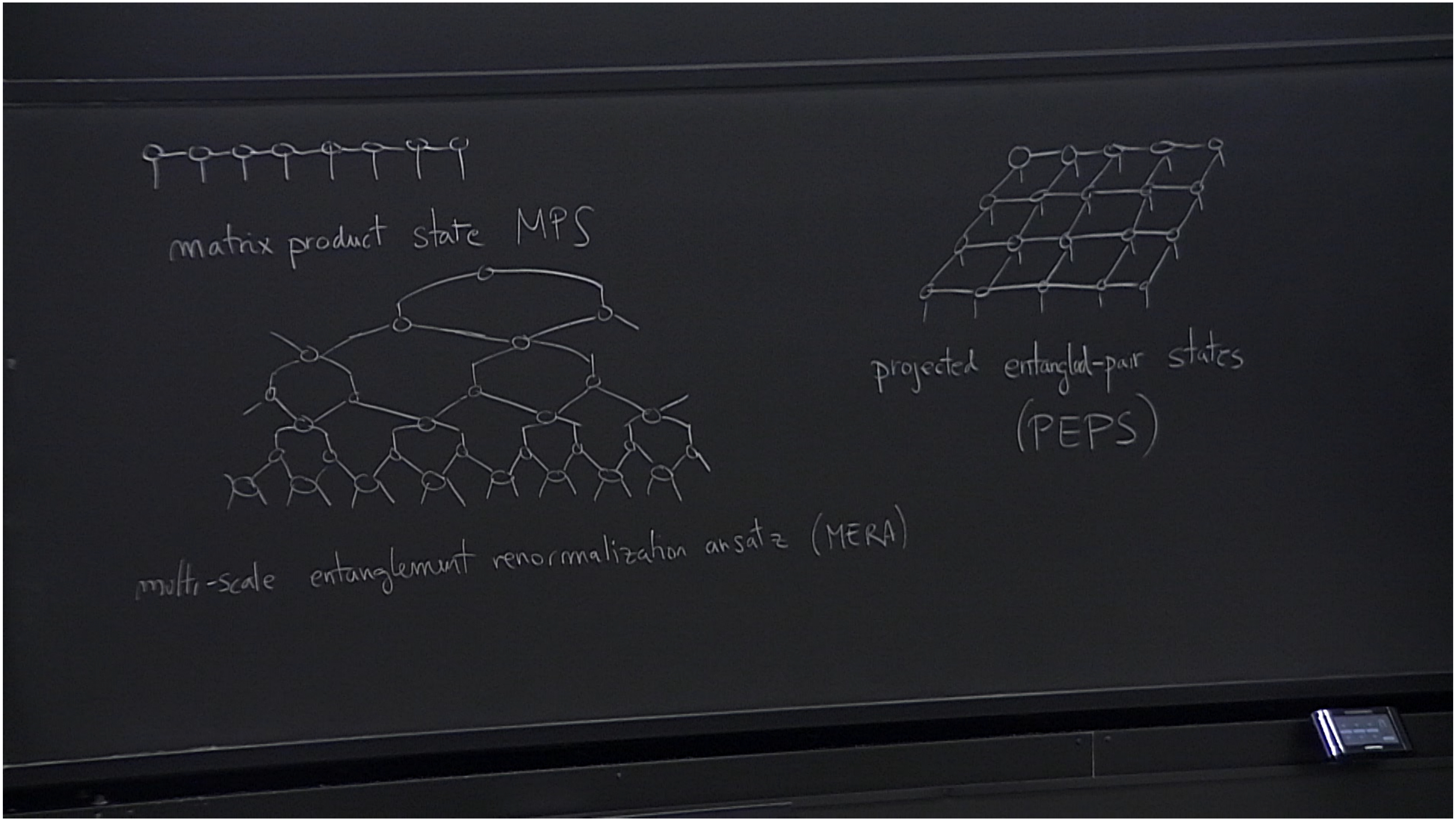
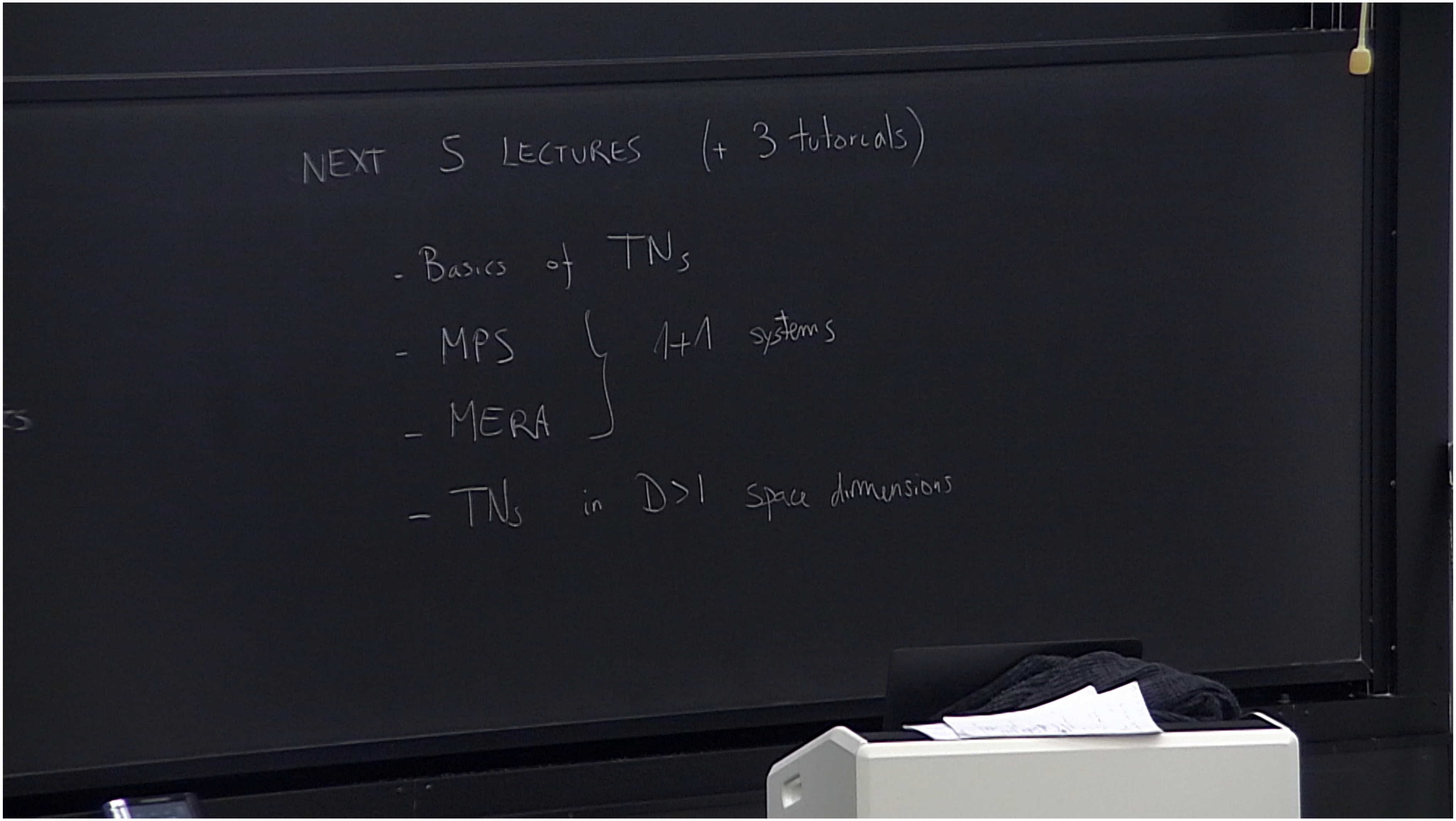
Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 8

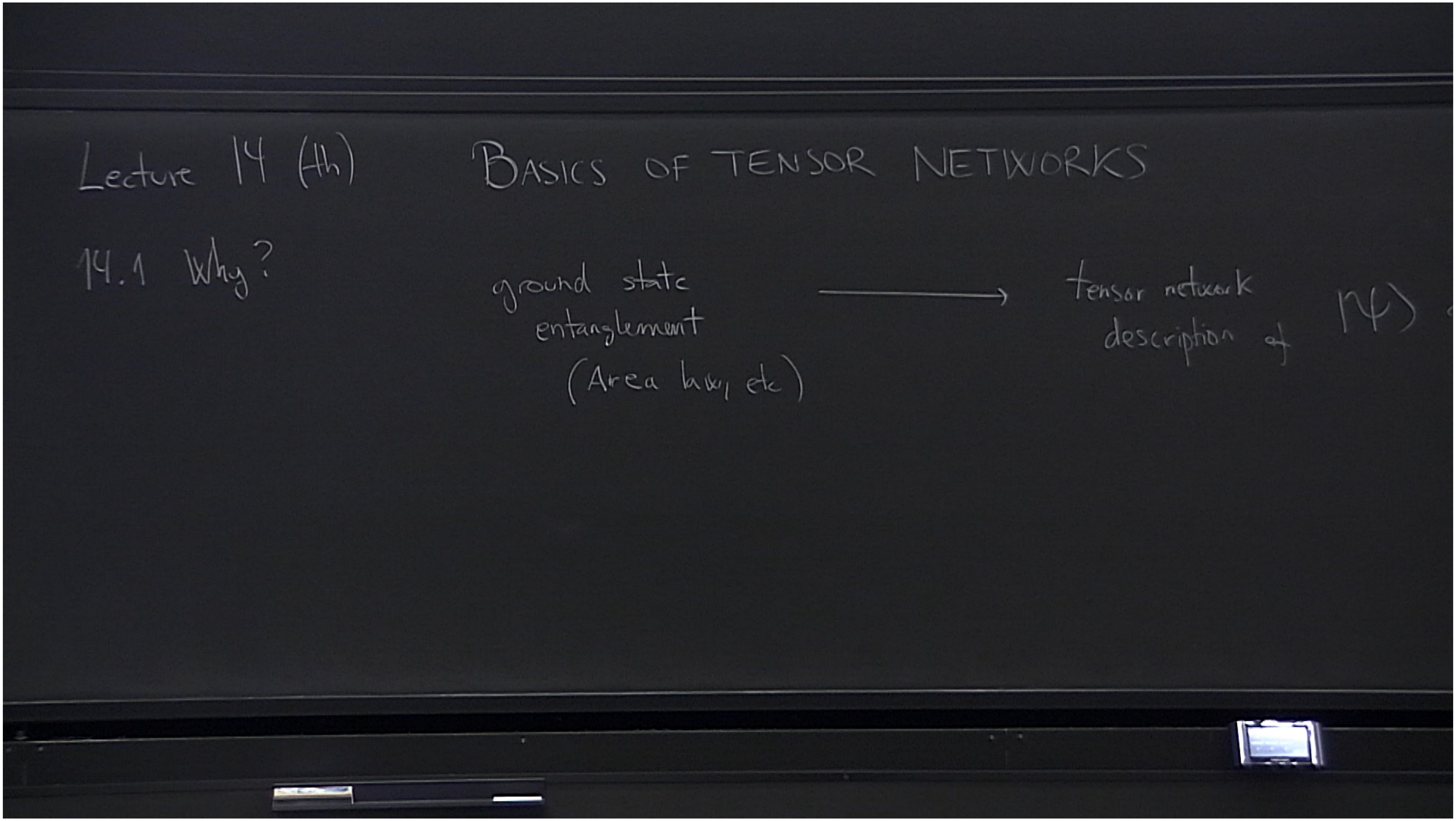
Date: Feb 10, 2017 09:00 AM

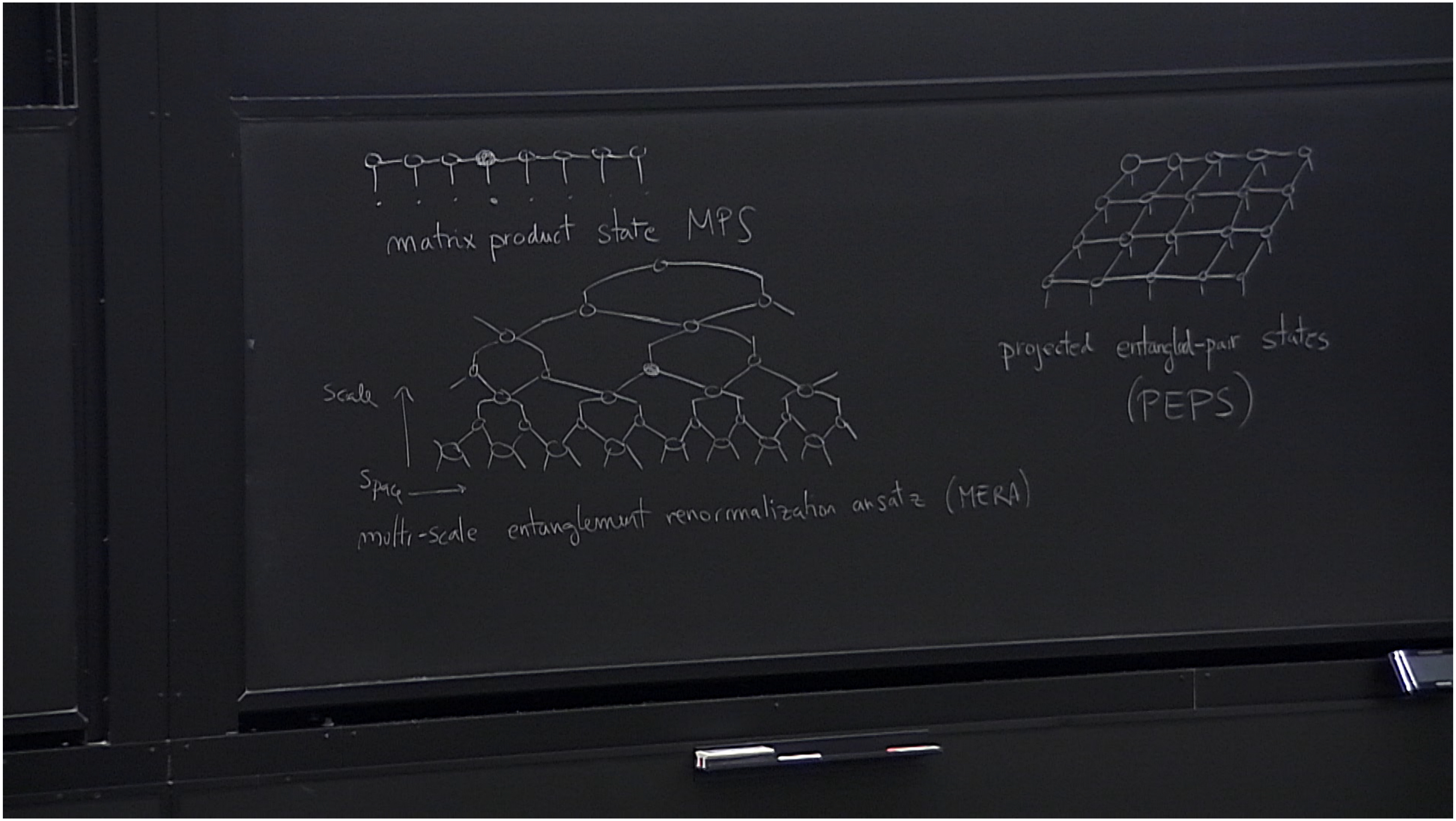
URL: http://pirsa.org/17020045

Abstract:

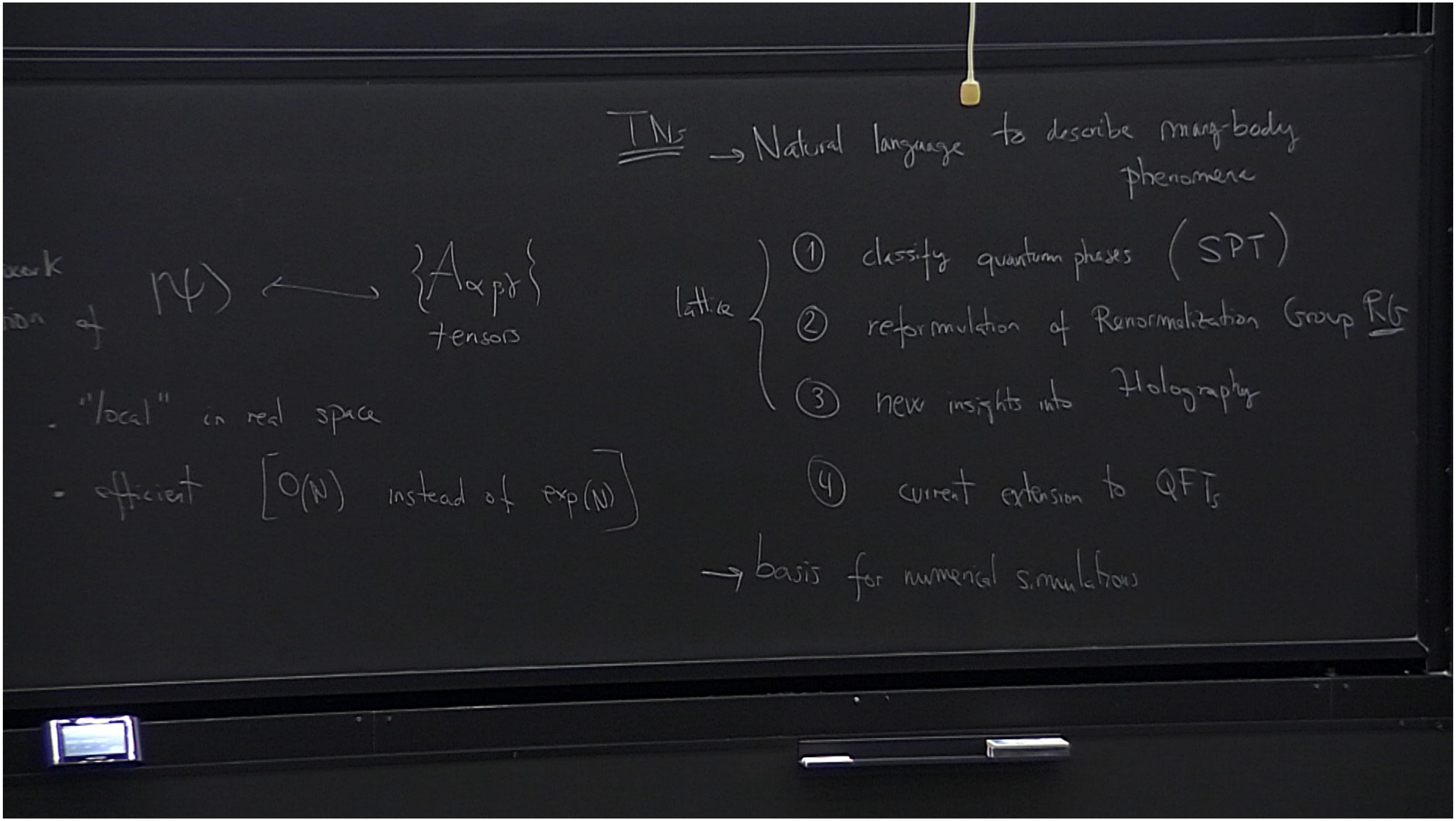

matrix product state MPS
multi-scale entanglement renormalization ansatz (MERA)
projected entangled-pair states
(PEPS)

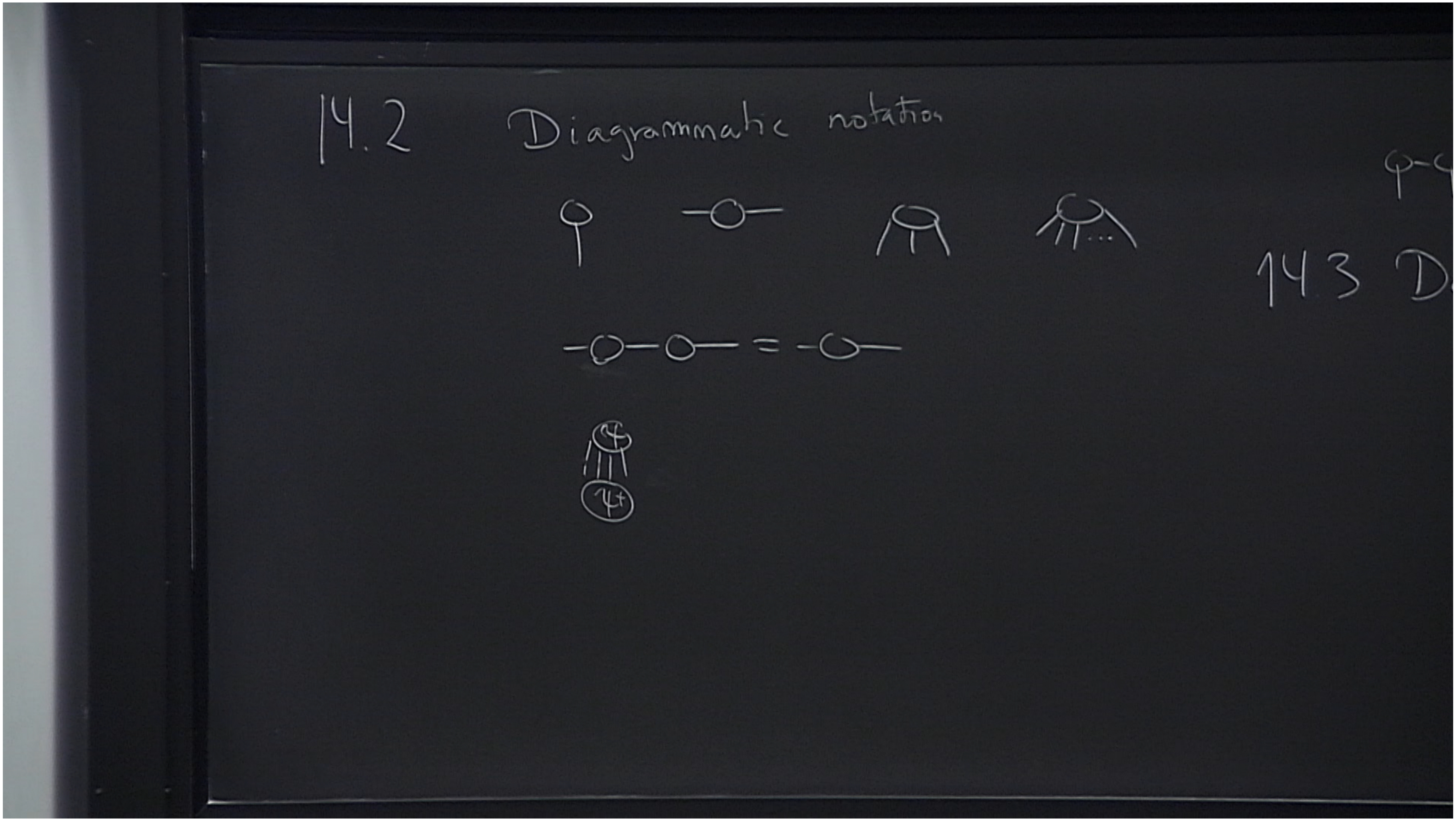

NEXT 5 LECTURES (+ 3 tutorials)
- Basics of TNs
- MPS
- MERA
1+1 systems
- TNs in D>1 space dimensions

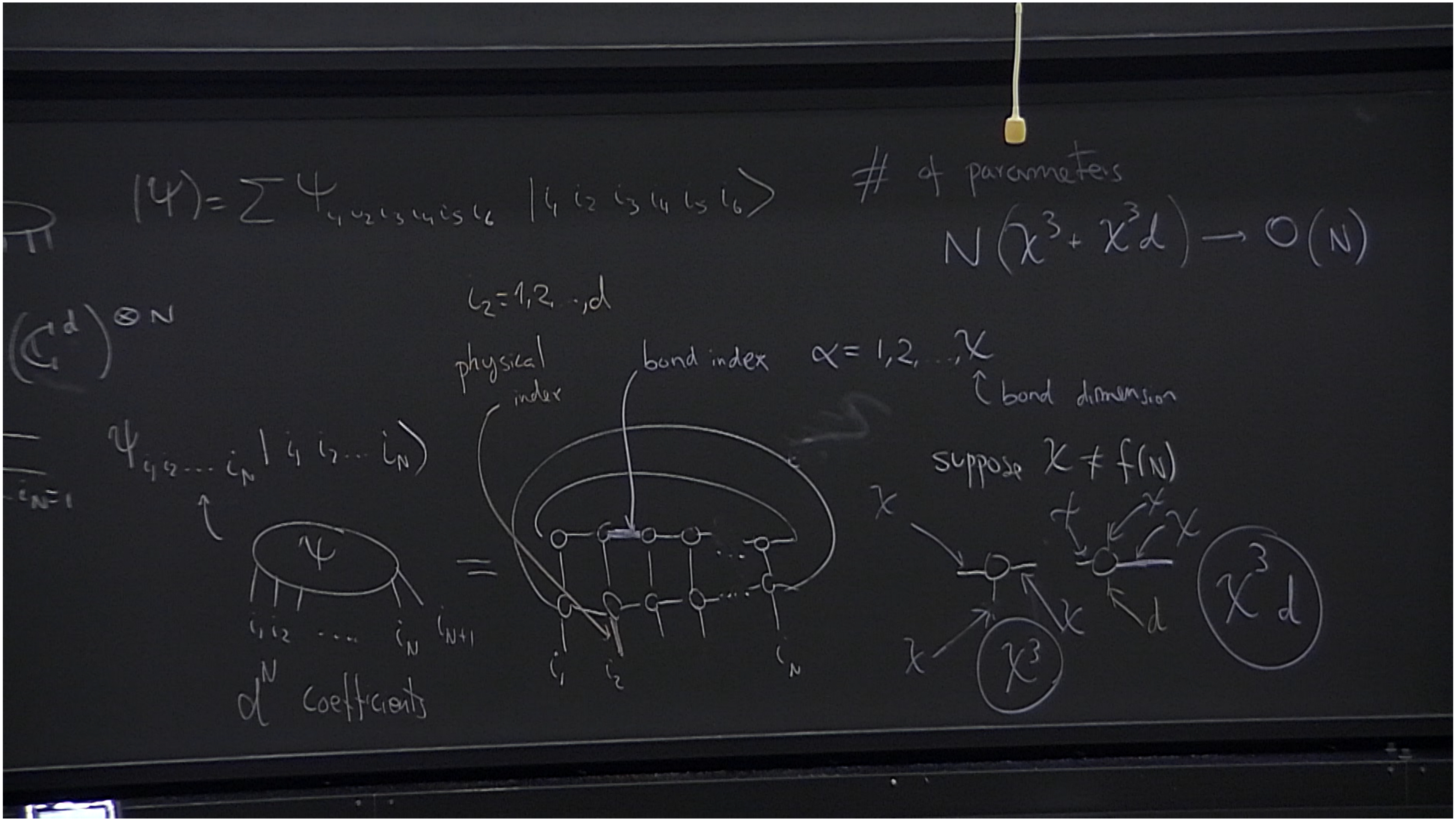

Lecture 14 (4th)
BASICS OF TENSOR NETWORKS
14.1 Why?
ground state
entanglement
(Area law, etc)
tensor network
description of |ψ⟩



matrix product state MPS
projected entangled-pair states
(PEPS)
scale
space
multi-scale entanglement renormalization ansatz (MERA)


TNs → Natural language to describe many-body phenomena
lattice
① classify quantum phases (SPT)
② reformulation of Renormalization Group RG
③ new insights into Holography
④ current extension to QFTs
→ basis for numerical simulations
|ψ⟩ ⟷ {A_αβγ}
tensors
· "local" in real space
· efficient [O(N) instead of exp(N)]


14.2 Diagrammatic notation
14.3

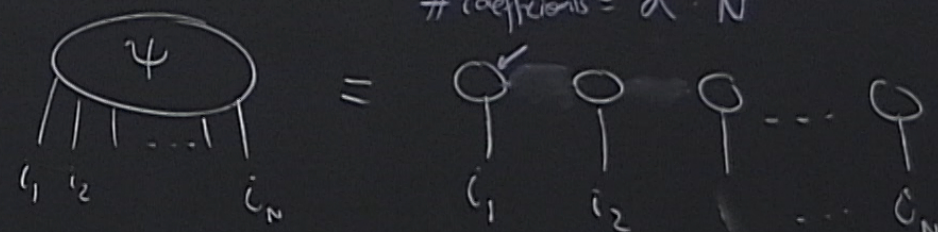

|Ψ⟩ = Σ Ψ_{i1 i2 i3 i4 i5 i6} |i1 i2 i3 i4 i5 i6⟩
# of parameters
N(χ³ + χ³d) → O(N)
i2 = 1, 2, .., d
(C^d)^⊗N
physical index
bond index
α = 1, 2, ..., χ
bond dimension
suppose χ ≠ f(N)
Ψ_{i1 i2 ... iN} |i1 i2 ... iN⟩
Ψ
i1 i2 ... iN iN+1
i1 i2 iN
d^N coefficients
χ³
χ³d

Example 1

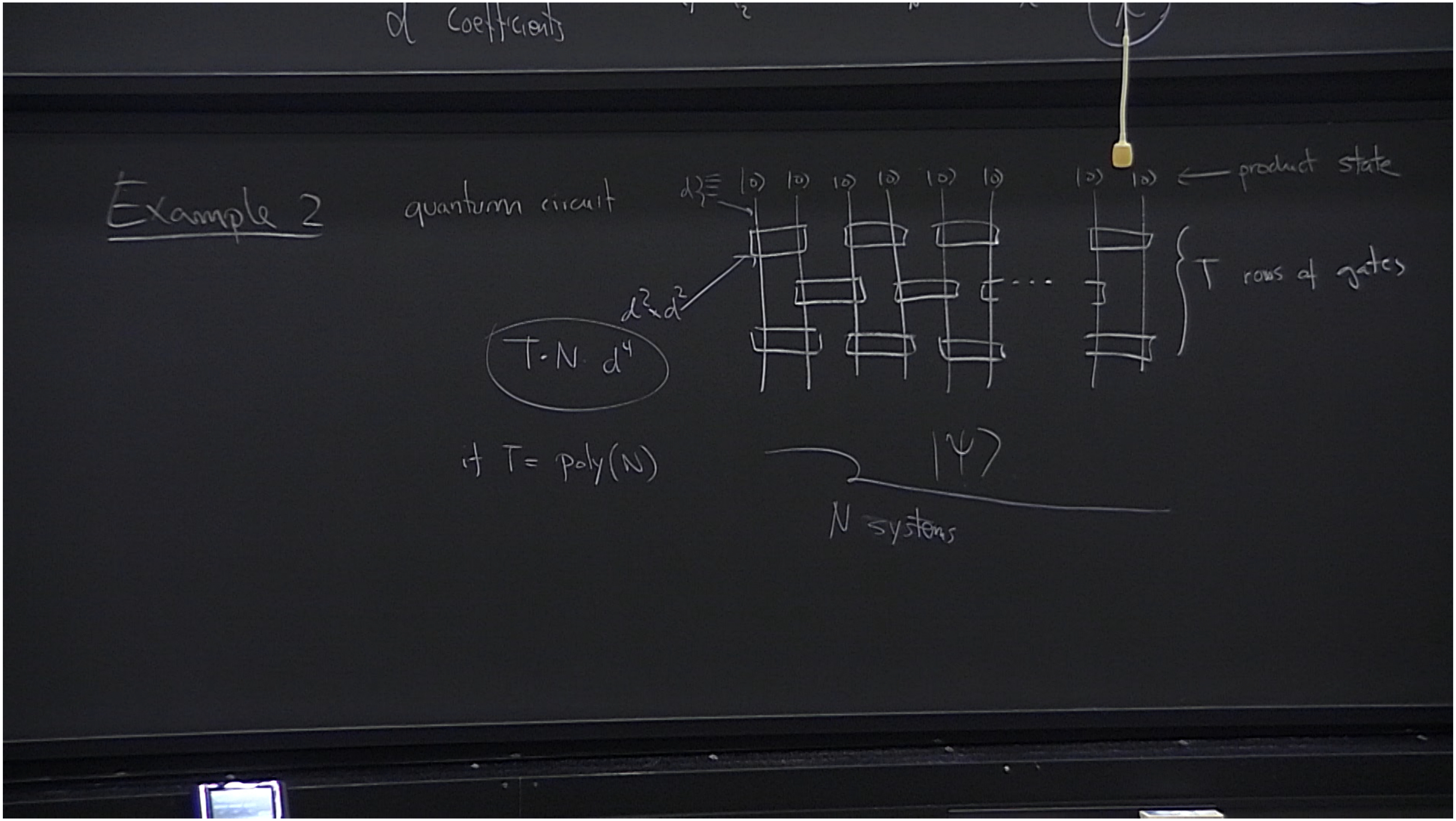
$$|\psi\rangle = |\varphi^1\rangle \otimes |\varphi^2\rangle \otimes \dots \otimes |\varphi^N\rangle$$

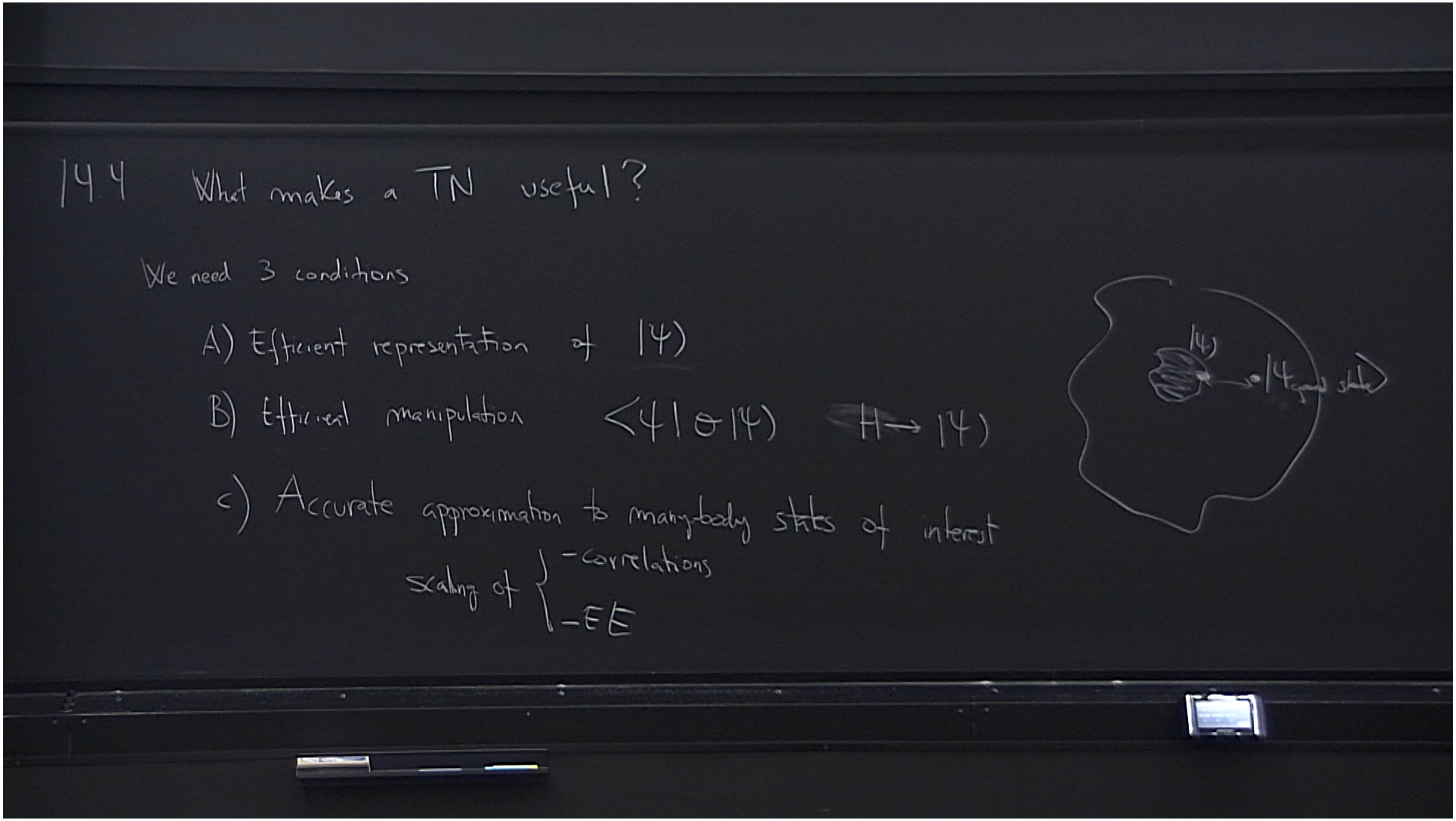
$$\psi_{i_1 i_2 \dots i_N} = \left(\varphi^{(1)}\right)_{i_1} \cdot \left(\varphi^{(2)}\right)_{i_2} \cdots \left(\varphi^{(N)}\right)_{i_N}$$

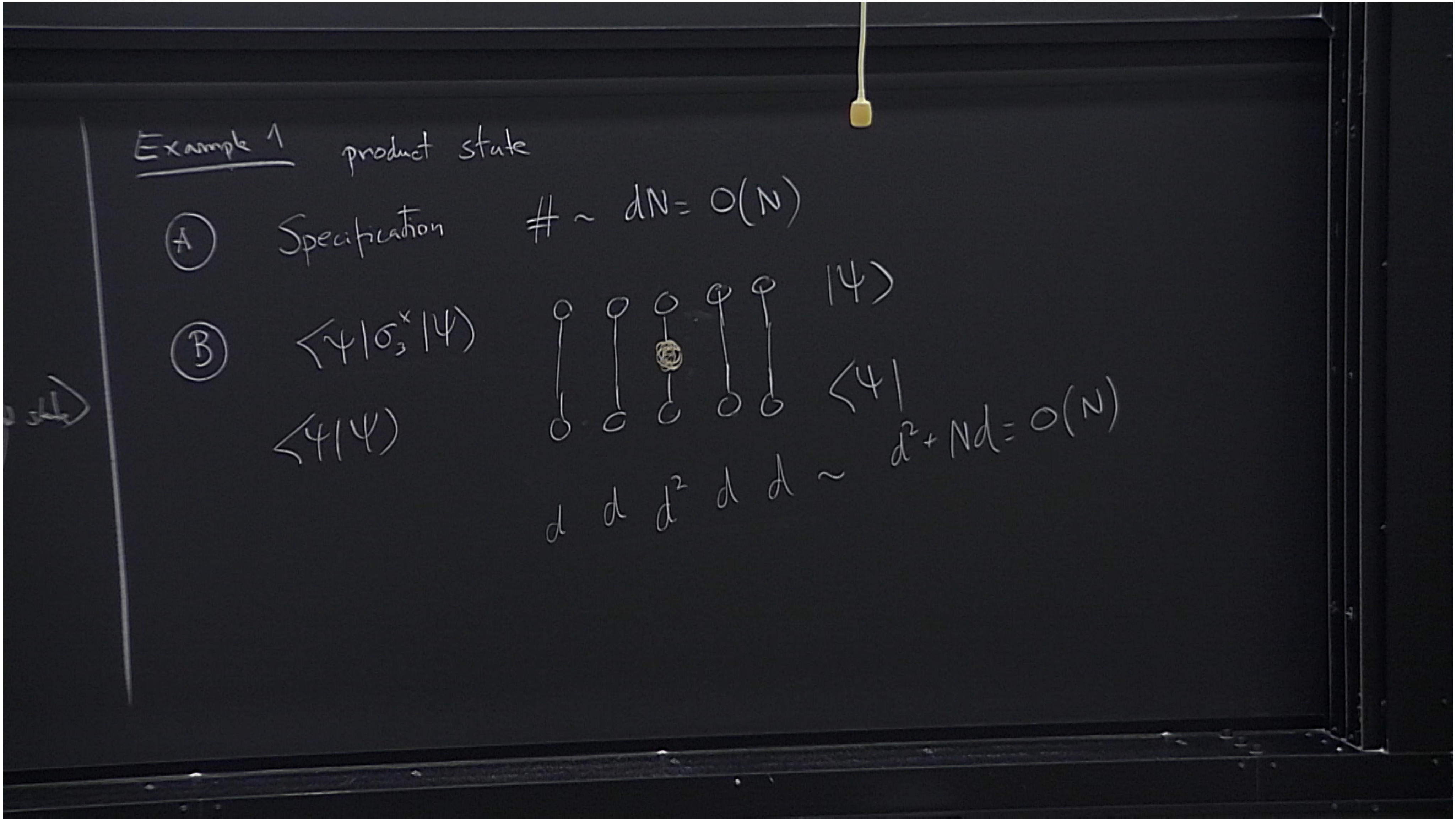
coefficients = $d \cdot N$

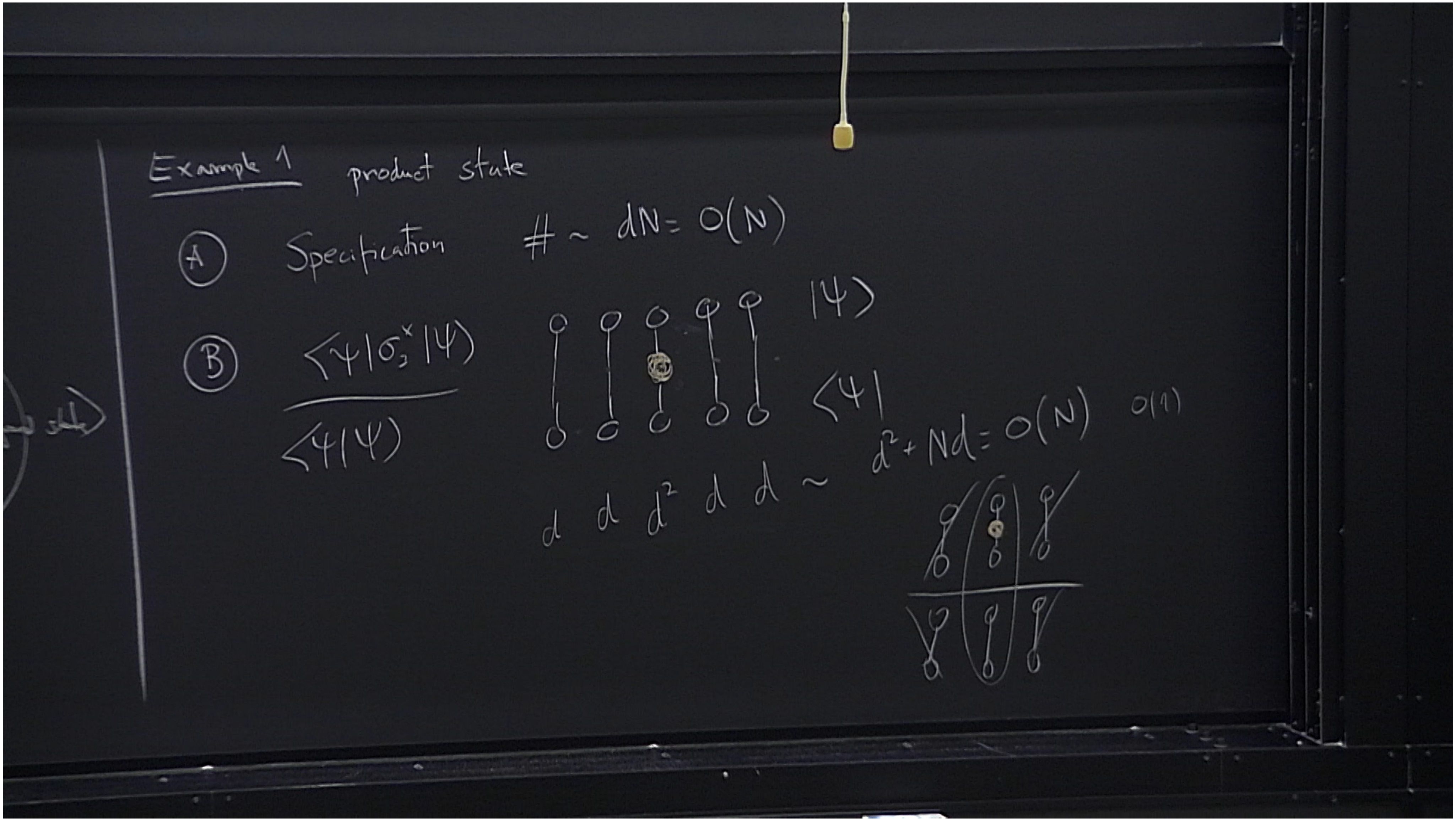


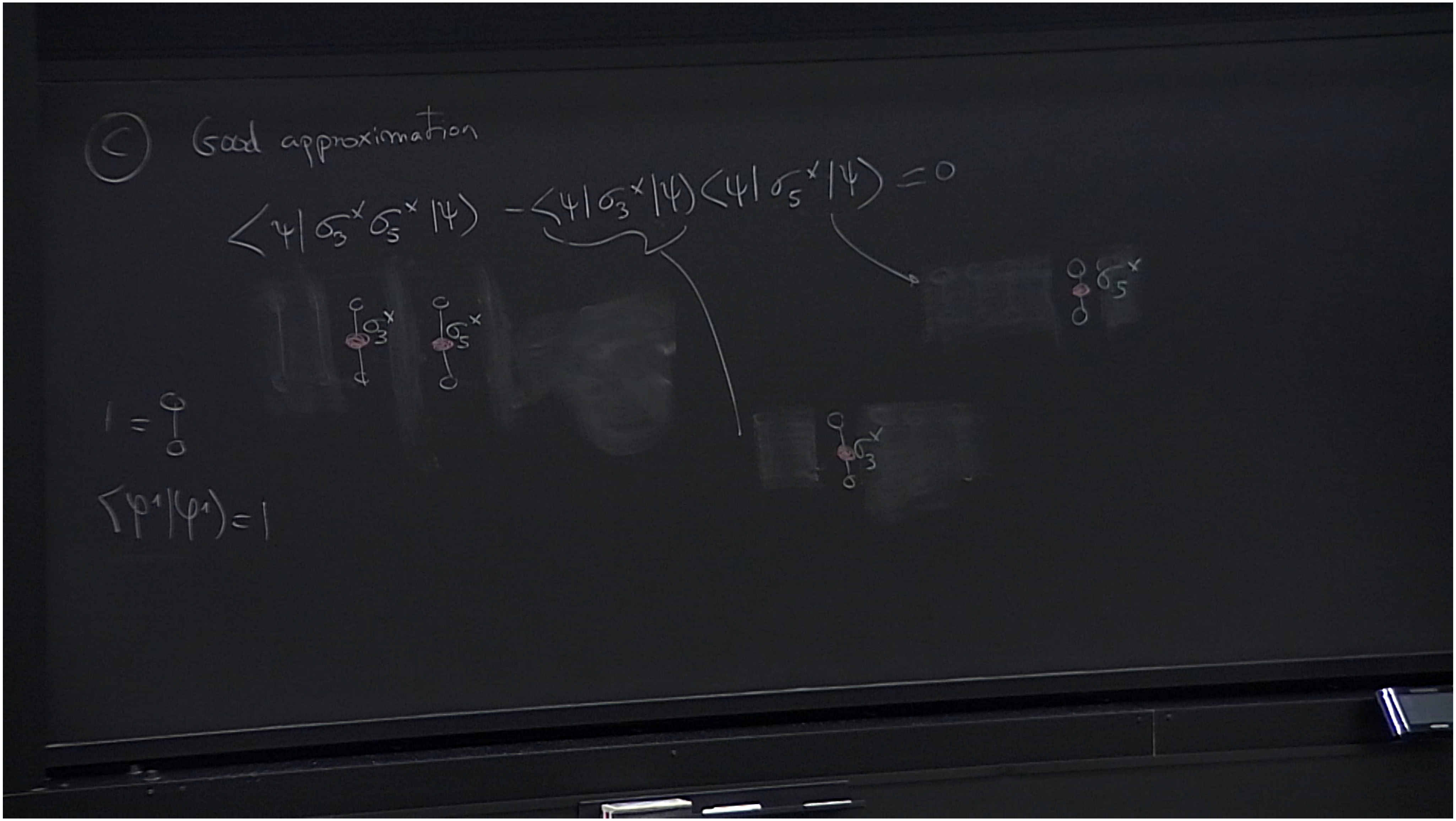
$$|\varphi^1\rangle = \sum_{i=1}^{d} c_i |i\rangle$$

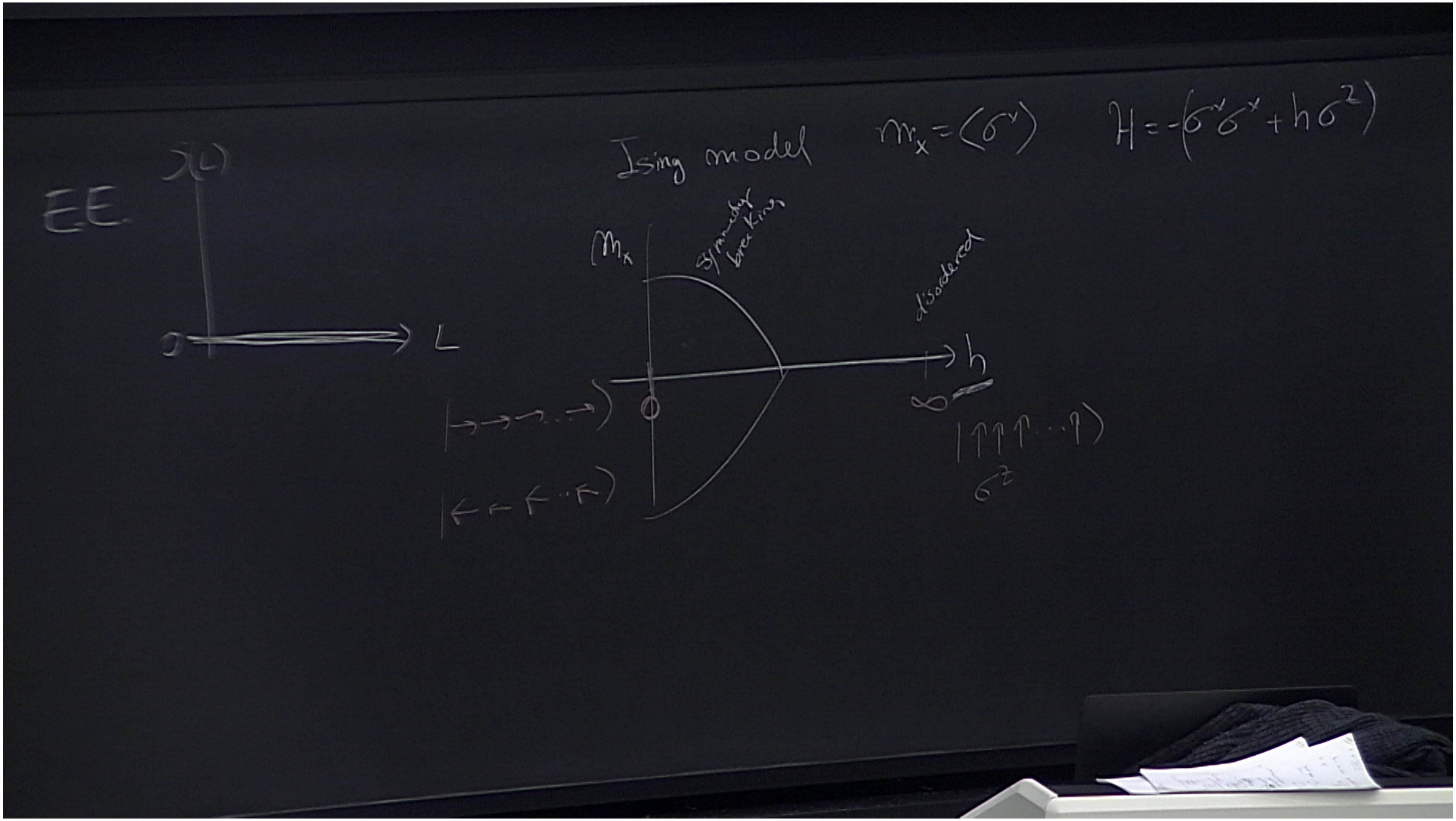

d Coefficients
Example 2
quantum circuit
|0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩
← product state
T rows of gates
d²×d²
T·N·d⁴
if T = poly(N)
|Ψ⟩
N systems


144 What makes a TN useful?
We need 3 conditions
A) Efficient representation of |ψ⟩
B) Efficient manipulation ⟨ψ|O|ψ⟩ H → |ψ⟩
C) Accurate approximation to many-body states of interest
scaling of
- correlations
- EE
|ψ⟩
|ψ ground state⟩


Example 1 product state
A Specification # ~ dN = O(N)
B ⟨ψ|σ$_3^x$|ψ⟩
⟨ψ|ψ⟩
|ψ⟩
⟨ψ|
d d d^2 d d ~ d^2 + Nd = O(N)


Example 1 product state
(A) Specification # ~ dN = O(N)
(B) ⟨ψ|σ_3^x|ψ⟩ / ⟨ψ|ψ⟩
|ψ⟩
⟨ψ|
d d d² d d ~ d² + Nd = O(N) O(1)


(c) Good approximation
$\langle\psi|\sigma_3^x\sigma_5^x|\psi\rangle - \langle\psi|\sigma_3^x|\psi\rangle\langle\psi|\sigma_5^x|\psi\rangle = 0$
σ_3^x
σ_5^x
σ_5^x
σ_3^x
1 =
$\langle\varphi^1|\varphi^1\rangle = 1$


E.E.
S(L)
L
Ising model
symmetry breaking
disordered
h
σ^z

In order to discuss tensor network representations of quantum many-body states, it is useful to introduce a diagrammatic notation. Fig. 2 shows how to represent tensors and tensor multiplications.

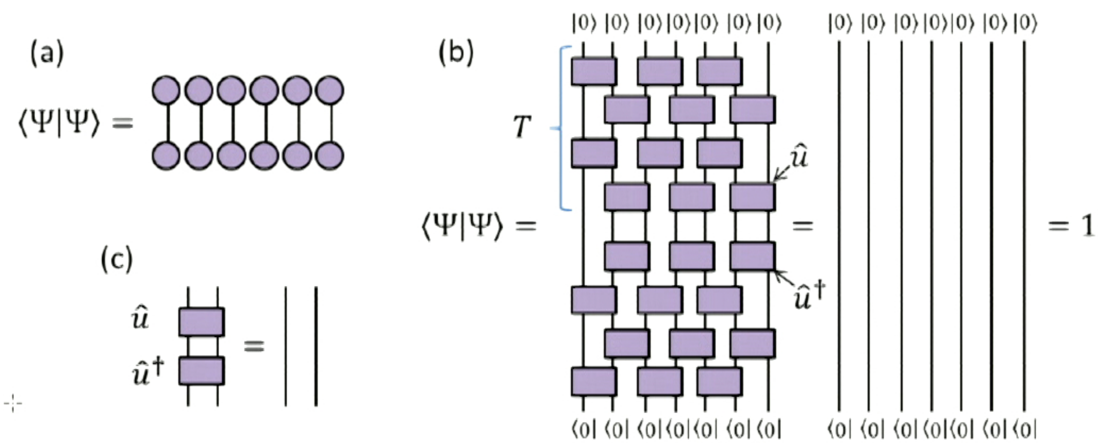


FIG. 4: (a) Scalar product of a product state. (b) Scalar product for a tensor network based on a quantum circuit of depth T. (c) Diagrammatical representation of the expression $\hat{u}^\dagger \hat{u} = \hat{\mathbb{I}} \otimes \hat{\mathbb{I}}$.

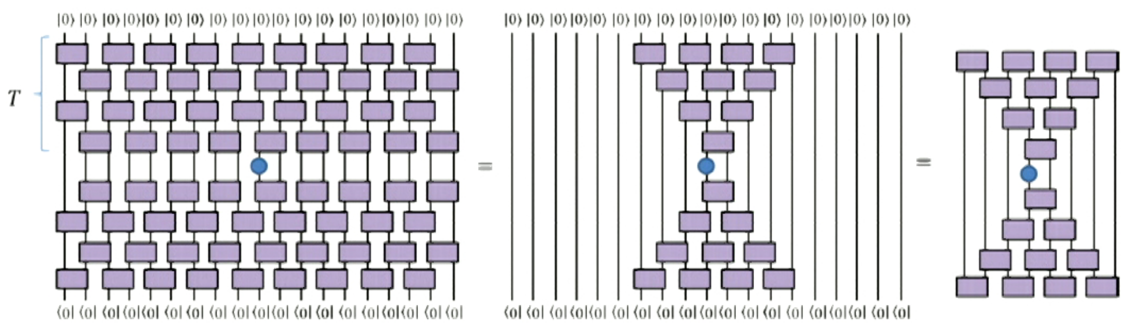
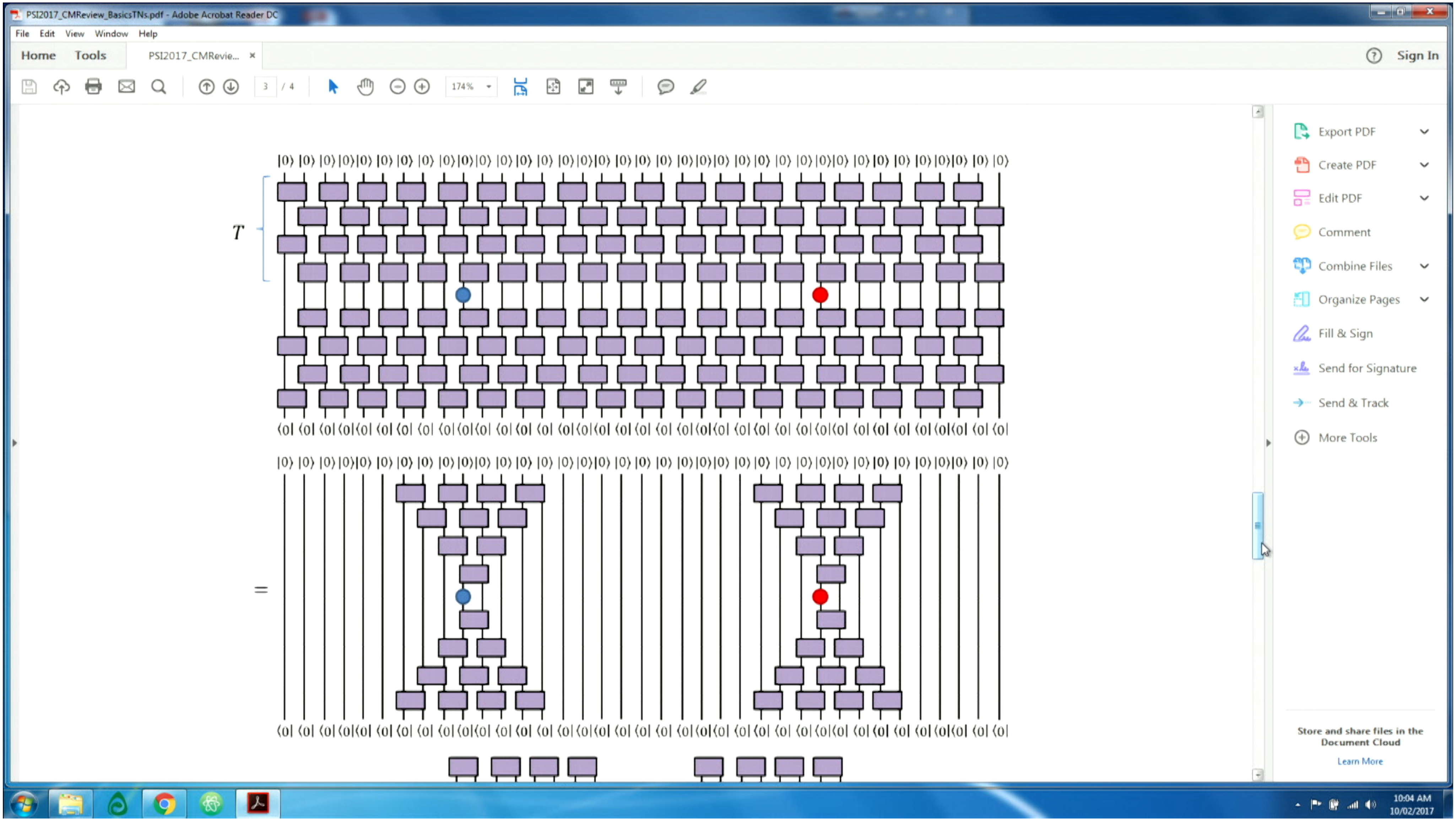


FIG. 5: Expectation value of a local observable, $\langle\Psi|\hat{o}|\Psi\rangle$, for a tensor network based on a depth-T quantum circuit. The local observable is represented by a blue circle on a central site. Notice that the *past causal cone* of this site has a width proportional to T. This results in a computational cost that grows exponentially in T, $C \approx e^{T}$.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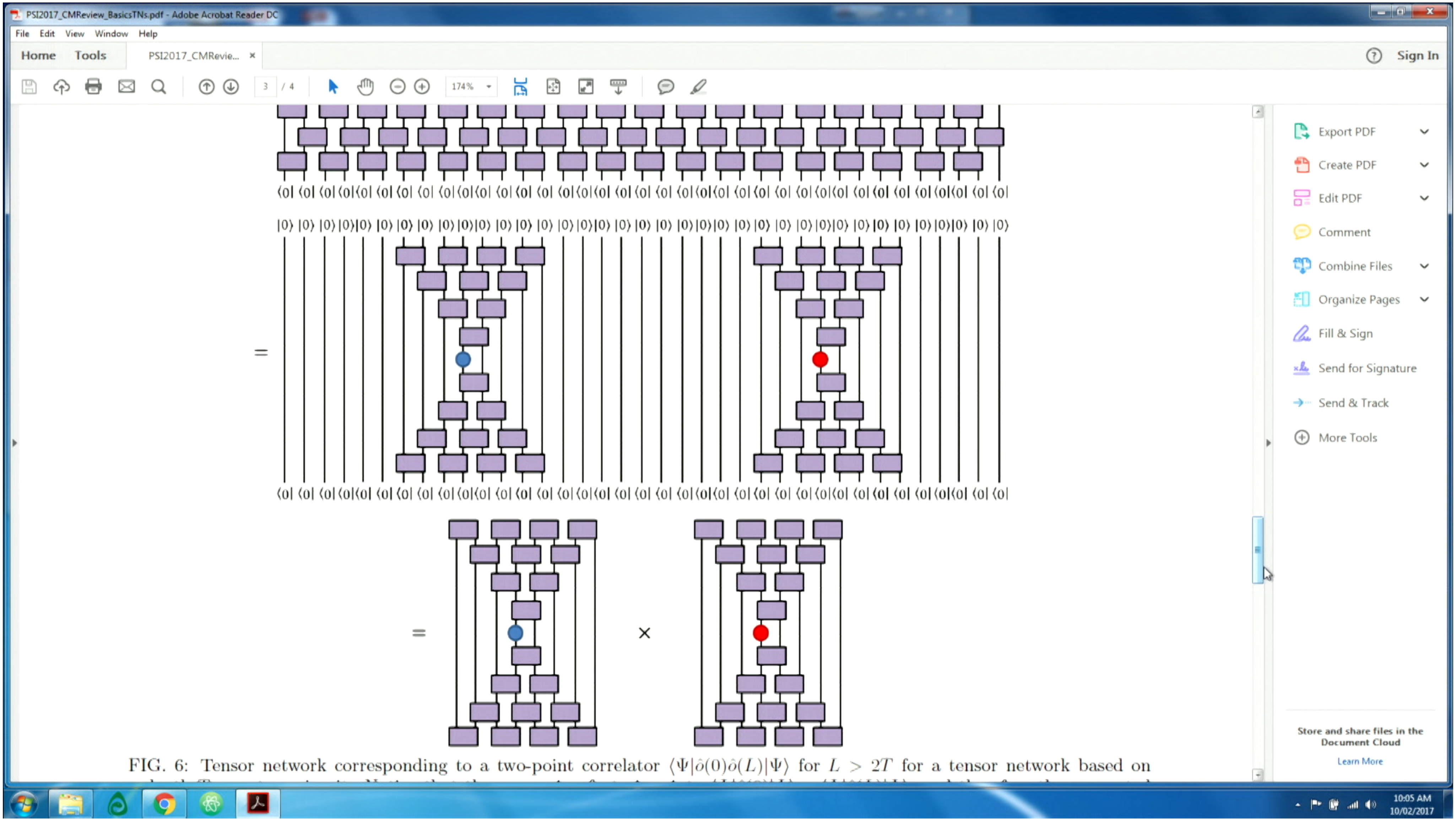
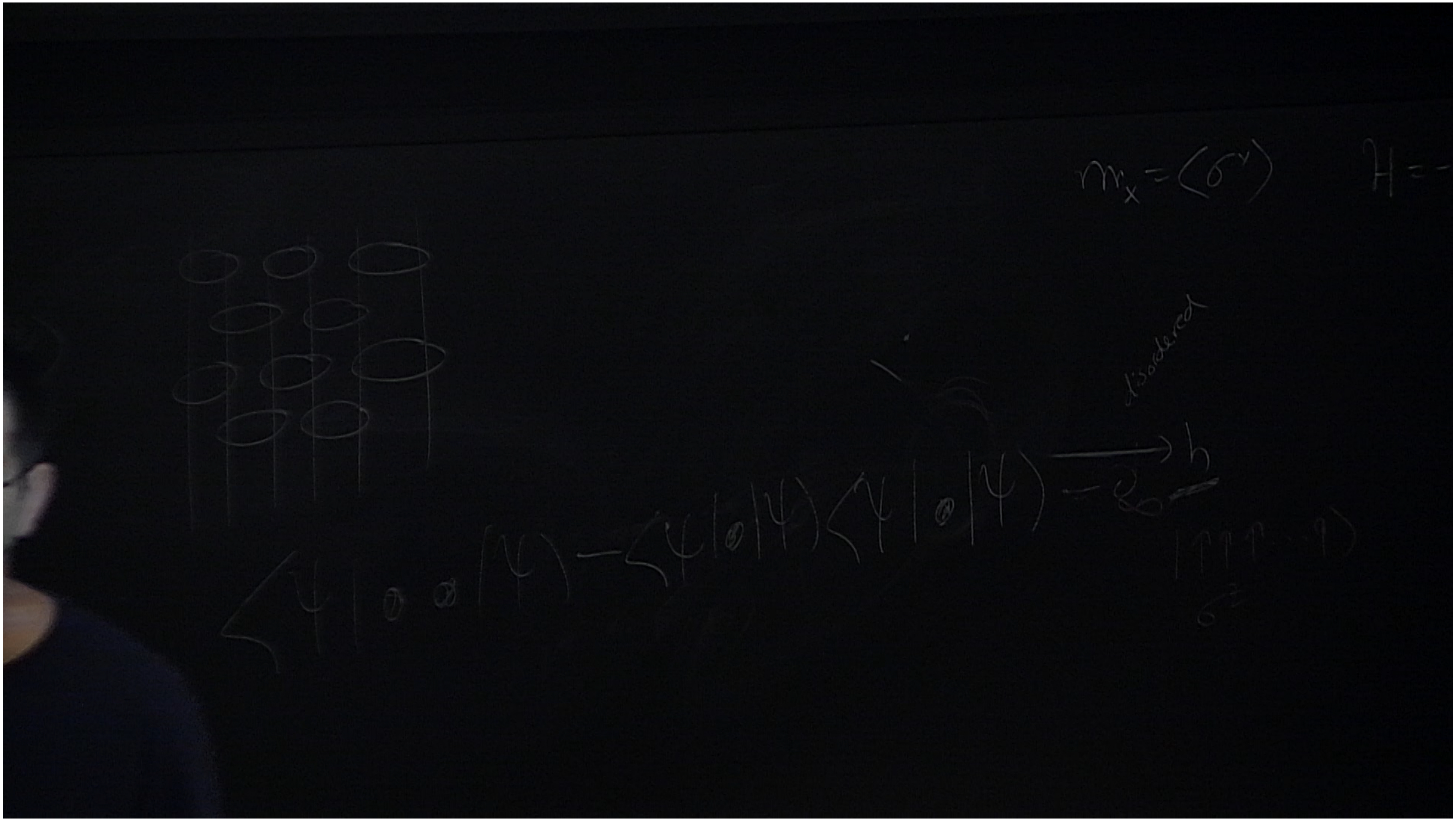
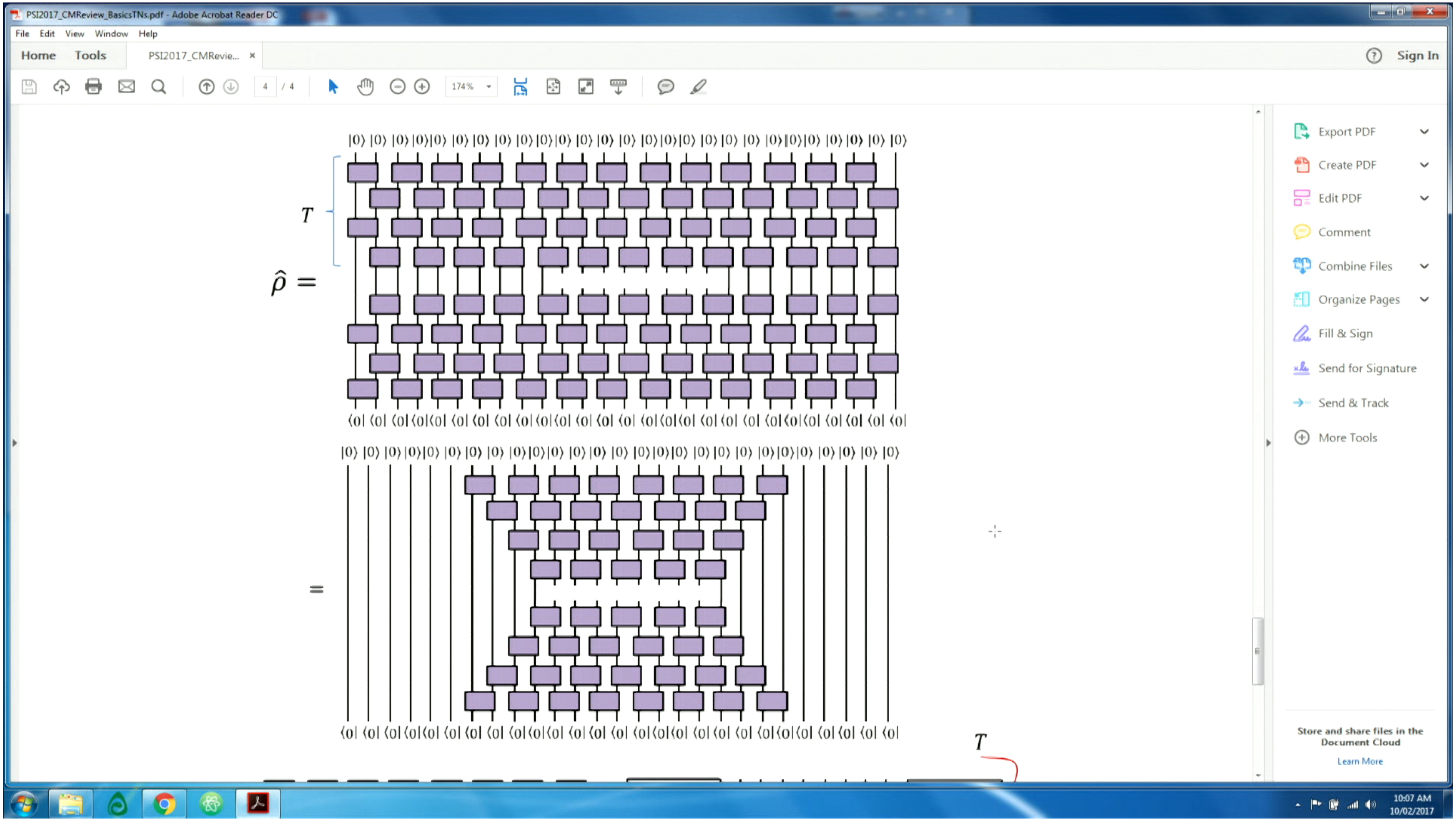


FIG. 6: Tensor network corresponding to a two-point correlator $\langle\Psi|\hat{o}(0)\hat{o}(L)|\Psi\rangle$ for $L > 2T$ for a tensor network based on


disordered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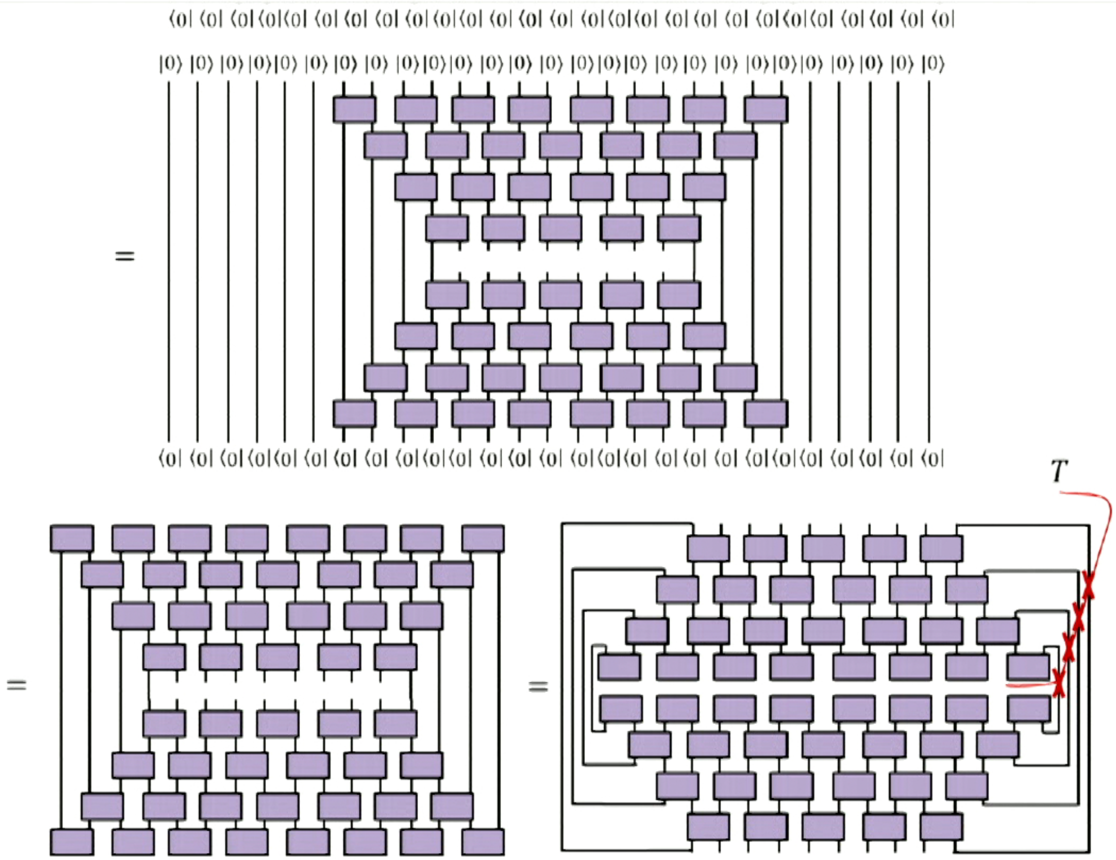

FIG. 7: Tensor network corresponding to the reduced density matrix $\hat{\rho}$ for L contiguous sites, when the state of the whole lattice is described by a tensor network based on a depth-T quantum circuit. Notice that, for large L, the incoming and outgoing indices of $\hat{\rho}$ can be spitted by cutting $2T$ indices – which means that the entropy of $\hat{\rho}$ can at most be $2T \log_2(d)$.