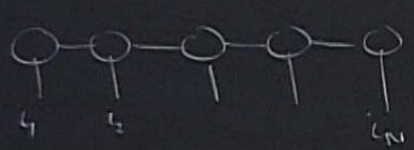
Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 10

Date: Feb 13, 2017 09:00 AM

URL: http://pirsa.org/17020046

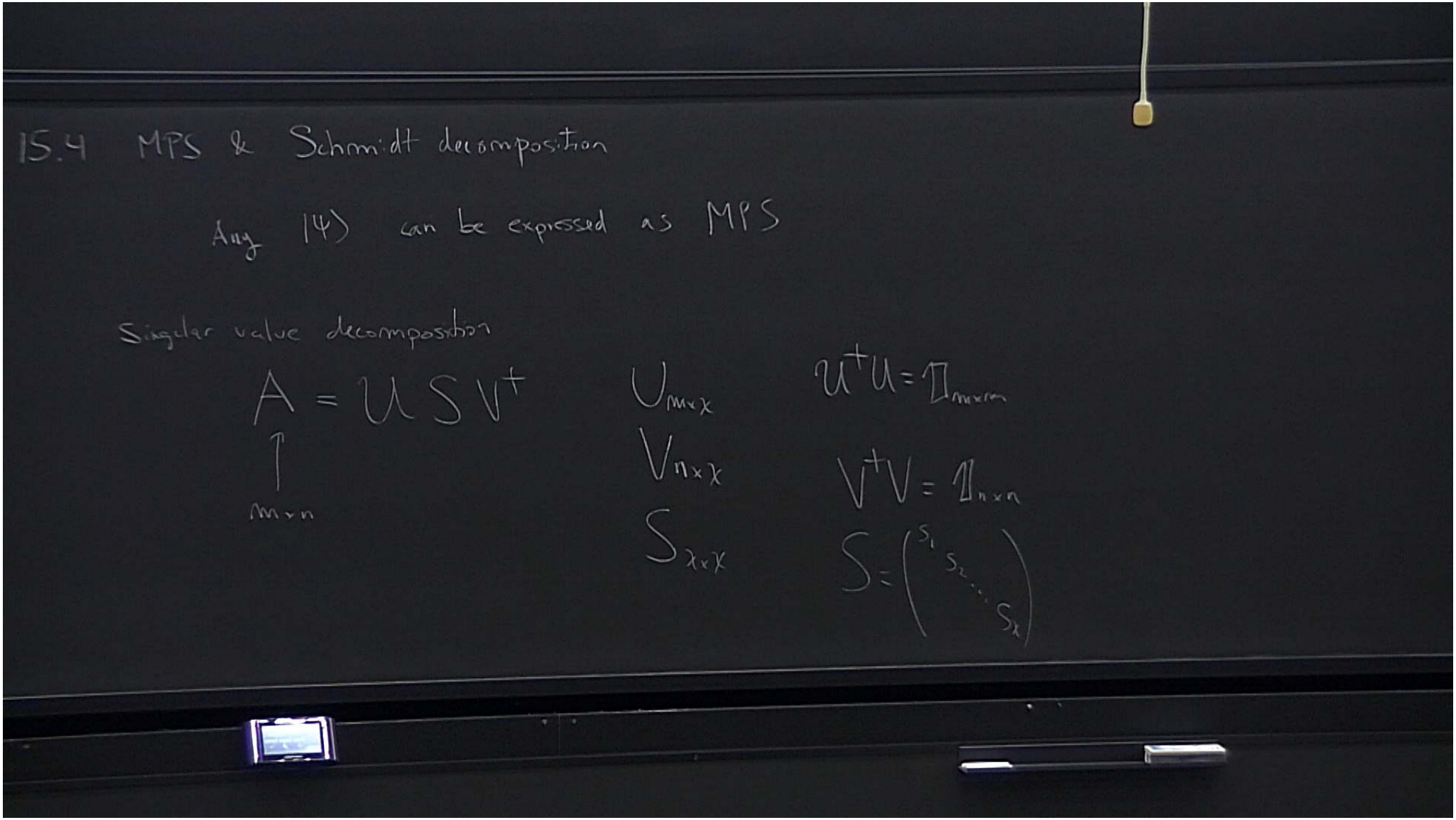
Abstract:

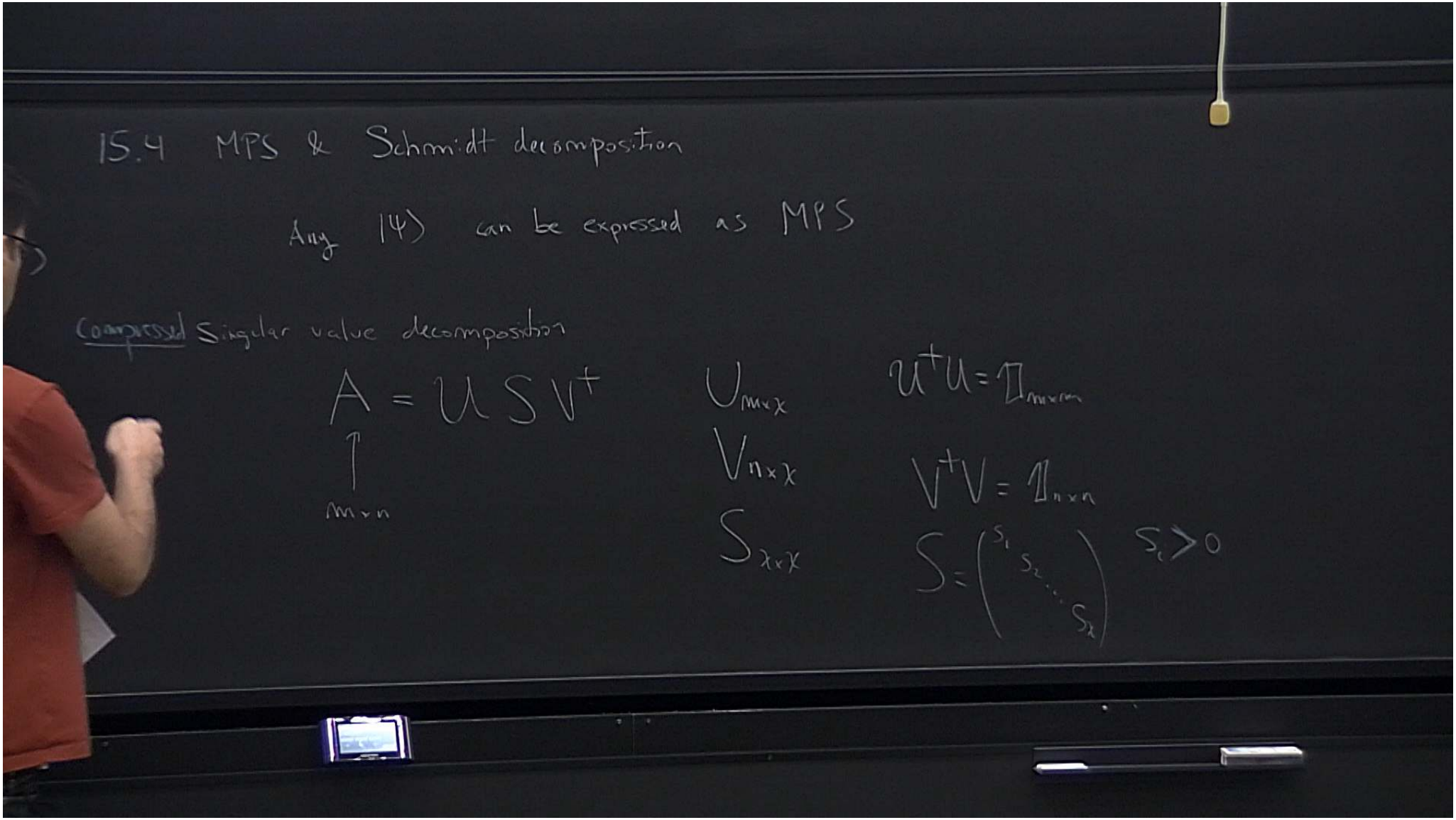
lecture 16? MPS continued 15.4 MPS & Schmidt decomposition

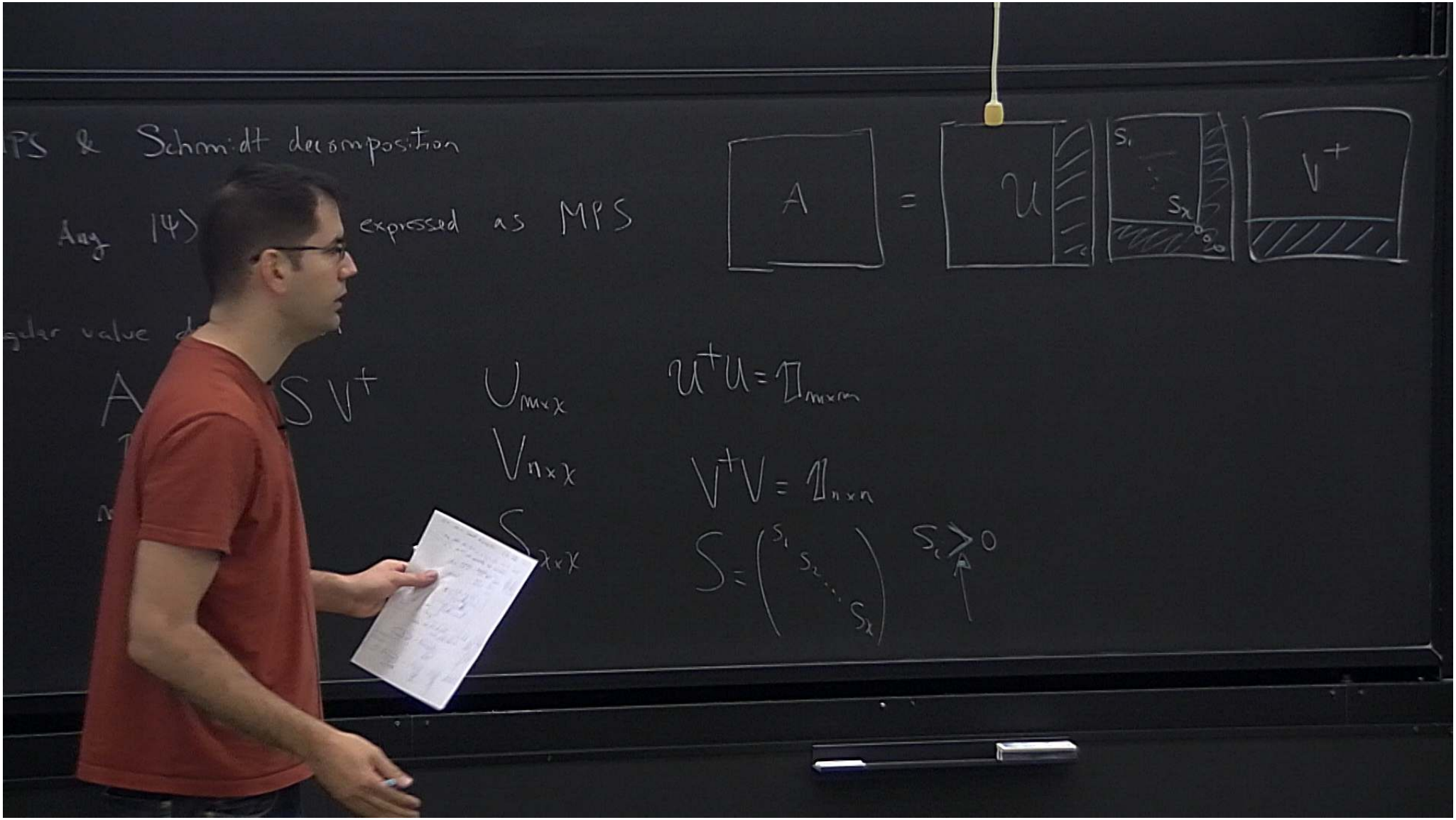


$$|\psi\rangle = \sum_{i_1, i_2 \cdots i_N} \psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

$$|\psi\rangle = \sum_{\alpha=1}^{\chi} \lambda_\alpha |\psi_\alpha^L\rangle |\psi_\alpha^R\rangle$$

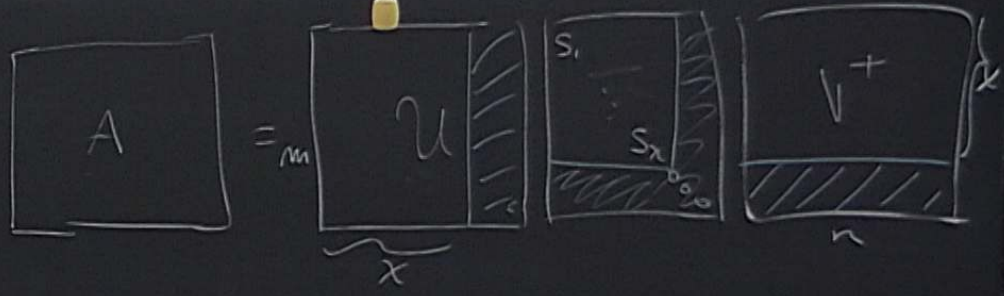

15.4 MPS & Schmidt decomposition
Any |ψ⟩ can be expressed as MPS
Singular value decomposition
A = U S V†
m × n
U_{m×χ}
V_{n×χ}
S_{χ×χ}
U†U = 𝟙_{m×m}
V†V = 𝟙_{n×n}
S = (s1 s2 ... sχ)


15.4 MPS & Schmidt decomposition
Any |Ψ⟩ can be expressed as MPS
Compressed Singular value decomposition
A = U S V†
m×n
U_{m×χ}
V_{n×χ}
S_{χ×χ}
U†U = 𝟙_{m×m}
V†V = 𝟙_{n×n}
S = (s1 s2 ... sχ)
s_i > 0


PS & Schmidt decomposition
Any |ψ⟩ expressed as MPS
A = U S V⁺
$U_{m\times\chi}$
$V_{n\times\chi}$
$S_{\chi\times\chi}$
$U^\dagger U = \mathbb{1}_{\chi\times\chi}$
$V^\dagger V = \mathbb{1}_{\chi\times\chi}$
$S = \begin{pmatrix} s_1 & & \\ & s_2 & \\ & & \ddots \\ & & & s_\chi \end{pmatrix}$
$s_i \geq 0$
A = U S V⁺

PS & Schmidt decomposition

Any $|\psi\rangle$ can be expressed as MPS



gular value decomposition

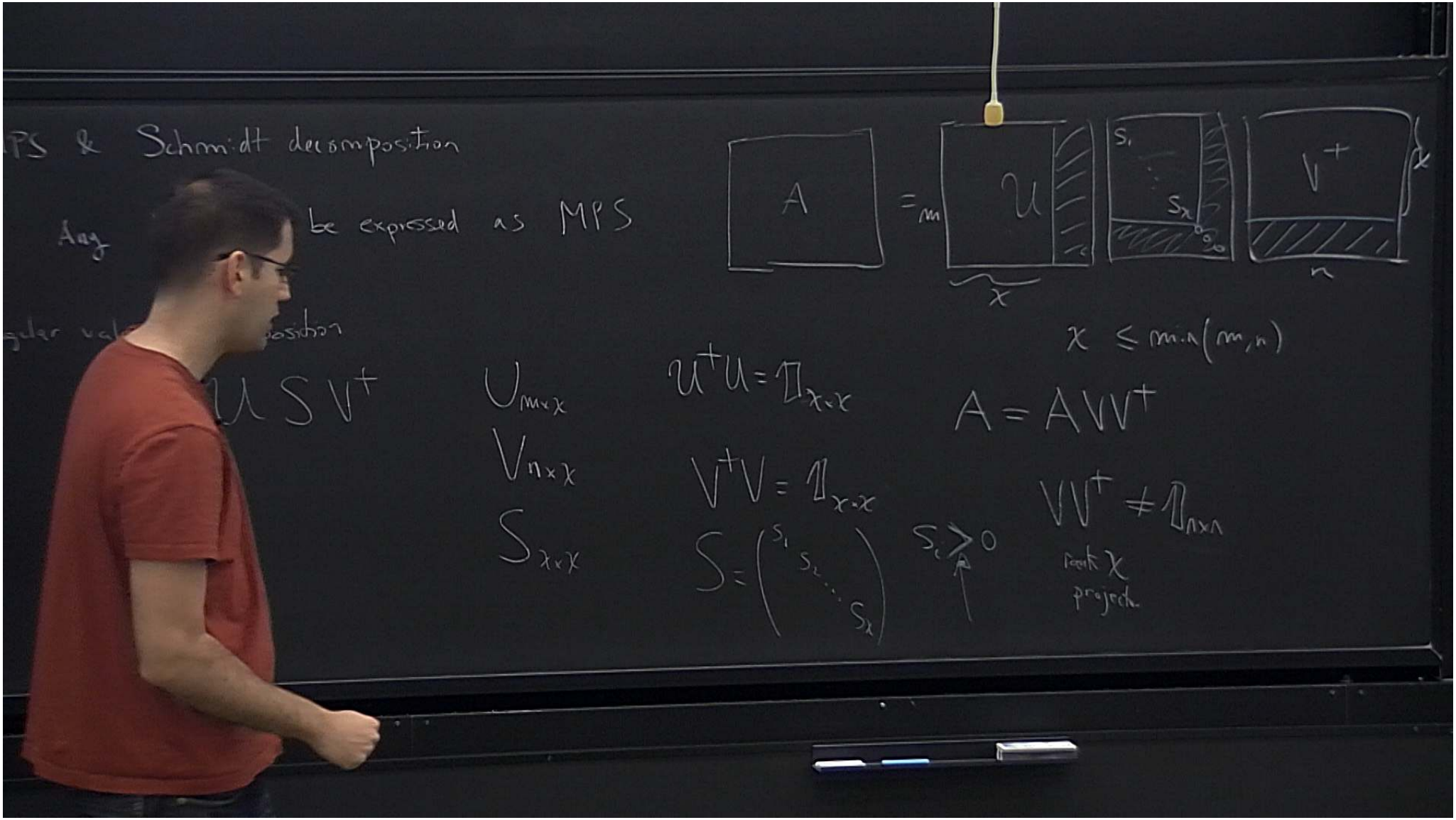
$$A = USV^\dagger \qquad \chi \leq \min(m,n)$$

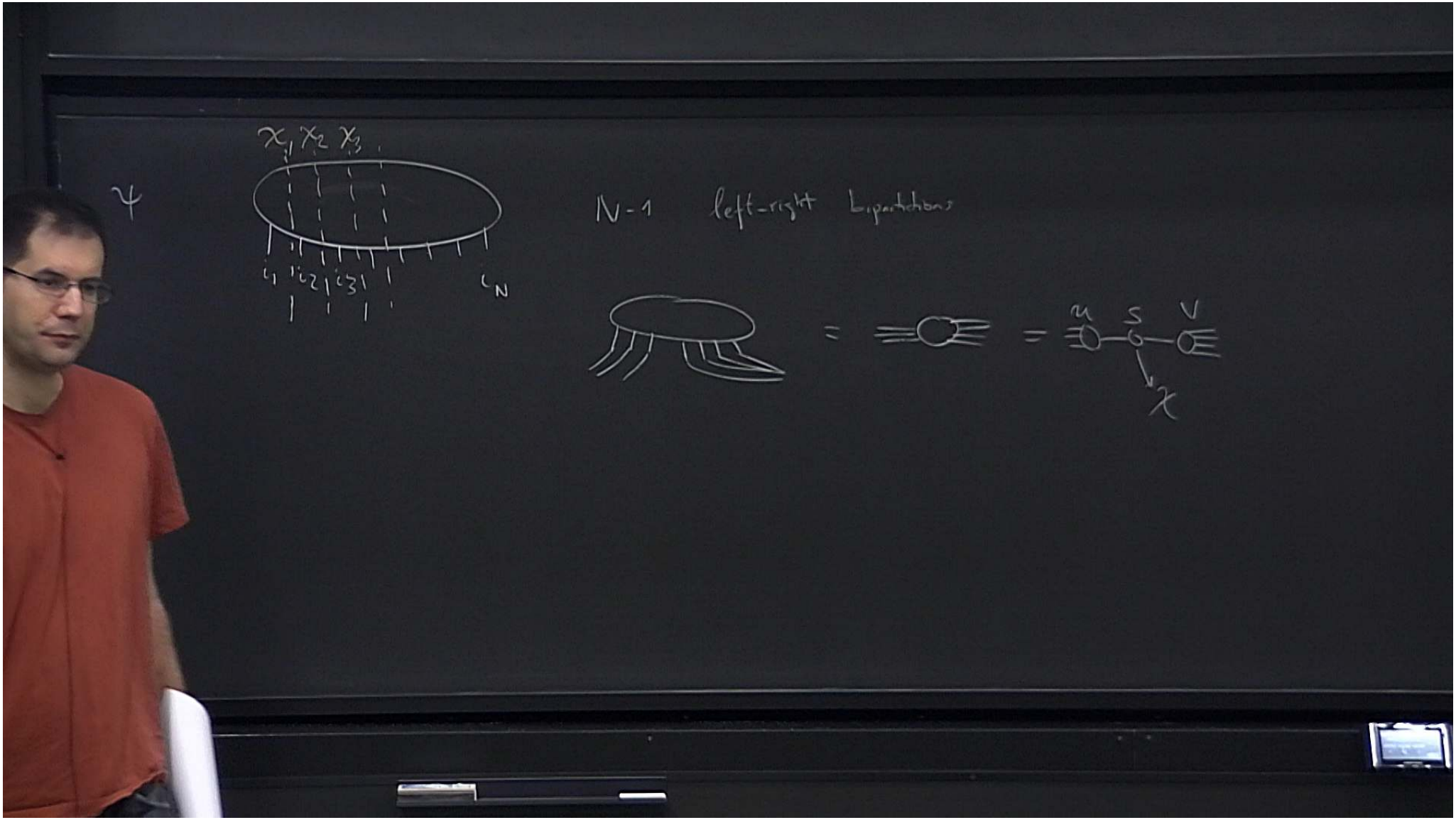
A: $m \times n$

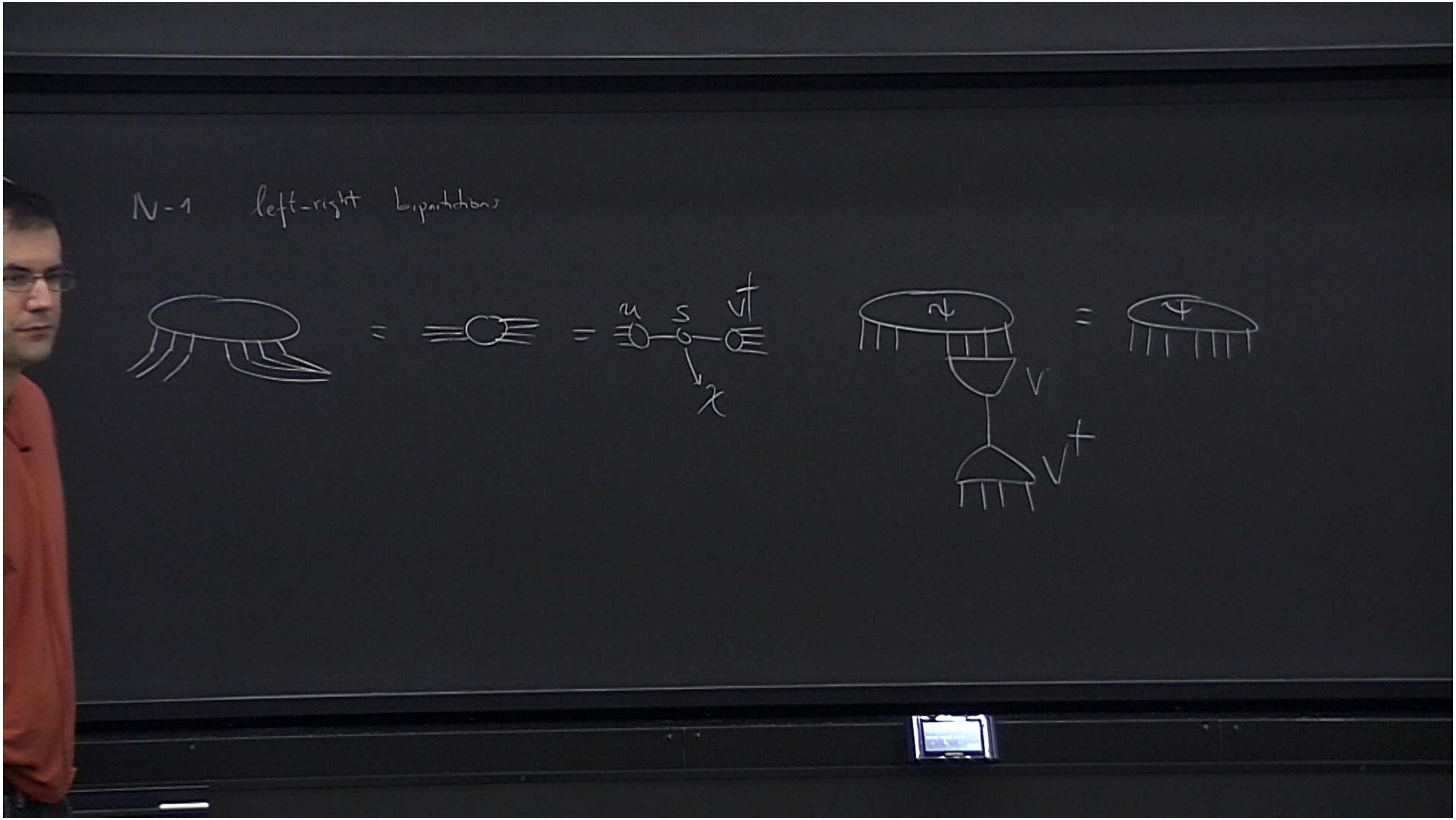
$U_{m\times\chi}$ $\quad U^\dagger U = \mathbb{1}_{m\times m}$

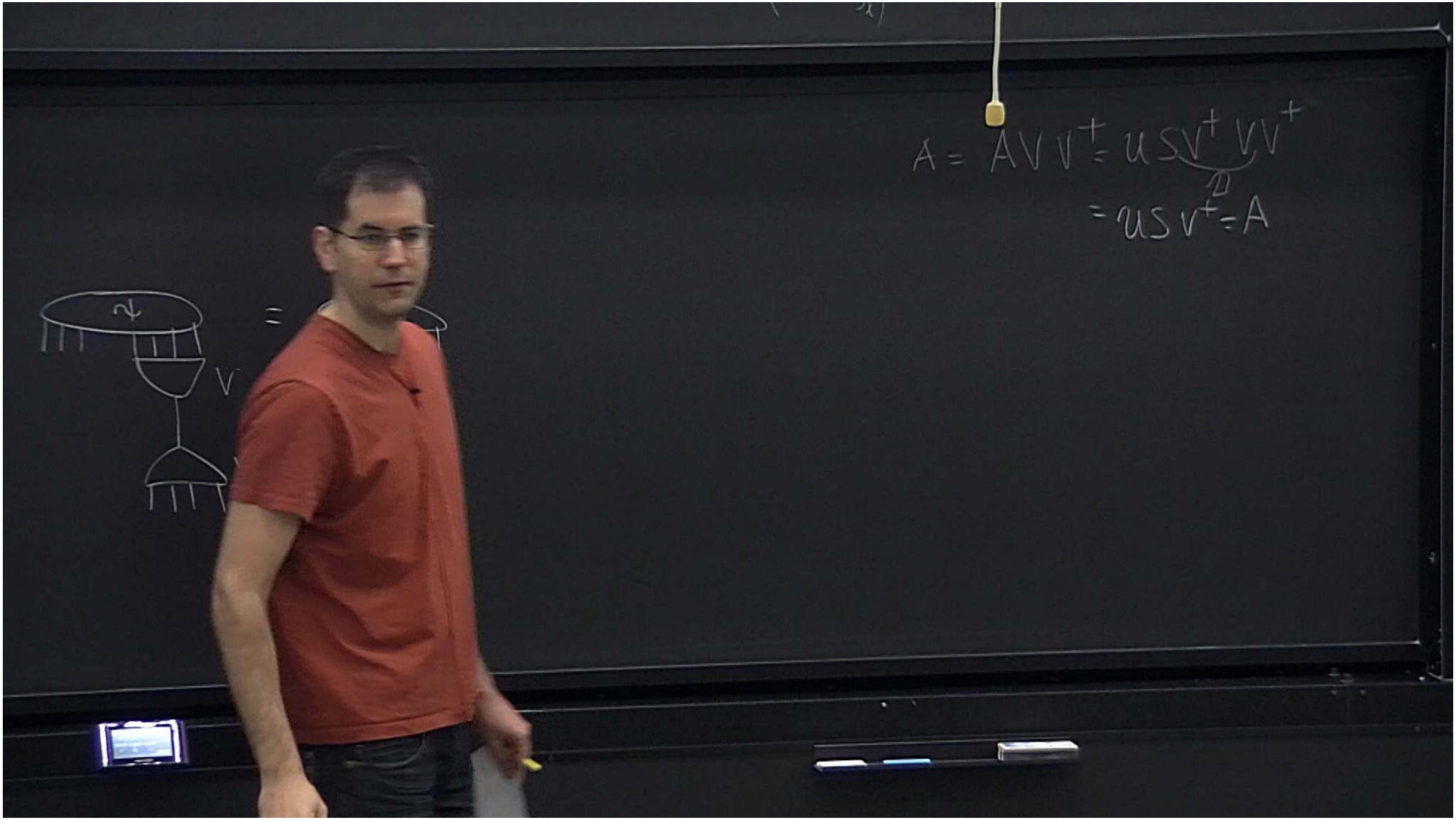
$V_{n\times\chi}$ $\quad V^\dagger V = \mathbb{1}_{n\times n}$

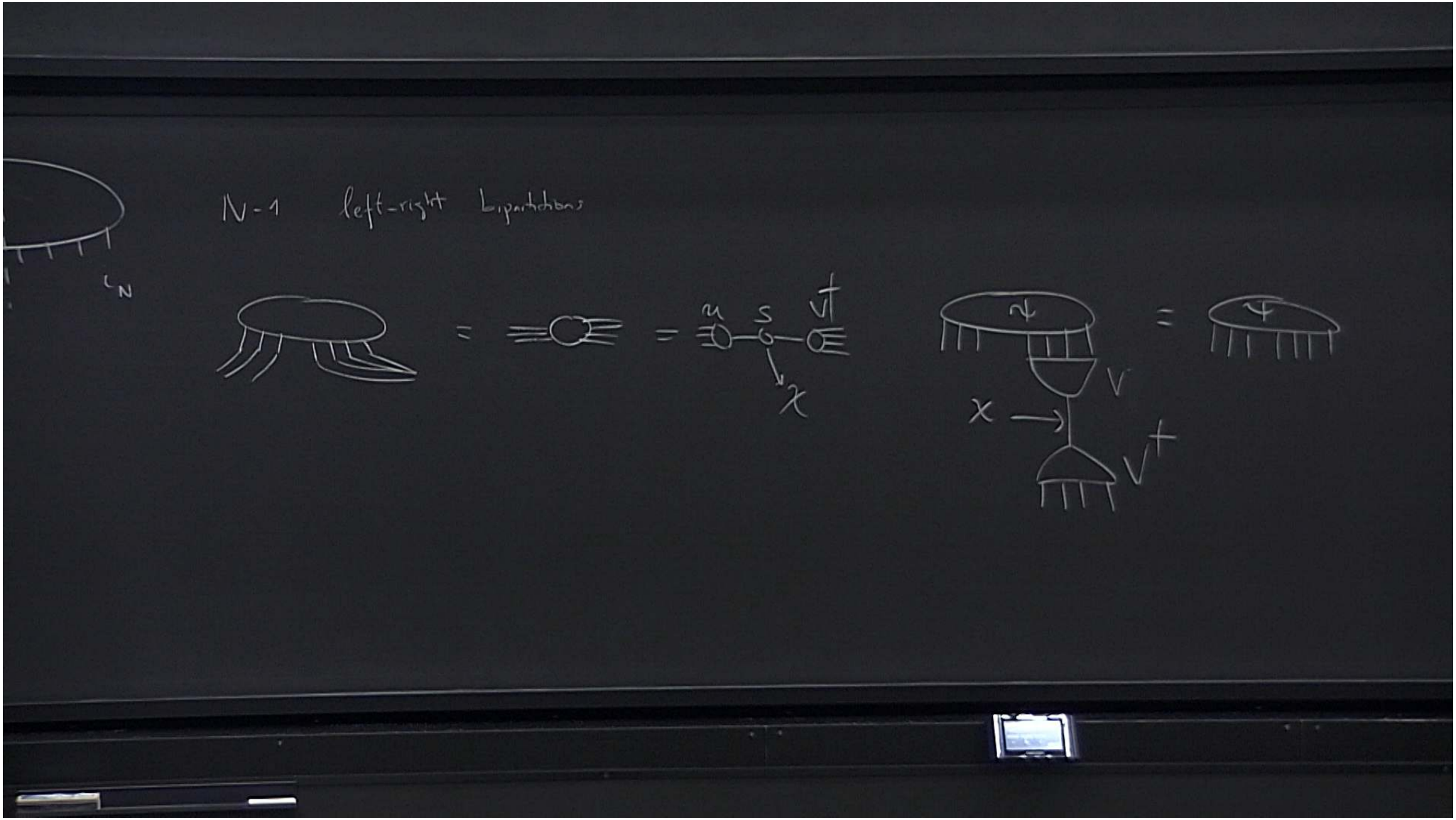
$S_{\chi\times\chi}$ $\quad S = \begin{pmatrix} s_1 & & \\ & s_2 & \\ & & \ddots \\ & & & s_\chi \end{pmatrix}$ $\quad s_i \geq 0$

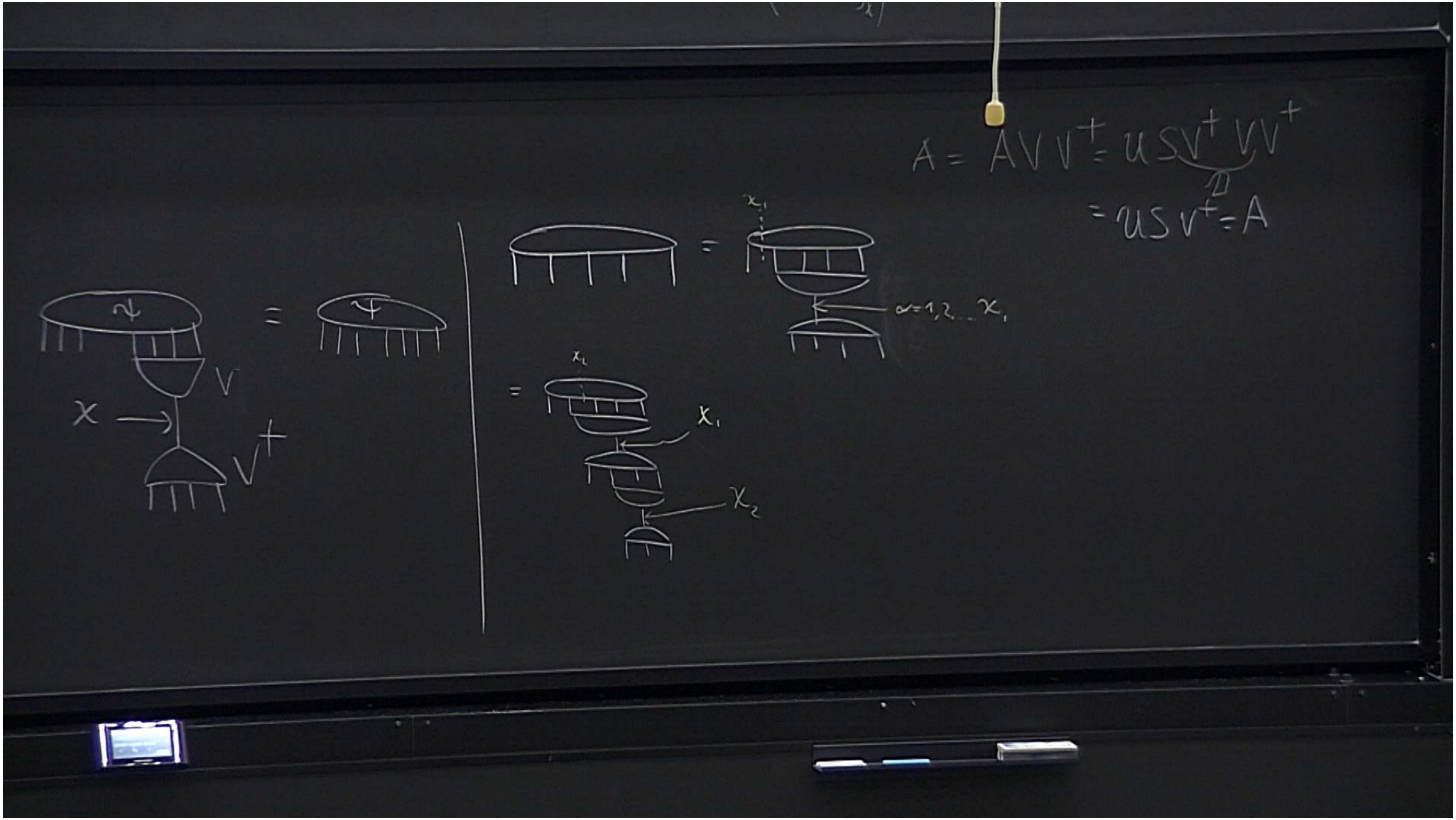

PS & Schmidt decomposition
Any
be expressed as MPS
A
U
V+
χ ≤ min(m,n)
U S V+
U_{m×χ}
V_{n×χ}
S_{χ×χ}
U+U = 1_{χ×χ}
V+V = 1_{χ×χ}
A = AVV+
VV+ ≠ 1_{n×n}
rank χ
projection
S_i > 0

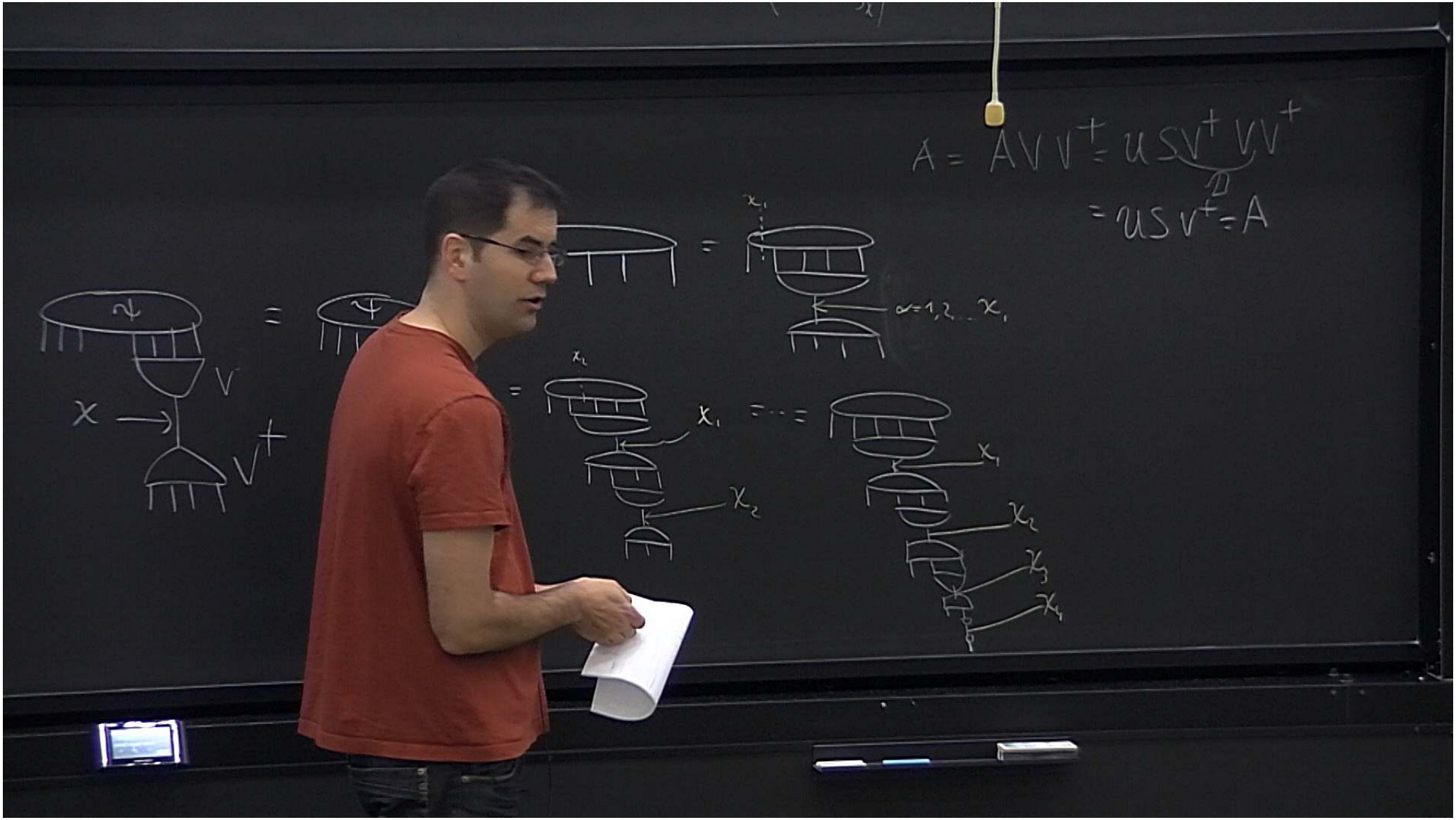

Ψ
x_1 x_2 x_3
i_1 i_2 i_3 i_N
N-1 left-right bipartitions
=
=
u
s
v
χ

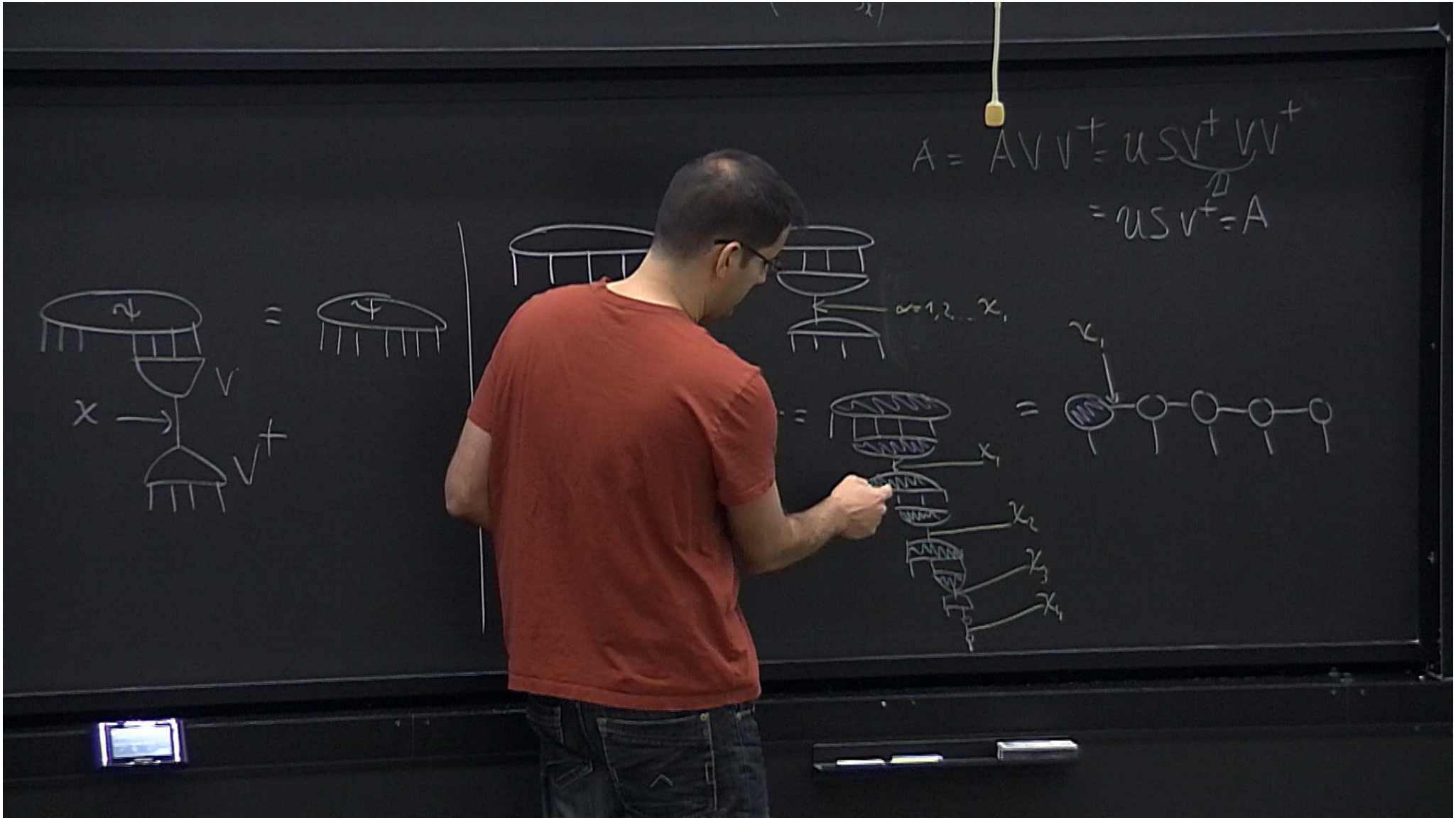

N-1 left-right bipartitions
u
s
v†
χ
ψ
V
V†
Ψ

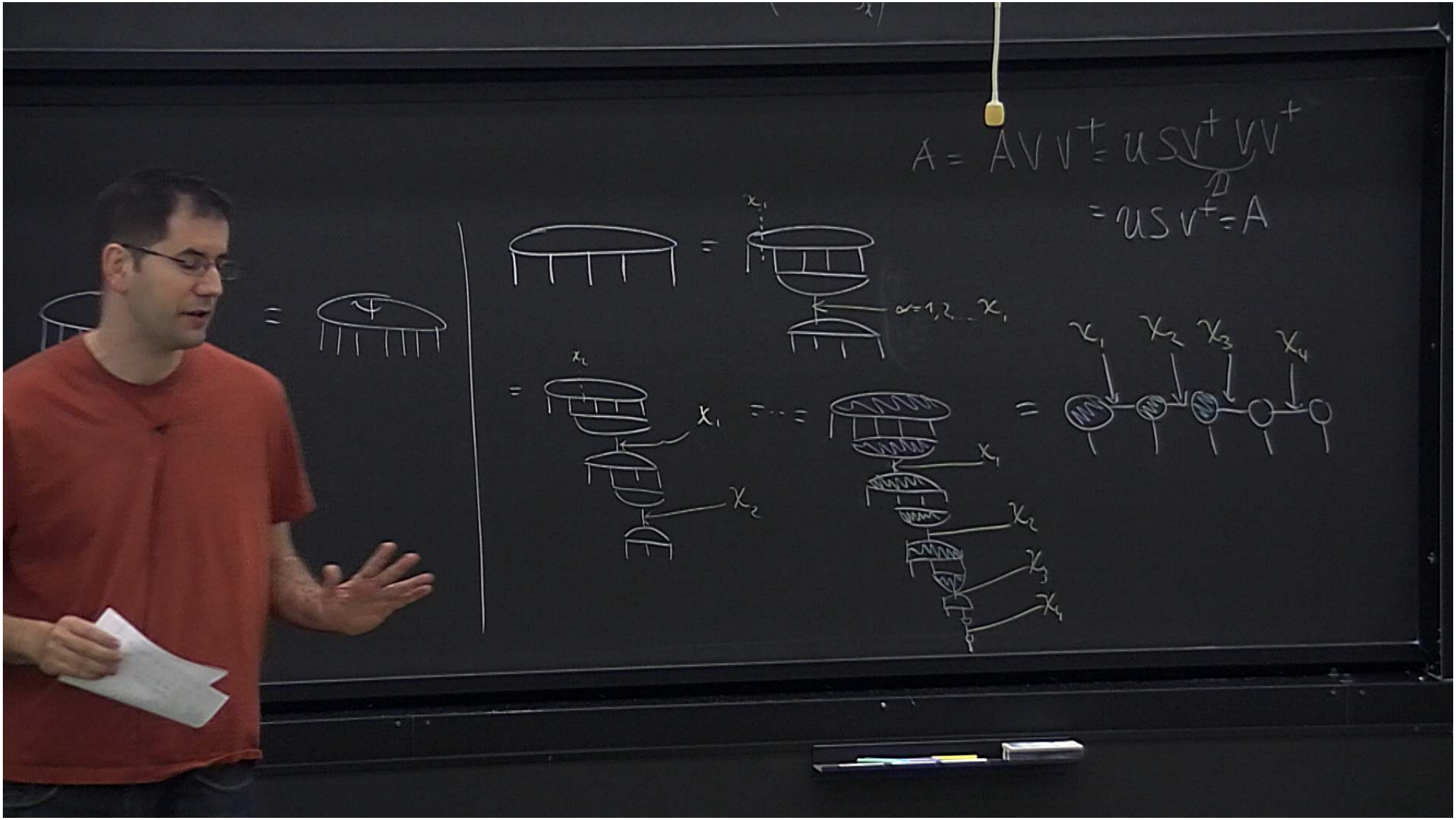

A = AVV† = USV† VV†
= USV† = A
V


N-1 left-right bipartitions
N
=
=
u
s
v†
χ
N
=
V
x
V†

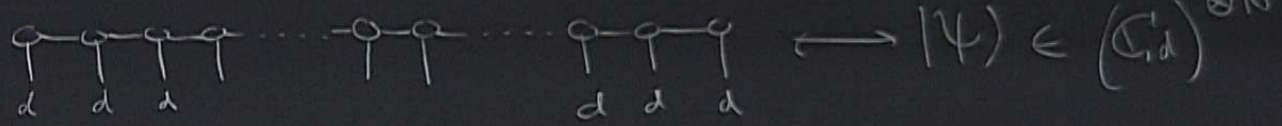

A = AVV† = USV† VV†
= USV† = A
V
X
V†
α=1,2... χ1
χ1
χ2

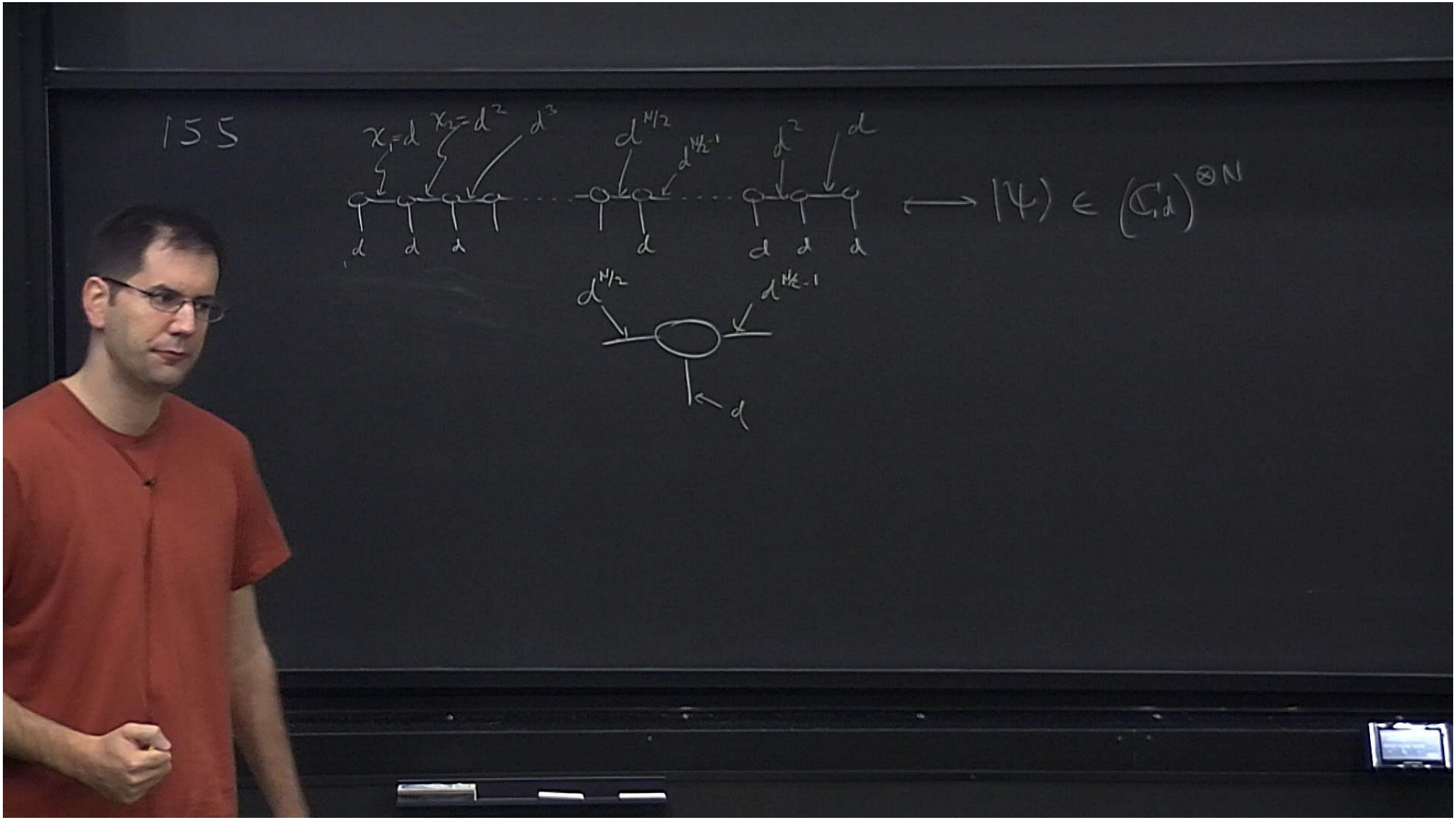

A = AVV† = USV† VV†
= USV† = A
α=1,2,... χ1
χ1
χ2
χ3
χ4
V
V†
χ

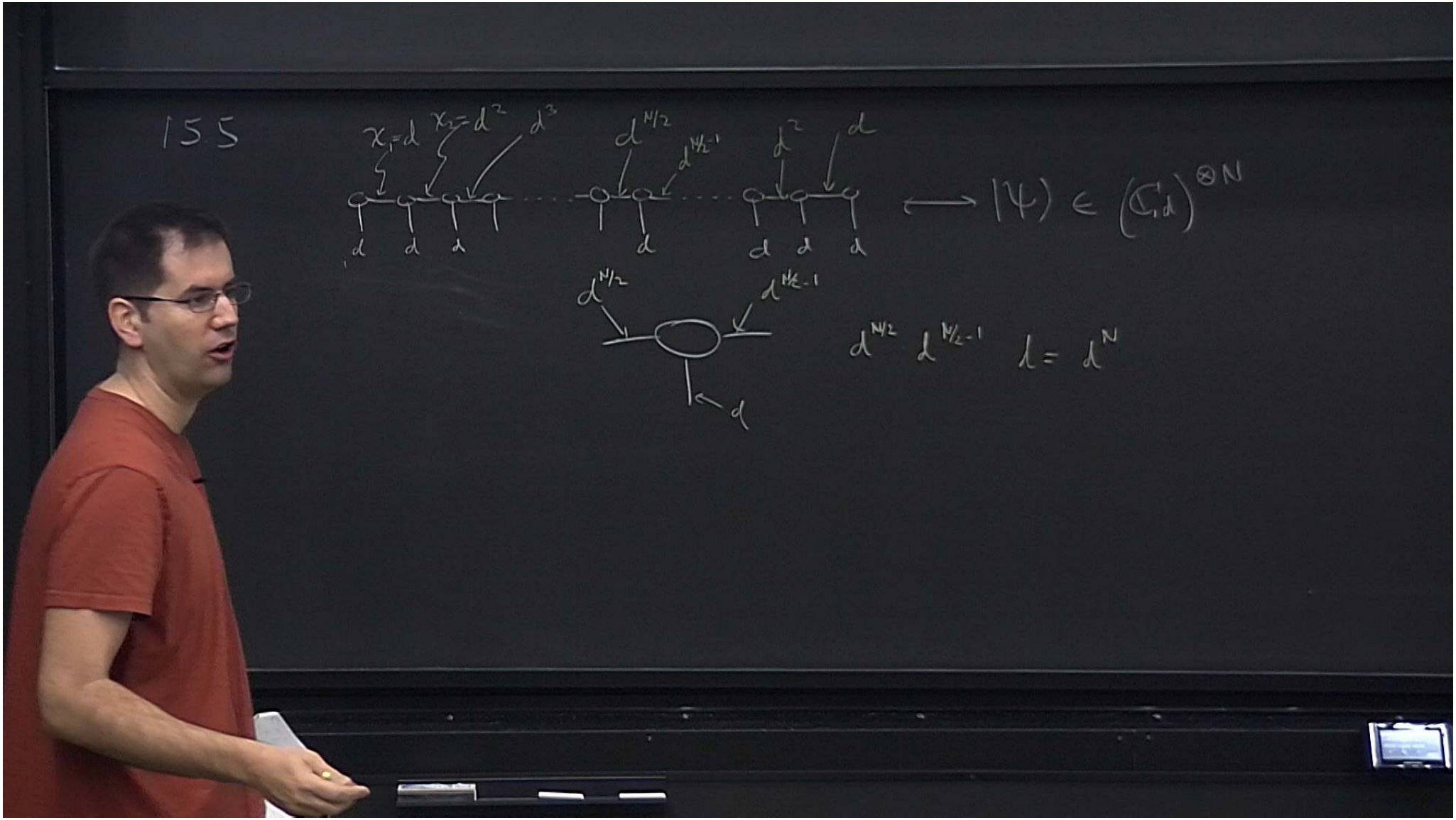

A = AVV† = USV† VV†
= USV† = A
V
V†
x
α=1,2,... χ1
χ1
χ2
χ3
χ4


A = AVV† = USV† VV†
= USV† = A
α=1,2 ... χ1
χ1 χ2 χ3 χ4

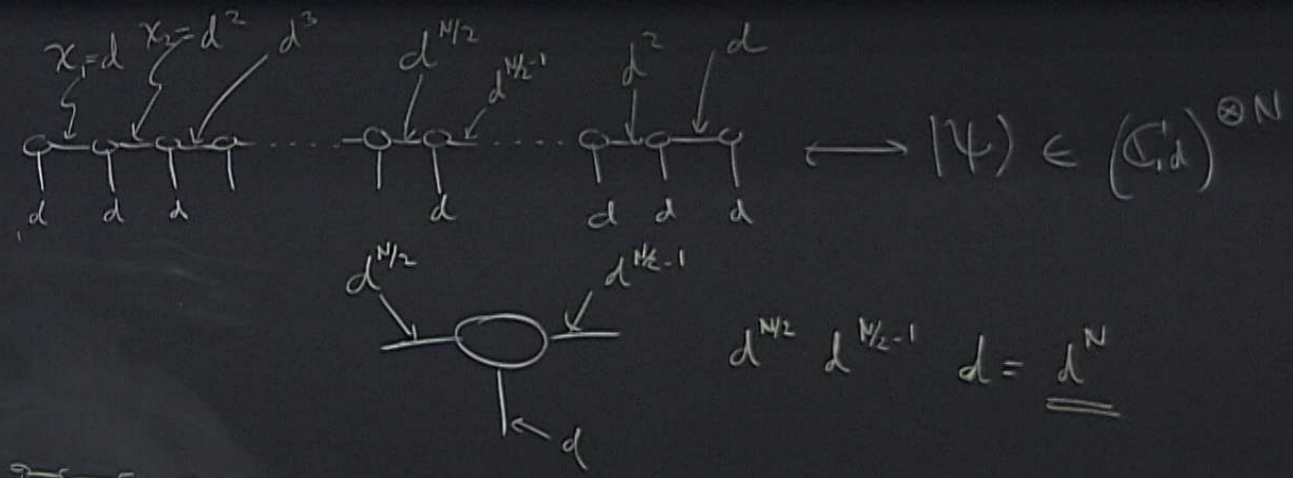
155

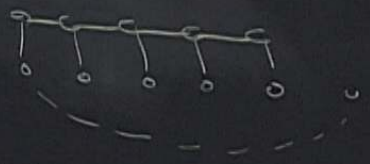

d d d
d d d
$\longleftrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$

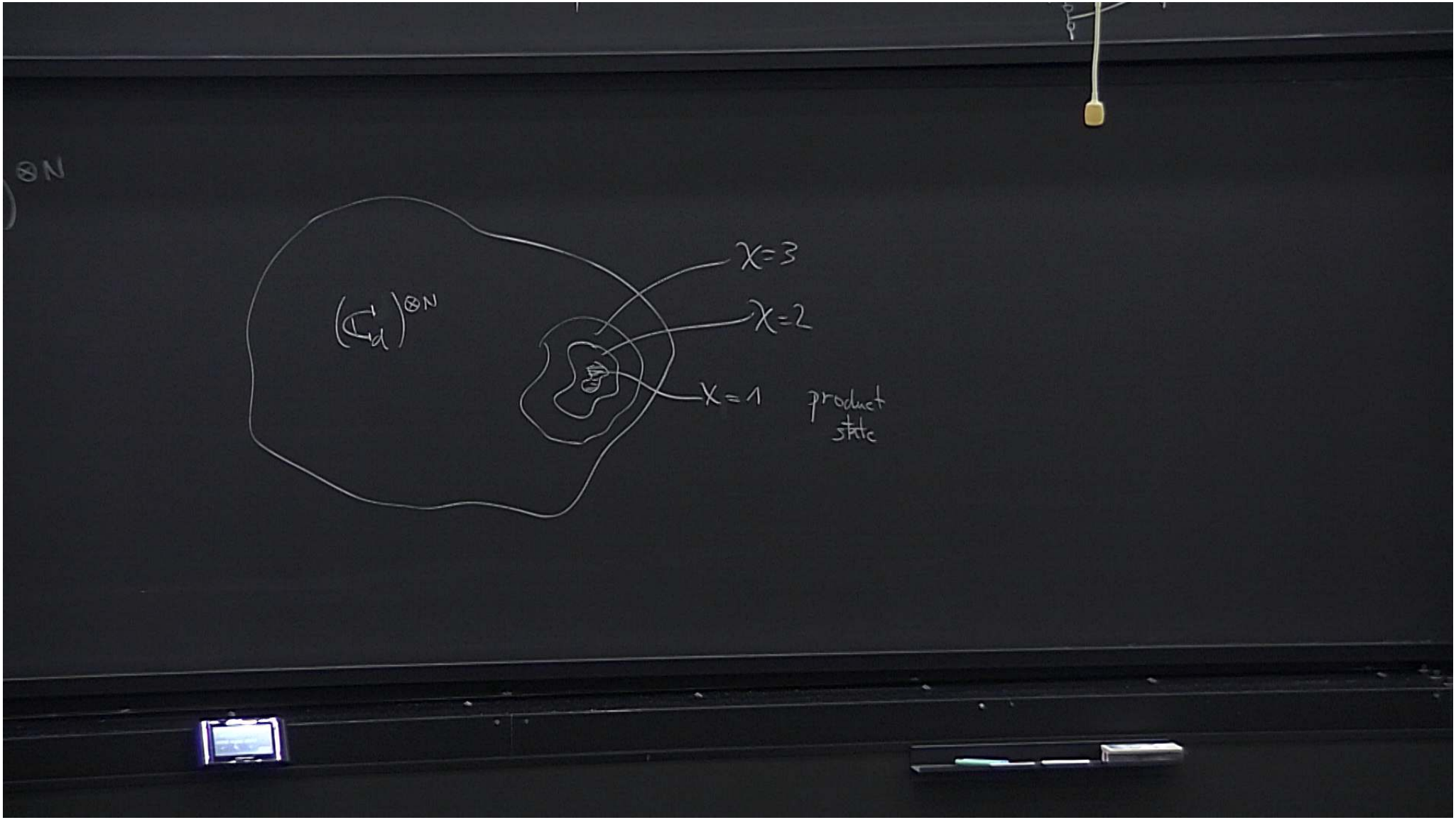

155
$\chi_1 = d$
$\chi_2 = d^2$
d^3
$d^{N/2}$
$d^{N/2-1}$
d^2
d
$\longleftrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
$d^{N/2}$
$d^{N/2-1}$
d

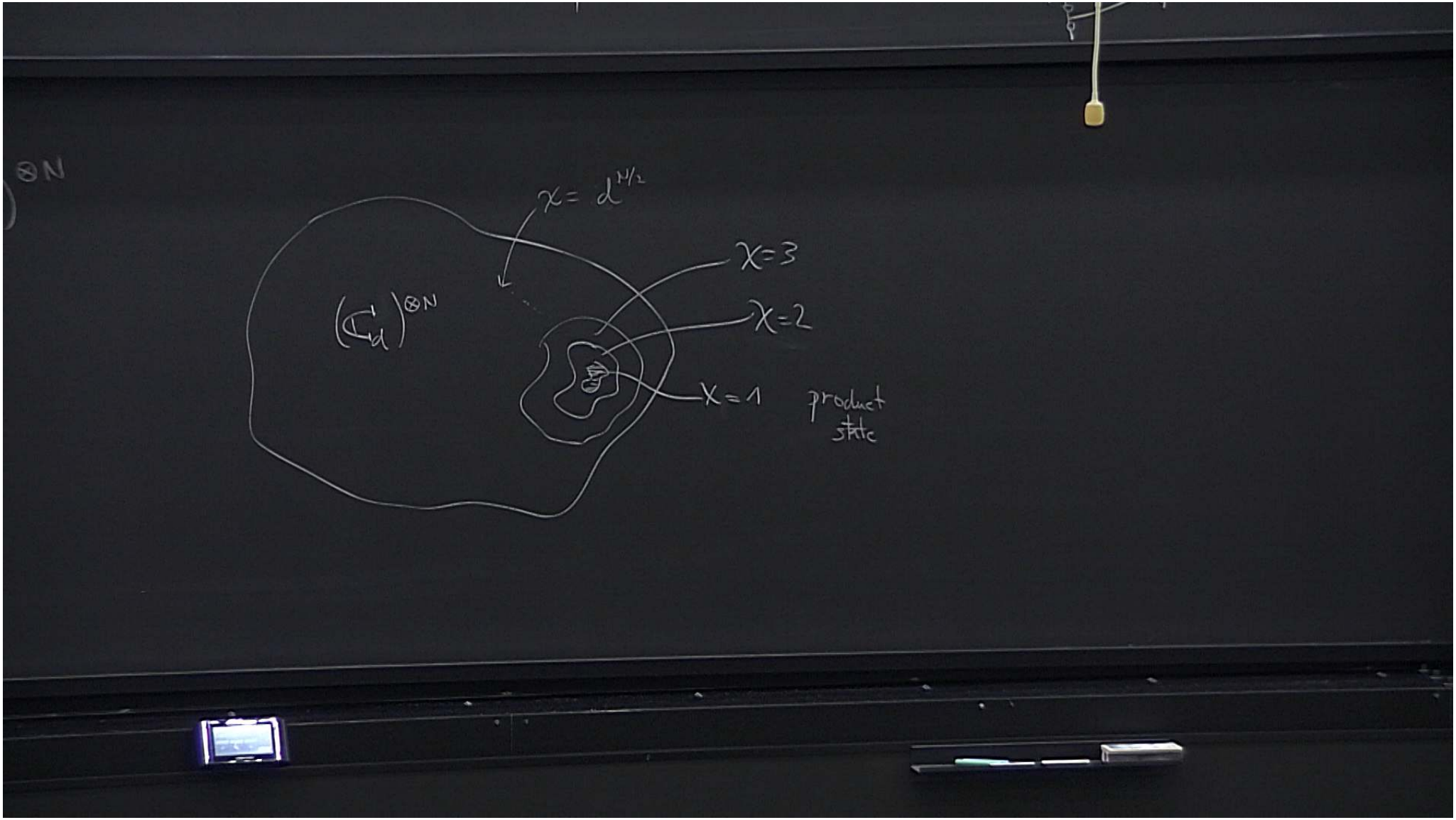

155
$\chi_1 = d$
$\chi_2 = d^2$
d^3
$d^{N/2}$
$d^{N/2-1}$
d^2
d
$\longleftrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
$d^{N/2}$
$d^{N/2-1}$
$d^{N/2}\, d^{N/2-1}\, 1 = d^N$

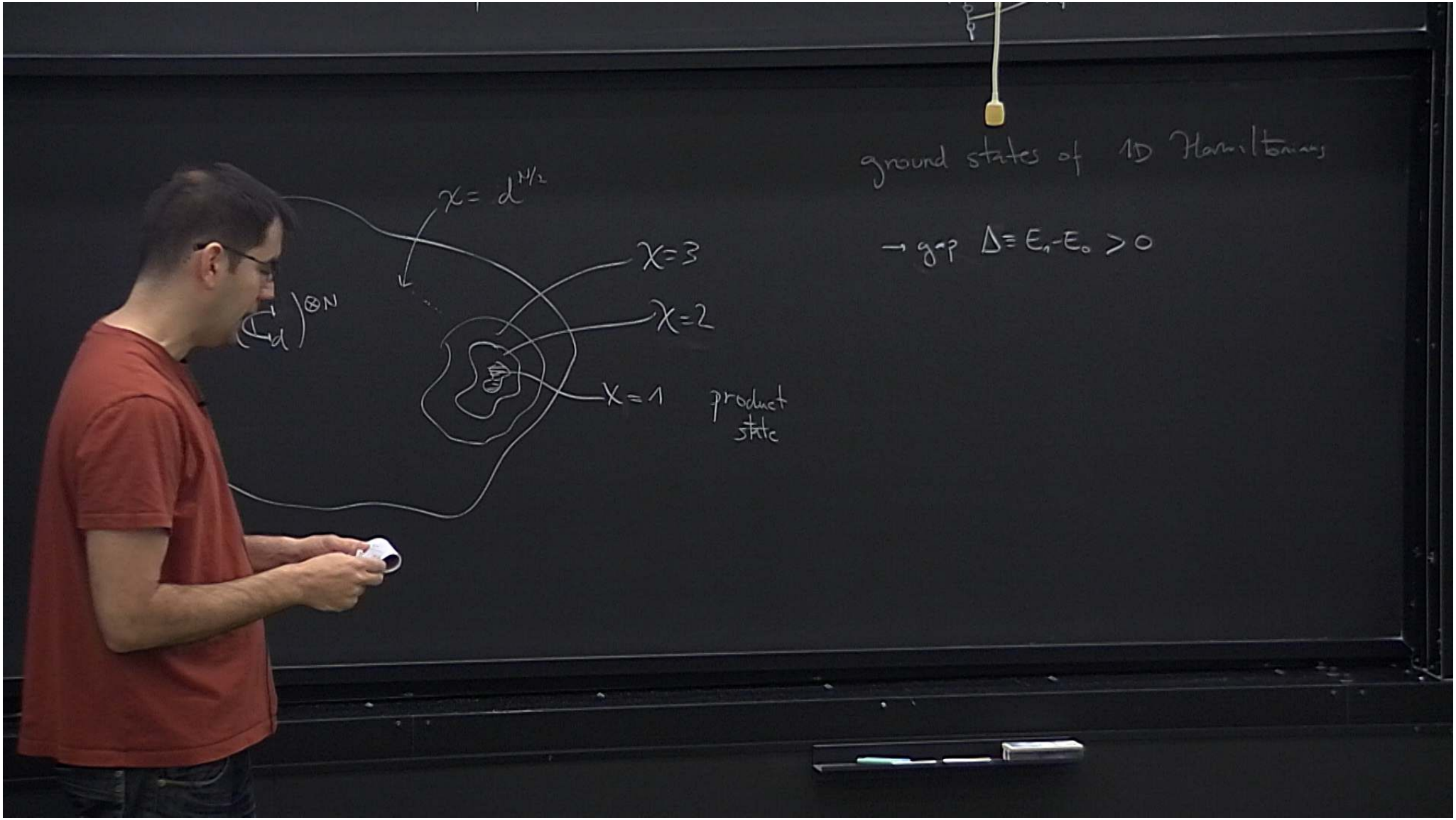
155

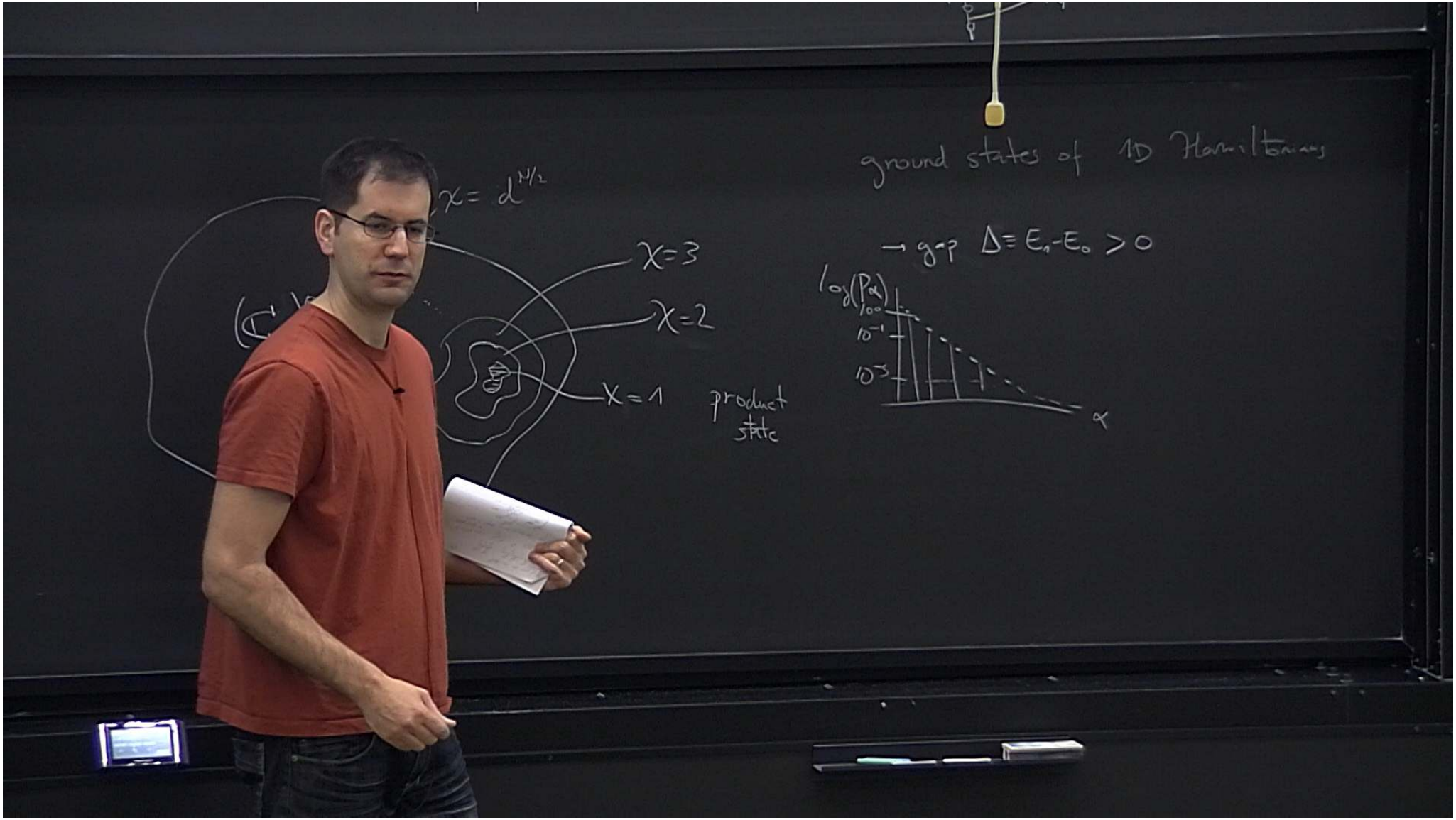

$x_1 = d$
$x_2 = d^2$
d^3
$d^{N/2}$
$d^{N/2-1}$
d^2
d
$\longleftrightarrow |\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
$d^{N/2}$
$d^{N/2-1}$
d
$d^{N/2}\, d^{N/2-1}\, d = d^N$

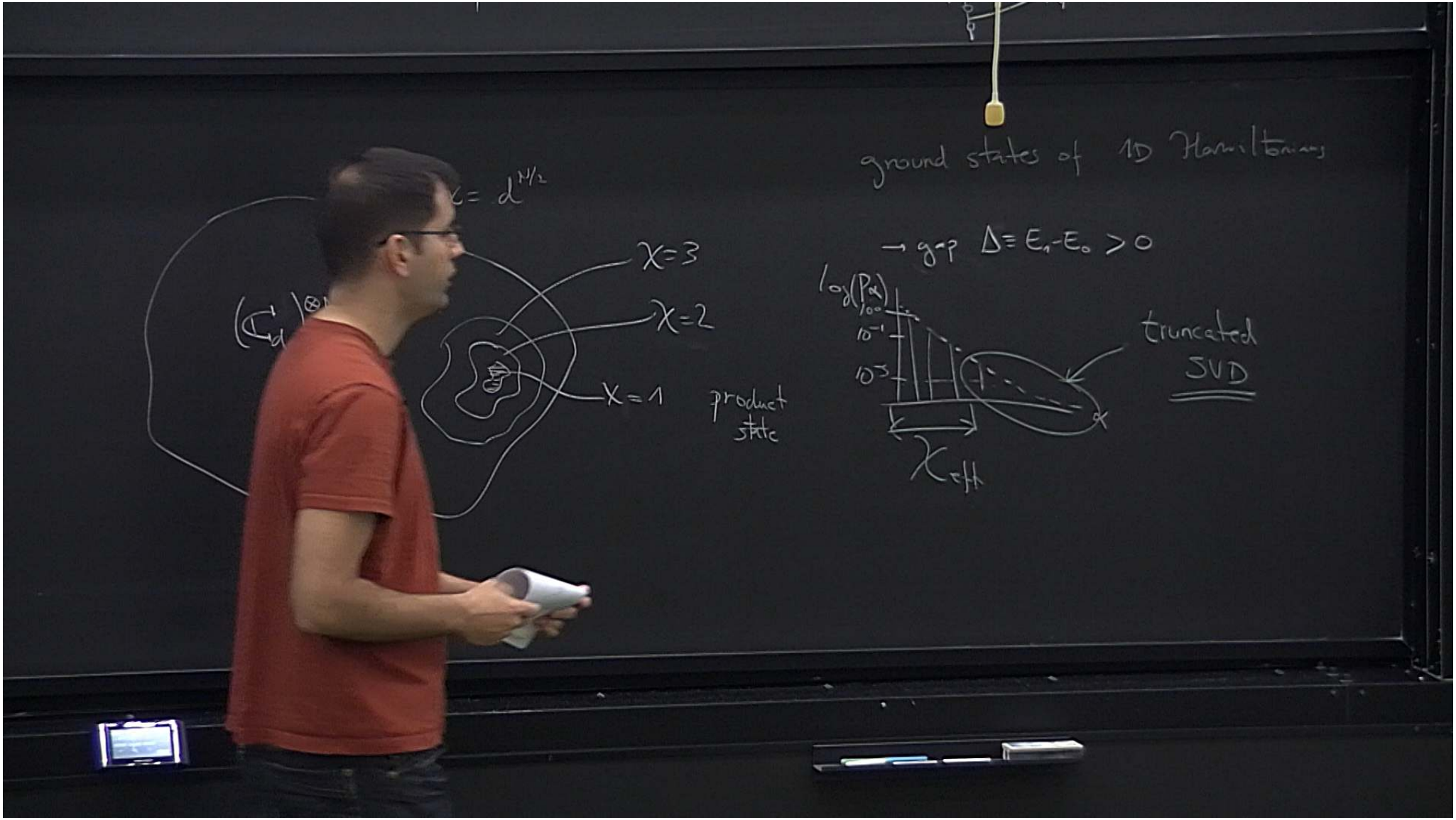



$(\mathbb{C}^4)^{\otimes N}$
$\chi=3$
$\chi=2$
$\chi=1$
product state

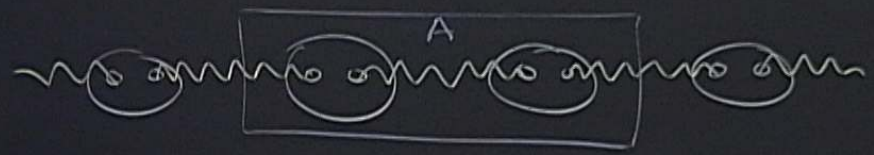

$\otimes N$
$\chi = d^{N/2}$
$\chi = 3$
$(\mathbb{C}^d)^{\otimes N}$
$\chi = 2$
$\chi = 1$
product
state


ground states of 1D Hamiltonians
→ gap Δ ≡ E₁ - E₀ > 0
χ = d^{N/2}
χ = 3
χ = 2
χ = 1
product state
(C_d)^{⊗N}

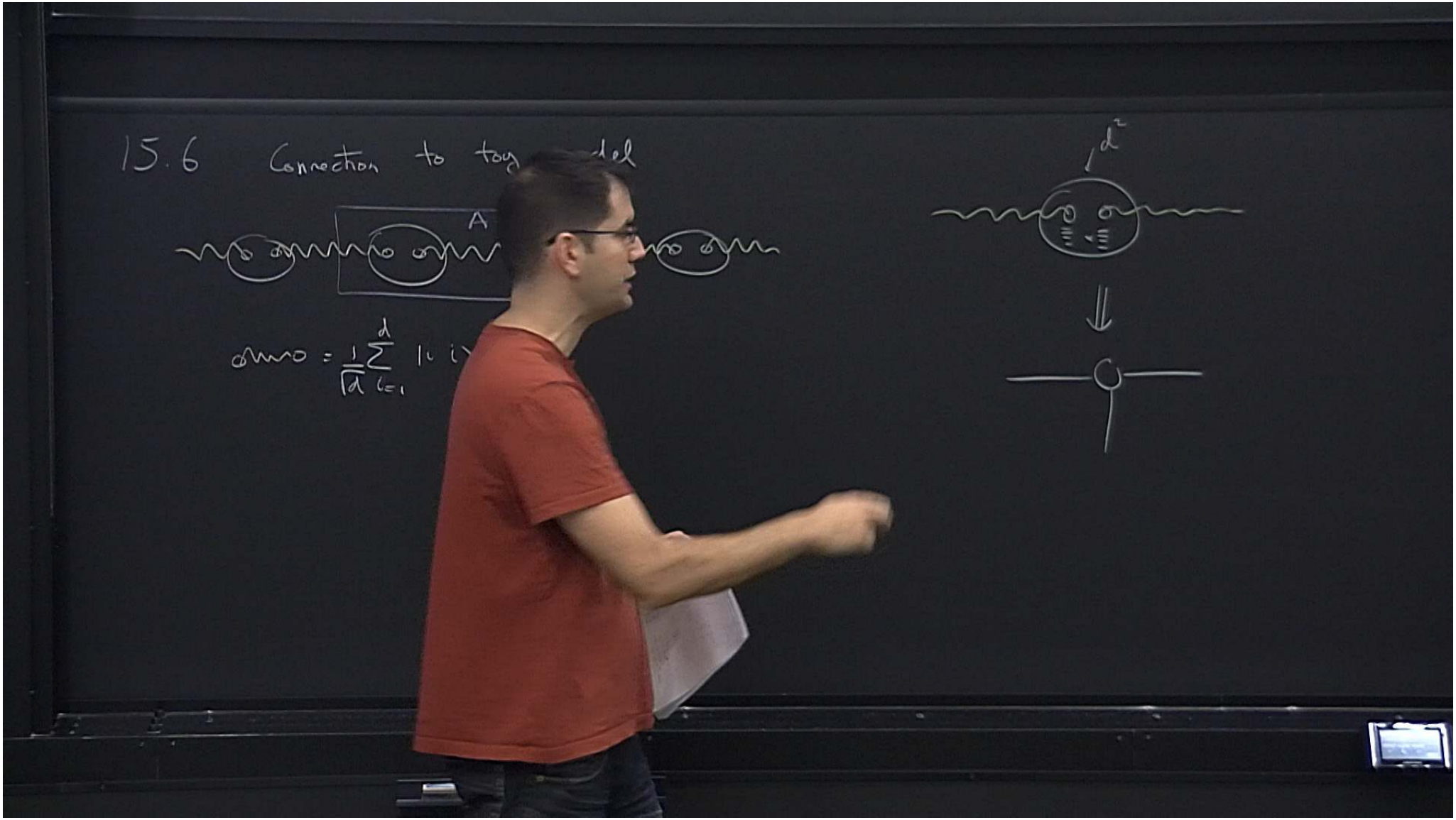

ground states of 1D Hamiltonians
→ gap Δ ≡ E₁ - E₀ > 0
χ = d^{M/2}
χ = 3
χ = 2
χ = 1
product state
log(p_α)
10^{-1}
10^{-3}
α

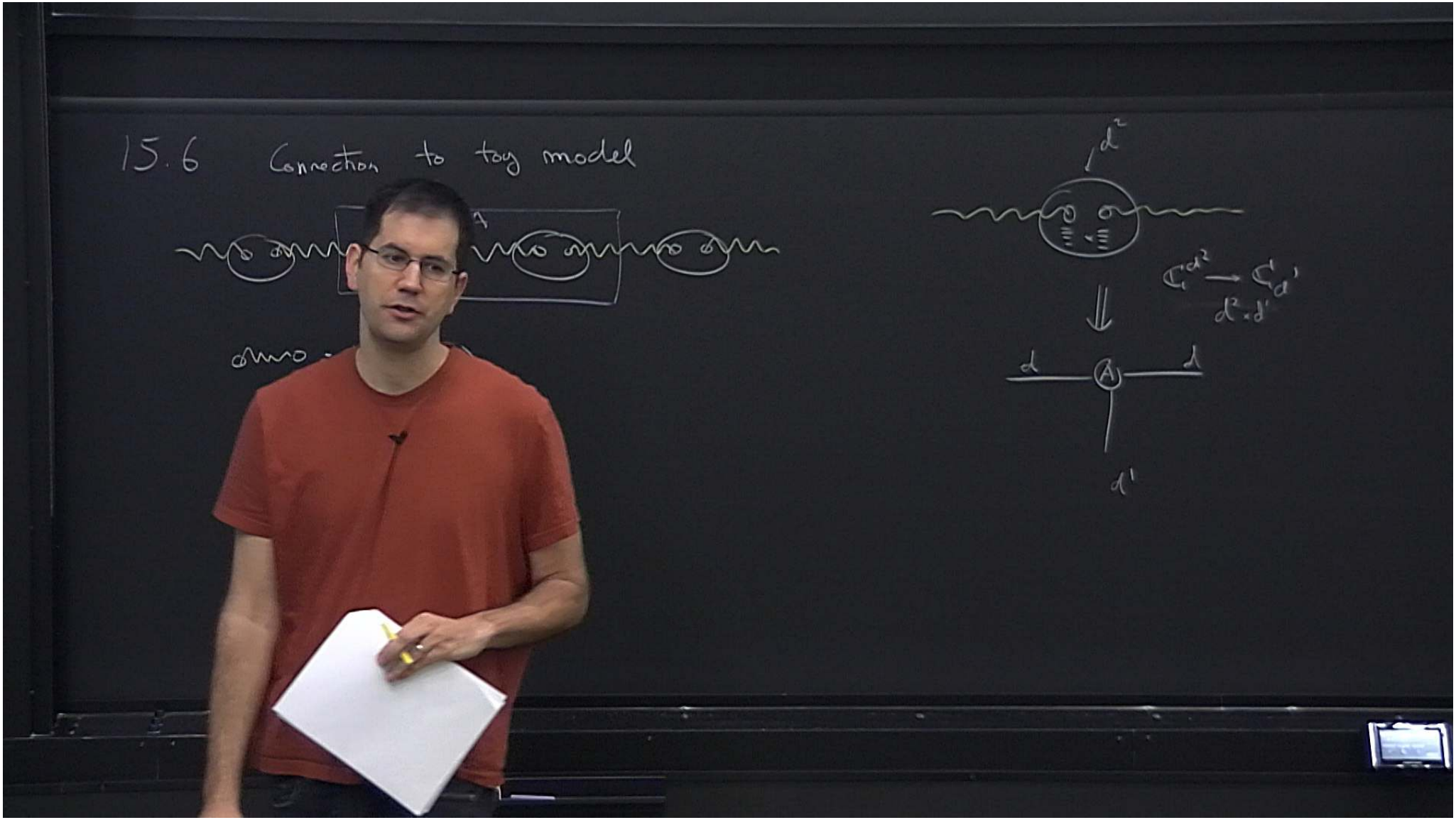

ground states of 1D Hamiltonians
→ gap Δ ≡ E₁ - E₀ > 0
χ = 3
χ = 2
χ = 1
product state
truncated SVD

15.6 Connection to toy model

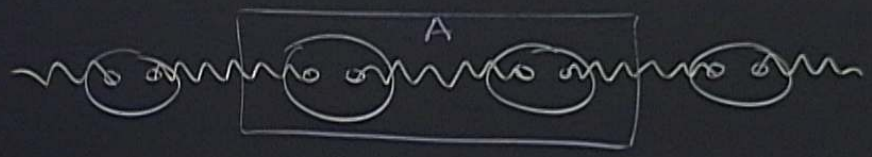


$$|\text{two}\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i\ i\rangle$$

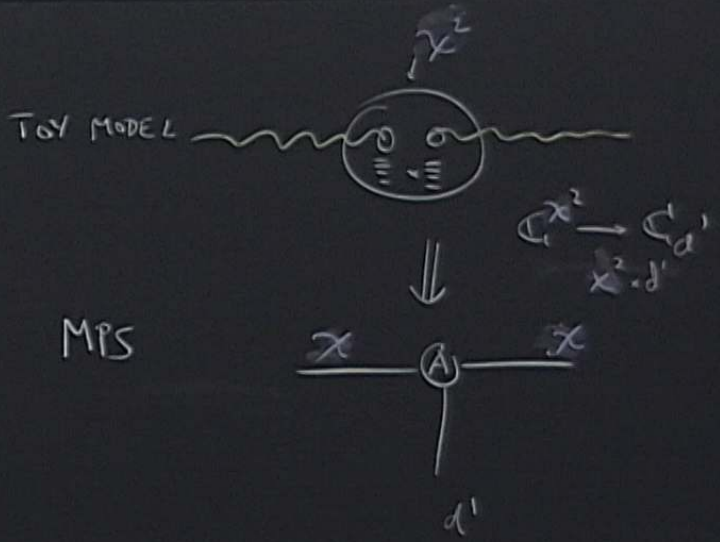

15.6 Connection to toy
A
d^2


15.6 Connection to toy model

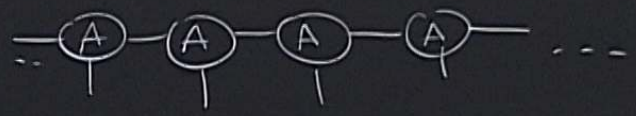
15.6 Connection to toy model



$$\omega_{MPS} = \frac{1}{\sqrt{\chi}} \sum_{i=1}^{\chi} |i\,i\rangle$$

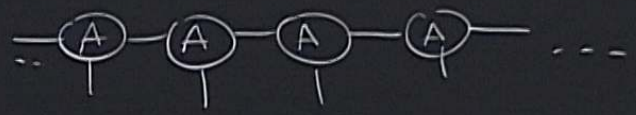


15.7 Low energy excited states

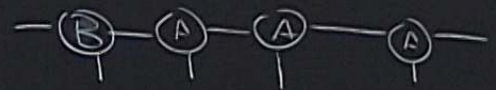
MPS ground state 

15.7 Low energy excited states

MPS ground state

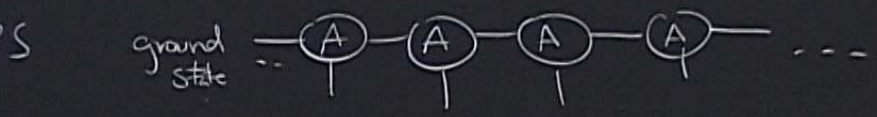


ansatz for excited states

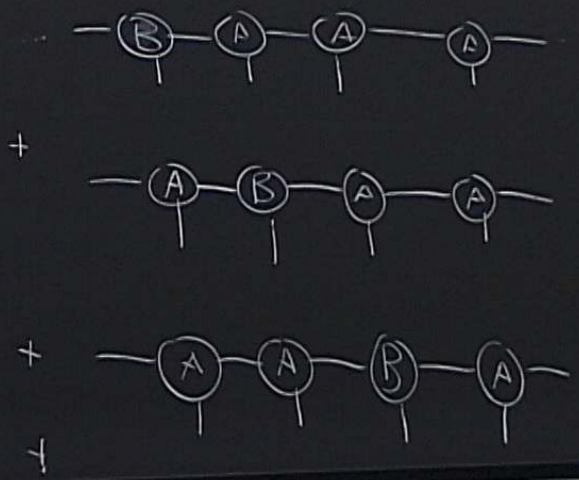


15.7 Low energy excited states

MPS ground state

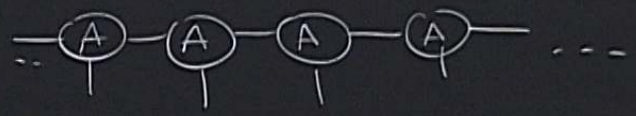


ansatz for excited states

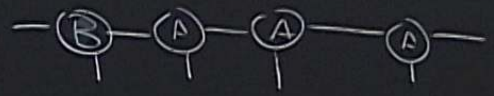


15.7 Low energy excited states

MPS ground state



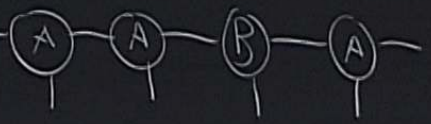
ansatz for excited states



$|\Psi(k)\rangle =$

$+ e^{ik}$

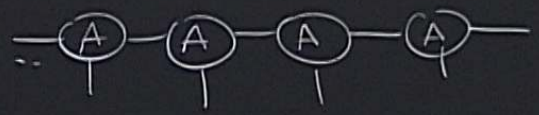
$+ e^{ik_2}$

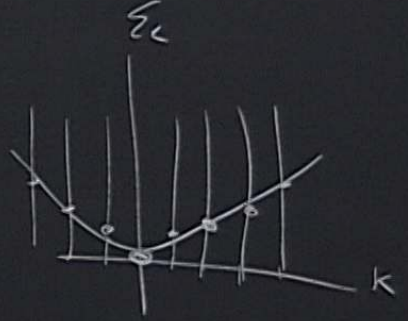


$+ \ldots$

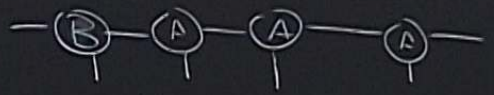
15.7 Low energy excited states

MPS ground state



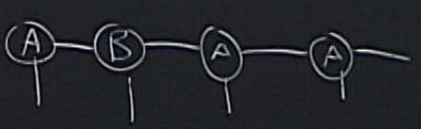


ansatz for excited states

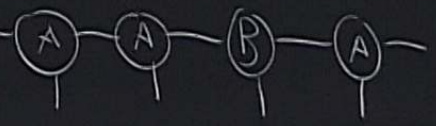


$|\Psi(k)\rangle =$

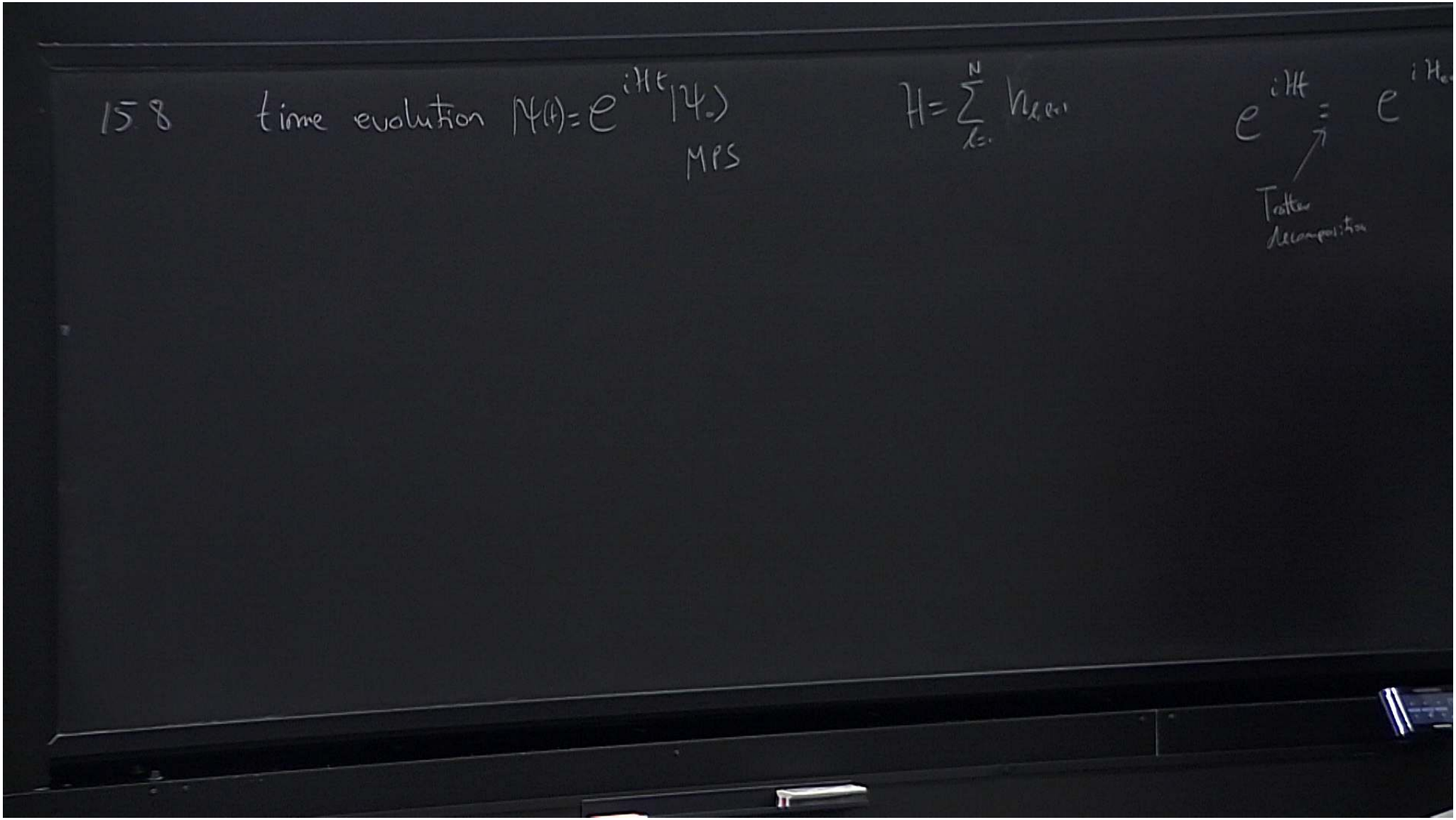
$+ e^{ik}$

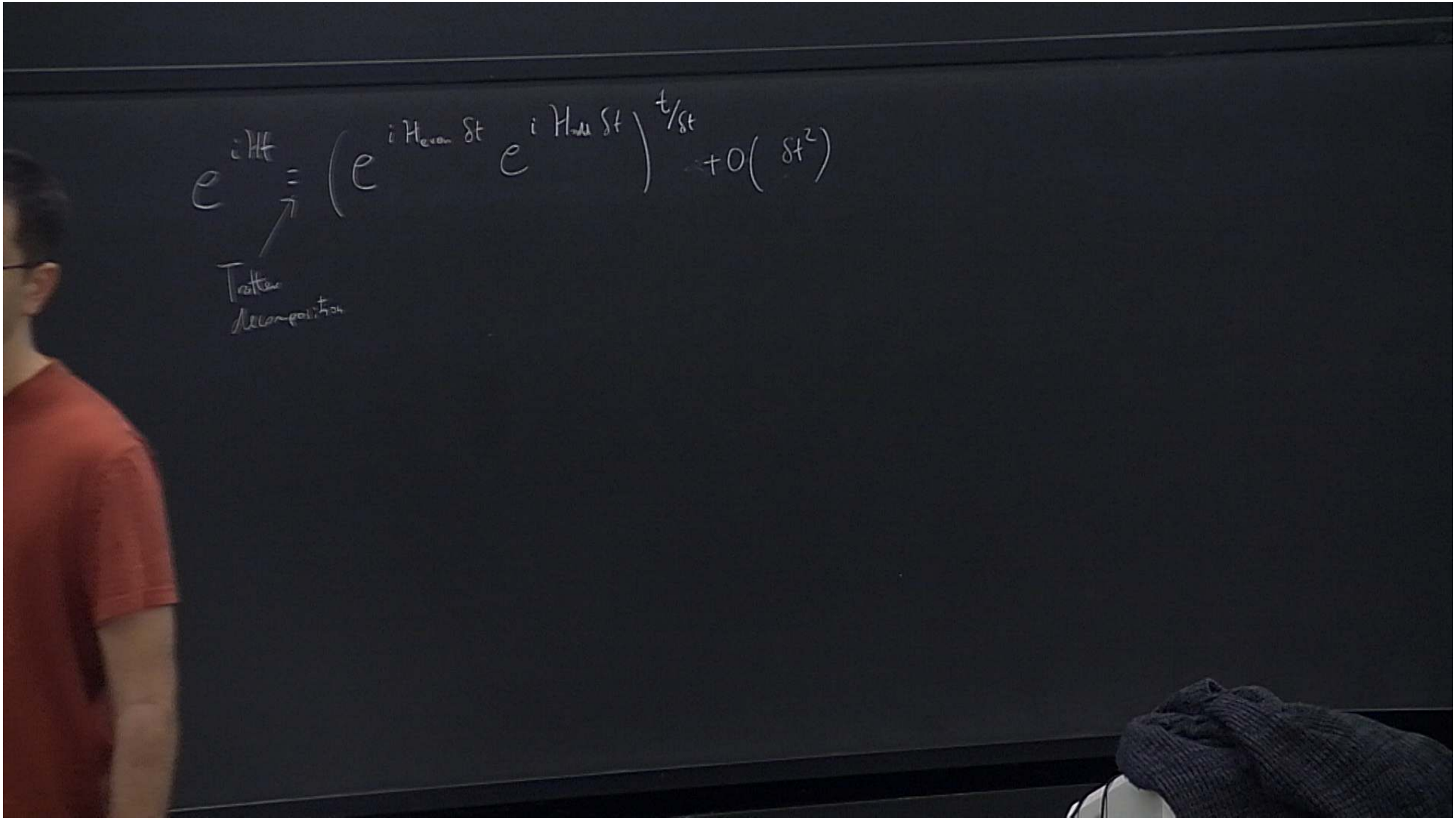


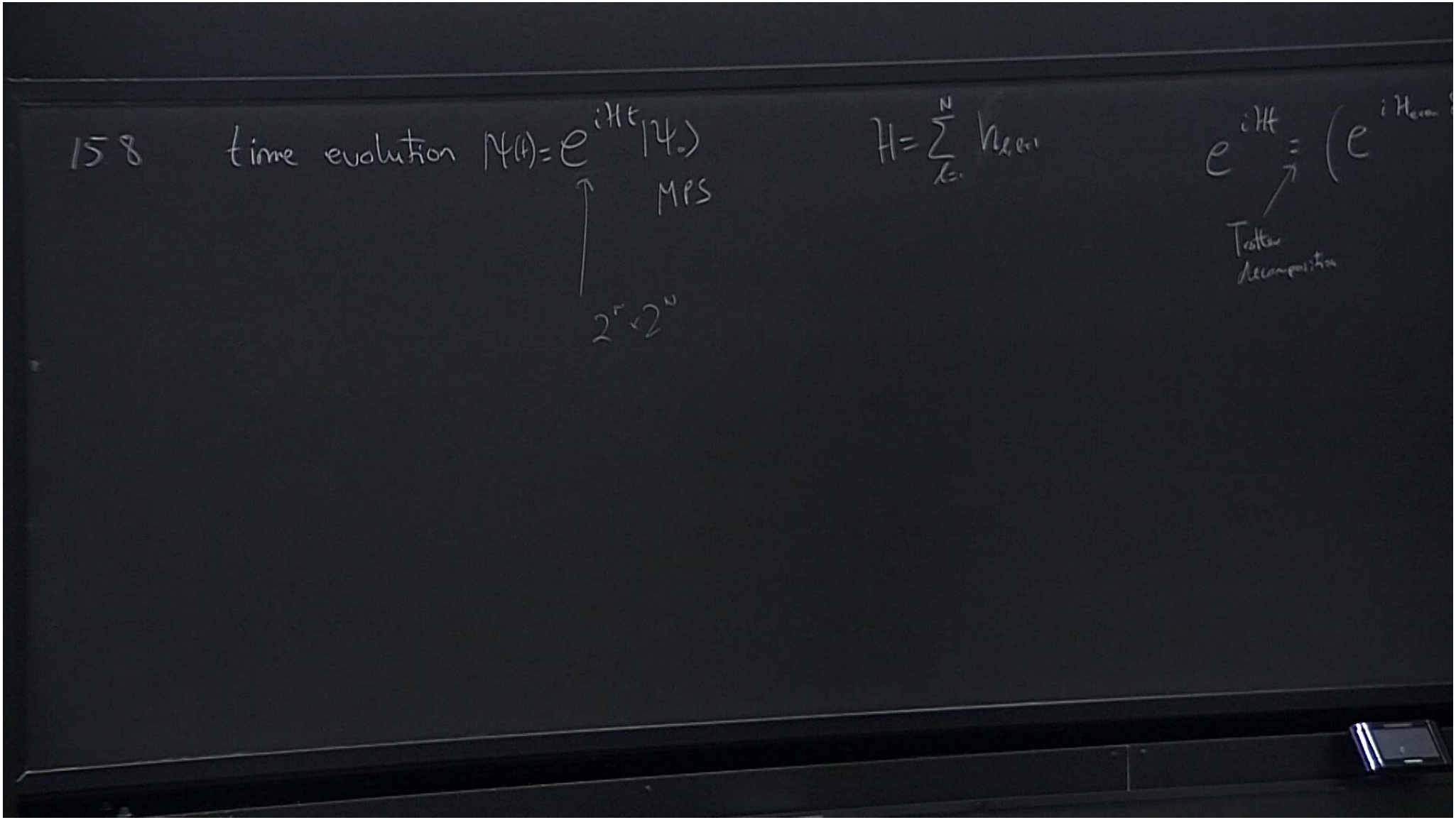
$+ e^{ik_2}$

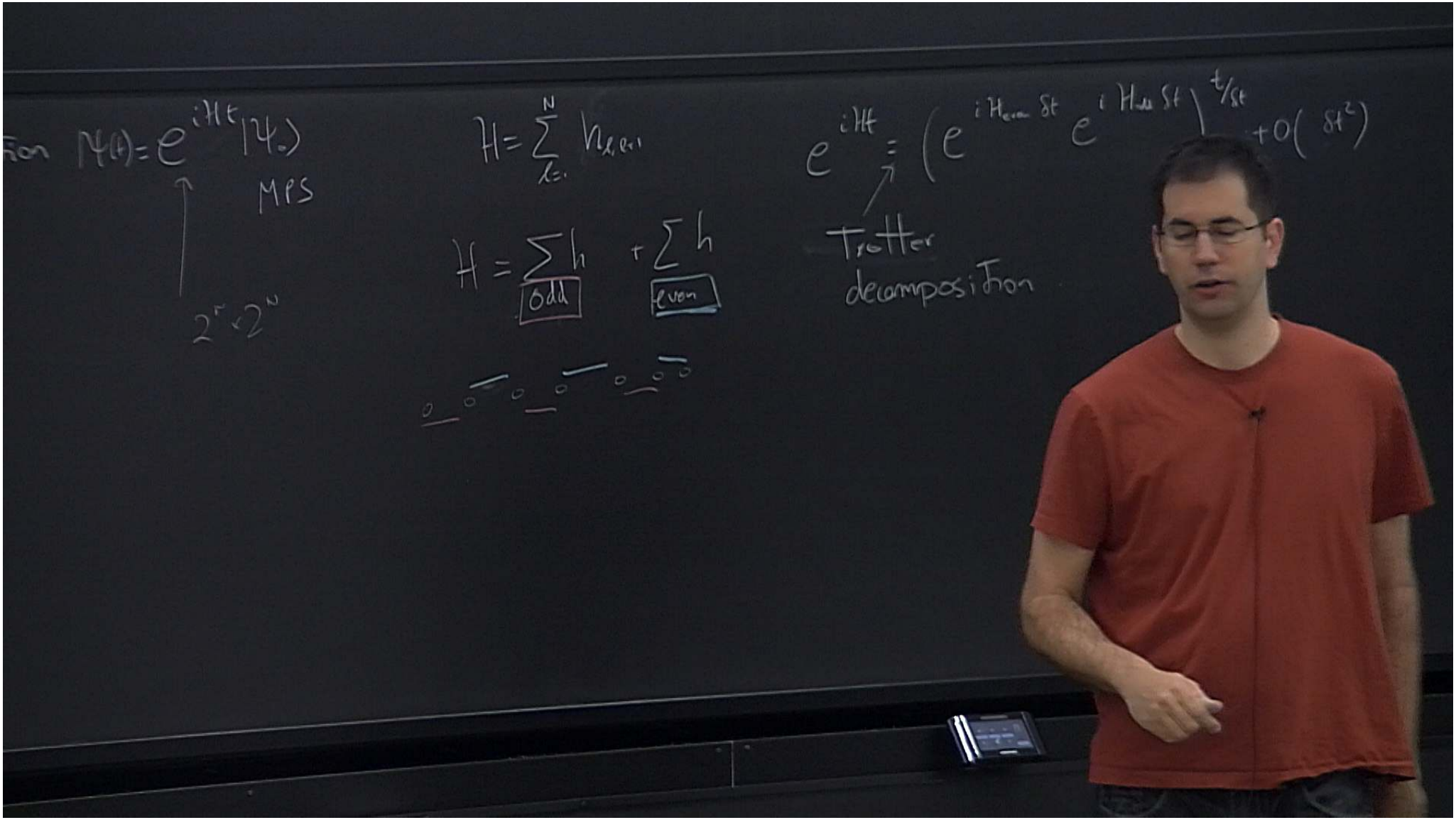


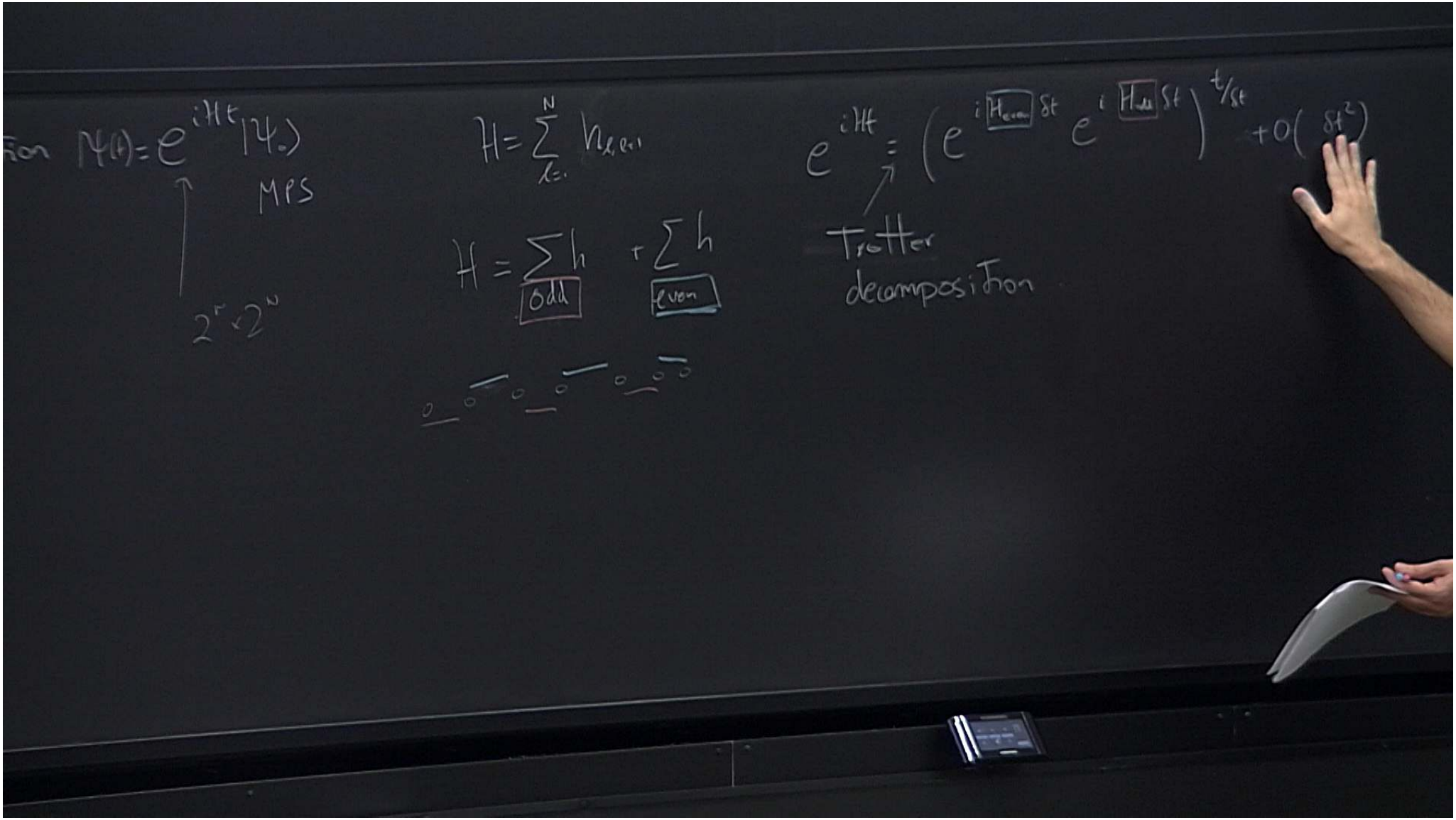
+ ...


158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$
MPS
$H = \sum_{i=1}^{N} h_{i,i+1}$
$e^{iHt} = e^{ih_{1,2}t}$
Trotter decomposition


$e^{iHt} = \left(e^{iH_{even}\delta t} e^{iH_{odd}\delta t}\right)^{t/\delta t} + O(\delta t^2)$
Trotter decomposition


158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$
MPS
$2^r \times 2^N$
$H = \sum_{k=1}^{N} h_{k,k+1}$
$e^{iHt} = (e^{iH_{even}}$
Trotter decomposition


MPS
Trotter
decomposition
odd
even


MPS
Trotter
decomposition
Odd
Even

158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

MPS

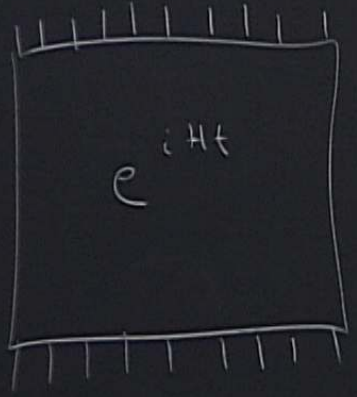
$2^r \times 2^N$

$H = \sum_{i=1}^{N} h_{i,i+1}$

$H = \sum h_{\text{odd}} + \sum h_{\text{even}}$

$e^{iHt} = (e^{i H_{\text{even}}}$

Trotter decomposition



158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$ MPS

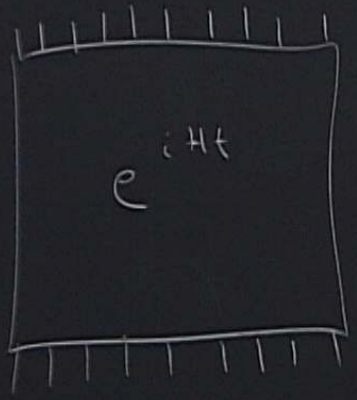
$H = \sum_{k=1}^{N} h_{k,k+1}$

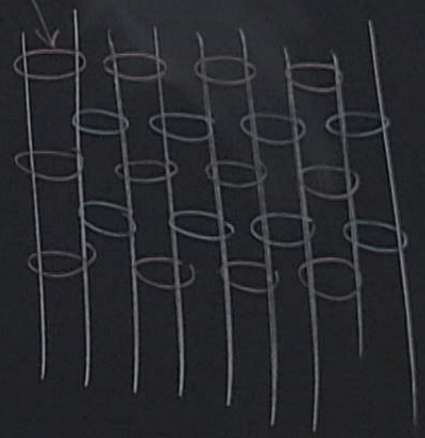
$e^{iHt} = (e^{i H_{even}}$

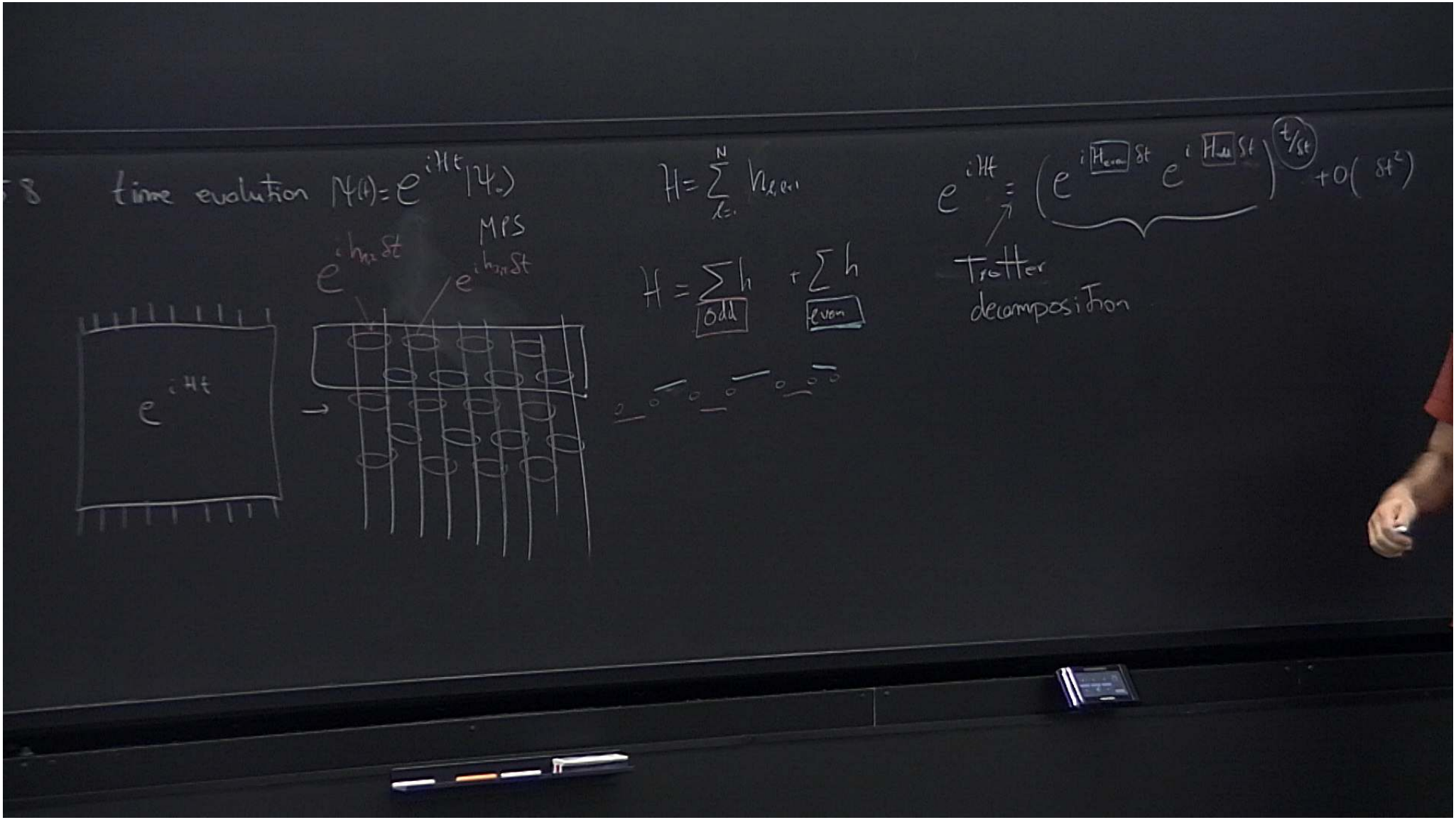
Trotter decomposition

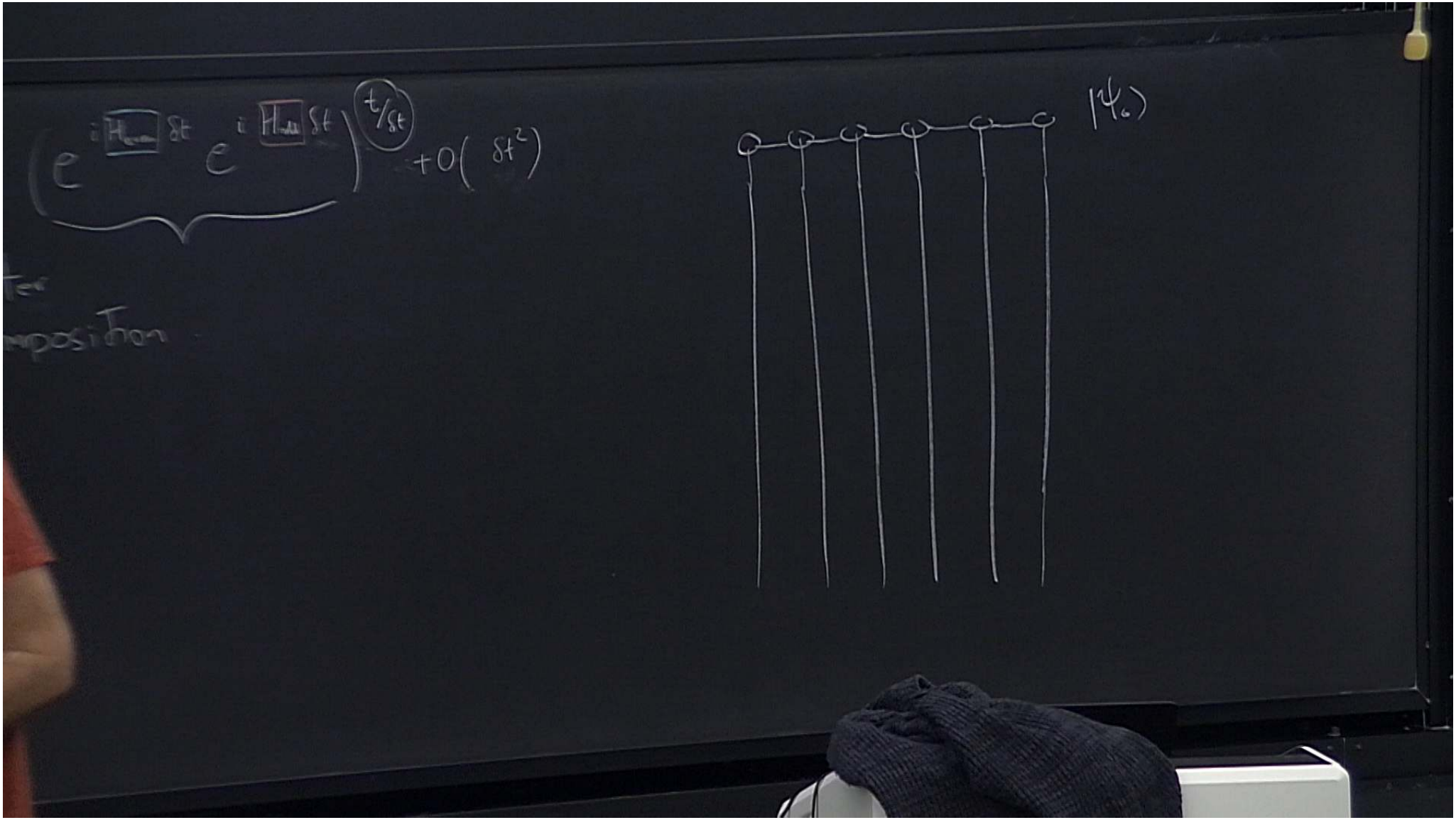
$e^{i h_{1,2} \delta t}$

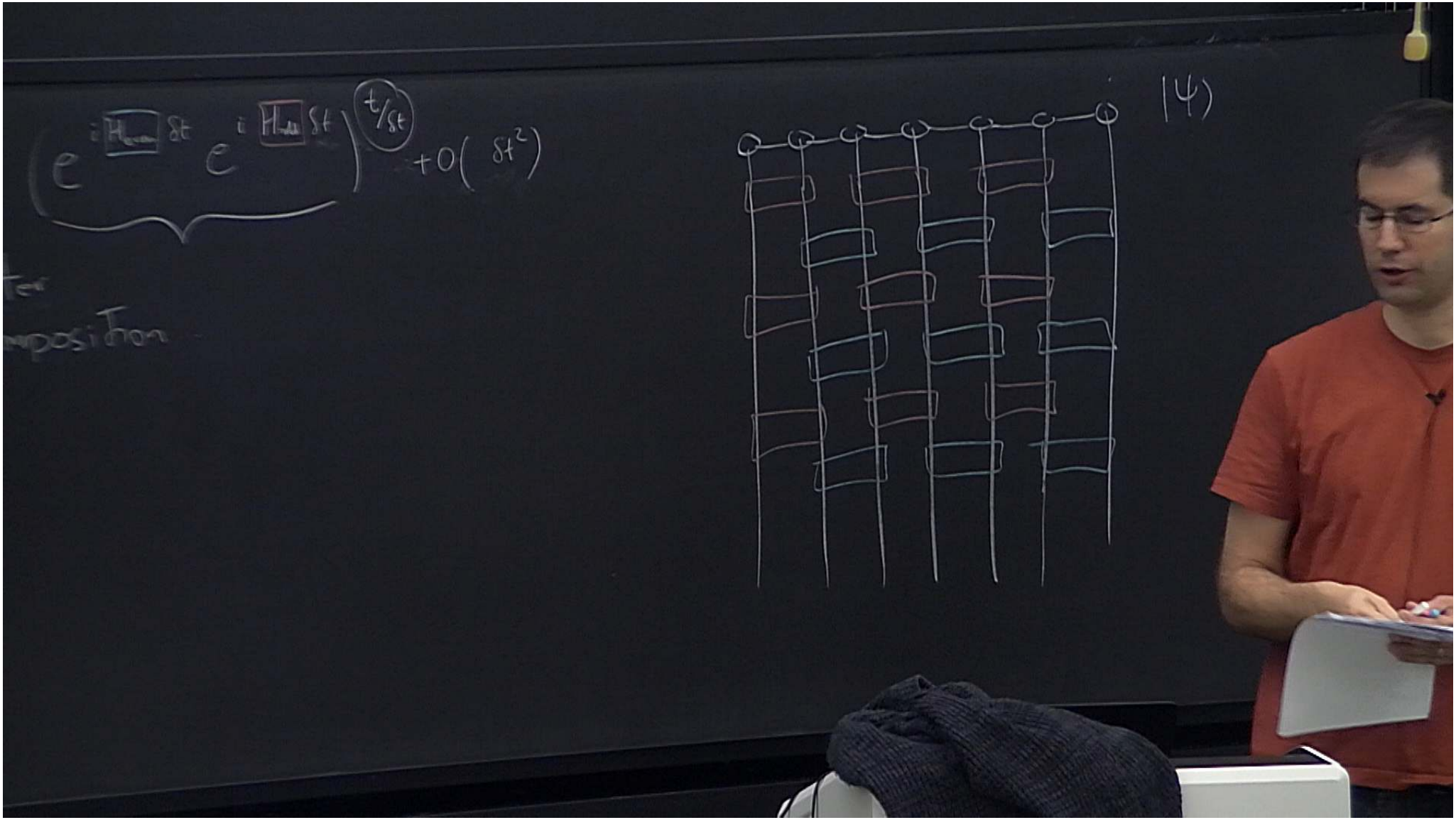
$H = \sum_{\text{odd}} h + \sum_{\text{even}} h$

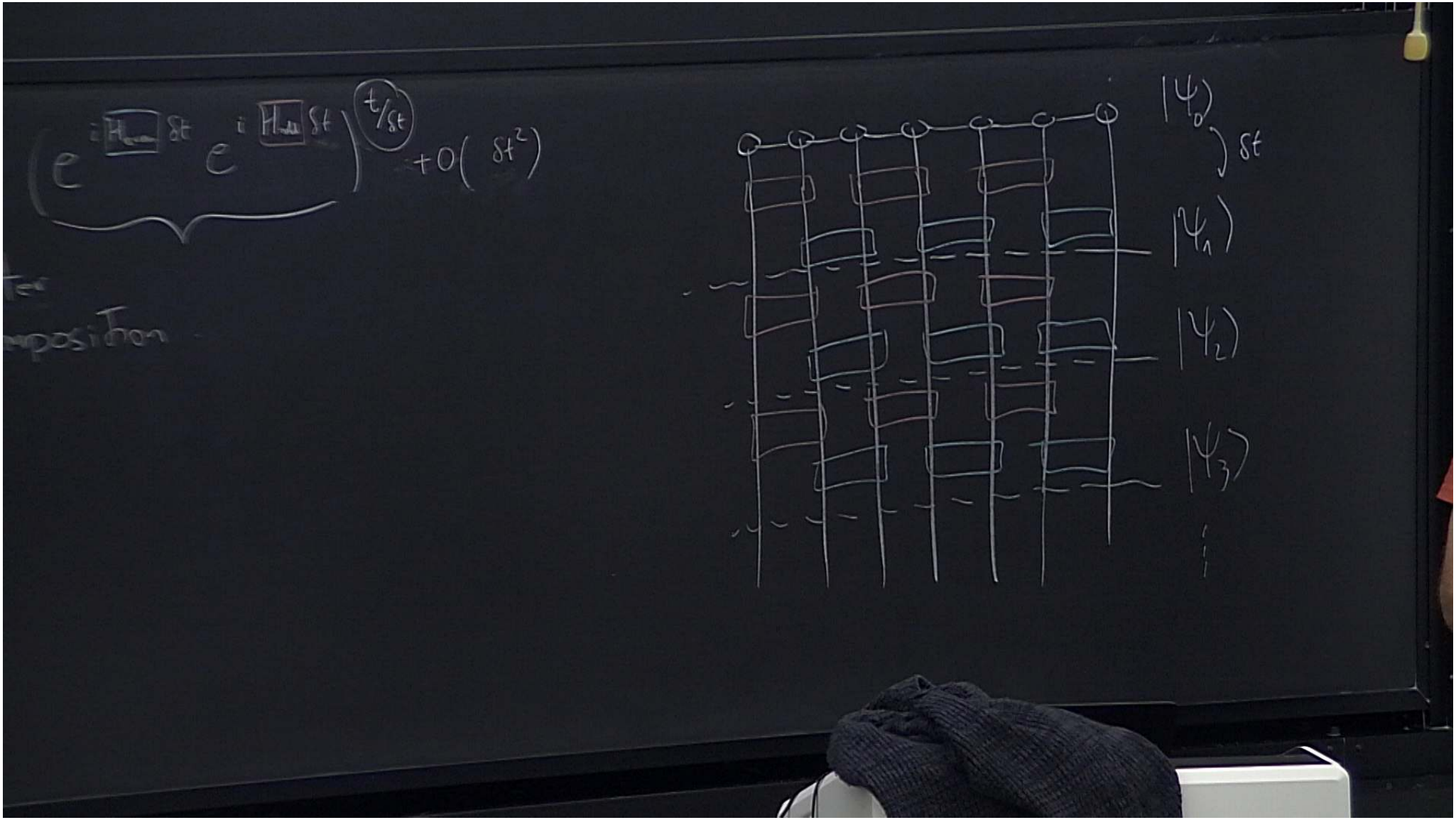


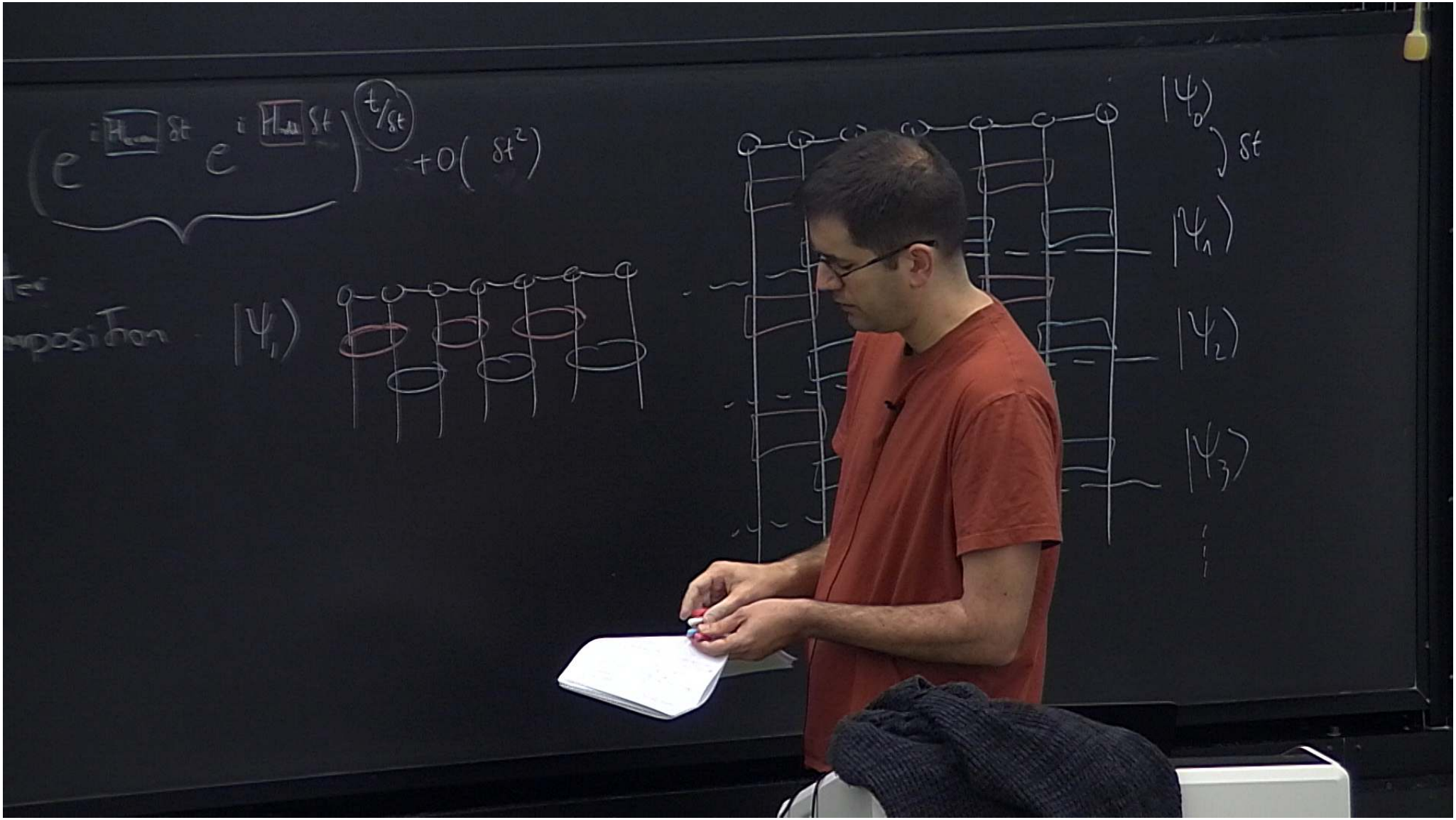


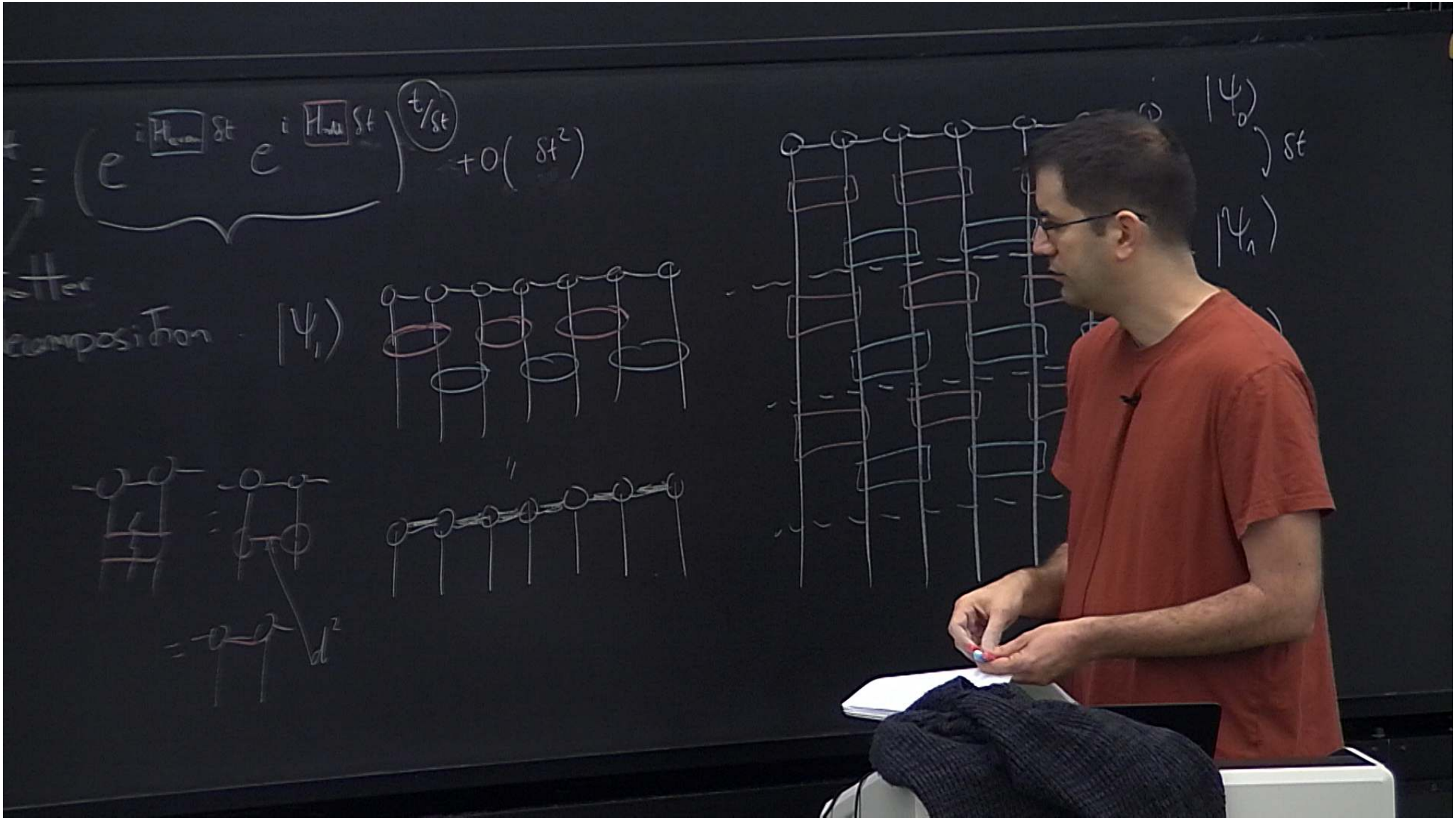

time evolution
MPS
Trotter
decomposition
odd
even


$\left(e^{i H_{even} \delta t} e^{i H_{odd} \delta t}\right)^{t/\delta t} + O(\delta t^2)$
$|\psi_0\rangle$

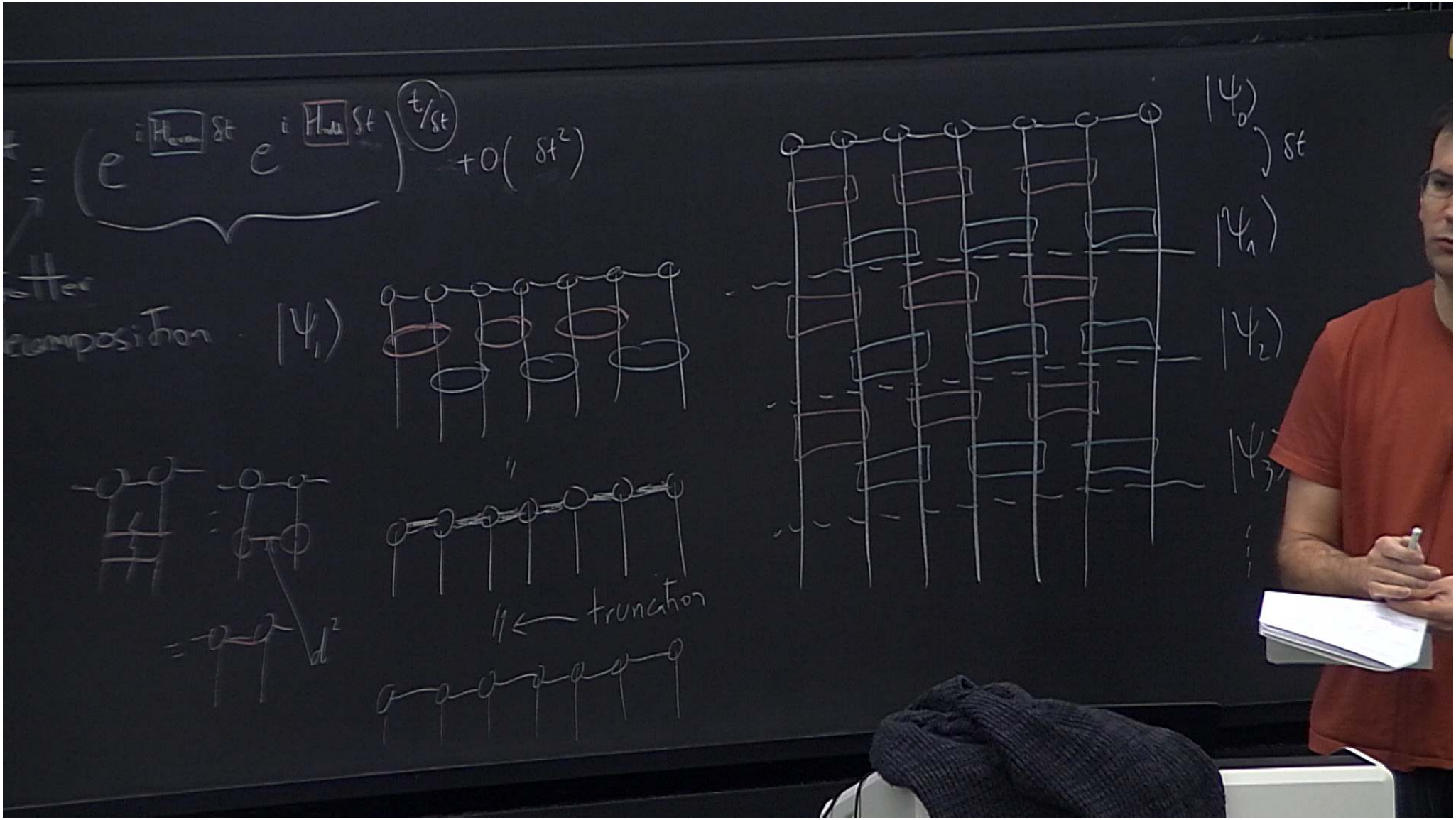

$\left(e^{i H_{even} \delta t} e^{i H_{odd} \delta t}\right)^{t/\delta t} + O(\delta t^2)$
$|\psi\rangle$


$\left(e^{i H_{even} \delta t} e^{i H_{odd} \delta t}\right)^{t/\delta t} + O(\delta t^2)$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$


$+O(\delta t^2)$
$|\psi_1\rangle$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$



$+O(\delta t^2)$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_1\rangle$
d^2


$e^{i H_{even} \delta t} e^{i H_{odd} \delta t}$
$t/\delta t$
$+O(\delta t^2)$
$|\psi_0\rangle$
δt
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
$|\psi_1\rangle$
d^2
truncation

158 time evolution $|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$

$H = \sum_{i=1}^{N} h_{i,i+1}$

$e^{iHt} = \left(e^{i H_{even}}\right)$

Trotter decomposition

MPS

$e^{i h_{1,2} \delta t}$ $e^{i h_{3,4} \delta t}$

e^{iHt} →

$H = \sum_{\text{odd}} h + \sum_{\text{even}} h$