSITY
MATTER (GALAXIES)
P≈0
0
ρ∼1/a³
RADIATION (PHOTONS)
P≈⅓ρ
⅓
ρ∼1/a⁴
COSMOLOGICAL CONST (Λ)
P=−ρ
−1
ρ=CONST
CURVATURE
P=−⅓ρ
−⅓
ρ∼1/a²


REMARK ON COSMOLOGICAL CONSTANT
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$
$T_{\mu\nu}^{(\Lambda)} = -\frac{\Lambda g_{\mu\nu}}{8\pi} = -\rho_\Lambda g_{\mu\nu}$
$\rho_\Lambda = -P_\Lambda = \frac{\Lambda}{8\pi}$


VACUUM FLUCTUATIONS SHOULD CONTRIBUTE TO VALUE OF Λ.
$T_{(VAC)\mu\nu} = \rho_{(VAC)} g_{\mu\nu}$
$\frac{\rho_{(VAC)}}{\rho_{(OBS\ \Lambda)}} = 10^{120}$
OLD COSMOLOGICAL PROBLEM


$\frac{\rho(VAC)}{\rho(COSM\ \Lambda)} = 10^{120}$
QCD COSMOLOGICAL PROBLEM
$\chi > \chi_{BB}$
PAST (PARTICLE) HORIZON
χ_{BB}
$\chi = 0$
χ
χ_{US}
FUTURE HORIZON

- PHYSICAL DERIVATION OF $a = a(\varrho)$

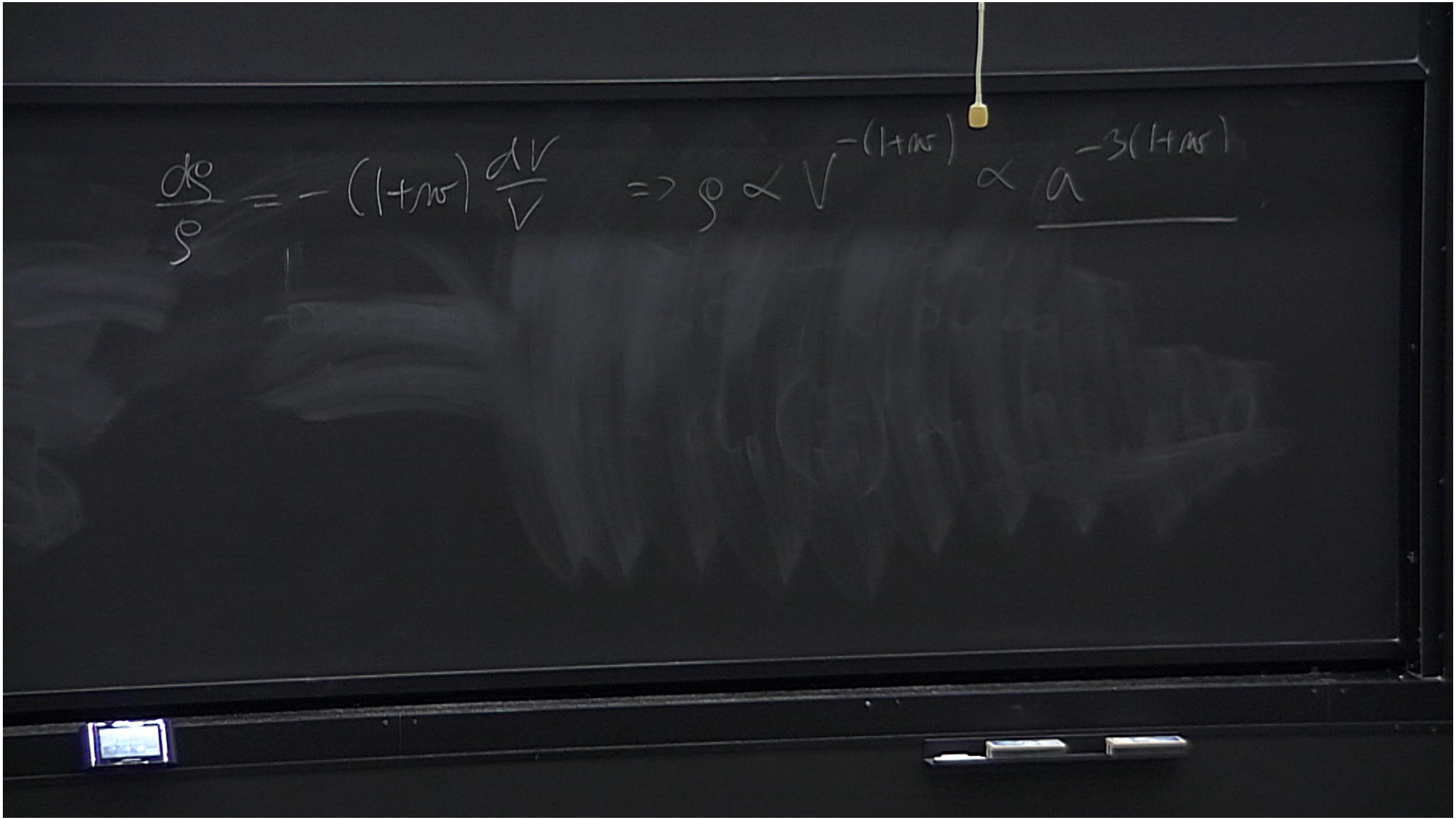


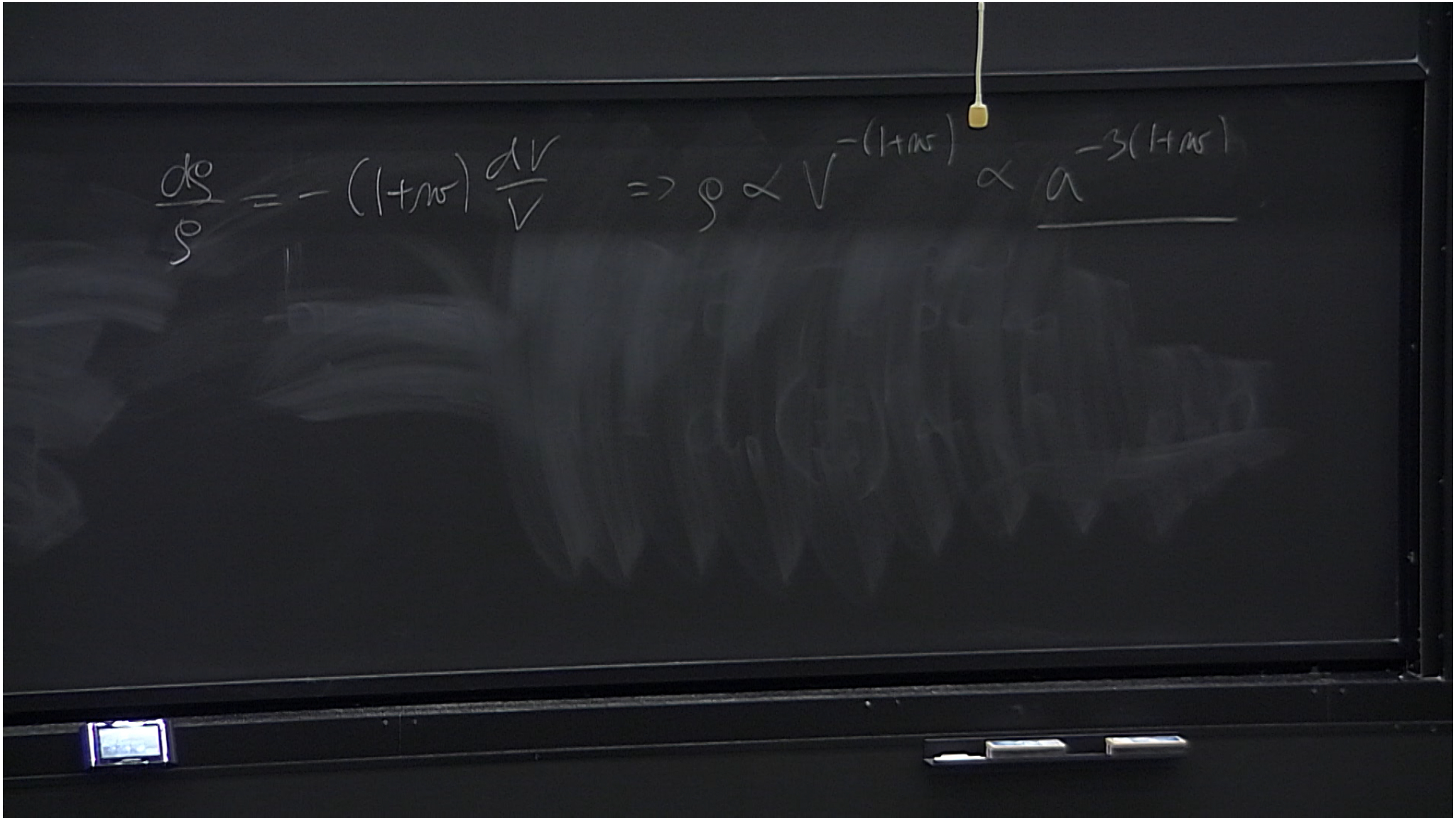
$$dE = dQ - P\,dV$$

NO NET HEAT FLOW $dQ = 0$

$$d(\varrho V) = -P\,dV = -n\varrho\,dV$$

$$d\varrho\, V + \varrho\, dV$$


$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Rightarrow \rho \propto V^{-(1+w)} \propto a^{-3(1+w)}$


$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Rightarrow \rho \propto V^{-(1+w)} \propto a^{-3(1+w)}$