Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 9

Date: Feb 10, 2017 11:30 AM

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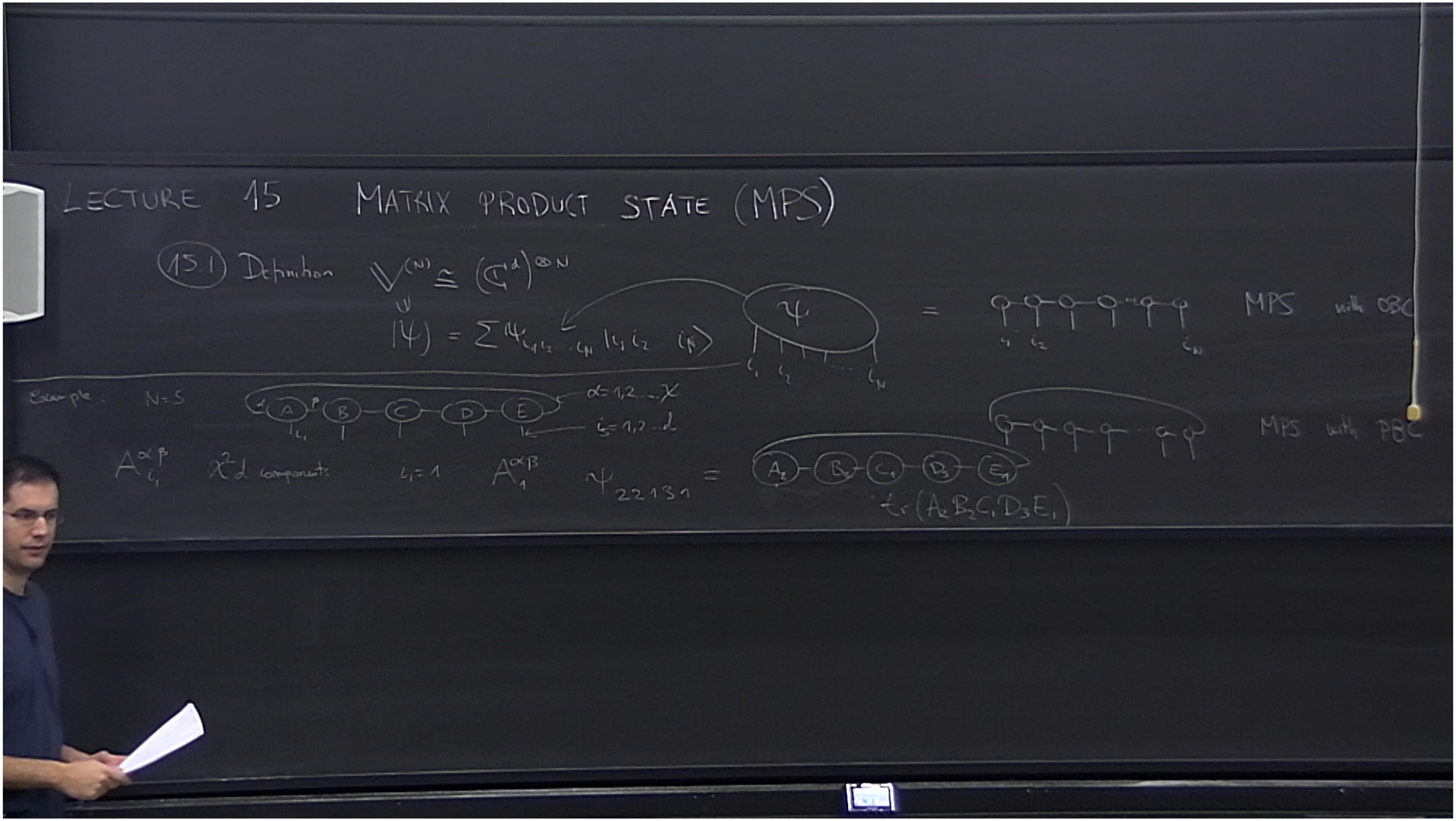
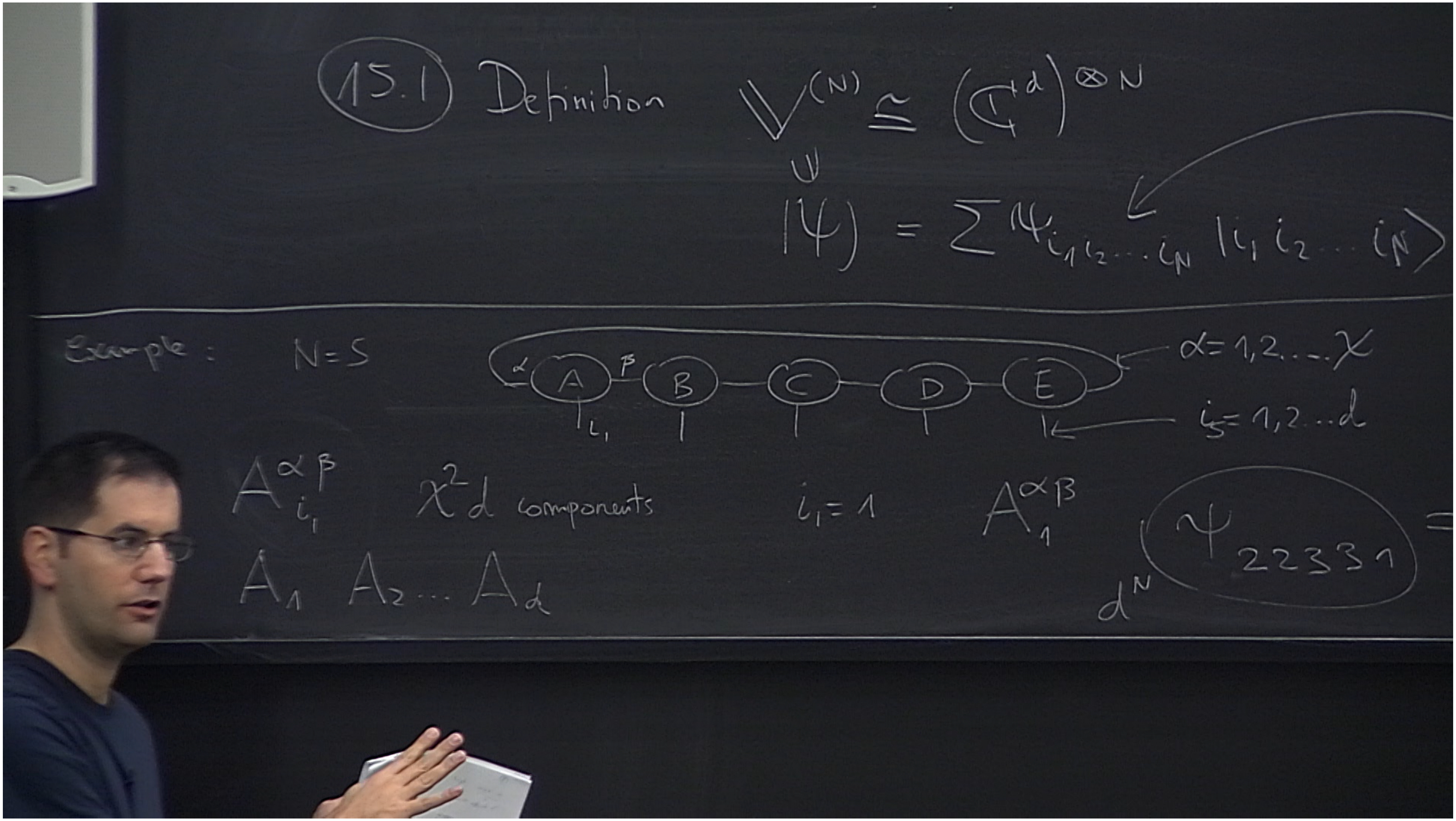
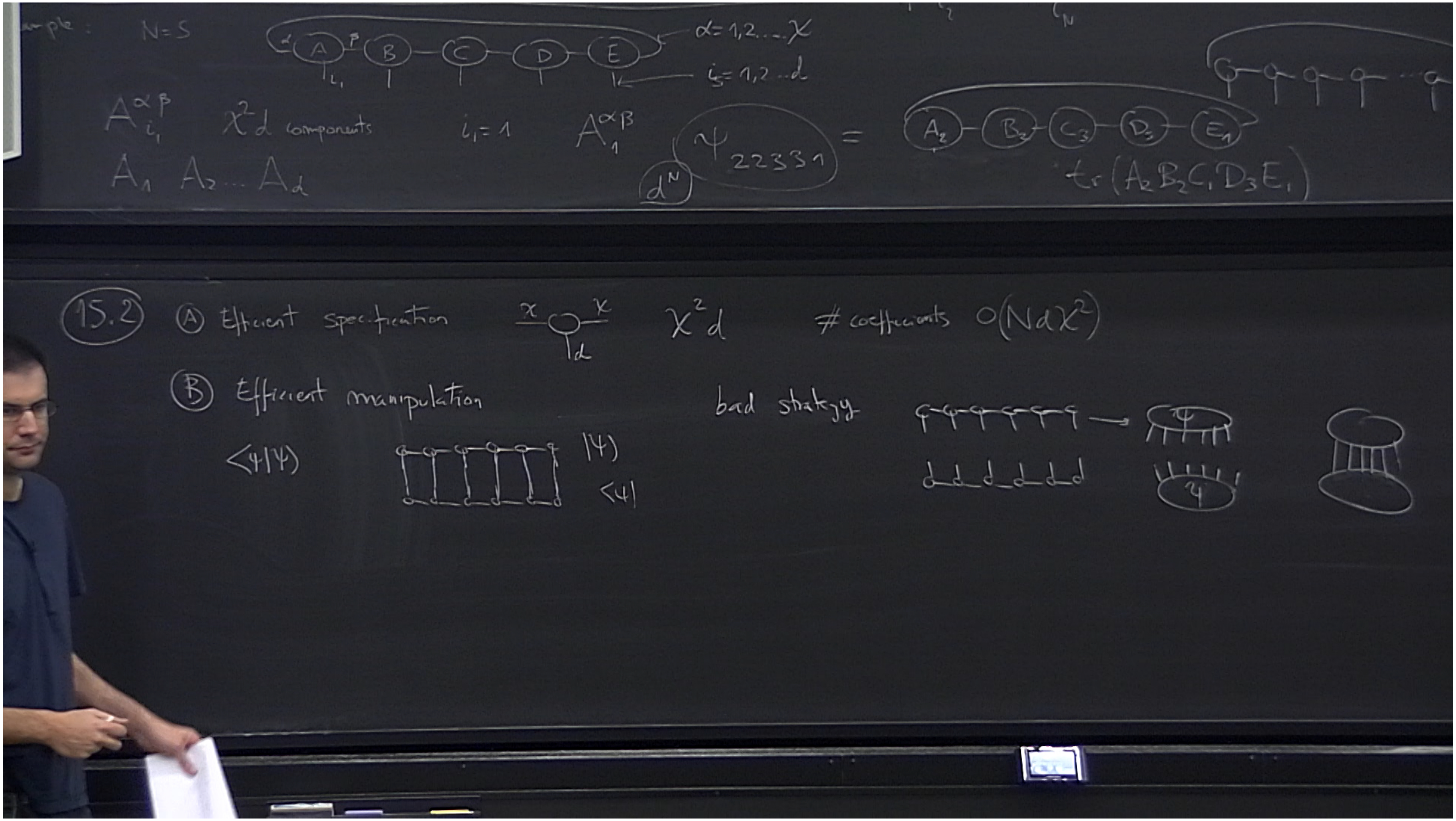
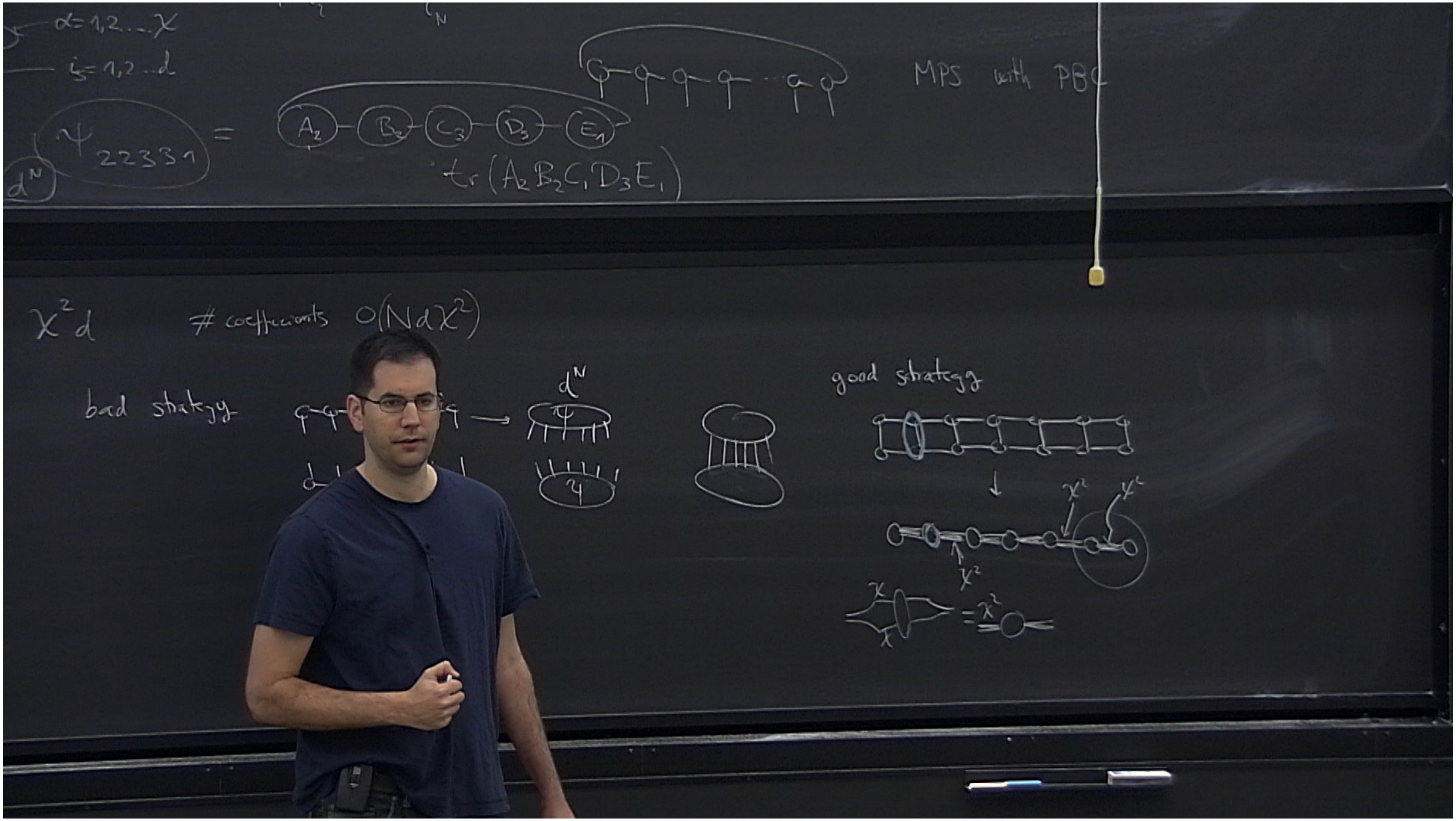
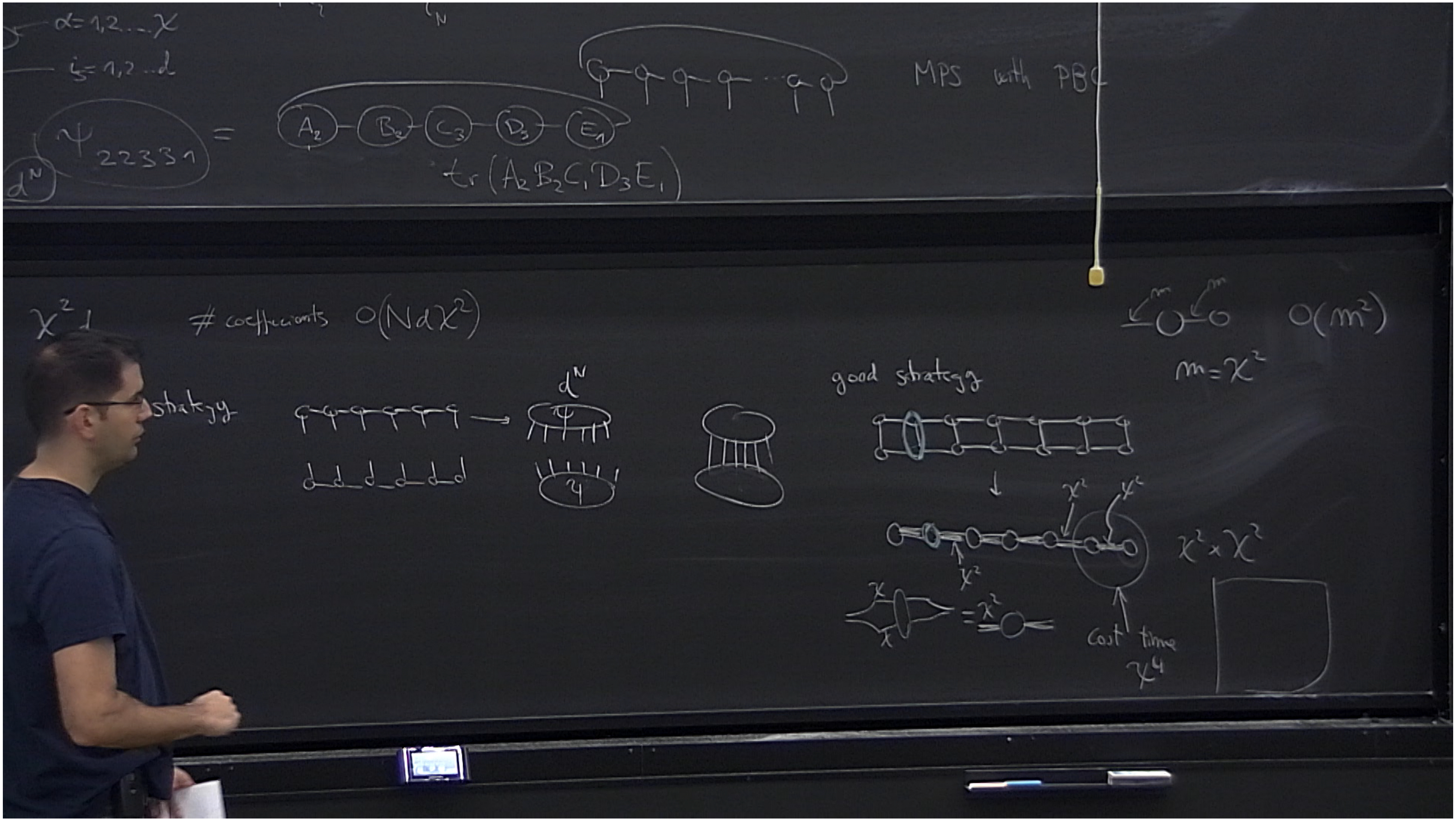
Abstract:

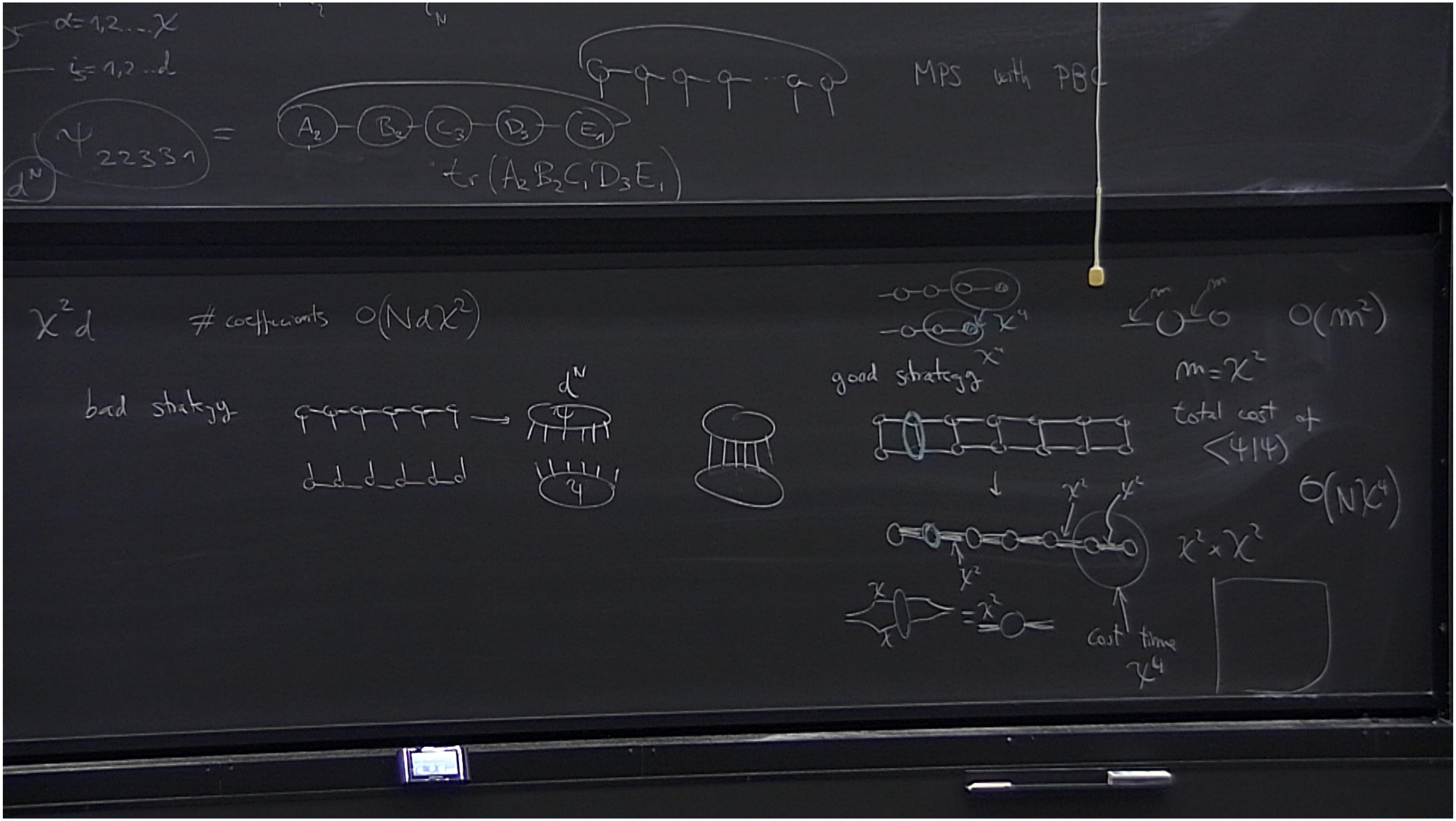

LECTURE 15 MATRIX PRODUCT STATE (MPS)
15.1 Definition
MPS with OBC
Example: N=5
α = 1,2 ... χ
i = 1,2 ... d
χ²d components
MPS with PBC

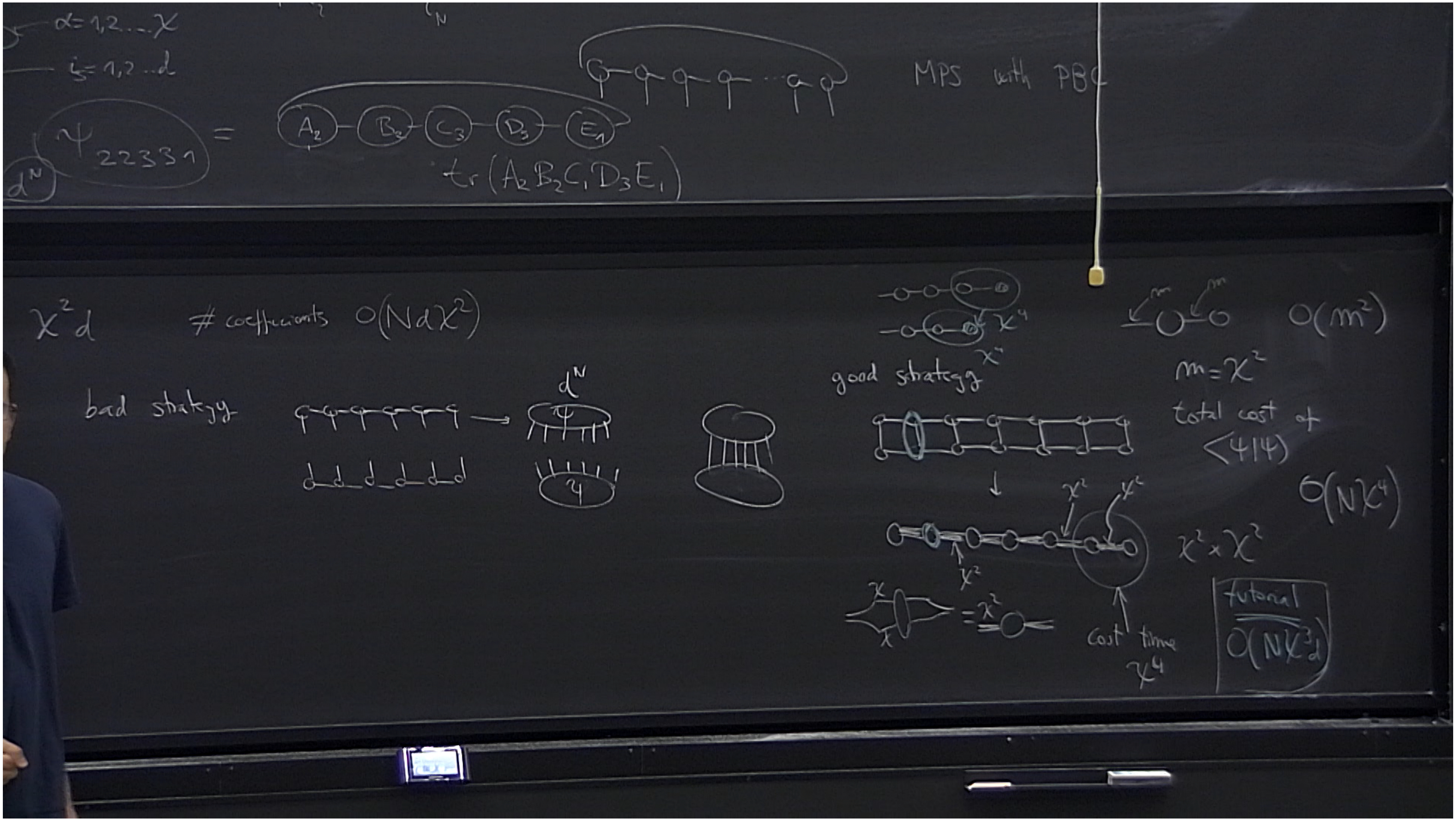

15.1 Definition
Example : N=5
A B C D E
α=1,2...χ
i5=1,2..d
χ²d components
i1=1
d^N
Ψ22331

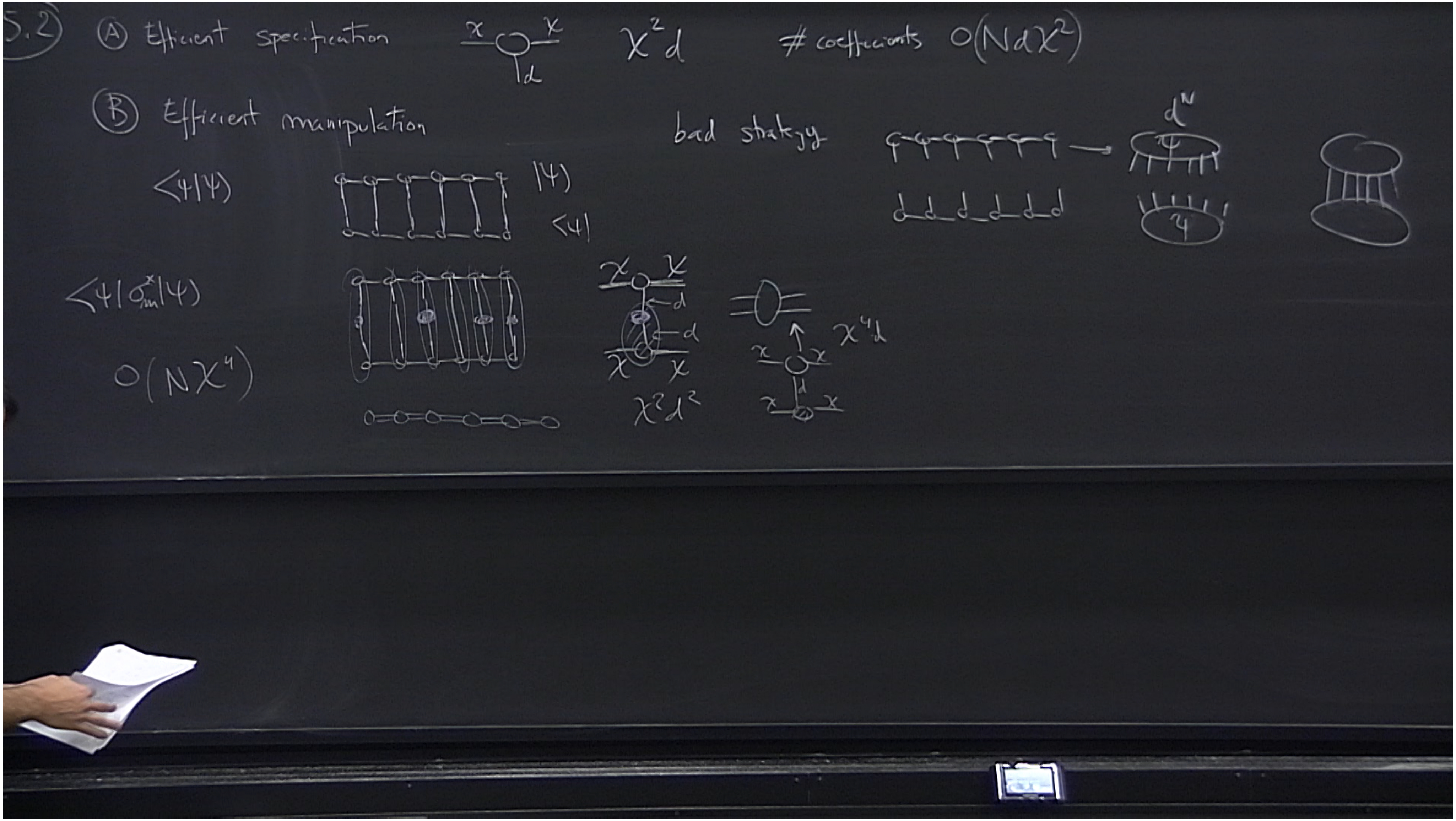

N=5
α=1,2...χ
i=1,2..d
χ²d components
$i_1=1$
ψ_{22331}
d^N
$tr(A_2 B_2 C_1 D_3 E_1)$
15.2
(A) Efficient specification
χ²d
coefficients O(Ndχ²)
(B) Efficient manipulation
bad strategy
⟨ψ|ψ⟩
|ψ⟩
⟨ψ|

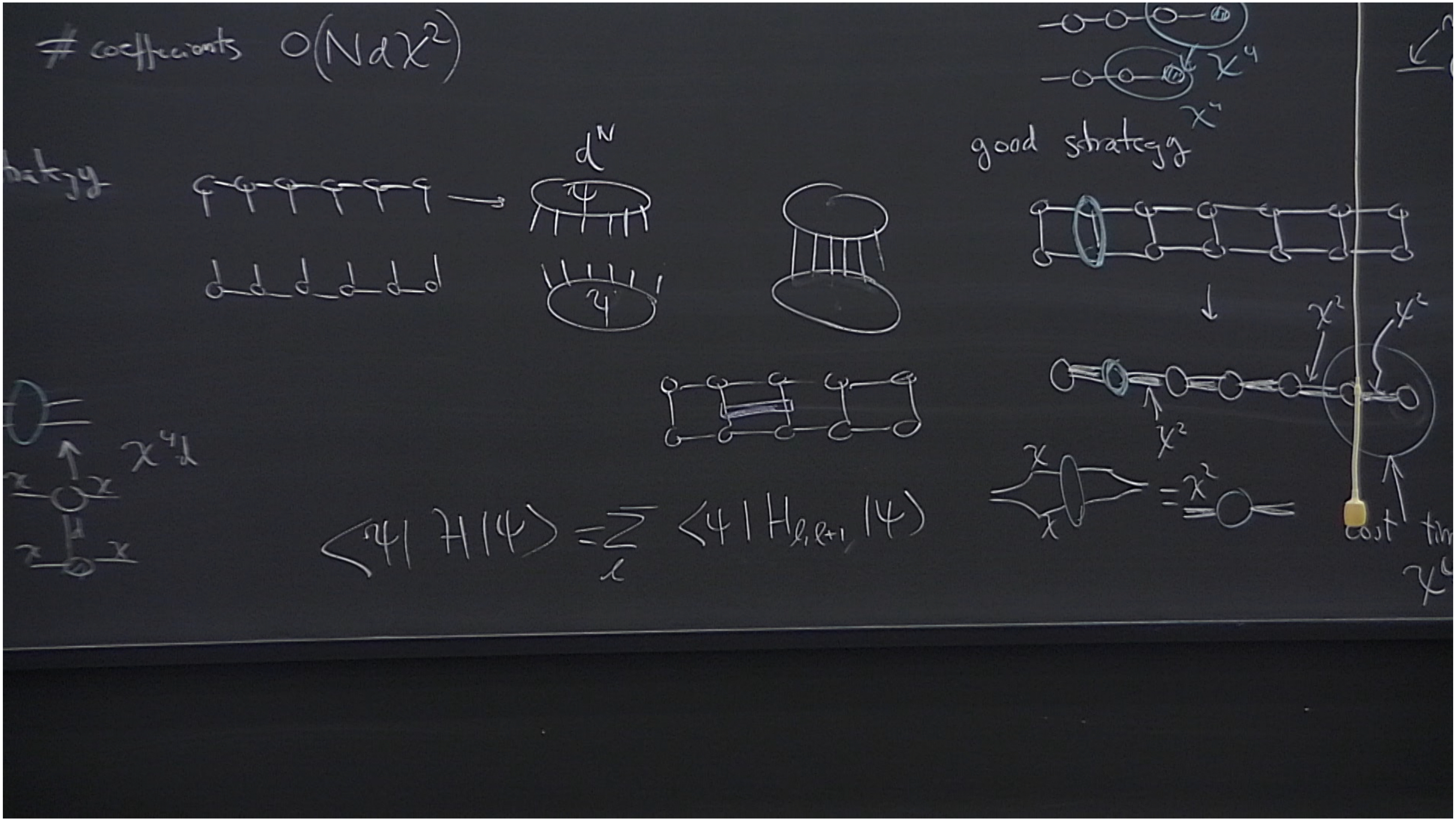

α = 1,2 ... χ
j = 1,2 .. d
MPS with PBC
ψ 22331 =
d^N
tr(A2 B2 C1 D3 E1)
χ² d
coefficients O(N d χ²)
bad strategy
good strategy
χ²

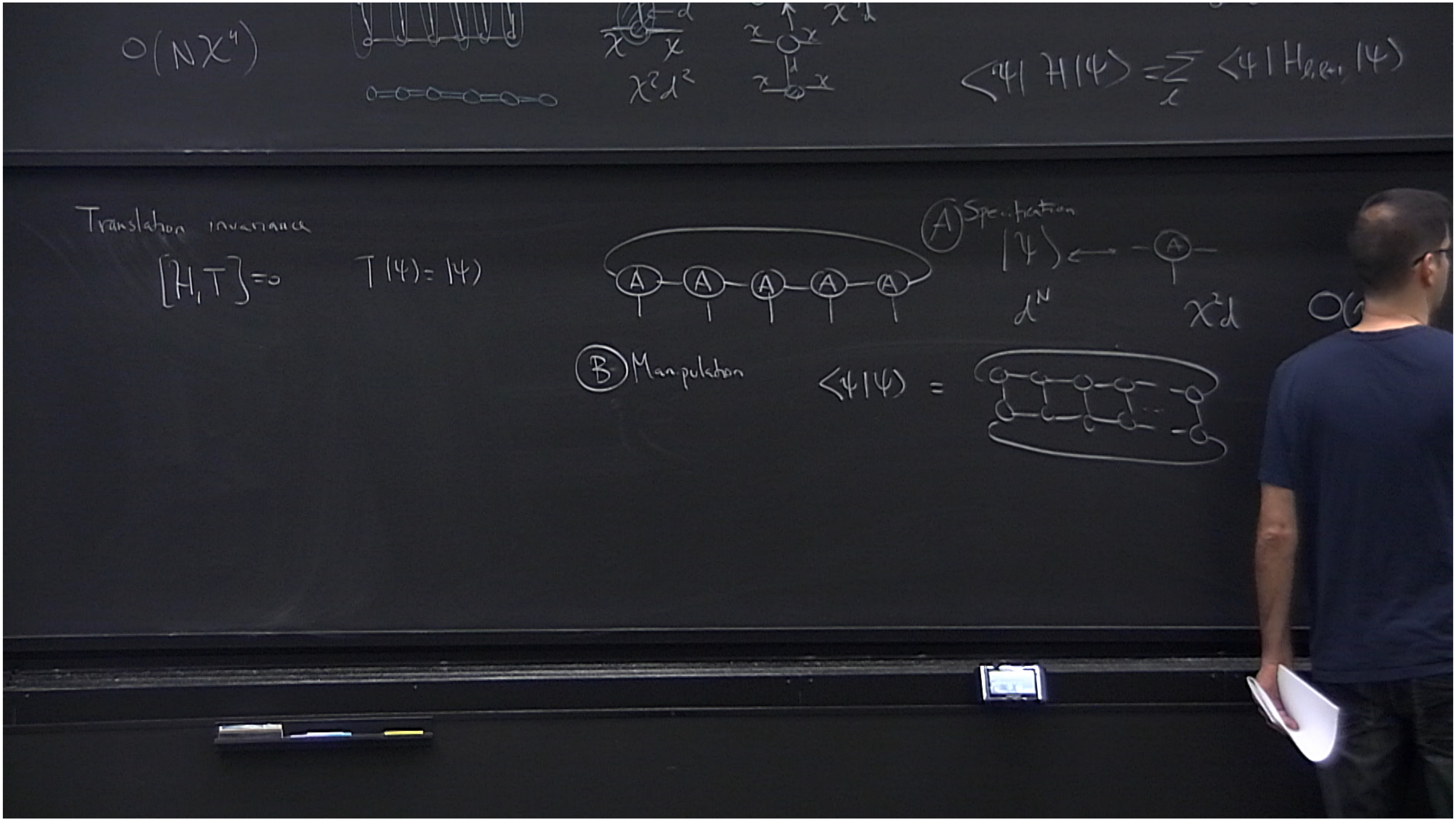

MPS with PBC
tr(A2 B2 C1 D3 E1)
# coefficients O(N d χ^2)
good strategy
m = χ^2
O(m^2)
χ^2 × χ^2
cost time
χ^4


α=1,2...χ
i=1,2...d
MPS with PBC
tr(A2B2C1D3E1)
χ²d
coefficients O(Ndχ²)
bad strategy
good strategy
O(m²)
m=χ²
total cost of
<ψ|ψ>
O(Nχ⁴)
χ²×χ²
cost time
χ⁴

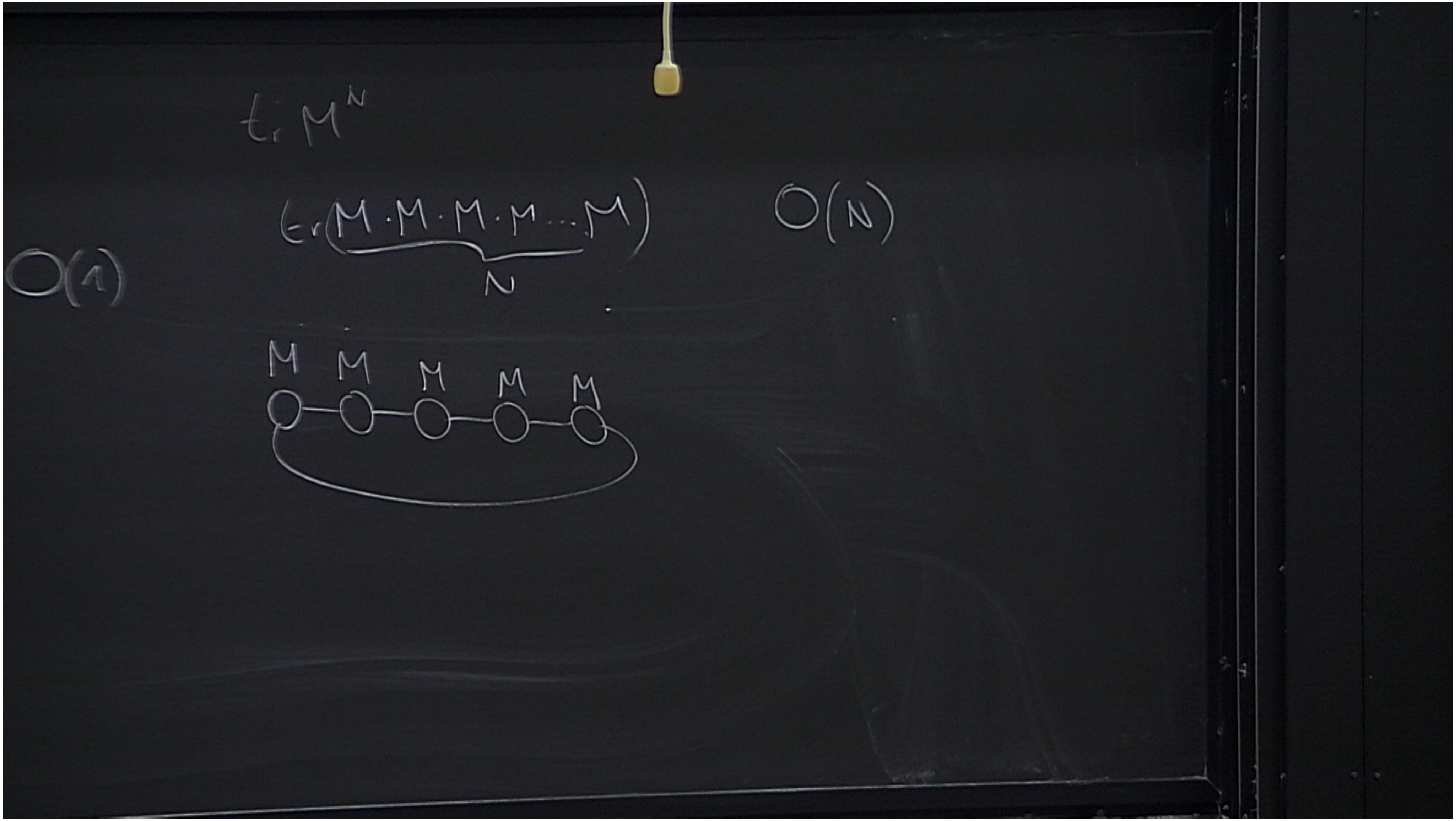

α=1,2...χ
i=1,2...d
MPS with PBC
ψ 22331 =
A2 B2 C3 D3 E1
tr(A2 B2 C1 D3 E1)
d^N
χ²d
coefficients O(Ndχ²)
bad strategy
good strategy
χ⁴
O(m²)
m=χ²
total cost of
<ψ|ψ>
O(Nχ⁴)
χ²×χ²
tutorial
O(Nχ³d)
cost time
χ⁴

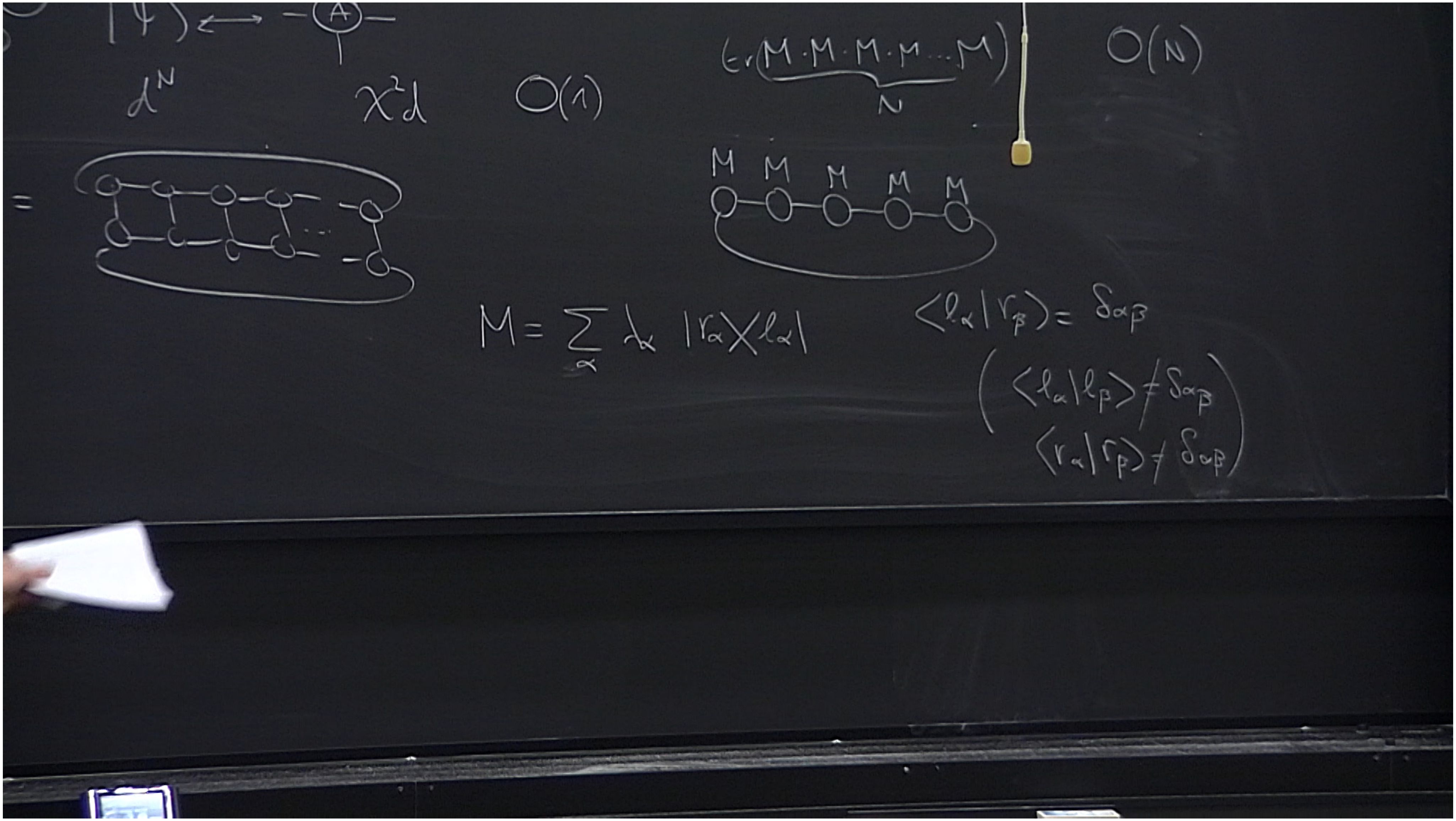

5.2
(A) Efficient specification
χ²d
coefficients O(Ndχ²)
(B) Efficient manipulation
bad strategy
d^N
⟨ψ|ψ⟩
|ψ⟩
⟨ψ|
⟨ψ|σ^x_m|ψ⟩
O(Nχ⁴)
χ²d²
χ⁴d

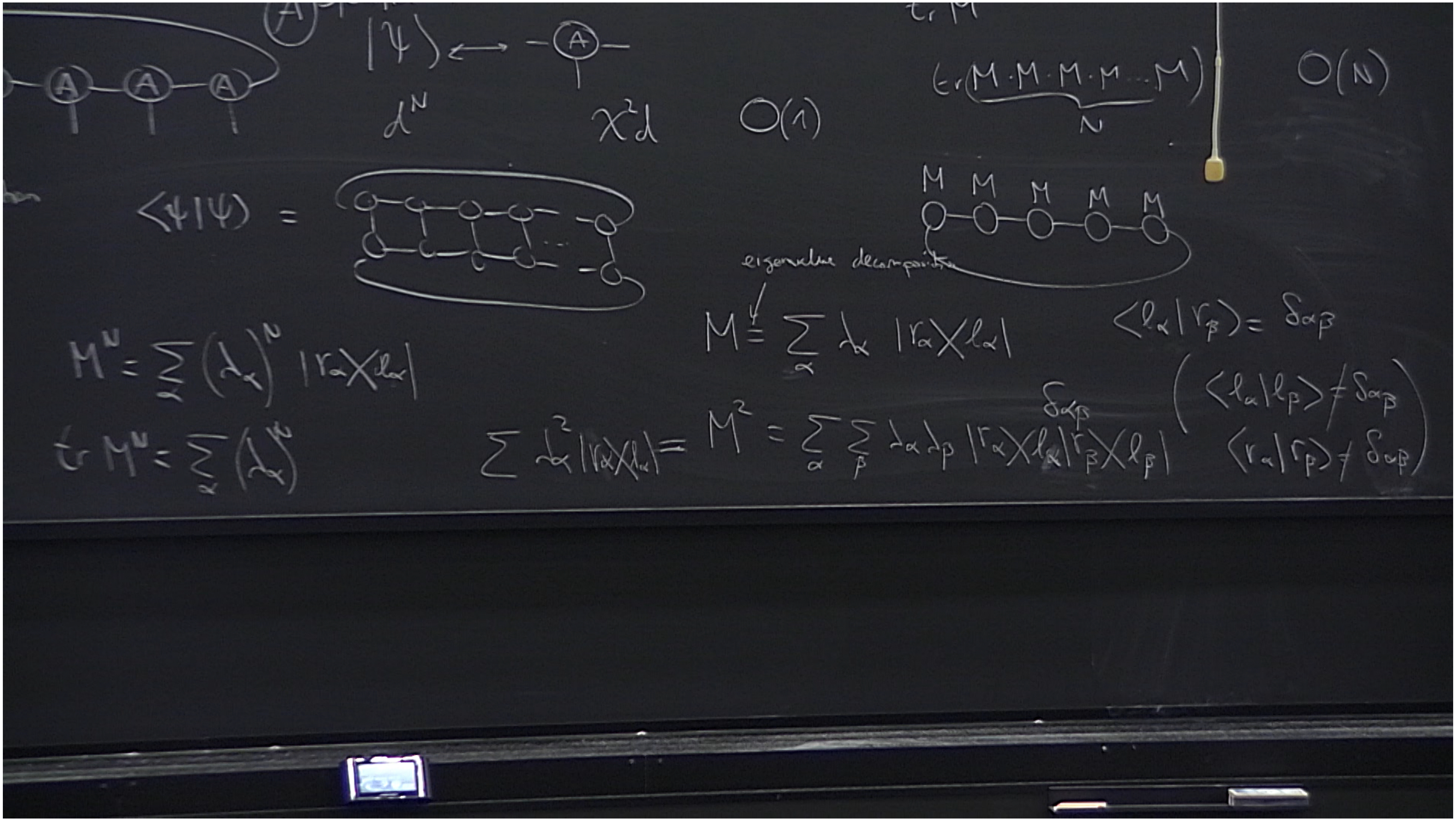

coefficients O(Nd χ²)
d^N
good strategy
χ⁴
χ²
⟨ψ| H |ψ⟩ = Σ_l ⟨ψ| H_{l,l+1} |ψ⟩
cost
χ⁴d

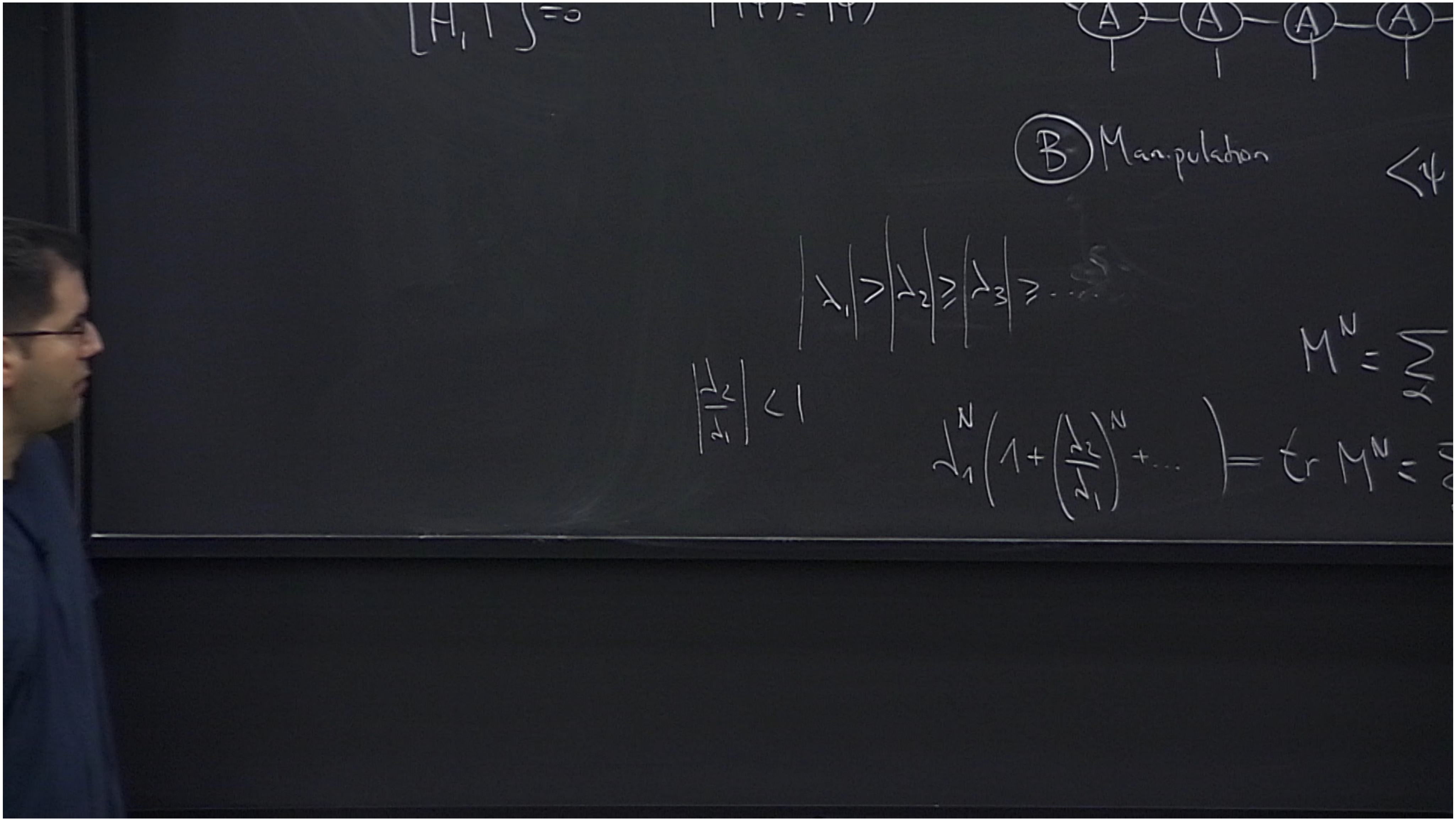


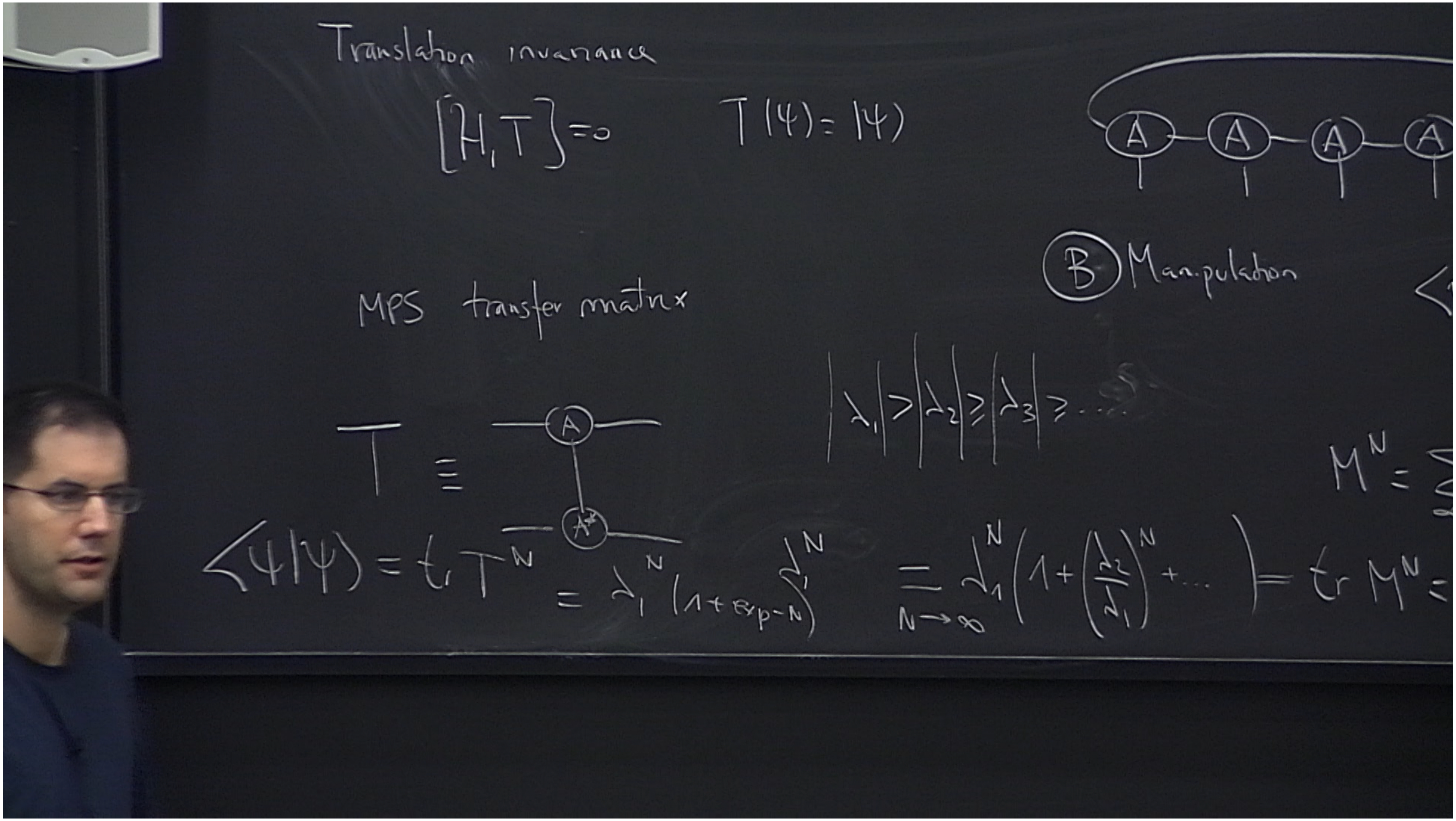
O(Nχ⁴)
⟨ψ| H |ψ⟩ = Σ ⟨ψ| H_{l,l+1} |ψ⟩
Translation invariance
[H,T]=0
T|ψ⟩=|ψ⟩
A Specification
|ψ⟩ ⟵
d^N
χ²d
B Manipulation
⟨ψ|ψ⟩ =

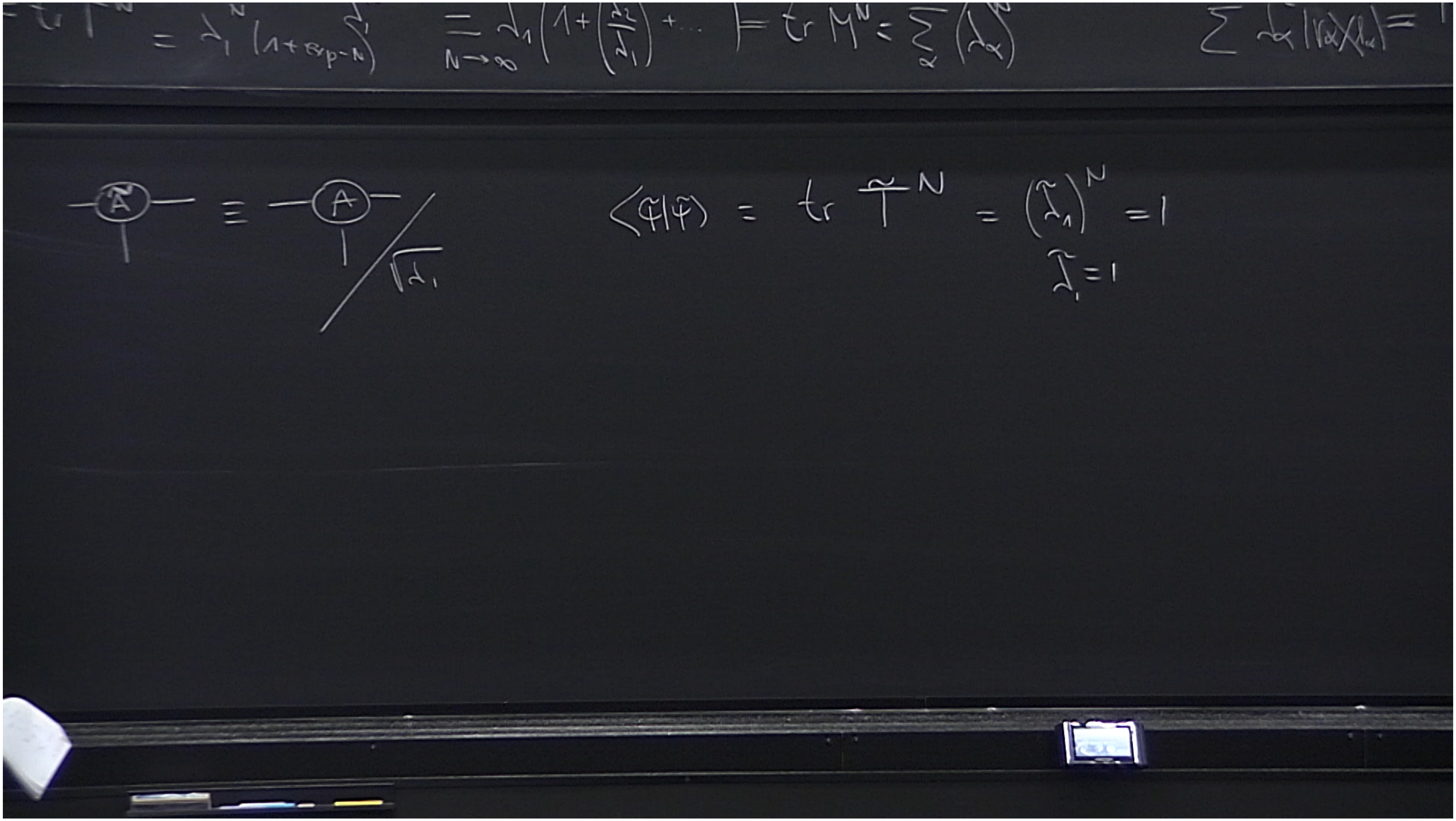

tr M^N
tr(M·M·M·M…M)
N
O(N)
O(1)
M M M M M

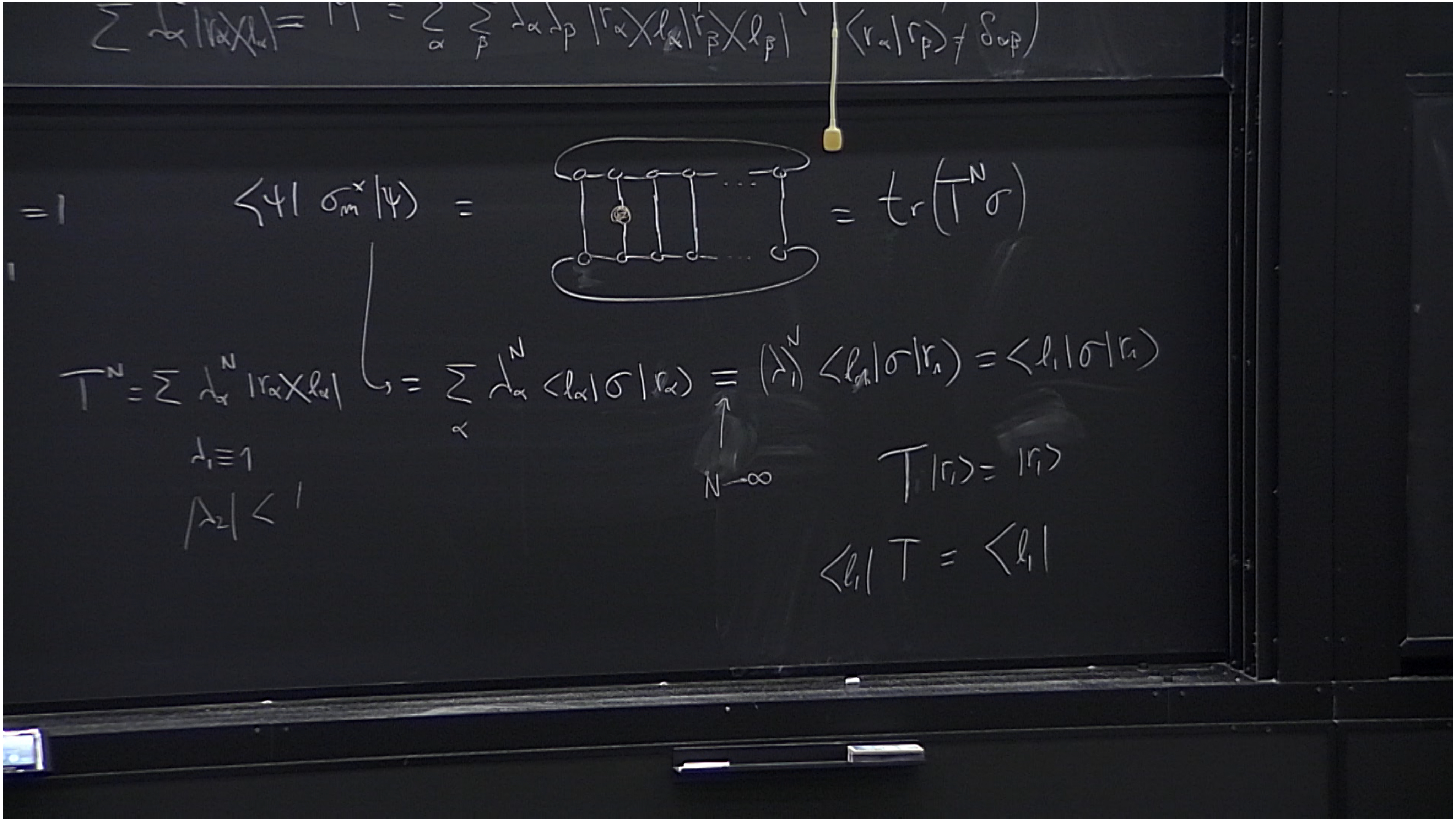

|ψ⟩ ⟷ A
d^N
χ²d
O(1)
tr(M·M·M·M…M)
N
O(N)
M M M M M
M = ∑_α λ_α |r_α⟩⟨ℓ_α|
⟨ℓ_α|r_β⟩ = δ_αβ
(⟨ℓ_α|ℓ_β⟩ ≠ δ_αβ
⟨r_α|r_β⟩ ≠ δ_αβ)

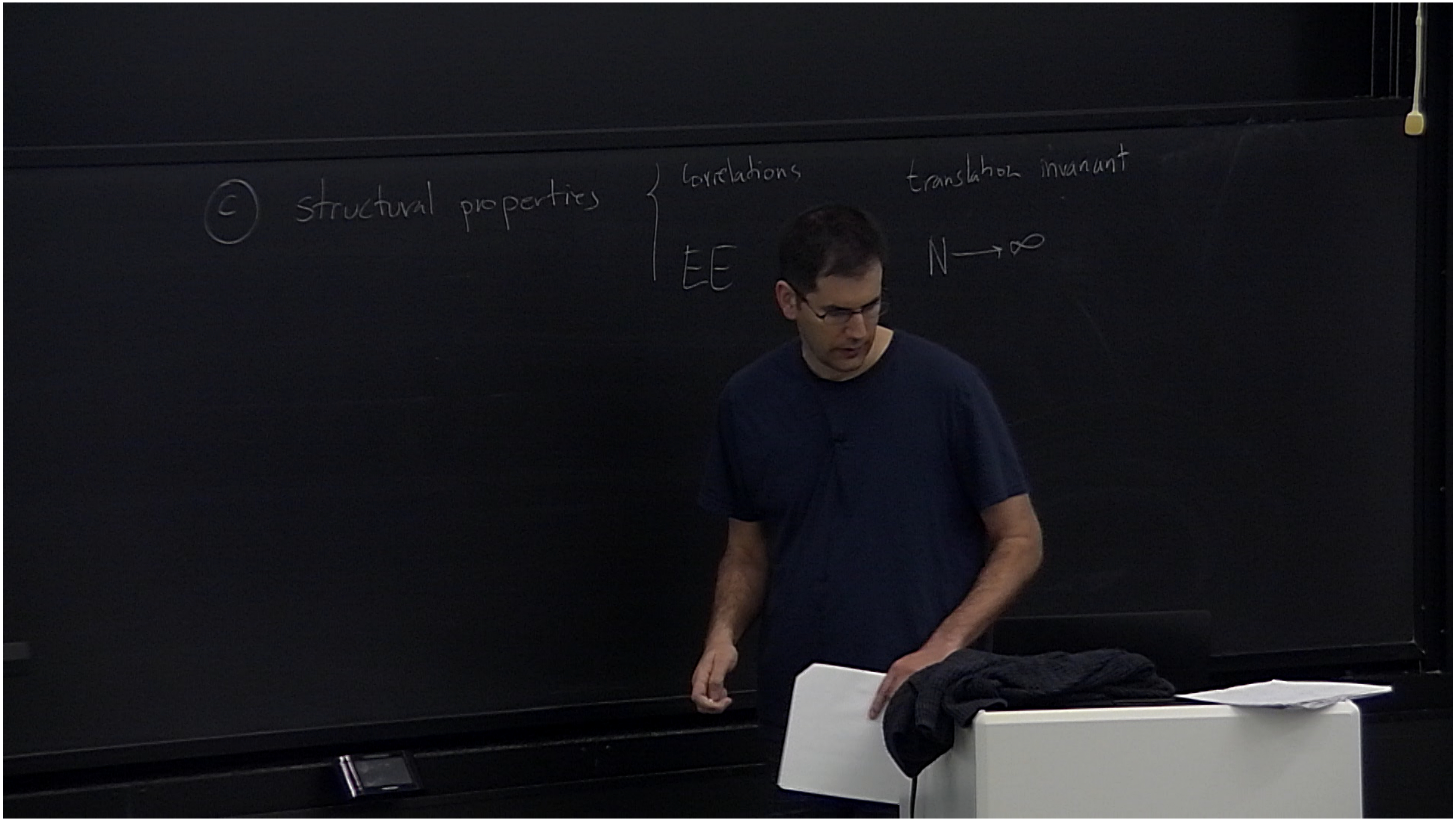

|ψ⟩ ⟷ —(A)—
d^N
χ²d
O(1)
tr(M·M·M·M…M)
N
O(N)
⟨ψ|ψ⟩ =
M M M M M
eigenvalue decomposition
M = Σ_α λ_α |r_α⟩⟨l_α|
⟨l_α|r_β⟩ = δ_αβ
M^N = Σ_α (λ_α)^N |r_α⟩⟨l_α|
tr M^N = Σ_α (λ_α)^N
Σ λ_α² |r_α⟩⟨l_α| = M² = Σ_α Σ_β λ_α λ_β |r_α⟩⟨l_α|r_β⟩⟨l_β|
δ_αβ
⟨l_α|l_β⟩ ≠ δ_αβ
⟨r_α|r_β⟩ ≠ δ_αβ

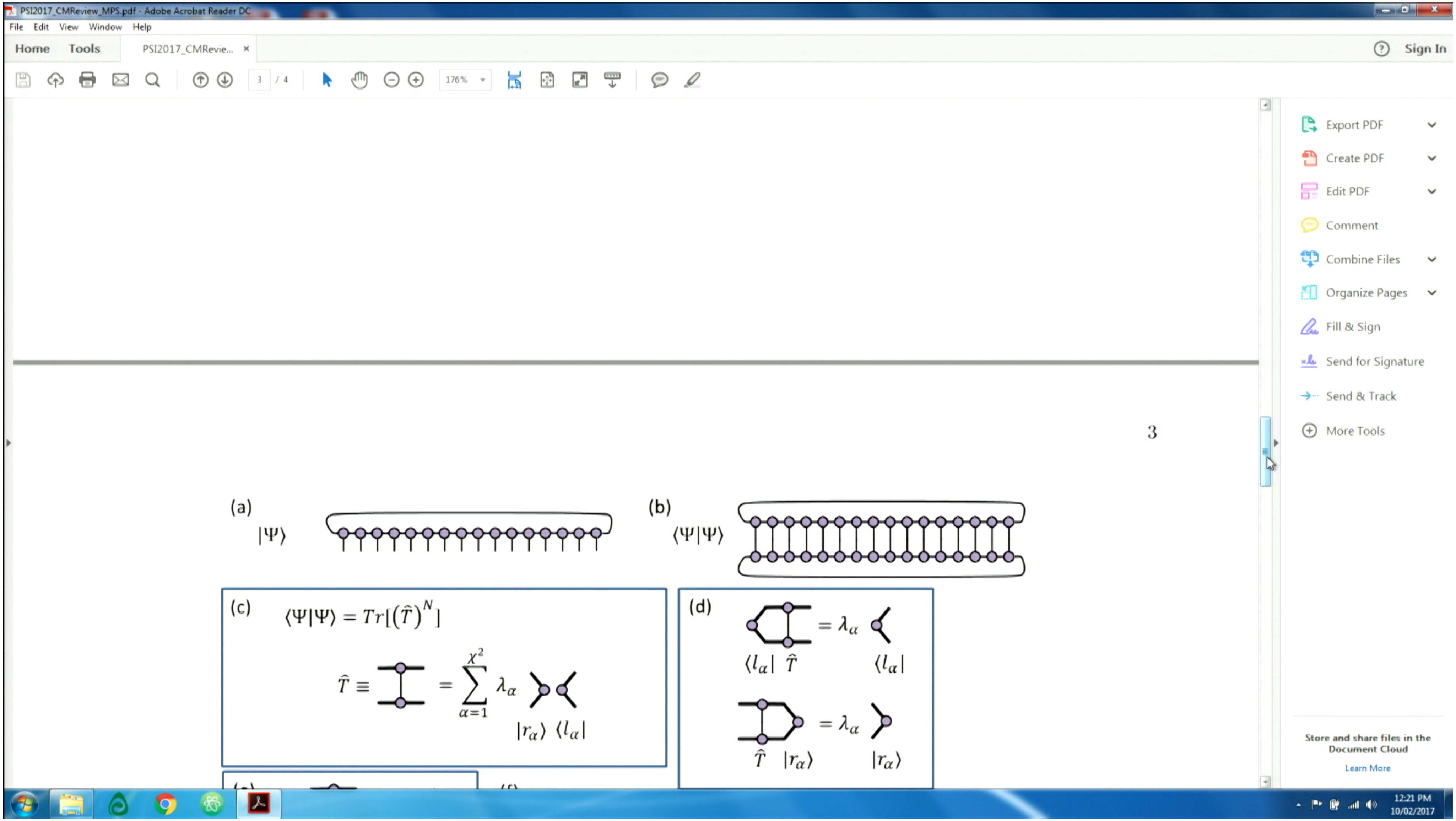

B Manipulation
A A A A
|λ1| > |λ2| ≥ |λ3| ≥ ...
|λ2/λ1| < 1
λ1^N (1 + (λ2/λ1)^N + ...) = tr M^N =
M^N =


Translation invariance
[H, T] = 0
T|ψ) = |ψ)
A
B Manipulation
MPS transfer matrix
|λ1| > |λ2| ≥ |λ3| ≥ ...
T ≡
A*
M^N =
⟨ψ|ψ) = tr T^N = λ1^N (1 + exp-N)
= λ1^N (1 + (λ2/λ1)^N + ...) = tr M^N =
N→∞


= λ_1^N (1 + exp−N)
= λ_1 (1 + (λ_2/λ_1) + …)
N→∞
tr M^N = Σ_α (λ_α)^N
⟨φ̃|φ̃⟩ = tr T̃^N = (λ̃_1)^N = 1
λ̃_1 = 1
√λ_1


⟨ψ| σ_m^x |ψ⟩ =
= tr(T^N σ)
T^N = Σ λ_α^N |r_α⟩⟨l_α|
= Σ_α λ_α^N ⟨l_α| σ |r_α⟩ = (λ_1)^N ⟨l_1| σ |r_1⟩ = ⟨l_1| σ |r_1⟩
λ_1 ≡ 1
|λ_2| < 1
N → ∞
T |r_1⟩ = |r_1⟩
⟨l_1| T = ⟨l_1|
⟨r_α|r_β⟩ ≠ δ_αβ


(c) structural properties
correlations
translation invariant
EE
N ⟶ ∞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3
(a) $|\Psi\rangle$
(b) $\langle\Psi|\Psi\rangle$
(c) $\langle\Psi|\Psi\rangle = Tr[(\hat{T})^N]$
$\hat{T} \equiv \;=\; \sum_{\alpha=1}^{\chi^2} \lambda_\alpha$
$|r_\alpha\rangle$ $\langle l_\alpha|$
(d) $= \lambda_\alpha$
$\langle l_\alpha|$ $\hat{T}$ $\langle l_\alpha|$
$= \lambda_\alpha$
$\hat{T}$ $|r_\alpha\rangle$ $|r_\alpha\rangle$
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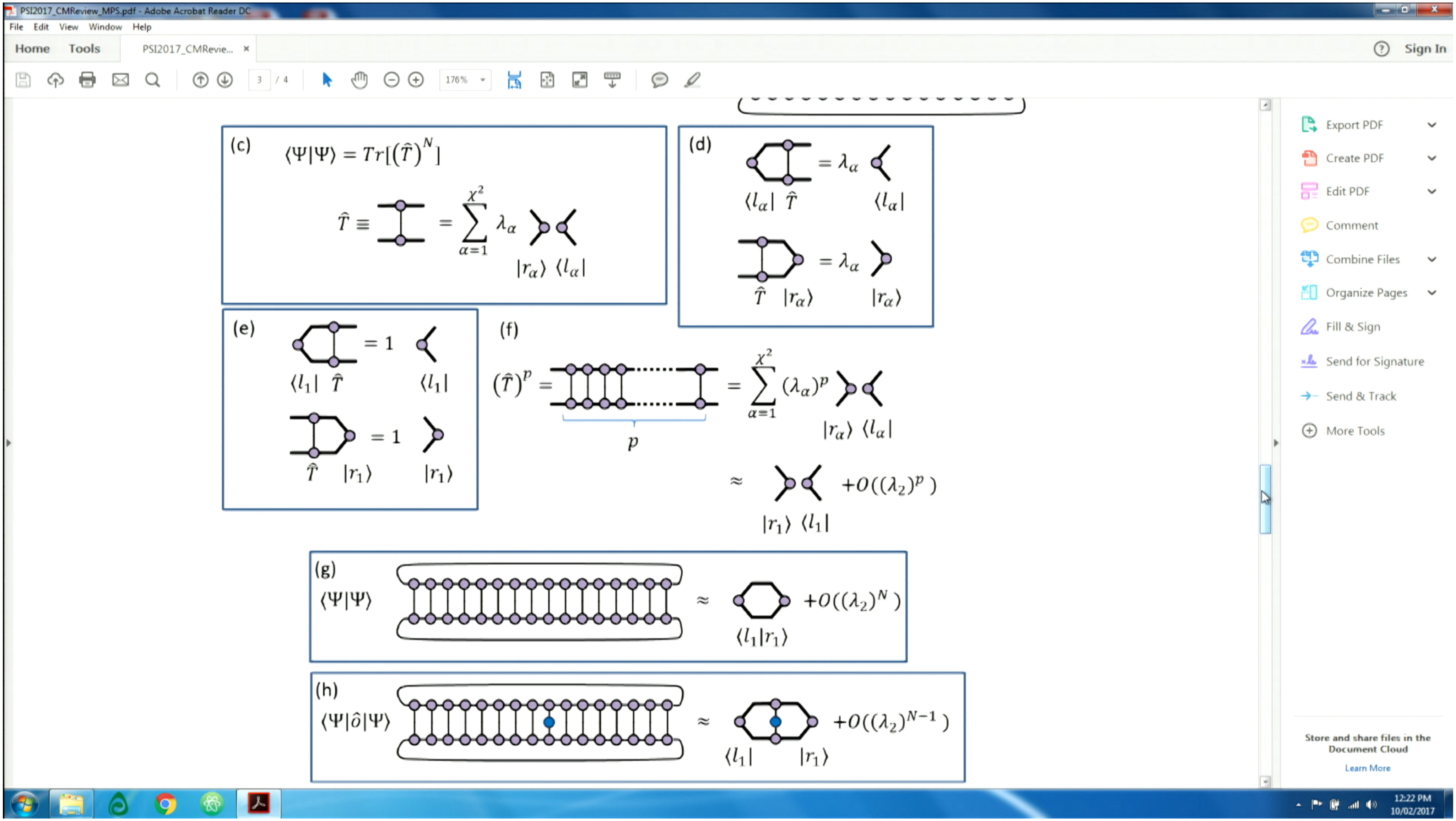
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(c) ⟨Ψ|Ψ⟩ = Tr[(T̂)^N]
T̂ ≡ = Σ_{α=1}^{χ²} λ_α
|r_α⟩ ⟨l_α|
(d) = λ_α
⟨l_α| T̂ ⟨l_α|
= λ_α
T̂ |r_α⟩ |r_α⟩
(e) = 1
⟨l_1| T̂ ⟨l_1|
= 1
T̂ |r_1⟩ |r_1⟩
(f) (T̂)^p = = Σ_{α=1}^{χ²} (λ_α)^p
|r_α⟩ ⟨l_α|
p
≈ +O((λ_2)^p)
|r_1⟩ ⟨l_1|
(g) ⟨Ψ|Ψ⟩ ≈ +O((λ_2)^N)
⟨l_1|r_1⟩
(h) ⟨Ψ|ô|Ψ⟩ ≈ +O((λ_2)^{N−1})
⟨l_1| |r_1⟩
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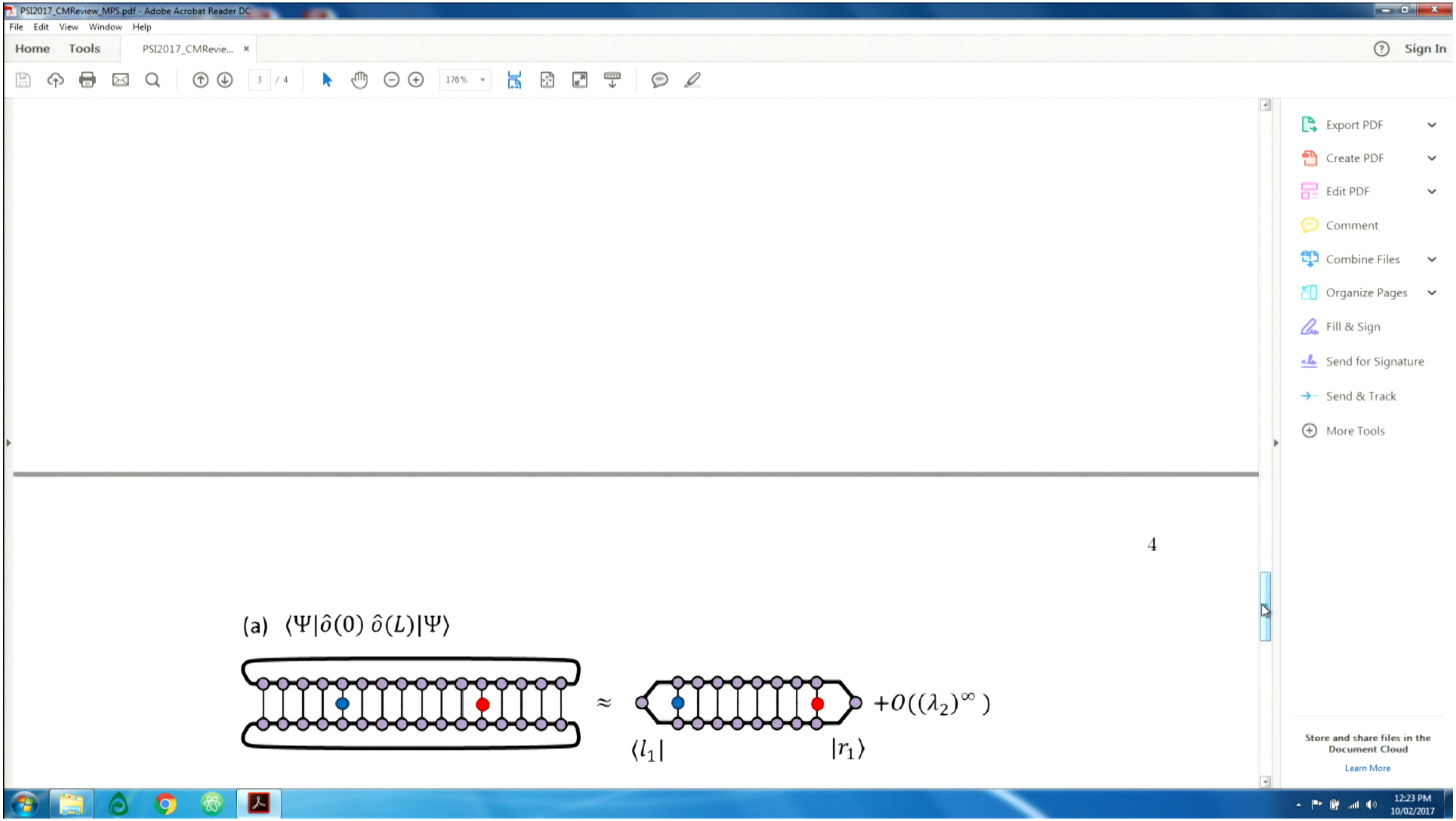
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4
(a) $\langle\Psi|\hat{o}(0)\,\hat{o}(L)|\Psi\rangle$
$\approx$
$+O((\lambda_2)^{\infty})$
$\langle l_1|$
$|r_1\rangle$
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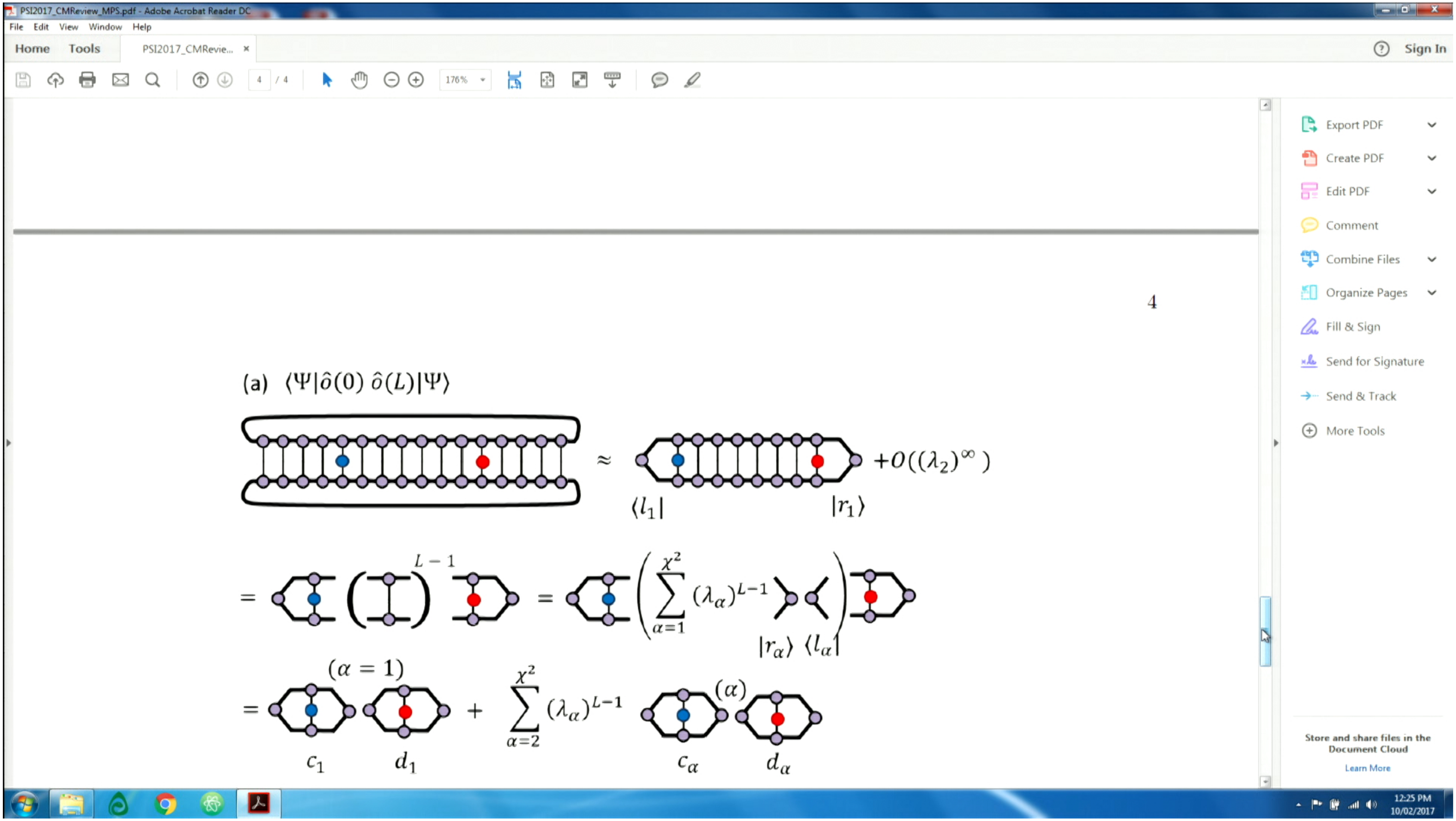
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4
(a) $\langle\Psi|\hat{o}(0)\,\hat{o}(L)|\Psi\rangle$
$\approx$
$+O((\lambda_2)^\infty)$
$\langle l_1|$
$|r_1\rangle$
$L-1$
$\sum_{\alpha=1}^{\chi^2}(\lambda_\alpha)^{L-1}$
$|r_\alpha\rangle$ $\langle l_\alpha|$
$(\alpha=1)$
$\sum_{\alpha=2}^{\chi^2}(\lambda_\alpha)^{L-1}$
(α)
c_1
d_1
c_α
d_α
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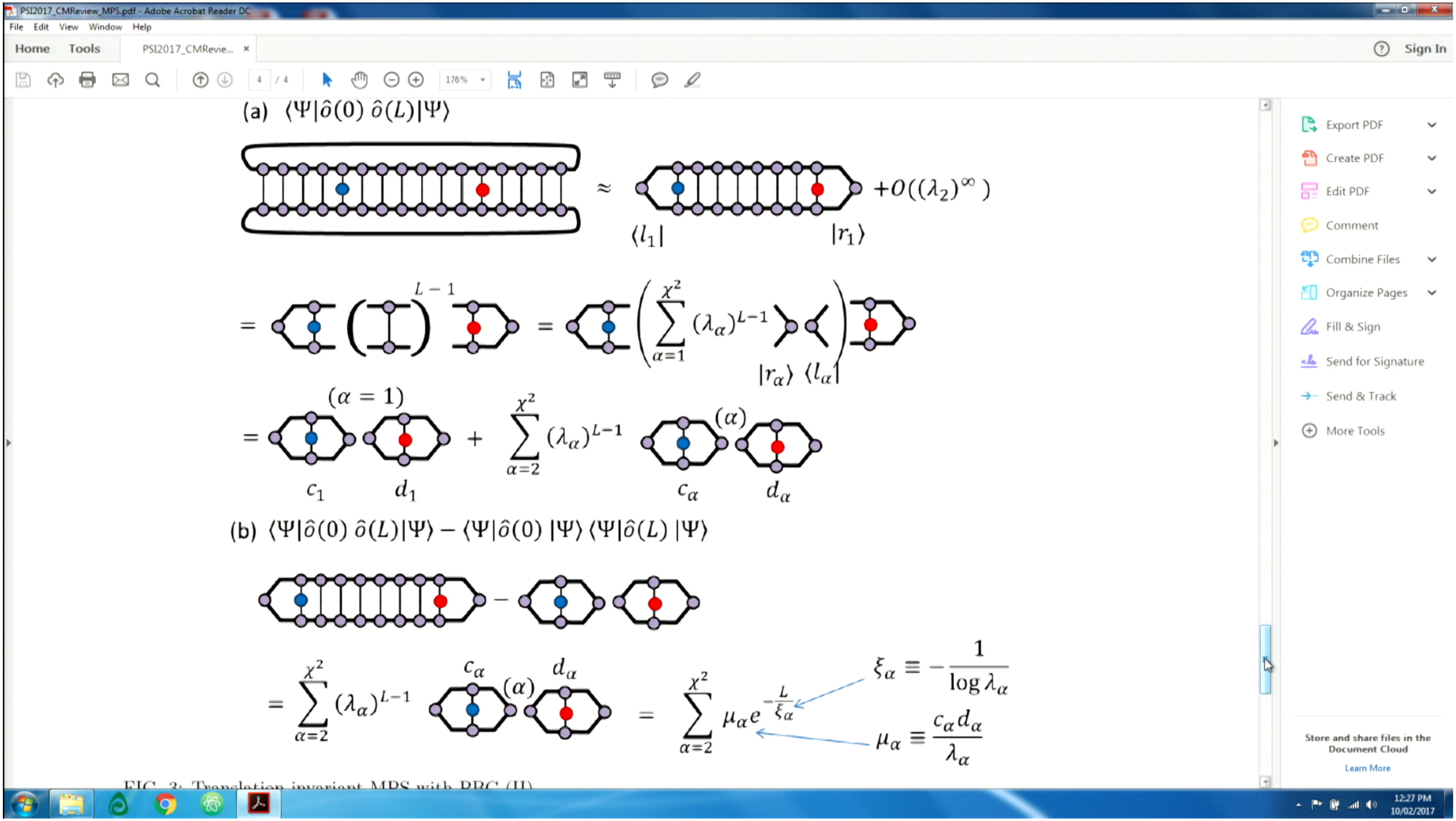
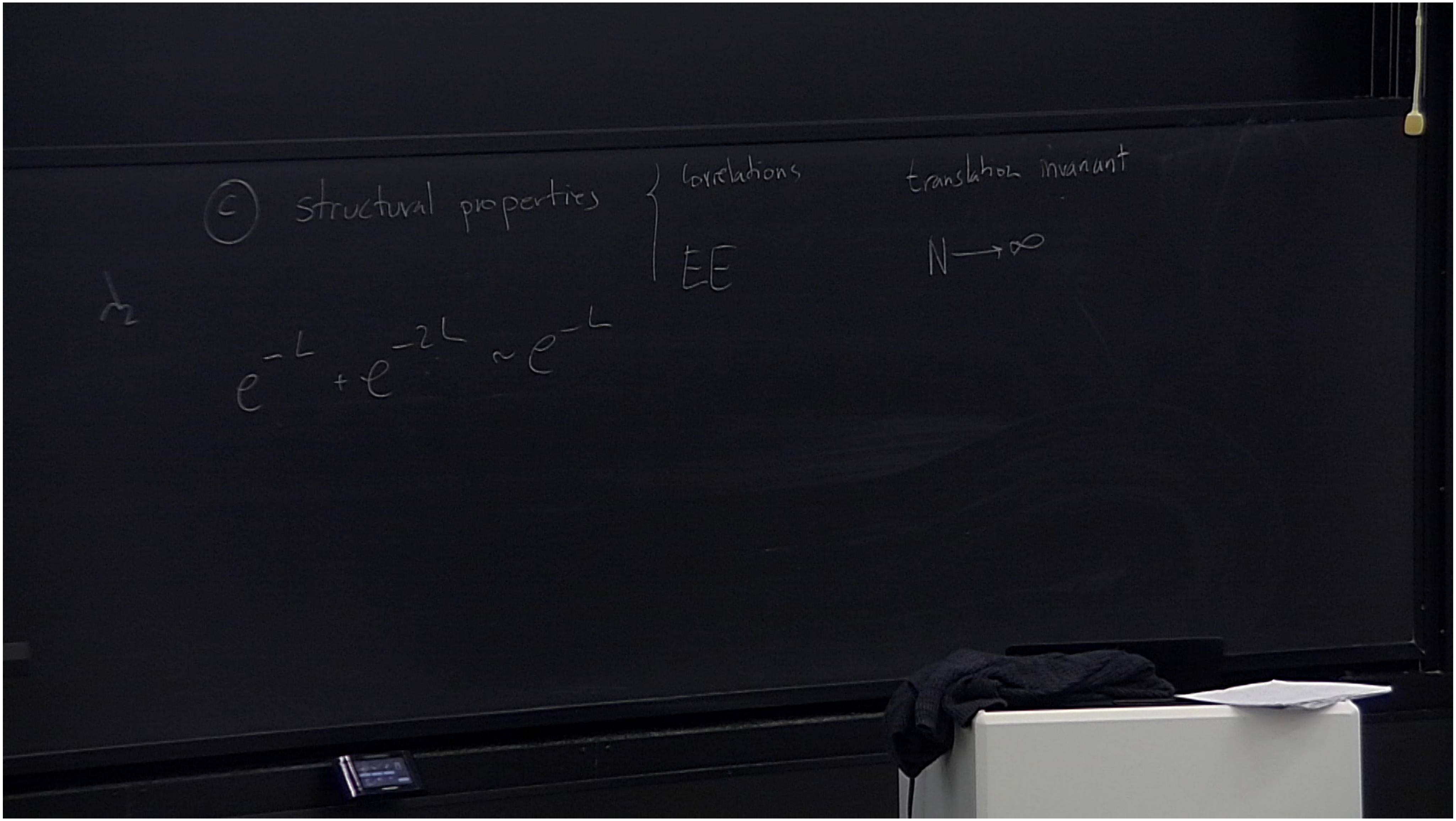

(a) $\langle\Psi|\hat{o}(0)\,\hat{o}(L)|\Psi\rangle$
$\approx$ $+O((\lambda_2)^\infty)$
$\langle l_1|$ $|r_1\rangle$
$L-1$
$\sum_{\alpha=1}^{\chi^2}(\lambda_\alpha)^{L-1}$
$|r_\alpha\rangle\ \langle l_\alpha|$
$(\alpha=1)$
c_1 d_1
$\sum_{\alpha=2}^{\chi^2}(\lambda_\alpha)^{L-1}$
(α)
c_α d_α
(b) $\langle\Psi|\hat{o}(0)\,\hat{o}(L)|\Psi\rangle - \langle\Psi|\hat{o}(0)\,|\Psi\rangle\,\langle\Psi|\hat{o}(L)\,|\Psi\rangle$
$=\sum_{\alpha=2}^{\chi^2}(\lambda_\alpha)^{L-1}$
$=\sum_{\alpha=2}^{\chi^2}\mu_\alpha e^{-\frac{L}{\xi_\alpha}}$
$\xi_\alpha \equiv -\frac{1}{\log\lambda_\alpha}$
$\mu_\alpha \equiv \frac{c_\alpha d_\alpha}{\lambda_\alpha}$


(c) structural properties
correlations
EE
translation invariant
N ⟶ ∞
e^{-L} + e^{-2L} ~ e^{-L}

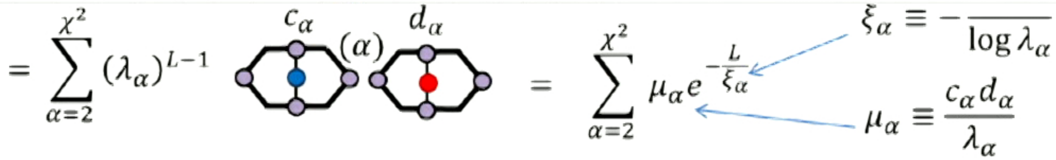


FIG. 3: Translation invariant MPS with PBC (II)
(a) The computation of a two-point correlator $\langle\Psi|\hat{o}(0)\hat{o}'(L)|\Psi\rangle$ can be reduced to computing a collection of complex coefficients c_α and d_β, each one resulting from a simple tensor network made of 5 tensors. Namely, c_α is obtained multiplying together $(\hat{o}, \hat{A}, \hat{A}^*, |r_1\rangle, \langle l_\alpha|)$, whereas d_α is obtained multiplying together $(\hat{o}', \hat{A}, \hat{A}^*, |r_\alpha\rangle, \langle l_1|)$. (b) The connected correlator $C(L) \equiv \langle\Psi|\hat{o}(0)\hat{o}'(L)|\Psi\rangle - \langle\Psi|\hat{o}(0)|\Psi\rangle\langle\Psi|\hat{o}'(L)|\Psi\rangle$ reduces to a sum of exponentials $e^{-\frac{L}{\xi_\alpha}}$, where by construction $\xi_2 \geq \xi_3 \geq \cdots$. Thus, for large L, the correlator is dominated by the largest correlation length and we find $C(L) \approx \mu_1 e^{-\frac{L}{\xi_2}}$, so that correlations decay exponentially.

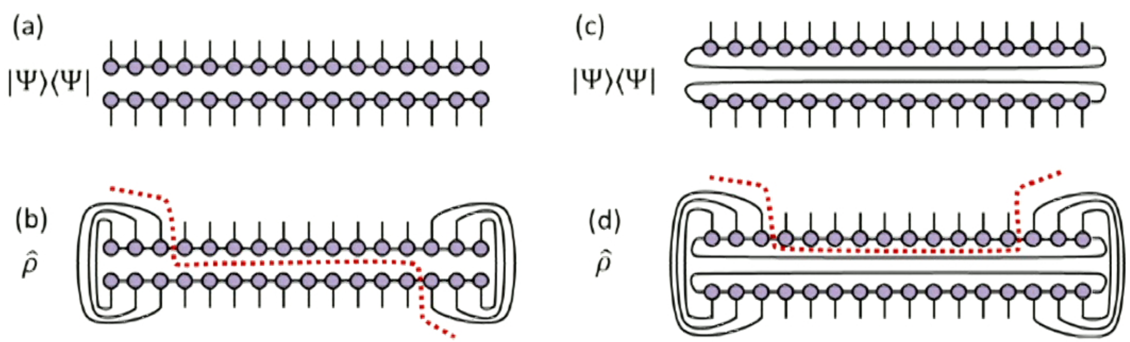
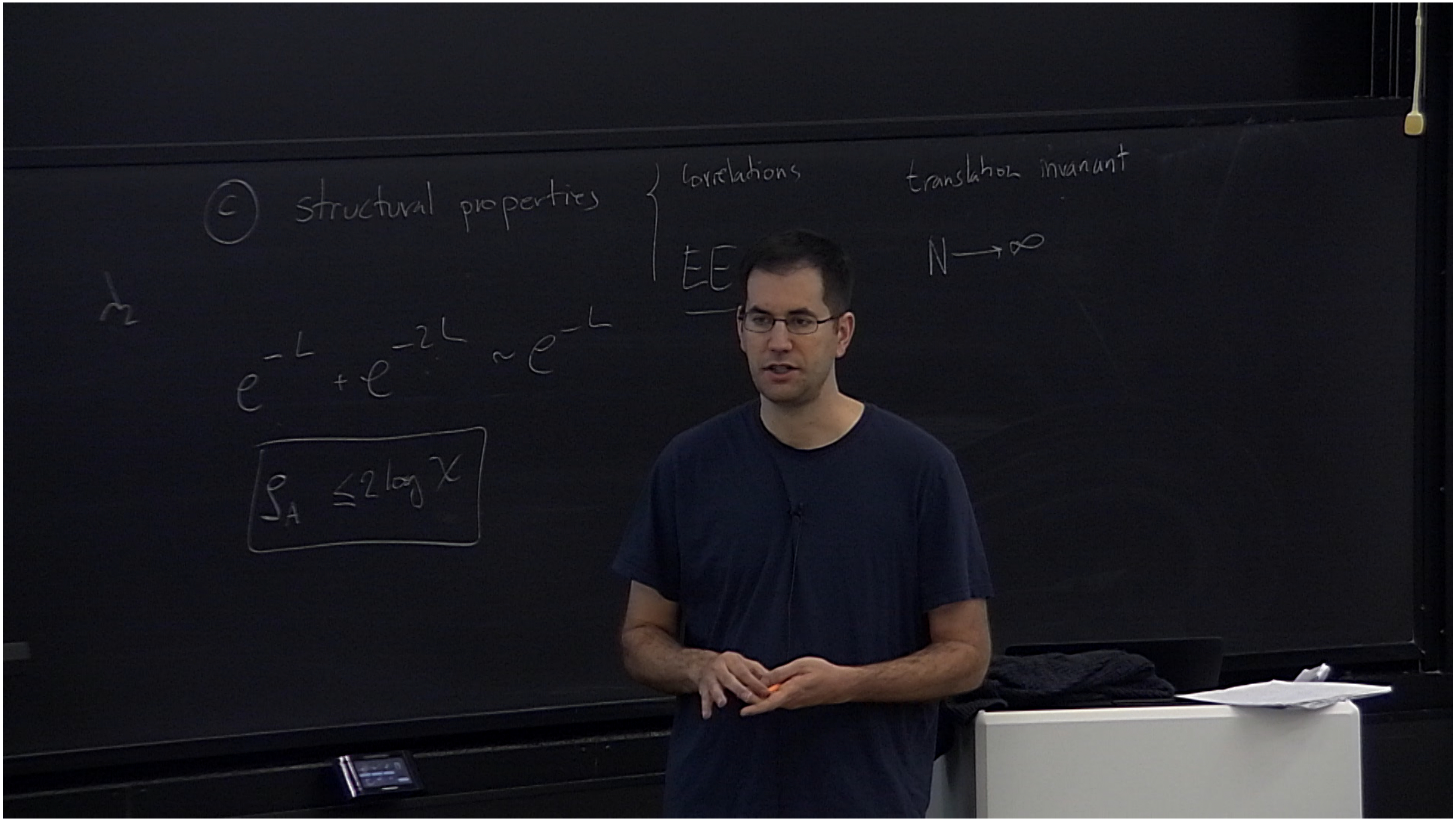


FIG. 4: Scaling of entanglement entropy
(a) Projector $|\Psi\rangle\langle\Psi|$ onto state $|\Psi\rangle \in \mathbb{V}^{\otimes 16}$ represented with an MPS with OBC. (b) Reduced density matrix $\hat{\rho}$ for 10 sites obtained after tracing out 6 sites. The dashed line splits the bra-indices from the ket-indices by cutting two bond indices. This means that the rank of $\hat{\rho}$ is at most χ^2, and its entropy at most $2\log_2(\chi)$, where χ is the bond dimension. (c) Projector $|\Psi\rangle\langle\Psi|$


(c) structural properties
correlations
translation invariant
EE
N → ∞
e^{-L} + e^{-2L} ~ e^{-L}
S_A ≤ 2 log χ