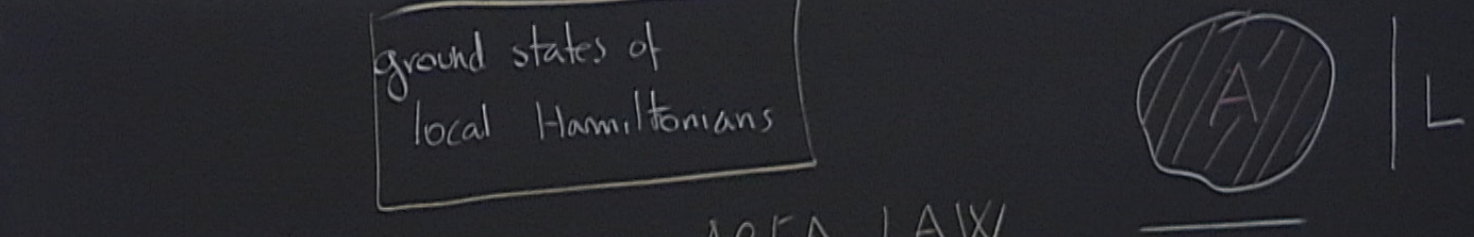
Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 7

Date: Feb 09, 2017 09:00 AM

URL: http://pirsa.org/17020044

Abstract:

ground states of
local Hamiltonians



AREA LAW

$S_L \sim |\partial A| \sim L^{D-1}$

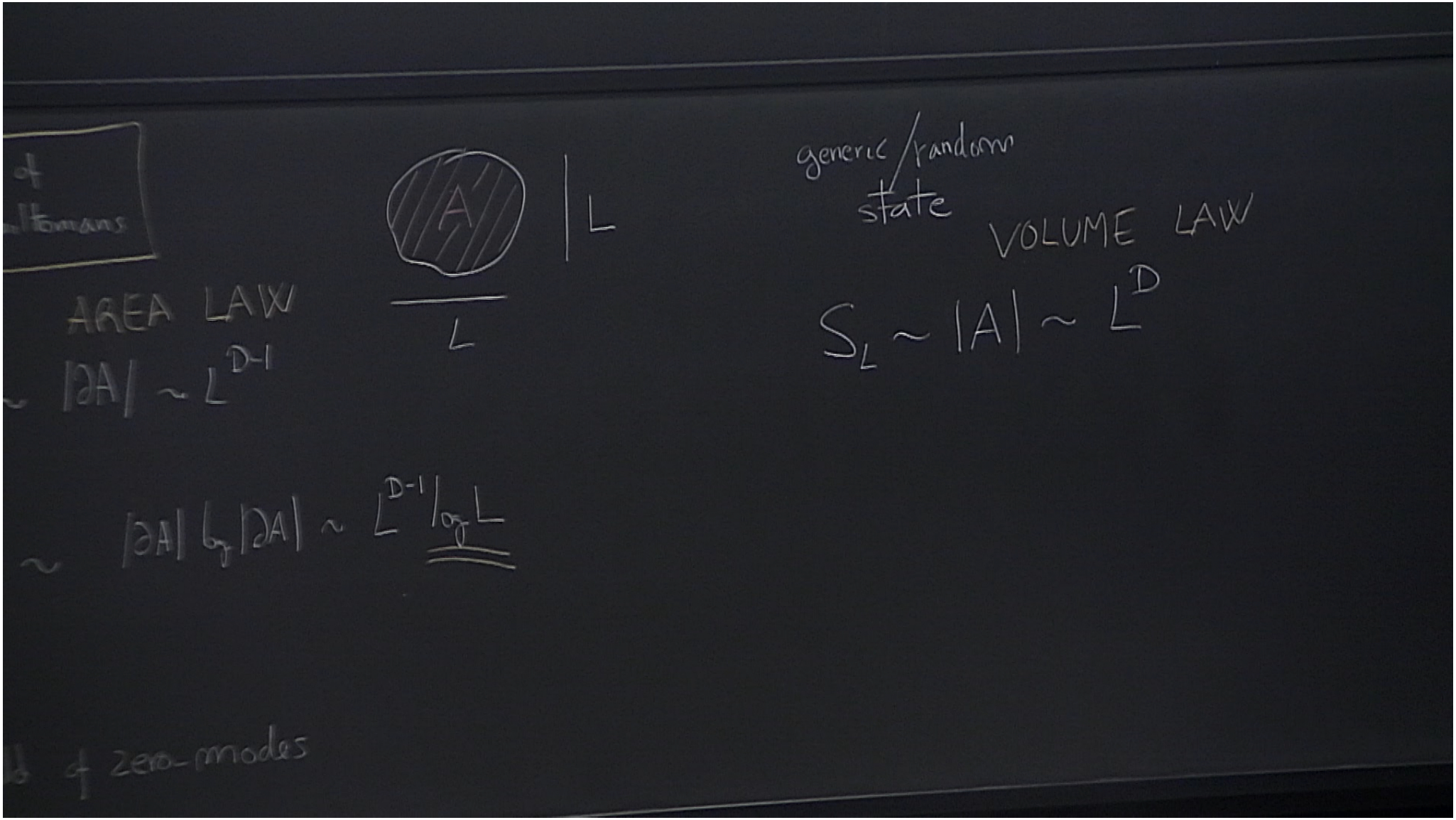
gapped
EK

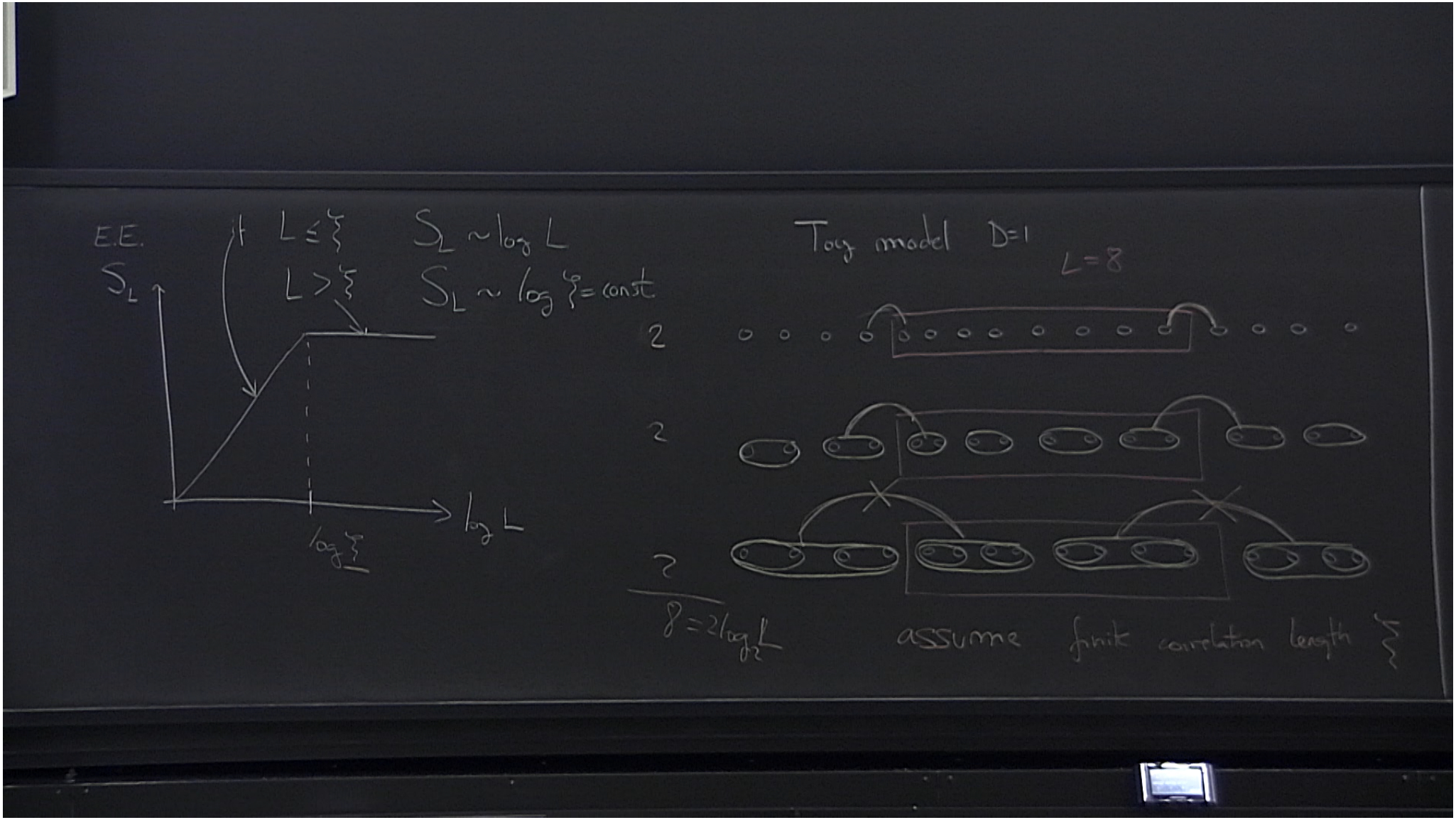
gapless
I

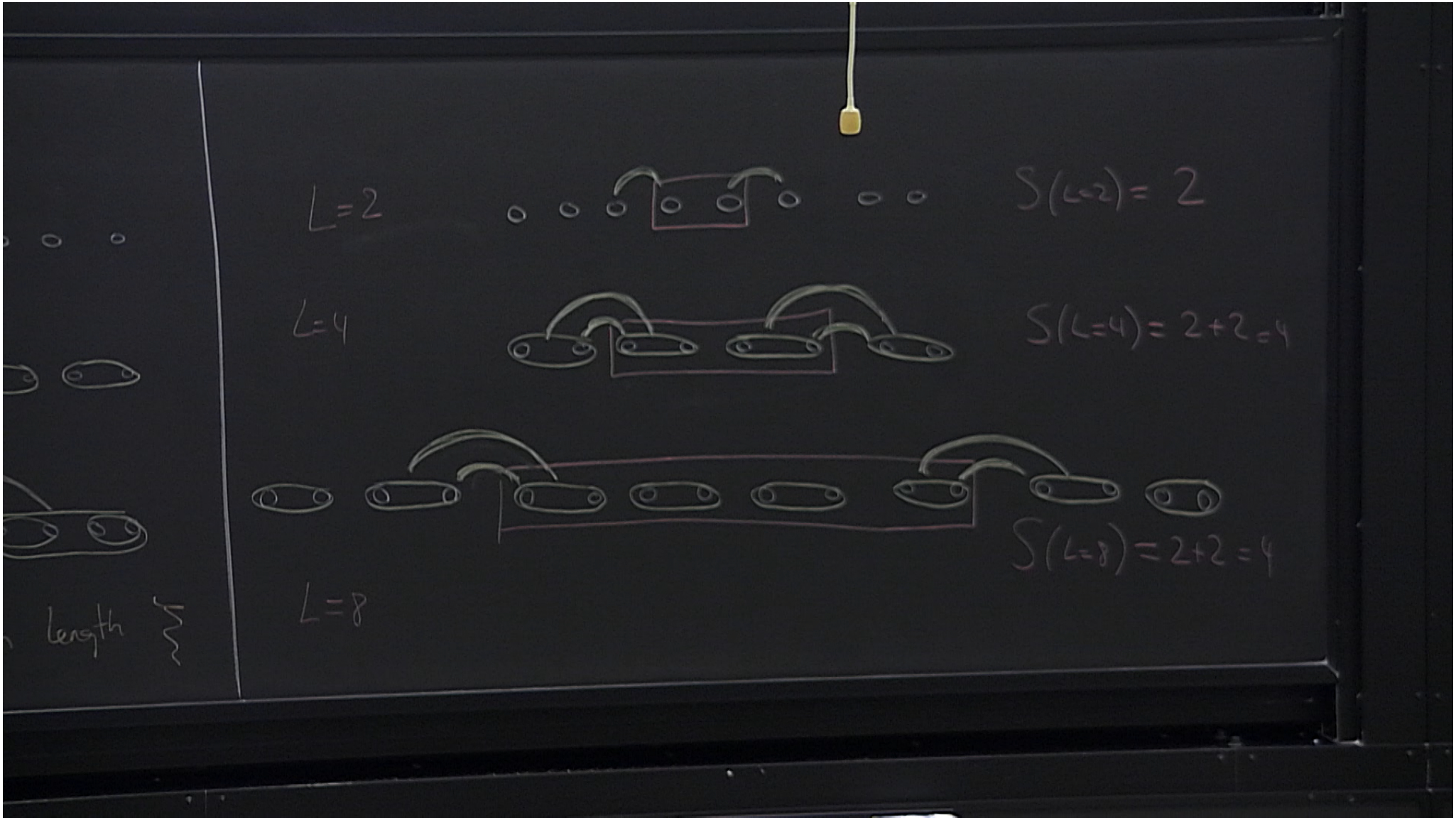
gapless II

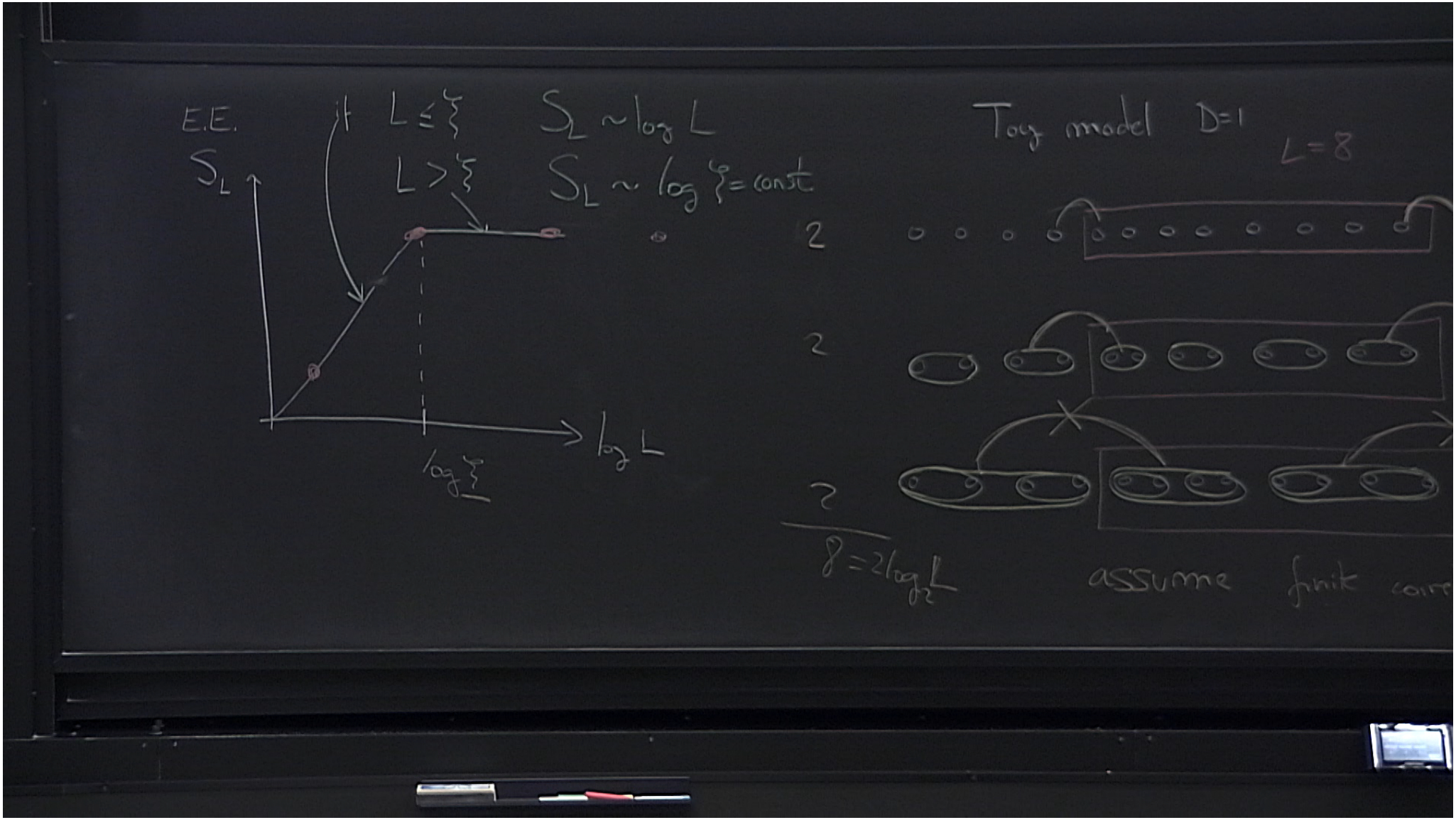
$S_L \sim |\partial A| \log |\partial A| \sim L^{D-1} \log L$

D−1 dimensional manifold of zero-modes


AREA LAW
$\sim |\partial A| \sim L^{D-1}$
$\sim |\partial A| \log |\partial A| \sim L^{D-1} \log L$
A
L
L
generic / random
state
VOLUME LAW
$S_L \sim |A| \sim L^D$
of zero-modes


E.E.
S_L
if $L \leq \xi$ $S_L \sim \log L$
$L > \xi$ $S_L \sim \log \xi = \text{const}$
$\log \xi$
$\log L$
Toy model D=1
L=8
2
2
2
$\delta = 2 \log_2 L$
assume finit correlation length ξ


L=2
S(L=2)= 2
L=4
S(L=4)= 2+2=4
S(L=8)=2+2=4
L=8
length


E.E.
if $L \leq \xi$ $S_L \sim \log L$
$L > \xi$ $S_L \sim \log \xi = \text{const}$
S_L
$\log L$
$\log \xi$
Toy model D=1
L=8
2
2
2
$\delta = 2 \log_2 L$
assume finit

Lect. 13 (th) Entanglement as a theoretical tool

13.1 Entanglement Entropy (EE) & universality

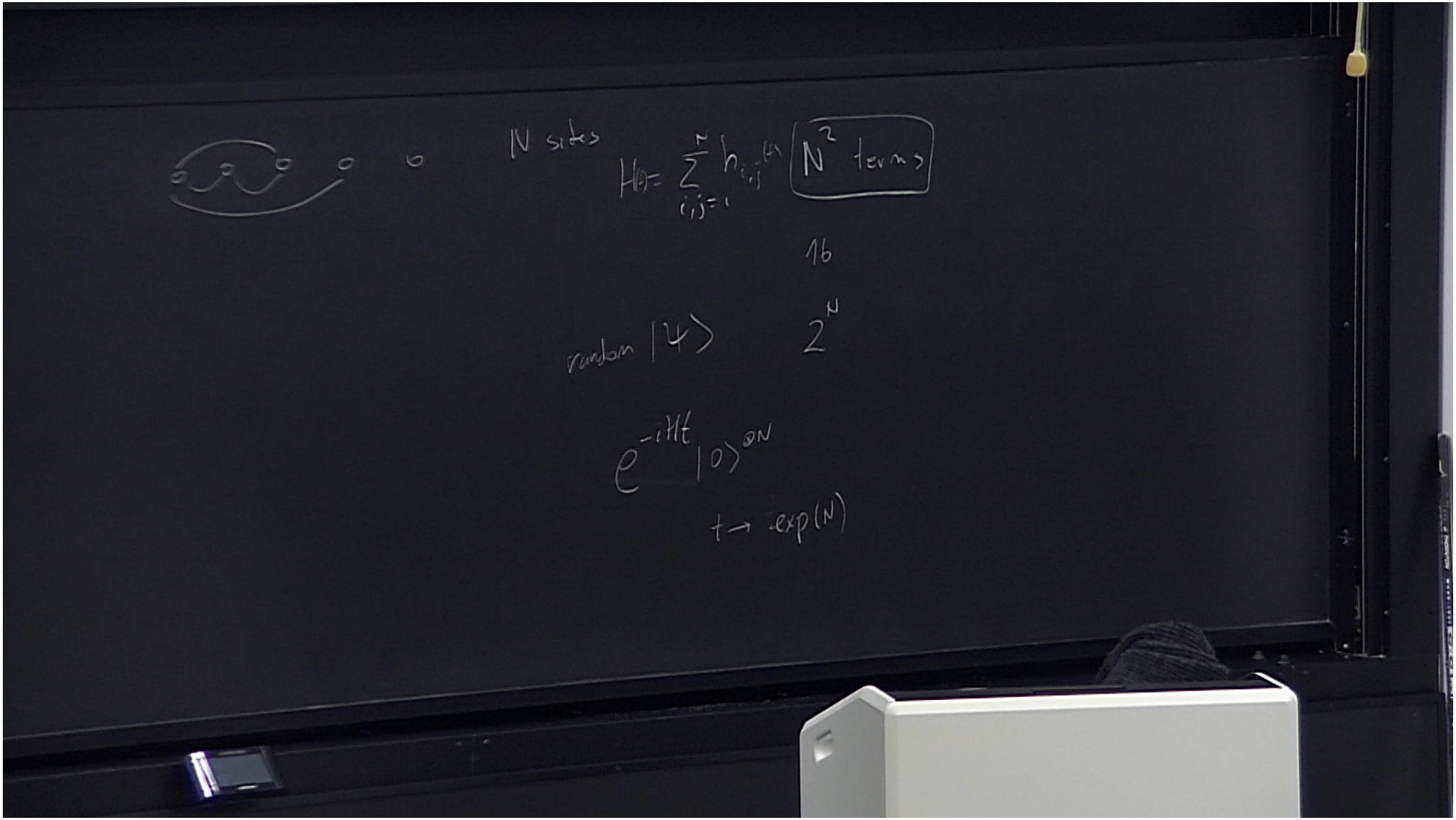
$\mathcal{H}$ local ⟵ ? ⟶ Area law

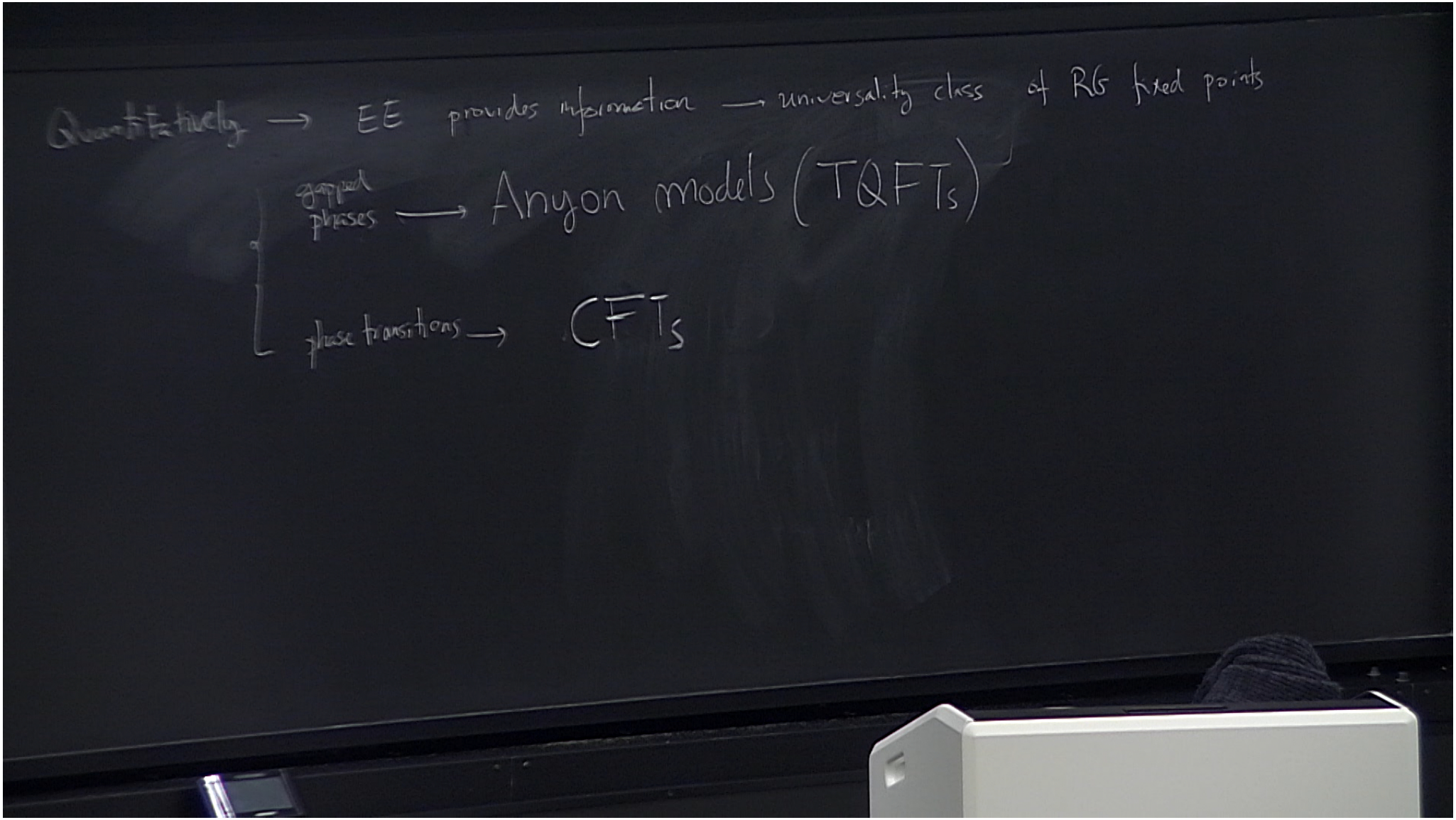
D-dimensions

lattice, QFT

$S_L \sim L^{D-1}$

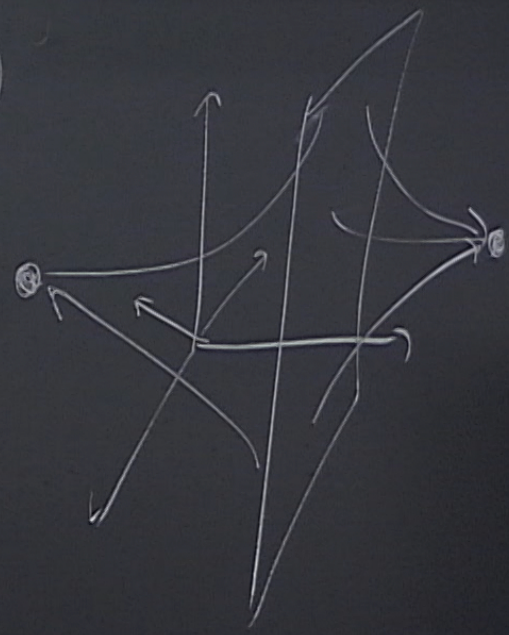
$(L^{D-1} \log L)$

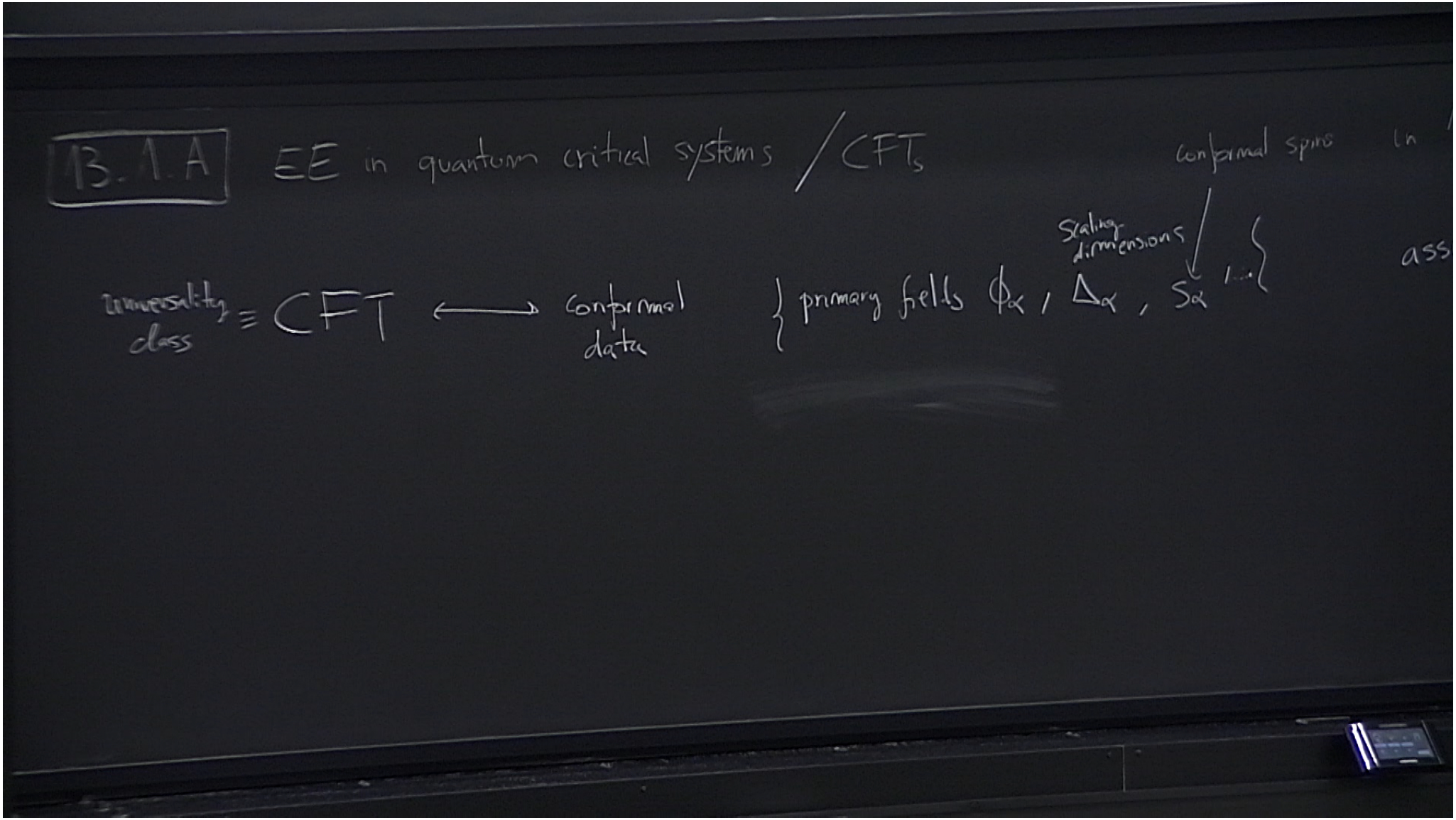

N sites
$H = \sum_{i,j=1}^{N} h_{i,j}$
N^2 terms
16
random $|4\rangle$
2^N
$e^{-iHt}|0\rangle^{\otimes N}$
$t \to \exp(N)$

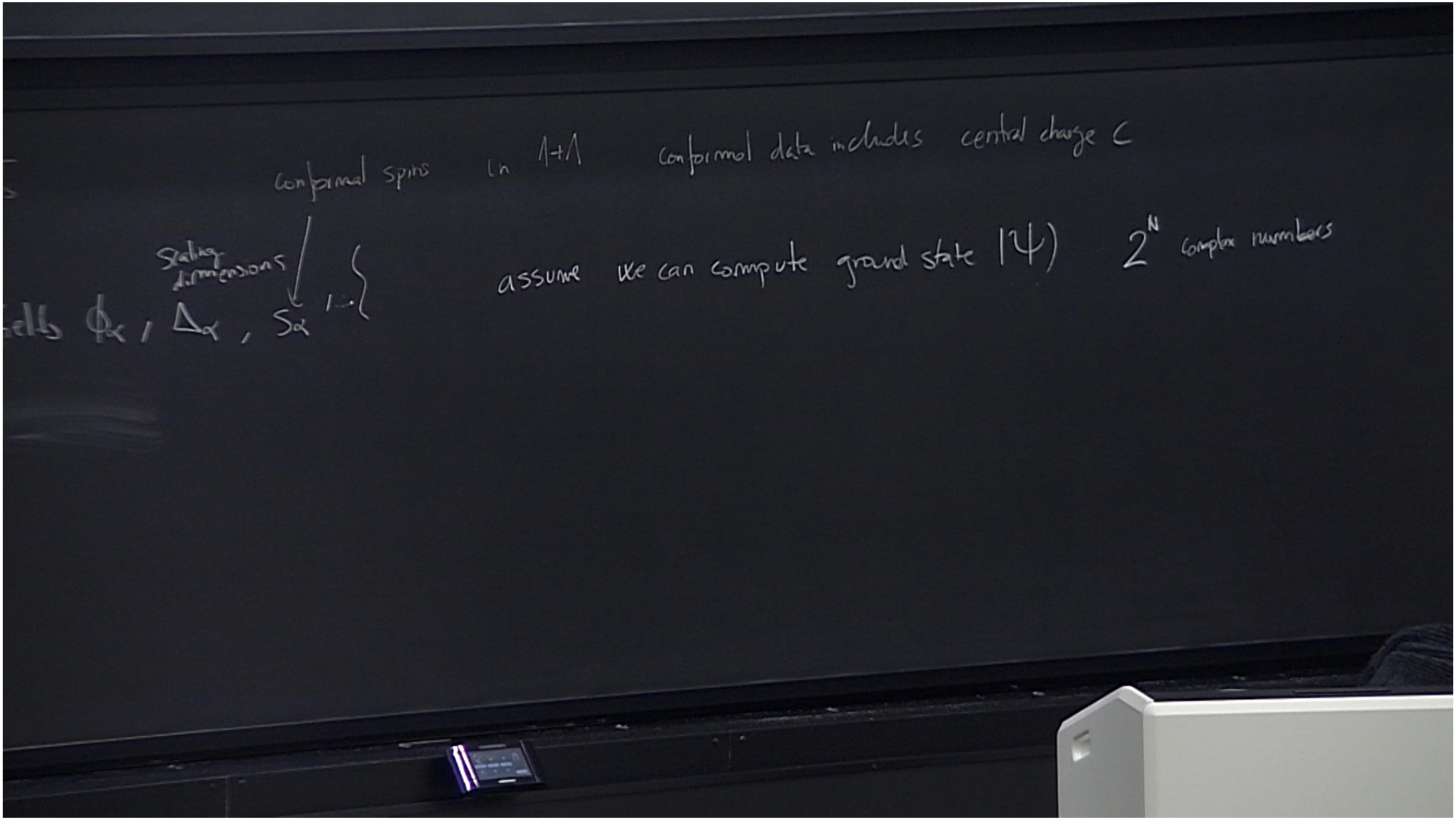

Quantitatively → EE provides information → universality class of RG fixed points
gapped phases → Anyon models (TQFTs)
phase transitions → CFTs

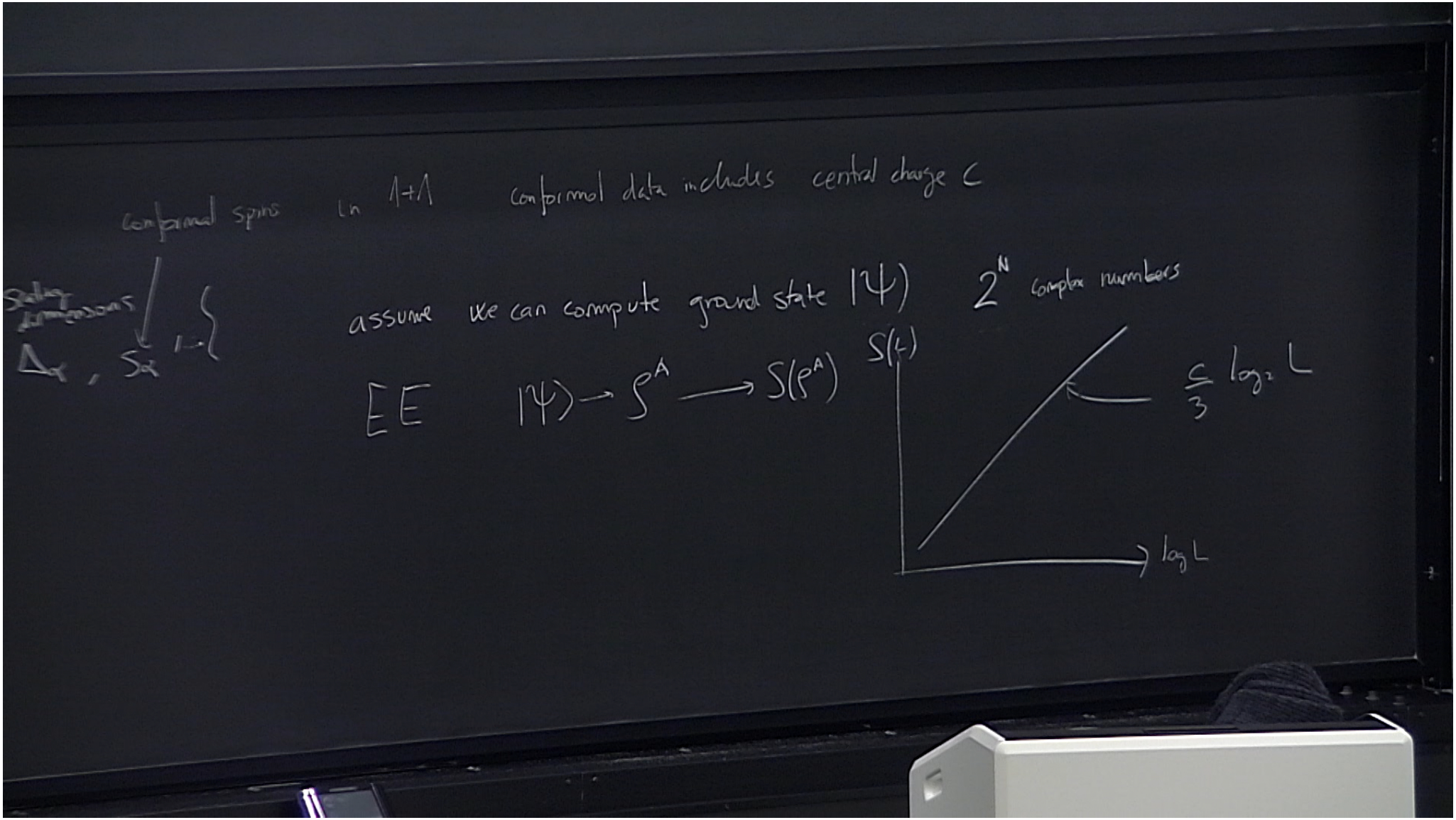
phases ⟶ Anyon models (TQFTs)

phase transitions ⟶ CFTs




B.1.A
EE in quantum critical systems / CFTs
conformal spins
scaling dimensions
universality class ≡ CFT ⟷ conformal data
{ primary fields ϕ_α, Δ_α, S_α, ... }


conformal spins in 1+1 conformal data includes central charge c
scaling dimensions
assume we can compute ground state |Ψ⟩ 2^N complex numbers


conformal spins in 1+1 conformal data includes central charge c
Scaling dimensions
Δ_i, S_i
assume we can compute ground state $|\psi\rangle$
2^N complex numbers
EE $|\psi\rangle \to \rho^A \to S(\rho^A)$
S(L)
$\frac{c}{3}\log_2 L$
log L

1.A EE in quantum critical systems / CFTs

Universality class $\equiv$ CFT $\longleftrightarrow$ conformal data $\{$ primary fields ϕ_α, Δ_α (scaling dimensions), s_α (conformal spins), ... $\}$ in 1+1

assume uc

EE

$c<1$ discrete set of unitary minimal models $c = 1 - \frac{6}{m(m+1)}$ $m = 3, 4, 5 \ldots$

$c = \frac{1}{2}$ Ising CFT

$c = 7/10$ tricritical Ising

$c = 4/5$ 3-level Potts model

For $c=1$
2 continuous families of CFT

/CFTs

conformal spins in 1+1 conformal data includes central charge

scaling dimensions

primary fields ϕ_α / Δ_α, S_α / ... }

assume we can compute ground state $|\psi\rangle$

EE $|\psi\rangle \to \rho^A \longrightarrow S(\rho^A)$

S(t)

rational models $c = 1 - \frac{6}{m(m+1)}$ $m = 3, 4, 5 \ldots$

For $c = 1$

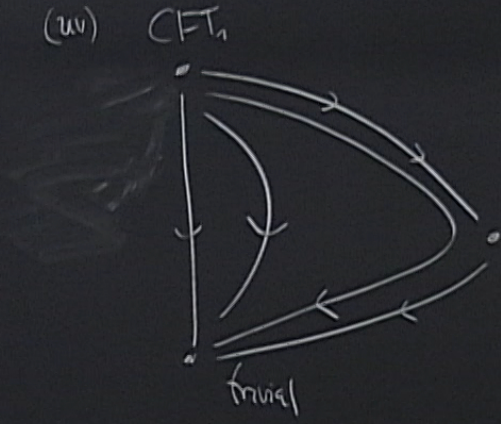
2 continuous families of CFT

compactified boson

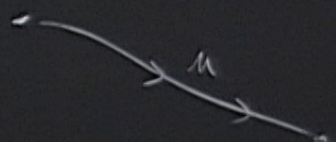
- circle/Coulomb models R_{circle}
- Orbifold models $R_{orbifold}$

C-theorem

Zamolodchikov 1986



$\exists\ c(g_i) \geq 0$ decreases monoton



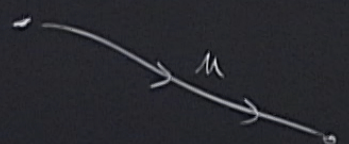
$\frac{dC(\mu)}{d\mu}$

Casini-Huerta 2006

$C(L) = 3L \frac{dS_L}{dL}$

P

$\exists\ c(g_i) \geq 0$ decreases monotonically along the RG flow



$\frac{dC(\mu)}{d\mu} < 0$ and $C(\mu)$ at an RG fixed-point is the central charge

$\| \; C$

Casini-Huerta 2006

proof requires:

SSA $S(ABC) + S(B) \leq S(AB) + S(BC)$

$C(L) = 3L \frac{dS_L}{dL}$

decreases monotonically along the RG flow

$\frac{dC(\mu)}{d\mu} < 0$ and $C(\mu)$ at an RG fixed-point is the central charge

$= C$



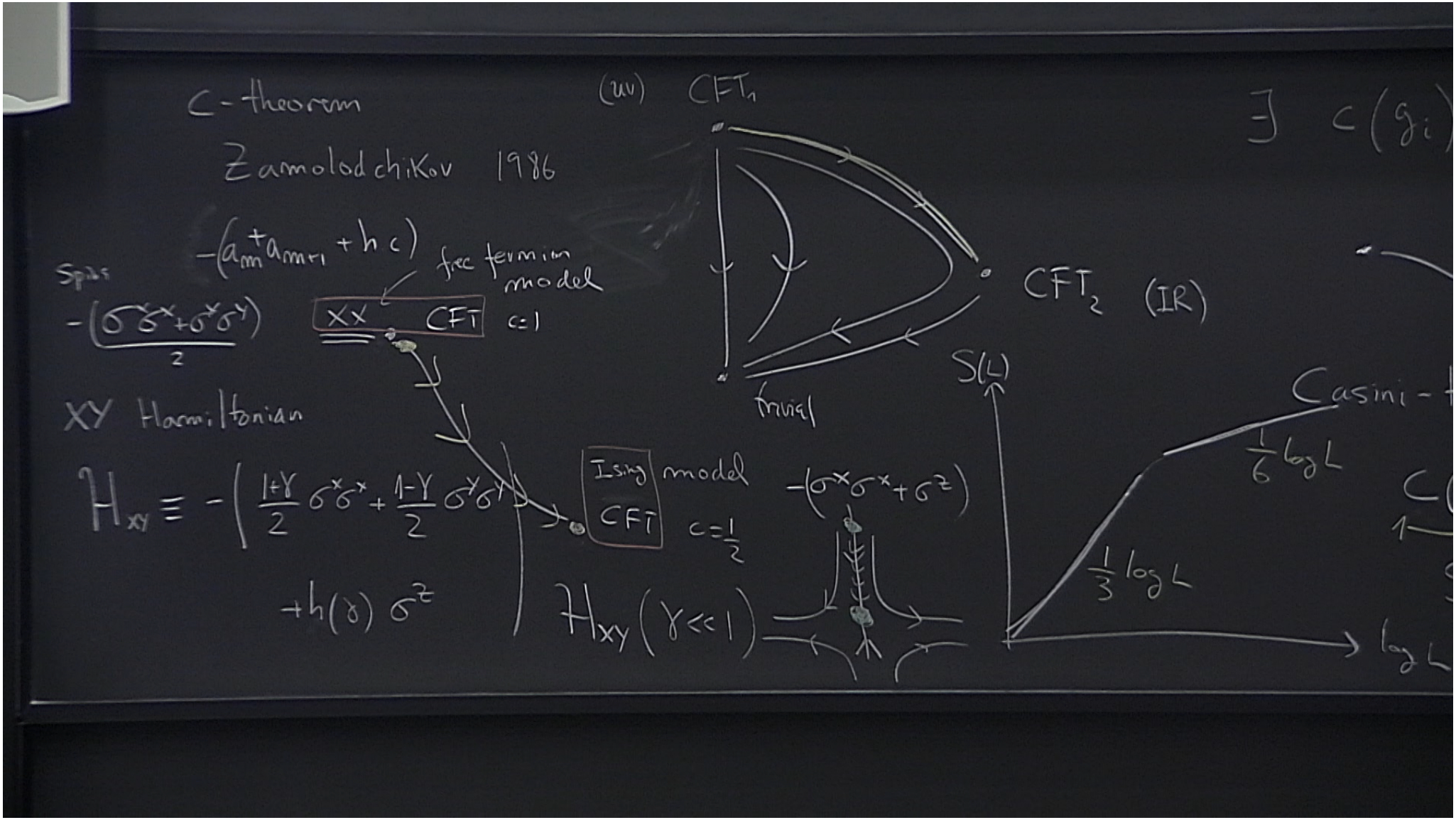
erta 2006

proof requires:

- SSA $S(ABC) + S(B) \leq S(AB) + S(BC)$

$= 3L \frac{dS_L}{dL}$

- Lorentz invariance
  Unitarity


C-theorem
Zamolodchikov 1986
(UV) CFT1
-J c(gi)
-(a_m^+ a_{m+1} + h.c)
free fermion model
Spins
-(σ^x σ^x + σ^y σ^y)/2
XX CFT c=1
CFT2 (IR)
trivial
XY Hamiltonian
H_XY ≡ -((1+γ)/2 σ^x σ^x + (1-γ)/2 σ^y σ^y)
Ising model
CFT c=1/2
-(σ^x σ^x + σ^z)
+h(γ) σ^z
H_XY (γ<<1)
S(L)
Casini-
1/6 log L
1/3 log L
log L

compute ground state $|\psi\rangle$

Entanglement spectrum

$|\psi\rangle \rightarrow \rho^A \longrightarrow S(\rho^A)$

$|\psi\rangle = \sum \sqrt{p_\alpha}\, |\psi_\alpha^A\rangle |\tilde{\psi}_\alpha^{\bar{A}}\rangle$

Circle

Orbifold

$\{p_\alpha\} \rightarrow \Delta_\alpha$

