Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 10

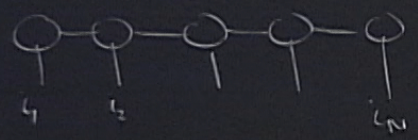
Date: Feb 13, 2017 09:00 AM

URL: http://pirsa.org/17020046

Abstract:

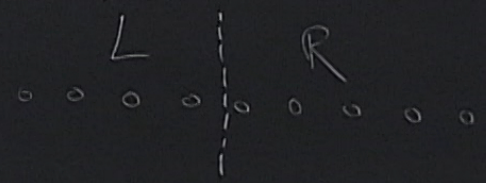
lecture 16? MPS continued

15.4 MPS & Schmidt decomposition



$$|\psi\rangle = \sum_{i_1, i_2, \ldots, i_N} \psi_{i_1 i_2 \ldots i_N} |i_1 i_2 \ldots i_N\rangle$$

$$|\psi\rangle = \sum_{\alpha=1}^{\chi} \lambda_\alpha |\psi_\alpha^L\rangle |\psi_\alpha^R\rangle$$



15.4 MPS & Schmidt decomposition

Any $|\psi\rangle$ can be expressed as MPS

Singular value decomposition

$$\underset{m\times n}{A} = U S V^{\dagger}$$

$U_{m\times\chi}$ $\quad U^{\dagger}U = \mathbb{1}_{m\times m}$

$V_{n\times\chi}$ $\quad V^{\dagger}V = \mathbb{1}_{n\times n}$

$S_{\chi\times\chi}$ $\quad S = \begin{pmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_\chi \end{pmatrix}$

15.4 MPS & Schmidt decomposition

Any $|\psi\rangle$ can be expressed as MPS

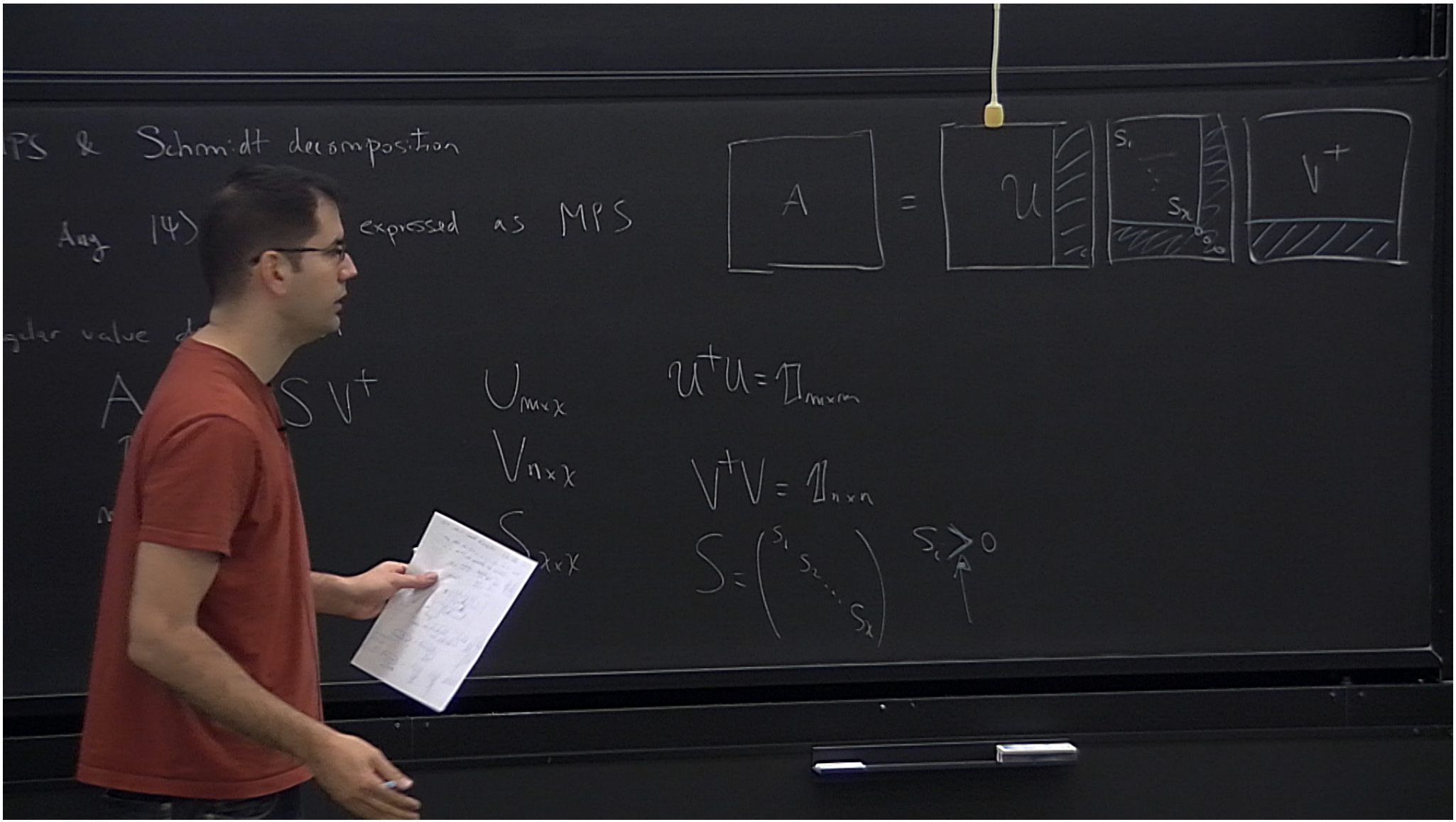
<u>Compressed</u> singular value decomposition

$$\underset{m\times n}{A} = U S V^\dagger$$

$U_{m\times\chi}$ $\quad U^\dagger U = \mathbb{1}_{m\times m}$

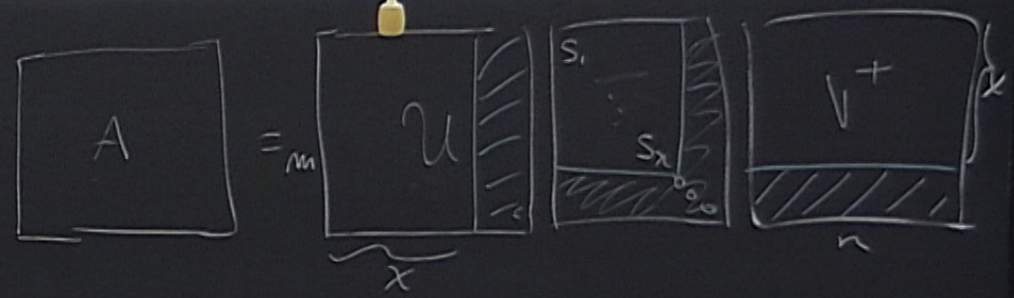
$V_{n\times\chi}$ $\quad V^\dagger V = \mathbb{1}_{n\times n}$

$S_{\chi\times\chi}$ $\quad S = \begin{pmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_\chi \end{pmatrix}$ $\quad s_i > 0$


PS & Schmidt decomposition
Any |ψ⟩ expressed as MPS
gular value d
A = S V†
U_{m×χ}
V_{n×χ}
S_{χ×χ}
U†U = 𝟙_{m×m}
V†V = 𝟙_{n×n}
S = (s1 s2 ... sχ)
s_i ≥ 0
A
=
U
s1
sχ
V†

PS & Schmidt decomposition

Any $|\psi\rangle$ can be expressed as MPS



$\chi \leq \min(m,n)$

gular value decomposition

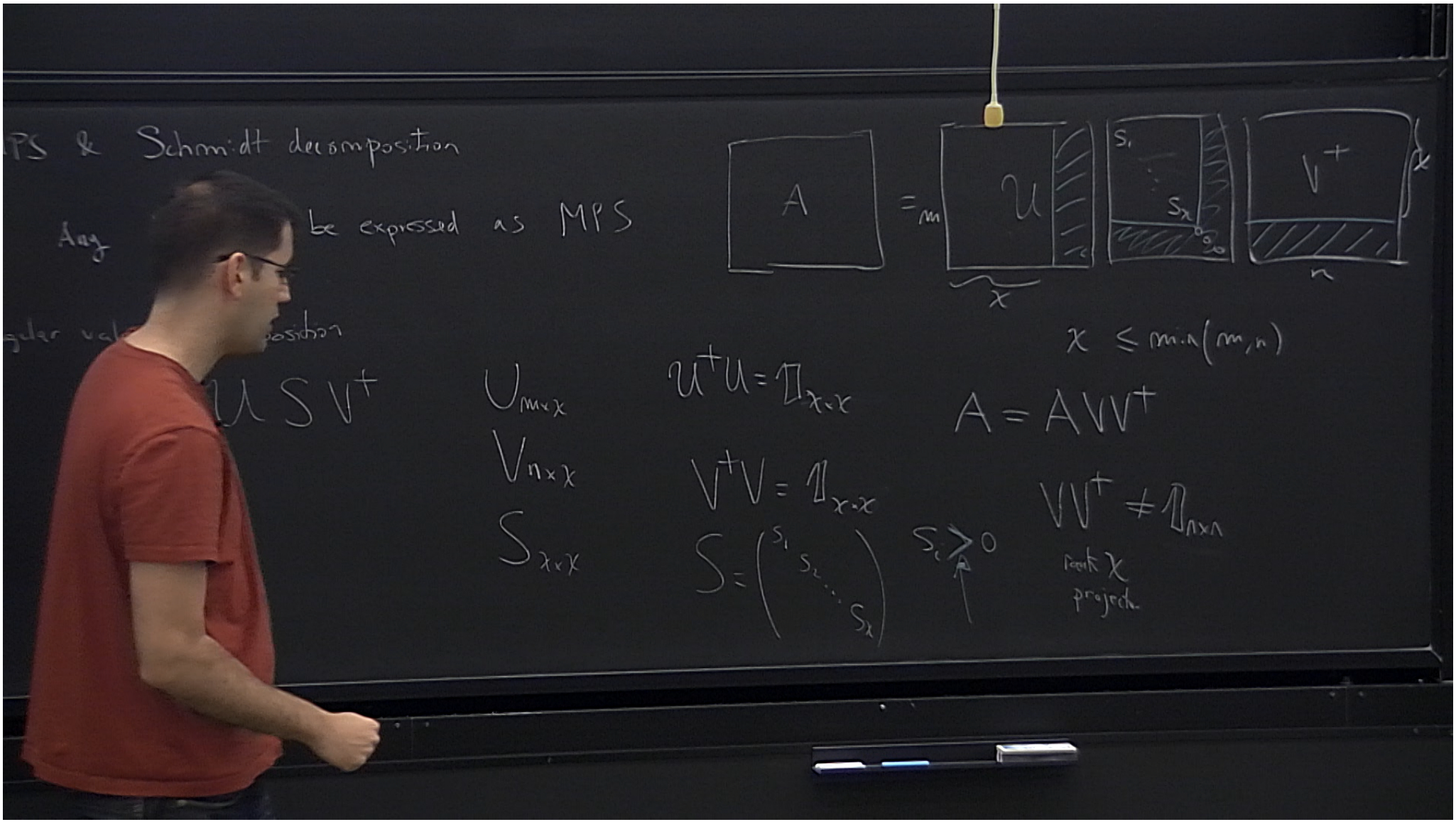
$$A = U S V^{\dagger}$$

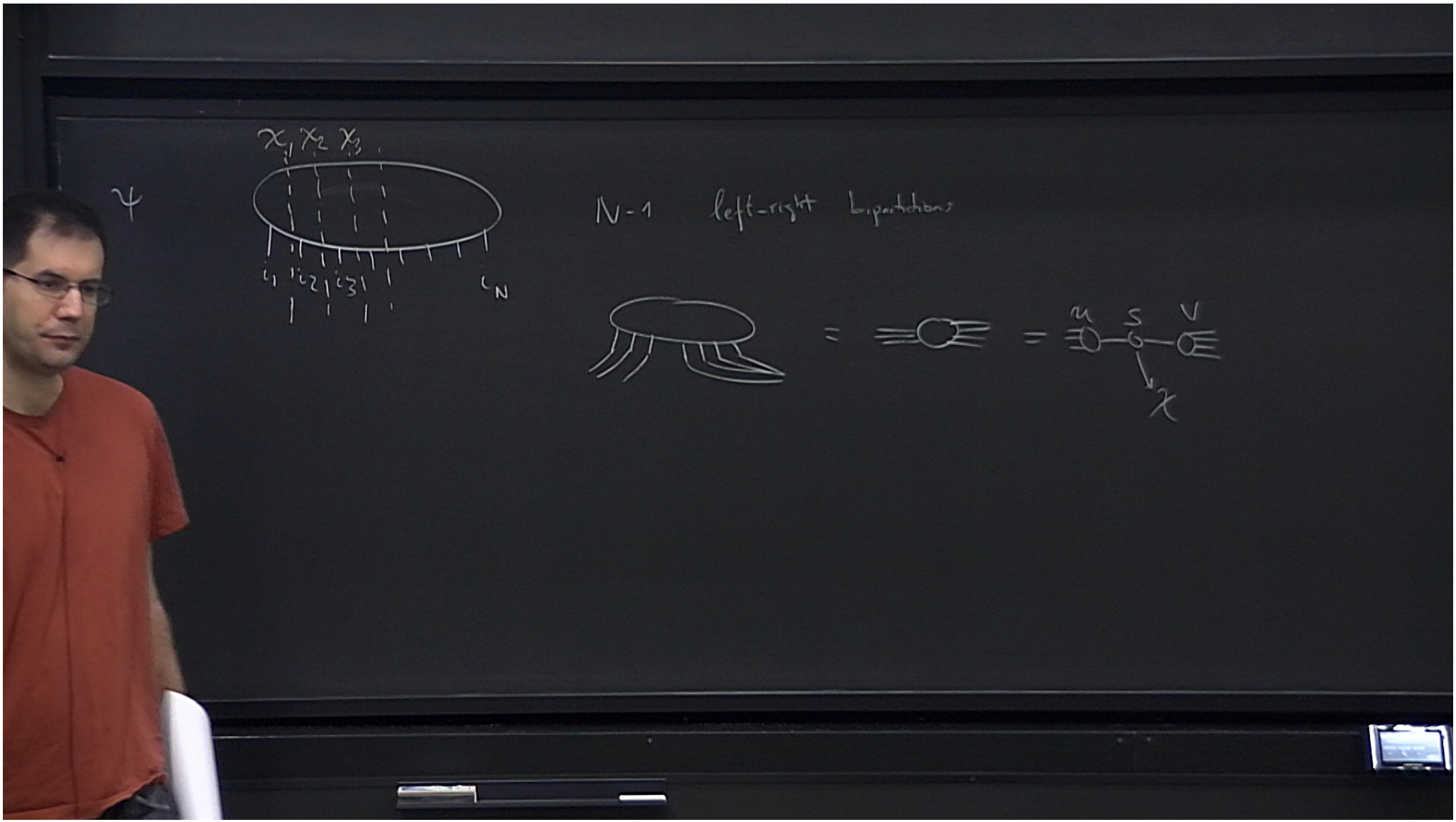
$m \times n$

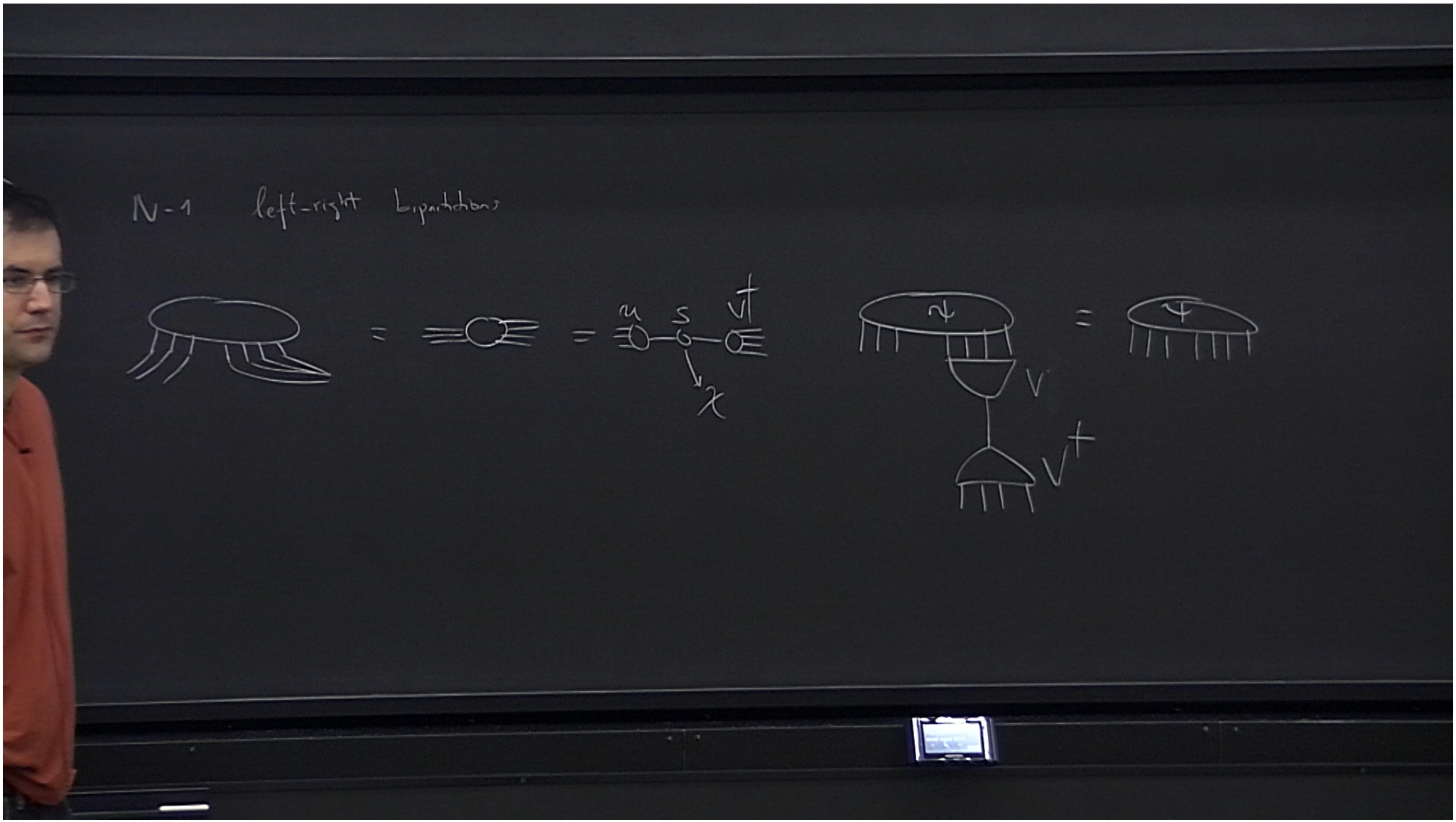
$U_{m\times\chi}$ $\quad U^{\dagger}U = \mathbb{1}_{m\times m}$

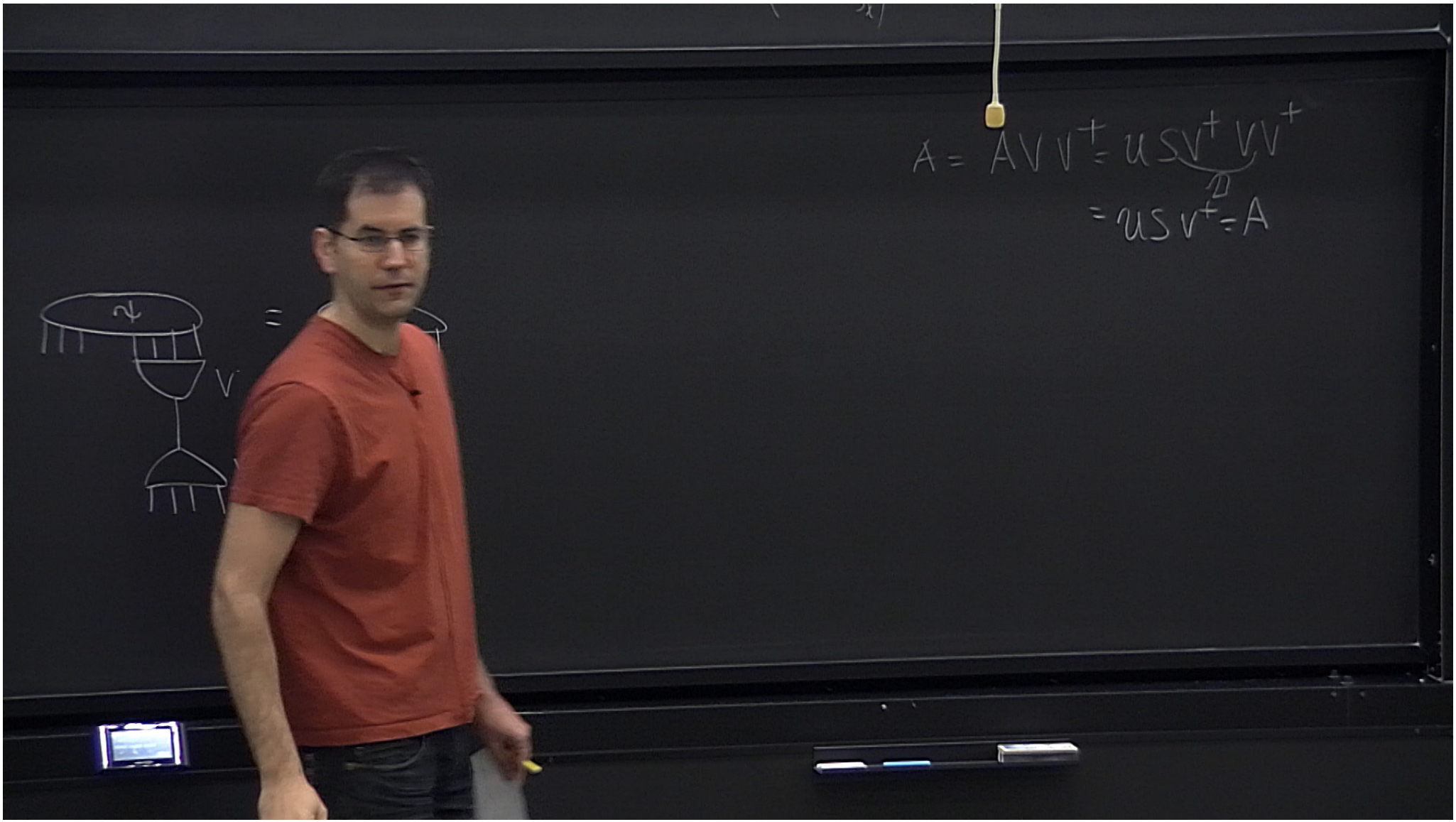
$V_{n\times\chi}$ $\quad V^{\dagger}V = \mathbb{1}_{n\times n}$

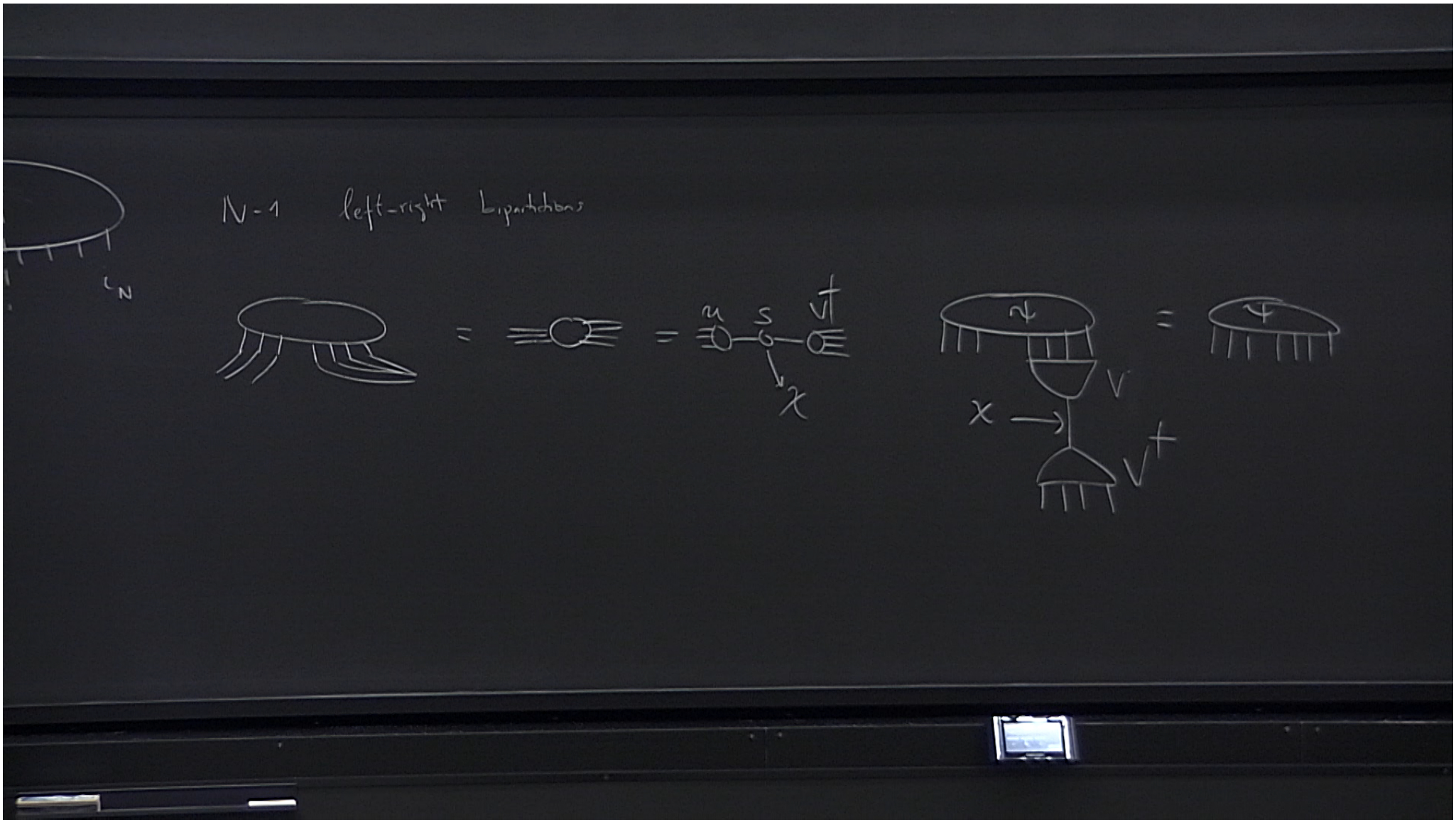
$S_{\chi\times\chi}$ $\quad S = \begin{pmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_\chi \end{pmatrix}$ $\quad s_i \geq 0$

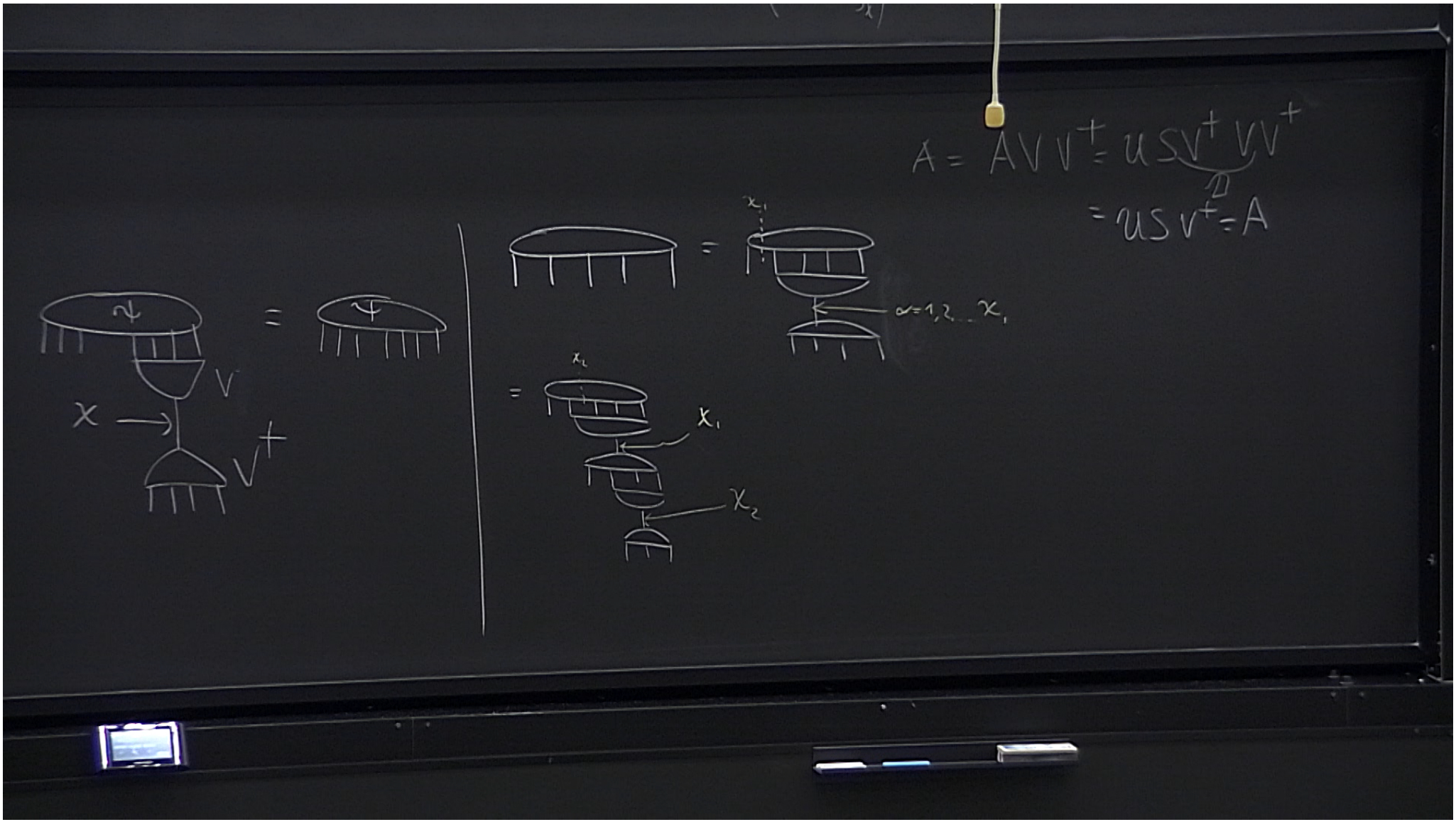

PS & Schmidt decomposition
be expressed as MPS
A
= m
U
S1
Sχ
V+
χ
n
χ ≤ min(m,n)
U S V+
U_{m×χ}
V_{n×χ}
S_{χ×χ}
U+U = 𝟙_{χ×χ}
V+V = 𝟙_{χ×χ}
S = (S1 S2 ... Sχ)
Si ≥ 0
A = AVV+
VV+ ≠ 𝟙_{n×n}
rank χ
projech.

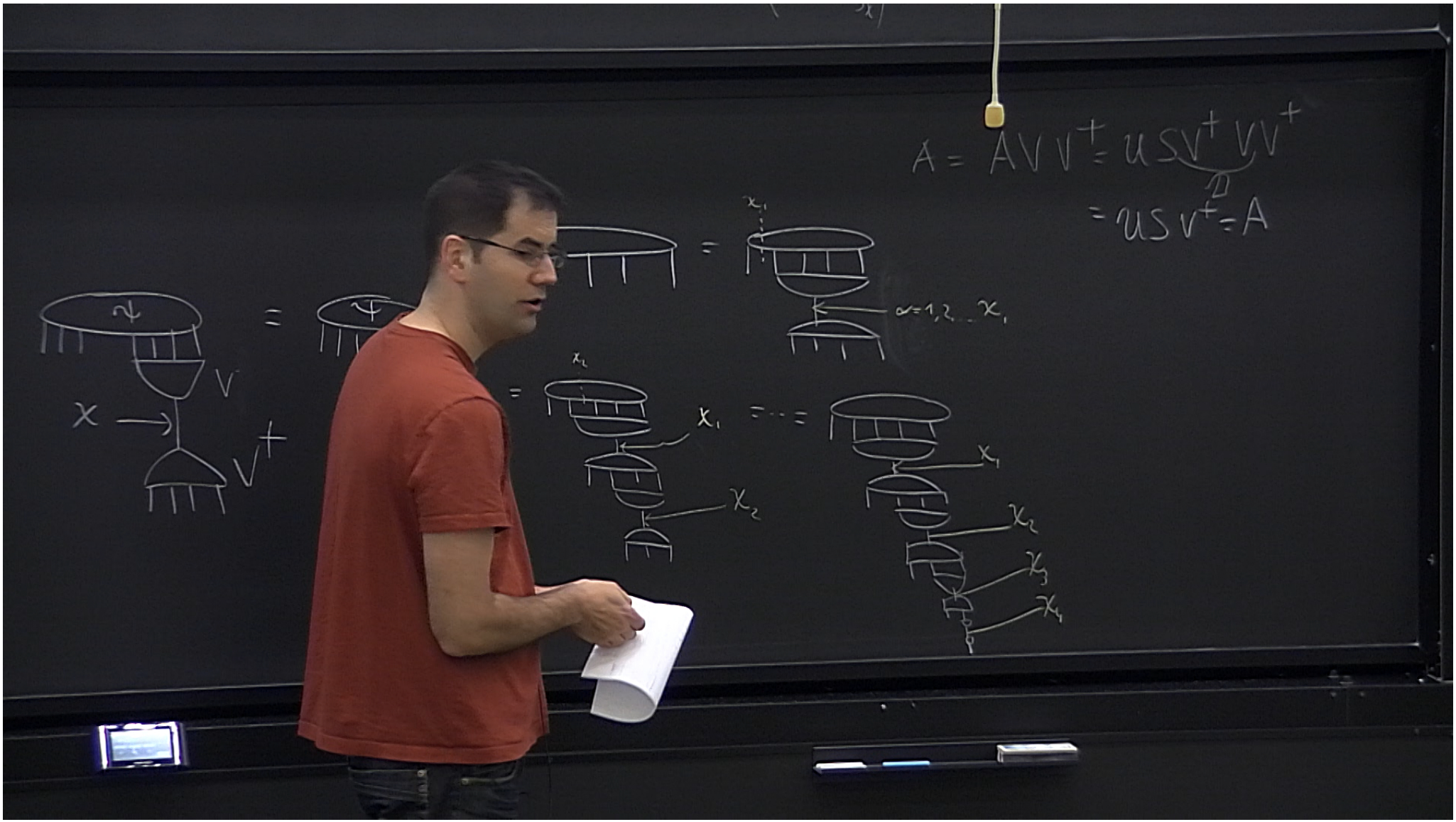

ψ
x_1 x_2 x_3
i_1 i_2 i_3 i_N
N-1 left-right bipartitions
u
s
v
χ

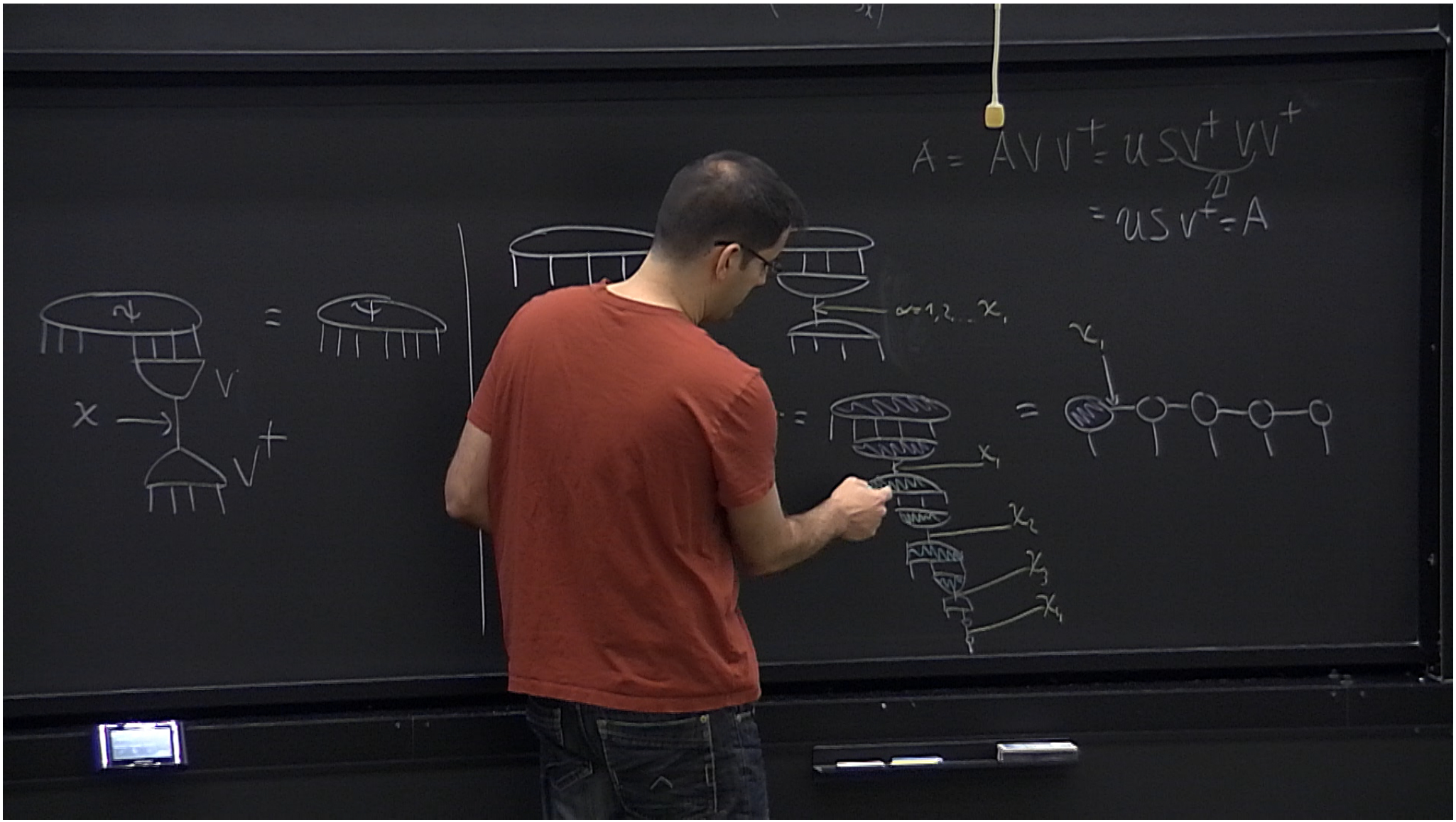

N-1 left-right bipartitions
u
s
v†
χ
ψ
V
V†
Φ

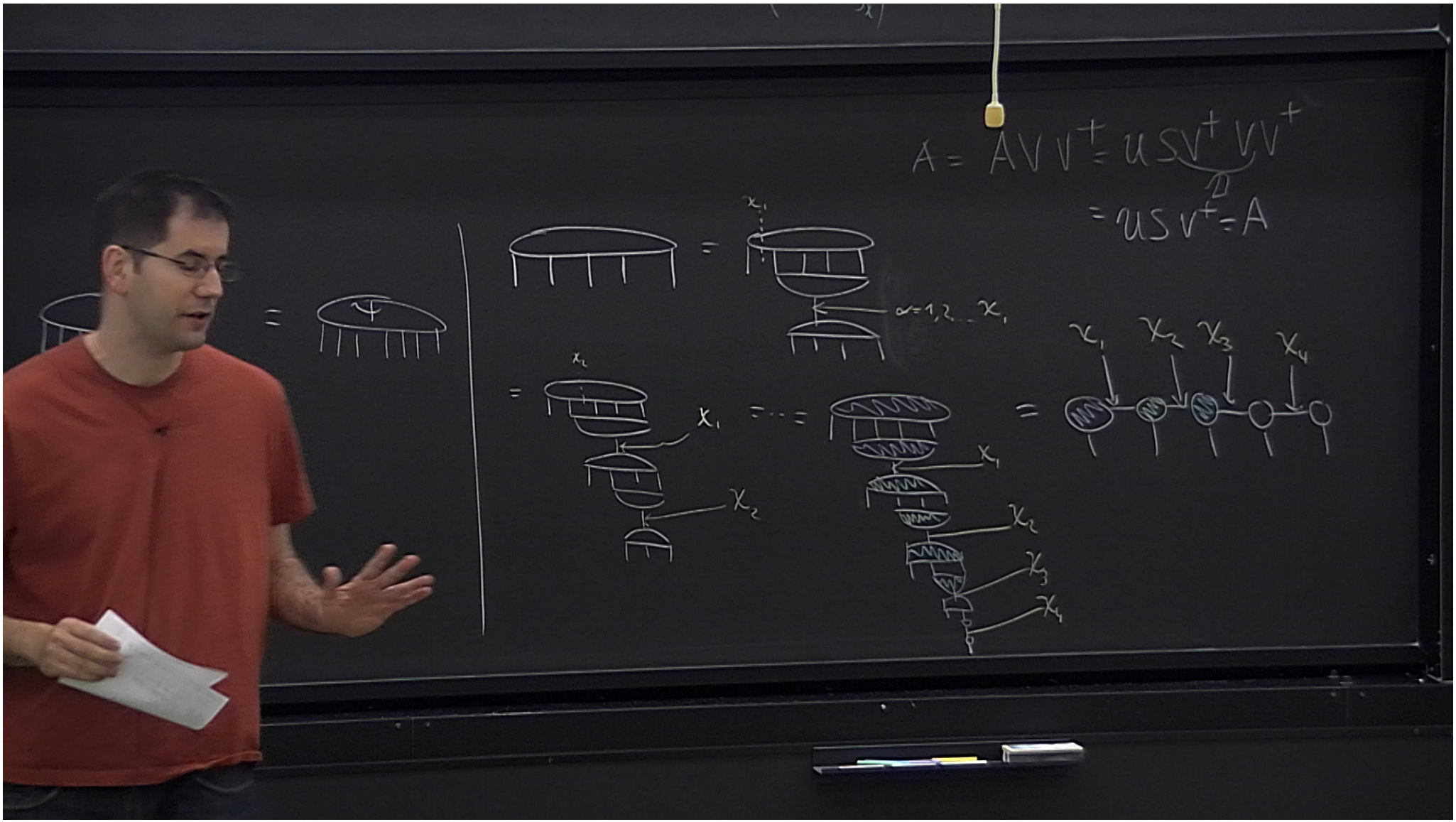

A = AVV† = USV† VV†
= USV† = A


N - 1 left-right bipartitions
N
u
s
v†
χ
ψ
V
χ
V†
ψ

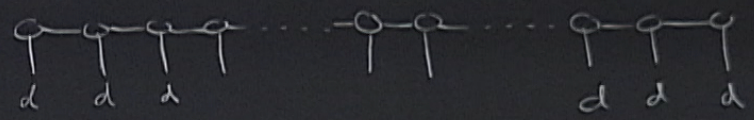

A = AVV† = USV† VV†
= USV† = A
V
X
V†
α=1,2...χ1
χ1
χ2


A = AVV† = USV† VV†
= USV† = A
V
V†
x
α=1,2..χ1
χ1
χ2
χ3
χ4

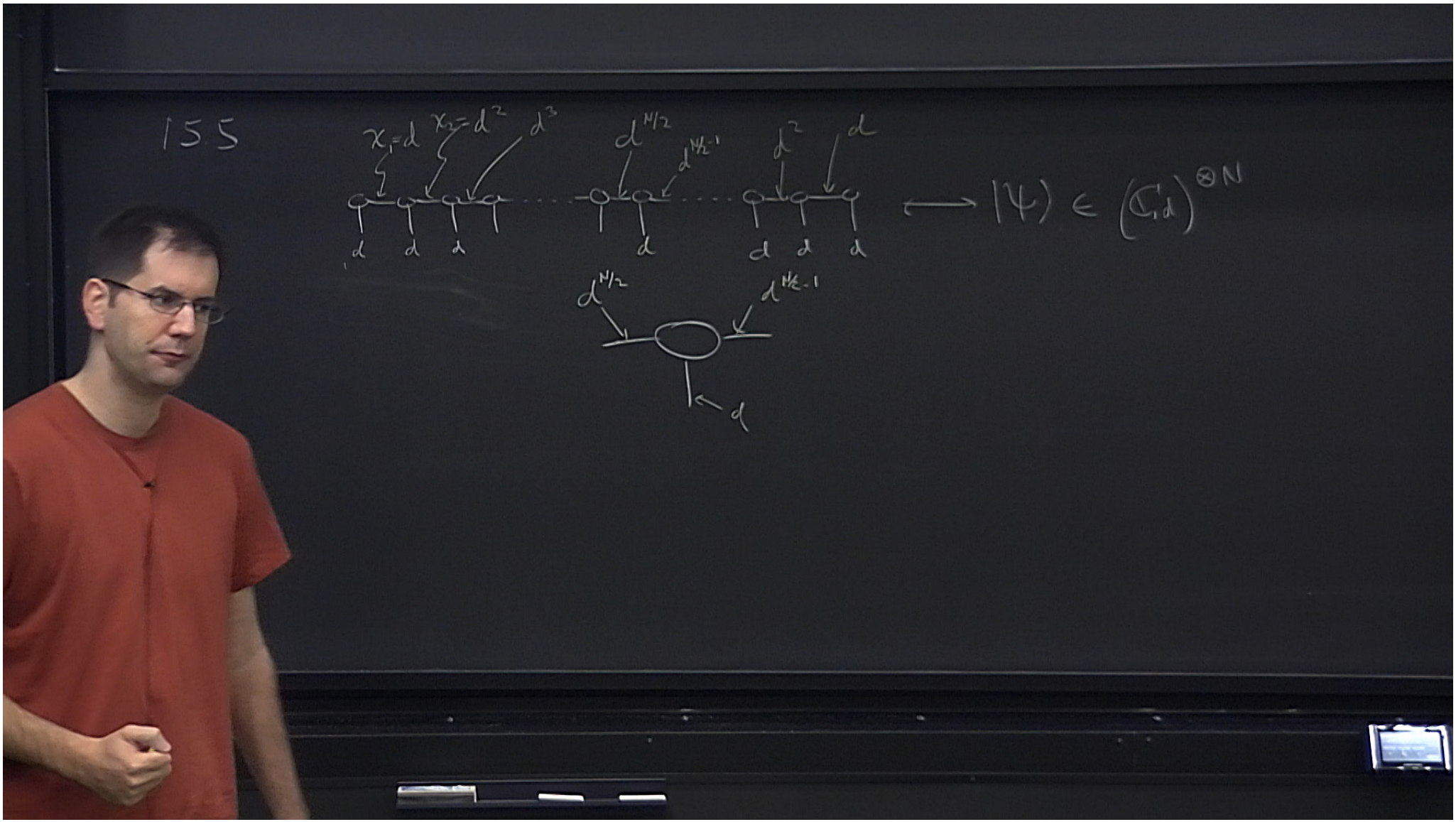

A = AVV† = USV† VV†
= USV† = A
V
V†
x
α=1,2...χ1
χ1
χ2
χ3
χ4

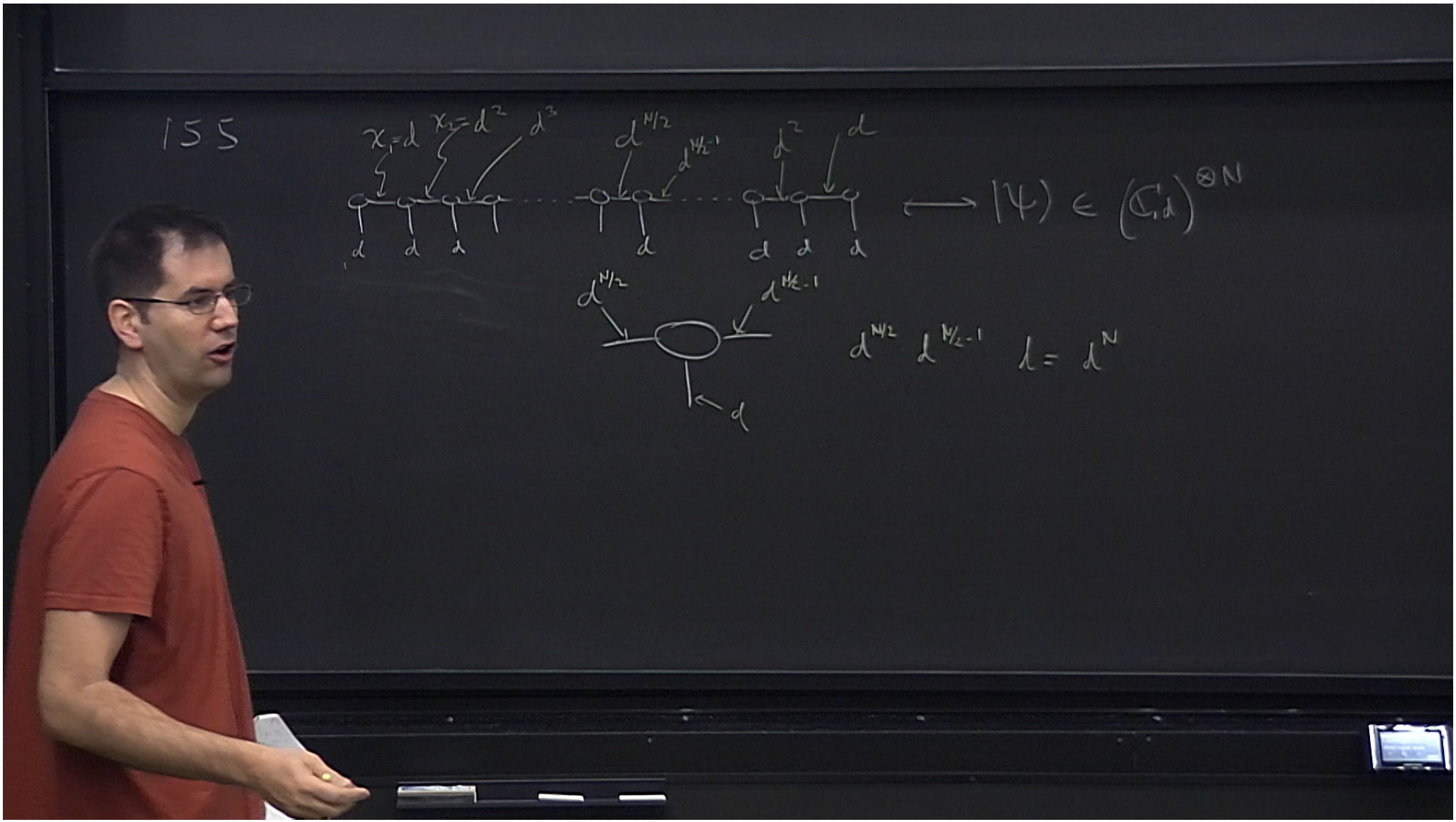

A = AVV† = USV† VV†
= USV† = A
α=1,2..χ1
χ1
χ2
χ3
χ4

15.5

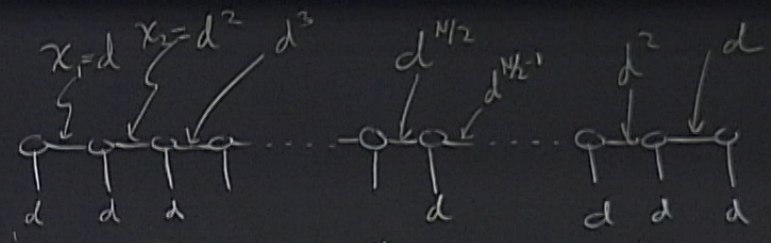


$$\longleftrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

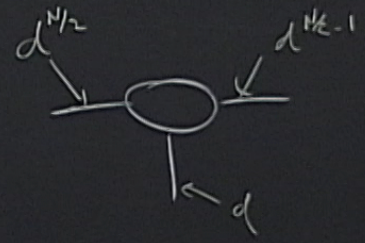

15.5
$\chi_1 = d$
$\chi_2 = d^2$
d^3
$d^{N/2}$
$d^{N/2-1}$
d^2
d
d d d
d
d d d
$\longleftrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
$d^{N/2}$
$d^{N/2-1}$
d


15.5
$\chi_1 = d$
$\chi_2 = d^2$
d^3
$d^{N/2}$
$d^{N/2-1}$
d^2
d
d d d
d
d d d
$\longrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
$d^{N/2}$
$d^{N/2-1}$
d
$d^{N/2} \cdot d^{N/2-1} \cdot d = d^N$

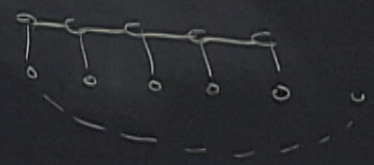
15.5

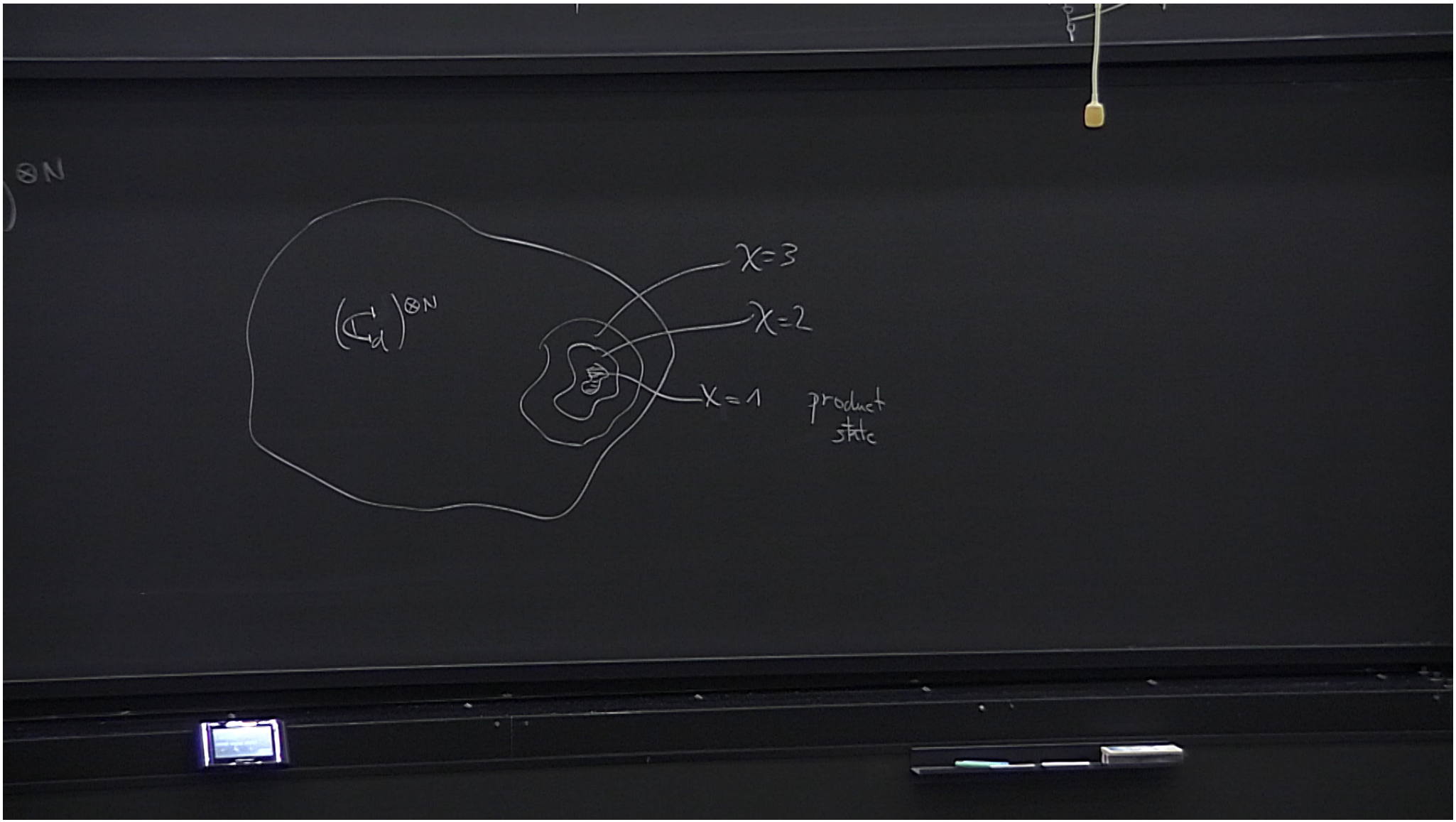


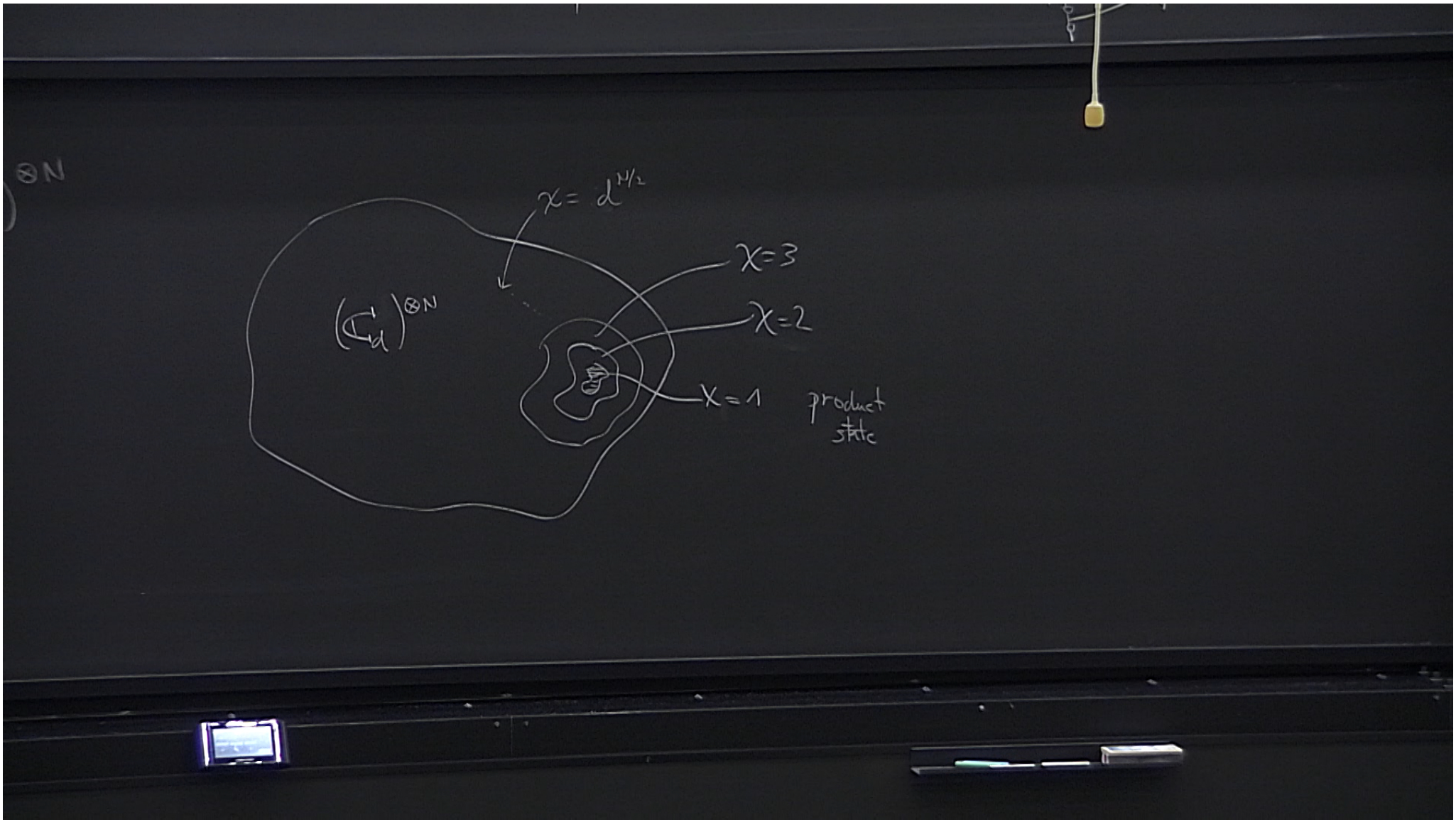
$\longleftrightarrow |\psi\rangle \in (\mathbb{C}, d)^{\otimes N}$

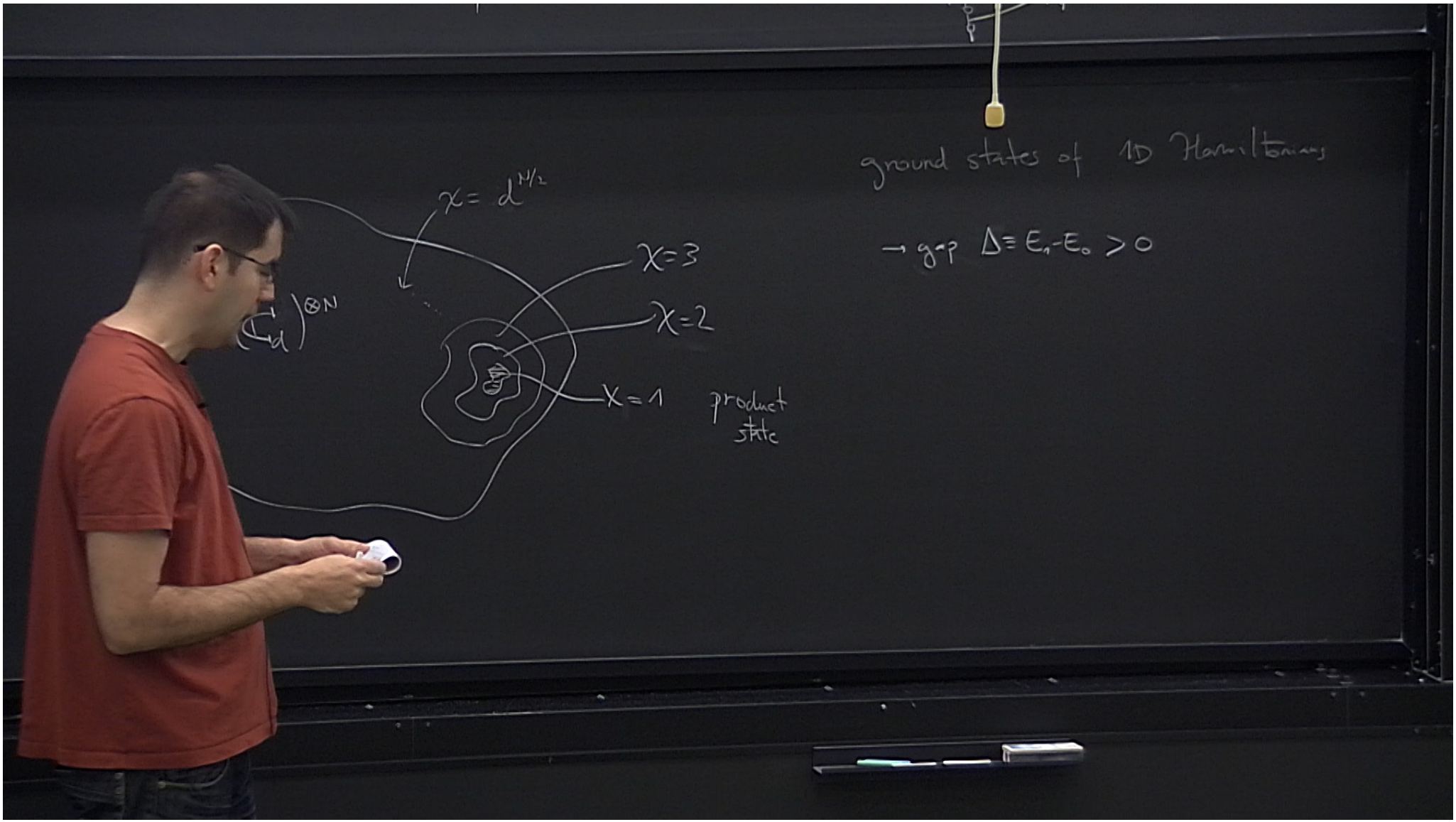


$d^{N/2}\, d^{N/2-1}\, d = \underline{\underline{d^N}}$

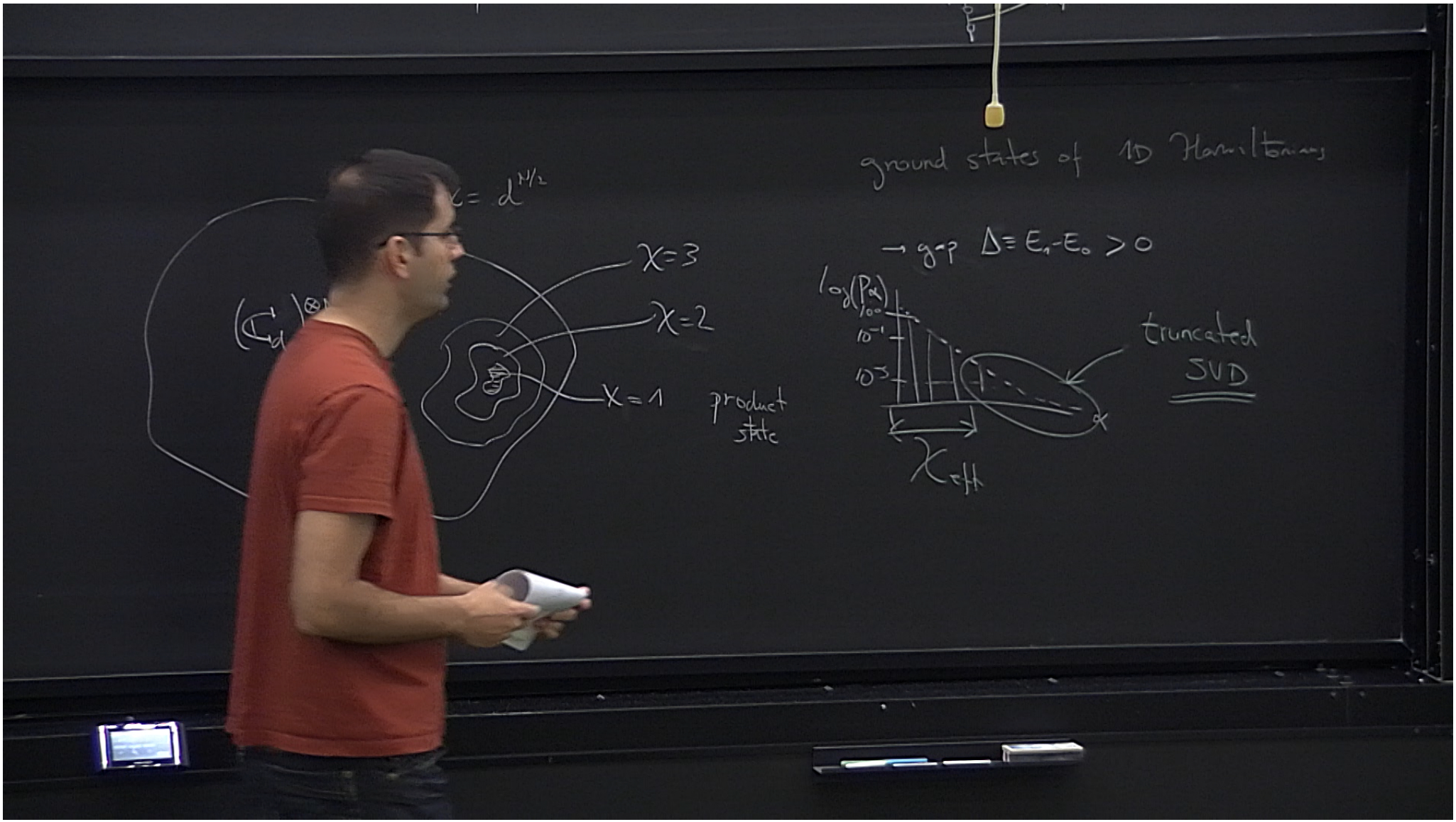


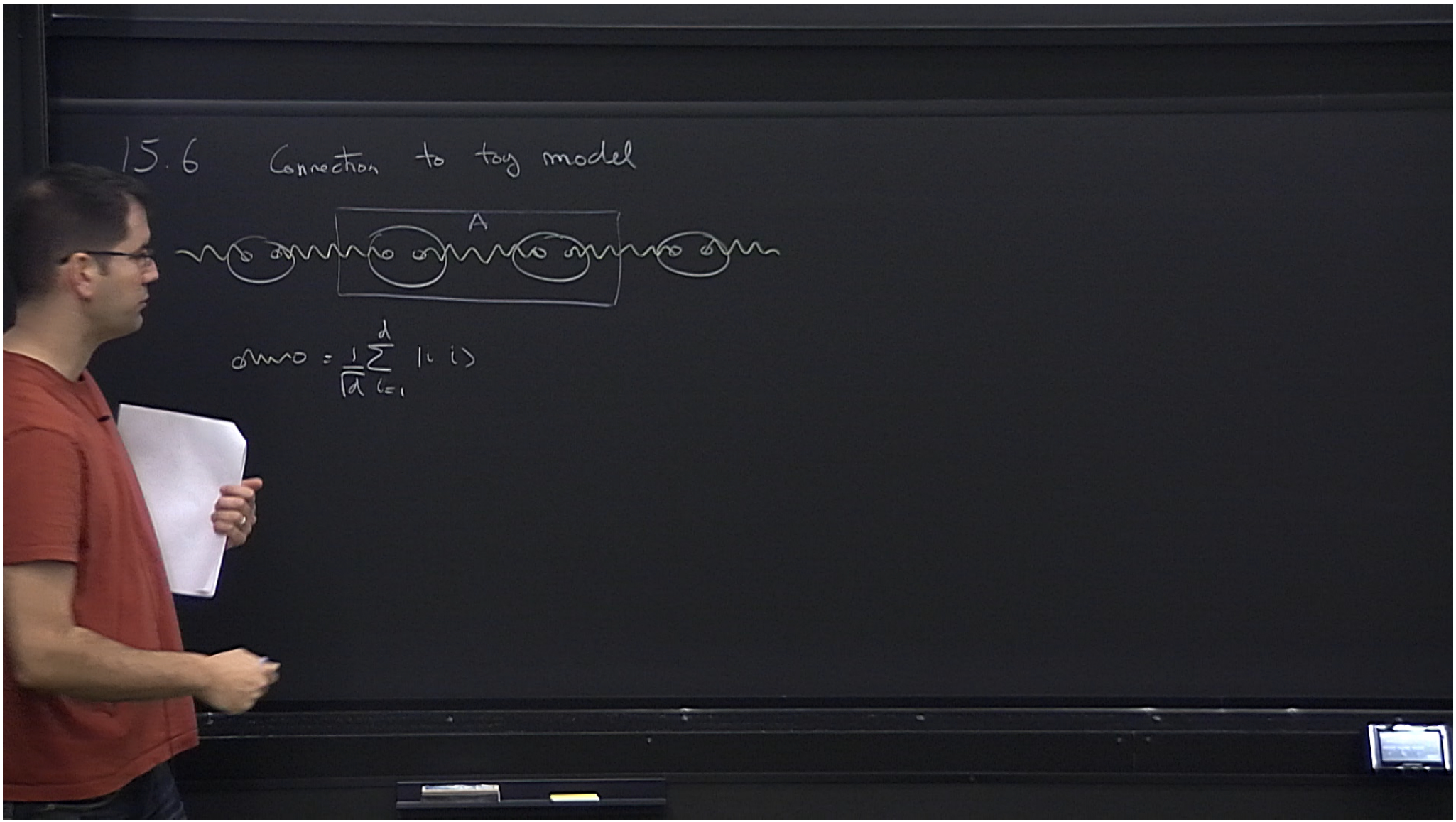

$(\mathbb{C}^d)^{\otimes N}$
$\chi=3$
$\chi=2$
$\chi=1$
product state

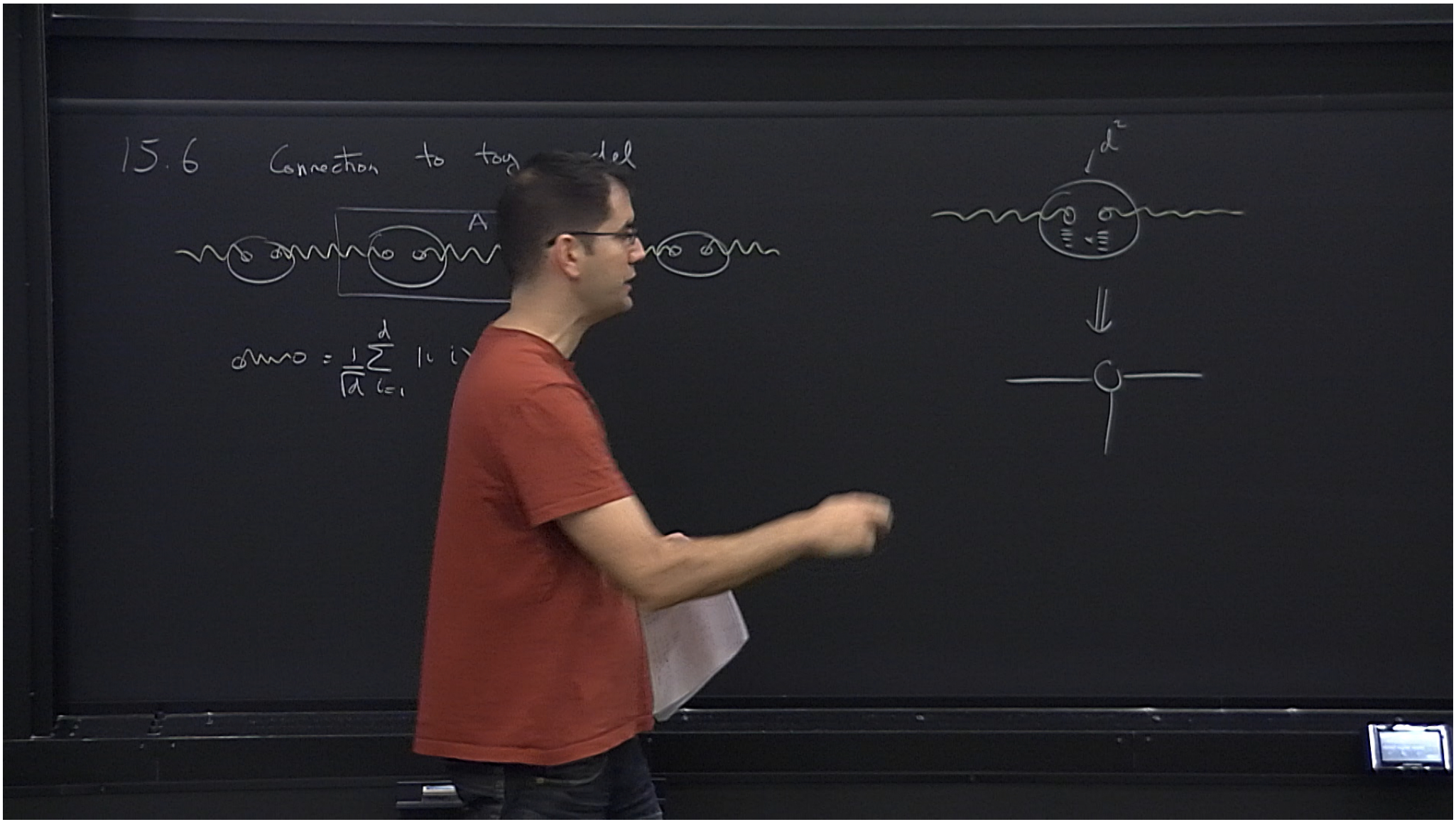

$\chi = d^{N/2}$
$(\mathbb{C}^d)^{\otimes N}$
$\chi = 3$
$\chi = 2$
$\chi = 1$
product state

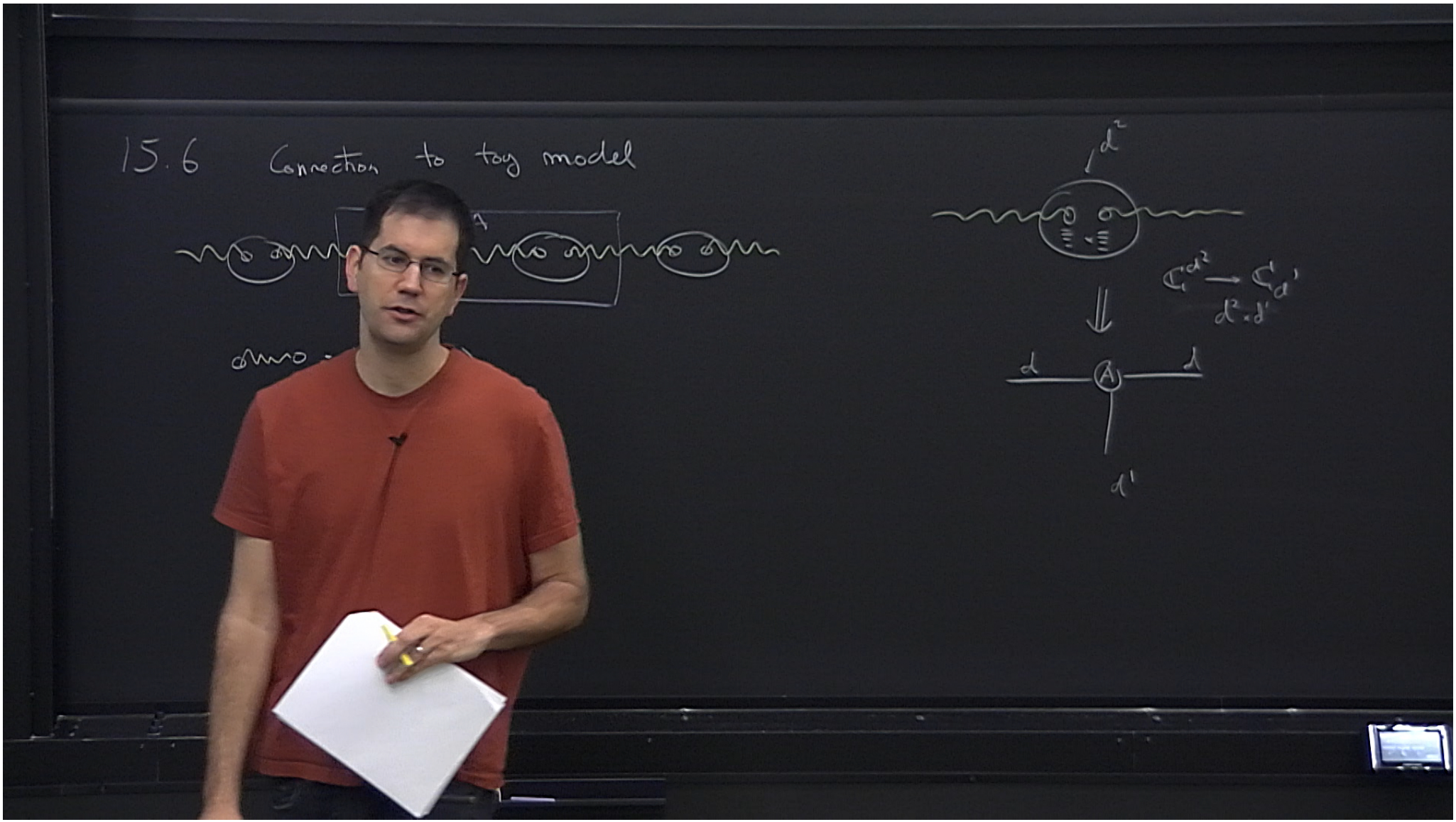

$\chi = d^{N/2}$
$(\mathbb{C}^d)^{\otimes N}$
$\chi = 3$
$\chi = 2$
$\chi = 1$
product state
ground states of 1D Hamiltonians
$\rightarrow$ gap $\Delta \equiv E_1 - E_0 > 0$

ground states of 1D Hamiltonians
→ gap Δ ≡ E₁-E₀ > 0
log(Pα)
10⁰
10⁻¹
10⁻³
α
χ = d^{N/2}
χ=3
χ=2
χ=1
product state

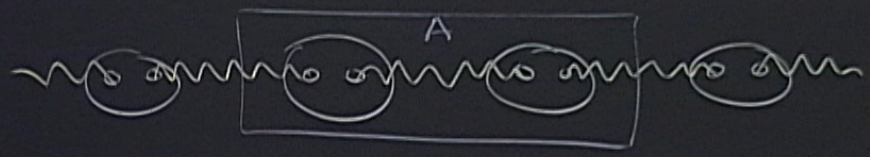

ground states of 1D Hamiltonians
→ gap Δ ≡ E₁-E₀ > 0
χ=3
χ=2
χ=1
product state
truncated SVD
χ_eff


15.6 Connection to toy model
A
$\sum_{i=1}^{d} |i\, i\rangle$


15.6 Connection to toy model
A
d^2


15.6 Connection to toy model
d^2
$\mathbb{C}_1^{d^2} \longrightarrow \mathbb{C}_{d'}^{1}$
$d^2 \cdot d'$
d
d
d'

15.6 Connection to toy model



$$\text{[illegible]} = \frac{1}{\sqrt{\chi}} \sum_{i=1}^{\chi} |i\, i\rangle$$

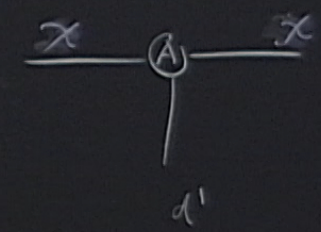
TOY MODEL

$$\mathbb{C}^{\chi^2} \longrightarrow \mathbb{C}^{d'}$$

$$\chi^2 \cdot d'$$

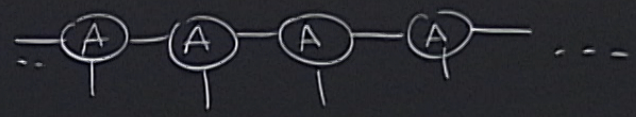
$$\Downarrow$$

MPS



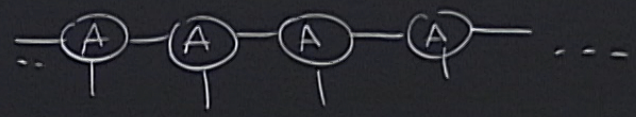
15.7 Low energy excited states

MPS ground state

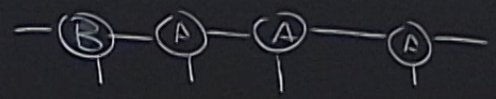


15.7 Low energy excited states

MPS ground state

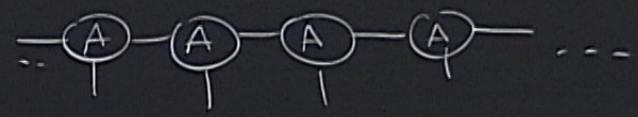


ansatz for excited states

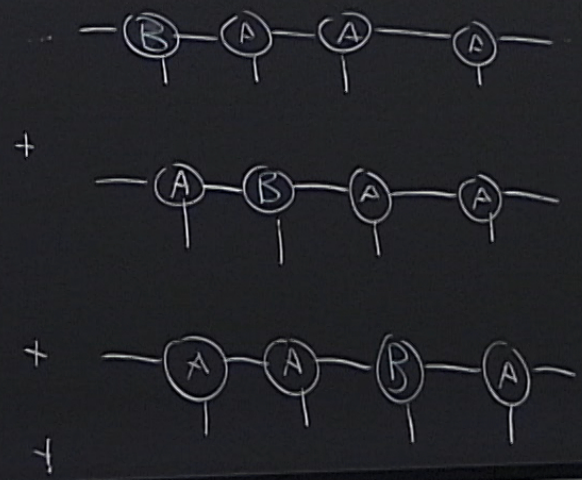


15.7 Low energy excited states

MPS ground state

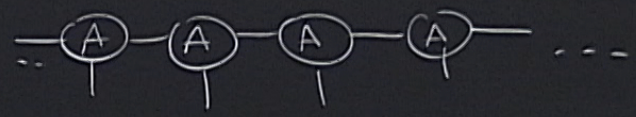


ansatz for excited states

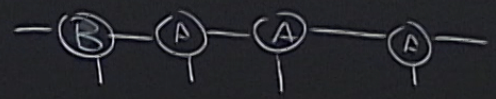


15.7 Low energy excited states

MPS ground state

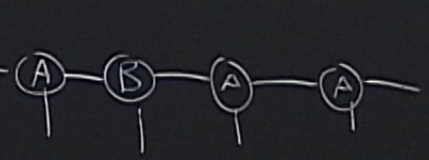


ansatz for excited states

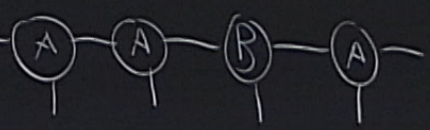


$|\psi(k)\rangle =$

$+ e^{ik}$



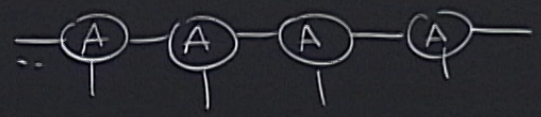
$+ e^{ik_2}$

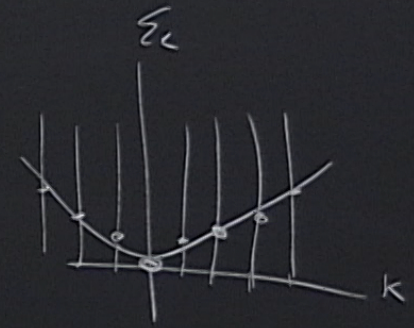


$+ \ldots$

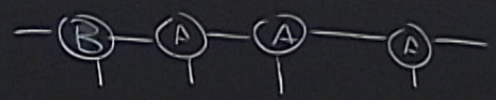
15.7 Low energy excited states

MPS ground state



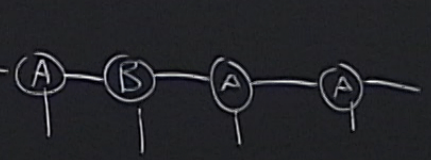


ansatz for excited states

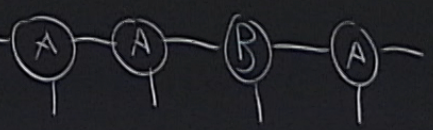


$|\psi(k)\rangle =$

$+ e^{ik}$



$+ e^{ik_2}$



$+ \ldots$

15.8 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

MPS

$$H = \sum_{l=1}^{N} h_{l,l+1}$$

$$e^{iHt} = e^{iH_{\ldots}}$$

Trotter decomposition

$$e^{iHt} = \left(e^{iH_{even}\,\delta t}\, e^{iH_{odd}\,\delta t}\right)^{t/\delta t} + O(\delta t^2)$$

Trotter decomposition

158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

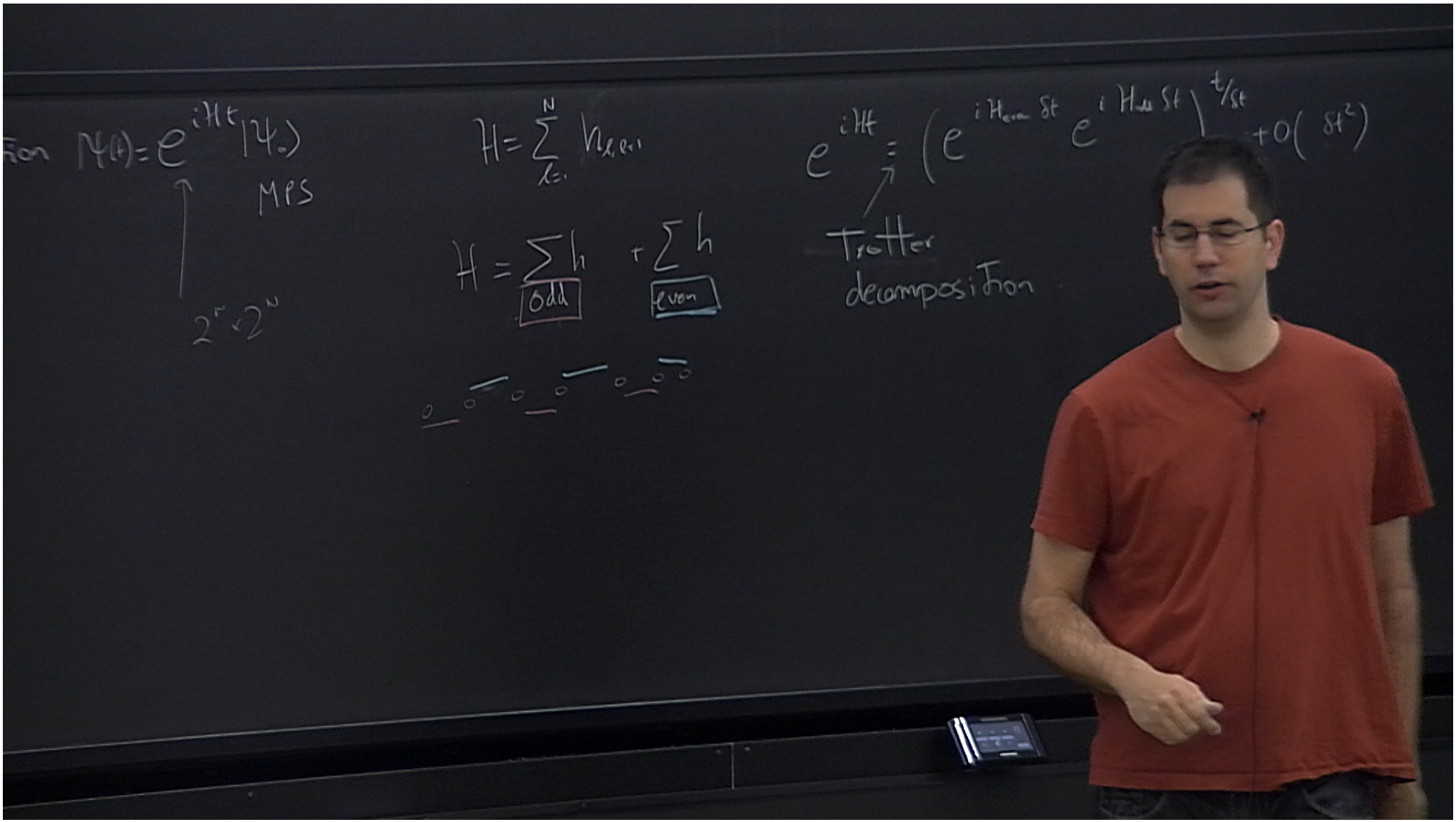
MPS

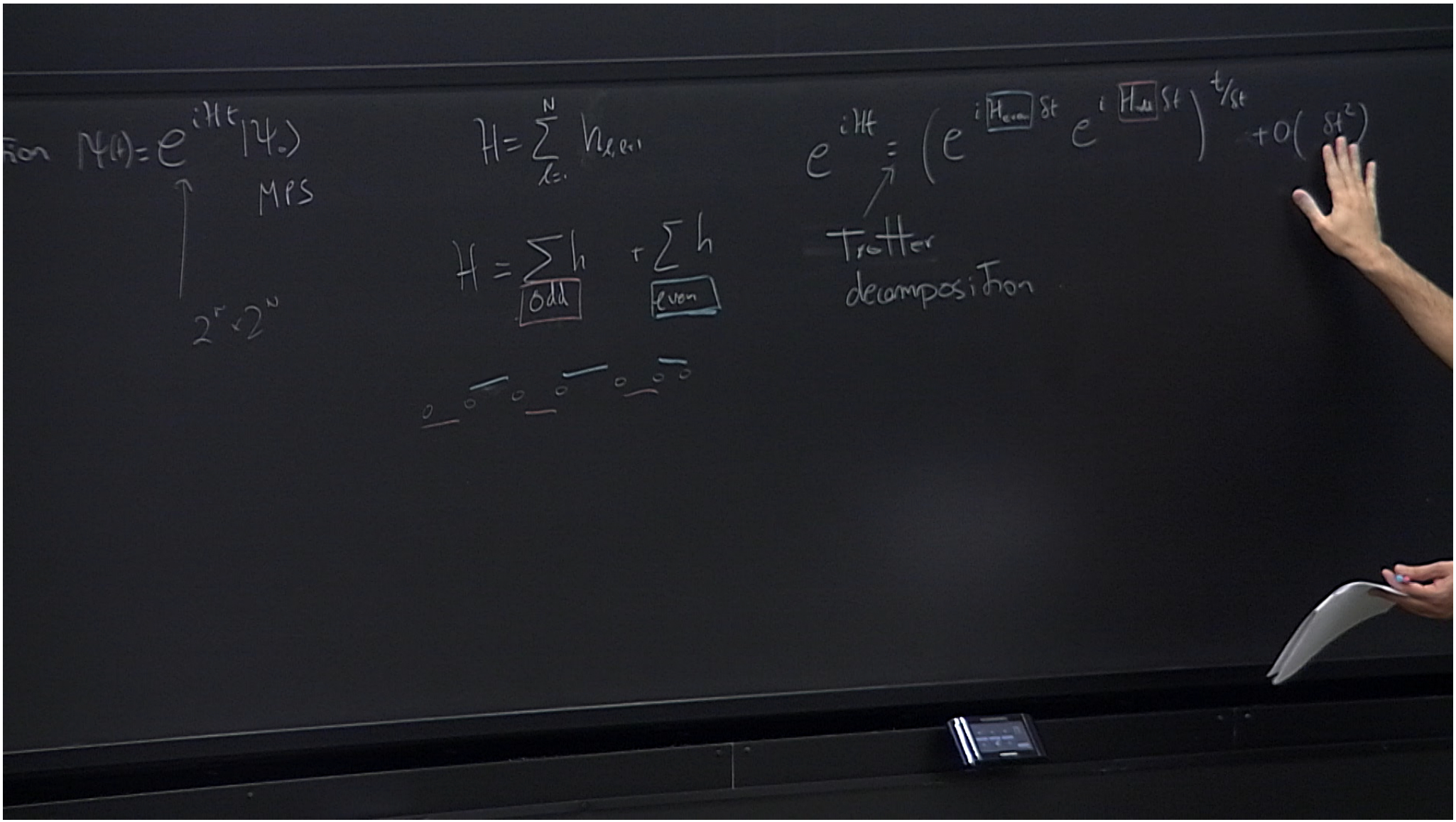
$2^N \times 2^N$

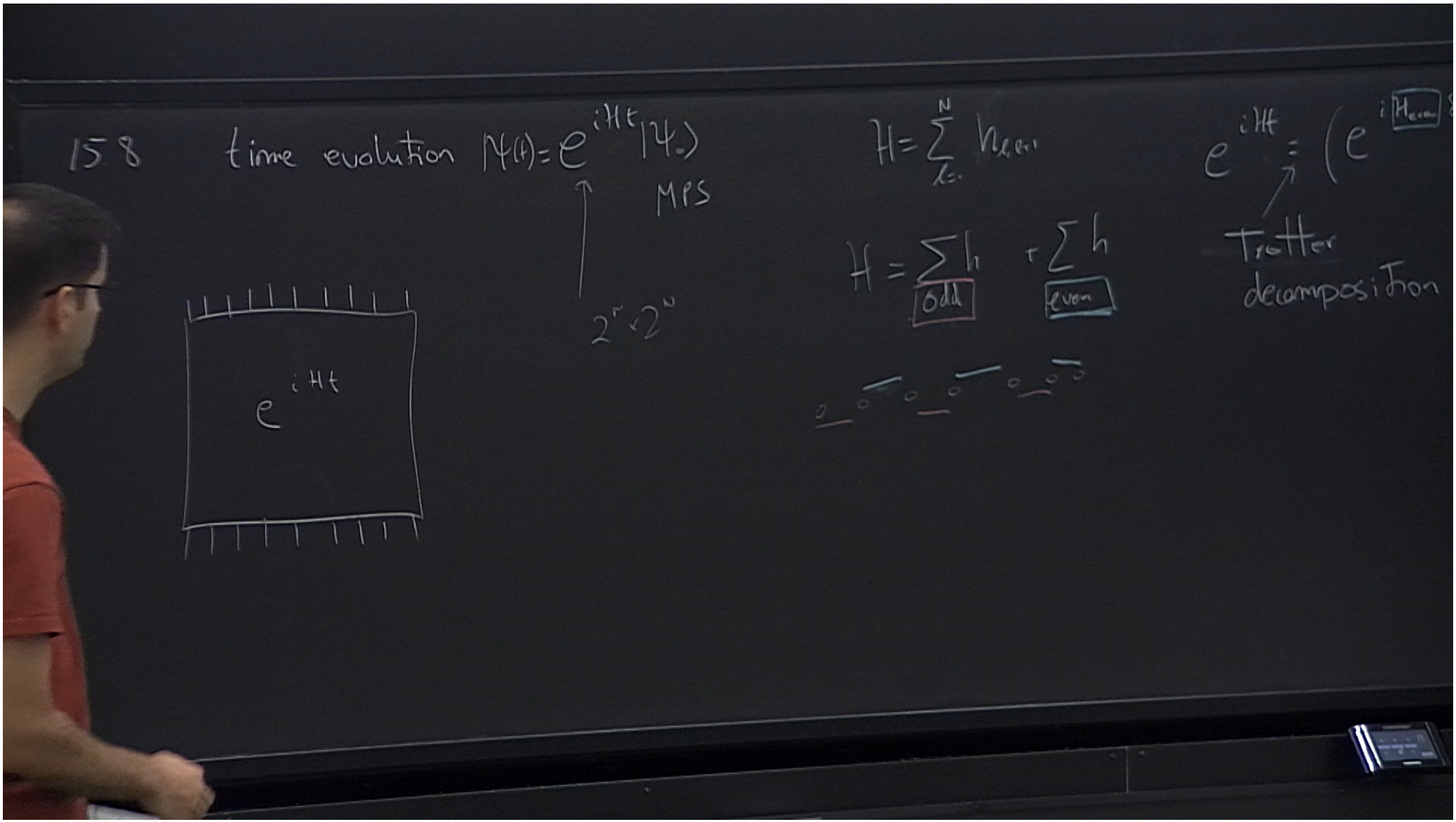
$H = \sum_{k=1}^{N} h_{k,k+1}$

$e^{iHt} = (e^{iH_{k,k+1}}$

Trotter decomposition


MPS
Trotter
decomposition


ion $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$
MPS
$2^N \times 2^N$
$H = \sum_{k=1}^{N} h_{k,k+1}$
$H = \sum_{\boxed{\text{odd}}} h + \sum_{\boxed{\text{even}}} h$
$e^{iHt} = \left(e^{i\boxed{H_{even}}\delta t} e^{i\boxed{H_{odd}}\delta t}\right)^{t/\delta t} + O(\delta t^2)$
Trotter decomposition


158
time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$
MPS
$2^N \times 2^N$
e^{iHt}
$H = \sum_{k=1}^{N} h_{k,k+1}$
$H = \sum_{\text{odd}} h + \sum_{\text{even}} h$
$e^{iHt} = (e^{iH_{\text{even}}}$
Trotter decomposition

158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

MPS

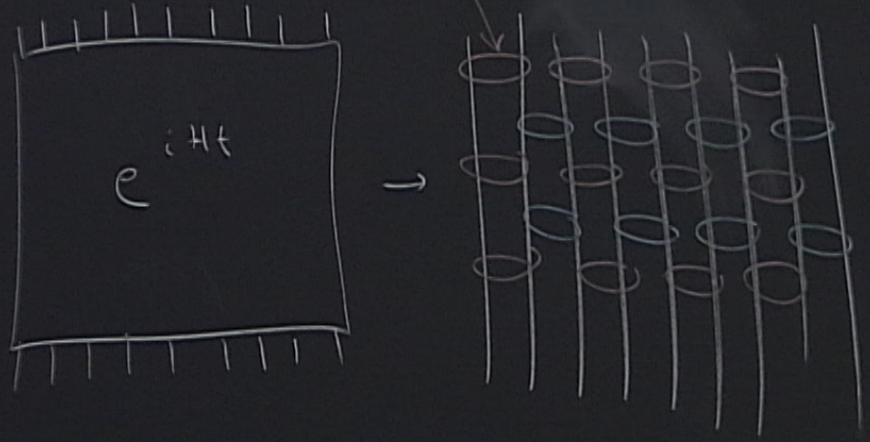
$H = \sum_{k=1}^{N} h_{k,k+1}$

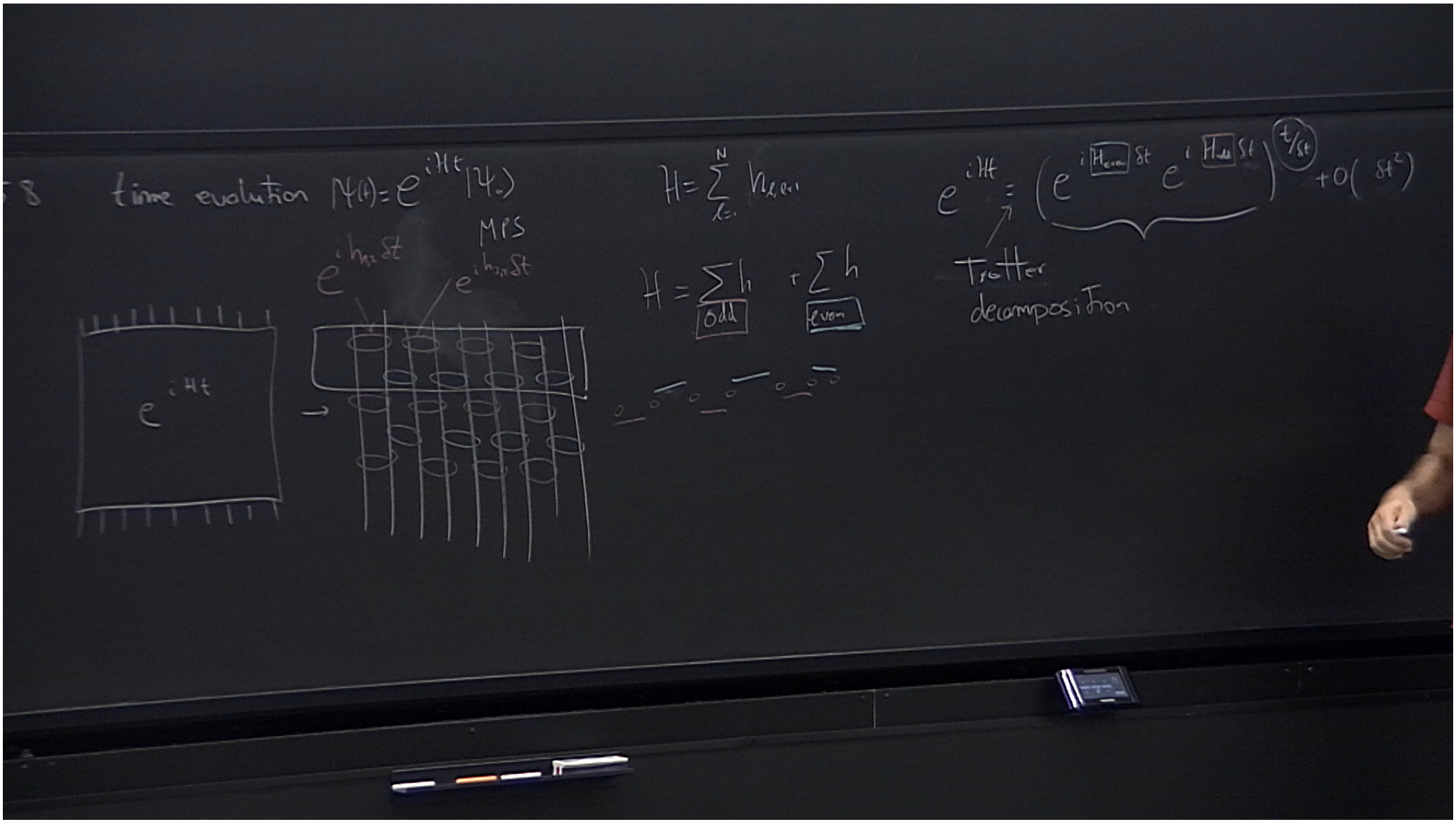
$e^{iHt} = (e^{i\boxed{H_{even}}}$

Trotter decomposition

$e^{ih_{1,2}\delta t}$

$H = \sum_{\boxed{odd}} h + \sum_{\boxed{even}} h$

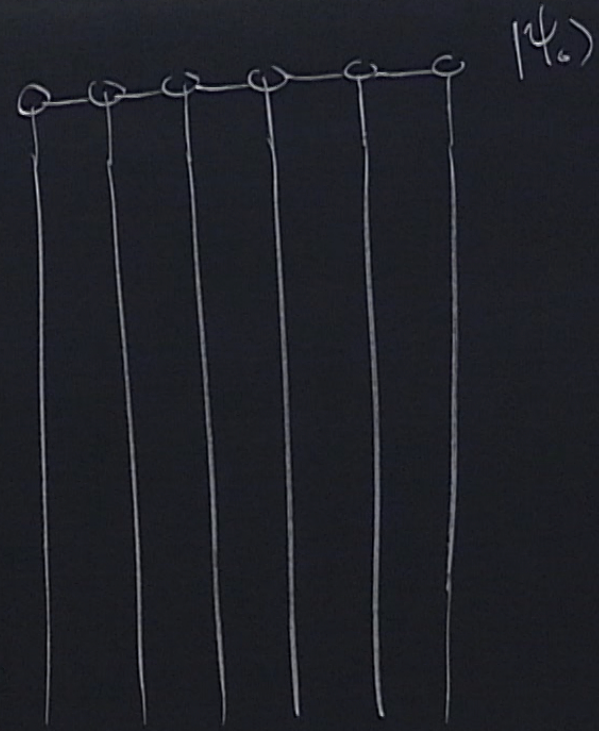


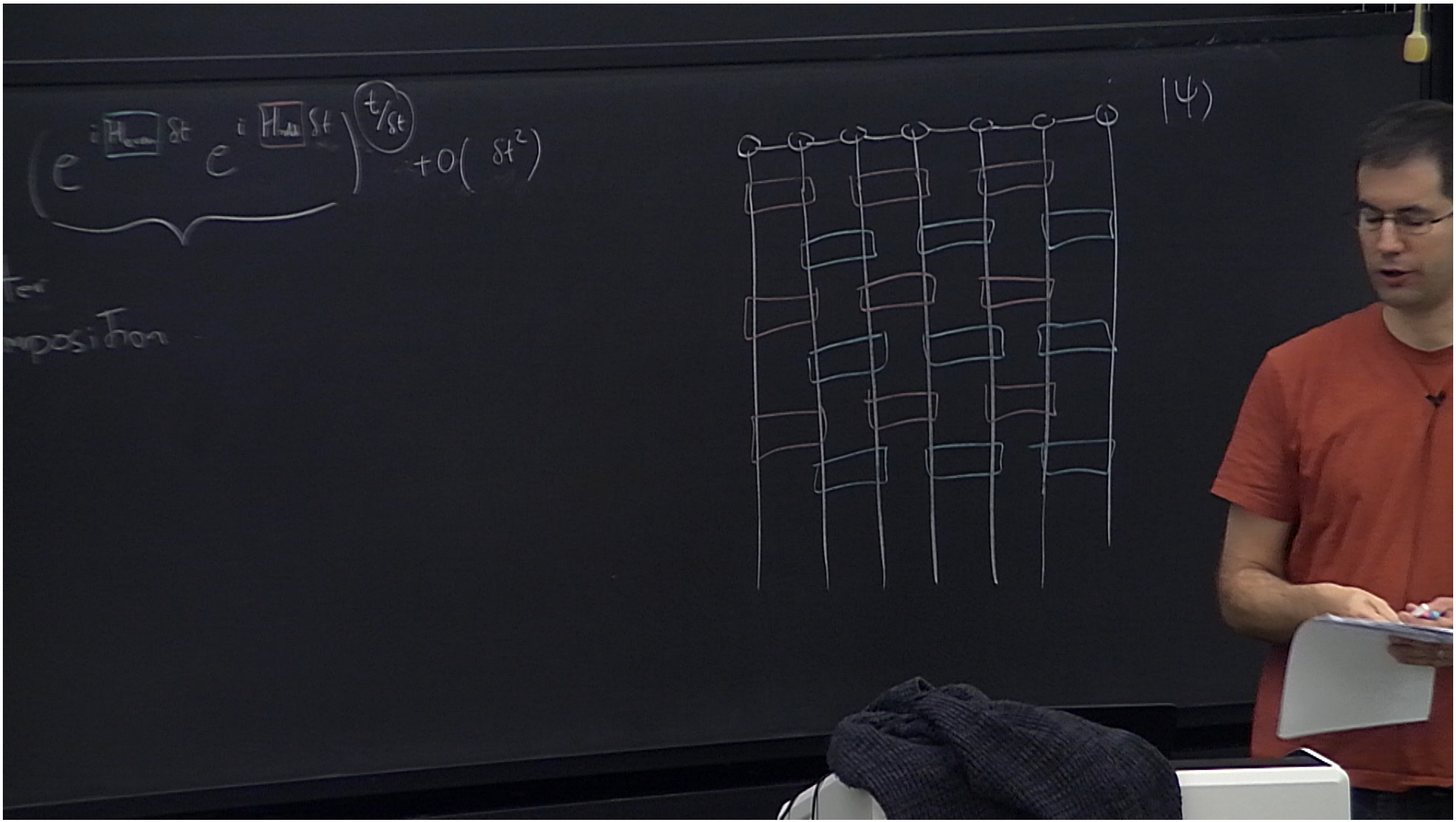

time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$
$H = \sum_{l=1}^{N} h_{l,l+1}$
$e^{iHt} = \left(e^{i H_{even} \delta t} e^{i H_{odd} \delta t}\right)^{t/\delta t} + O(\delta t^2)$
MPS
$e^{i h_{12} \delta t}$
$e^{i h_{23} \delta t}$
e^{iHt}
$H = \sum_{odd} h + \sum_{even} h$
Trotter decomposition

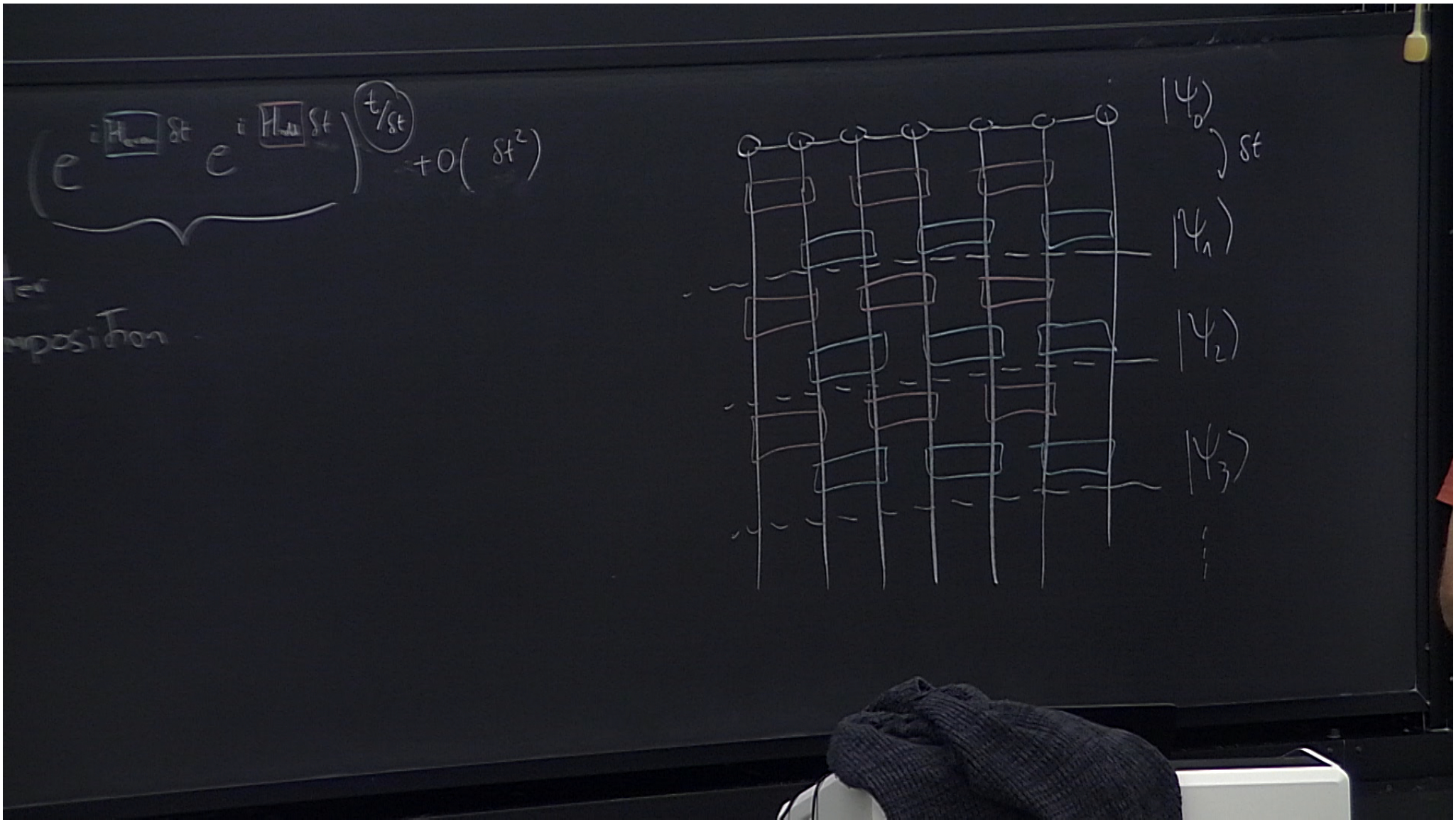
$$\left(\underbrace{e^{i\boxed{H_{\ldots}}\delta t}\, e^{i\boxed{H_{\ldots}}\delta t}}\right)^{t/\delta t} + O(\delta t^2)$$

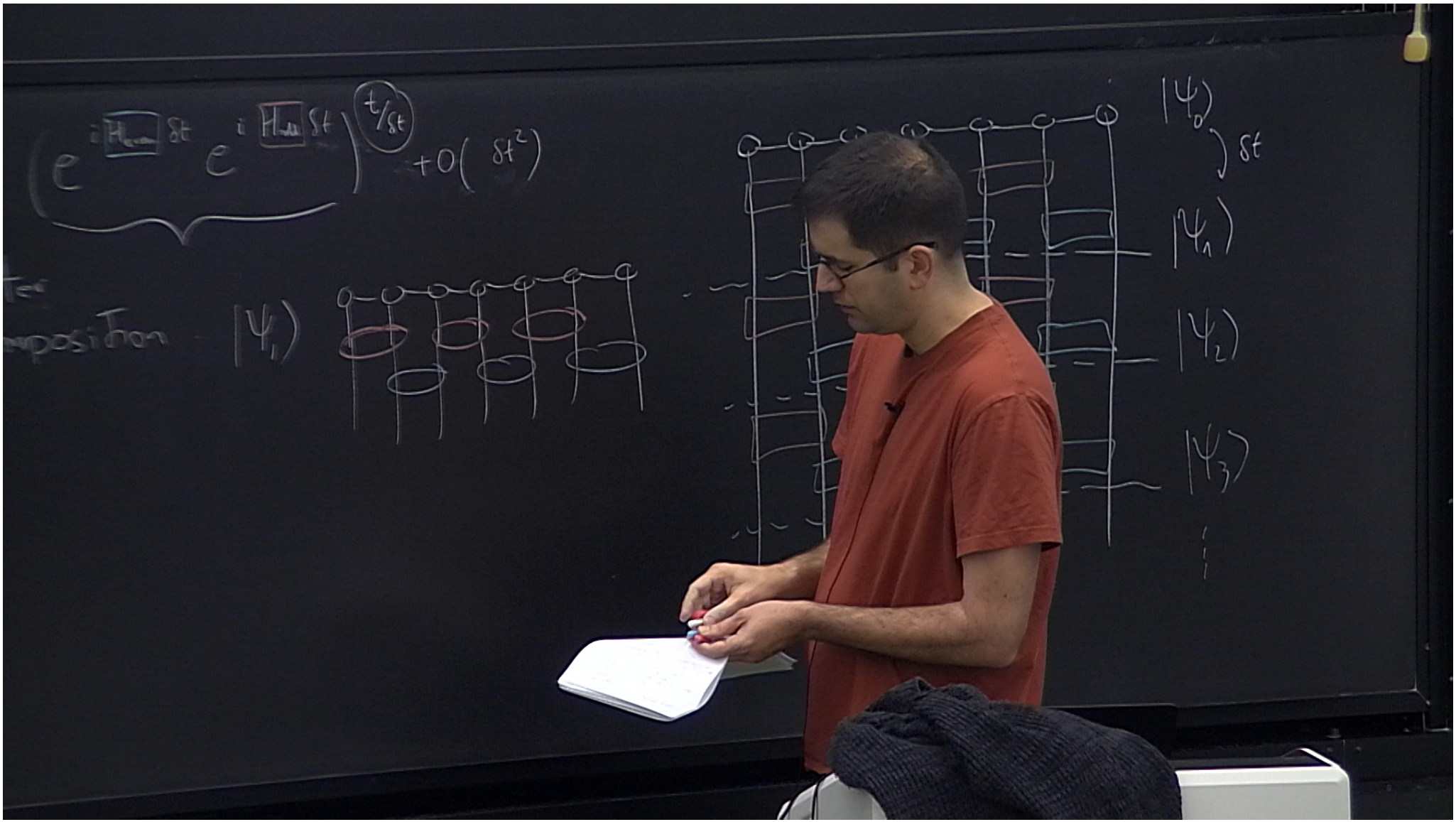
Ter

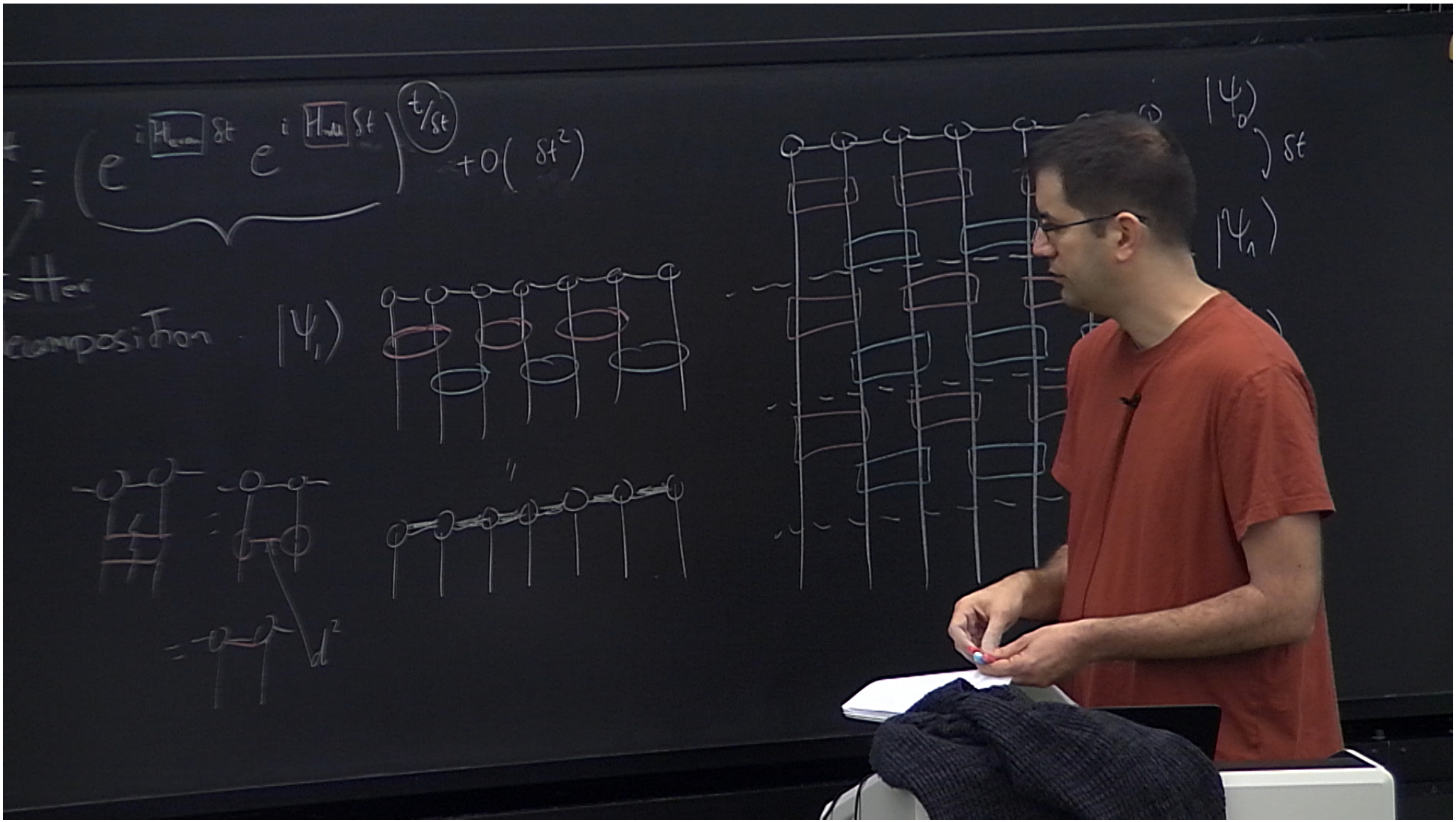
mposition

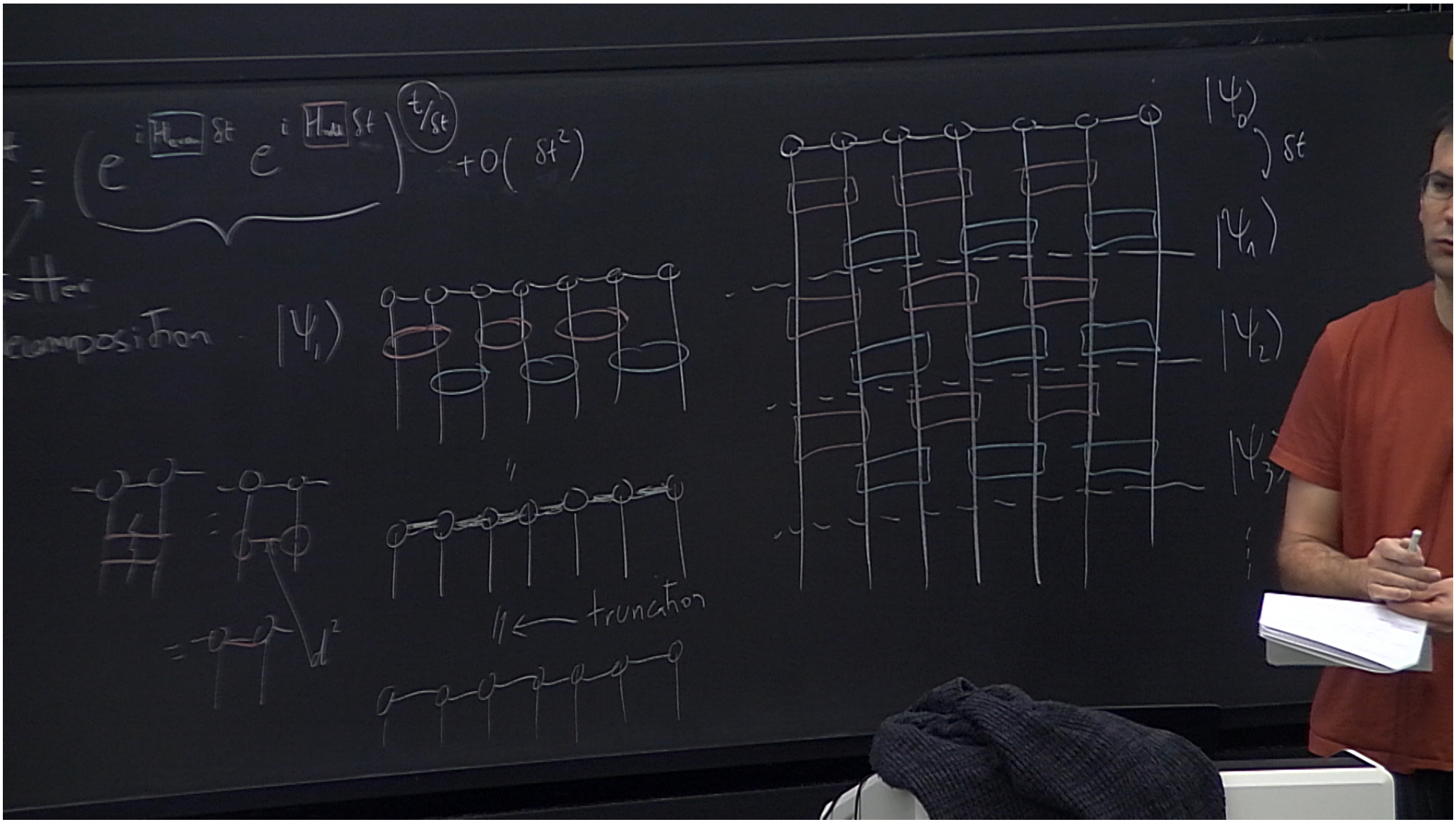



$\left(e^{i H_{\ldots}\delta t} e^{i H_{\ldots}\delta t}\right)^{t/\delta t} + O(\delta t^2)$
$|\psi\rangle$


$\left(e^{i H_{\ldots} \delta t} e^{i H_{\ldots} \delta t}\right)^{t/\delta t} + O(\delta t^2)$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$


$\left(e^{i H_{\ldots} \delta t} e^{i H_{\ldots} \delta t}\right)^{t/\delta t} + O(\delta t^2)$
$|\psi_1\rangle$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$


$= \left(e^{i H_{even} \delta t} e^{i H_{odd} \delta t} \right)^{t/\delta t} + O(\delta t^2)$
better
decomposition
$|\psi_1\rangle$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$

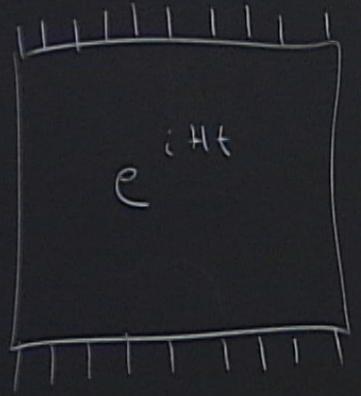

$= \left(e^{i H_{even} \delta t} e^{i H_{odd} \delta t} \right)^{t/\delta t} + O(\delta t^2)$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
$|\psi_1\rangle$
d^2
truncation

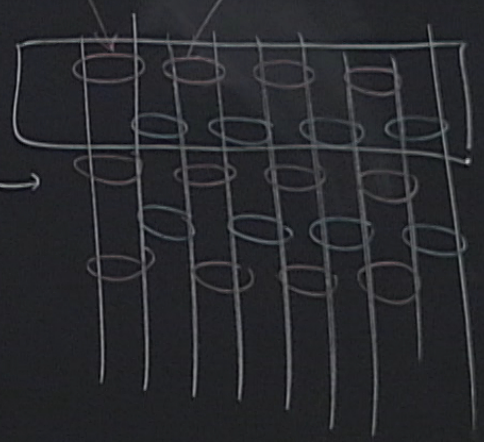
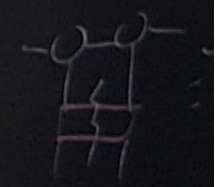
158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

$H = \sum_{k=1}^{N} h_{k,k+1}$

$e^{iHt} = \left(e^{i\boxed{H_{even}}}\right.$

Trotter decomposition





$H = \sum_{\boxed{odd}} h + \sum_{\boxed{even}} h$