

Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 11

Date: Feb 14, 2017 09:00 AM

URL: <http://pirsa.org/17020047>

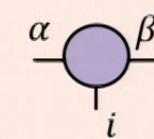
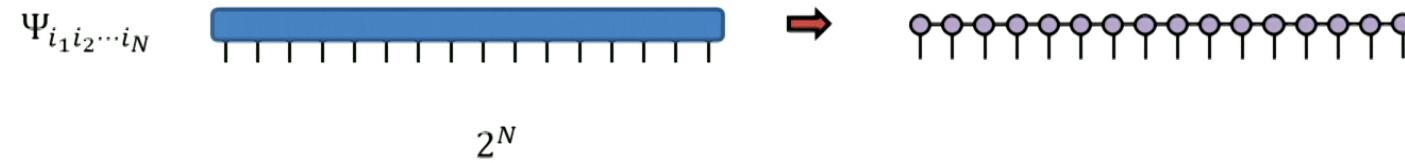
Abstract:

Summary of matrix product state (MPS)

$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

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$\Psi_{i_1 i_2 \cdots i_N}$

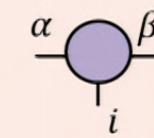


2^N



$O(d\chi^2)$

Efficient representation!



$|\alpha| = |\beta| = \chi$
 $|i| = d$

$O(d\chi^2)$ parameters

Summary of matrix product state (MPS)

$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

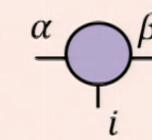
$\Psi_{i_1 i_2 \cdots i_N}$



2^N



$O(d\chi^2)$ parameters

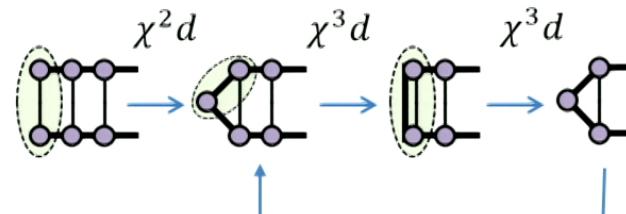
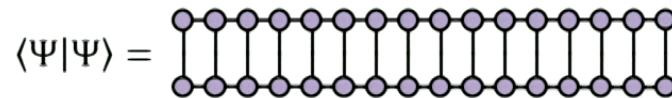


$|\alpha| = |\beta| = \chi$
 $|i| = d$

$O(Nd\chi^2)$

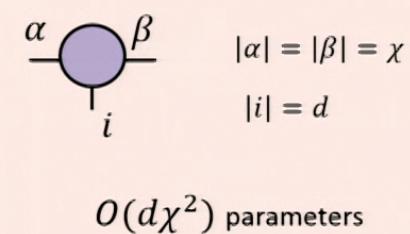
Efficient representation!

Efficient computation?



Summary of matrix product state (MPS)

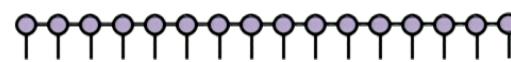
$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$



$\Psi_{i_1 i_2 \cdots i_N}$



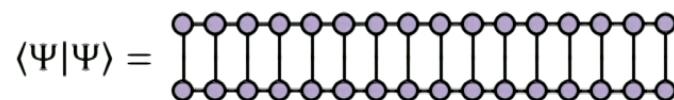
2^N



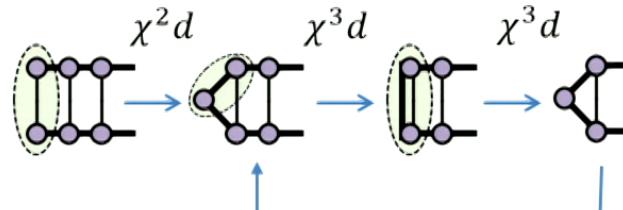
$O(Nd\chi^2)$

Efficient representation!

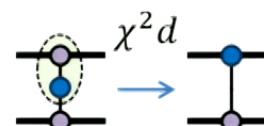
Efficient computation?



$O(Nd\chi^3) !!!$



$$\langle \Psi | \hat{o} | \Psi \rangle =$$



MPS physics? Structural properties:

- (a) entanglement entropy
- (b) correlations

➤ entanglement entropy

$$\rho_A = \text{tr}_B |\Psi\rangle\langle\Psi|$$

$$\rho_A =$$

MPS physics? Structural properties:

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- (a) entanglement entropy
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MPS physics? Structural properties:

➤ entanglement entropy

$$\rho_A = \text{tr}_B |\Psi\rangle\langle\Psi|$$

$\rho_A =$

$$= \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \mu = (\alpha, \beta)$$

- (a) entanglement entropy
- (b) correlations

$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

MPS physics? Structural properties:

➤ entanglement entropy

$$\begin{aligned} |\Psi\rangle\langle\Psi| &= \text{[Diagram of a 2D MPS tensor network with horizontal legs and vertical nodes]} \\ \downarrow & \\ \rho_A &= \text{tr}_B |\Psi\rangle\langle\Psi| \\ &= \text{[Diagram of the reduced density matrix } \rho_A \text{ obtained by tracing out sites from the original MPS. It shows two rectangular regions connected by a dashed red line, with blue bars below representing the bond dimension]} \\ &= \text{[Diagram showing the reduced density matrix } \rho_A \text{ as a bipartitioned system with boundary } \mu = (\alpha, \beta) \text{ indicated by a vertical line and red dots on the boundary] } \end{aligned}$$

(a) entanglement entropy
(b) correlations

$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

$$\begin{aligned} S(\rho_A) &= S(p_1, p_2, \dots, p_{\chi^2}) \\ &\leq S\left(\frac{1}{\chi^2}, \frac{1}{\chi^2}, \dots, \frac{1}{\chi^2}\right) \\ &= \log \chi^2 = 2 \log \chi \end{aligned}$$

MPS physics? Structural properties:

➤ entanglement entropy

$$|\Psi\rangle\langle\Psi| = \begin{array}{c} \text{Diagram of a 1D MPS with sites connected by vertical lines and horizontal bonds between adjacent sites. All sites are purple circles.} \\ | \Psi \rangle \langle \Psi | \end{array}$$

$$\rho_A = \text{tr}_B |\Psi\rangle\langle\Psi|$$

$$\rho_A = \begin{array}{c} \text{Diagram showing a central region of the MPS being traced over. Red dashed lines indicate the boundaries of the subsystem A.} \\ \rho_A = \dots \end{array}$$

$$= \begin{array}{c} \text{Diagram showing the reduced density matrix } \rho_A \text{ as a bipartitioned system. Two blue horizontal bars represent the left and right parts, with a vertical line connecting them at position } \mu = (\alpha, \beta). \\ = \dots \end{array}$$

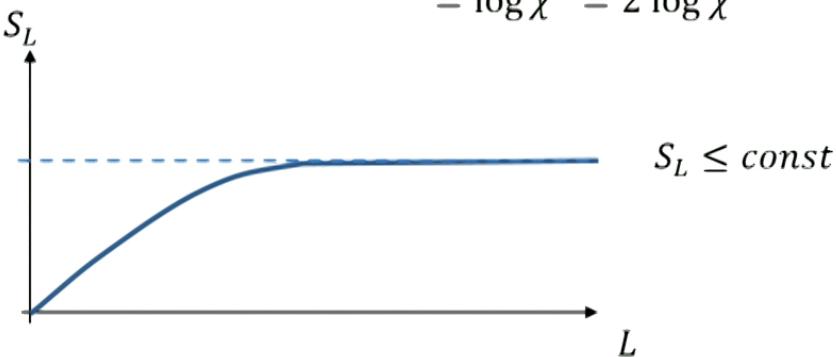
(a) entanglement entropy
 (b) correlations

$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

$$S(\rho_A) = S(p_1, p_2, \dots, p_{\chi^2})$$

$$\leq S\left(\frac{1}{\chi^2}, \frac{1}{\chi^2}, \dots, \frac{1}{\chi^2}\right)$$

$$= \log \chi^2 = 2 \log \chi$$



➤ correlations

$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle = \text{Diagram of a chain of } L \text{ sites with blue and red dots} = \text{Diagram of a chain of } L \text{ sites with blue and red dots}$$
$$= \text{Diagram of a single site with blue dot} \left(\text{Diagram of a single site with red dot} \right)^{L-1} \approx a \lambda^L = ae^{-L/\xi}$$
$$\xi \equiv -\frac{1}{\log \lambda}$$

➤ correlations

$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle = \text{Diagram of a 1D chain of sites with two marked points (blue and red).} = \text{Diagram of a 1D chain of sites with two marked points (blue and red).}$$
$$= \text{Diagram of a 1D chain of sites with two marked points (blue and red).} \left(\text{Diagram of a 1D chain of sites with two marked points (blue and red).} \right)^{L-1} \approx a \lambda^L = ae^{-L/\xi}$$
$$\xi \equiv -\frac{1}{\log \lambda}$$

Structural properties of MPS

correlations $C(L) \approx e^{-L/\xi}$

entanglement $S_L \leq 2 \log \chi$

➤ correlations

$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle = \text{Diagram of a 1D chain with two points highlighted in blue and red} = \text{Diagram of a 1D chain with two points highlighted in blue and red}$$
$$= \text{Diagram of a 1D chain with two points highlighted in blue and red} \left(\text{Diagram of a 1D chain with two points highlighted in blue and red} \right)^{L-1} \approx a \lambda^L = ae^{-L/\xi}$$
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Structural properties of MPS

correlations $C(L) \approx e^{-L/\xi}$

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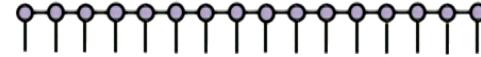
match with
ground states of 1D
gapped Hamiltonians

MERA: definition

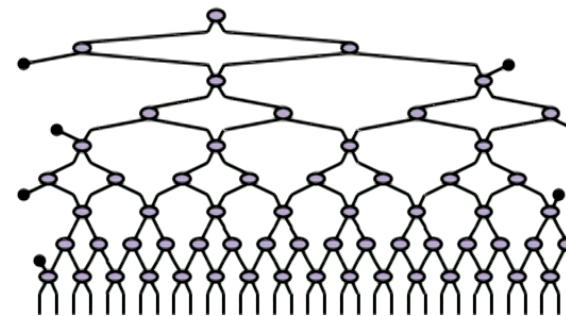
$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

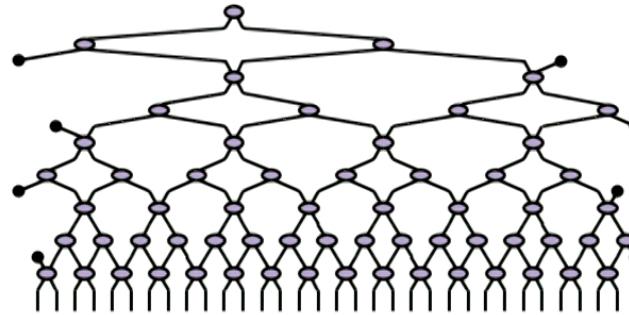
Matrix product state
(MPS)



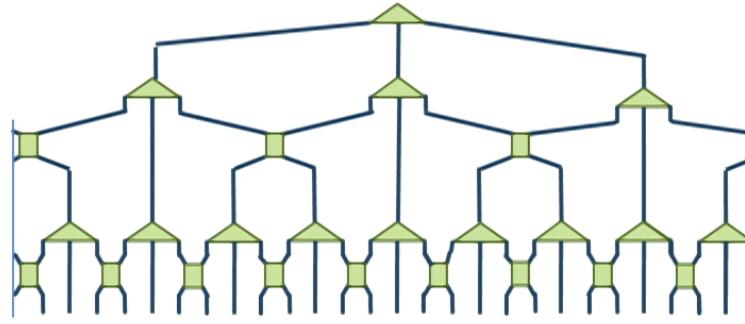
Multi-scale entanglement
renormalization ansatz
(MERA)



MERA



also MERA !

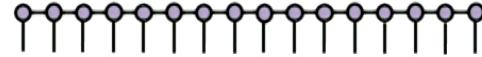


Efficient specification?

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

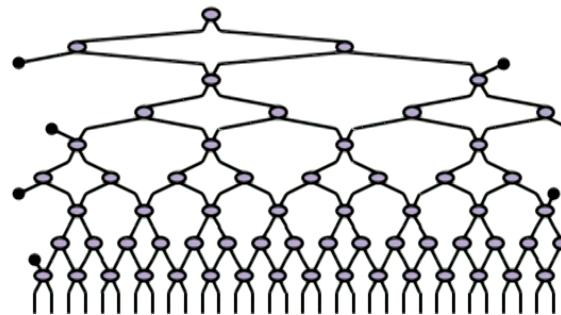
d^N complex numbers

Matrix product state
(MPS)



N spins

Multi-scale entanglement
renormalization ansatz
(MERA)



Efficient specification?

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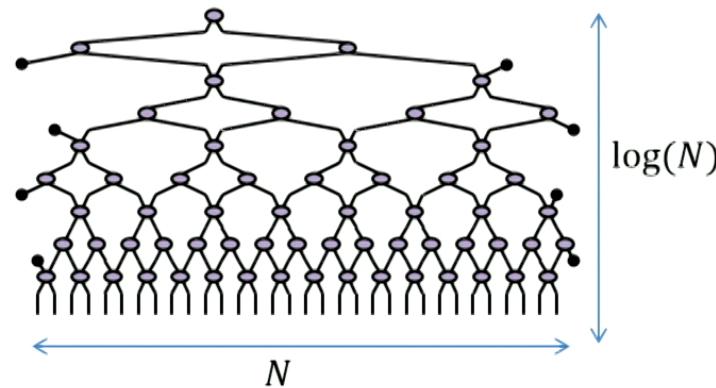
d^N complex numbers

Matrix product state
(MPS)



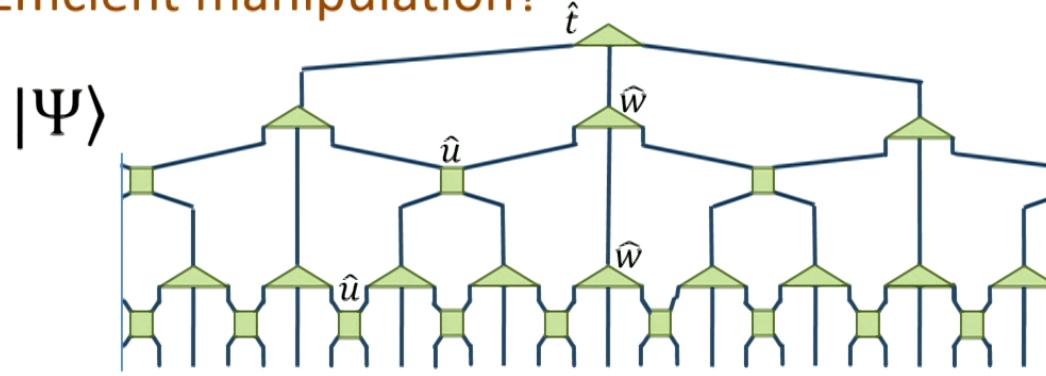
N spins $\Rightarrow N$ tensors
 $\Rightarrow O(N)$ parameters

Multi-scale entanglement
renormalization ansatz
(MERA)

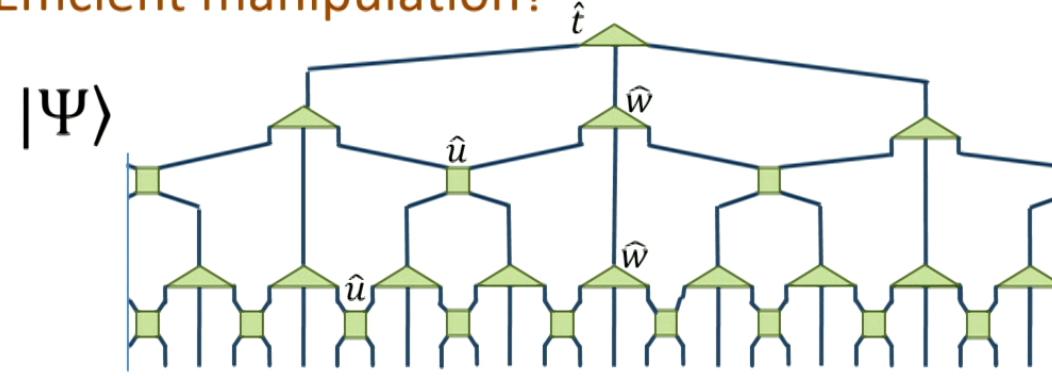


N spins $\Rightarrow N \log(N)$ tensors ?

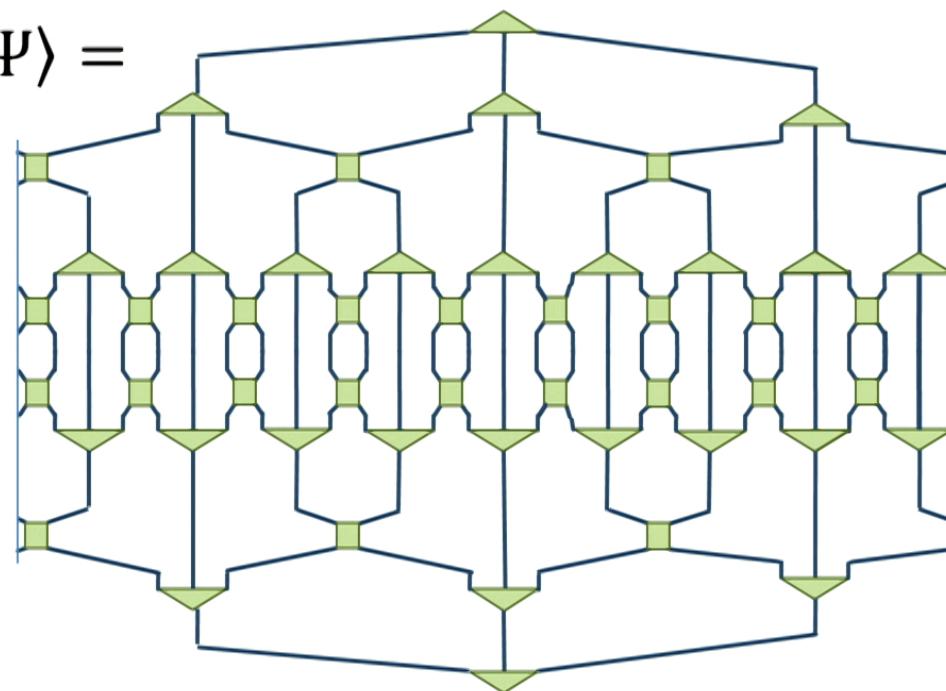
Efficient manipulation?



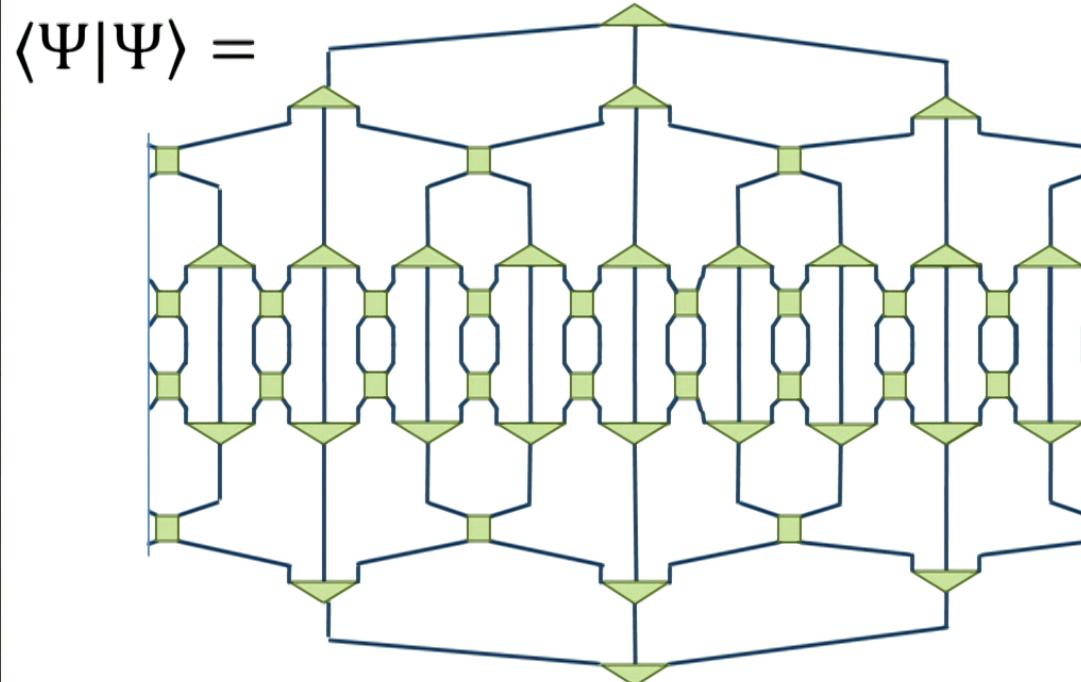
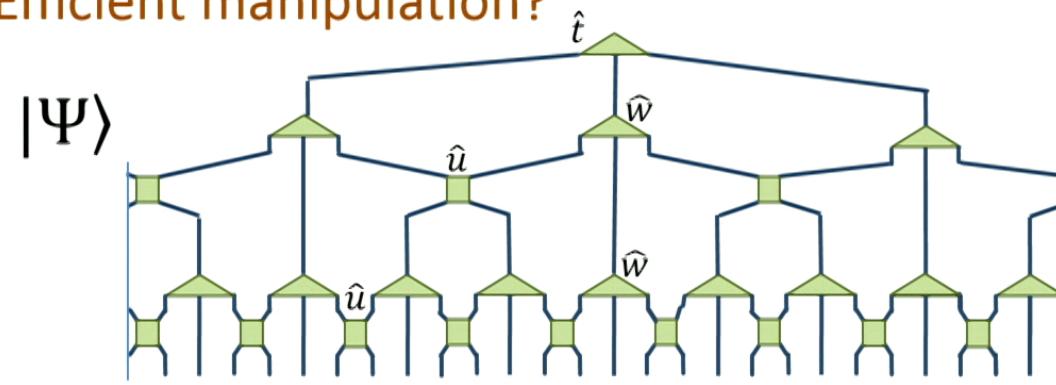
Efficient manipulation?



$\langle \Psi | \Psi \rangle =$



Efficient manipulation?



isometric tensors!

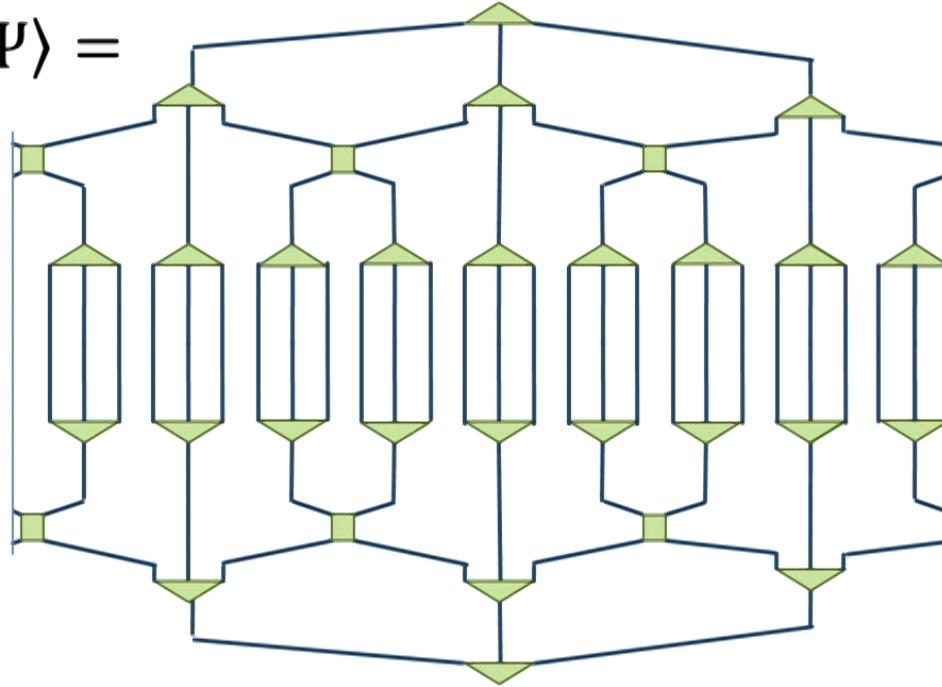
$$\hat{t} \quad \text{---} = 1$$

$$\hat{w} \quad \text{---} = \mid$$

$$\hat{u} \quad \text{---} = \mid \mid$$

Efficient manipulation?

$$\langle \Psi | \Psi \rangle =$$



isometric tensors!

$$\hat{t} \quad \begin{array}{c} \text{green diamond} \\ \text{blue rectangle} \end{array} = 1$$

$$\hat{w} \quad \begin{array}{c} \text{green diamond} \\ \text{blue rectangle} \end{array} = \parallel$$

$$\hat{u} \quad \begin{array}{c} \text{green diamond} \\ \text{blue rectangle} \end{array} = \parallel\parallel$$

Efficient manipulation?

$$\langle \Psi | \Psi \rangle = 1$$

isometric tensors!

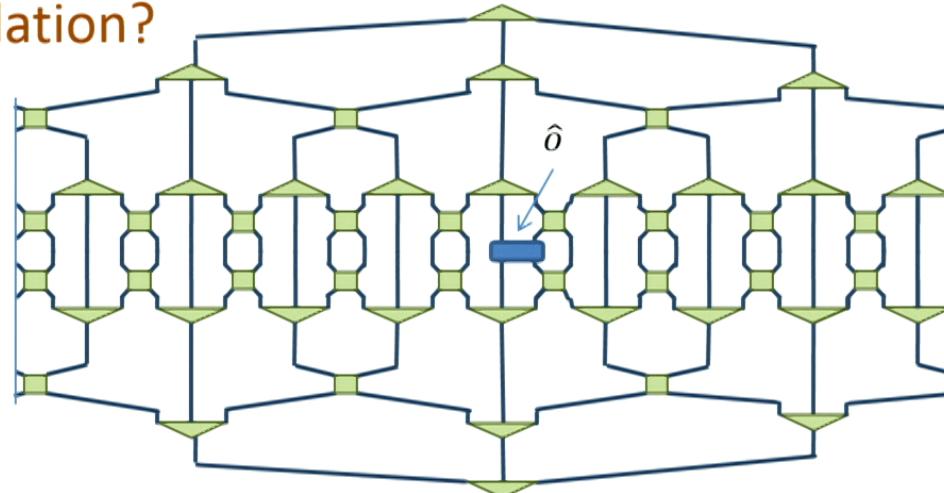
$$\hat{t} \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \quad = 1$$

$$\hat{w} \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \quad = \quad \mid$$

$$\hat{u} \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \quad = \quad \mid\mid$$

Efficient manipulation?

$$\langle \Psi | \hat{o} | \Psi \rangle =$$



Efficient manipulation?

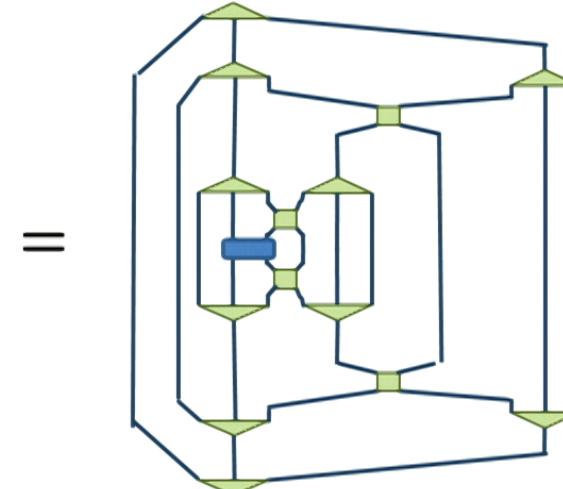
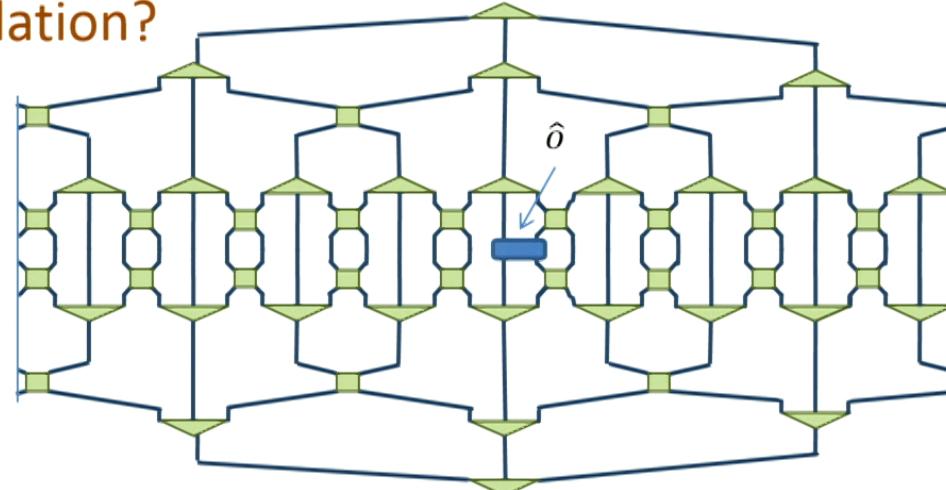
$$\langle \Psi | \hat{o} | \Psi \rangle =$$

isometric tensors!

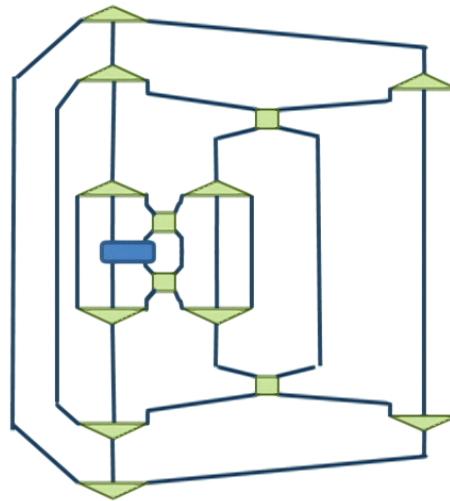
$$\hat{t} \quad \hat{t}^\dagger \quad = 1$$

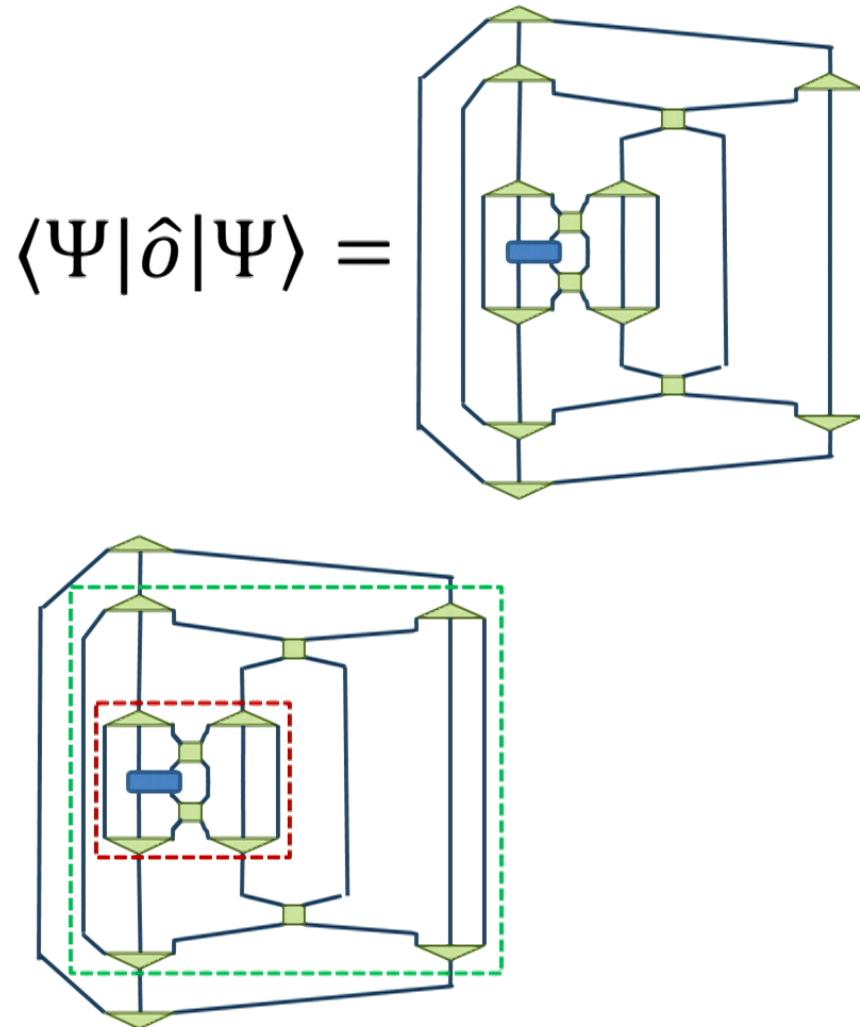
$$\hat{w} \quad \hat{w}^\dagger \quad = \quad |$$

$$\hat{u} \quad \hat{u}^\dagger \quad = \quad ||$$



$$\langle \Psi | \hat{o} | \Psi \rangle =$$



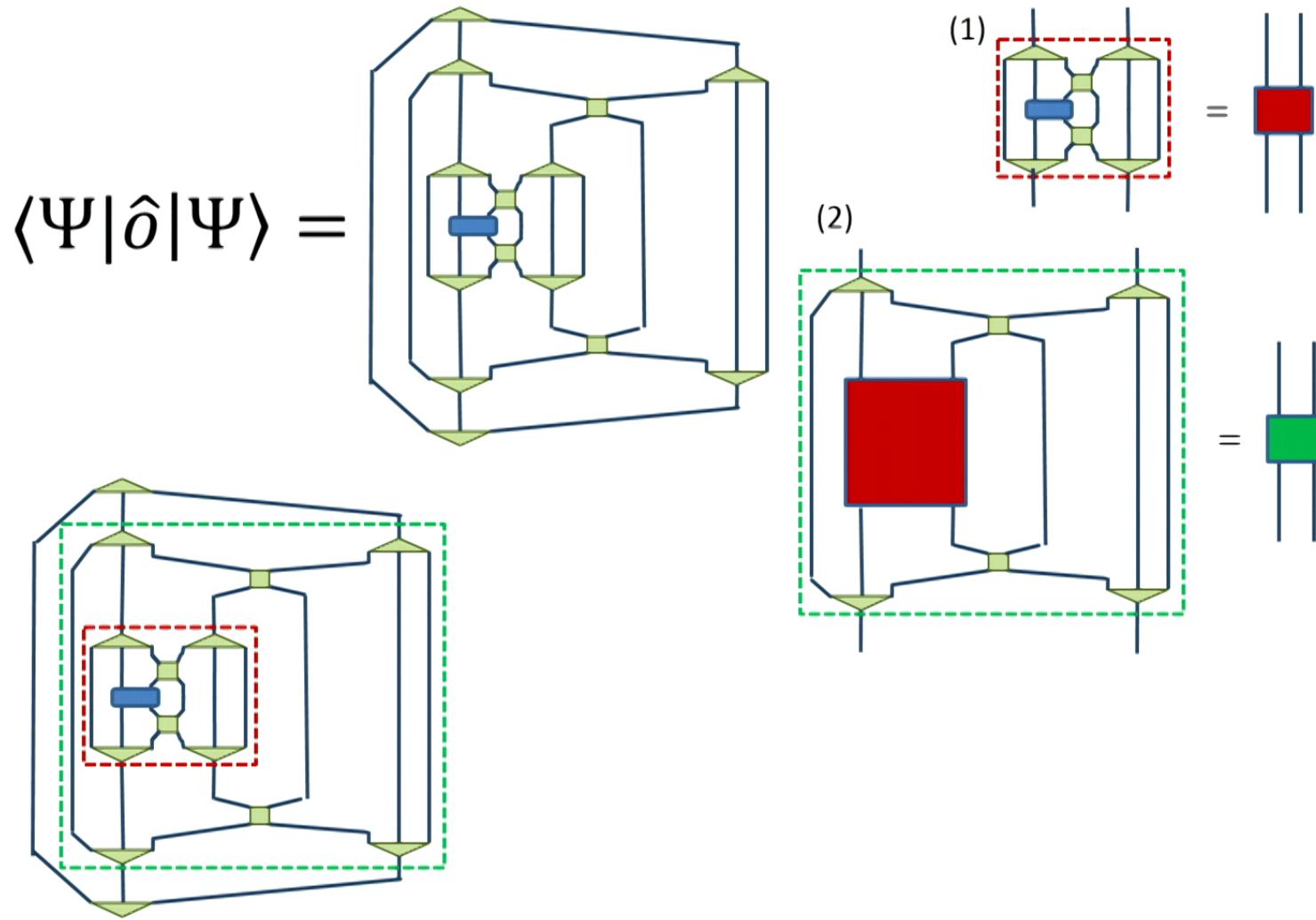


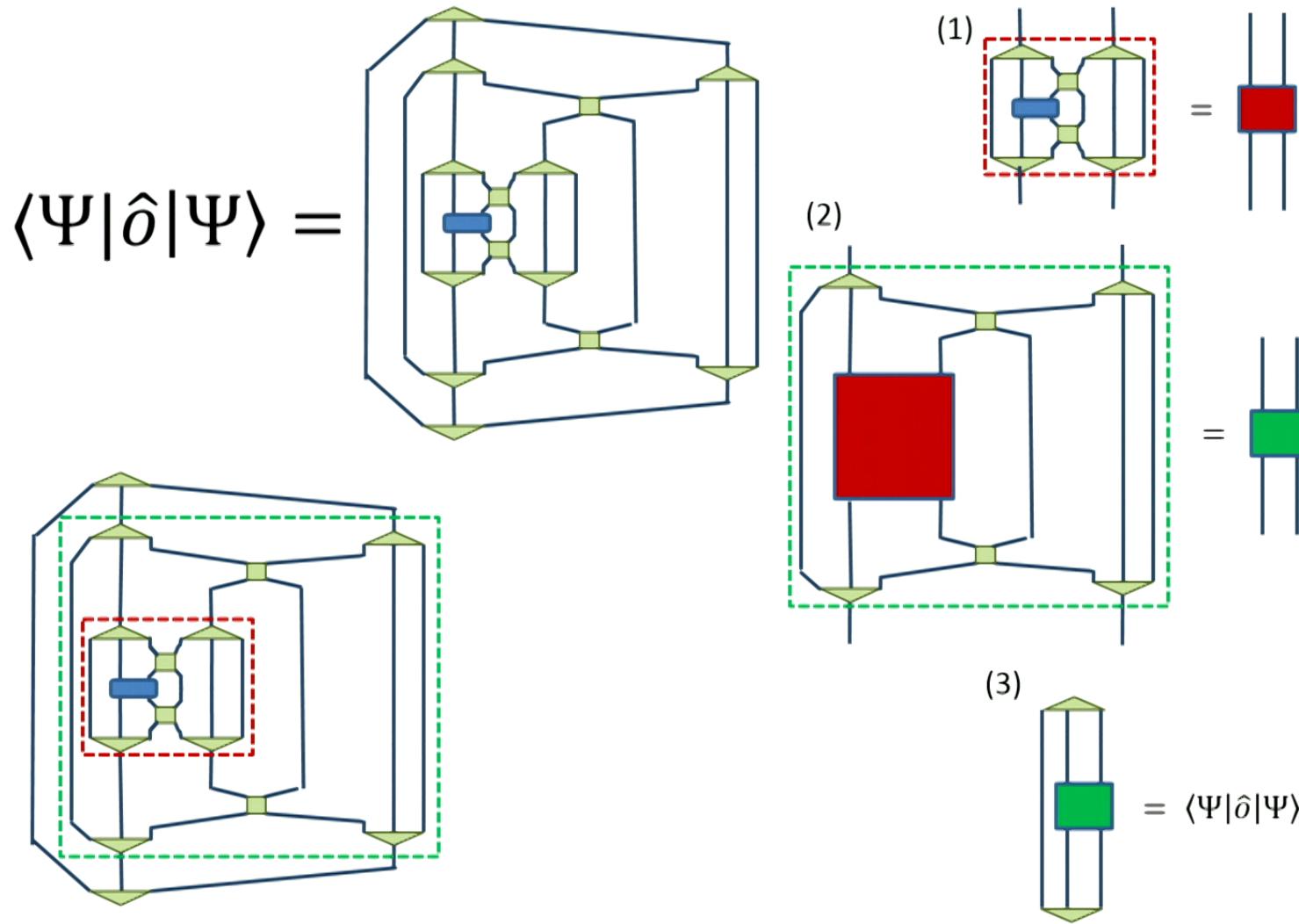
$$\langle \Psi | \hat{o} | \Psi \rangle =$$

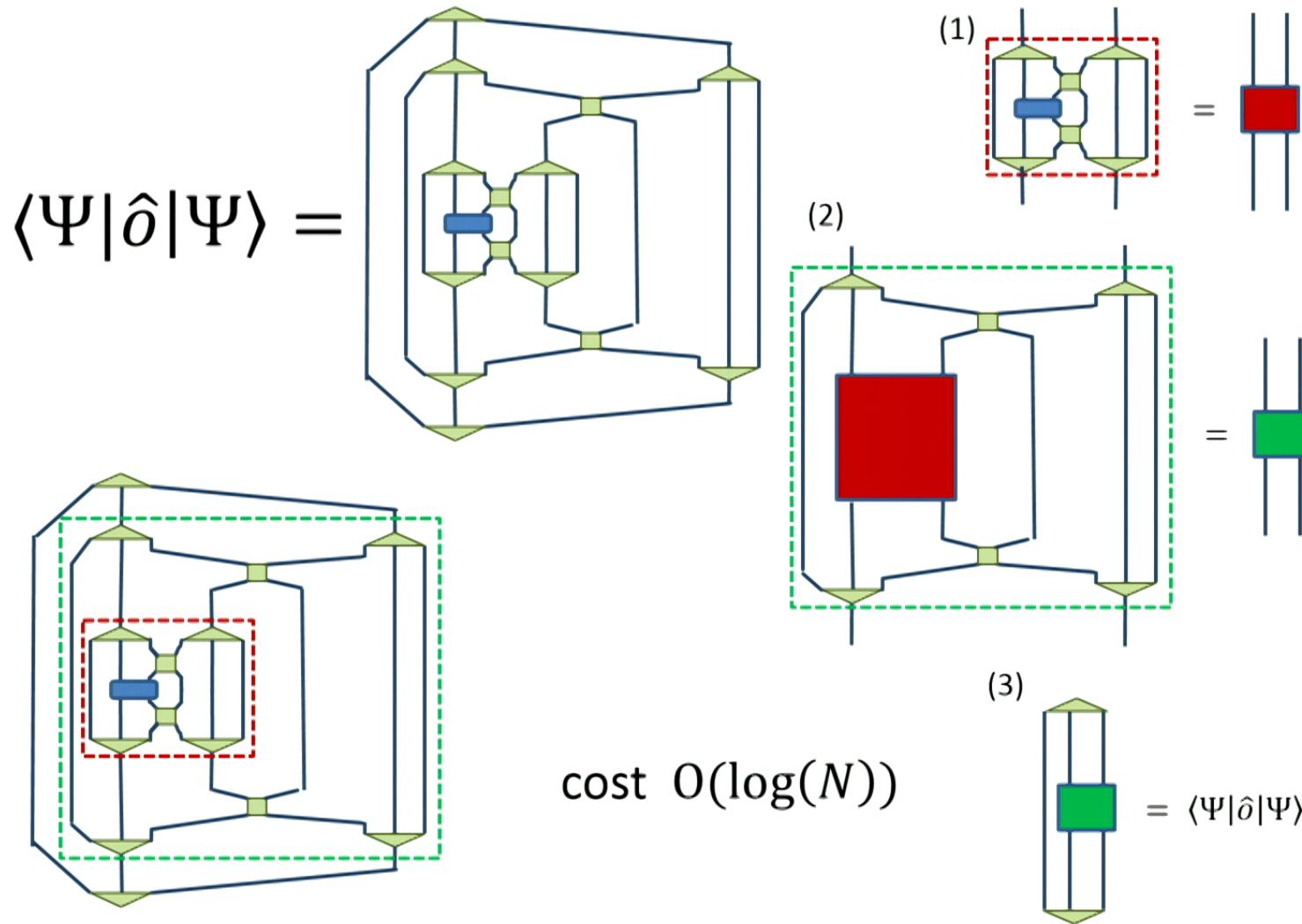
The top diagram shows a complex loop structure with a central blue rectangle and green vertices, enclosed in a blue octagon. The bottom diagram shows a similar structure but with a red dashed box highlighting a central part, enclosed in a green dashed octagon.

(1)

=



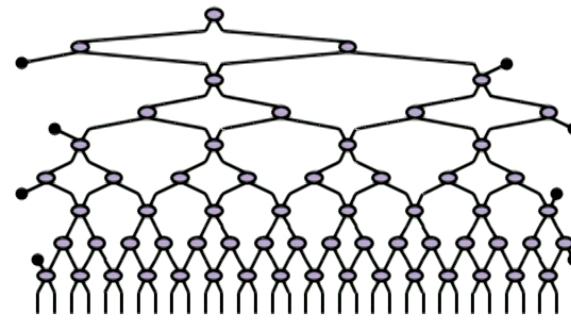




Structural properties

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers



- Decay of correlations
- Scaling of entanglement

$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

MPS

$$\begin{aligned} &= \text{Diagram of a 1D chain of sites (purple circles) with two operators (blue and red dots) inserted at positions 0 and L.} \\ &= \text{Diagram of a 1D chain of sites (purple circles) with two operators (blue and red dots) inserted at positions 0 and L, enclosed in a loop.} \\ &= \text{Diagram showing the MPS structure: a loop of sites (purple circles) with two operators (blue and red dots) inserted at positions 0 and L-1, followed by a bracketed product of local tensors (sites with two horizontal lines) from position 0 to L-1.} \\ &\approx a\lambda^L = ae^{-L/\xi} \end{aligned}$$
$$\xi \equiv -\frac{1}{\log \lambda}$$

$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

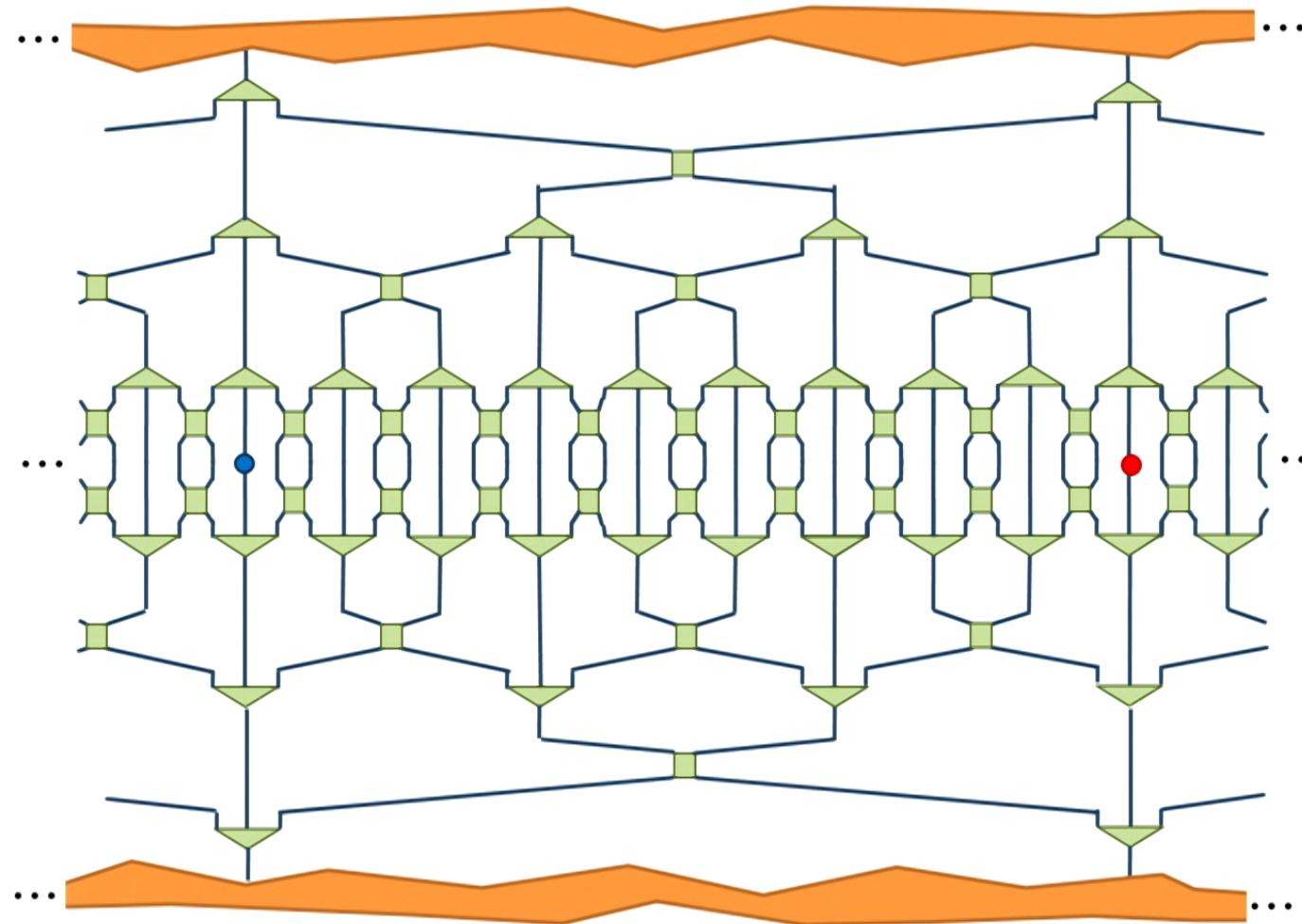
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$$\xi \equiv -\frac{1}{\log \lambda}$$

\Rightarrow exponential decay of correlations

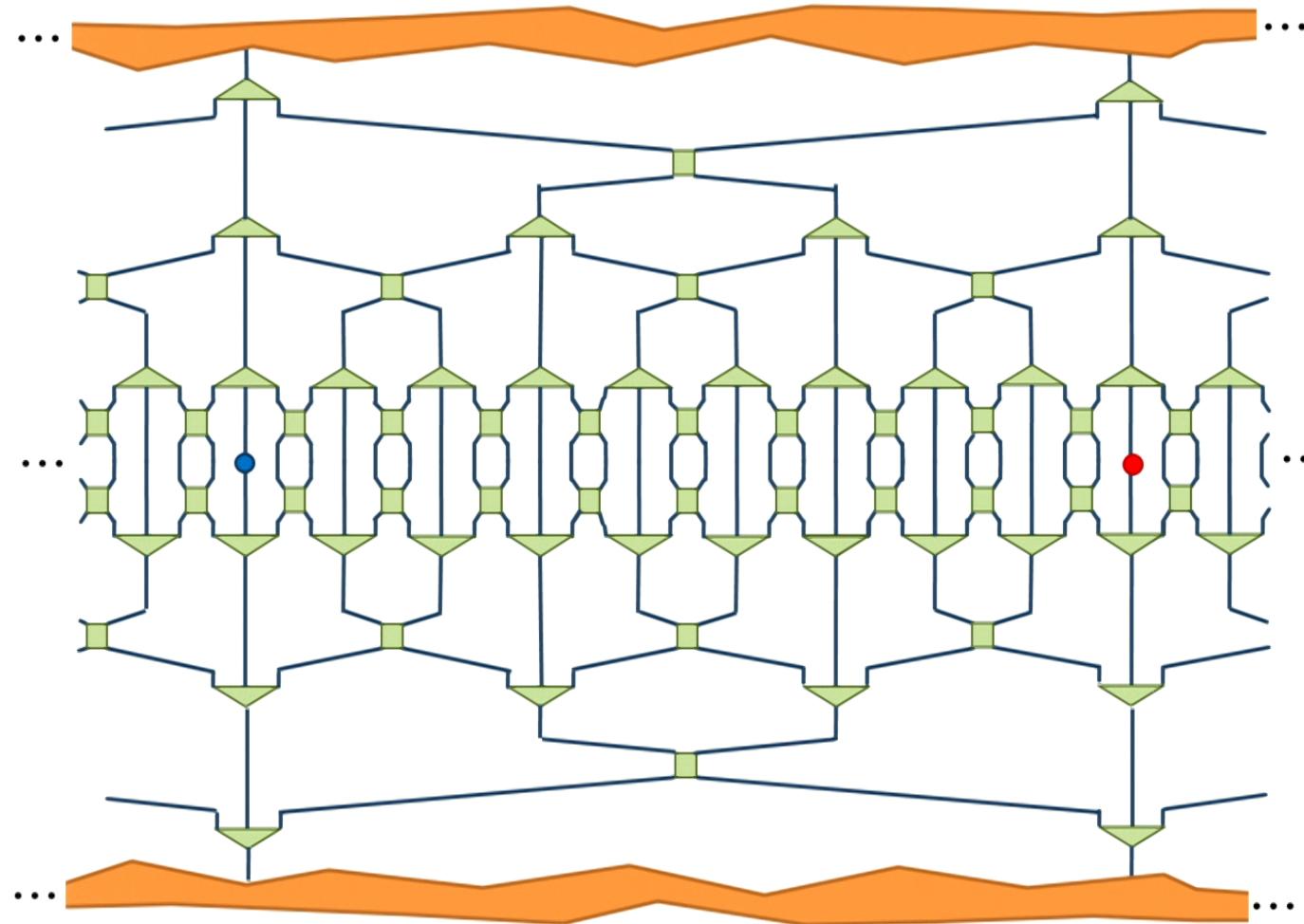
MERA

$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle =$$

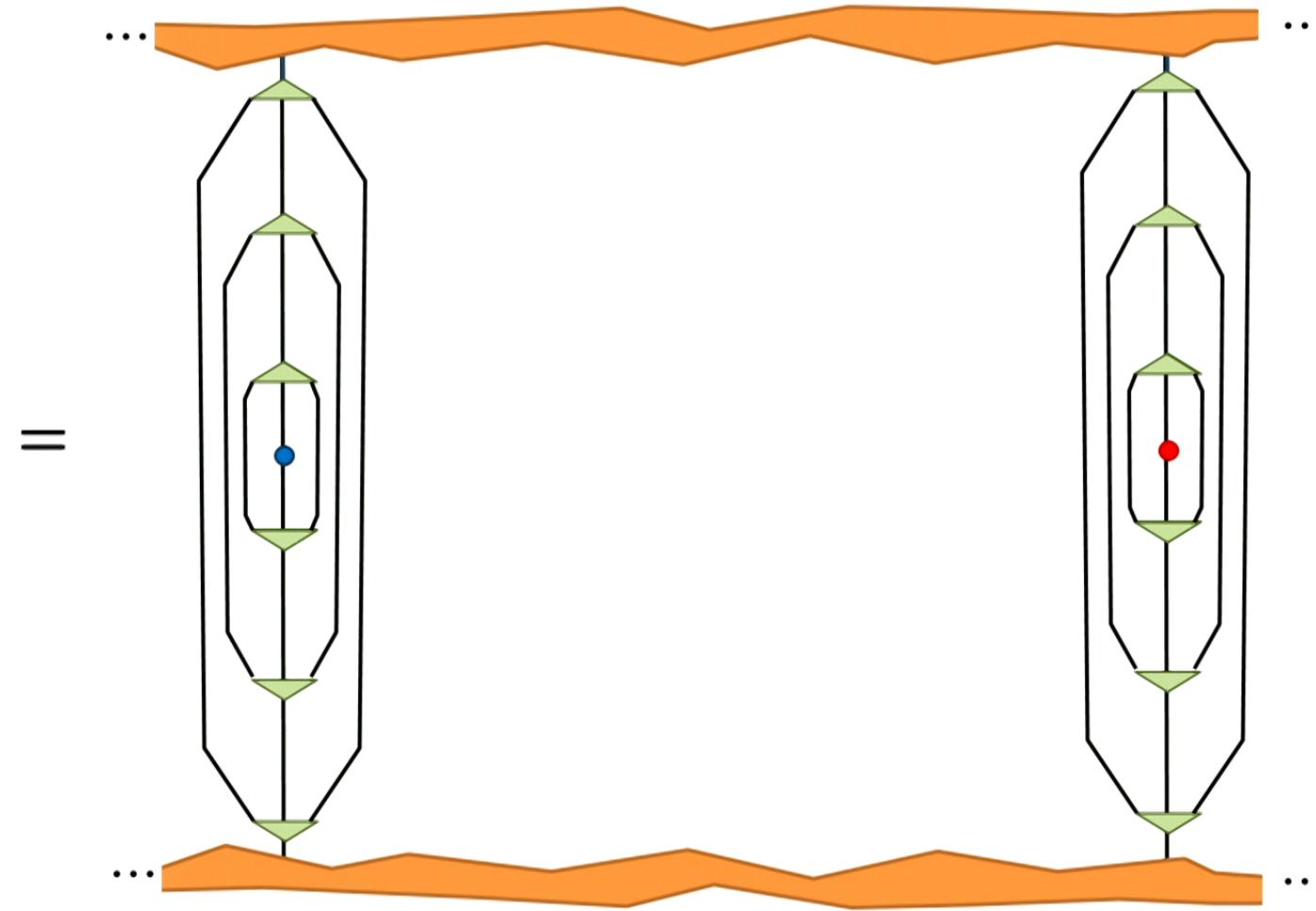


MERA

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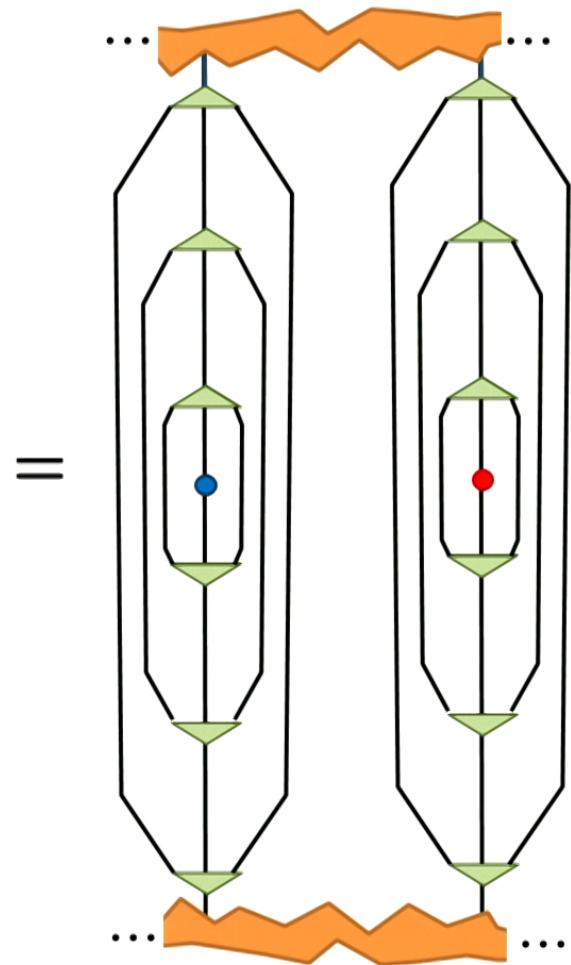


$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$$



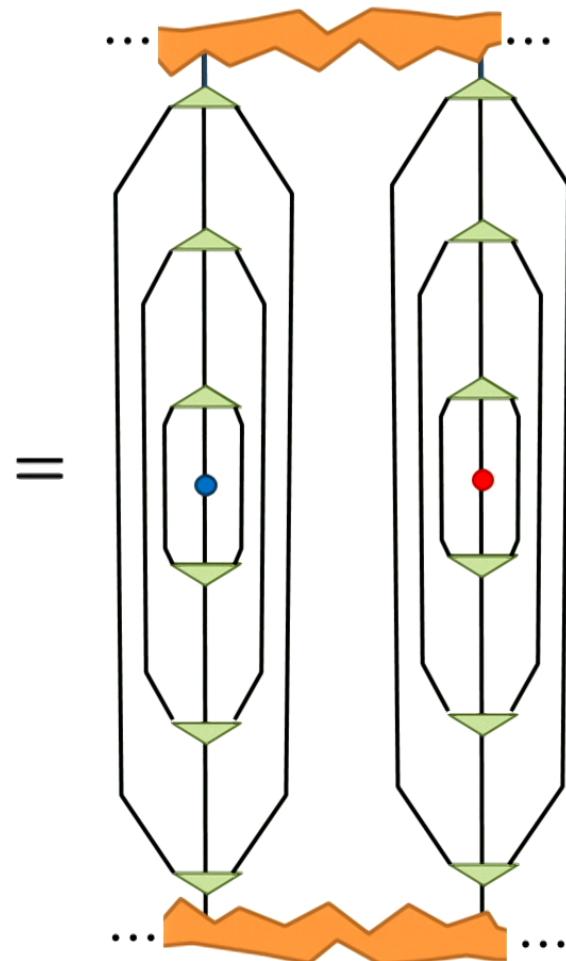
$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

MERA



$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

MERA



$$\approx (\lambda)^{\log_3(L)} (\lambda)^{\log_3(L)}$$

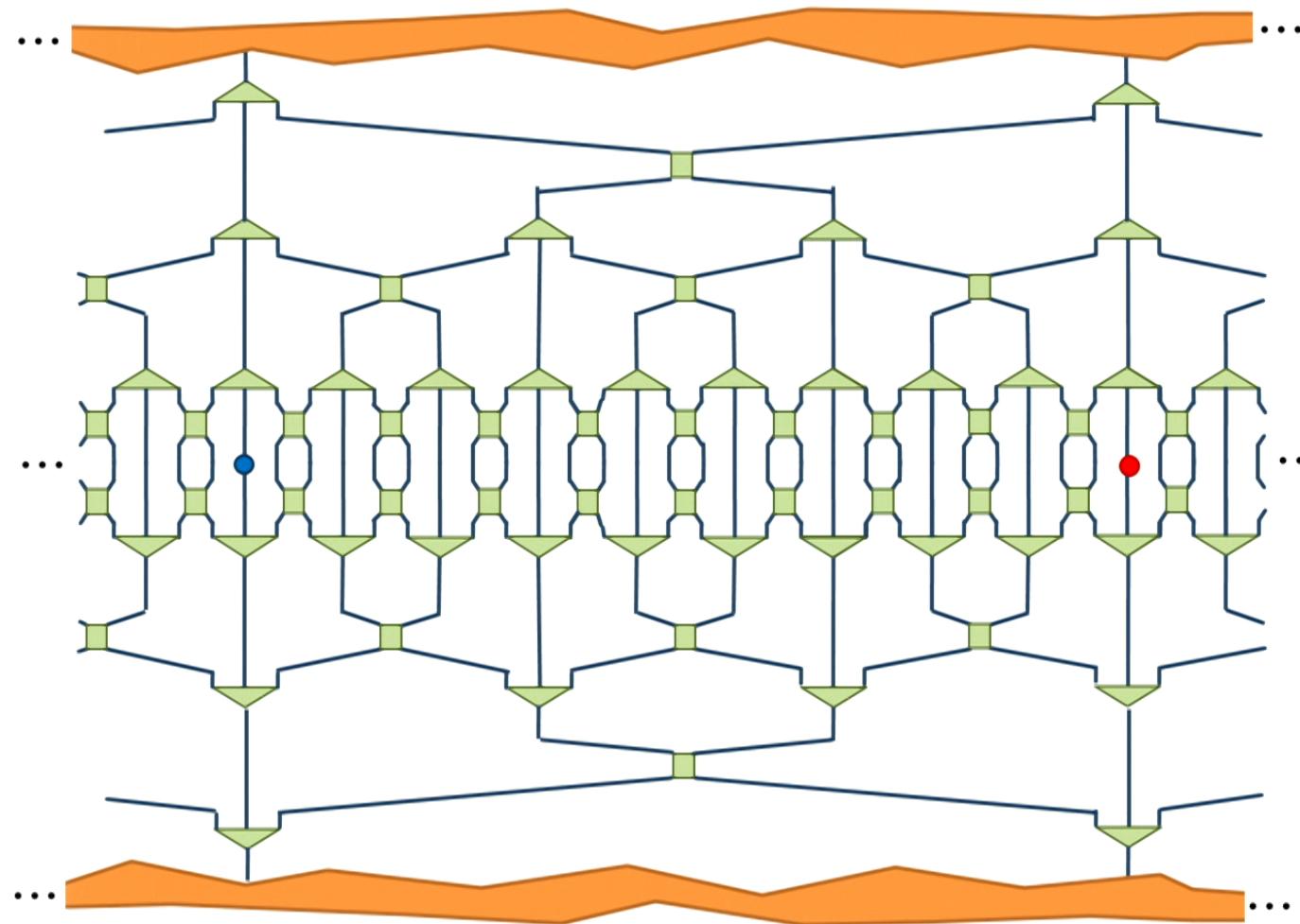
$$= \lambda^{2 \log_3(L)} = L^{2 \log_3(\lambda)} = L^{-p}$$

$$x^{\log_3(y)} = y^{\log_3(x)} \quad p \equiv -2 \log_3(\lambda)$$

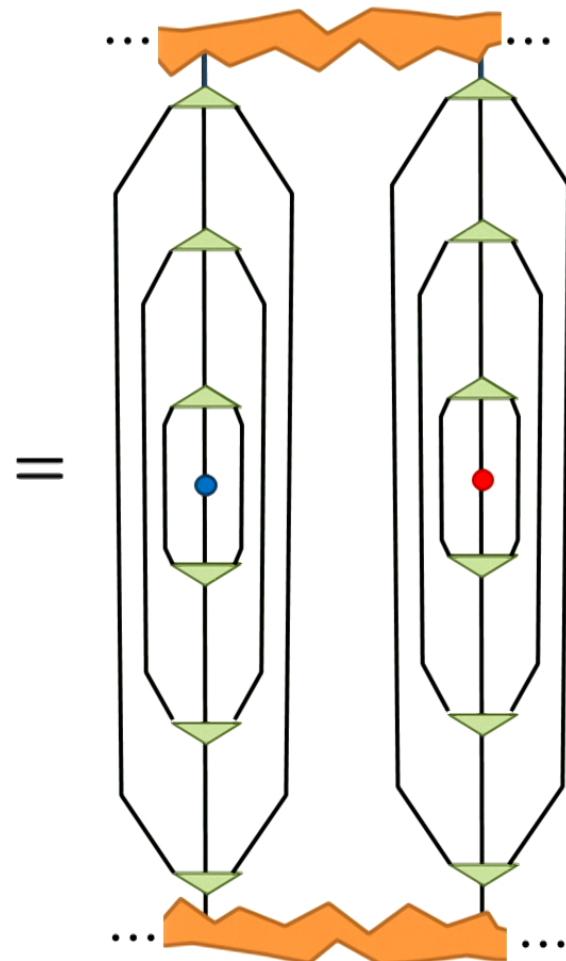
\Rightarrow polynomial decay of correlations

MERA

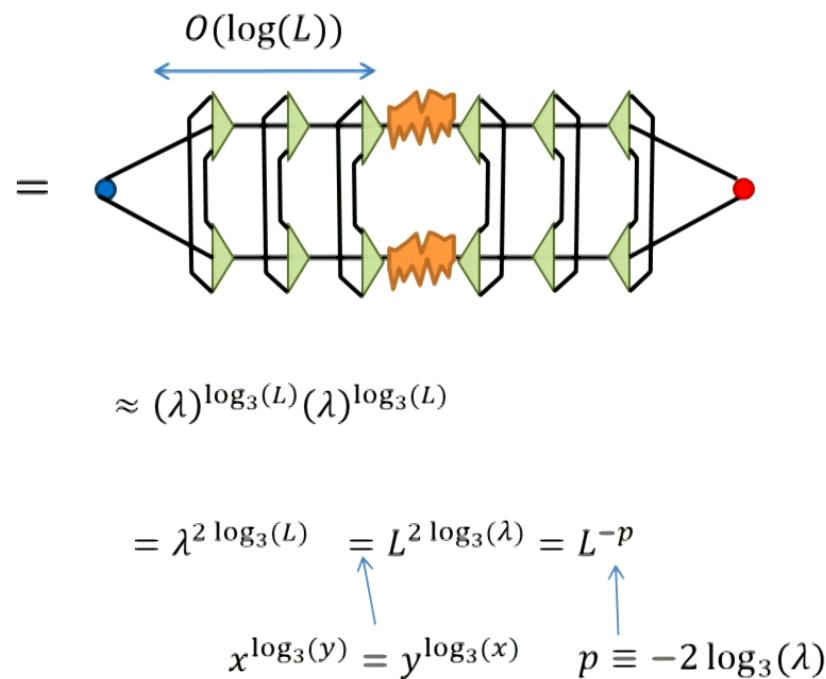
$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle =$$



$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

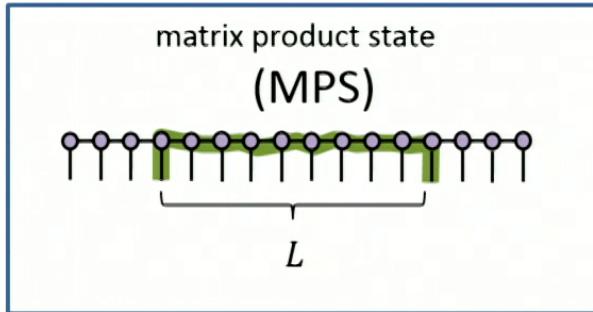


MERA



\Rightarrow polynomial decay of correlations

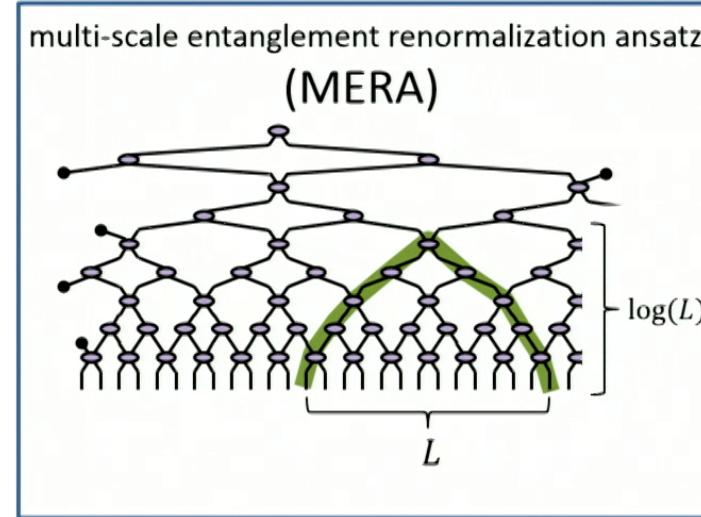
Correlations: summary and interpretation



structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L) \rangle \approx e^{-L/\xi}$$

exponential

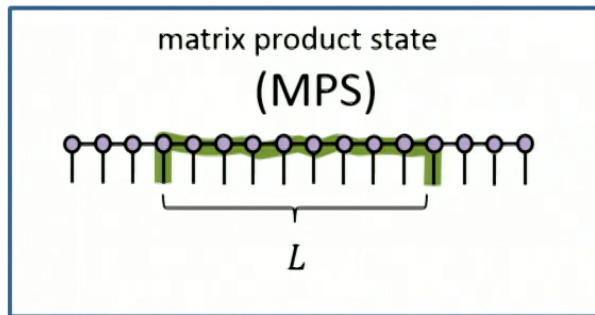


structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L) \rangle \approx L^{-p}$$

power-law

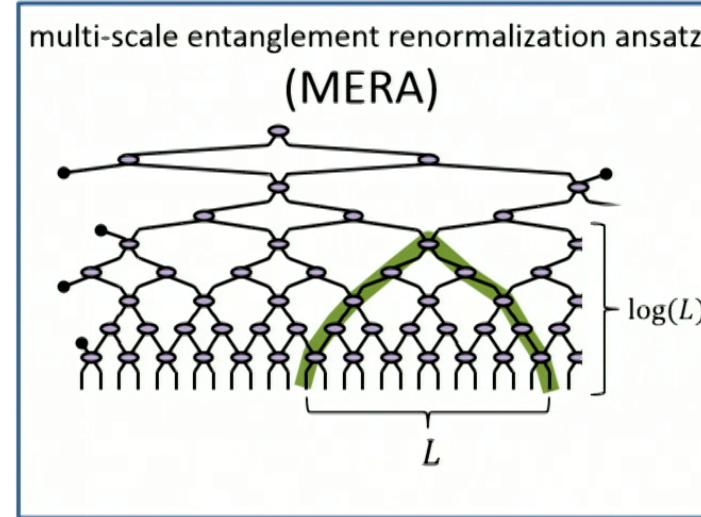
Correlations: summary and interpretation



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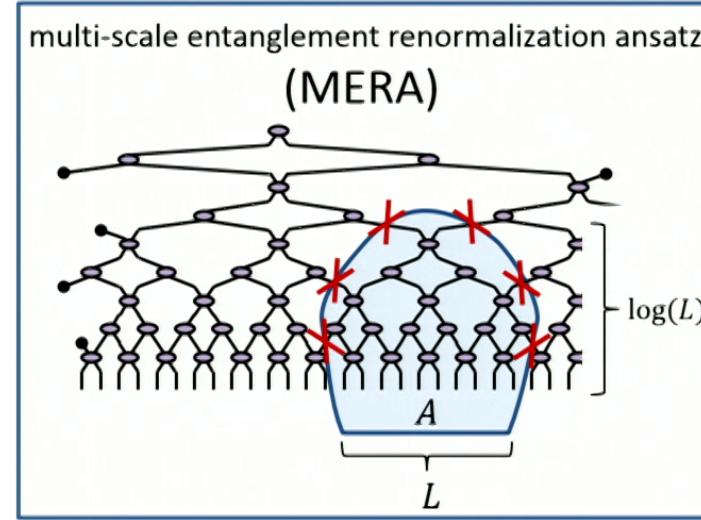
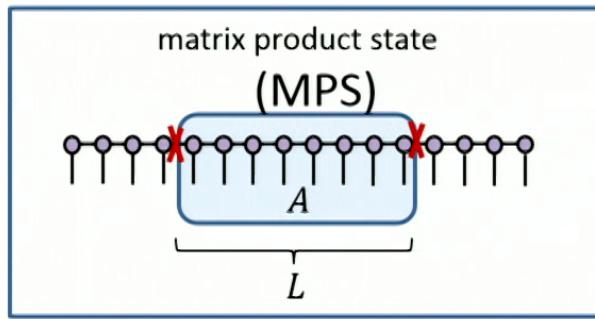


structure of geodesics:

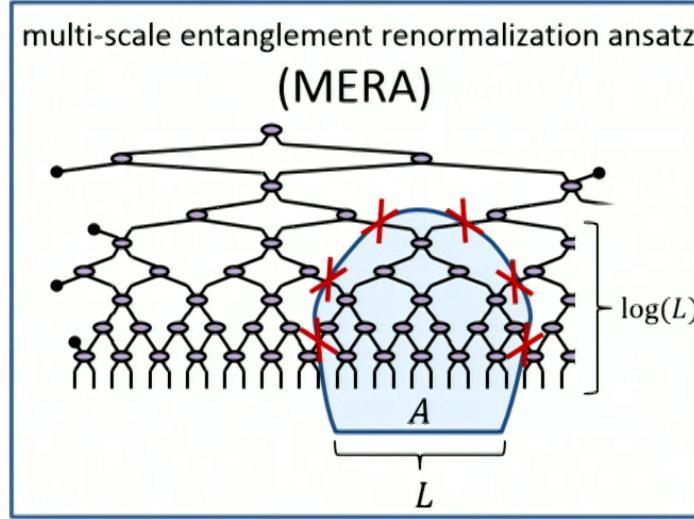
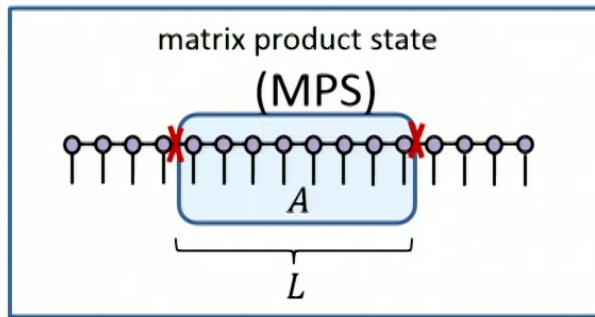
$$\langle \hat{o}(0)\hat{o}(L) \rangle \approx L^{-p}$$

power-law

Entanglement entropy



Entanglement entropy

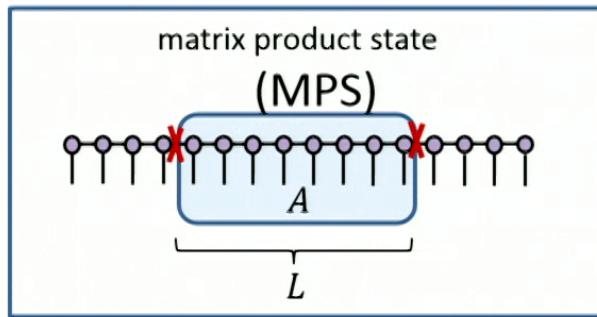


connectivity:

$$S(A) \leq \text{const}$$

boundary law!

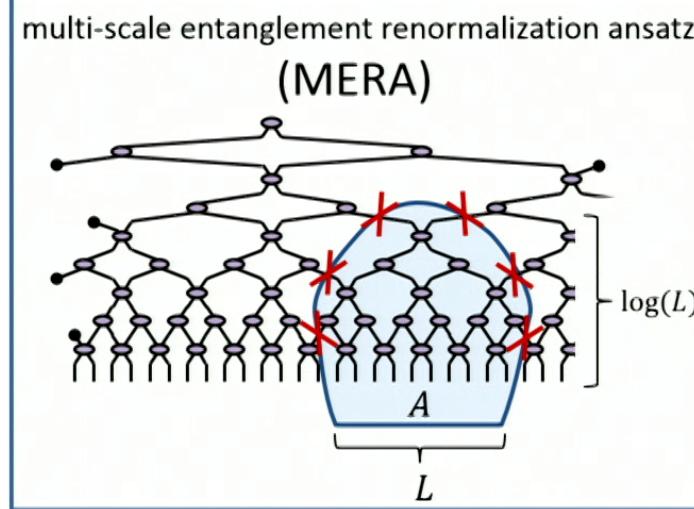
Entanglement entropy



connectivity:

$$S(A) \leq \text{const}$$

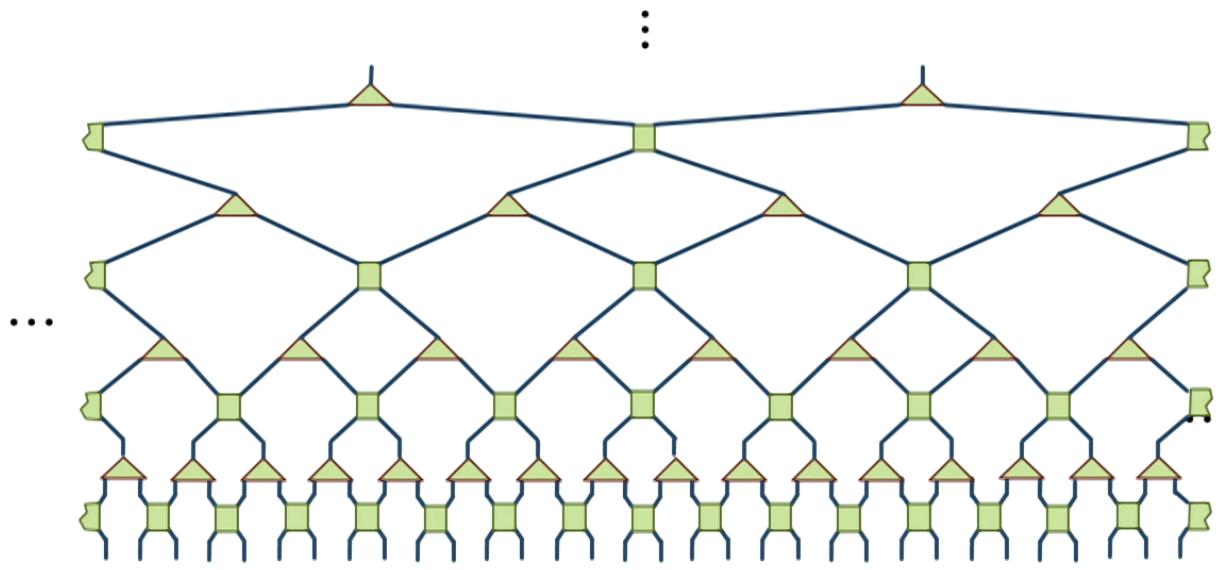
boundary law!



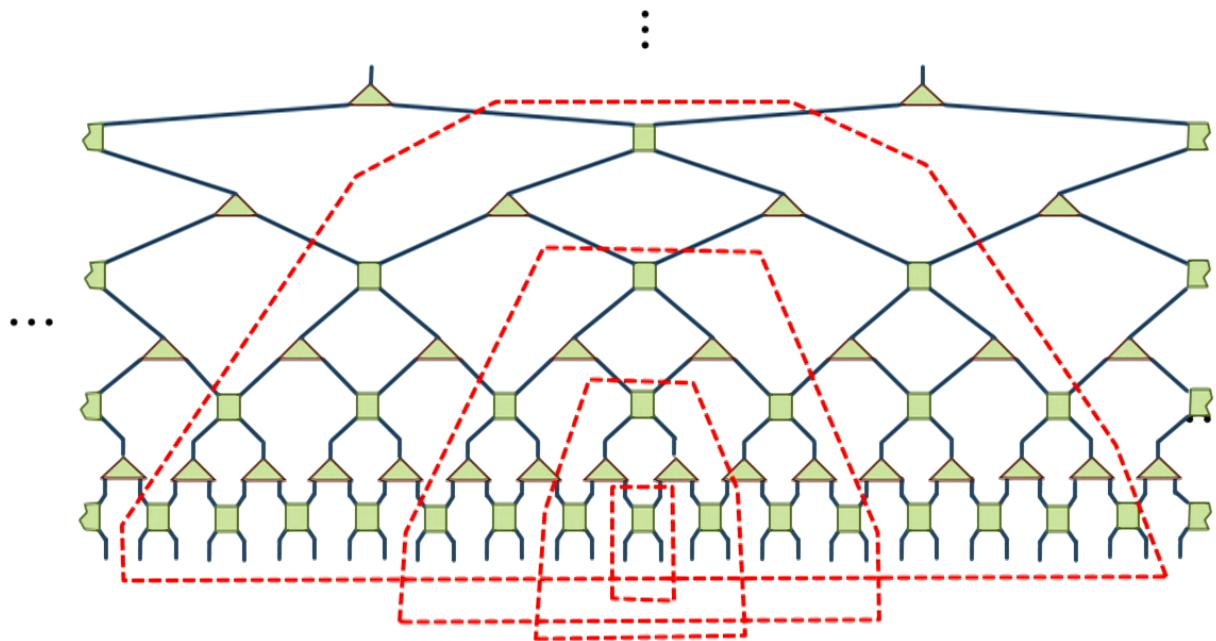
connectivity:

$$S(A) \leq \log L$$

logarithmic correction!



$$n(A) \approx \log L$$



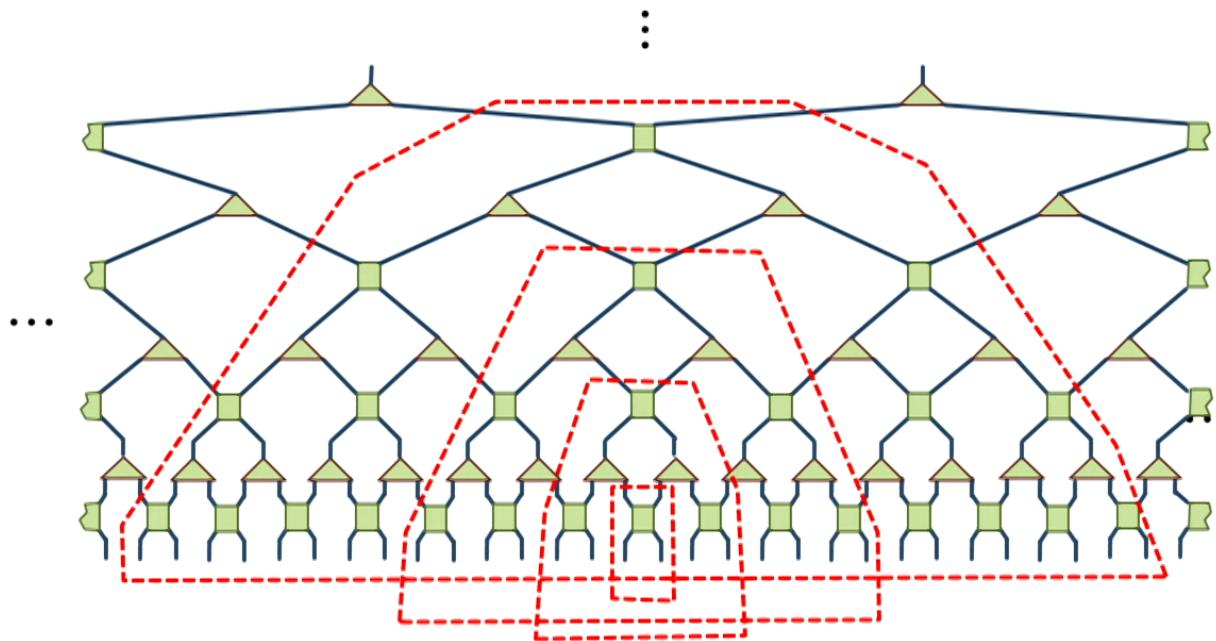
$$n(A) \approx \log L$$

$$L = 2, \quad n(A) = 2$$

$$L = 6, \quad n(A) = 4$$

$$L = 14, \quad n(A) = 6$$

$$L = 30, \quad n(A) = 8$$



$$n(A) \approx \log L$$

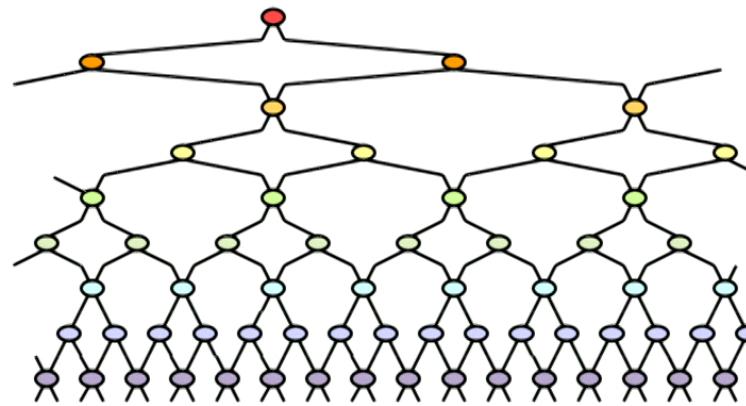
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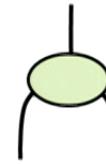
$$L = 30, \quad n(A) = 8$$

MERA as a quantum circuit



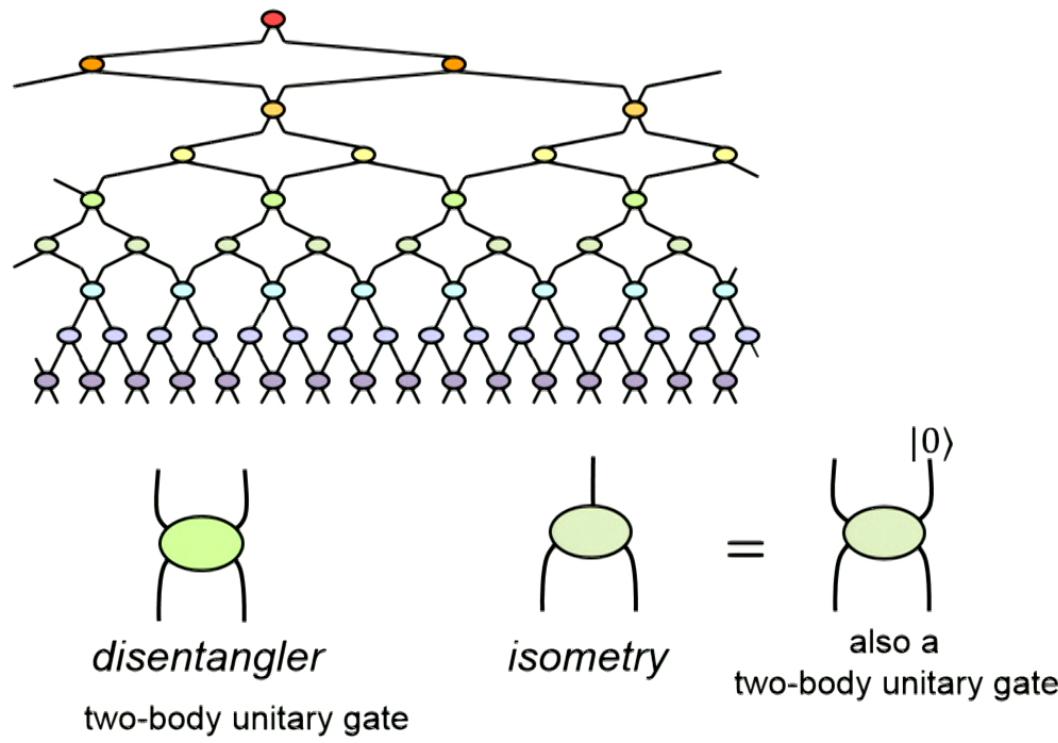
disentangler

two-body unitary gate

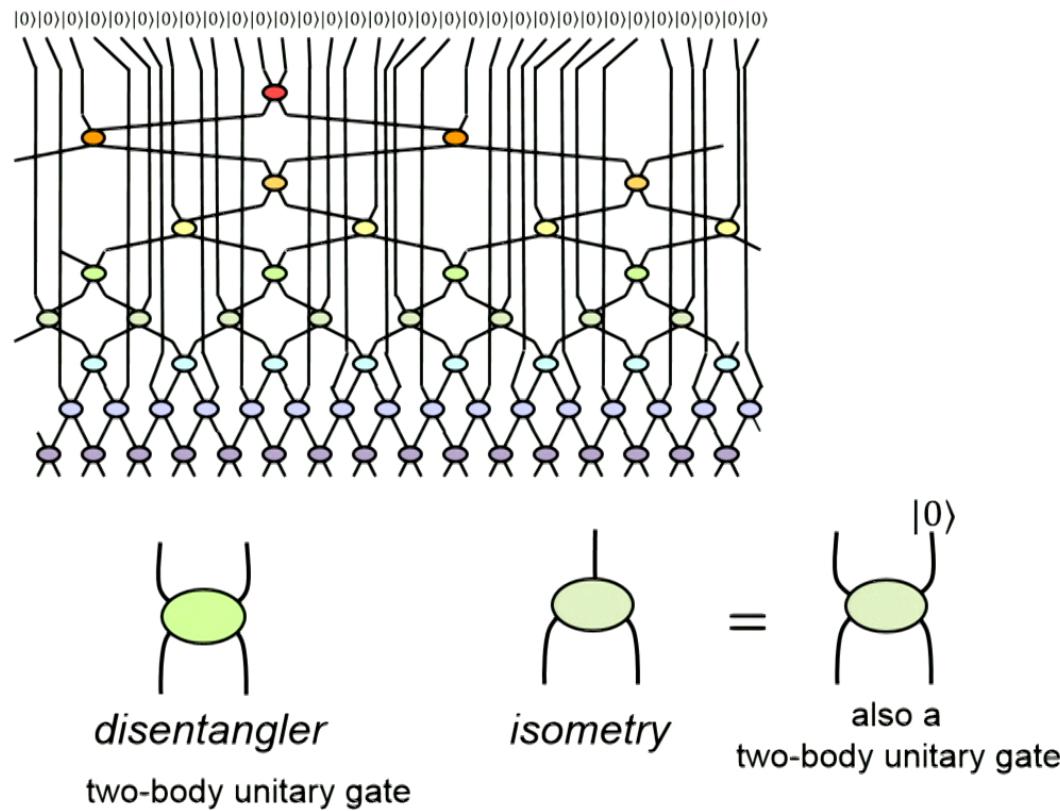


isometry

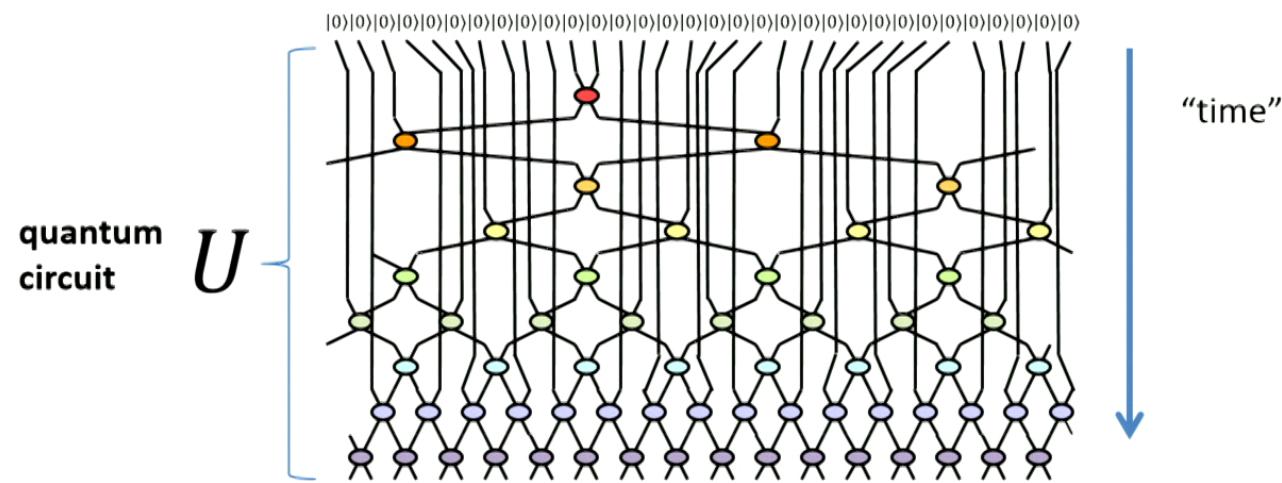
MERA as a quantum circuit



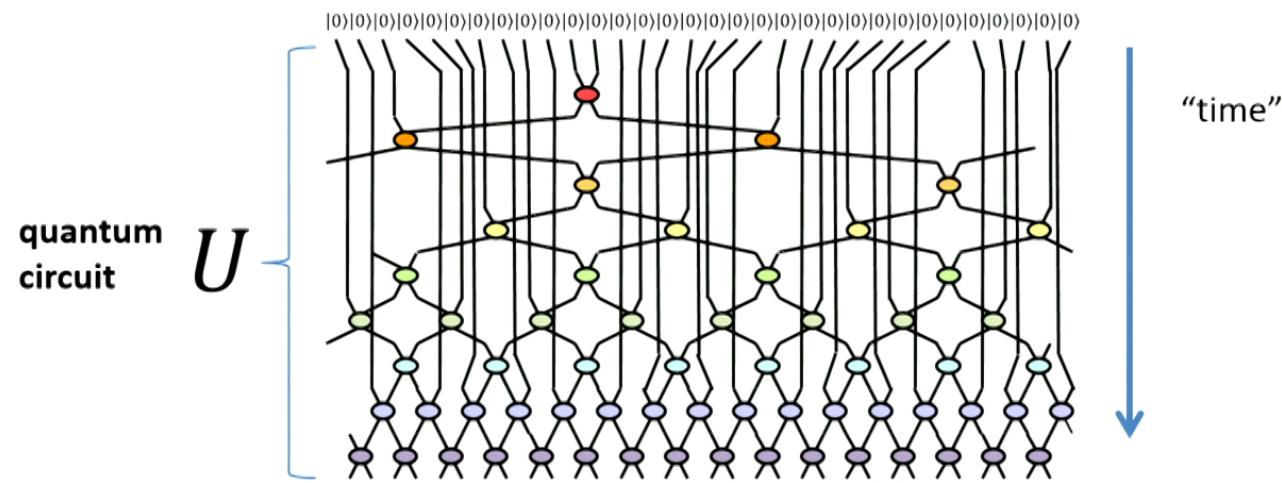
MERA as a quantum circuit



MERA as a quantum circuit

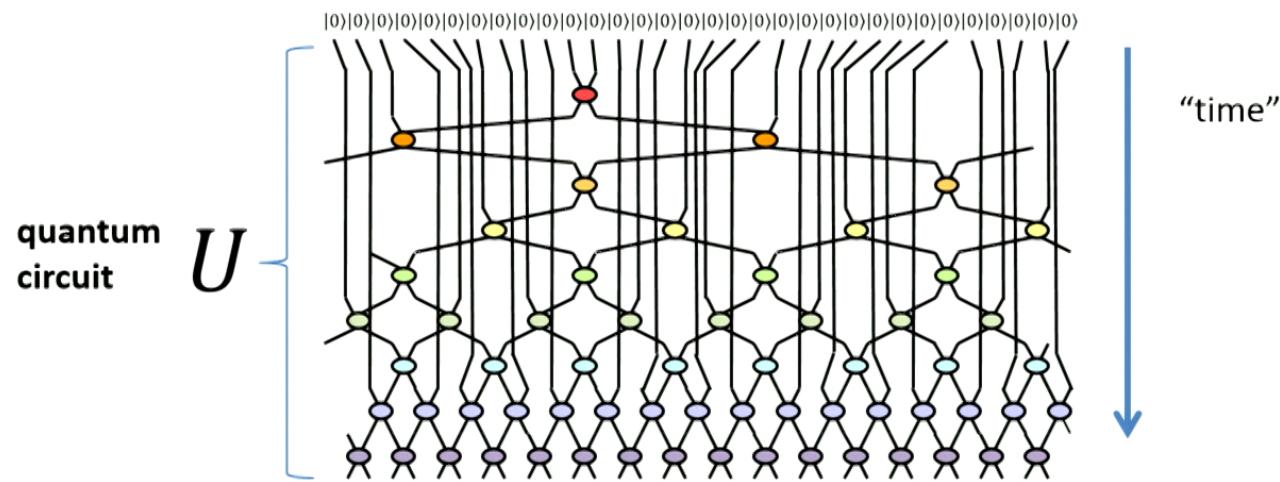


MERA as a quantum circuit



$$\text{ground state ansatz } |\Psi\rangle = U |0\rangle^{\otimes N}$$

MERA as a quantum circuit



$$\text{ground state ansatz } |\Psi\rangle = U |0\rangle^{\otimes N}$$

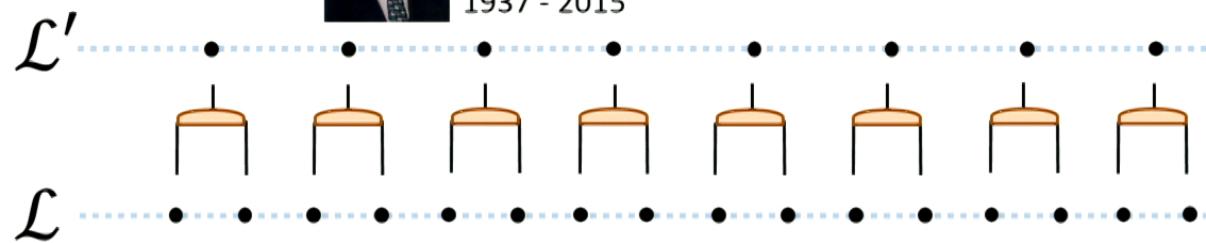
Entanglement introduced by gates at different “times” (= length scales)

MERA as a (real space) Renormalizatin Group transformation

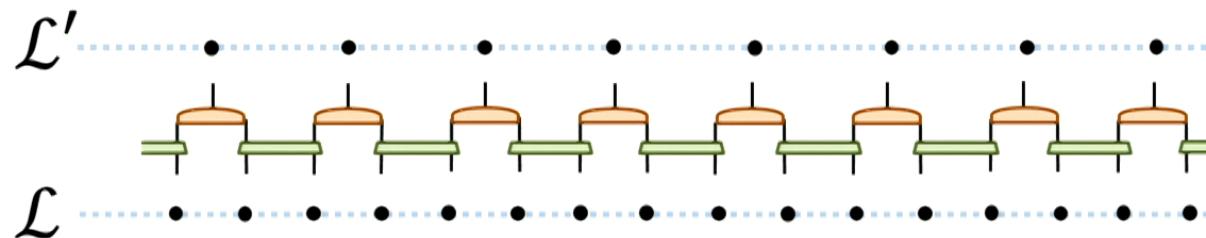
Spin blocking
(1966)



Leo Kadanoff
1937 - 2015



Entanglement renormalization (2005)

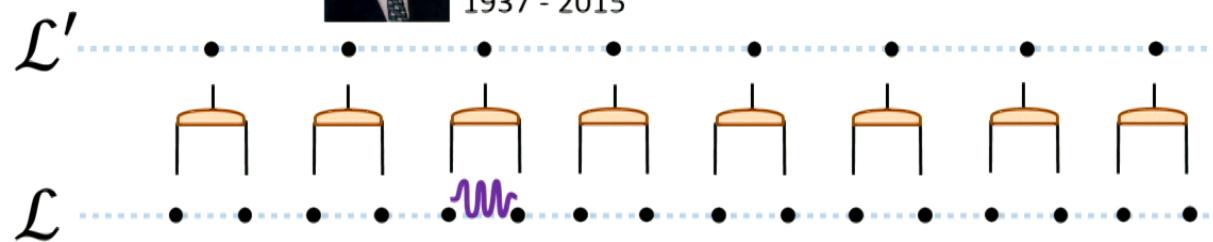


MERA as a (real space) Renormalizatin Group transformation

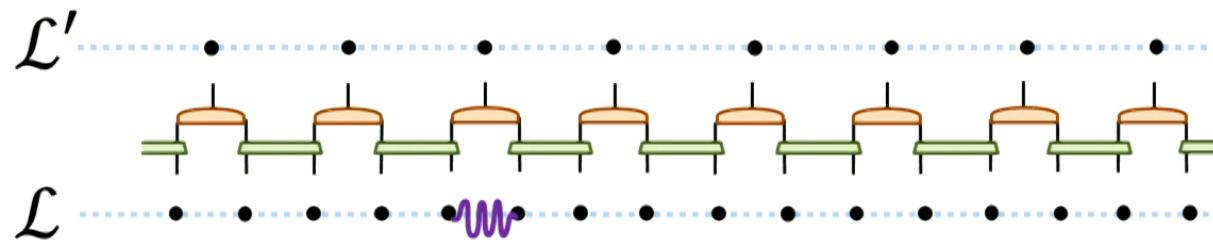
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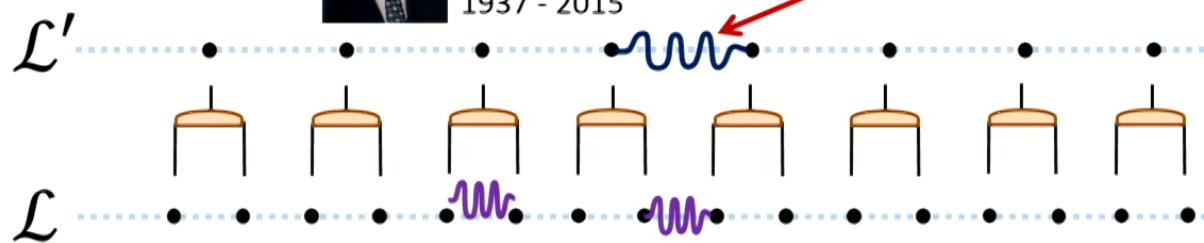
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Spin blocking
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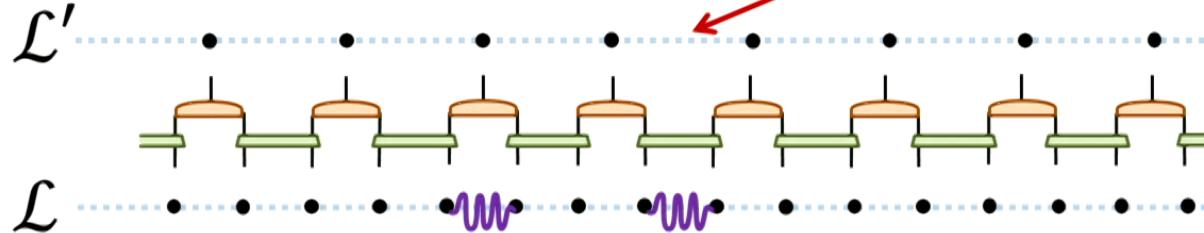
Leo Kadanoff
1937 - 2015

failure to remove
some short-range
entanglement !



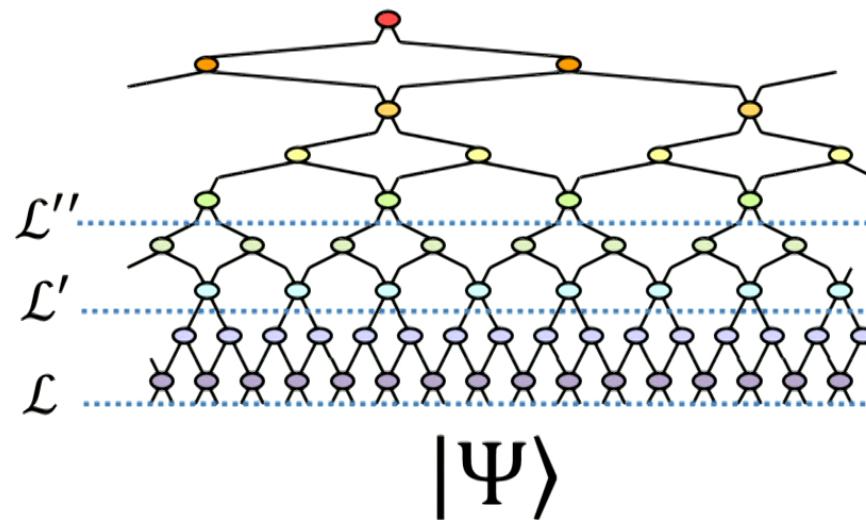
Entanglement renormalization (2005)

removal of *all*
short-range
entanglement



MERA as a (real space) Renormalizatin Group transformation

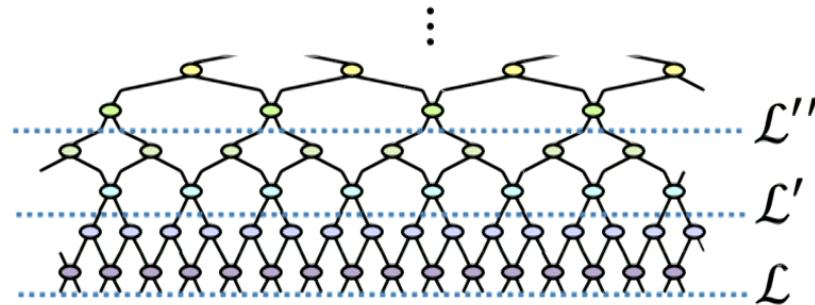
sequence of ground state wave-functions



$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$

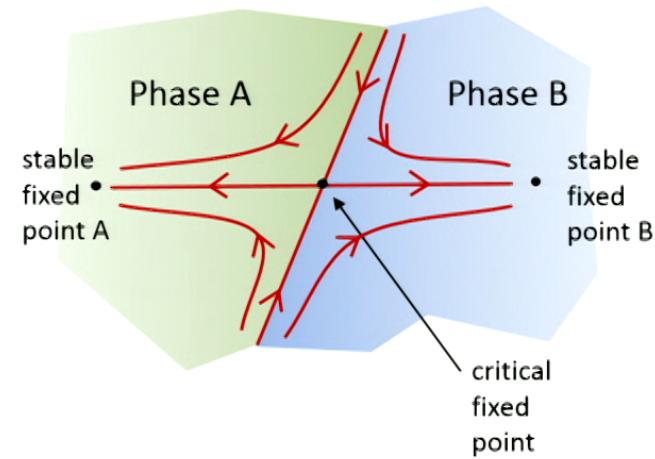
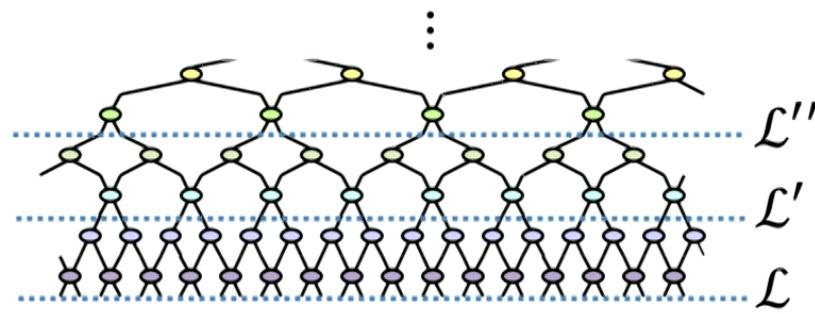
MERA defines an RG flow
in the space of wave-functions

$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$



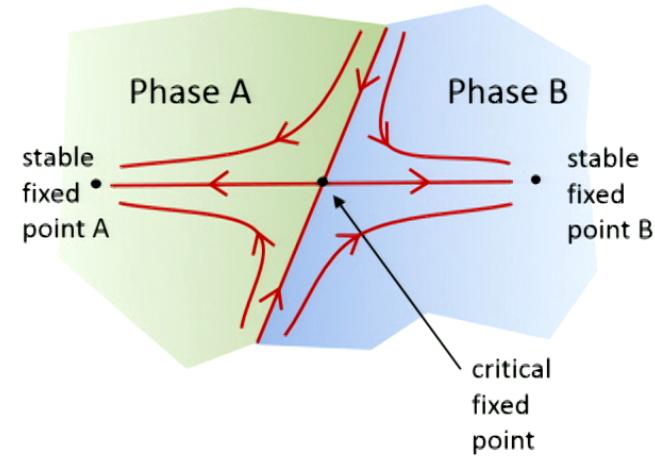
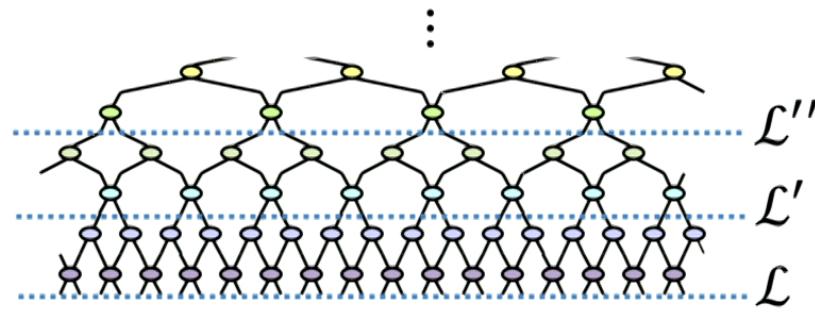
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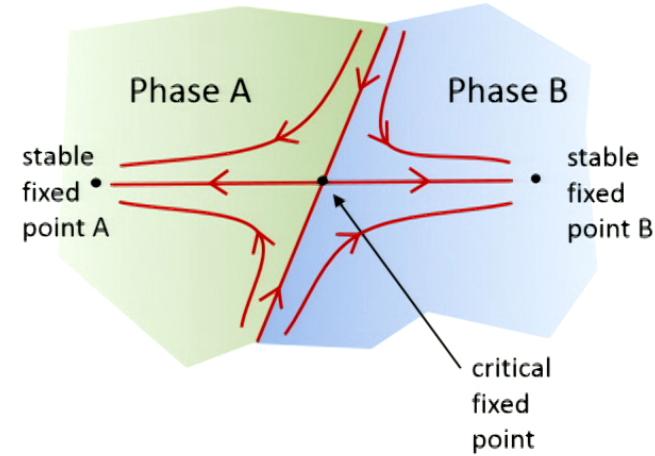
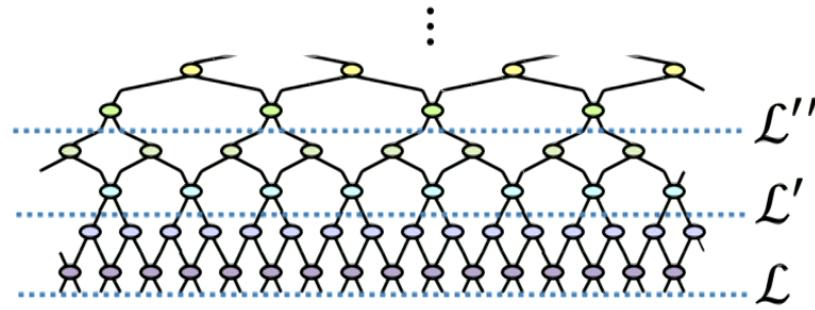
... and in the space of Hamiltonians

$$H \rightarrow H' \rightarrow H'' \rightarrow \dots$$



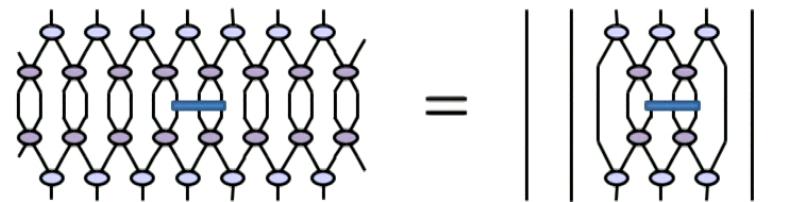
MERA defines an RG flow
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$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$



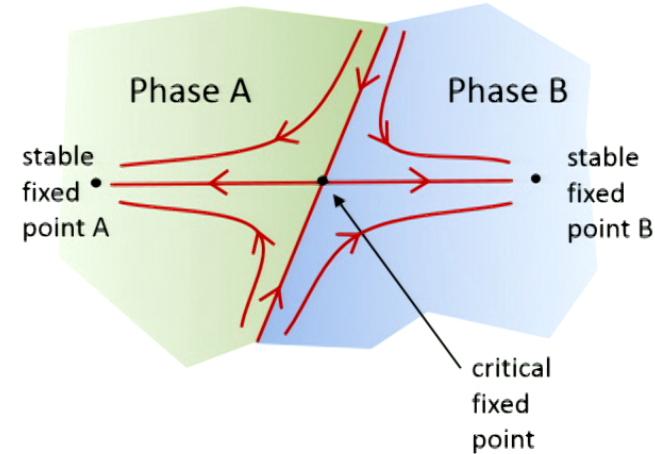
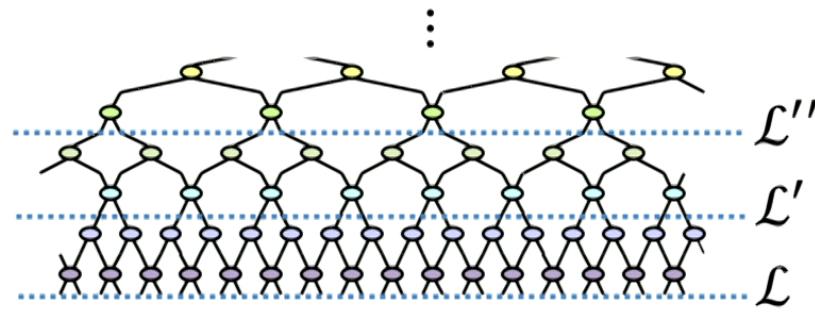
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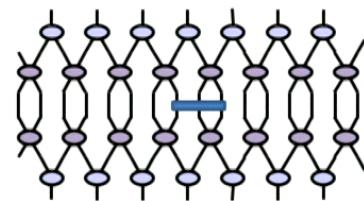
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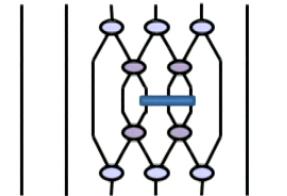


... and in the space of Hamiltonians

$$H \rightarrow H' \rightarrow H'' \rightarrow \dots$$



=



local operators
are mapped into
local operators !

Claim:

Entanglement renormalization
defines a *proper* scale transformation on the lattice

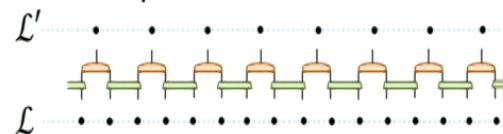
- Explicit scale invariance
at criticality !

input

1D quantum Hamiltonian on the lattice

- at a critical point

1 - optimization



2 - diagonalization



Claim:

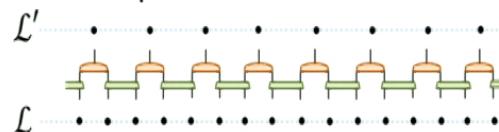
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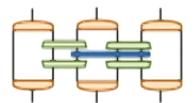
input

1D quantum Hamiltonian on the lattice
• at a critical point

1 - optimization



2 - diagonalization



→ scaling
operators

output

**Numerical extraction of conformal data
of underlying CFT:**

- central charge c
- scaling dimensions $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ and conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- OPE coefficients $C_{\alpha\beta\gamma}$

Claim:

Entanglement renormalization
defines a *proper* scale transformation on the lattice

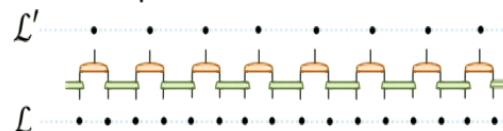
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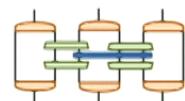
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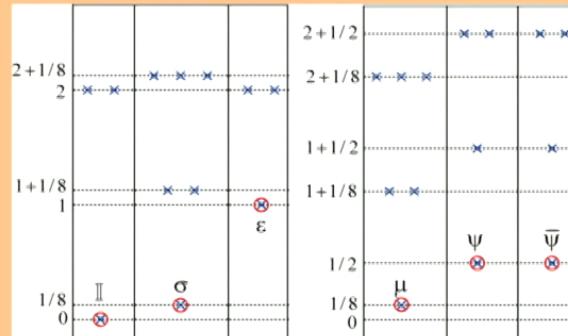


scaling
operators

e.g. critical Ising model

(approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$$\Delta_\sigma \approx 0.124997$$

$$\Delta_\varepsilon \approx 0.99993$$

$$\Delta_\mu \approx 0.125002$$

$$\Delta_\psi \approx 0.500001$$

$$\Delta_{\bar{\psi}} \approx 0.500001$$