Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 12

Date: Feb 15, 2017 09:00 AM

URL: http://pirsa.org/17020048

Abstract:

Condensed Matter Review – PSI 2016-2017

Lecture 18 (of 18)

Tensor Networks in

(i) D>1 spatial dimensions

(ii) Statistical Mechanics
(partition functions)

(iii) Holography

Guifre Vidal, Perimeter Institute

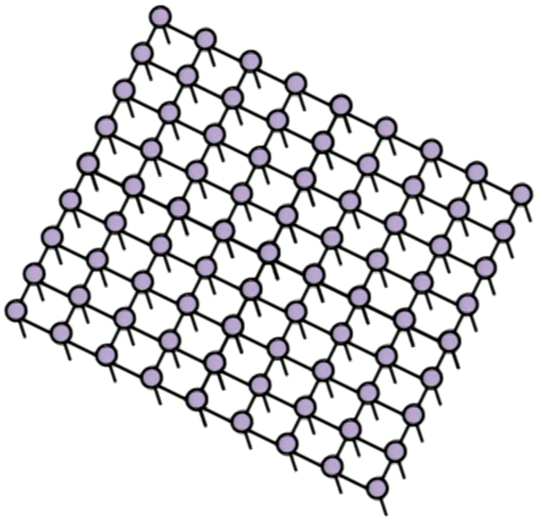
(i) Tensor Networks in D>1 spatial dimensions

area law

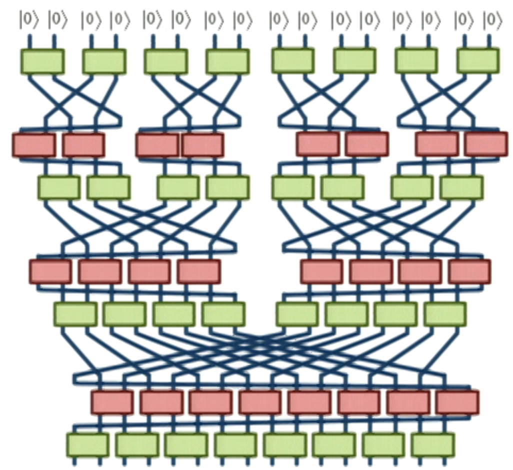
$S(L) = L^{D-1}$

logarithmic correction

$S(L) = L^{D-1}\log(L)$



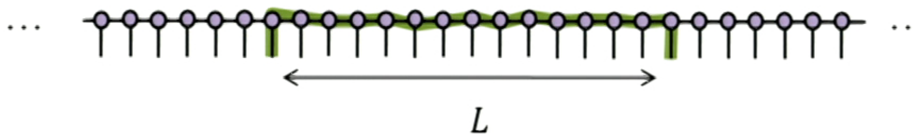
projected entangled pair states (PEPS)



branching MERA

CORRELATIONS and DISTANCE

matrix product state (MPS)

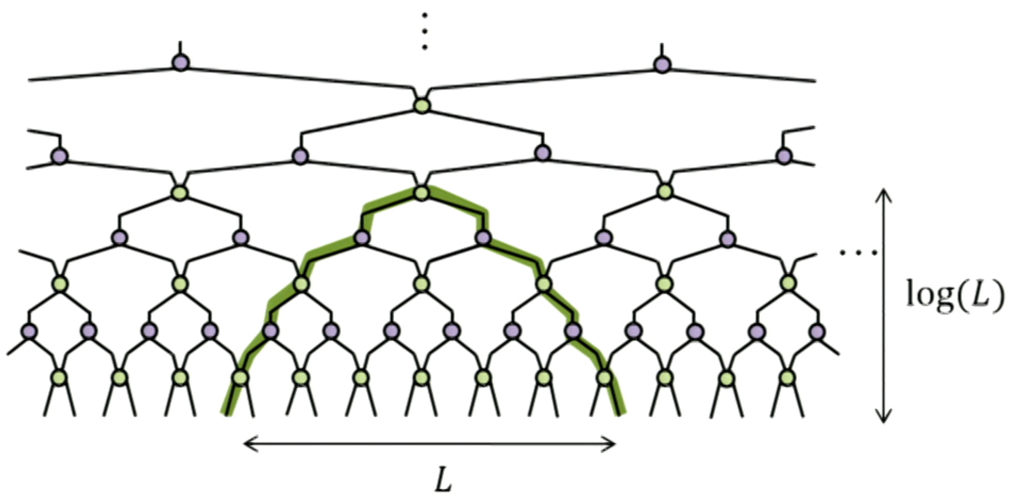


exponential correlators

$C(L) \approx e^{-L/\xi}$

$[C(L) \approx \lambda^{L}]$

multi-scale entanglement renormalization ansatz (MERA)



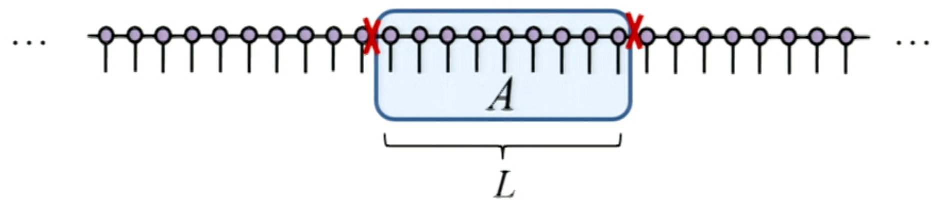
polynomial correlators

$C(L) \approx L^{-p}$

$[C(L) \approx \lambda^{\log(L)}]$

ENTANGLEMENT ENTROPY and CONNECTIVITY

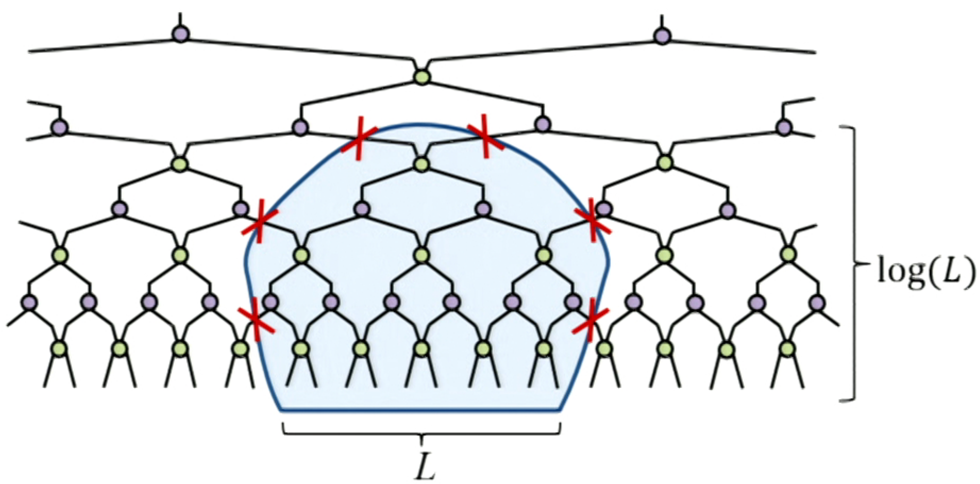
matrix product state (MPS)



constant entropy

$S_L \approx const$

multi-scale entanglement renormalization ansatz (MERA)

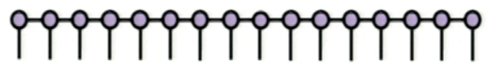


logarithmic entropy

$S_L \approx \log(L)$

D=1 spatial dimensions

matrix product state
(MPS)

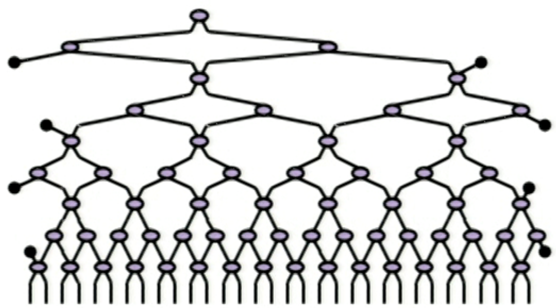


$$C(L) \approx e^{-L/\xi}$$

$$S_L \approx const \quad (= L^{D-1})$$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



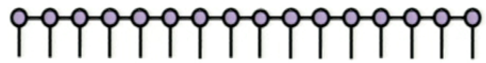
$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=1 spatial dimensions

matrix product state
(MPS)



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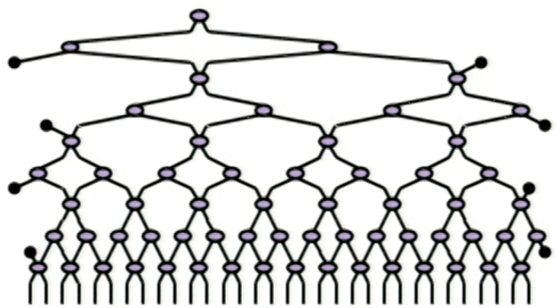
(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



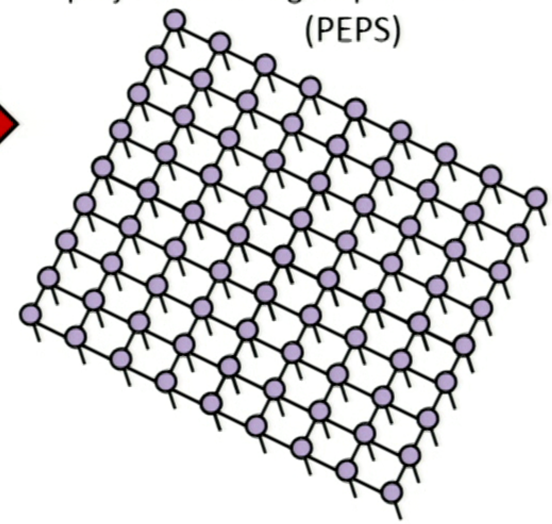
$C(L) \approx L^{-p}$

$S_L \approx \log L \quad (= L^{D-1} \log L)$

(critical systems)

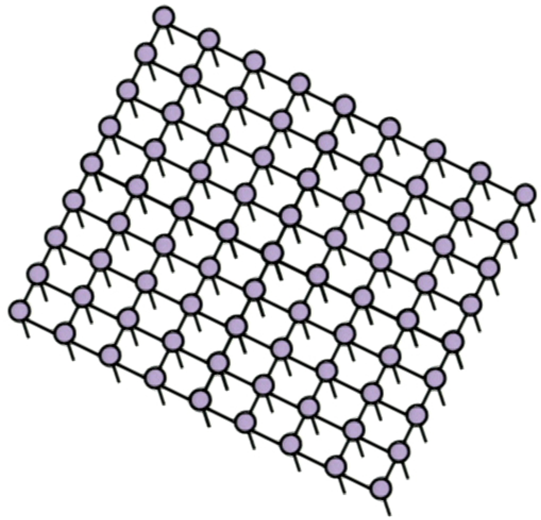
D=2 spatial dimensions

projected entangled pair states
(PEPS)



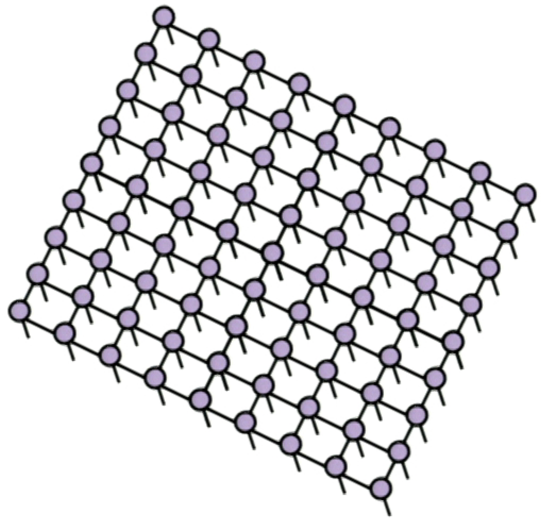
Projected entangled pair states (PEPS)

A - Efficient representation?



Projected entangled pair states (PEPS)

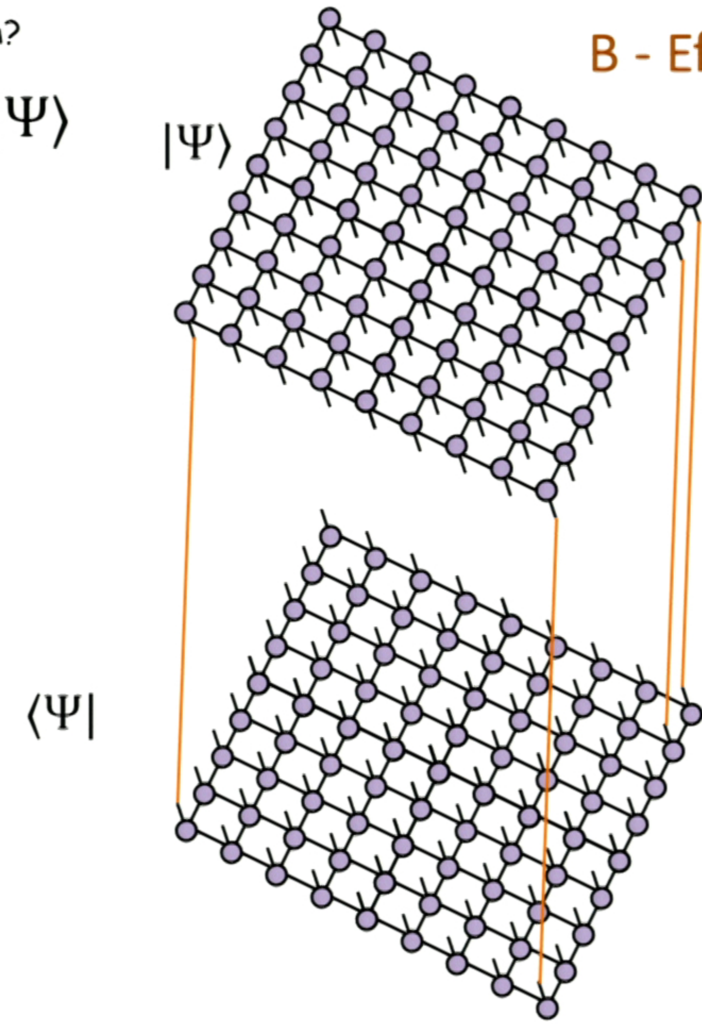
A - Efficient representation?



Projected entangled pair states (PEPS)

Norm?

$\langle\Psi|\Psi\rangle$



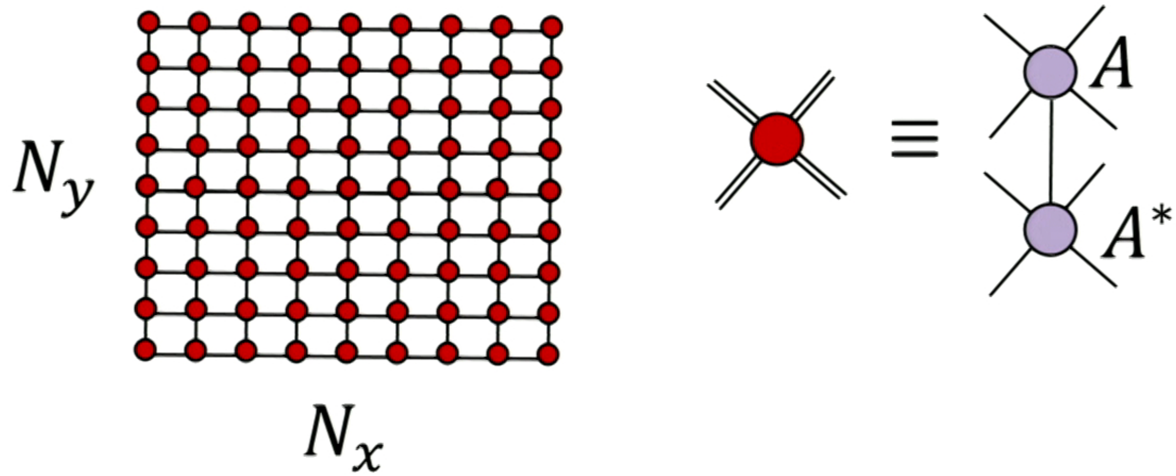
B - Efficient manipulation?

Projected entangled pair states (PEPS)

Norm?

B - Efficient manipulation?

$\langle\Psi|\Psi\rangle$

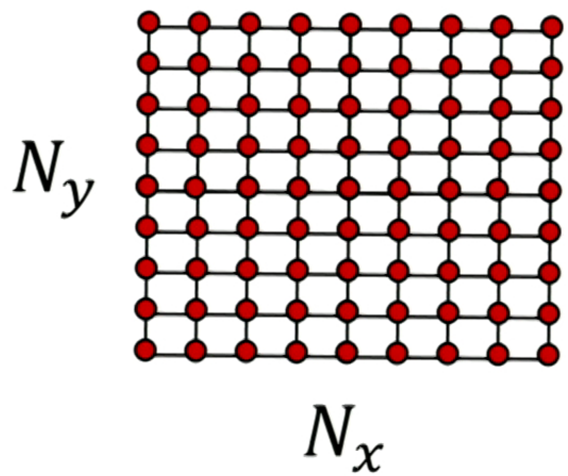


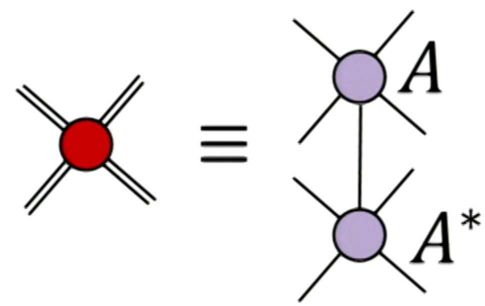
Projected entangled pair states (PEPS)

Norm?

B - Efficient manipulation?

$\langle\Psi|\Psi\rangle$



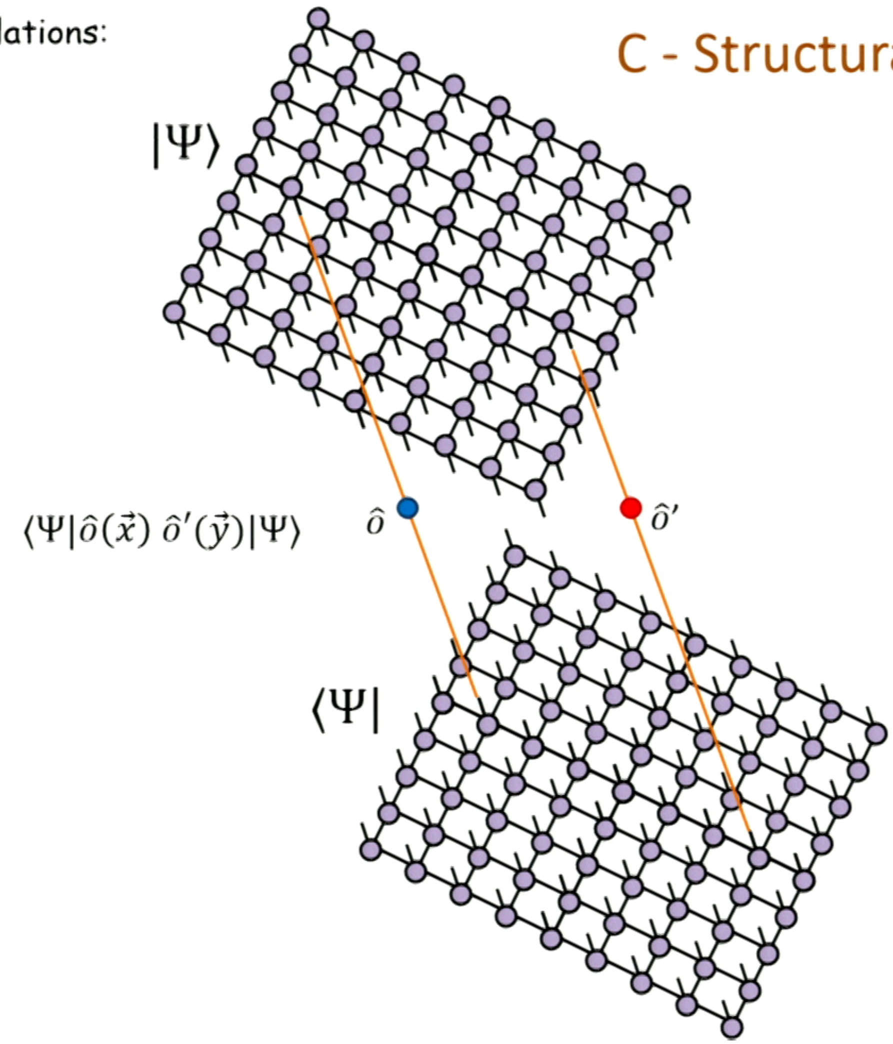


Cost exact contraction $\mathrm{N_x}\exp(N_y)$ (if $N_y < N_x$)

Projected entangled pair states (PEPS)

Correlations:

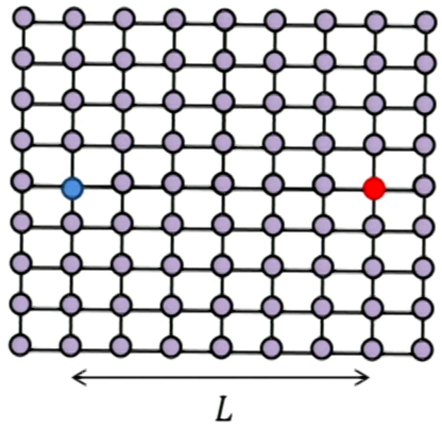
C - Structural properties?



Projected entangled pair states (PEPS)

Correlations:

$\langle\Psi|\hat{o}(\vec{x})\ \hat{o}'(\vec{y})|\Psi\rangle$

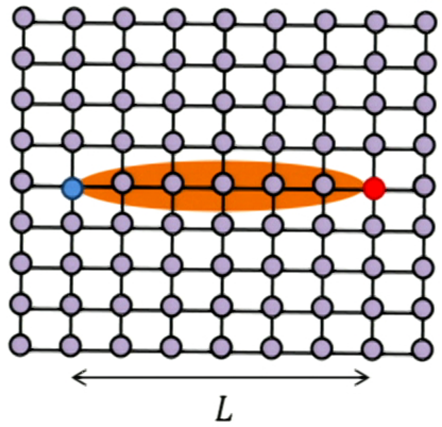


C - Structural properties?

Projected entangled pair states (PEPS)

Correlations:

$\langle \Psi | \hat{o}(\vec{x})\ \hat{o}'(\vec{y}) | \Psi \rangle$



C - Structural properties?

exponential correlations

$C(L) \approx e^{-L/\xi}$

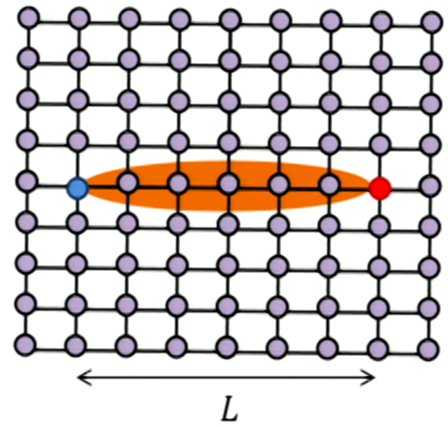
(generic)

Projected entangled pair states (PEPS)

Correlations:

$\langle\Psi|\hat{o}(\vec{x})\,\hat{o}'(\vec{y})|\Psi\rangle$

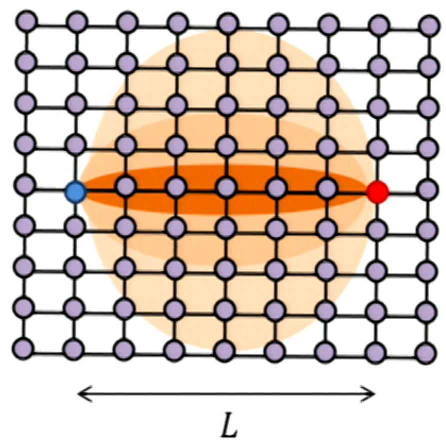
C - Structural properties?



exponential correlations

$C(L) \approx e^{-L/\xi}$

(generic)



polynomial correlations

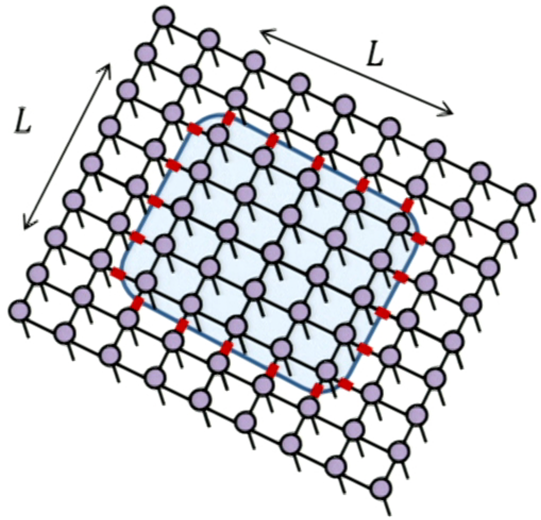
$C(L) \approx L^{-p}$

(fine-tuned)

Projected entangled pair states (PEPS)

Entanglement entropy:

C - Structural properties?

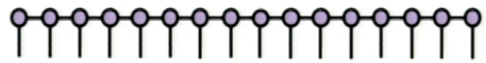


area law for entanglement entropy

$$S_L \leq 4L \log(\chi) \quad (= L^{D-1})$$

D=1 spatial dimensions

matrix product state
(MPS)

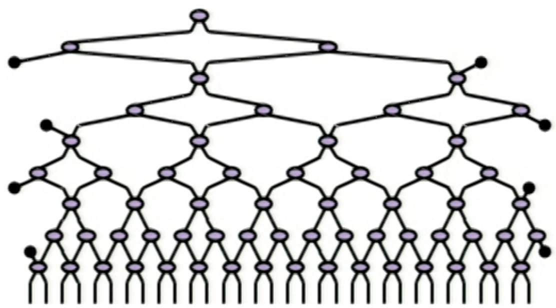


$C(L) \approx e^{-L/\xi}$

$S_L \approx const \;\; (= L^{D-1})$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



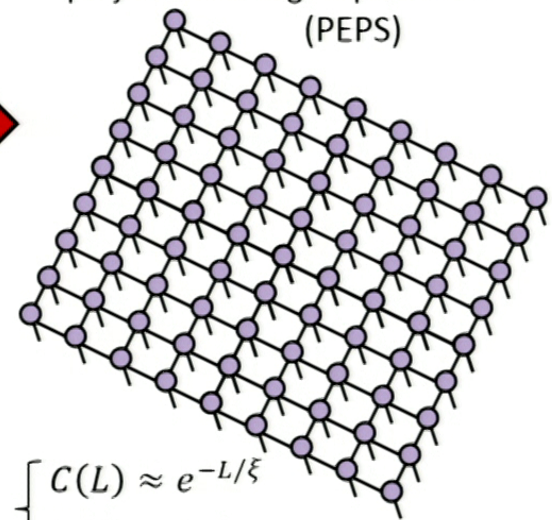
$C(L) \approx L^{-p}$

$S_L \approx \log L \quad (= L^{D-1} \log L)$

(critical systems)

D=2 spatial dimensions

projected entangled pair states
(PEPS)



$$\begin{cases} C(L) \approx e^{-L/\xi} \\ C(L) \approx L^{-p} \end{cases}$$

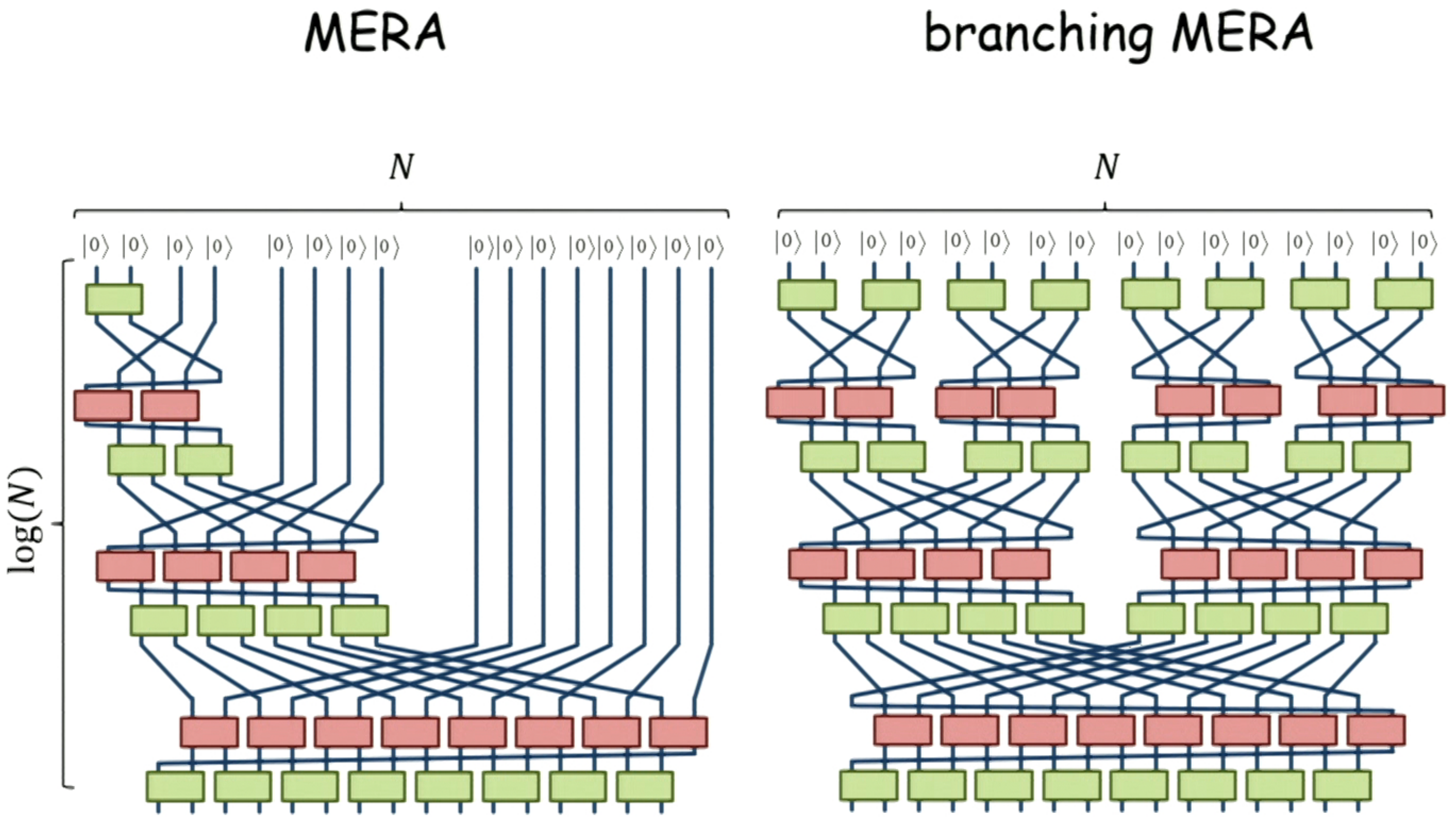
$S_L \approx L \quad (= L^{D-1})$

2D MERA

$C(L) \approx L^{-p}$

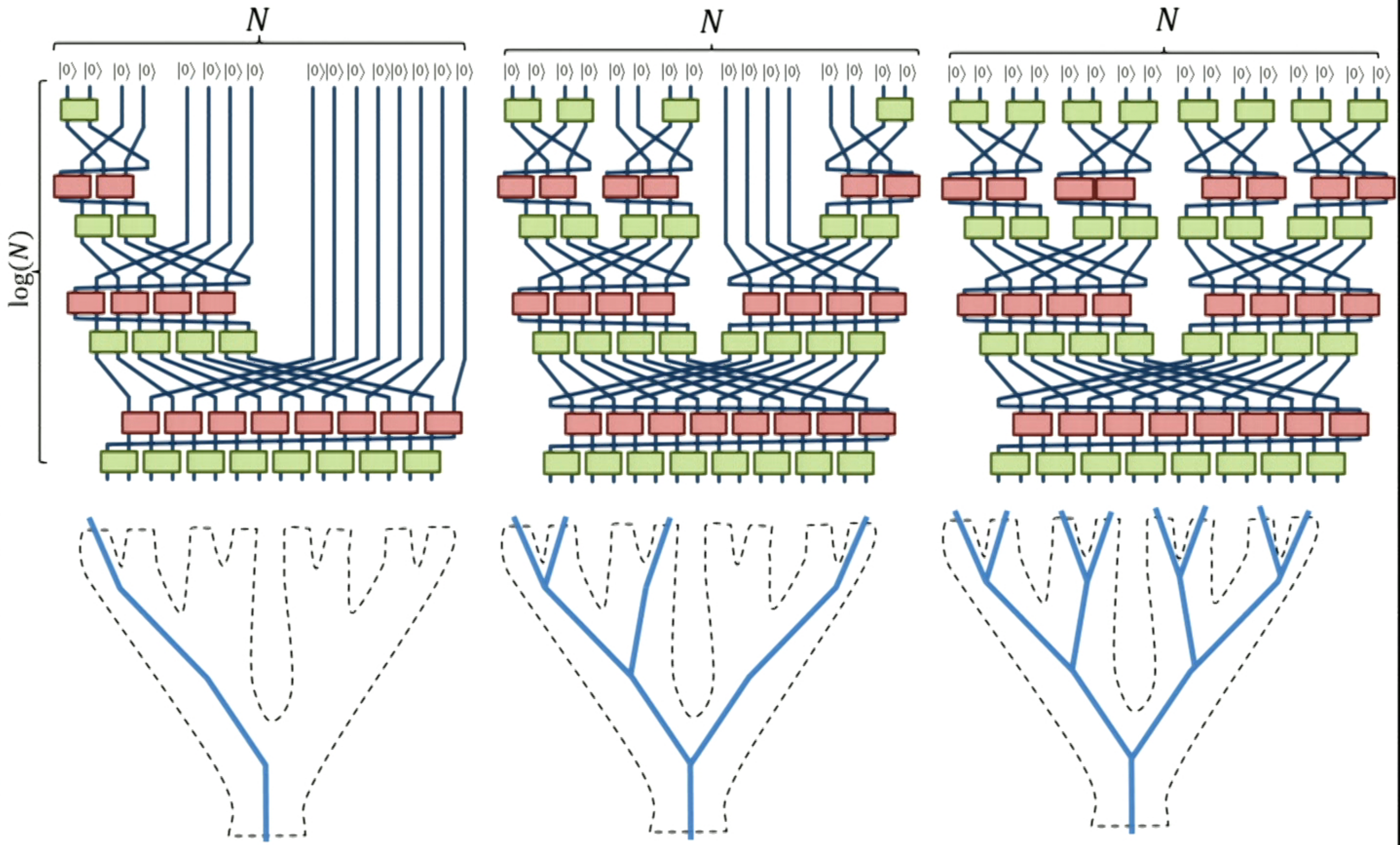
$S_L \approx L \quad (= L^{D-1})$

Branching MERA

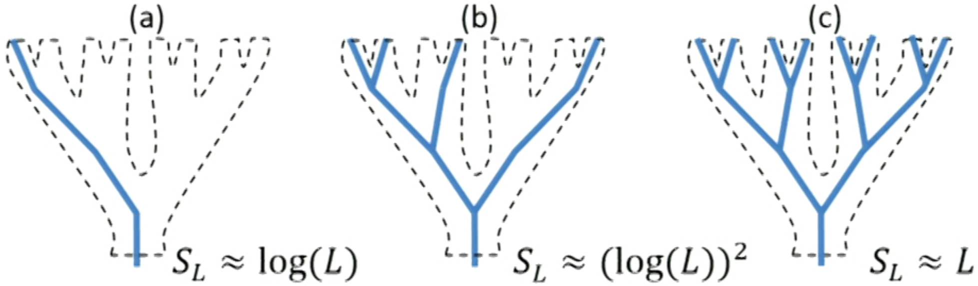


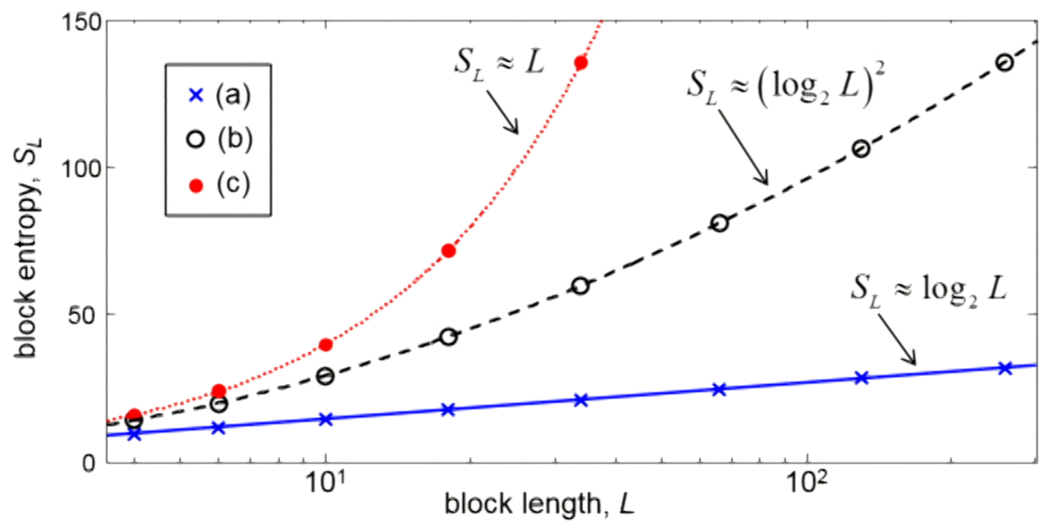
MERA

branching MERA

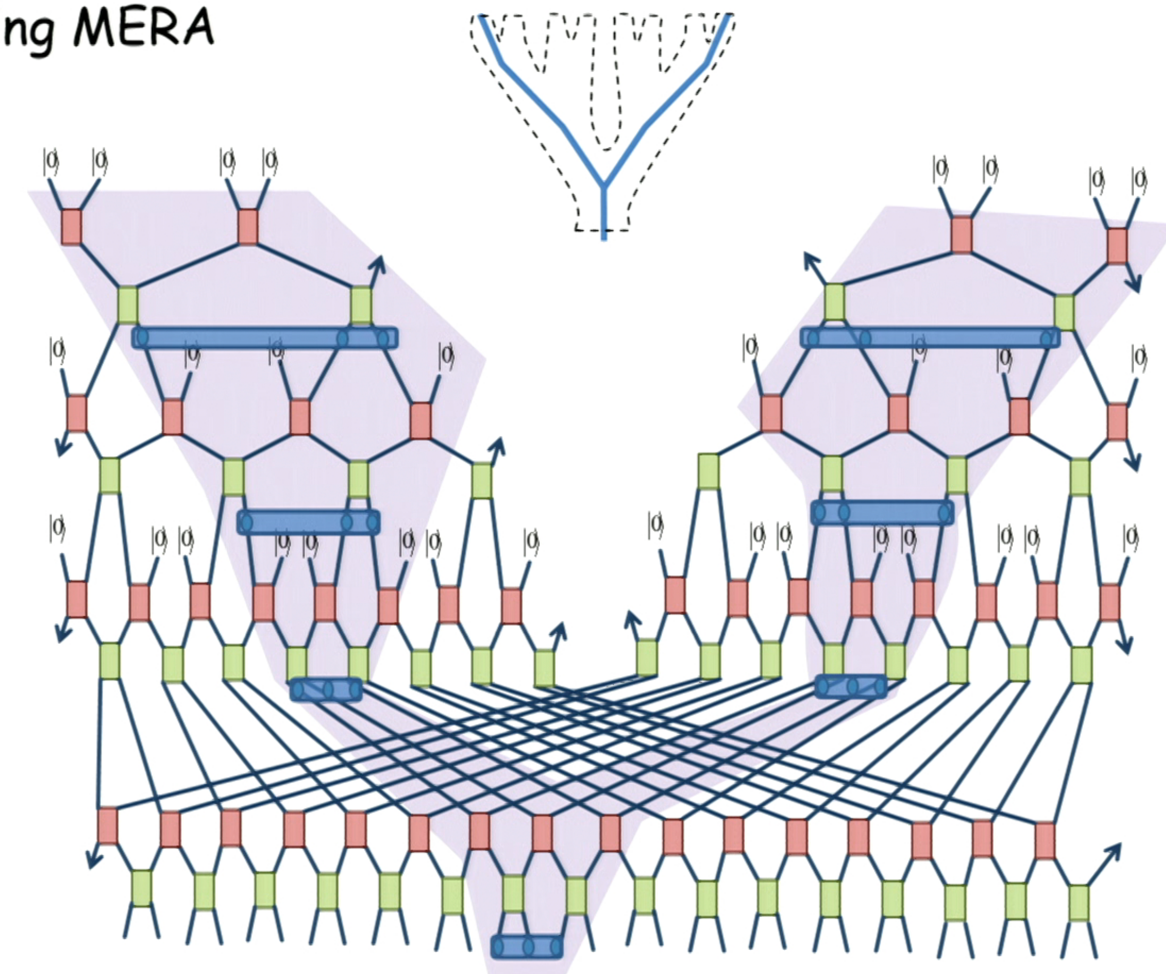


branching MERA





branching MERA

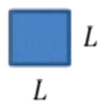
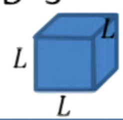


$$S_L \approx \log(L) + \log(L)$$

branching MERA

D=1 spatial dimensions	$S_L \approx \log(L)$		…		$S_L \approx L$
D>1 spatial dimensions	$S_L \approx L^{D-1}$	…	$S_L \approx L^{D-1}\log(L)$	…	$S_L \approx L^D$

Scaling of entanglement entropy (second week)

Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ — **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ — (D-1)- dimensional Fermi surface
D=1 	$S_L \approx const$	N/A	$S_L \approx \log(L)$
D=2 	$S_L \approx L$	$S_L \approx L$	$S_L \approx L\log(L)$
D=3 	$S_L \approx L^2$	$S_L \approx L^2$	$S_L \approx L^2\log(L)$

$S_L \approx L^{D-1}$

area law

$S_L \approx L^{D-1}\log(L)$

area law
with logarithmic
correction

Summary/outlook

Tensor networks (third week)

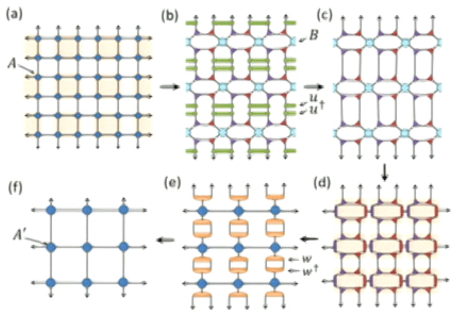
Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ — **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ — (D-1)- dimensional Fermi surface
D=1 (L)	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 (L, L)	$S_L \approx L$ PEPS	$S_L \approx L$ MERA	$S_L \approx L \log(L)$
D=3 (L, L, L)	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ MERA	$S_L \approx L^2 \log(L)$

$S_L \approx L^{D-1}$ area law

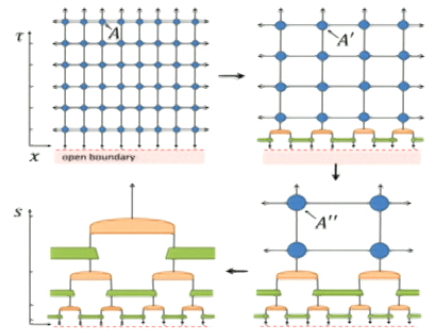
$S_L \approx L^{D-1}\log(L)$ area law with logarithmic correction

(ii) Tensor Networks in Statistical Mechanics (partition functions)

RG for statistical partition functions



MERA from Euclidean path integral



Euclidean path integral

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$

Statistical partition function

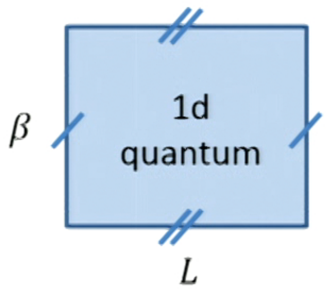
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

Euclidean path integral

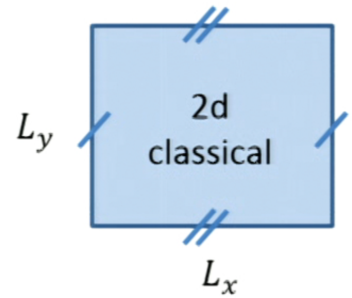
Statistical partition function

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

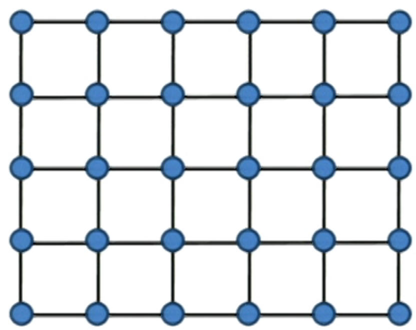


$\sim$



as a tensor network

$$Z =$$



Euclidean path integral

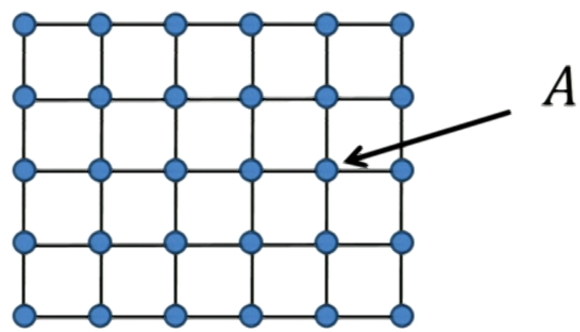
$$Z(\lambda) = tr\; e^{-\beta H_q^{1d}}$$

Statistical partition function

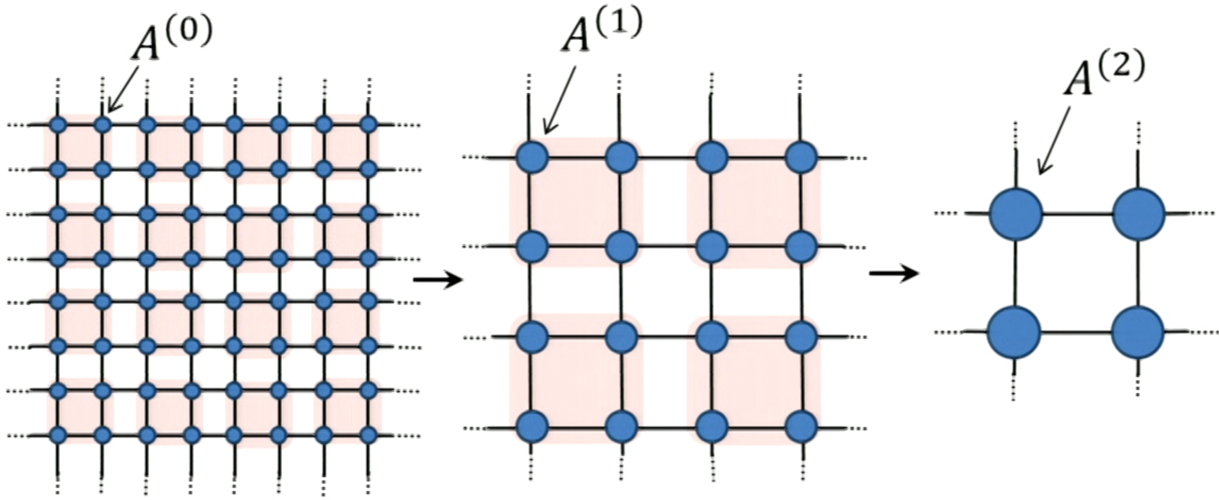
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

as a tensor network

$$Z =$$

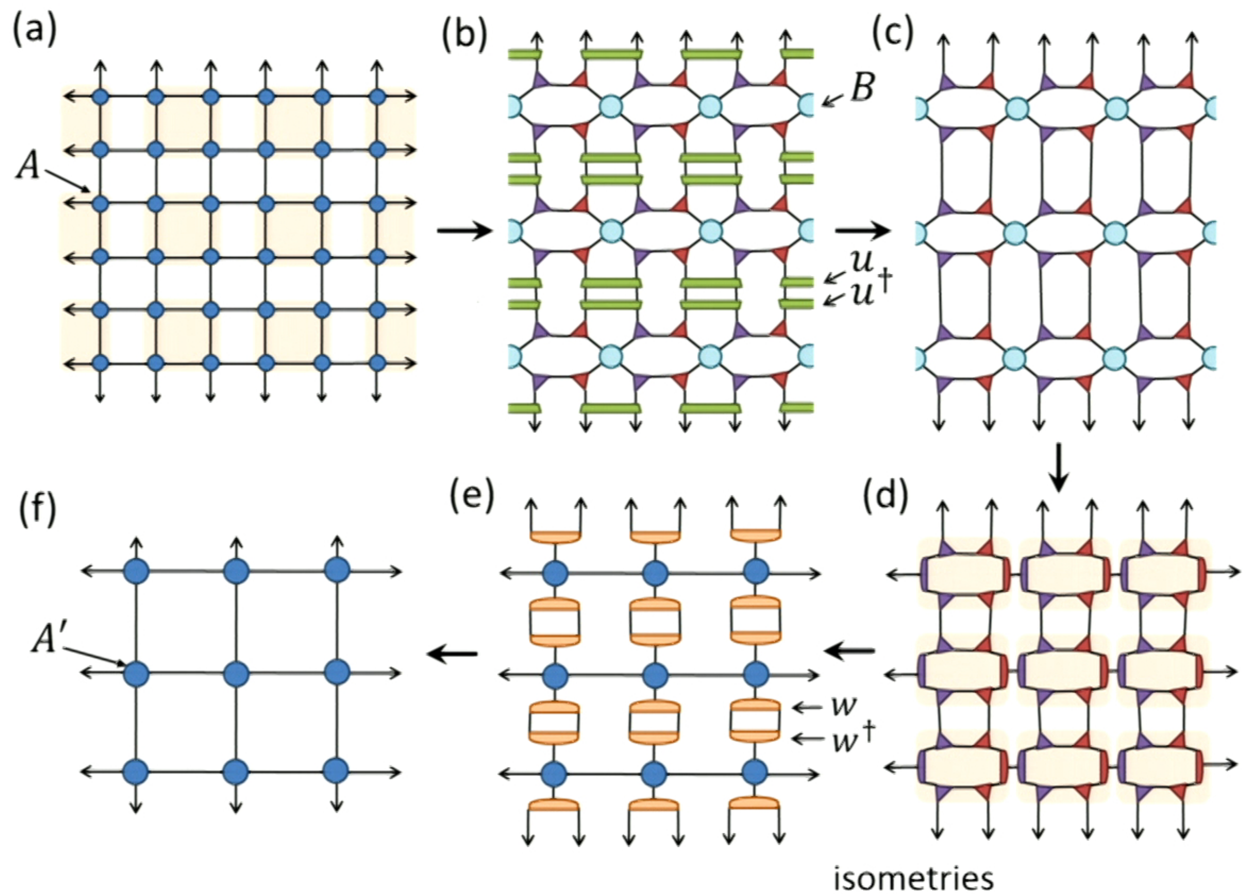


Goal: define an RG flow in the space of tensor networks



Tensor Network Renormalization (TNR)

[with Glen Evenbly, 2015 !]



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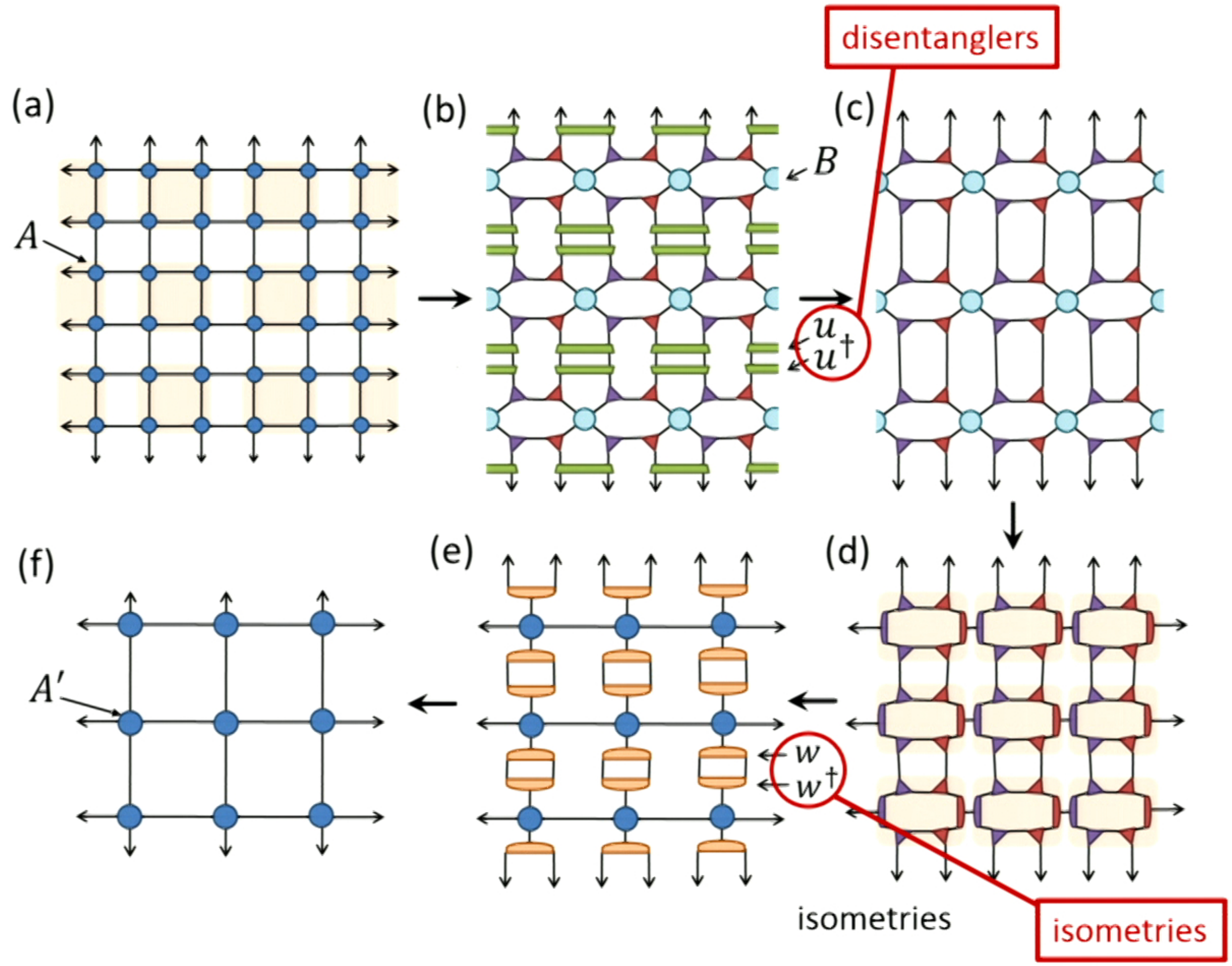

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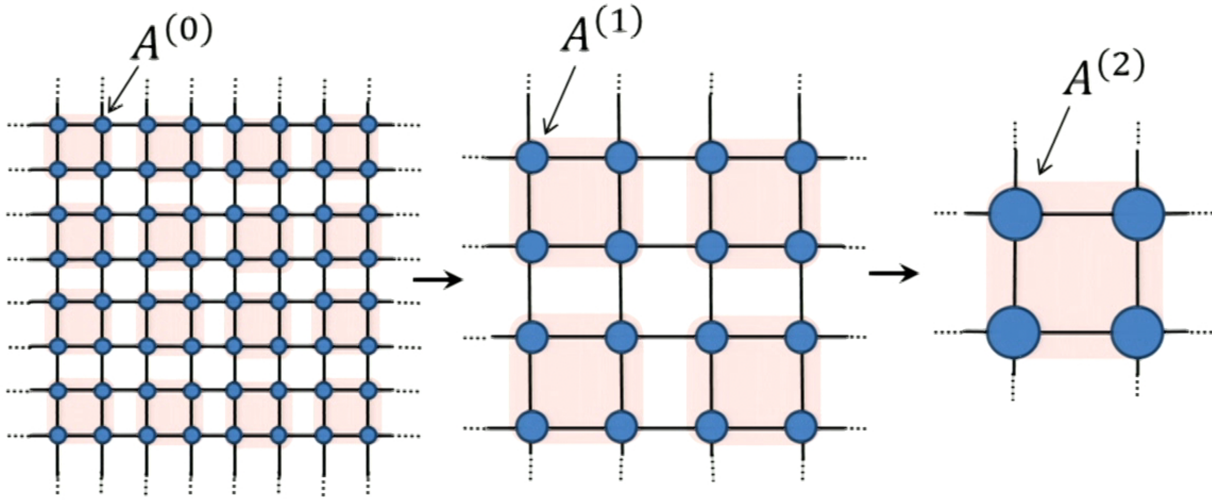
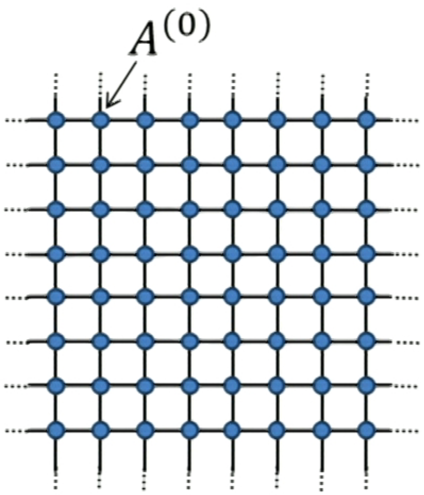


Many proposals, including

Tensor Renormalization Group (TRG) Levin-Nave, 2006

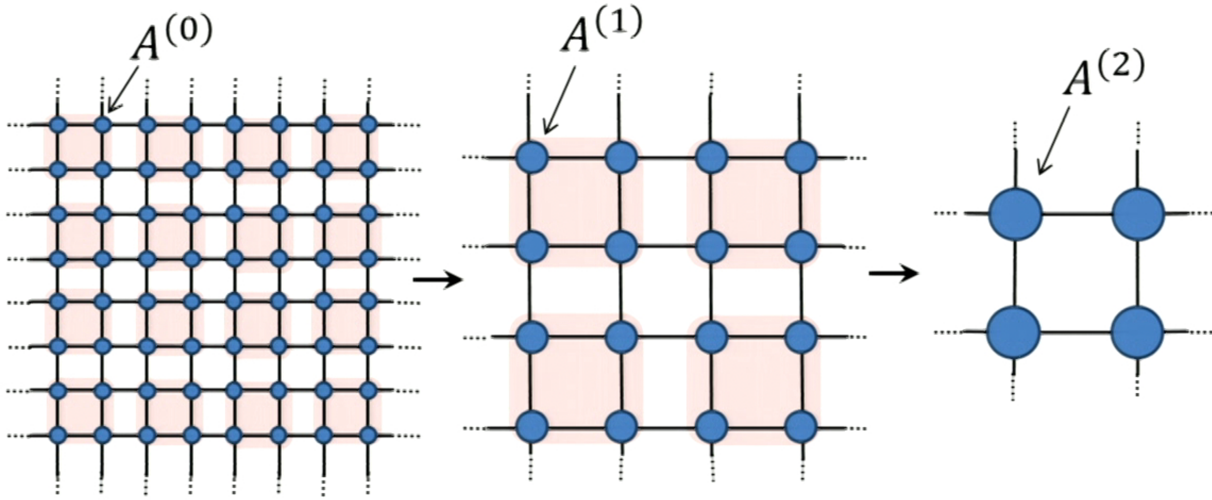
Net result: RG flow in the space of tensor networks



$$A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{fp}$$

Universal information of
the phase or phase transition

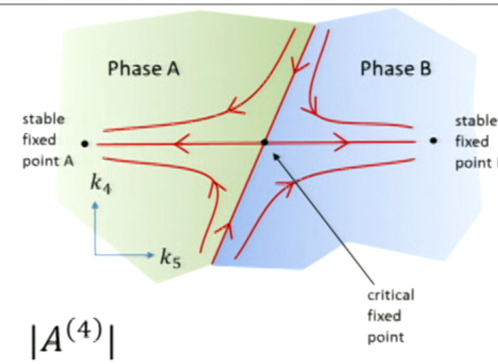
Net result: RG flow in the space of tensor networks



$$A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{fp}$$

Universal information of
the phase or phase transition

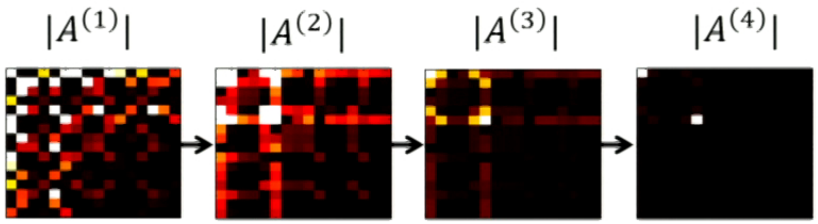
TNR -> proper RG flow
Example: 2D classical Ising



$$A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{fp}$$

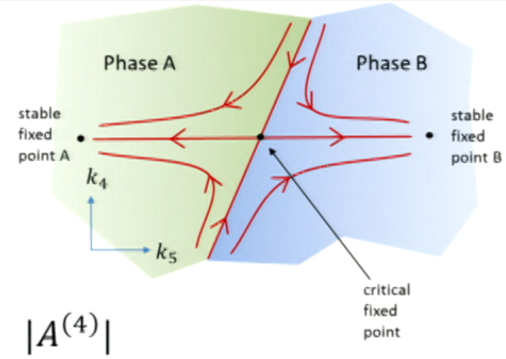
below critical

$T = 0.9\, T_c$



ordered (Z2) fixed point

TNR -> proper RG flow
Example: 2D classical Ising

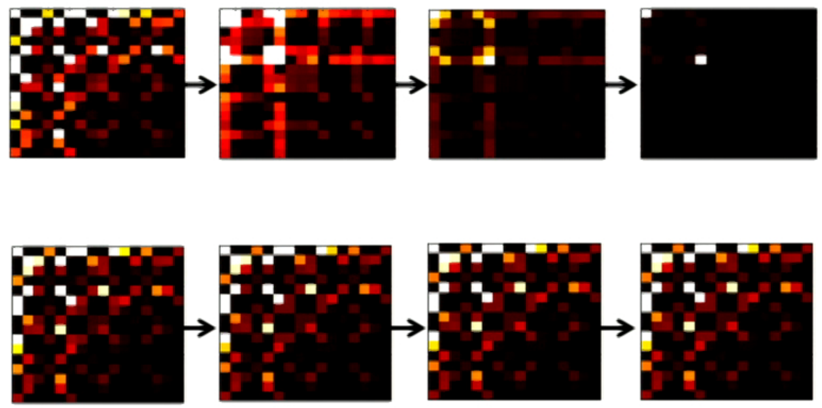


$$A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{fp}$$

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$

below critical
$T = 0.9\, T_c$

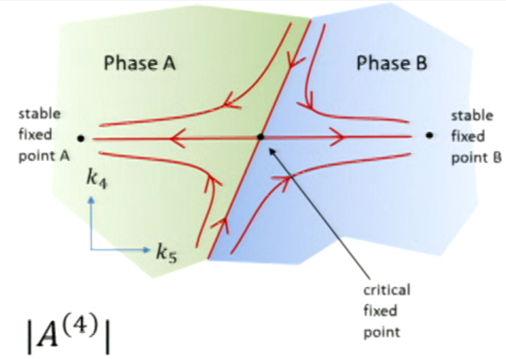
critical
$T = T_c$



ordered (Z2) fixed point

critical fixed point

TNR -> proper RG flow
Example: 2D classical Ising



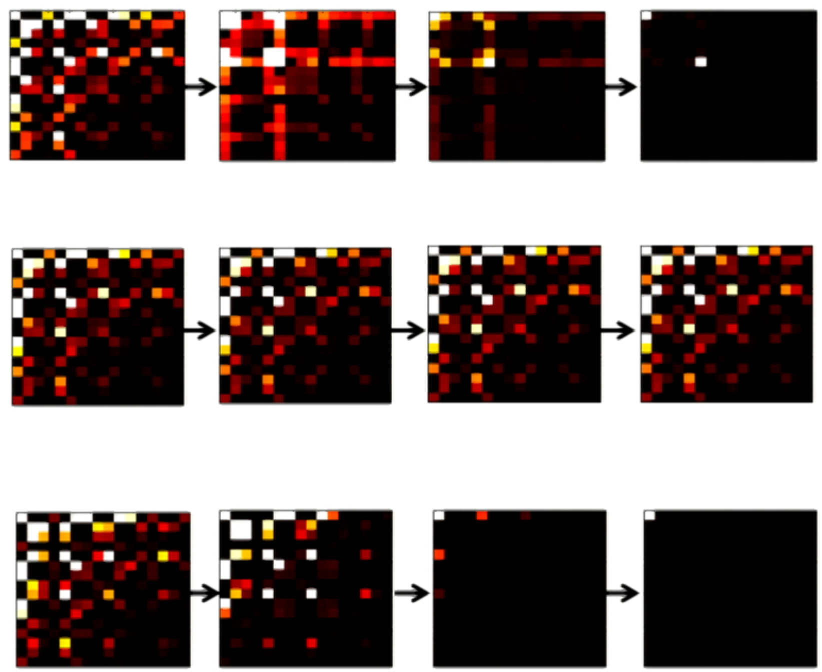
$$A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{fp}$$

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$

below critical
$T = 0.9\, T_c$

critical
$T = T_c$

above critical
$T = 1.1\, T_c$



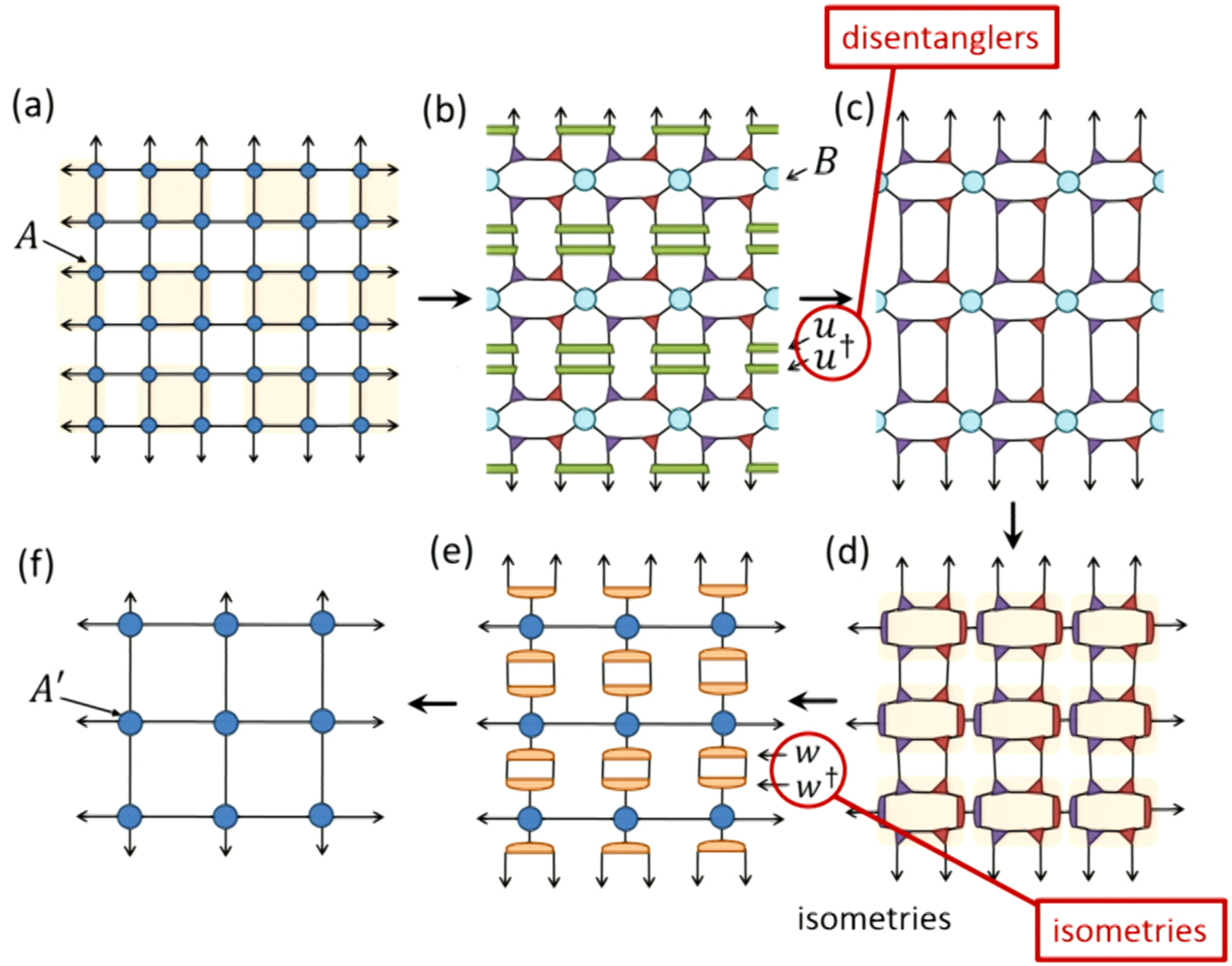
ordered (Z2) fixed point

critical fixed point

disordered (trivial) fixed point

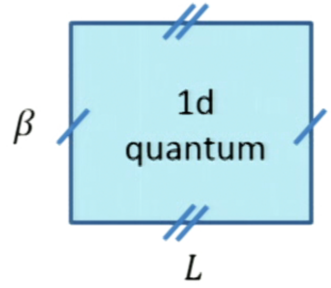
Tensor Network Renormalization (TNR)

[with Glen Evenbly, 2015 !]



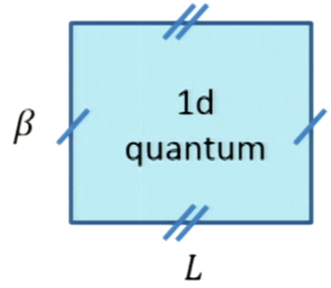
Euclidean path integral

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$



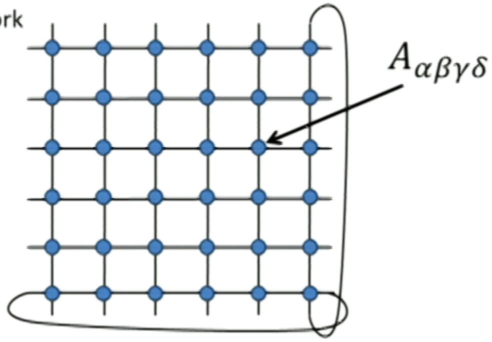
Euclidean path integral

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$



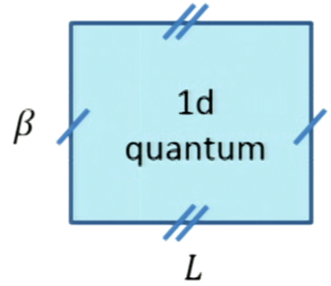
as a tensor network

$$Z =$$



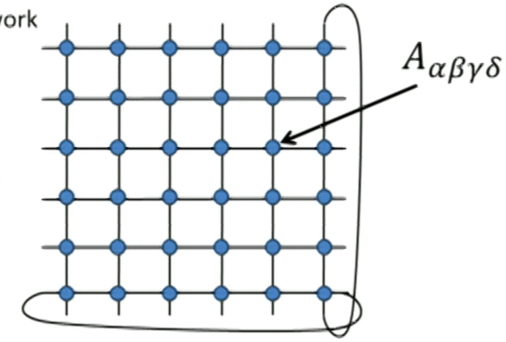
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



as a tensor network

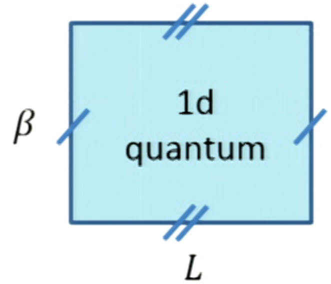
$$Z =$$



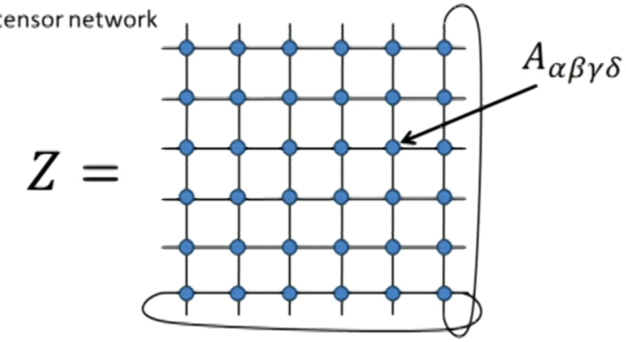
Euclidean time evolution on different geometries

Euclidean path integral

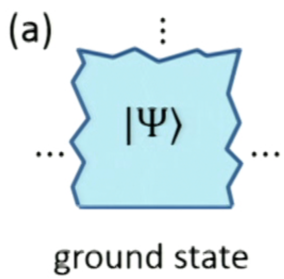
$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



as a tensor network

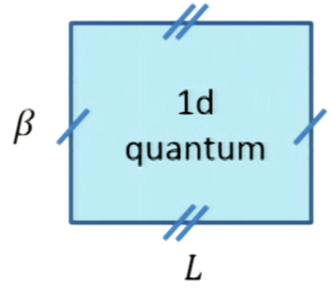


Euclidean time evolution on different geometries

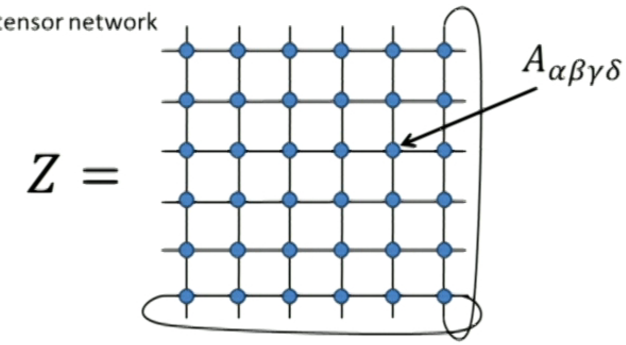


Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

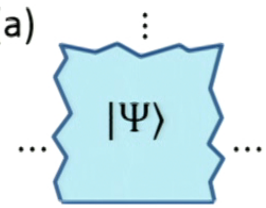


as a tensor network



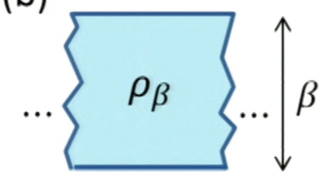
Euclidean time evolution on different geometries

(a)



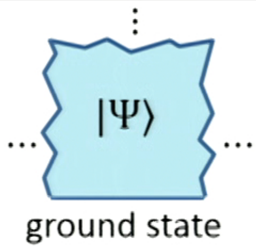
ground state

(b)

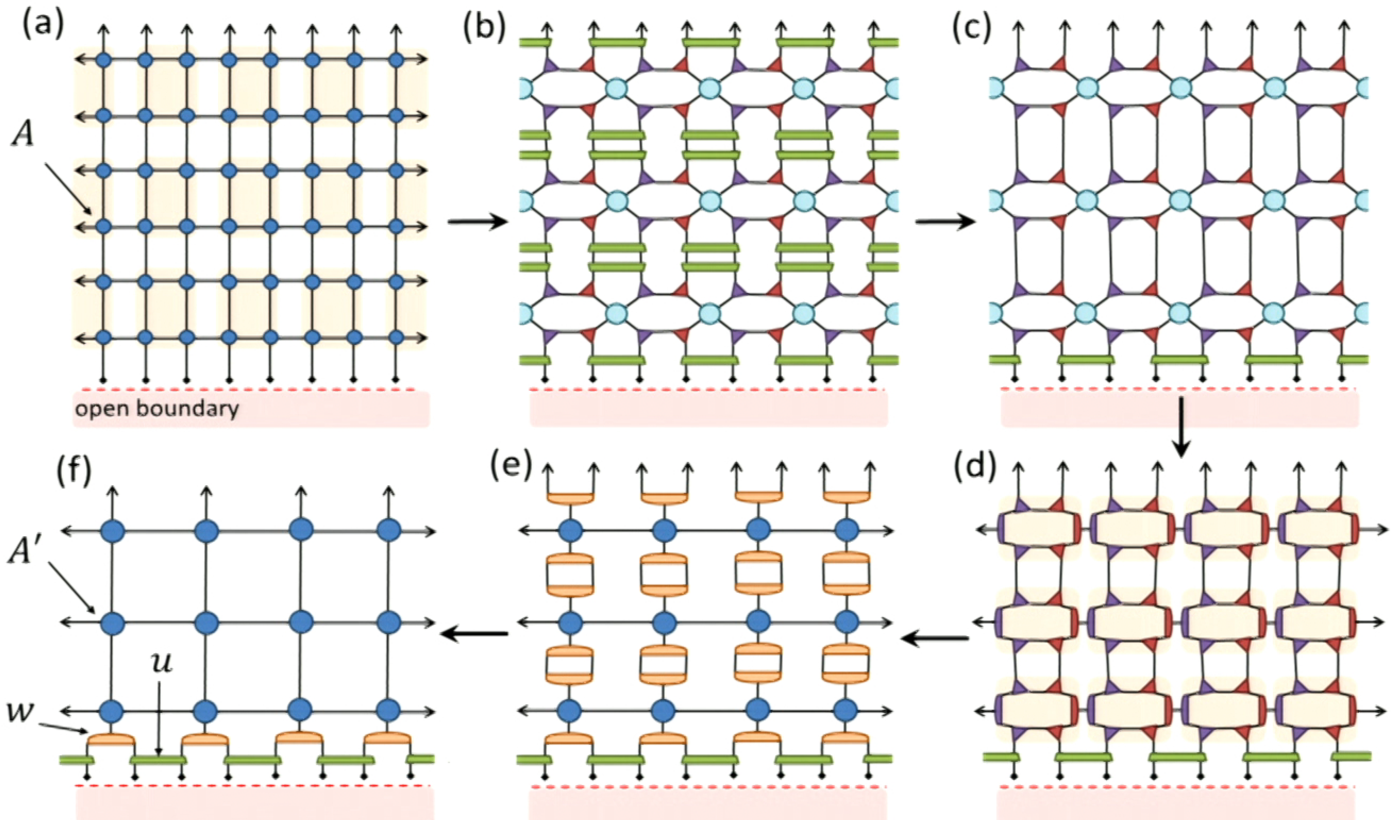


thermal state

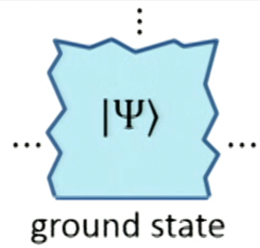
[with Glen Evenbly, 2015]



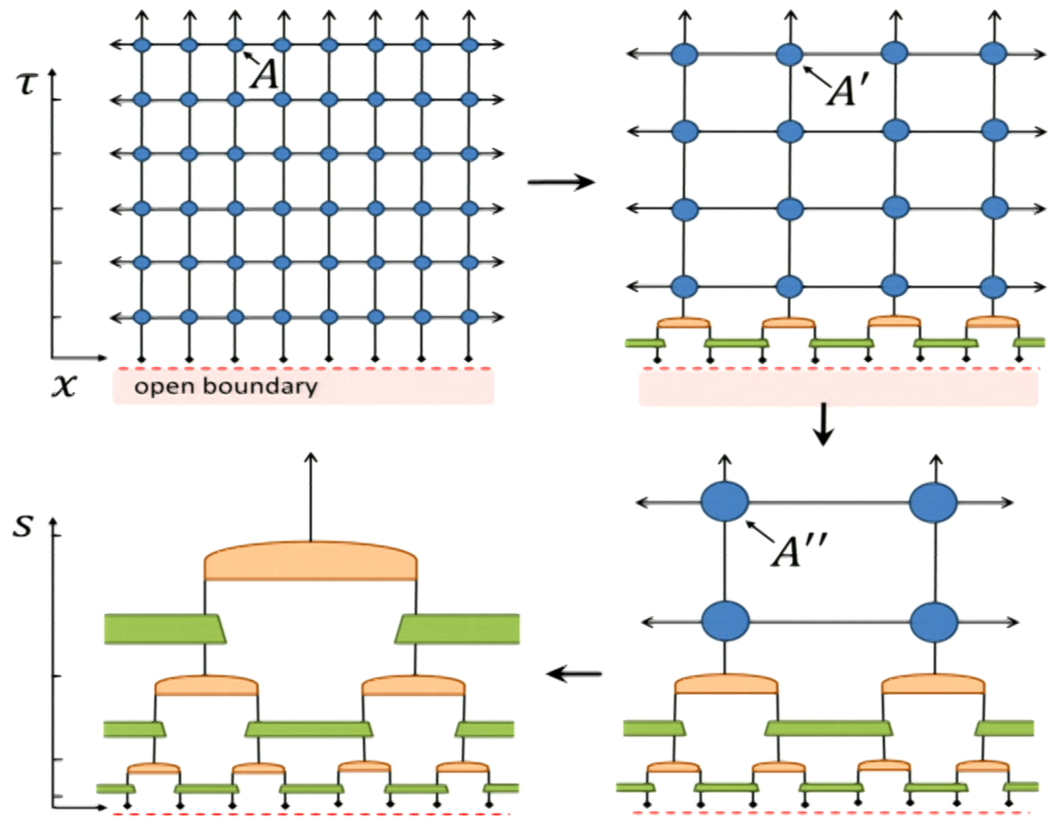
$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$

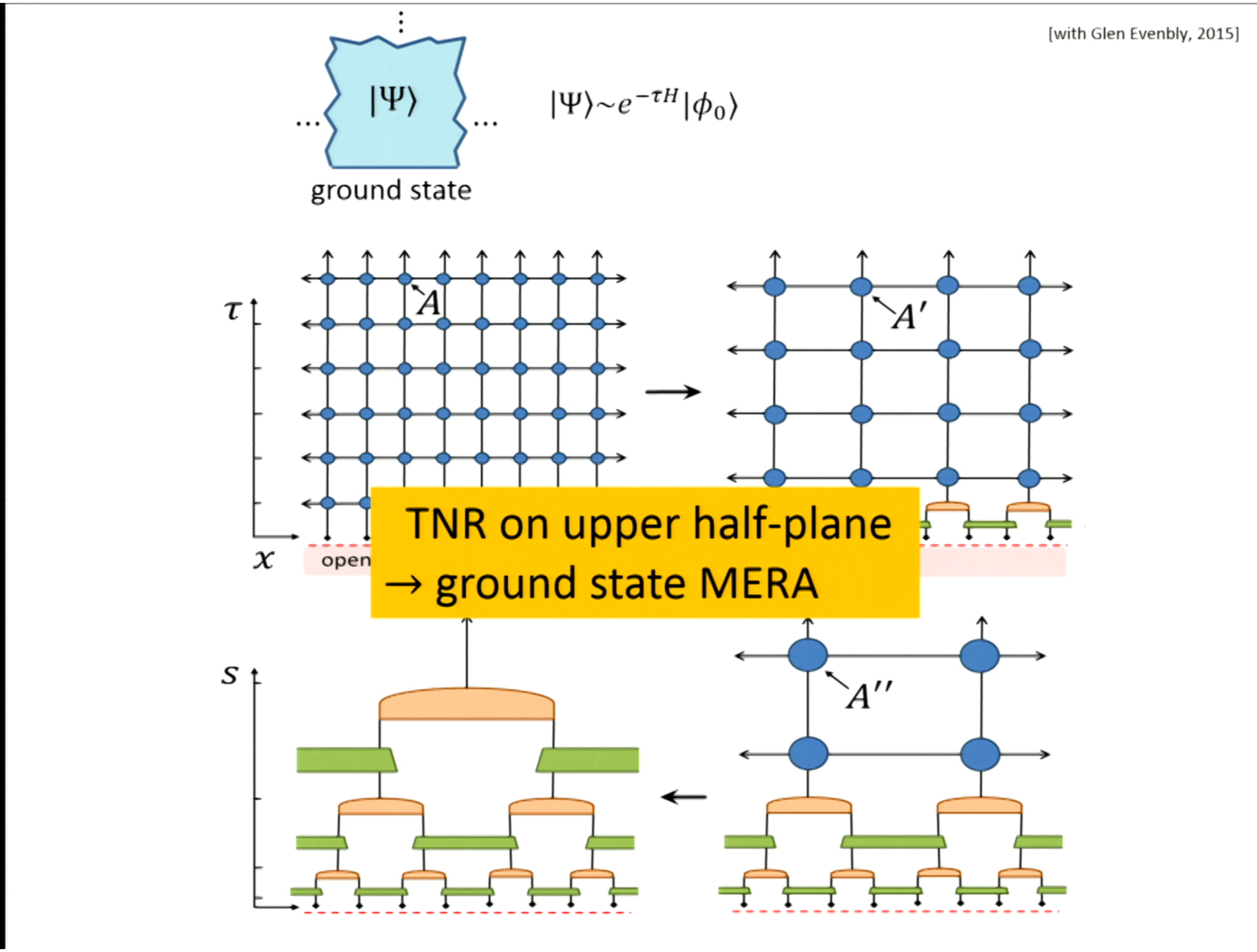


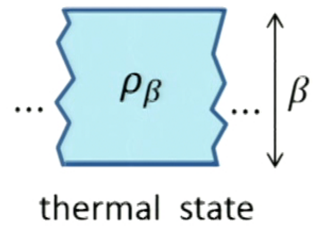
[with Glen Evenbly, 2015]



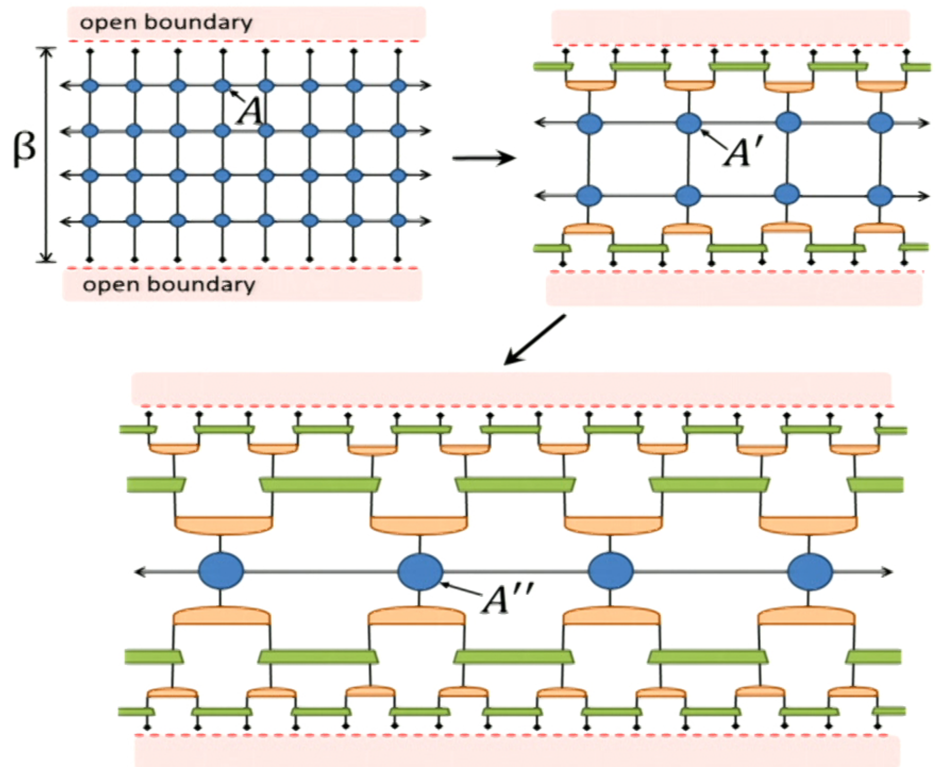
$$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$$

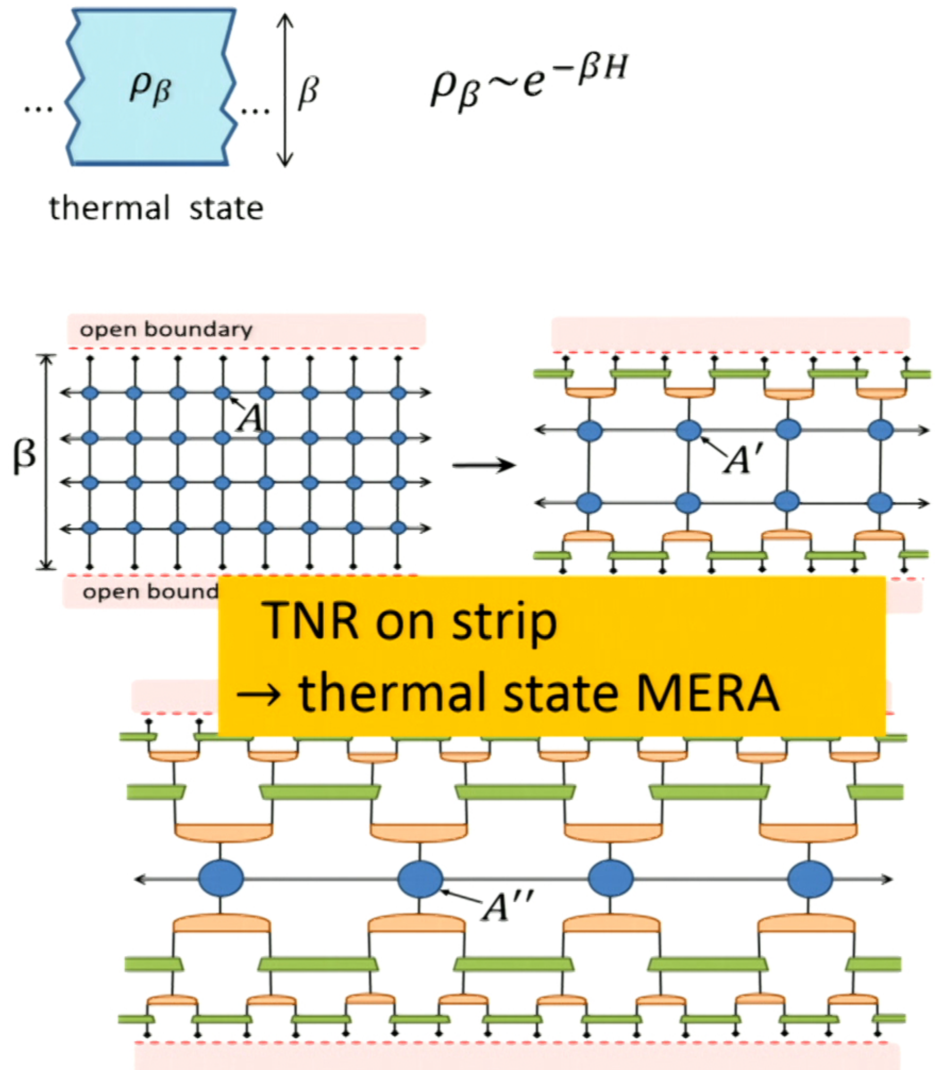



[with Glen Evenbly, 2015]
$|\Psi\rangle$
ground state
$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$
τ
A
A'
x
open
TNR on upper half-plane
→ ground state MERA
s
A''

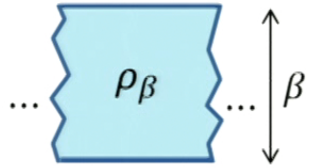


$\rho_\beta \sim e^{-\beta H}$



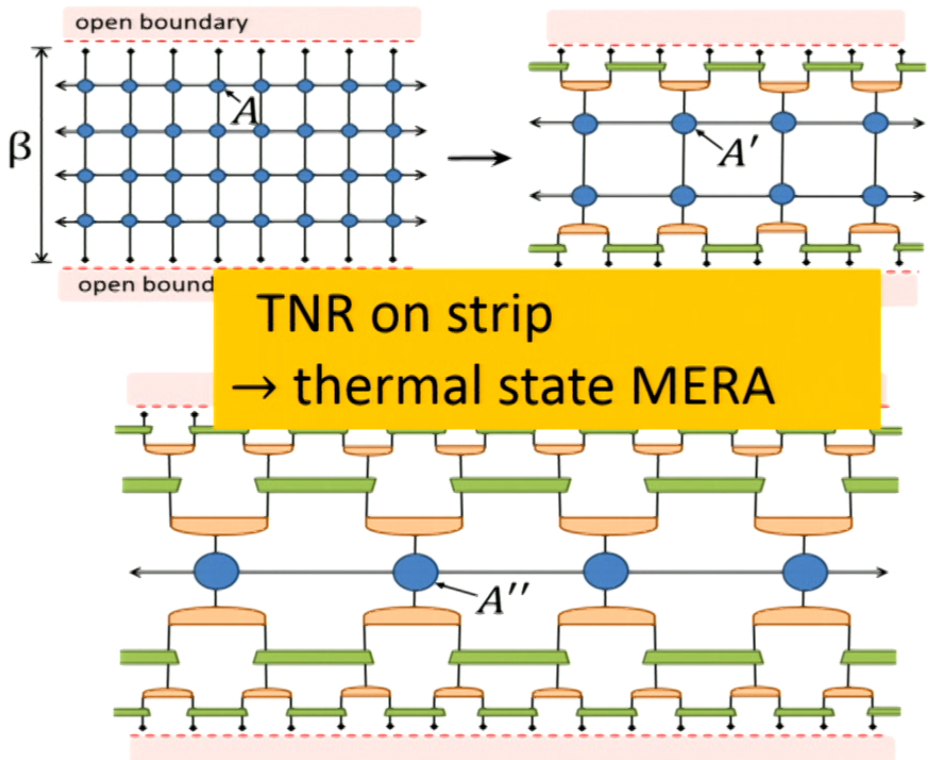


ρ_β
β
$\rho_\beta \sim e^{-\beta H}$
thermal state
open boundary
A
β
A'
open bound
TNR on strip
→ thermal state MERA
A''



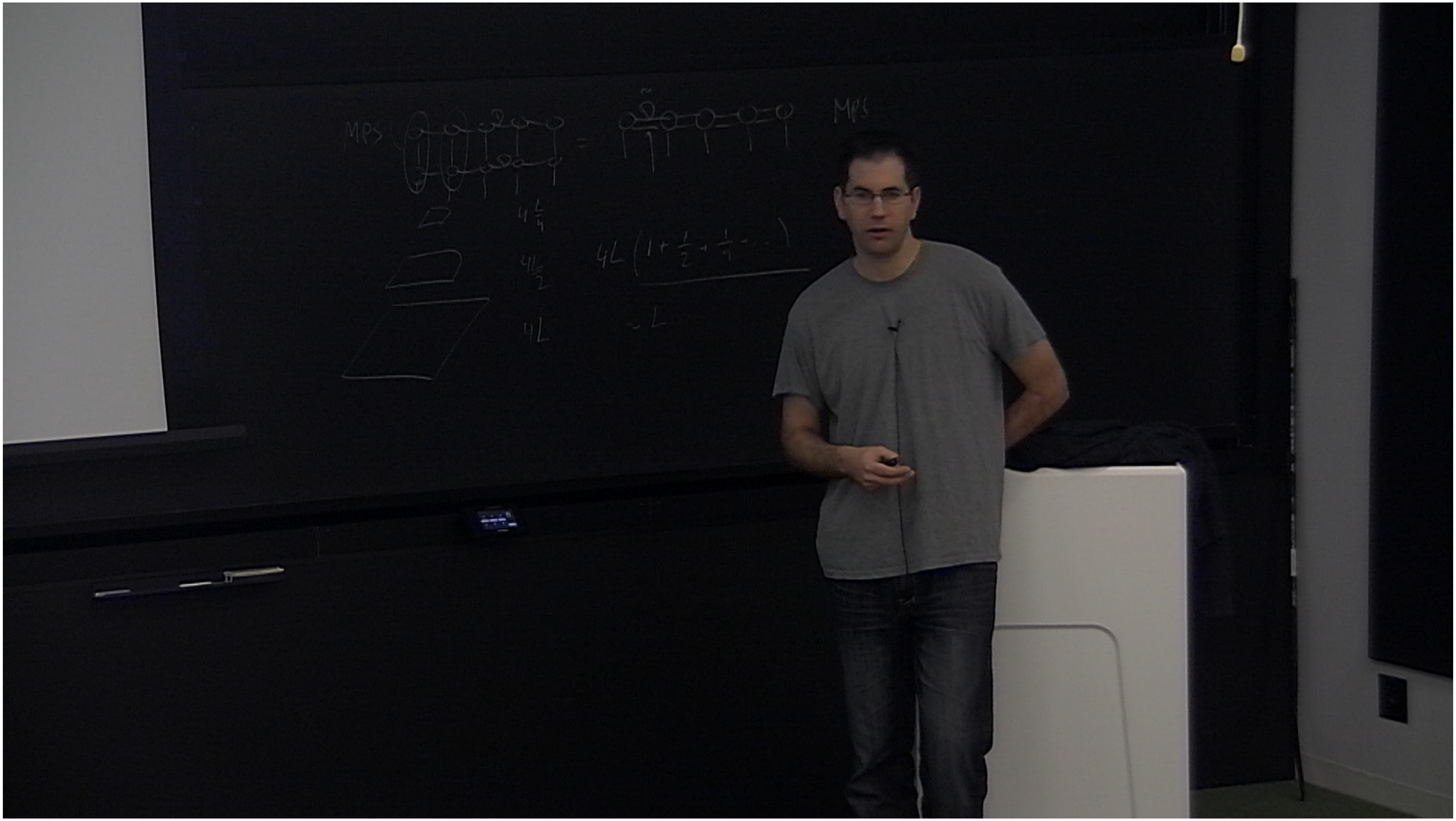
thermal state

$$\rho_\beta \sim e^{-\beta H}$$



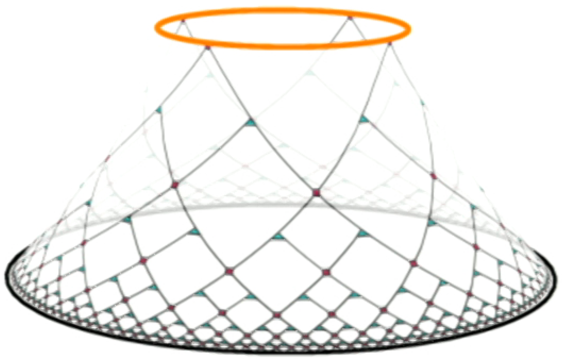
TNR on strip

→ thermal state MERA

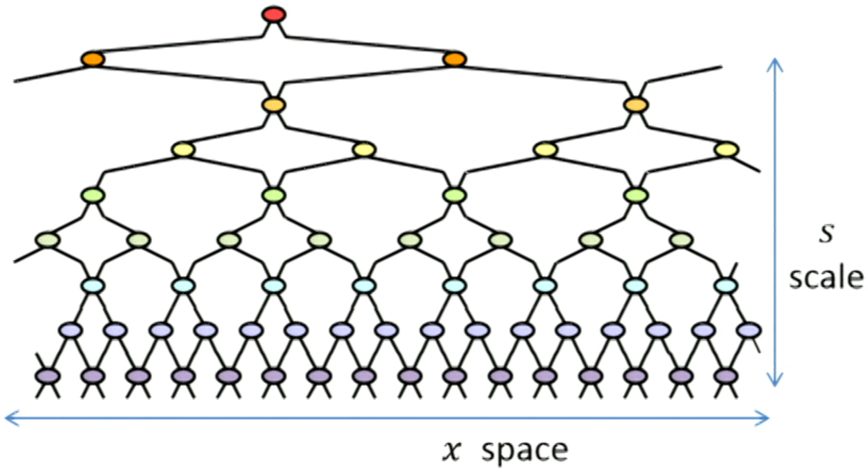


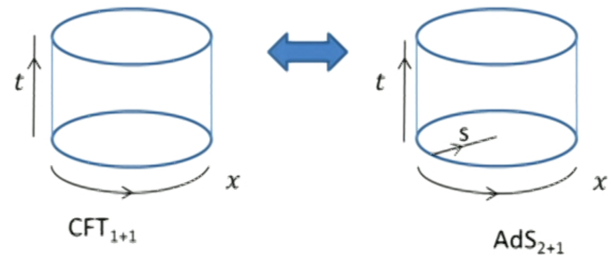
(iii) Tensor Networks in Holography

Toy models for holography/
emergent space-time

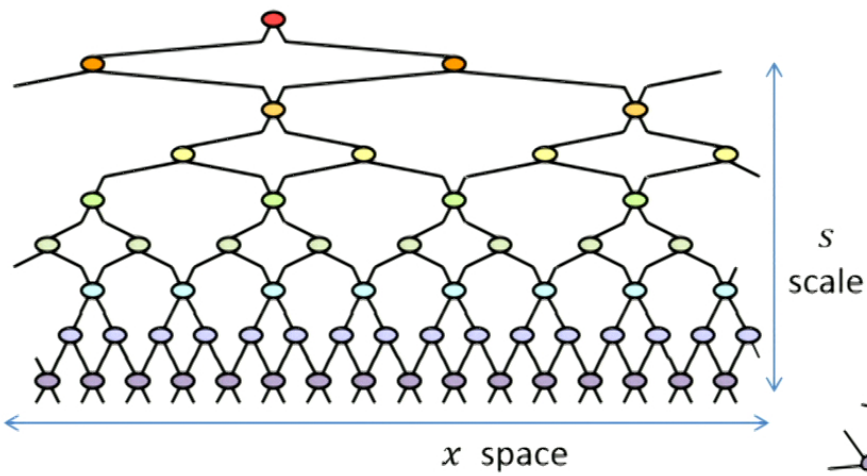


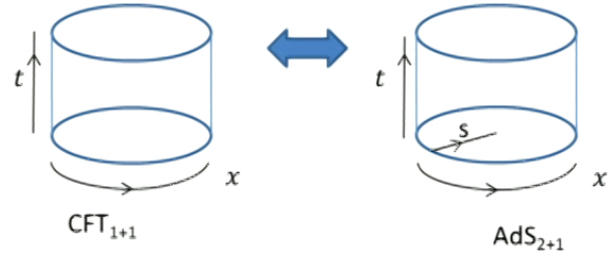
MERA and holography?

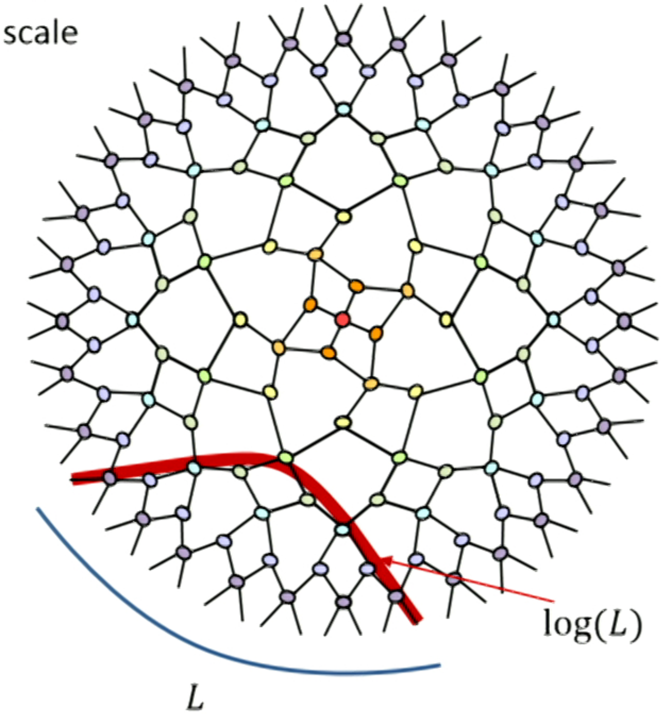




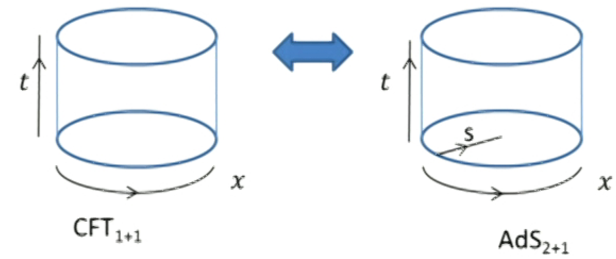
MERA and holography?

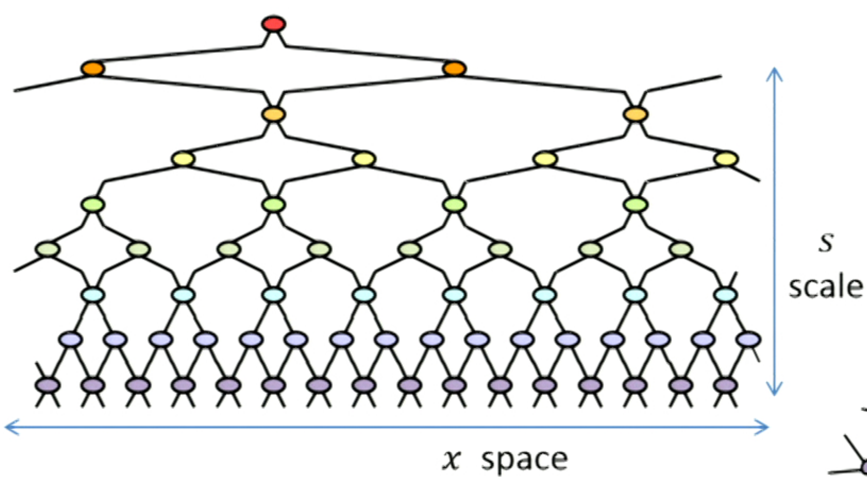






MERA and holography?





- entanglement entropy

$$S_L \approx \log(L)$$

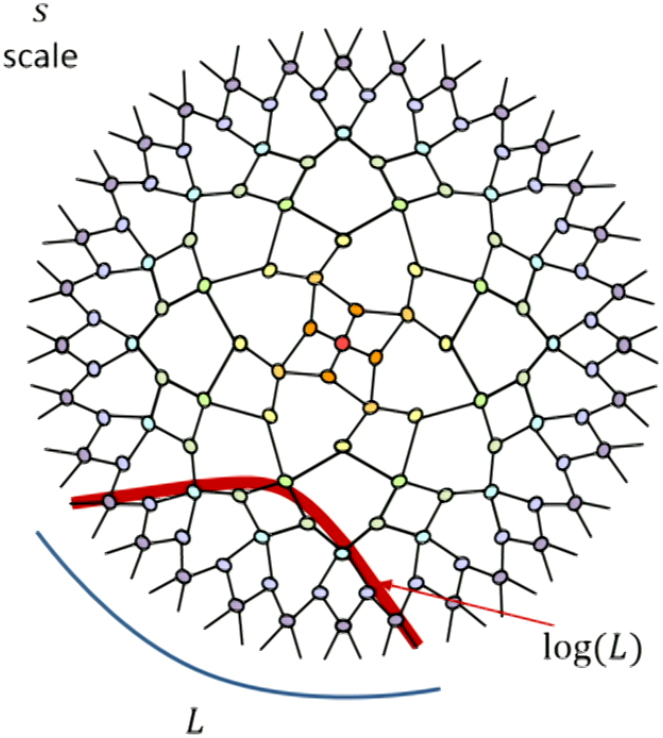
parallel to area of minimal surface in Ryu-Takayanagi

- two-point correlations

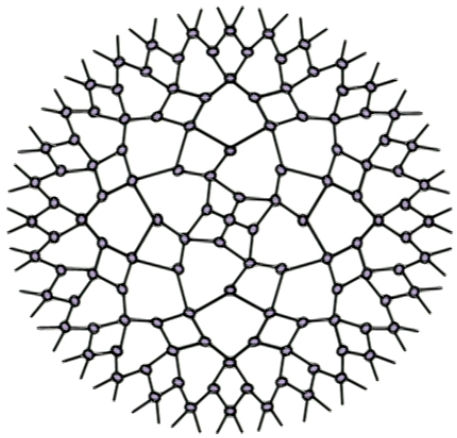
$$C(L) \approx L^{-2\Delta}$$

geodesic distance $D \approx \log(L)$ as in a hyperbolic geometry

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



multi-scale entanglement
renormalization ansatz (MERA)

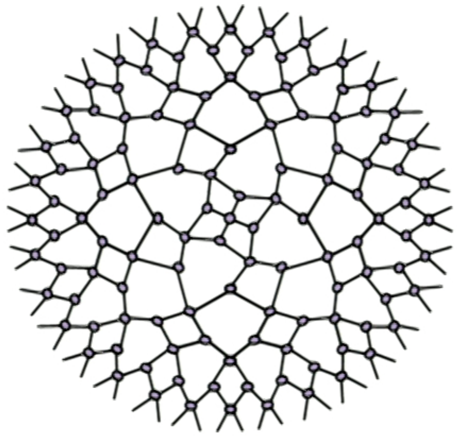


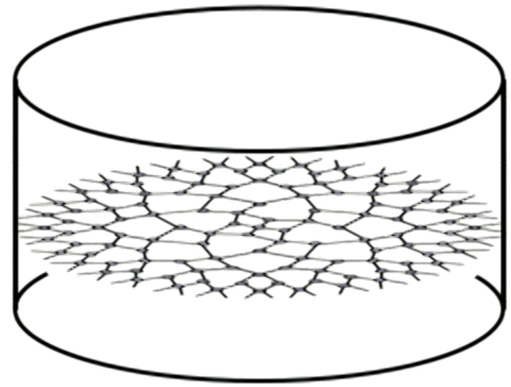
multi-scale entanglement
renormalization ansatz (MERA)

AdS_3/CFT_2

Swingle 2009, 2012

time slice of AdS_3 (hyperbolic plane H_2)



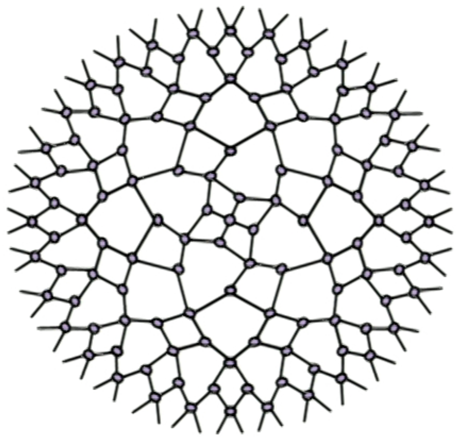


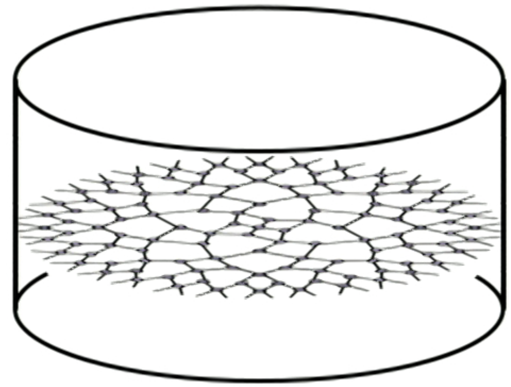
multi-scale entanglement renormalization ansatz (MERA)

AdS_3/CFT_2

Swingle 2009, 2012

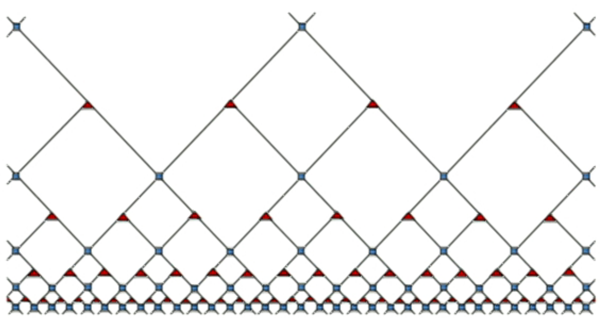
time slice of AdS_3 (hyperbolic plane H_2)

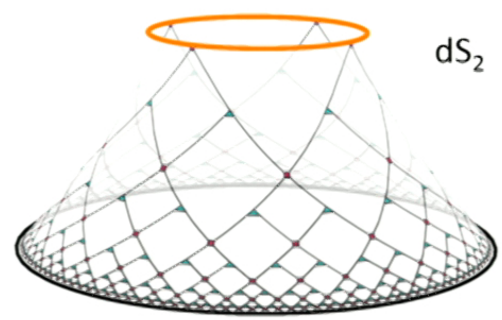




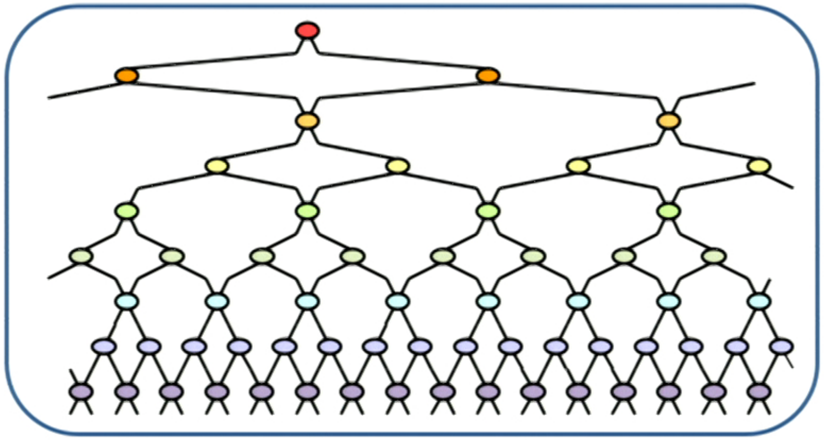
Czech, Lamprou, McCandlish, Sully, 2015-2016

kinematic space (integral transform of H_2)

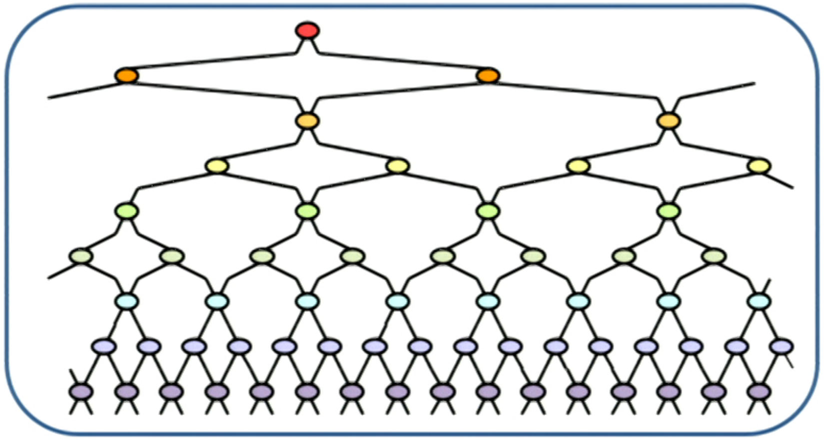




MERA = tensor network + isometric/unitary constraints



MERA = tensor network + isometric/unitary constraints

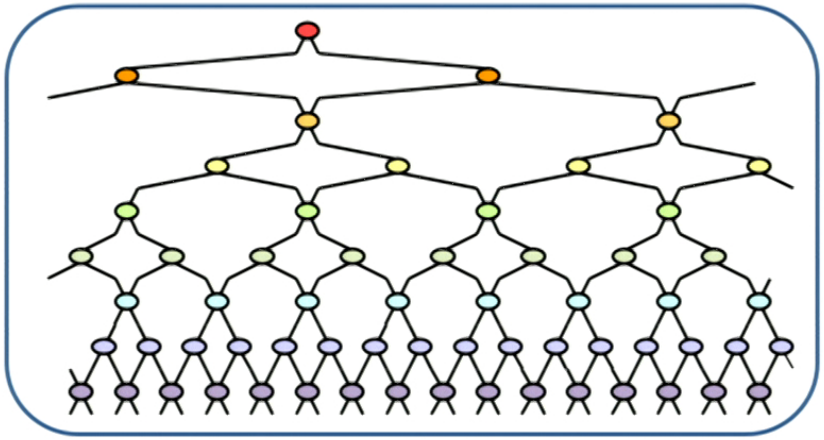


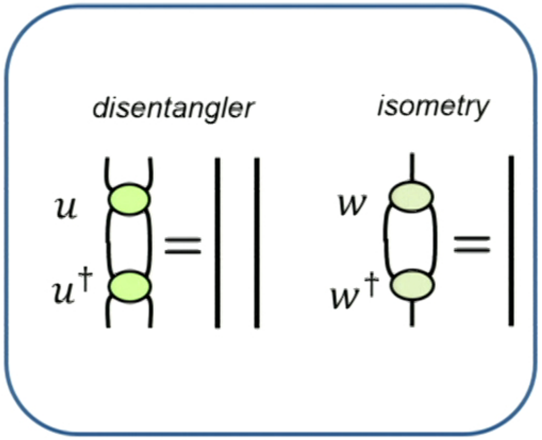
~ hyperbolic plane?

Euclidean metric signature

(Swingle 2009)

MERA = tensor network + isometric/unitary constraints





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Euclidean metric signature

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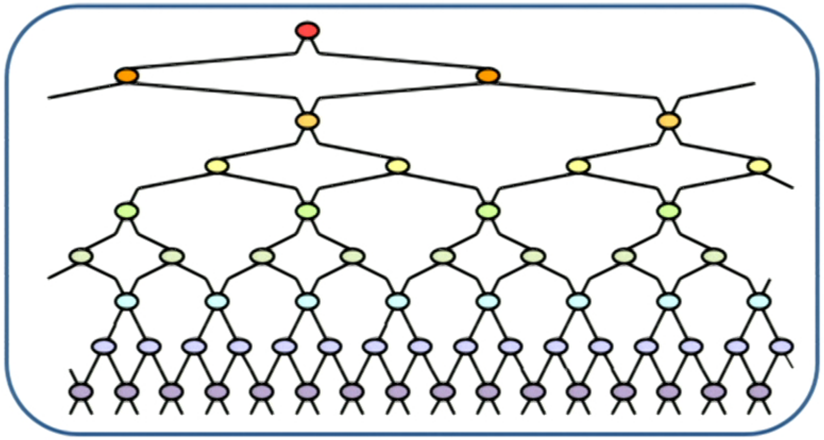
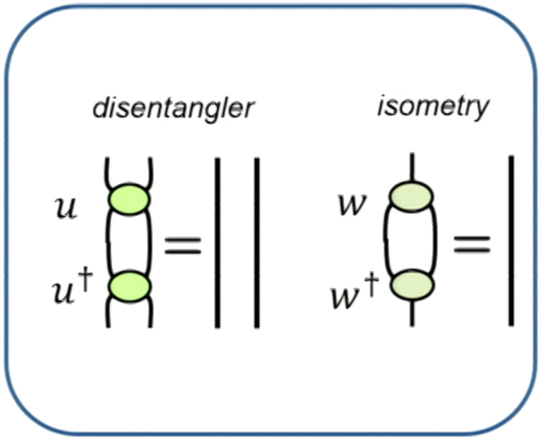


~ hyperbolic plane?

Euclidean metric signature

(Swingle 2009)

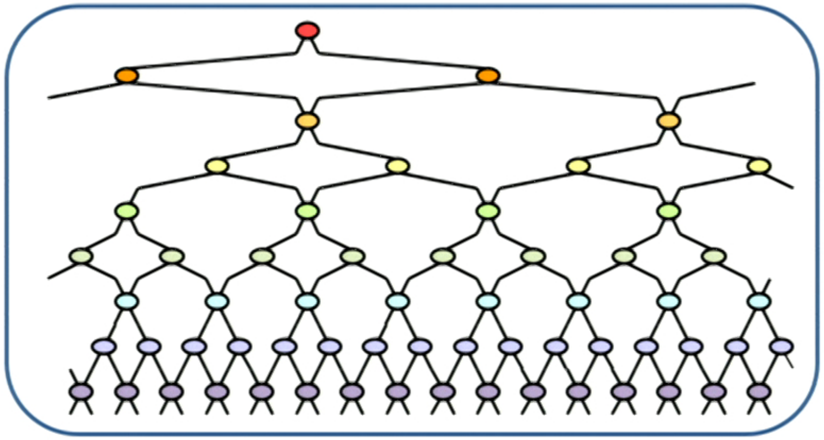


~ de Sitter space?

Lorentzian metric signature

(Beny 2011, Czech 2015)

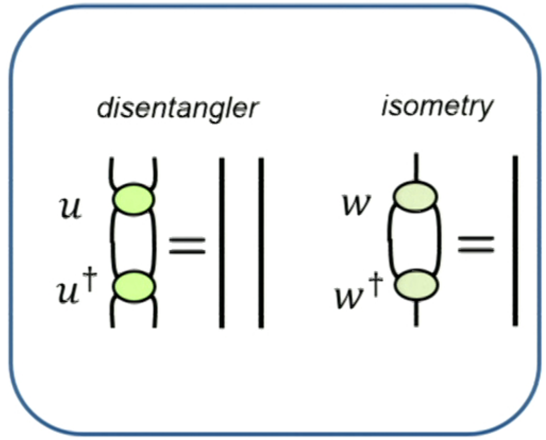
MERA = tensor network + isometric/unitary constraints



~ hyperbolic plane?

Euclidean metric signature

(Swingle 2009)



~ de Sitter space?

Lorentzian metric signature

(Beny 2011, Czech 2015)

MERA's
causal structure
= Lorentzian signature

MERA is a discretization
of Kinematic space - Czech

(space of geodesics of the hyperbolic plane,
and NOT the hyperbolic plane)

Many-body entanglement and tensor networks

the many-body computational challenge

simulation cost

$\exp N$

julia

exact diagonalization, Lanczos

free fermion formalism

ground state entanglement

Area law

$S(A) \sim |\partial A|$

computation of entanglement entropy

tensor networks

ground state representation

simulation cost

$O(N)$

structure of ground states

RG transformation

connection to holography, statistical mechanics