Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 11

Date: Feb 14, 2017 09:00 AM

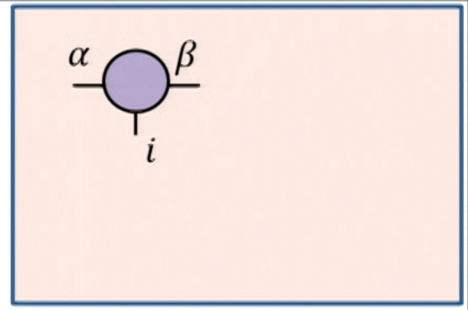
URL: http://pirsa.org/17020047

Abstract:

Summary of matrix product state (MPS)

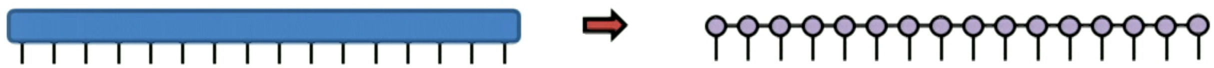
$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

Summary of matrix product state (MPS)



$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

$\Psi_{i_1 i_2 \cdots i_N}$



2^N

Summary of matrix product state (MPS)

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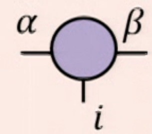
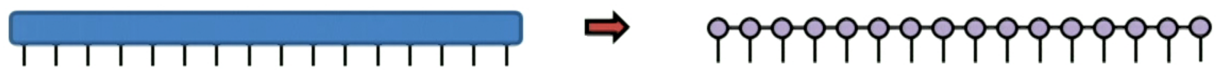


$|\alpha| = |\beta| = \chi$

$|i| = d$

$O(d\chi^2)$ parameters

$\Psi_{i_1 i_2 \cdots i_N}$



2^N

$O(Nd\chi^2)$

Efficient representation!

Summary of matrix product state (MPS)

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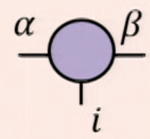

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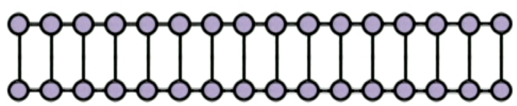
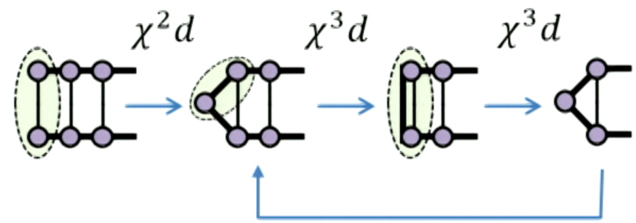
Efficient representation!

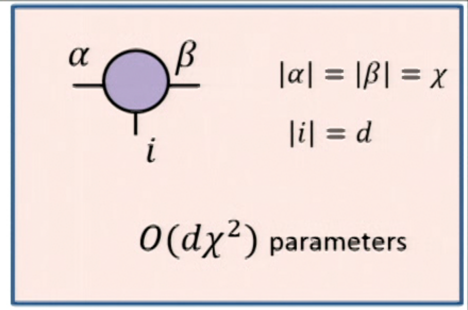
Efficient computation?

$\langle\Psi|\Psi\rangle =$





Summary of matrix product state (MPS)



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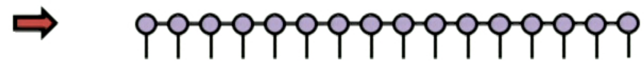

$O(N d \chi^2)$

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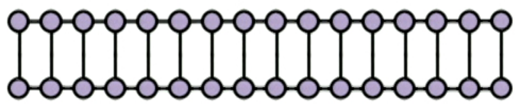
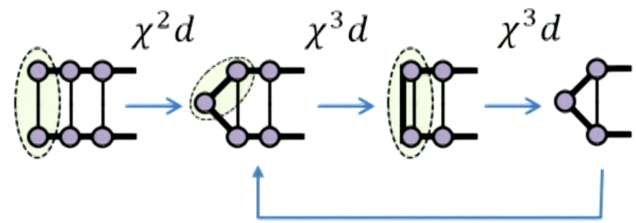
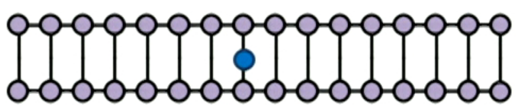
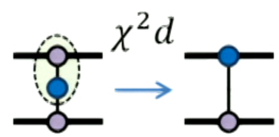




$O(N d \chi^3)$!!!

$\langle\Psi|\hat{o}|\Psi\rangle =$

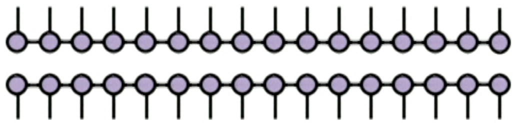




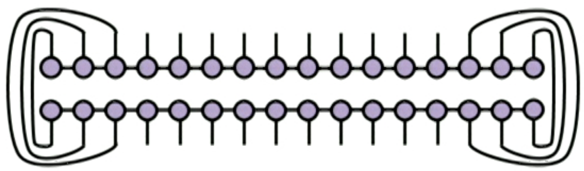
MPS physics? Structural properties:

(a) entanglement entropy
(b) correlations

- entanglement entropy

$|\Psi\rangle\langle\Psi| =$ 

$\rho_A = tr_B \, |\Psi\rangle\langle\Psi|$

$\rho_A =$ 

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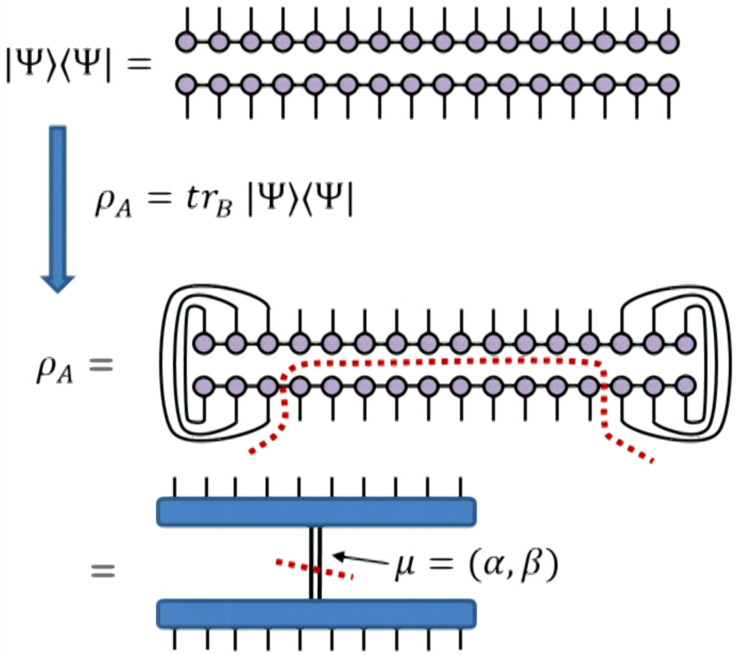

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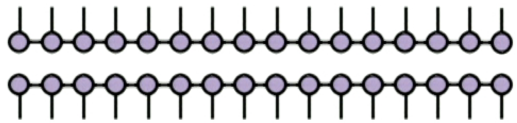

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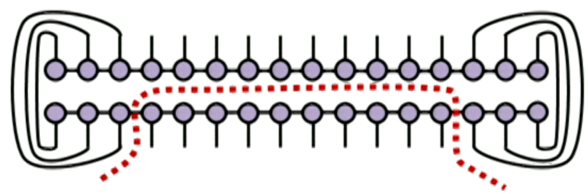

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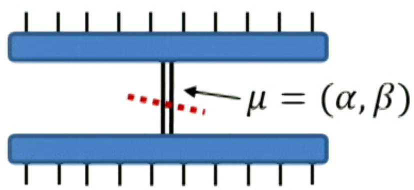

$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

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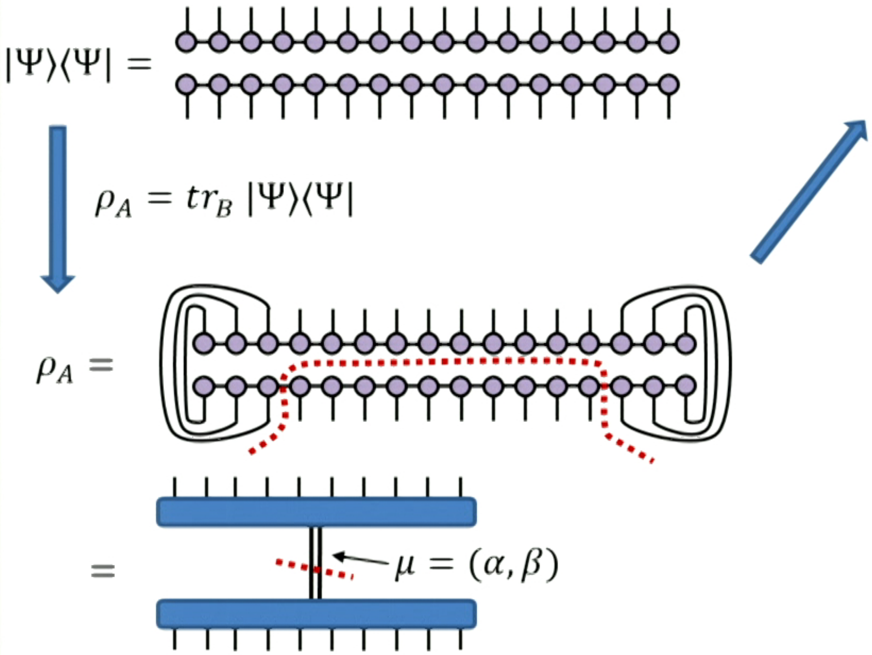

$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

$$S(\rho_A) = S(p_1, p_2, \cdots, p_{\chi^2})$$

$$\leq S\left(\frac{1}{\chi^2}, \frac{1}{\chi^2}, \cdots, \frac{1}{\chi^2}\right)$$

$$= \log \chi^2 = 2 \log \chi$$

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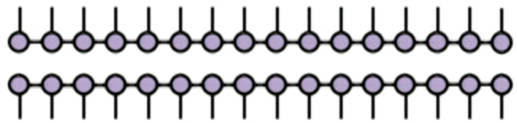

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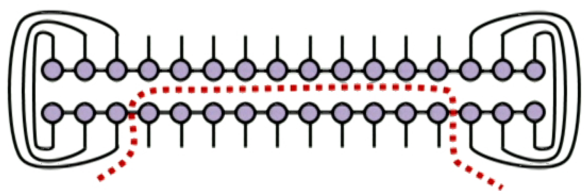

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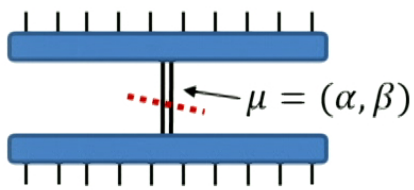

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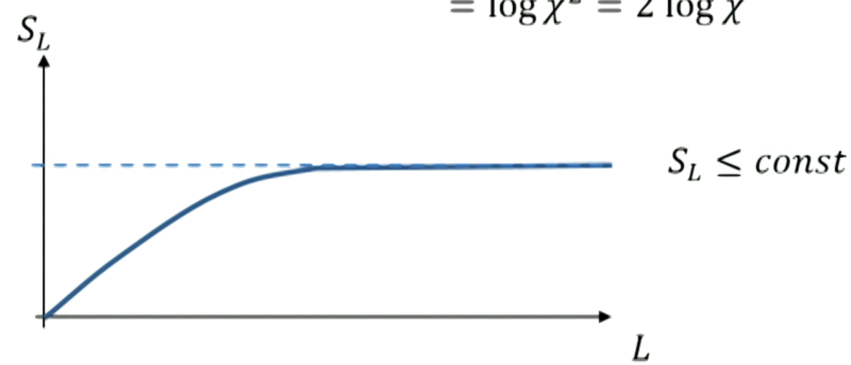
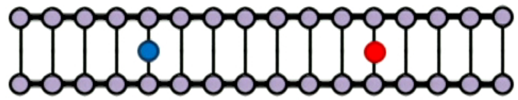
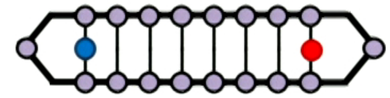
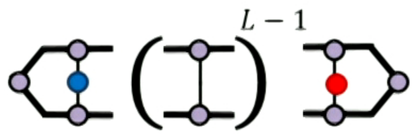


- correlations

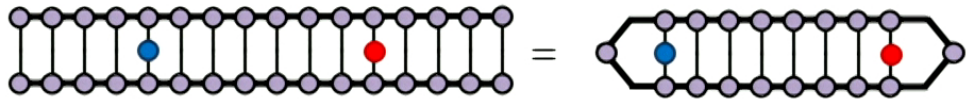
$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle =$  $=$ 

$=$  $\approx a\,\lambda^L = a e^{-L/\xi}$

$$\xi \equiv -\frac{1}{\log\lambda}$$

- correlations

$$\langle\Psi|\hat{o}(0)\,\hat{o}(L)|\Psi\rangle =$$



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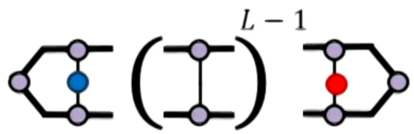

$$\approx a\,\lambda^{L} = a e^{-L/\xi}$$

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Structural properties of MPS

correlations	$C(L) \approx e^{-L/\xi}$
entanglement	$S_L \leq 2\log\chi$

- correlations

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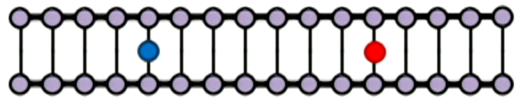

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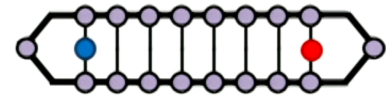

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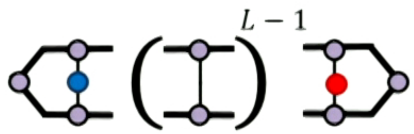

$\approx a\,\lambda^{L} = a e^{-L/\xi}$

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Structural properties of MPS

correlations	$C(L) \approx e^{-L/\xi}$
entanglement	$S_L \leq 2 \log \chi$

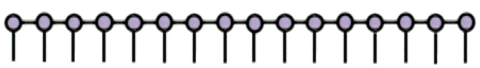
match with
ground states of 1D
gapped Hamiltonians

MERA: definition

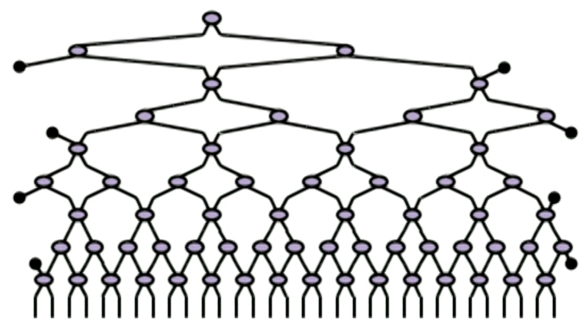
$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

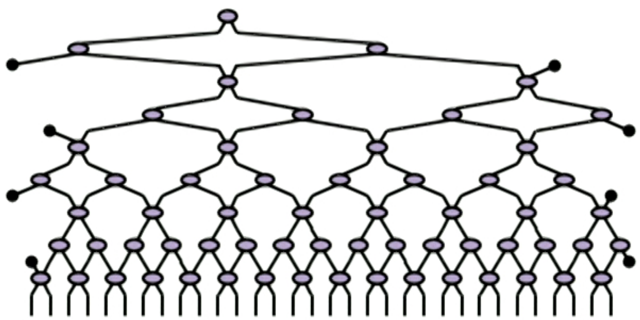
Matrix product state
(MPS)



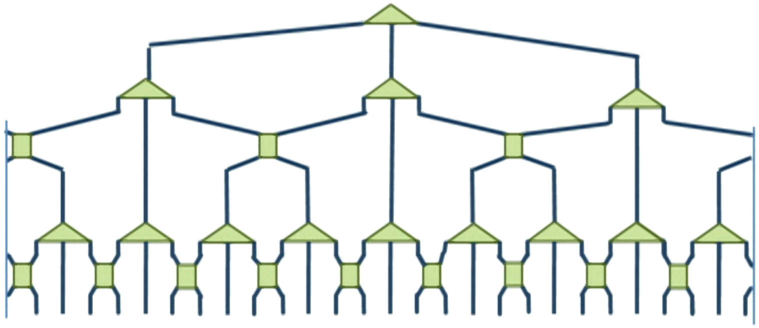
Multi-scale entanglement
renormalization ansatz
(MERA)



MERA



also MERA !

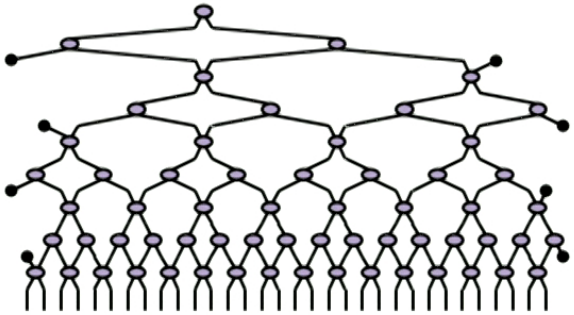


Efficient specification?

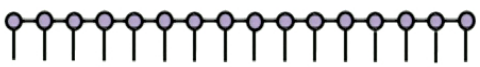
$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

Multi-scale entanglement renormalization ansatz (MERA)



Matrix product state (MPS)



N spins

Efficient specification?

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d^N complex numbers

Matrix product state
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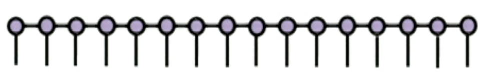

N spins $\Rightarrow$ N tensors
$\Rightarrow O(N)$ parameters

Multi-scale entanglement
renormalization ansatz
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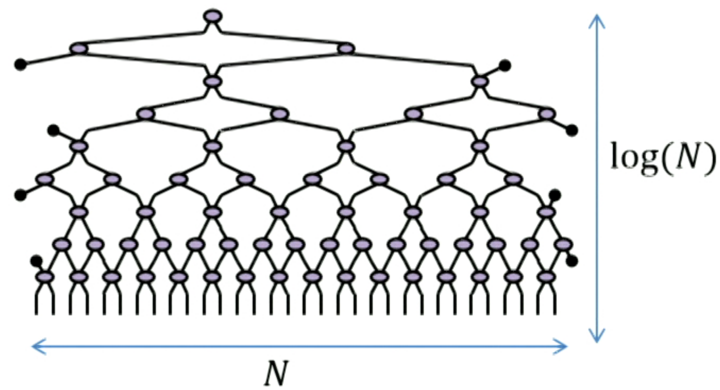
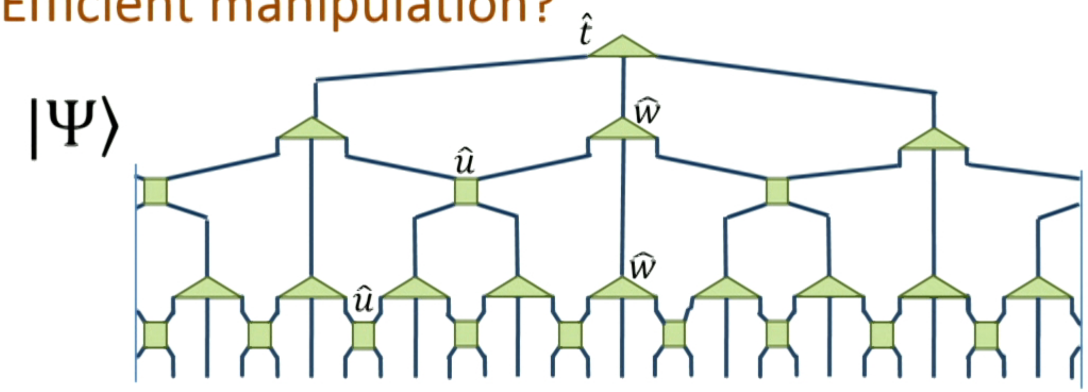


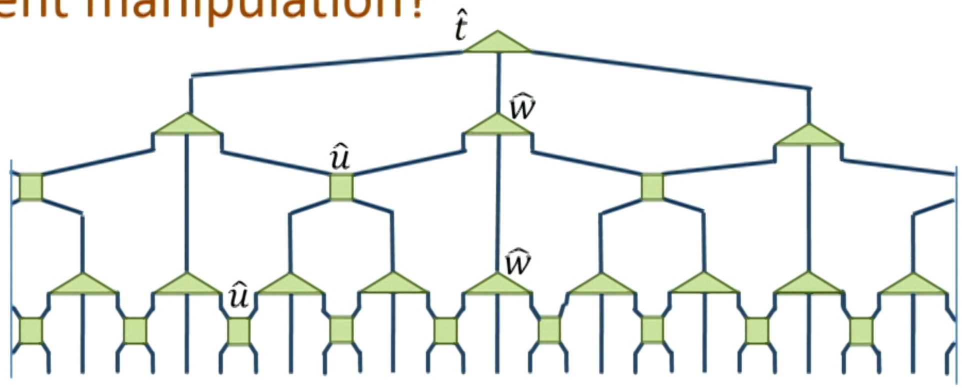
N spins $\Rightarrow N\log(N)$ tensors ?

Efficient manipulation?



Efficient manipulation?

$|\Psi\rangle$



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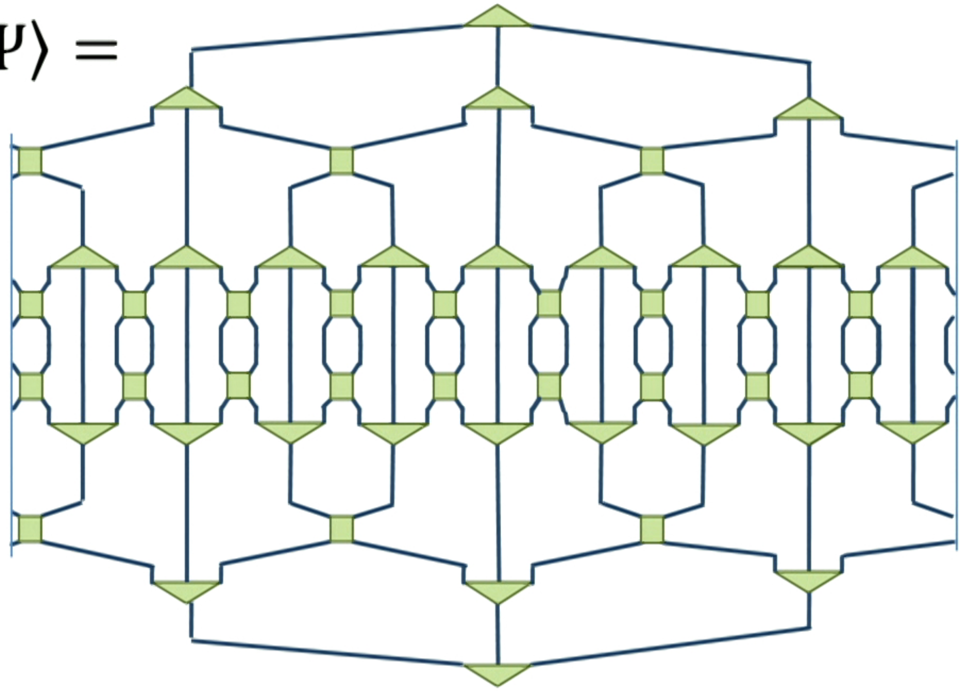
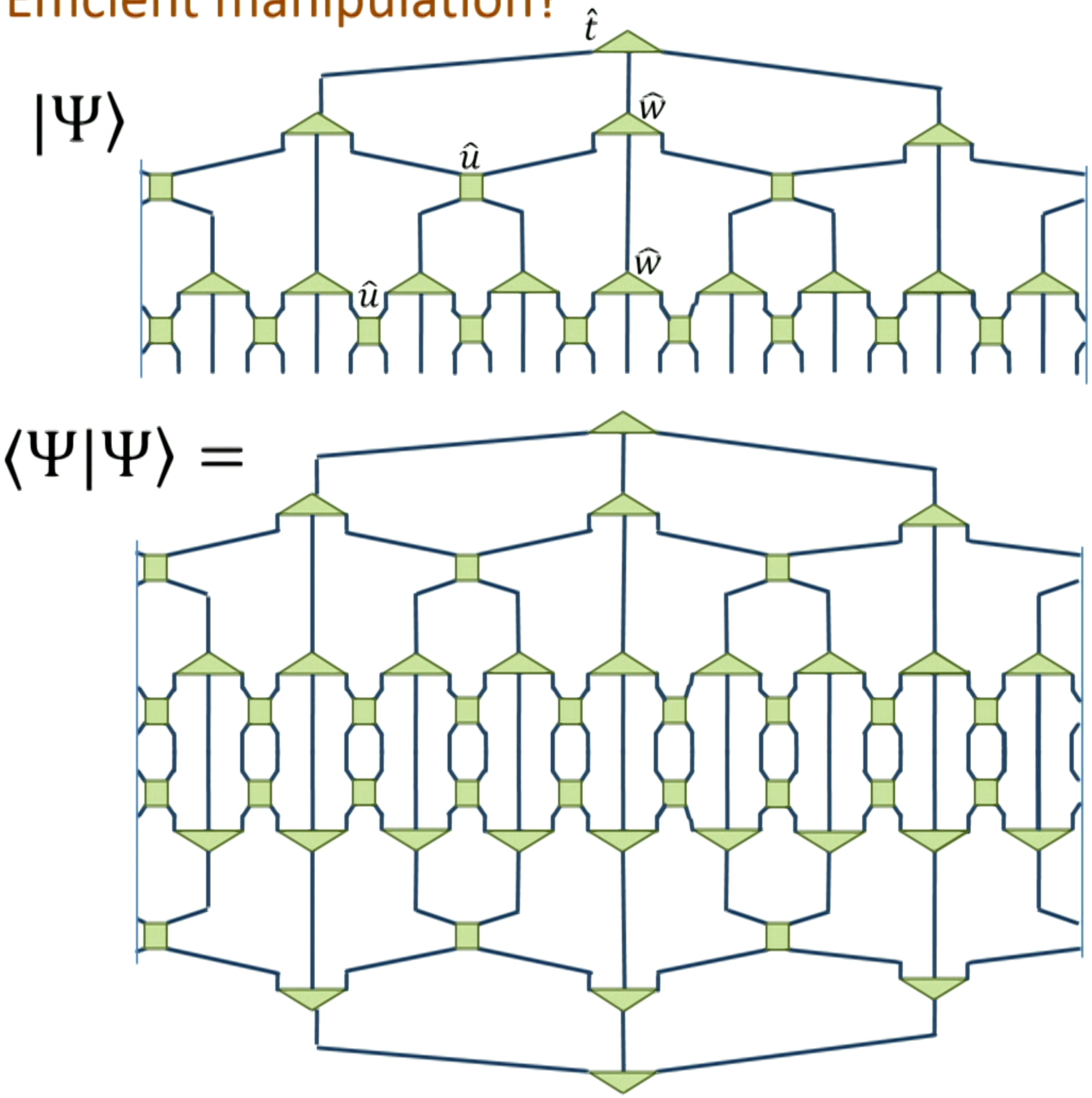
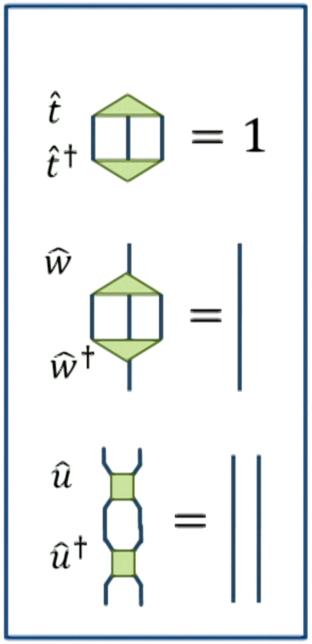


Efficient manipulation?

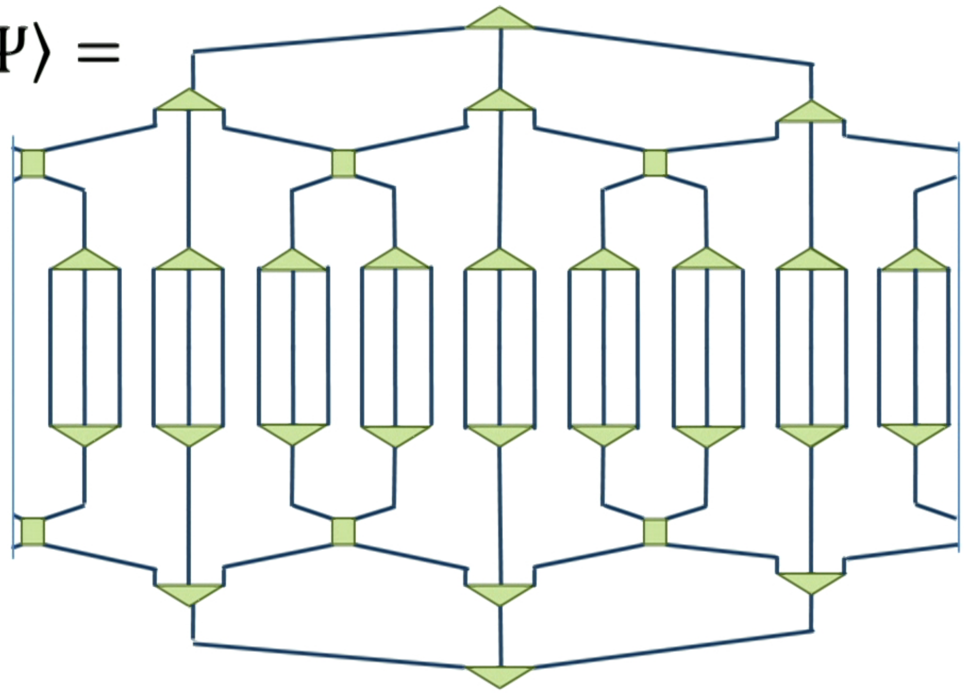


isometric tensors!



Efficient manipulation?

$$\langle \Psi | \Psi \rangle =$$



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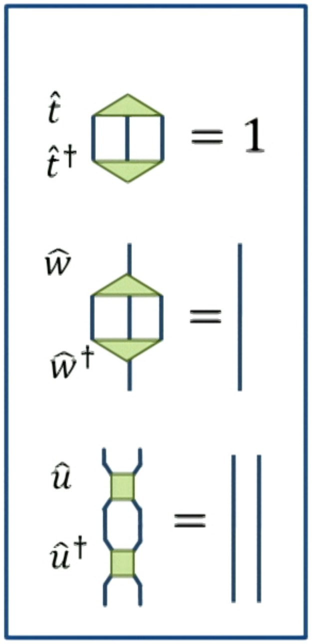
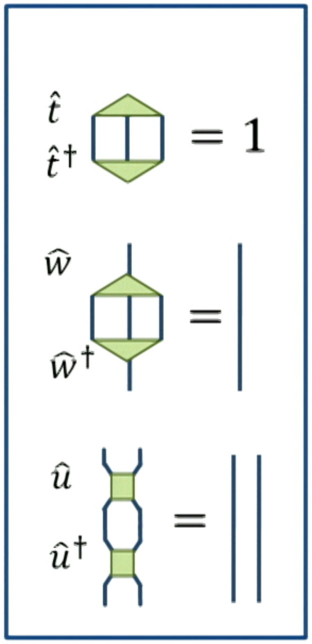


Efficient manipulation?

isometric tensors!

$$\langle \Psi | \Psi \rangle = 1$$



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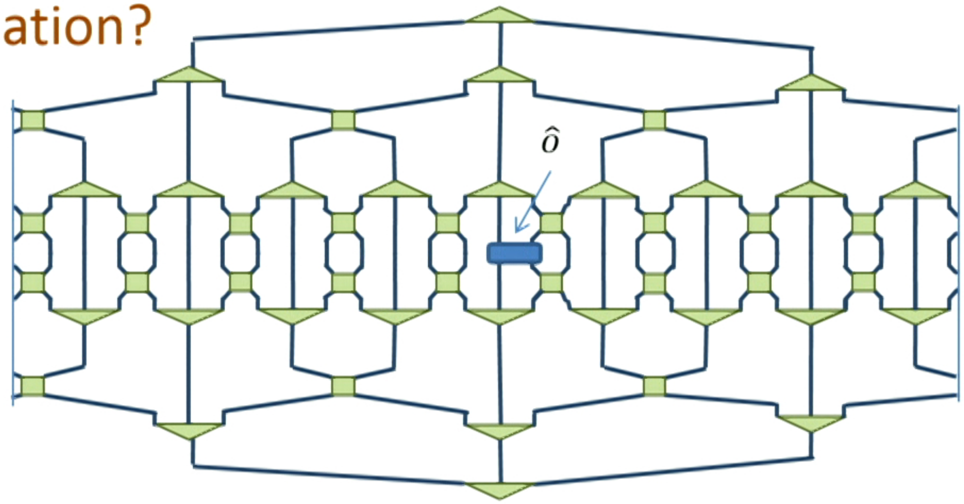
$\langle\Psi|\hat{o}|\Psi\rangle =$ 

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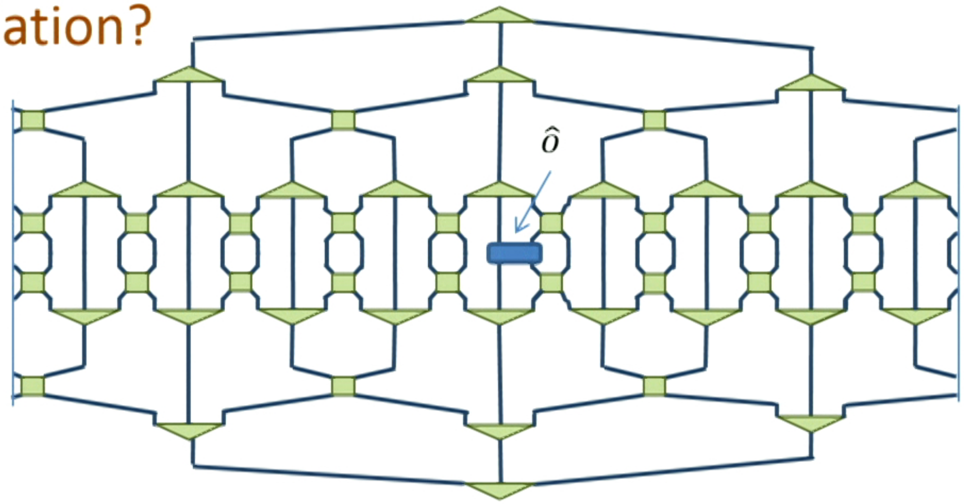

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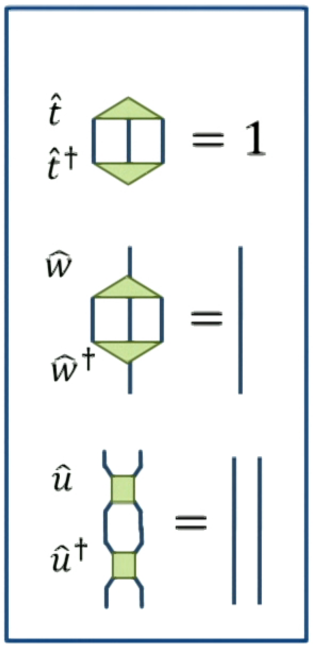

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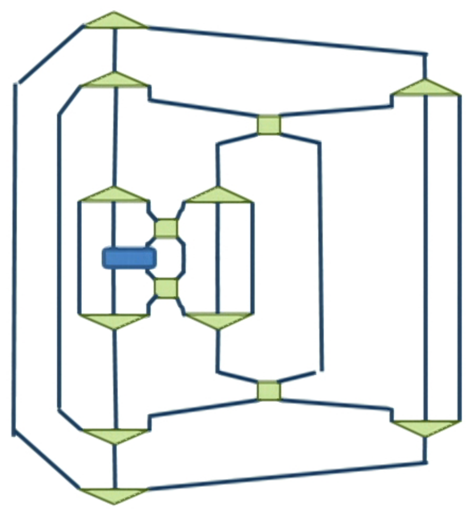

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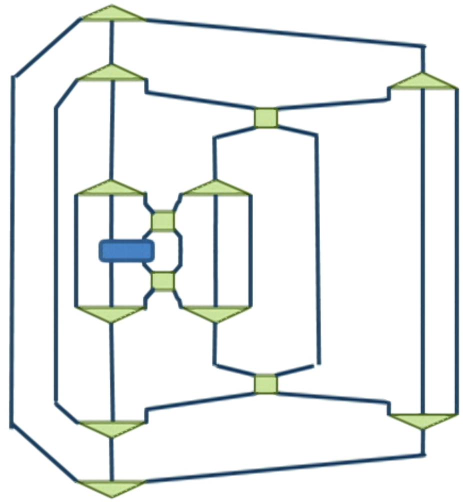

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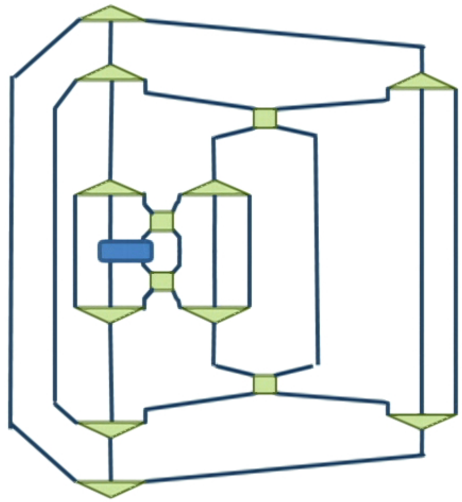

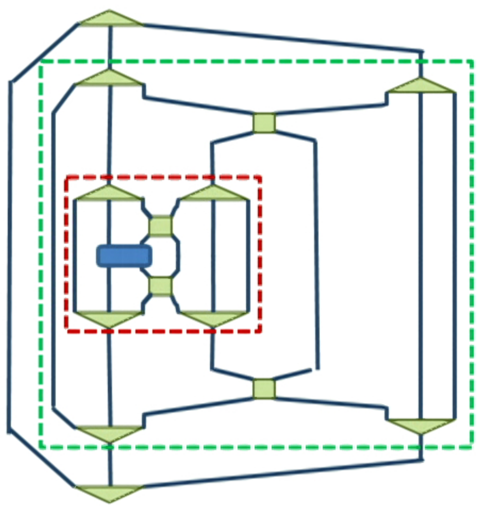

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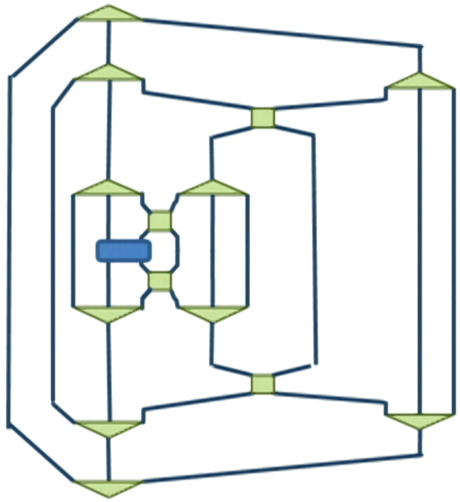
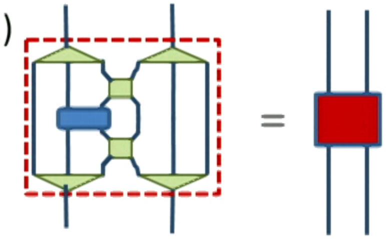
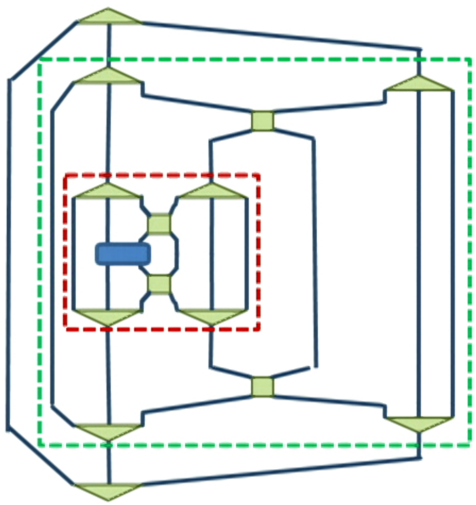


(1)





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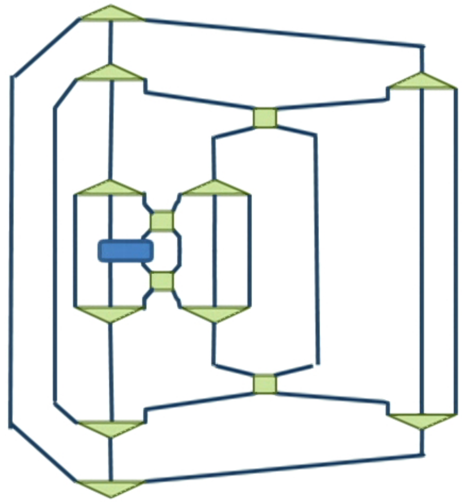
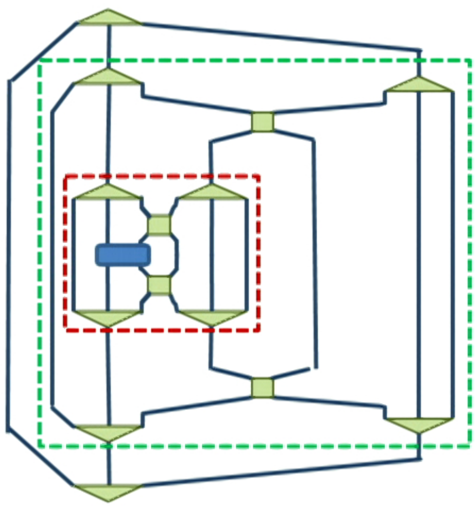
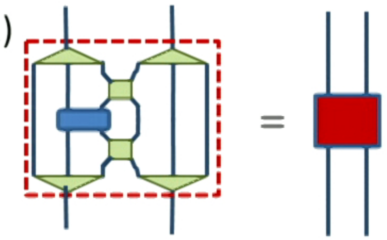
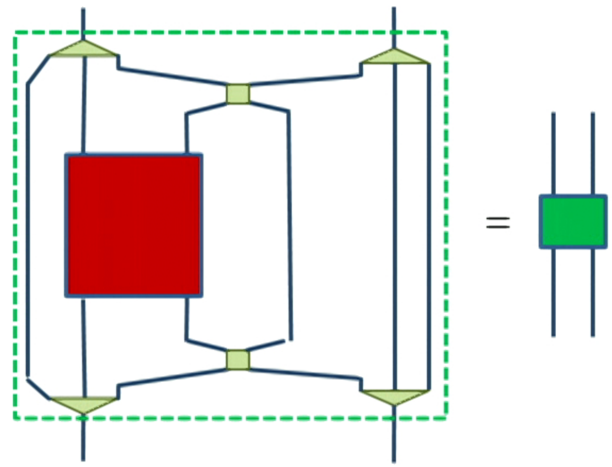


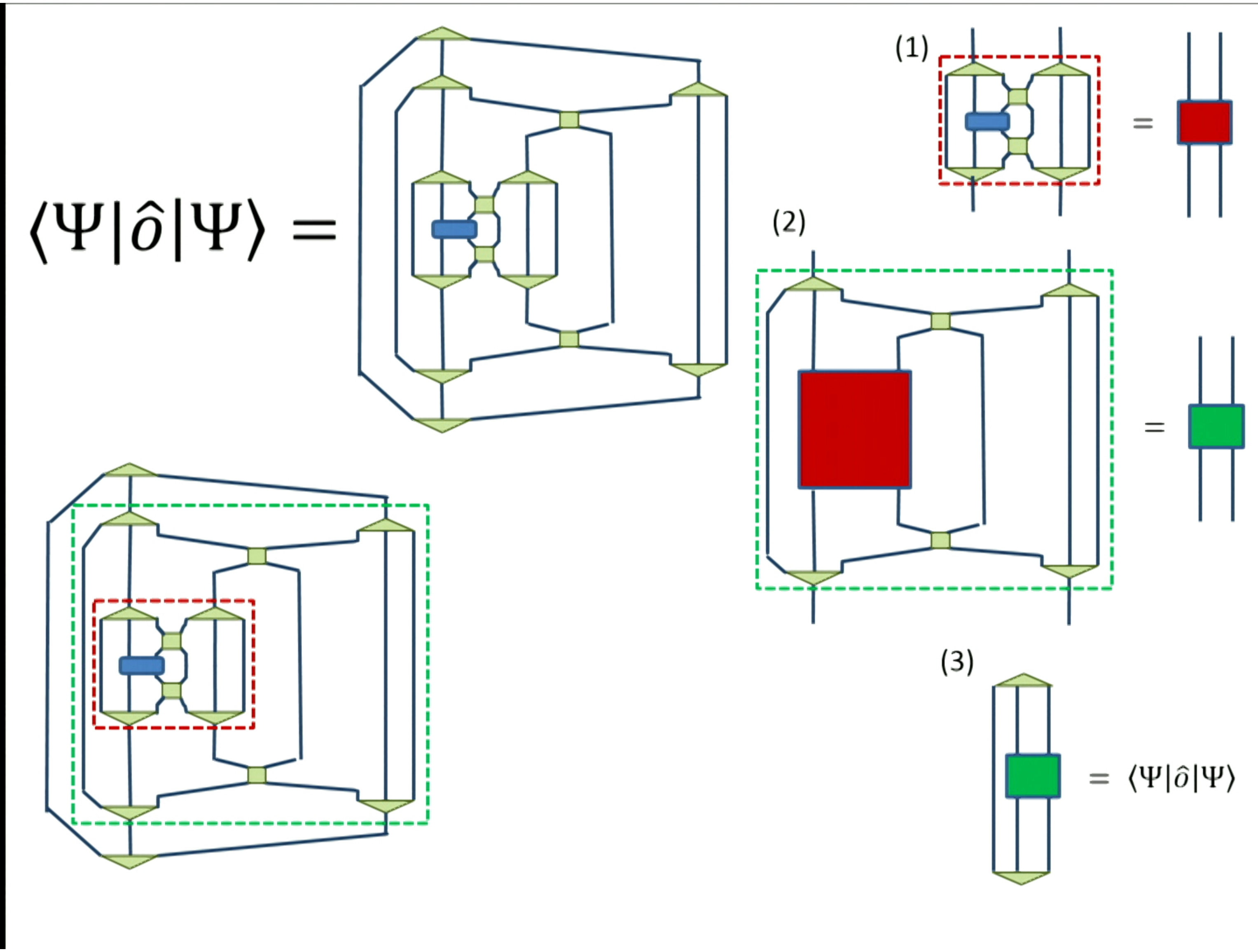


(1)

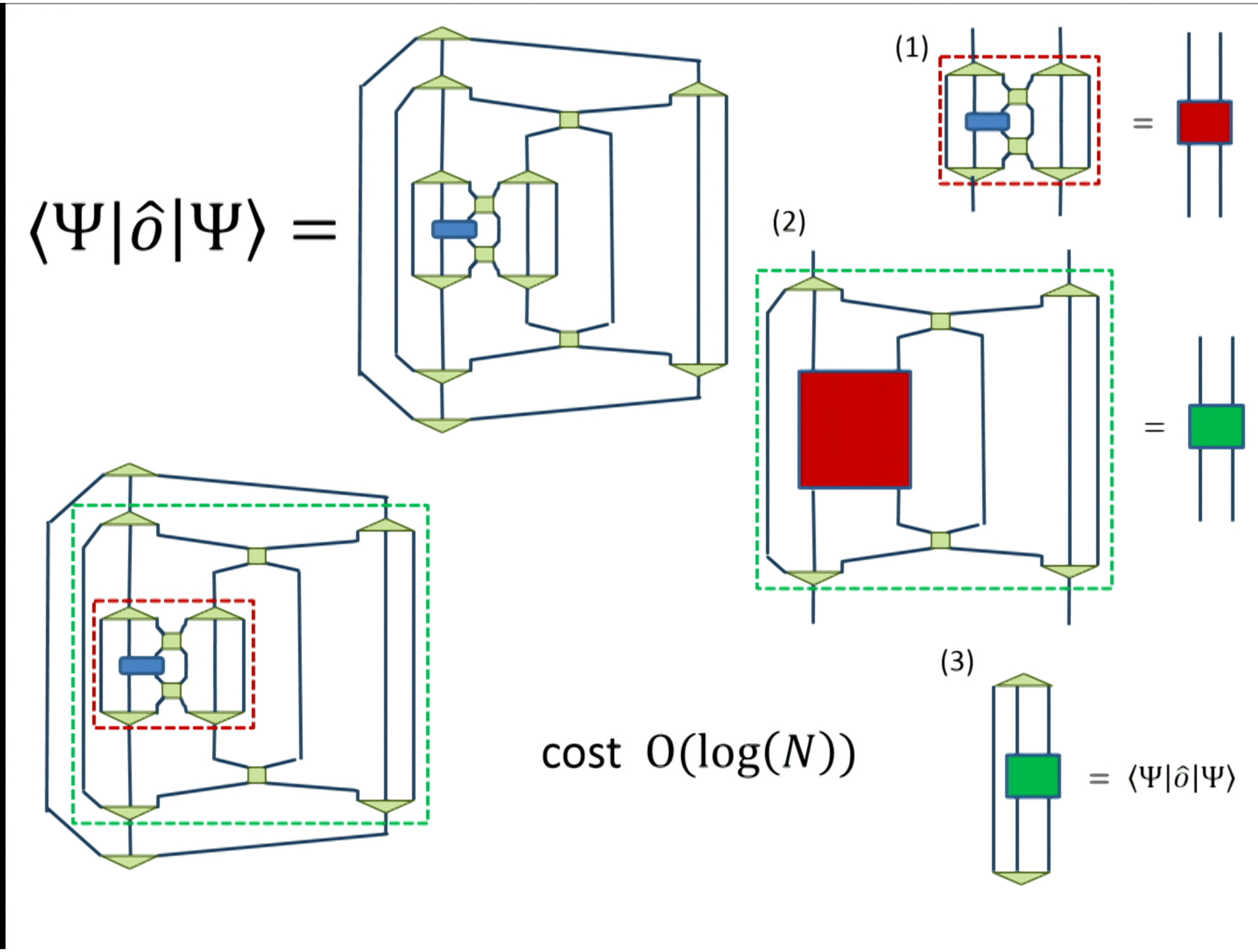


(2)





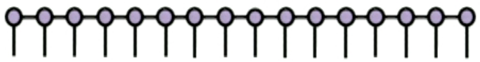
$\langle\Psi|\hat{o}|\Psi\rangle =$
(1)
=
(2)
=
(3)
$= \langle\Psi|\hat{o}|\Psi\rangle$

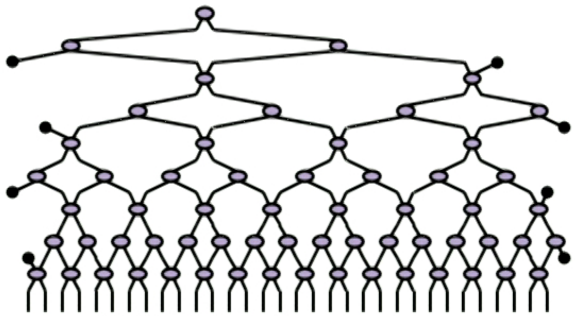

$\langle\Psi|\hat{o}|\Psi\rangle =$
(1)
=
(2)
=
(3)
$= \langle\Psi|\hat{o}|\Psi\rangle$
cost $O(\log(N))$

Structural properties

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

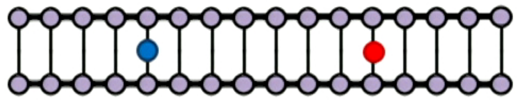
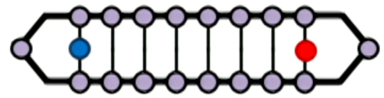


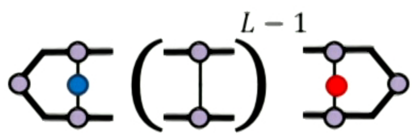


- Decay of correlations
- Scaling of entanglement

$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

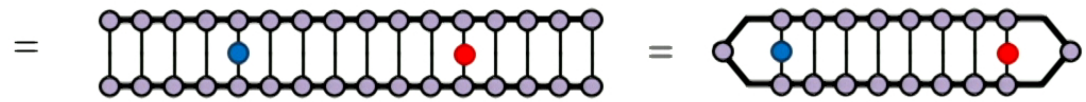
MPS

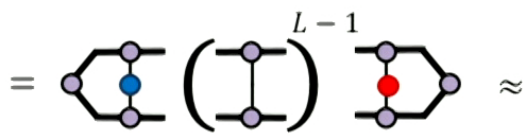
$=$  $=$ 

$=$  $\approx\ a\lambda^L = ae^{-L/\xi}$

$\xi \equiv -\dfrac{1}{\log \lambda}$

$\langle \Psi | \hat{o}(0)\ \hat{o}(L) | \Psi \rangle$ MPS





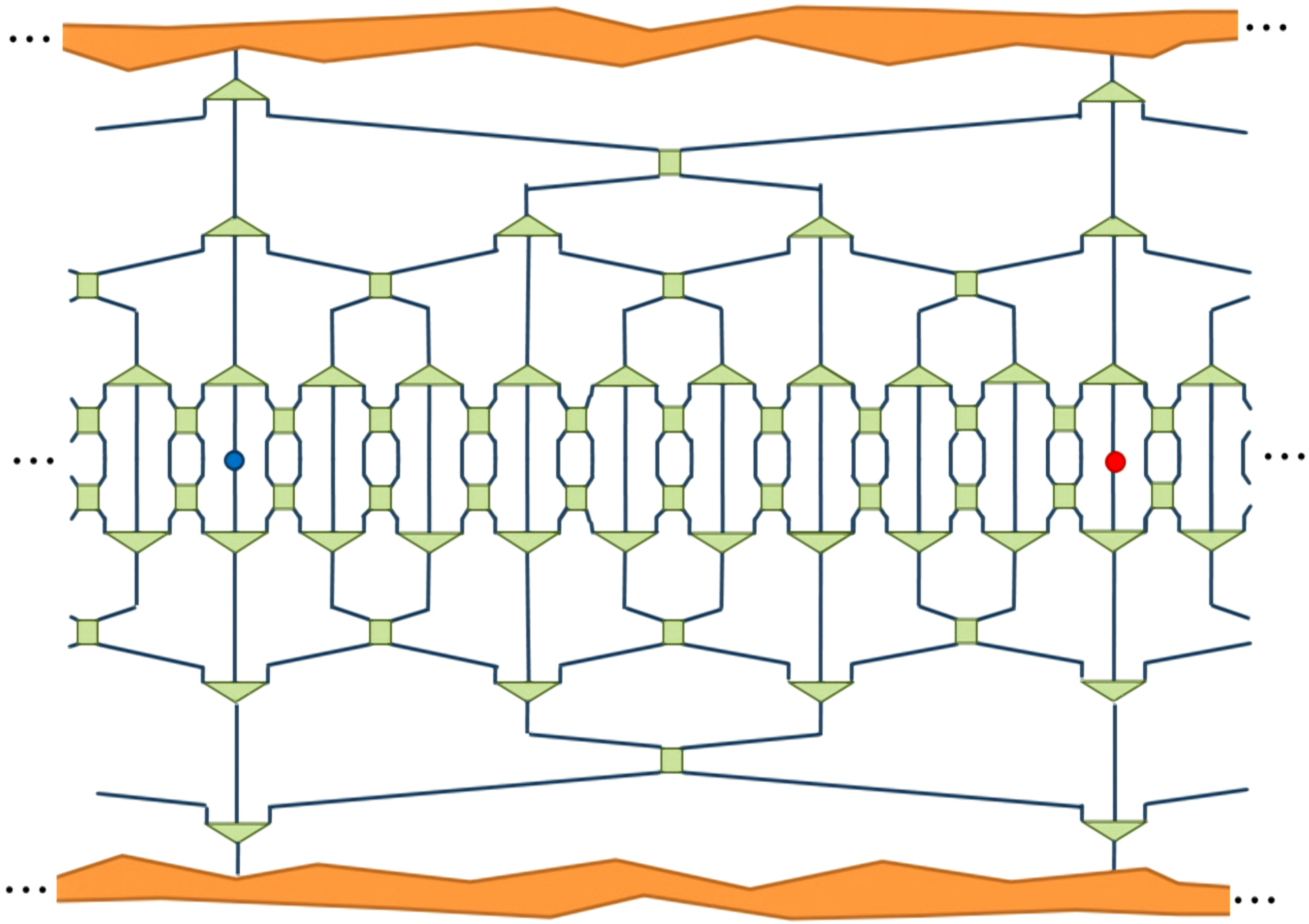
$$\approx a\lambda^L = a e^{-L/\xi}$$

$$\xi \equiv -\frac{1}{\log \lambda}$$

$\Rightarrow$ exponential decay of correlations

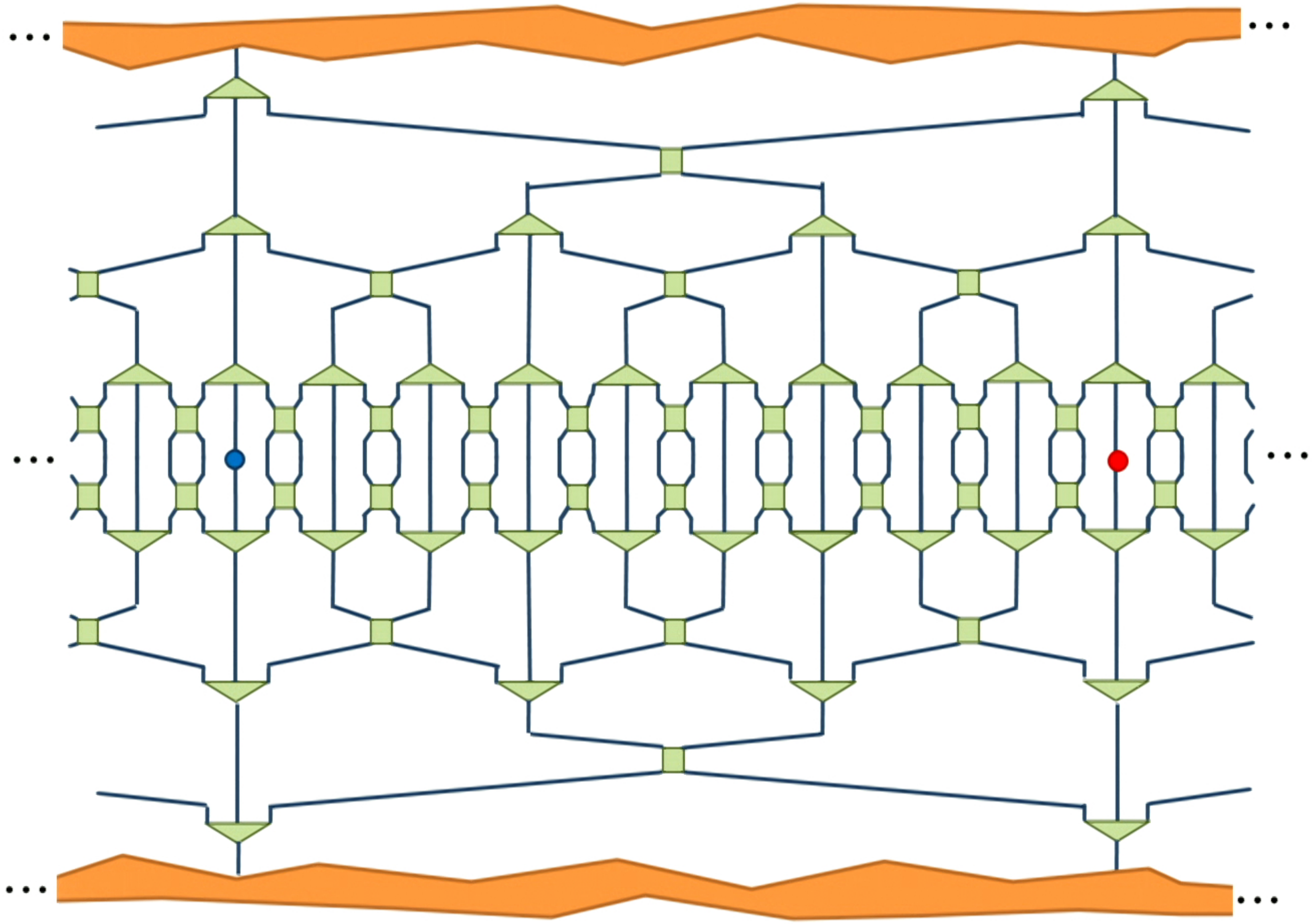
MERA

$$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle\ =$$



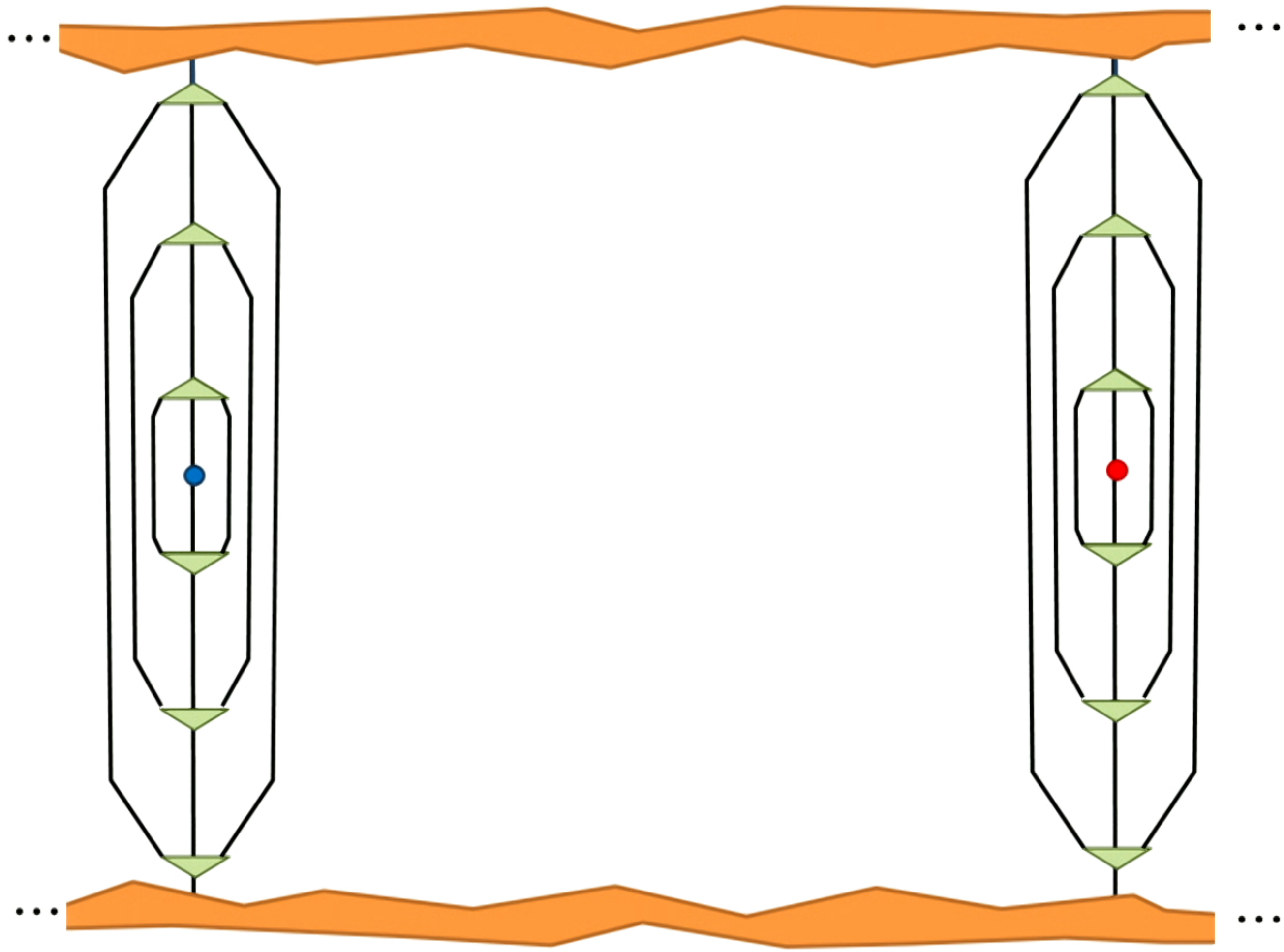
MERA

$$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle\ =$$



$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

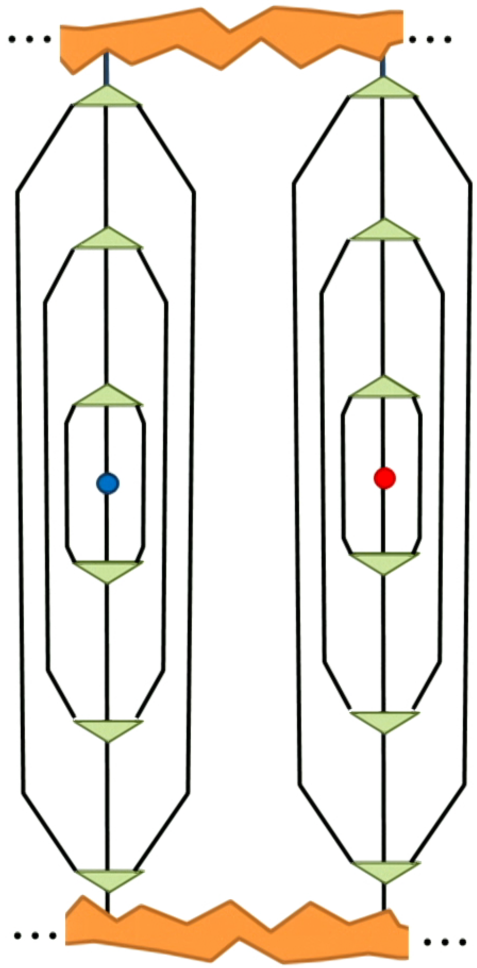
$=$



$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

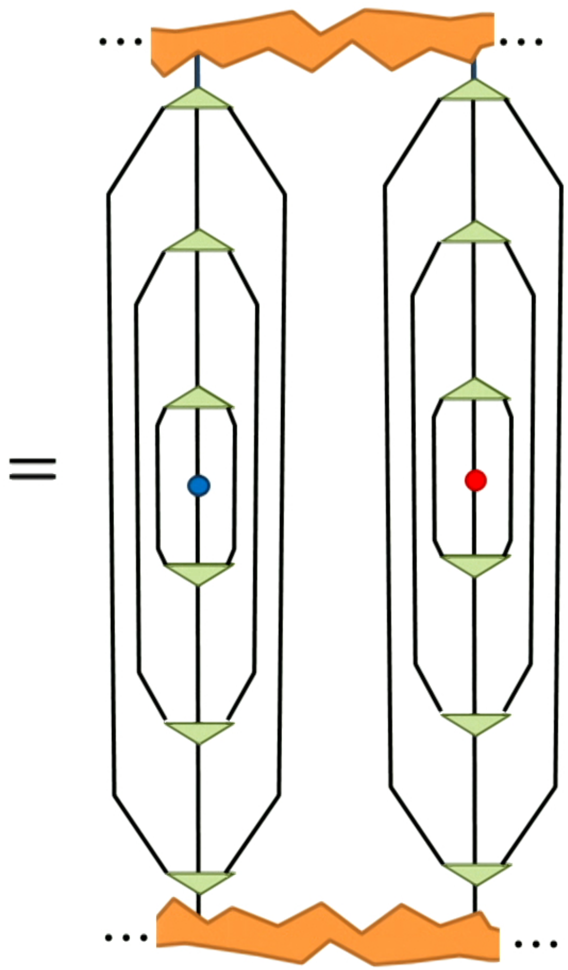
MERA

$=$



$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

MERA



$$\approx (\lambda)^{\log_3(L)}(\lambda)^{\log_3(L)}$$

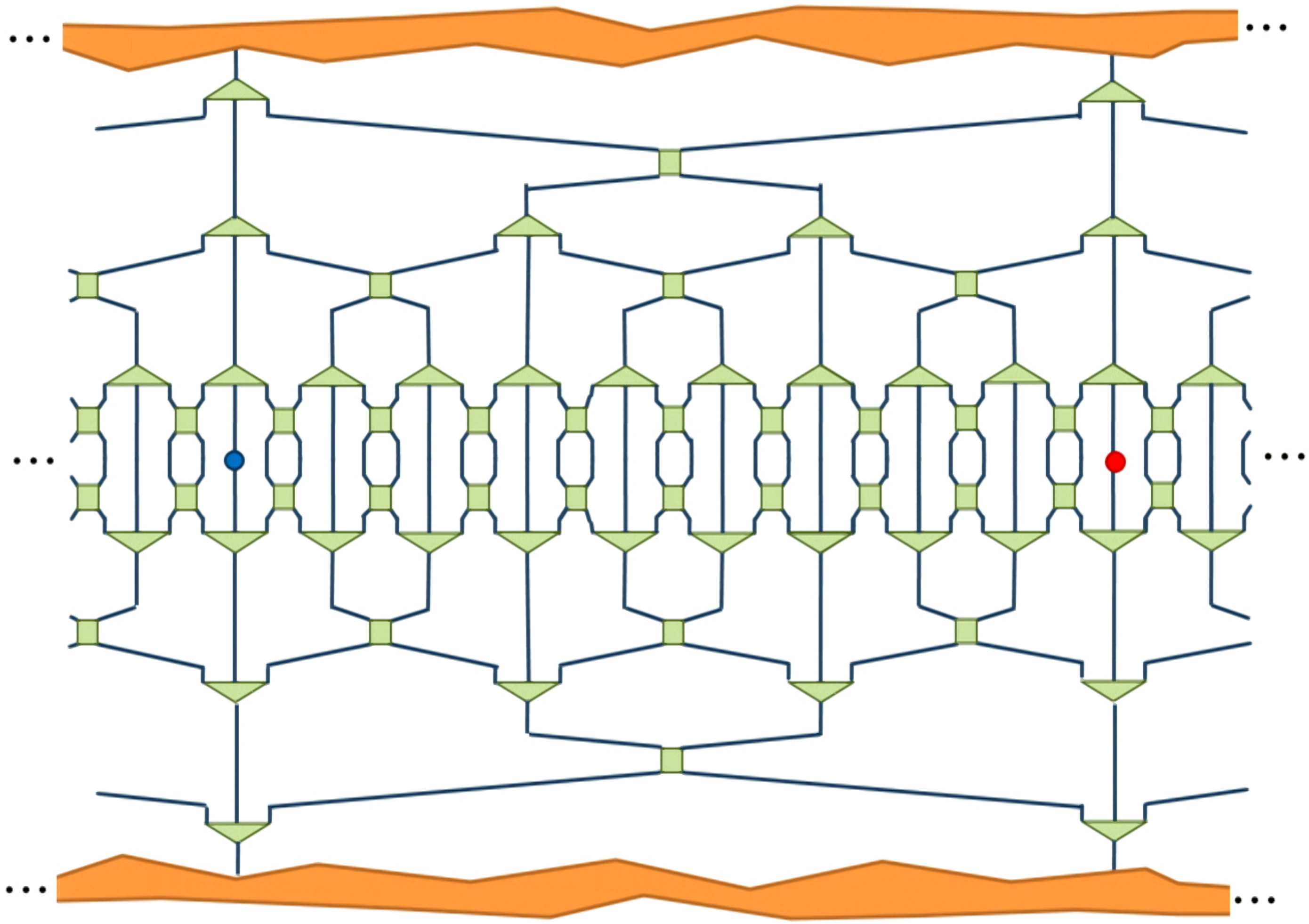
$$= \lambda^{2\log_3(L)} = L^{2\log_3(\lambda)} = L^{-p}$$

$$x^{\log_3(y)} = y^{\log_3(x)} \qquad p \equiv -2\log_3(\lambda)$$

$\Rightarrow$ polynomial decay of correlations

MERA

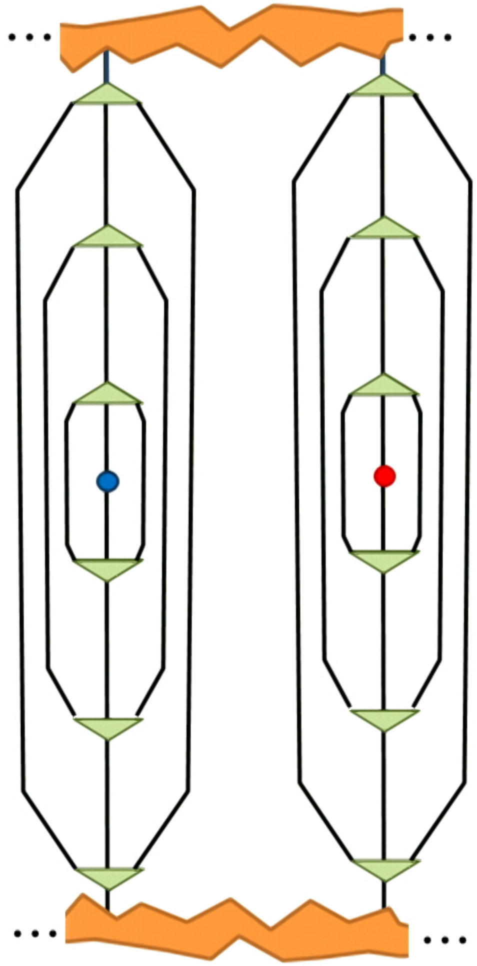
$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle \ =$



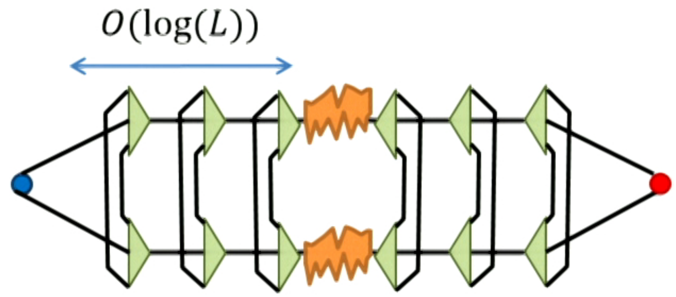
$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

MERA

$=$



$=$



$\approx (\lambda)^{\log_3(L)}(\lambda)^{\log_3(L)}$

$= \lambda^{2\log_3(L)} = L^{2\log_3(\lambda)} = L^{-p}$

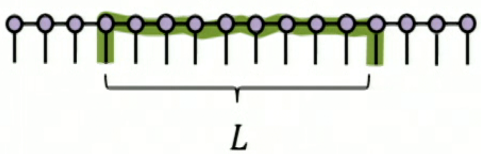
$x^{\log_3(y)} = y^{\log_3(x)}$ $\quad$ $p \equiv -2\log_3(\lambda)$

$\Rightarrow$ polynomial decay of correlations

Correlations: summary and interpretation

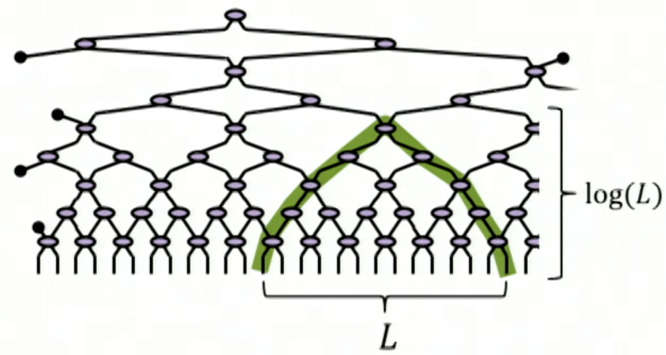
matrix product state

(MPS)



multi-scale entanglement renormalization ansatz

(MERA)



structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$$

exponential

structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx L^{-p}$$

power-law

Correlations: summary and interpretation

matrix product state

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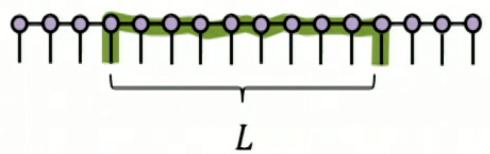

multi-scale entanglement renormalization ansatz

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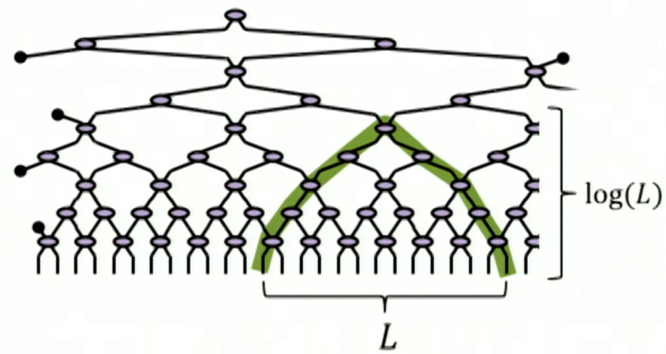

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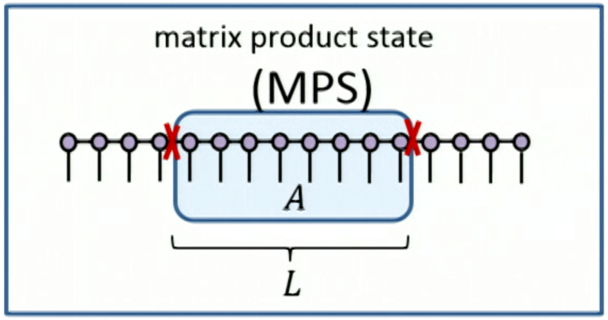
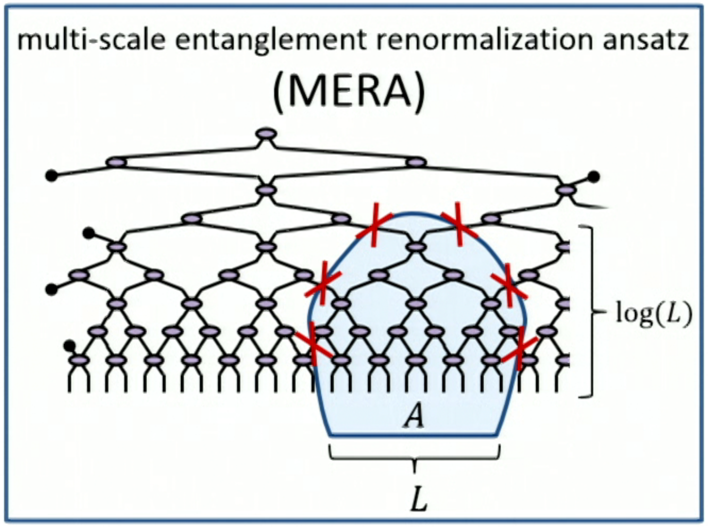
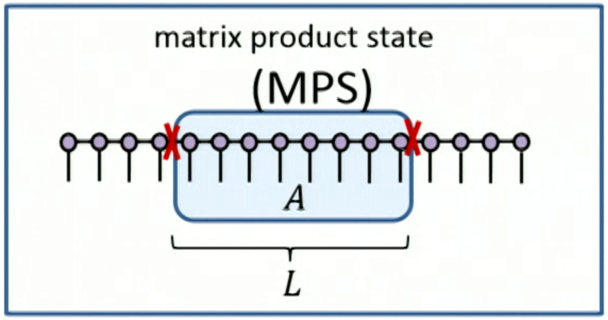
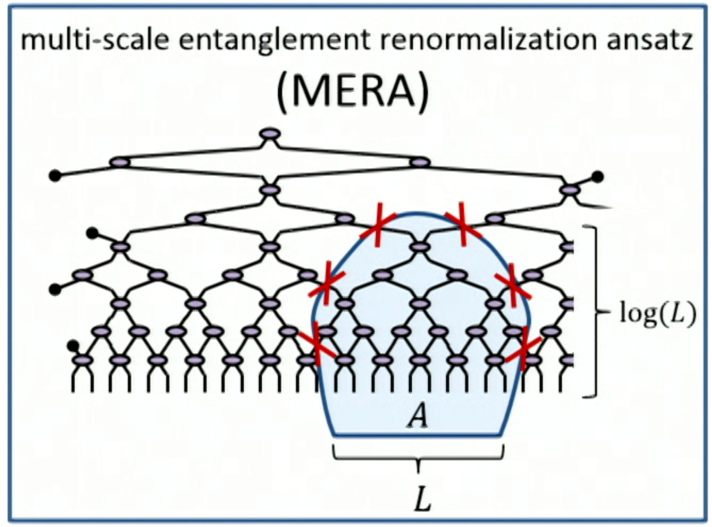
Entanglement entropy





Entanglement entropy



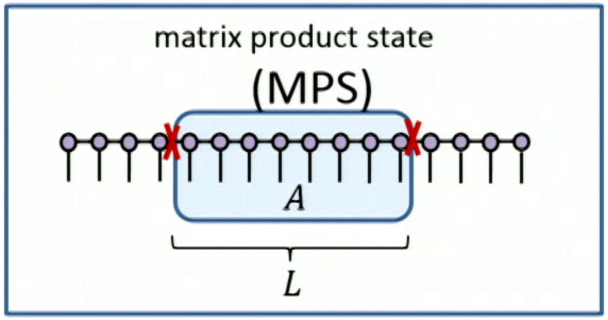


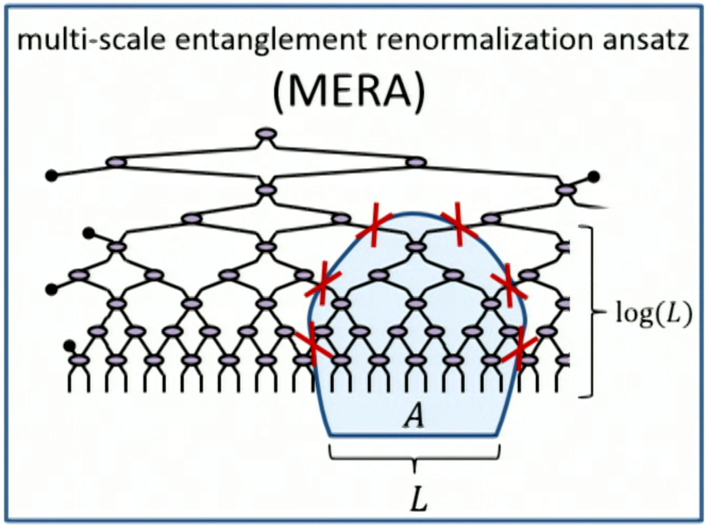
connectivity:

$S(A) \leq const$

boundary law!

Entanglement entropy





connectivity:

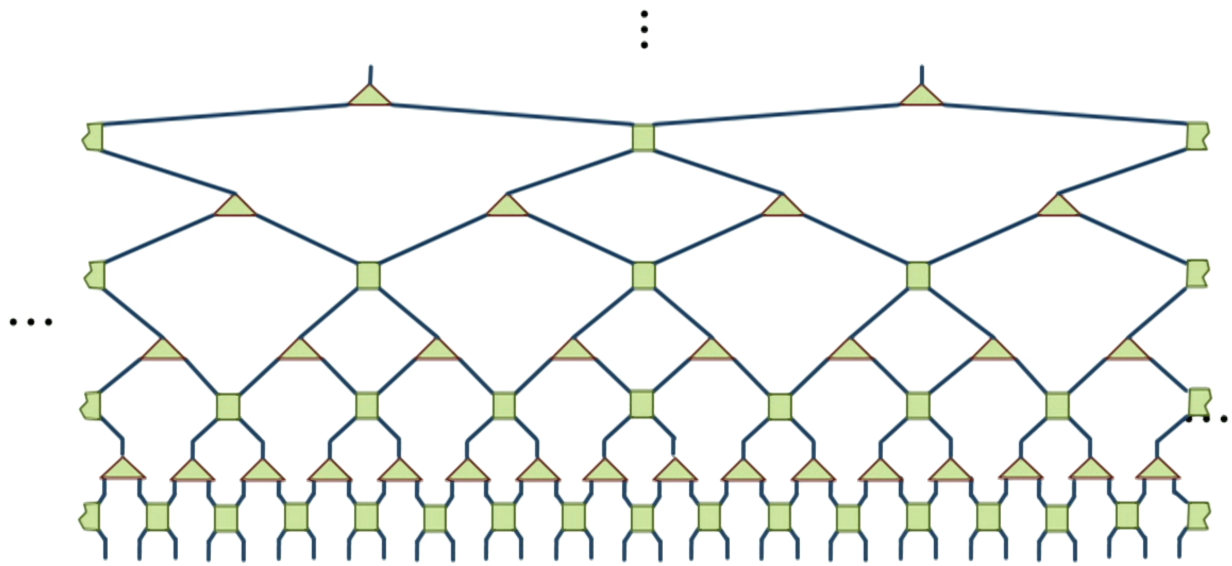
$S(A) \leq const$

boundary law!

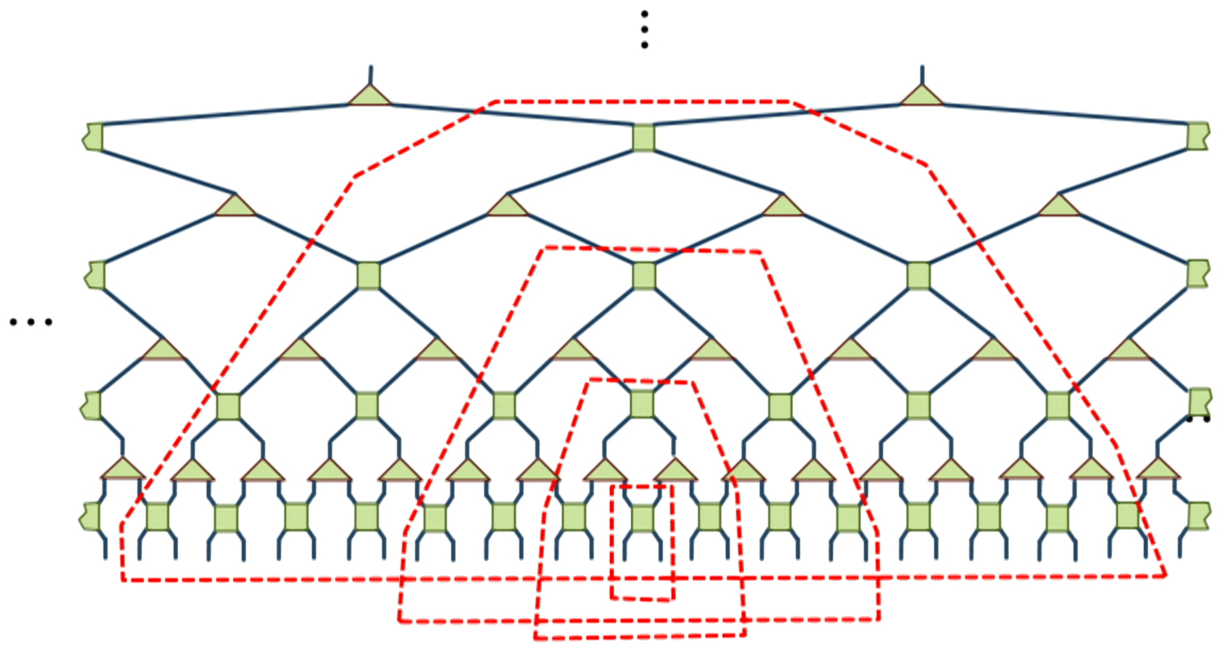
connectivity:

$S(A) \leq \log L$

logarithmic correction!



$$n(A) \approx \log L$$



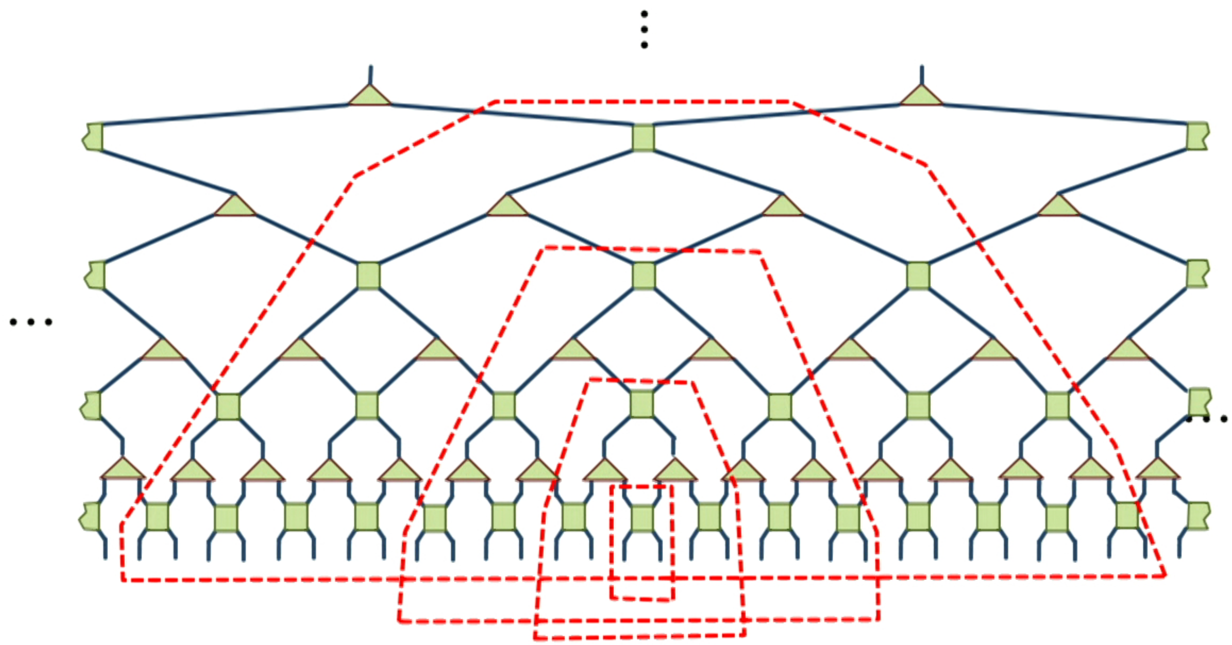
$n(A) \approx \log L$

$L = 2, \quad n(A) = 2$

$L = 6, \quad n(A) = 4$

$L = 14, \quad n(A) = 6$

$L = 30, \quad n(A) = 8$



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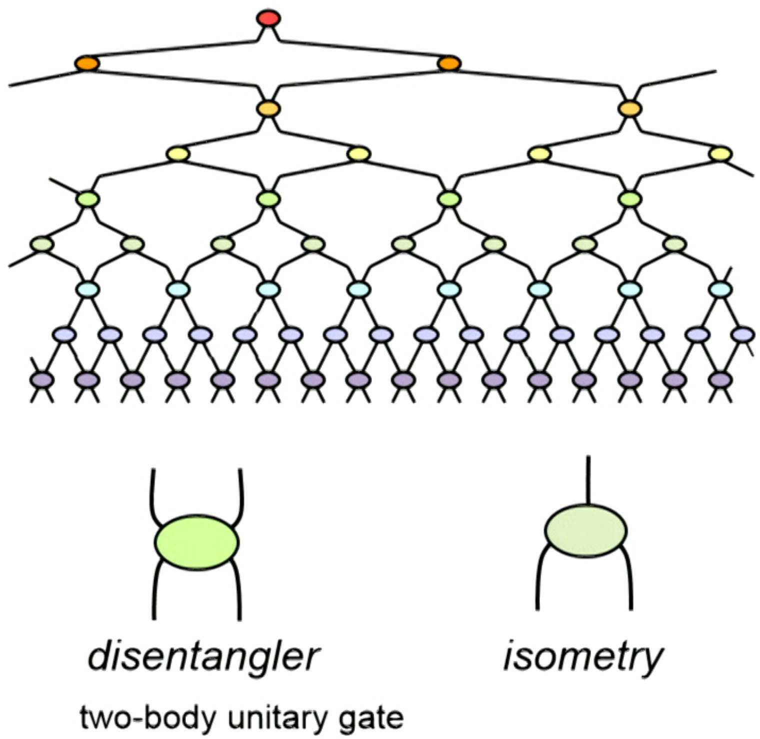
MERA as a quantum circuit



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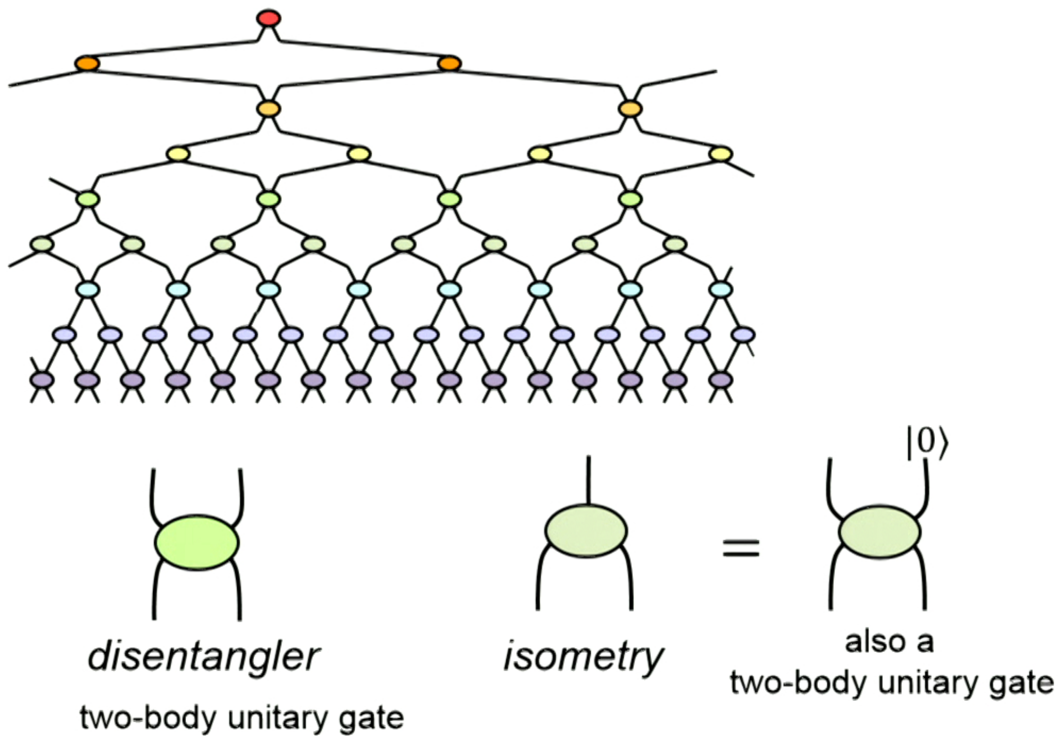

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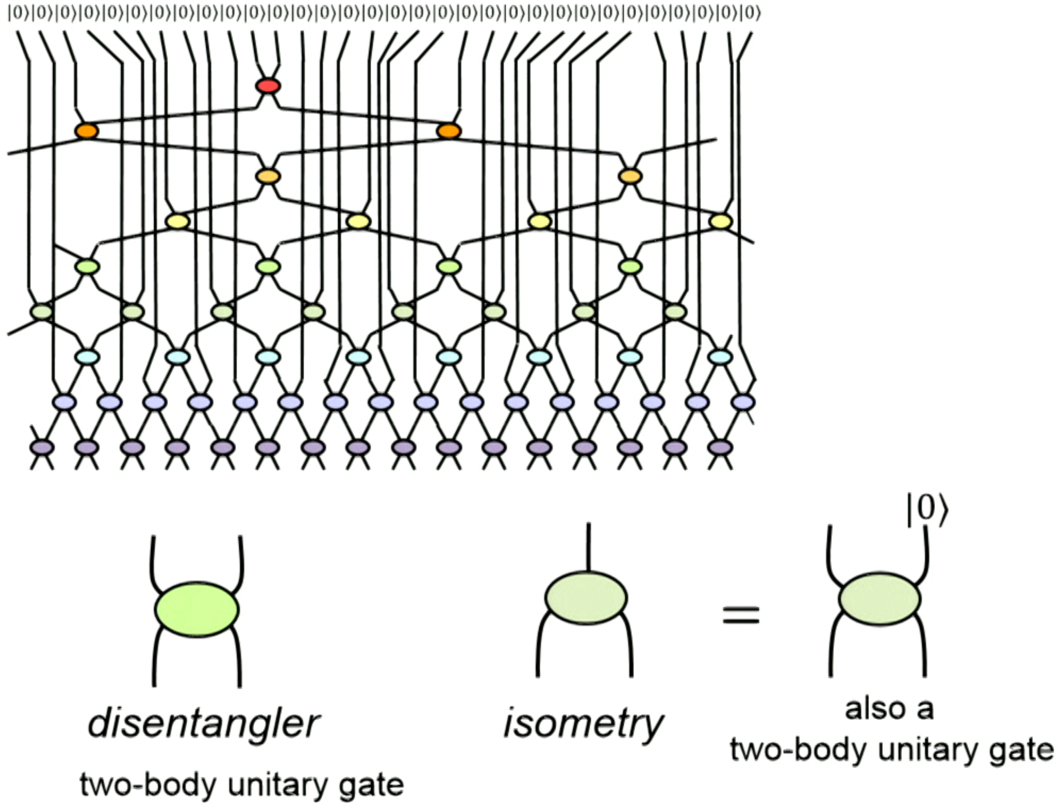

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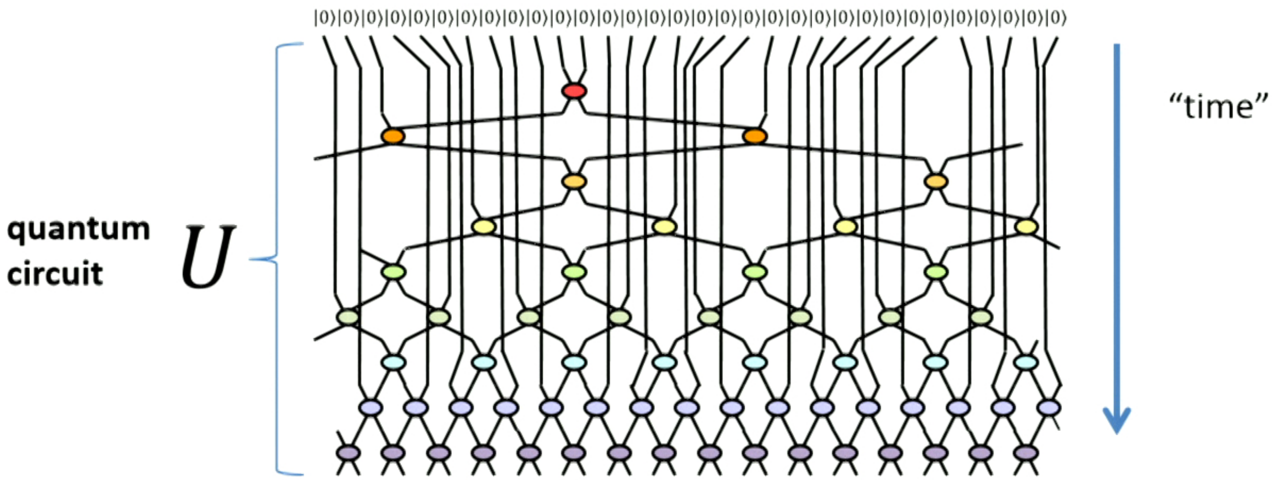
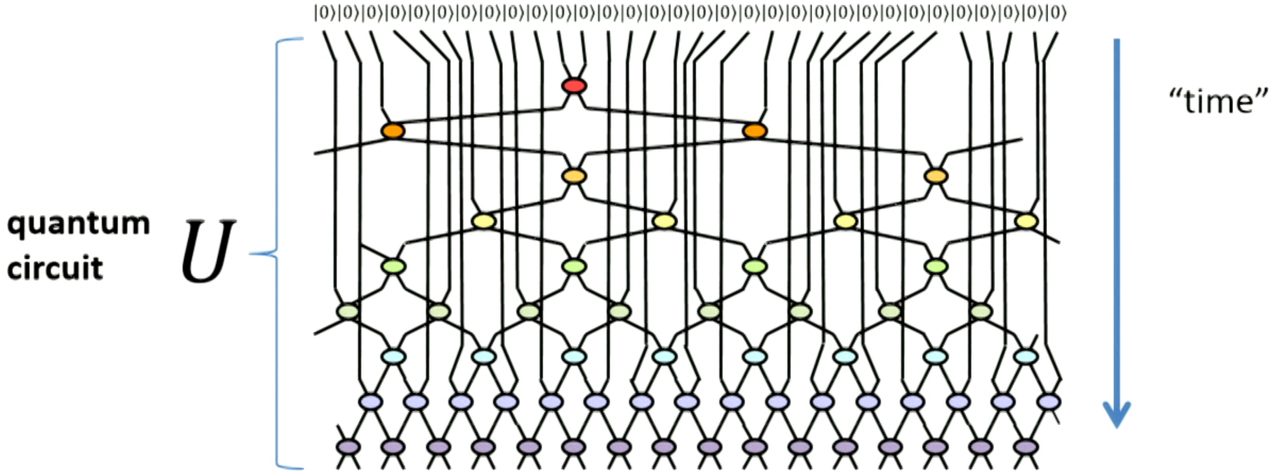
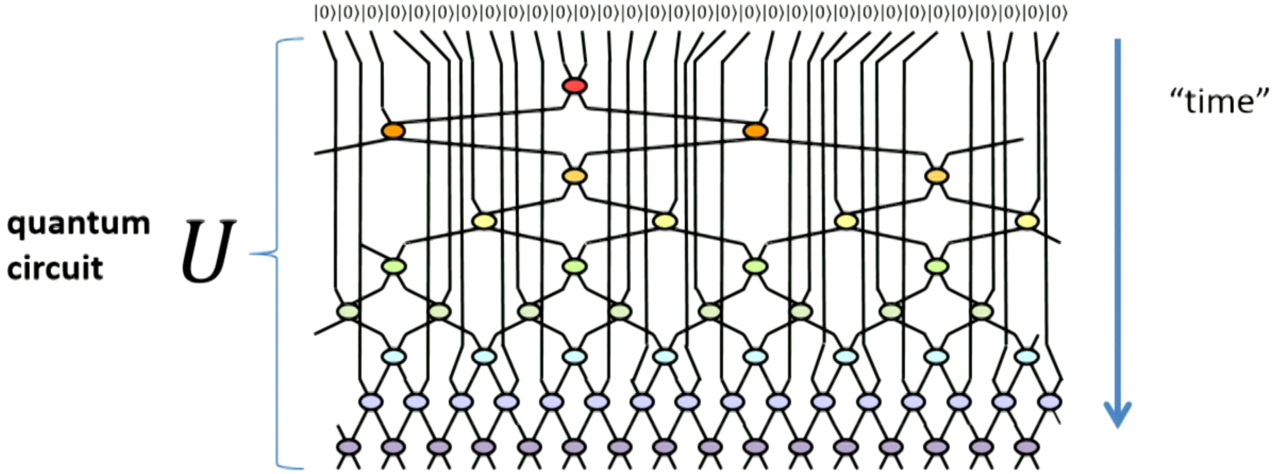


MERA as a quantum circuit



ground state ansatz $|\Psi\rangle = U\ |0\rangle^{\otimes N}$

MERA as a quantum circuit



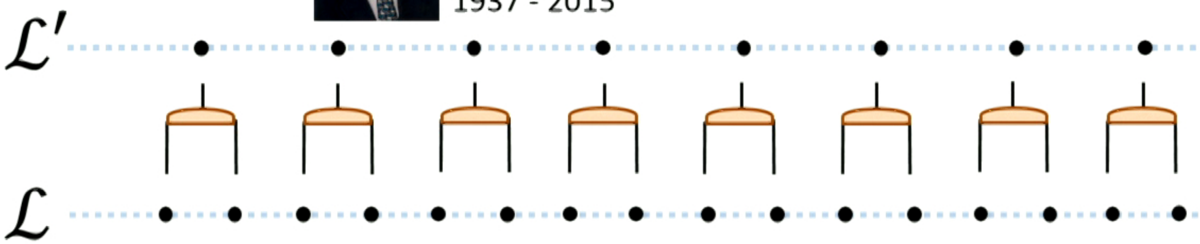
ground state ansatz $|\Psi\rangle = U\ |0\rangle^{\otimes N}$

Entanglement introduced by gates at different "times" (= length scales)

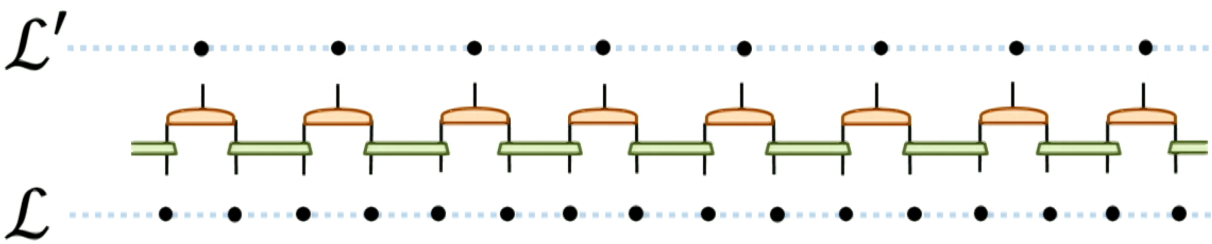
MERA as a (real space) Renormalizatin Group transformation

Spin blocking
(1966)

Leo Kadanoff
1937 - 2015



Entanglement renormalization (2005)



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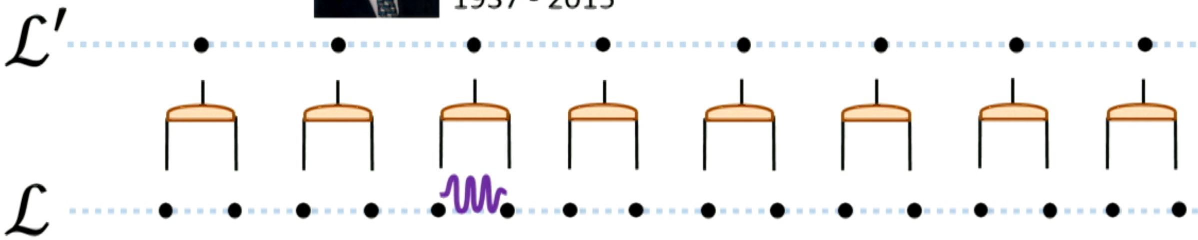

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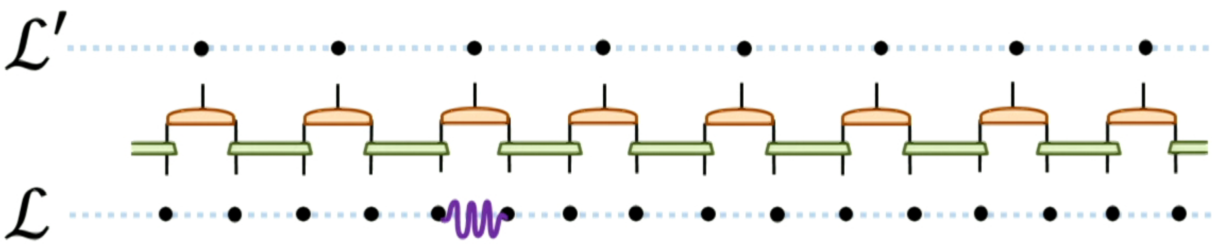
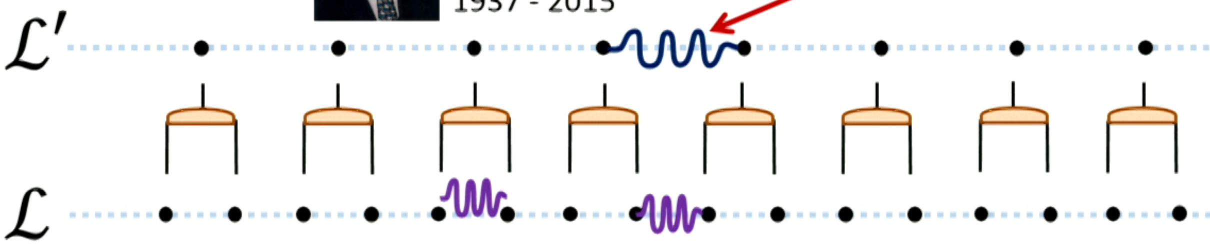


MERA as a (real space) Renormalizatin Group transformation

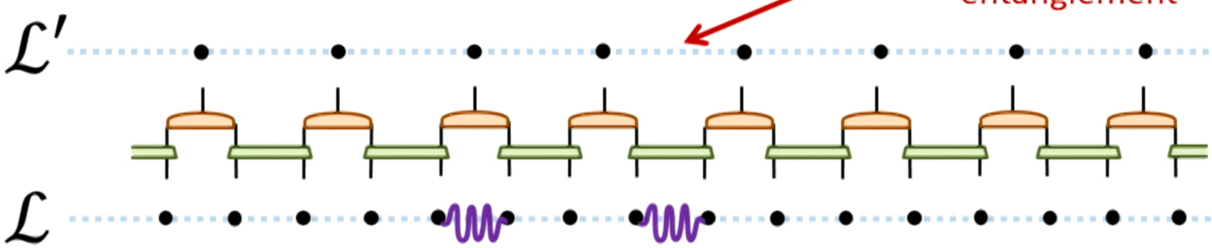
Spin blocking (1966)

failure to remove ***some*** short-range entanglement !



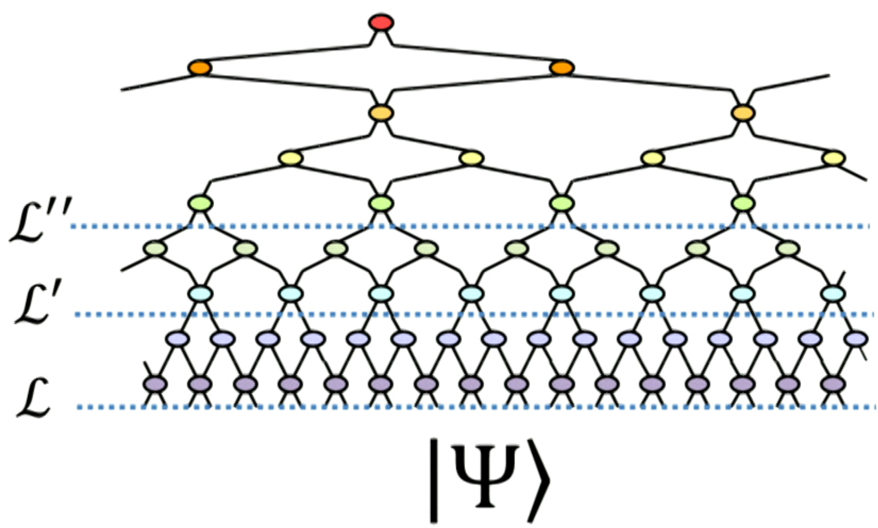
Entanglement renormalization (2005)

removal of ***all*** short-range entanglement



MERA as a (real space) Renormalizatin Group transformation

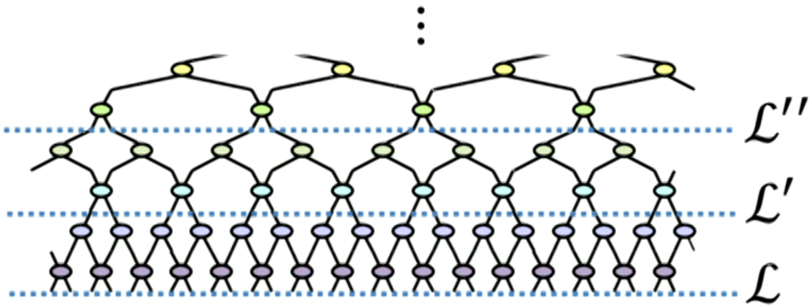
sequence of ground state wave-functions



$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

MERA defines an RG flow in the space of wave-functions

$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$



MERA defines an RG flow in the space of wave-functions

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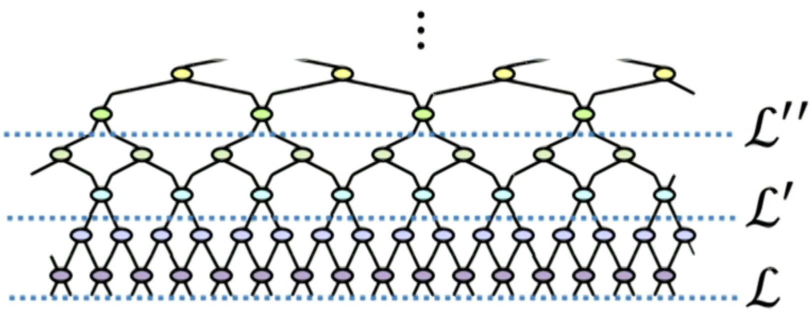
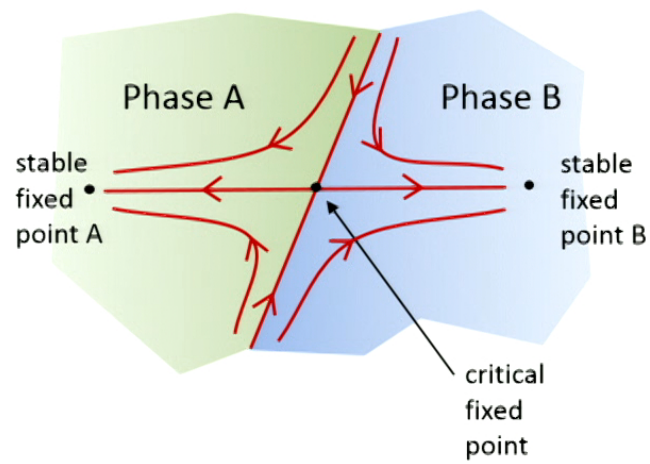
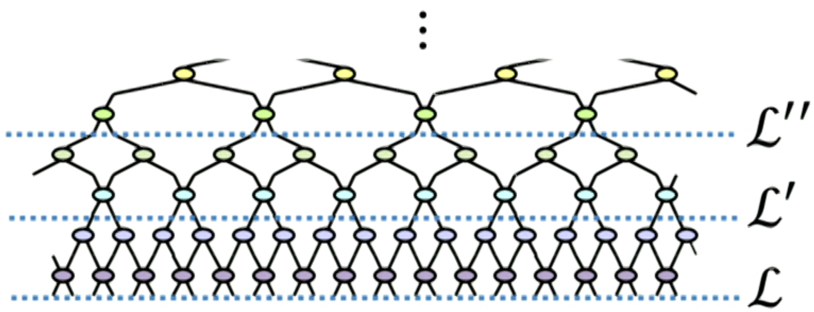
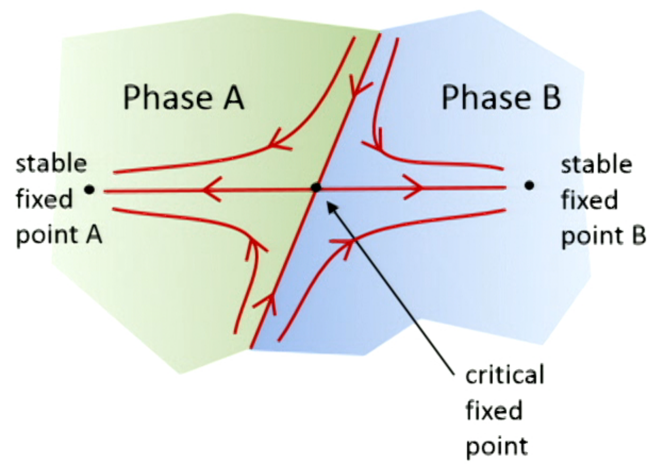




MERA defines an RG flow
in the space of wave-functions

$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$



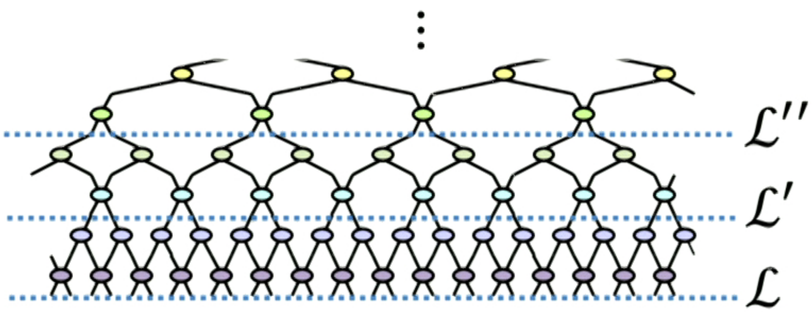


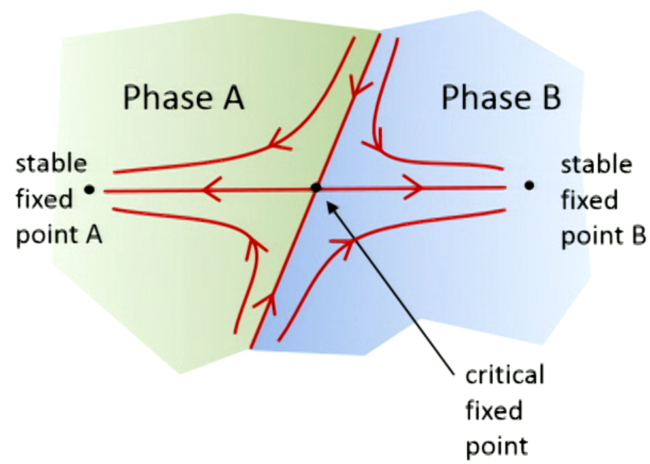
... and in the space of Hamiltonians

$H \rightarrow H' \rightarrow H'' \rightarrow \cdots$

MERA defines an RG flow
in the space of wave-functions

$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$





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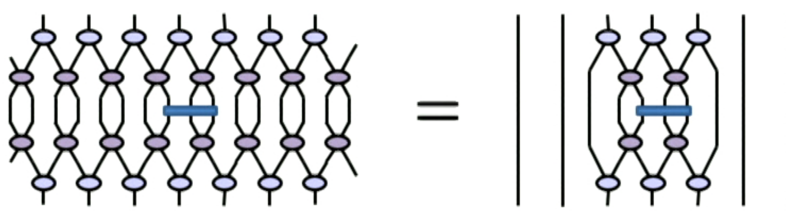

MERA defines an RG flow
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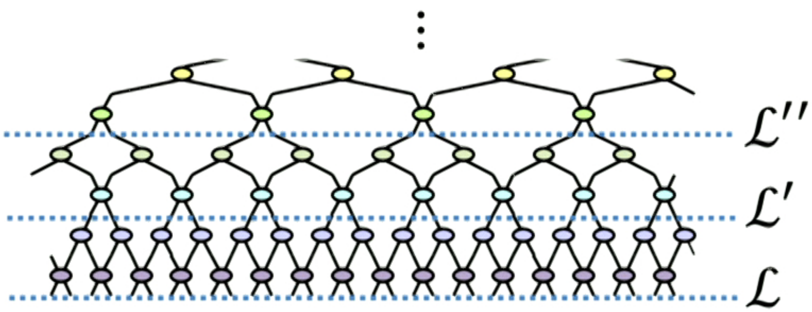
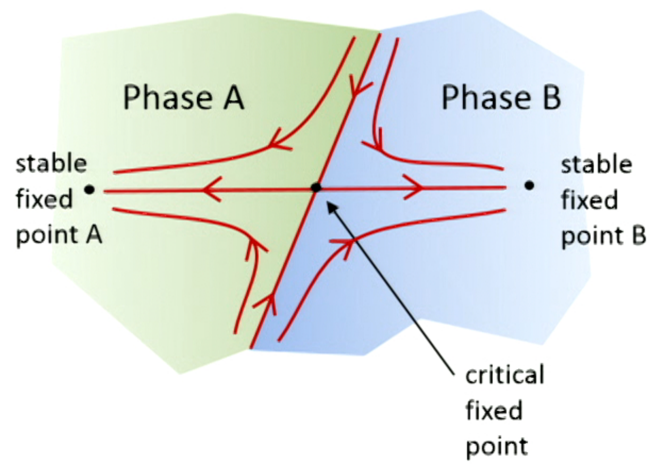
$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$





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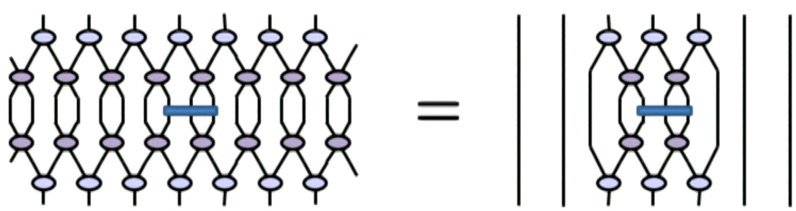


local operators are mapped into local operators !

Claim: Entanglement renormalization defines a *proper* scale transformation on the lattice

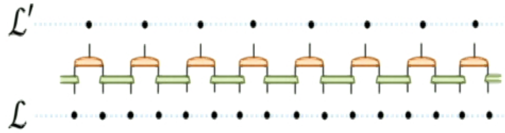
- Explicit scale invariance at criticality !

input

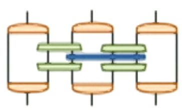
1D quantum Hamiltonian on the lattice

- **at a critical point**

1 - optimization



2 - diagonalization



→ scaling operators

Claim: Entanglement renormalization defines a *proper* scale transformation on the lattice

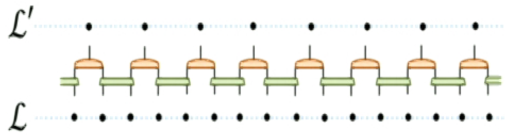
- Explicit scale invariance at criticality !

input

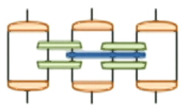
1D quantum Hamiltonian on the lattice

- **at a critical point**

1 - optimization



2 - diagonalization



→ scaling operators

output

Numerical extraction of conformal data of underlying CFT:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

Claim: Entanglement renormalization defines a *proper* scale transformation on the lattice

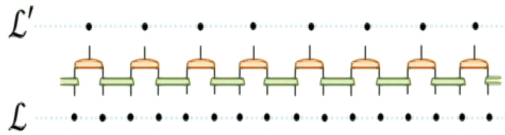
- Explicit scale invariance at criticality !

input

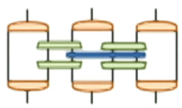
1D quantum Hamiltonian on the lattice

- **at a critical point**

1 - optimization



2 - diagonalization



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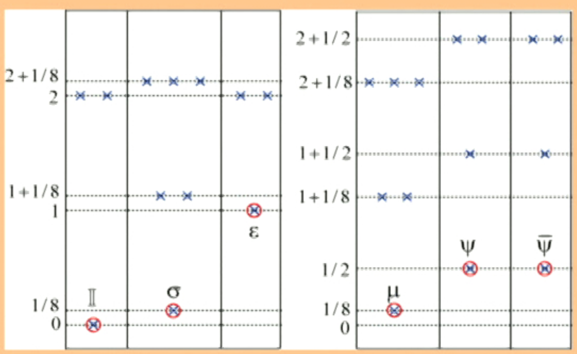
e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$