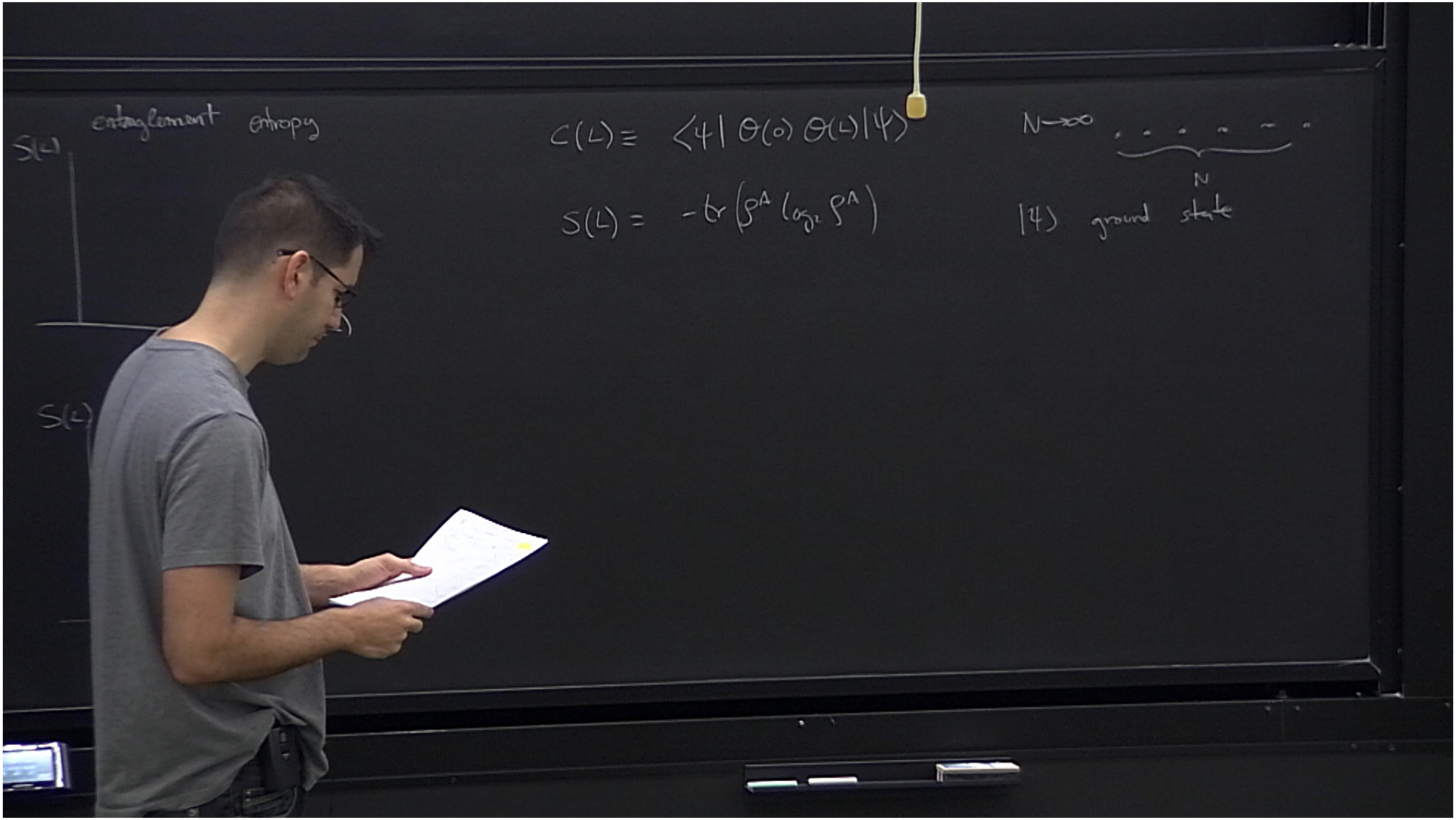
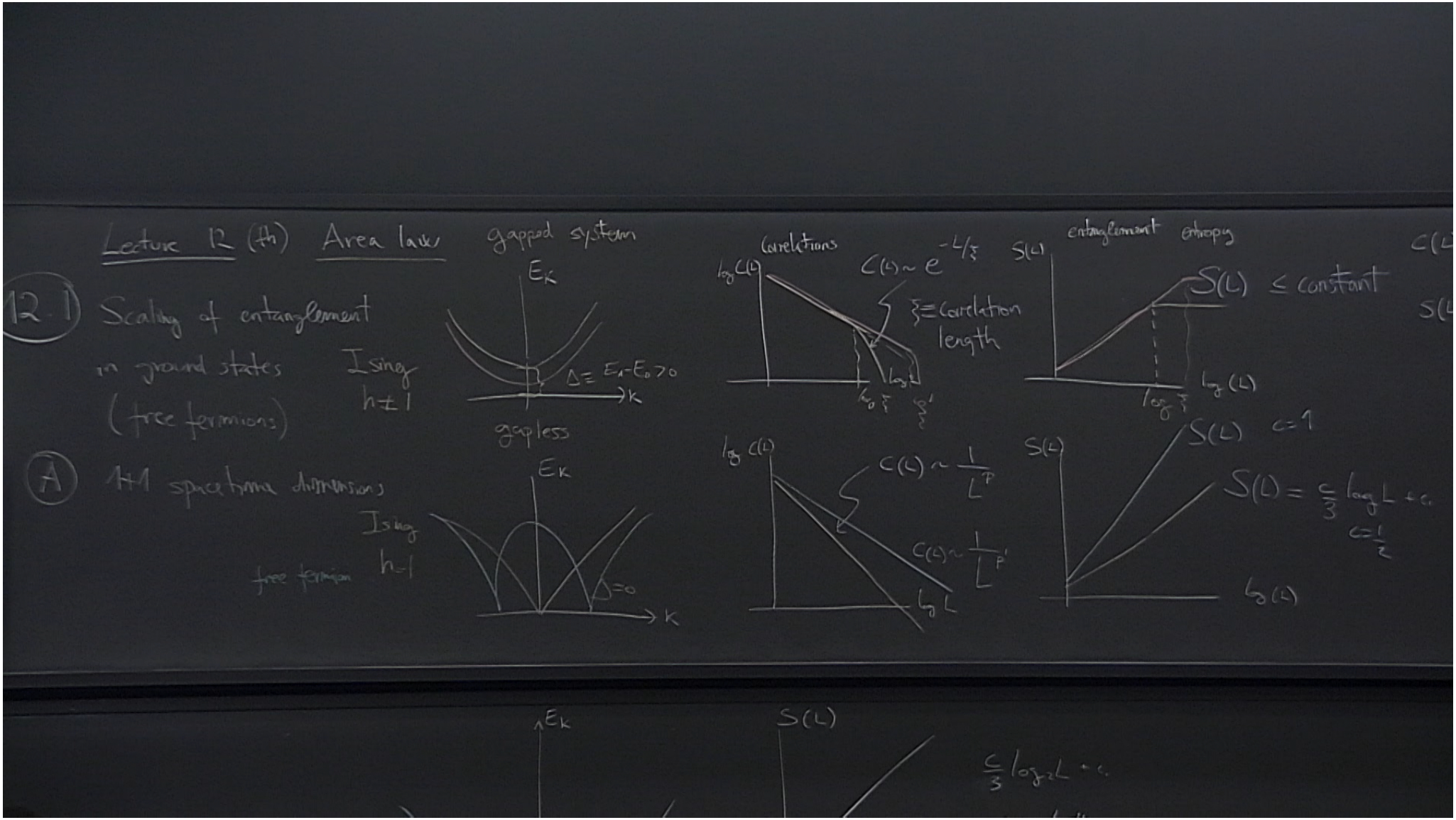
Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 6

Date: Feb 08, 2017 09:00 AM

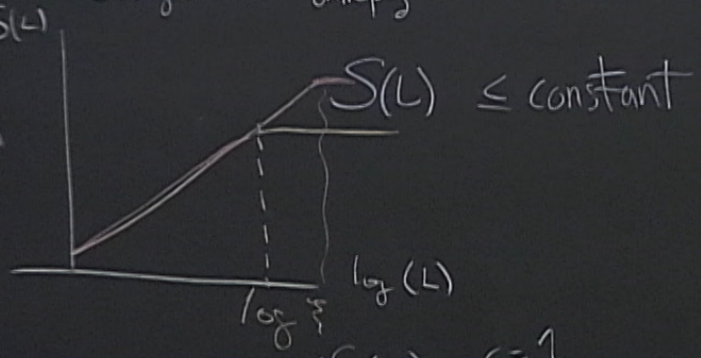
URL: http://pirsa.org/17020043

Abstract:


entanglement entropy
S(L)
$C(L) \equiv \langle 4 | \theta(0)\, \theta(L) | 4 \rangle$
$S(L) = -\mathrm{tr}\left(\rho^A \log_2 \rho^A\right)$
N→∞
N
|4⟩ ground state


Lecture 12 (th) Area laws
gapped system
12.1 Scaling of entanglement
in ground states
(free fermions)
Ising
h≠1
Ek
Δ = E1 - E0 > 0
k
gapless
correlations
C(L) ~ e^{-L/ξ}
ξ ≡ correlation length
log C(L)
log ξ
entanglement entropy
S(L)
S(L) ≤ constant
log L
A 1+1 space time dimensions
Ising
free fermion
h=1
Δ=0
C(L) ~ 1/L^p
S(L) c=1
S(L) = c/3 log L + ...
c = 1/2
S(L)
log2 L

entanglement entropy

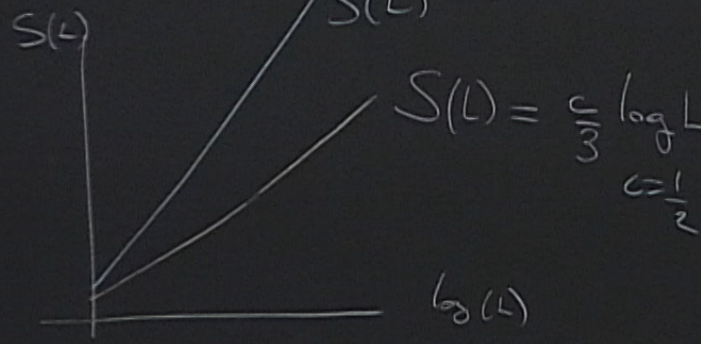


$$C(L) \equiv \langle 4 | \Theta(0)\, \Theta(L) | \psi \rangle$$

$$S(L) = -\mathrm{tr}\left(\rho^A \log_2 \rho^A\right)$$

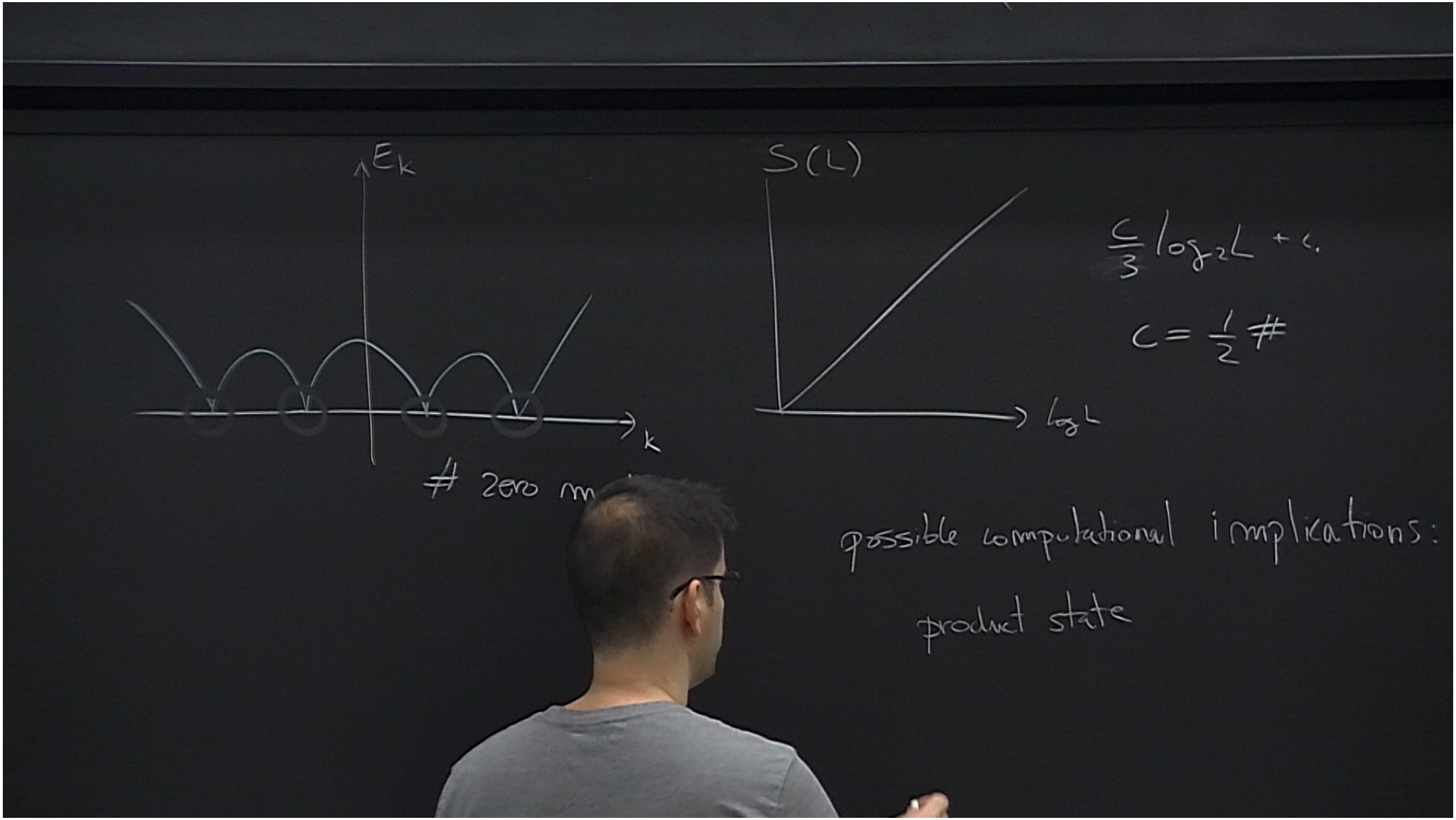
$N \to \infty$ . . . . . (N)

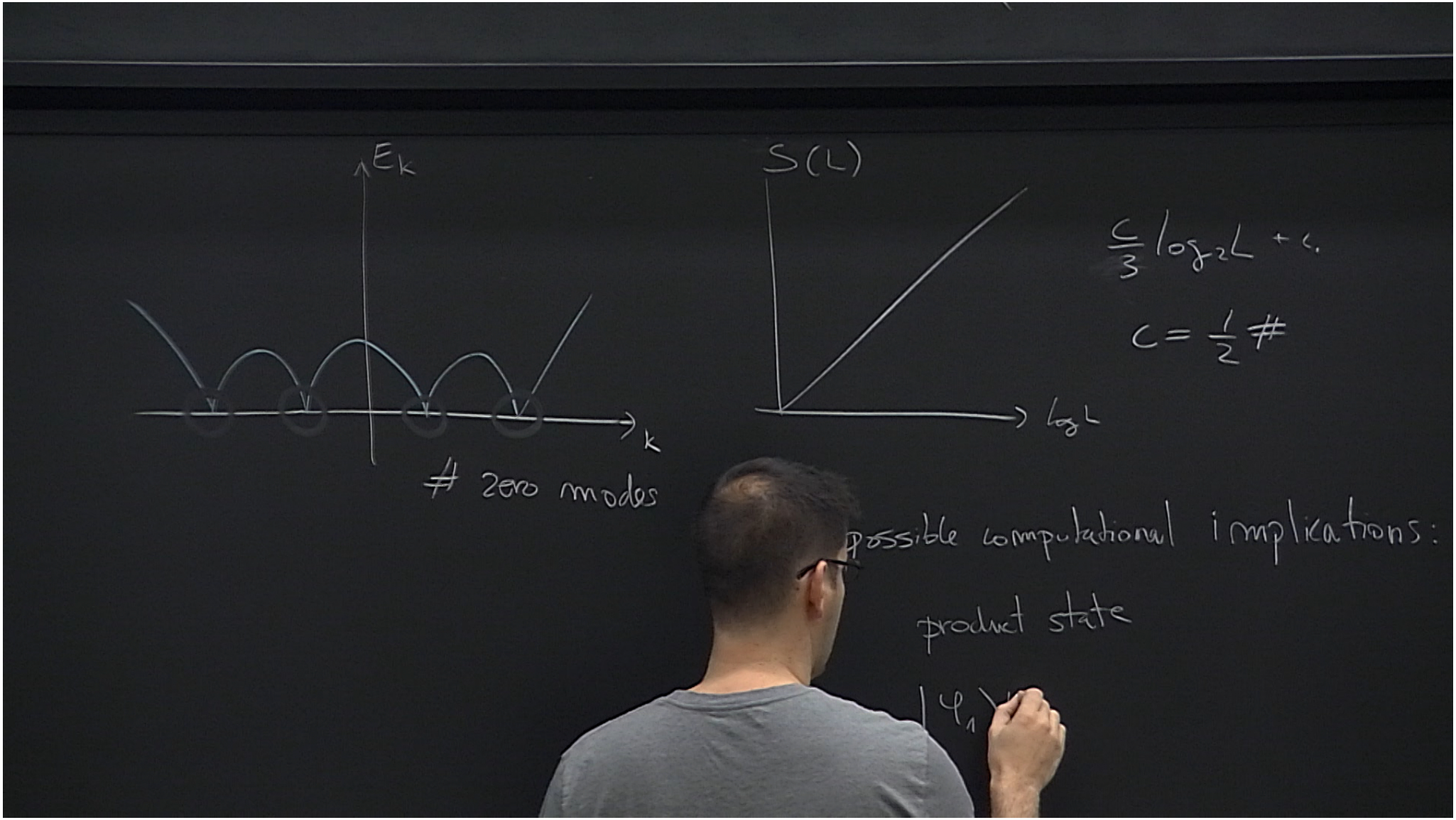
$|4\rangle$ ground state

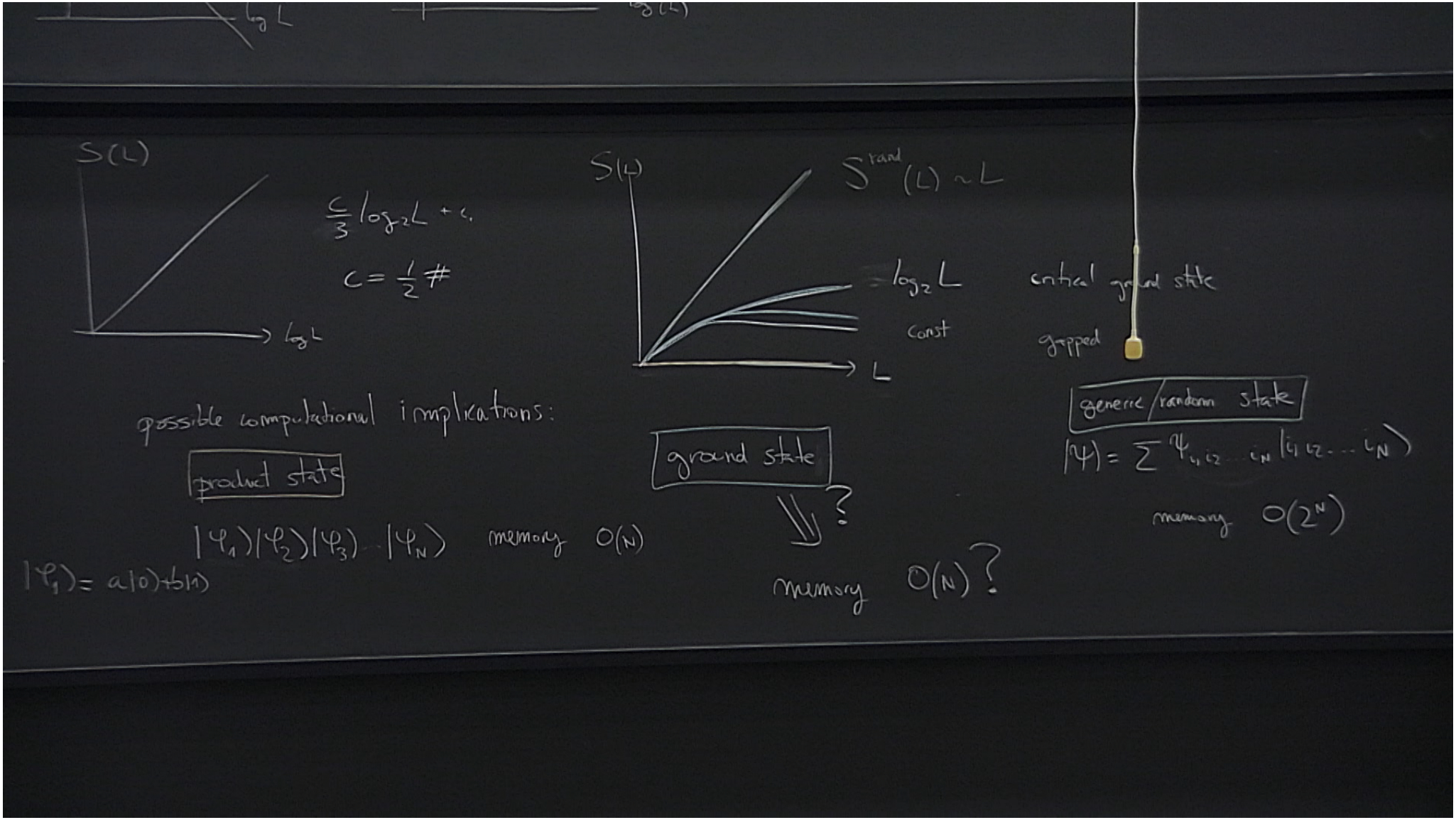


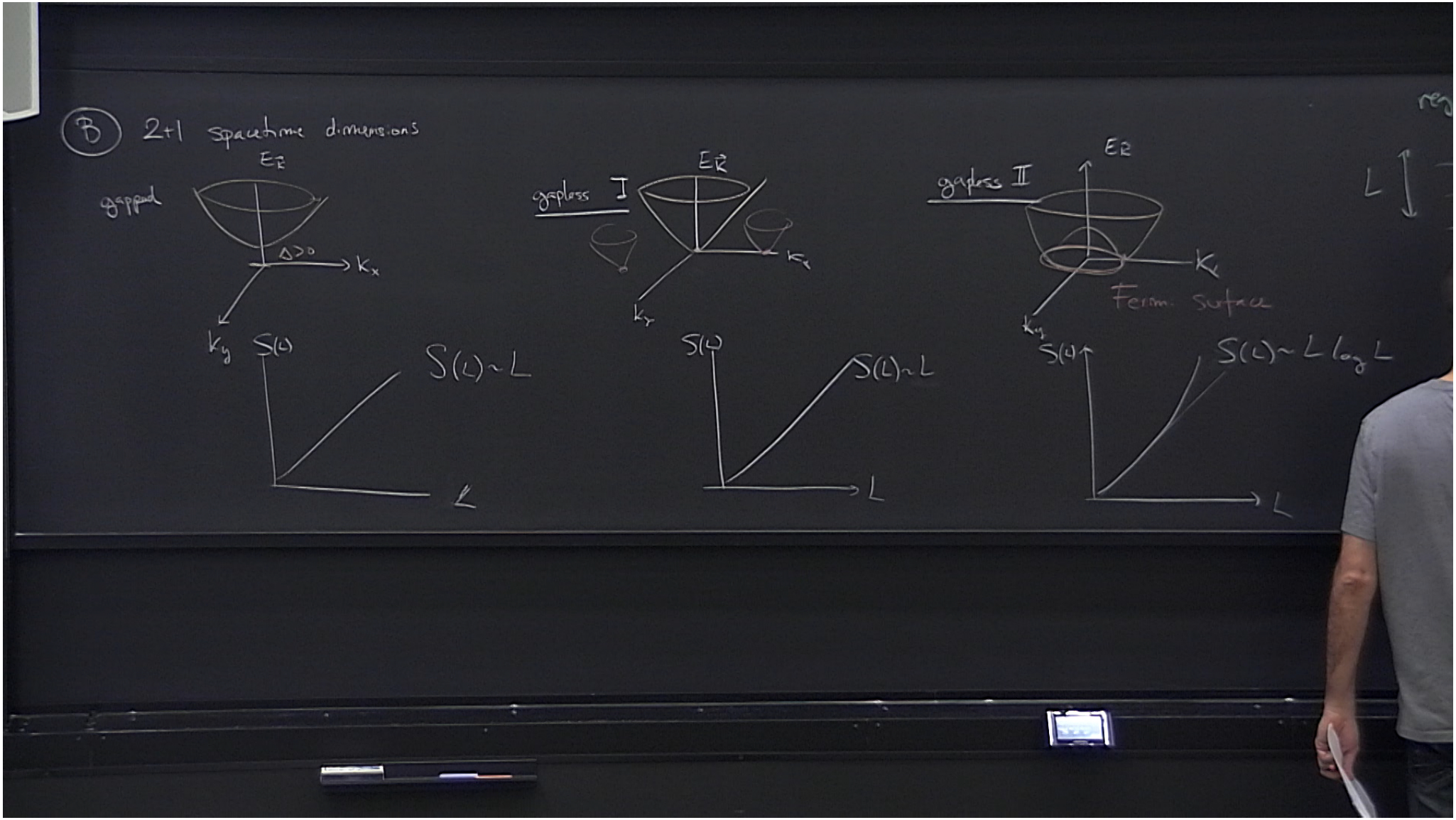
$\frac{1}{L^p}$

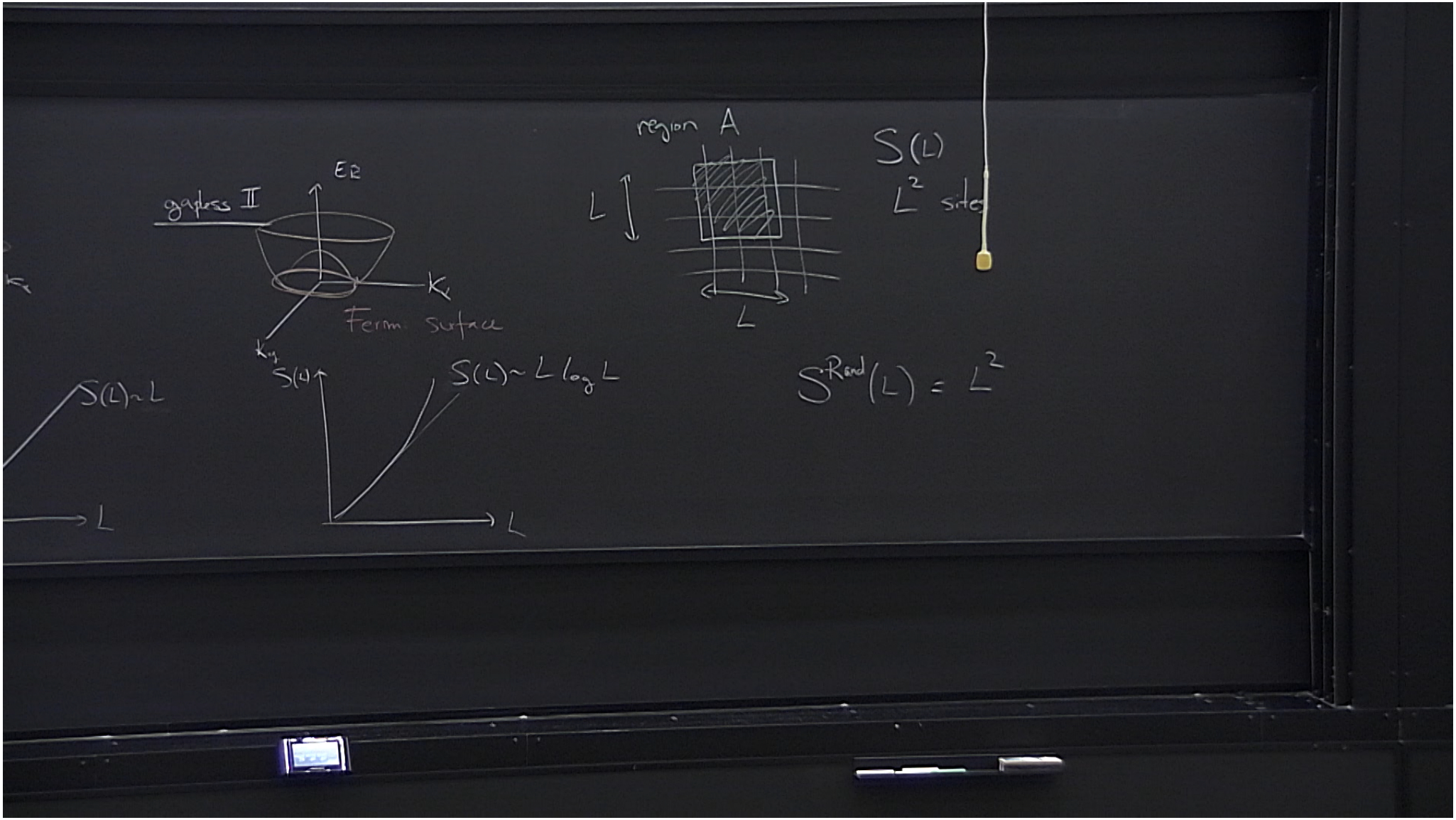
$\frac{1}{L^{p'}}$

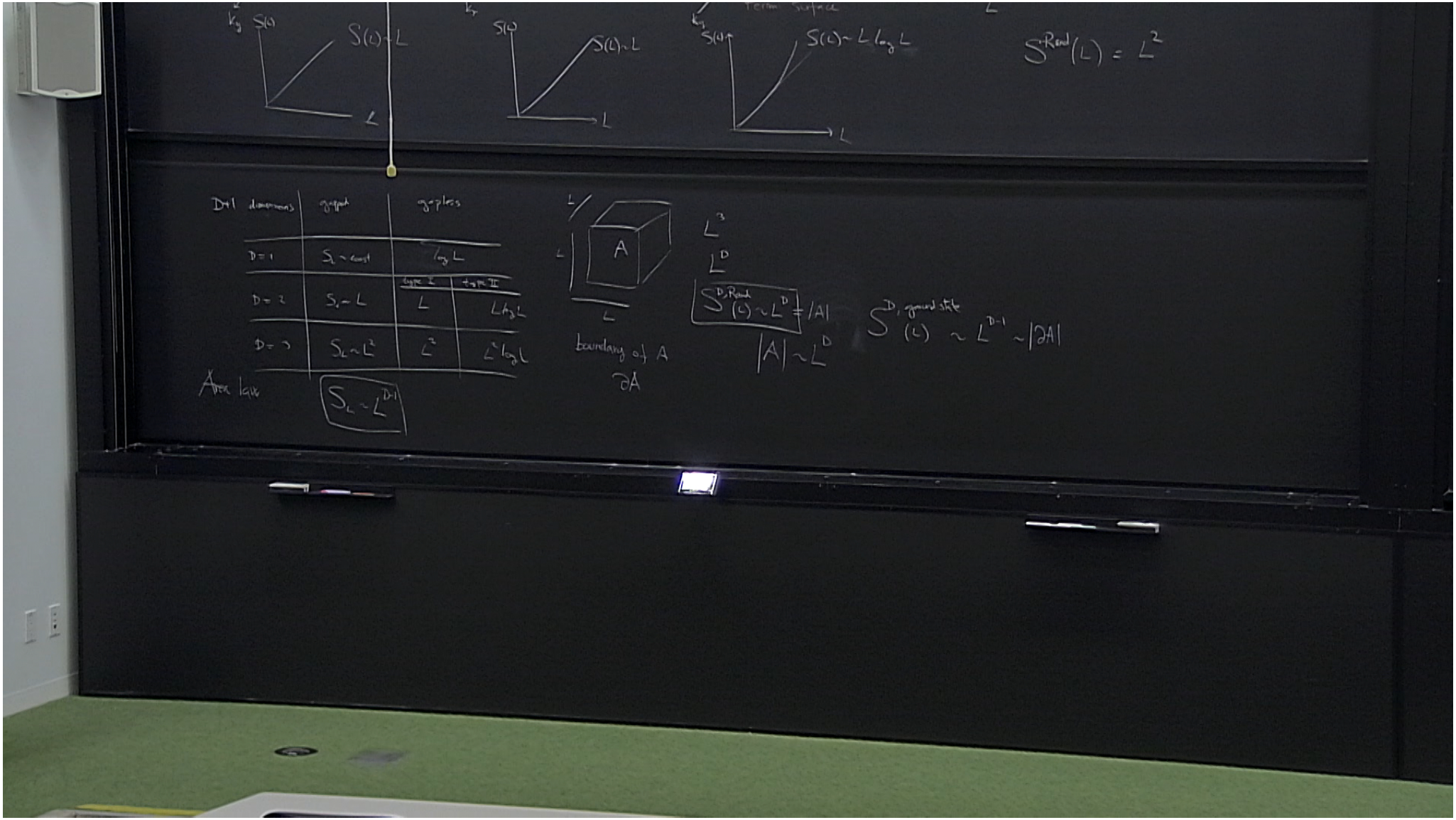

E_k
k
# zero m
S(L)
log L
$\frac{c}{3}\log_2 L + c$
$c = \frac{1}{2}$ #
possible computational implications:
product state

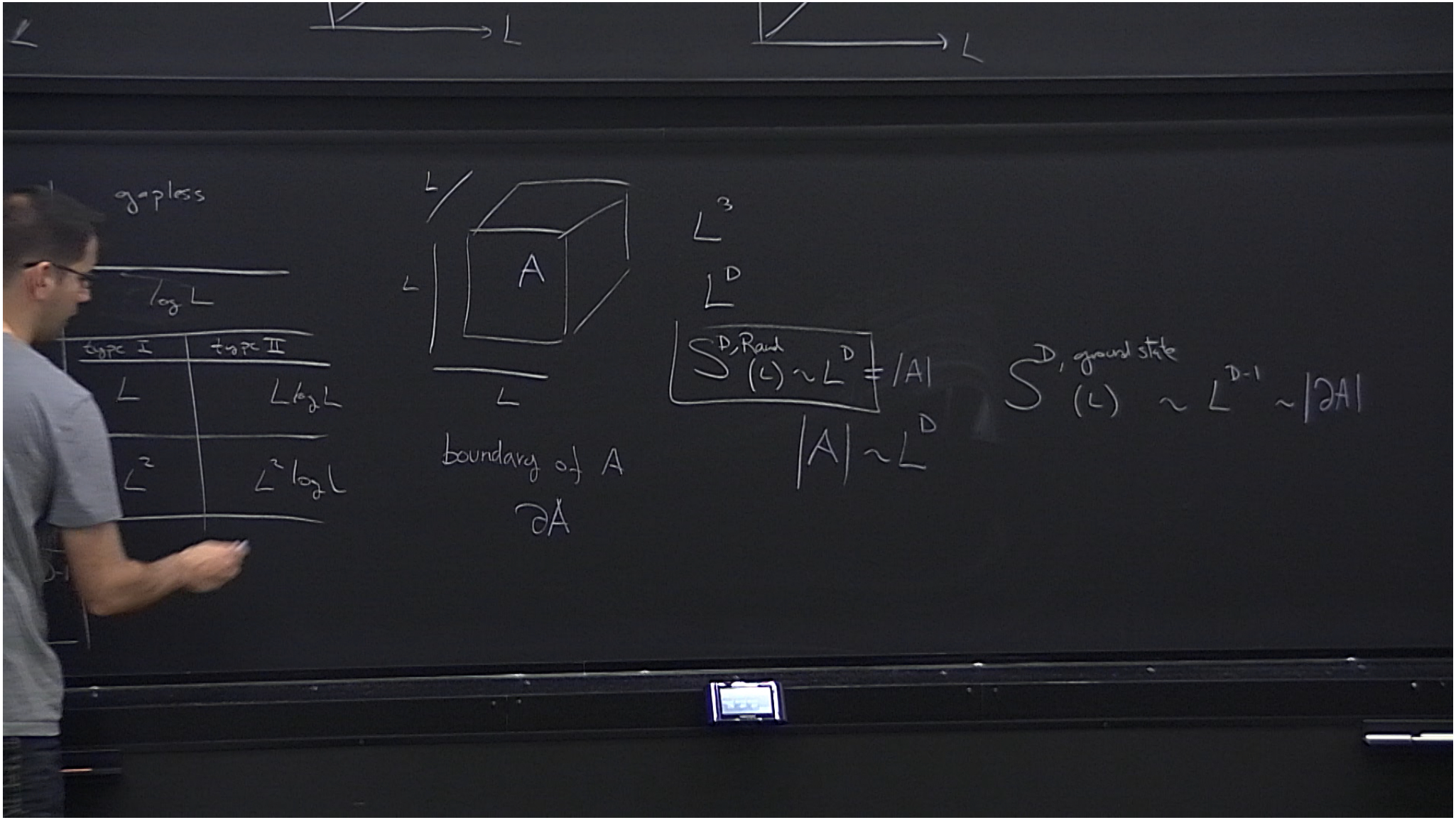

E_k
k
# zero modes
S(L)
$\frac{c}{3}\log_2 L + c_1$
$c = \frac{1}{2}$ #
log L
possible computational implications:
product state
$|\psi_1\rangle$

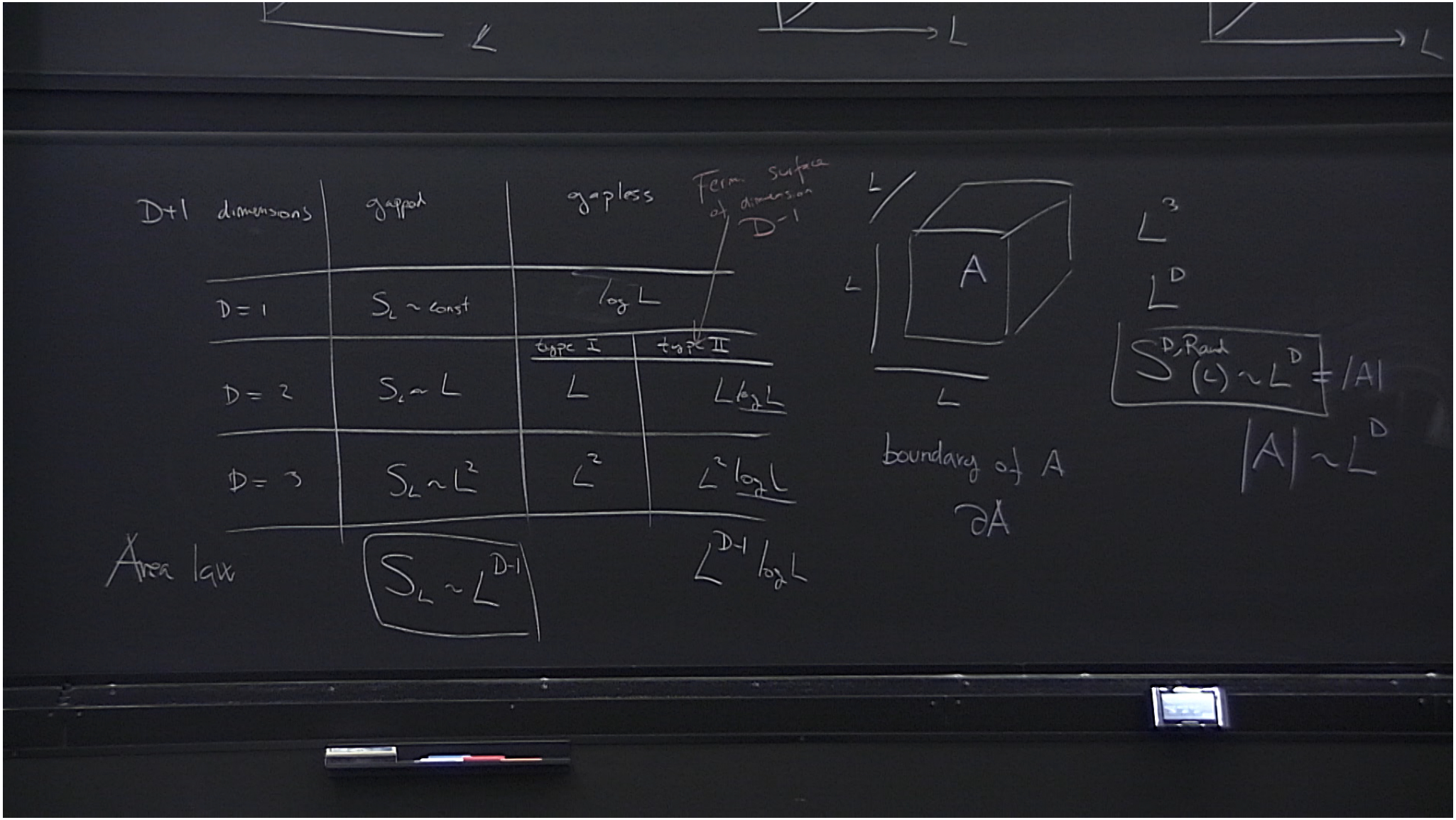

S(L)
log L
$\frac{c}{3}\log_2 L + c_1$
$c = \frac{1}{2}$ #
S(L)
$S^{rand}(L) \sim L$
$\log_2 L$
const
L
critical ground state
gapped
possible computational implications:
product state
$|\psi_1\rangle|\psi_2\rangle|\psi_3\rangle \ldots |\psi_N\rangle$ memory O(N)
$|\psi_1\rangle = a|0\rangle + b|1\rangle$
ground state
?
memory O(N)?
generic/random state
$|\psi\rangle = \sum \psi_{i_1 i_2 \ldots i_N} |i_1 i_2 \ldots i_N\rangle$
memory $O(2^N)$

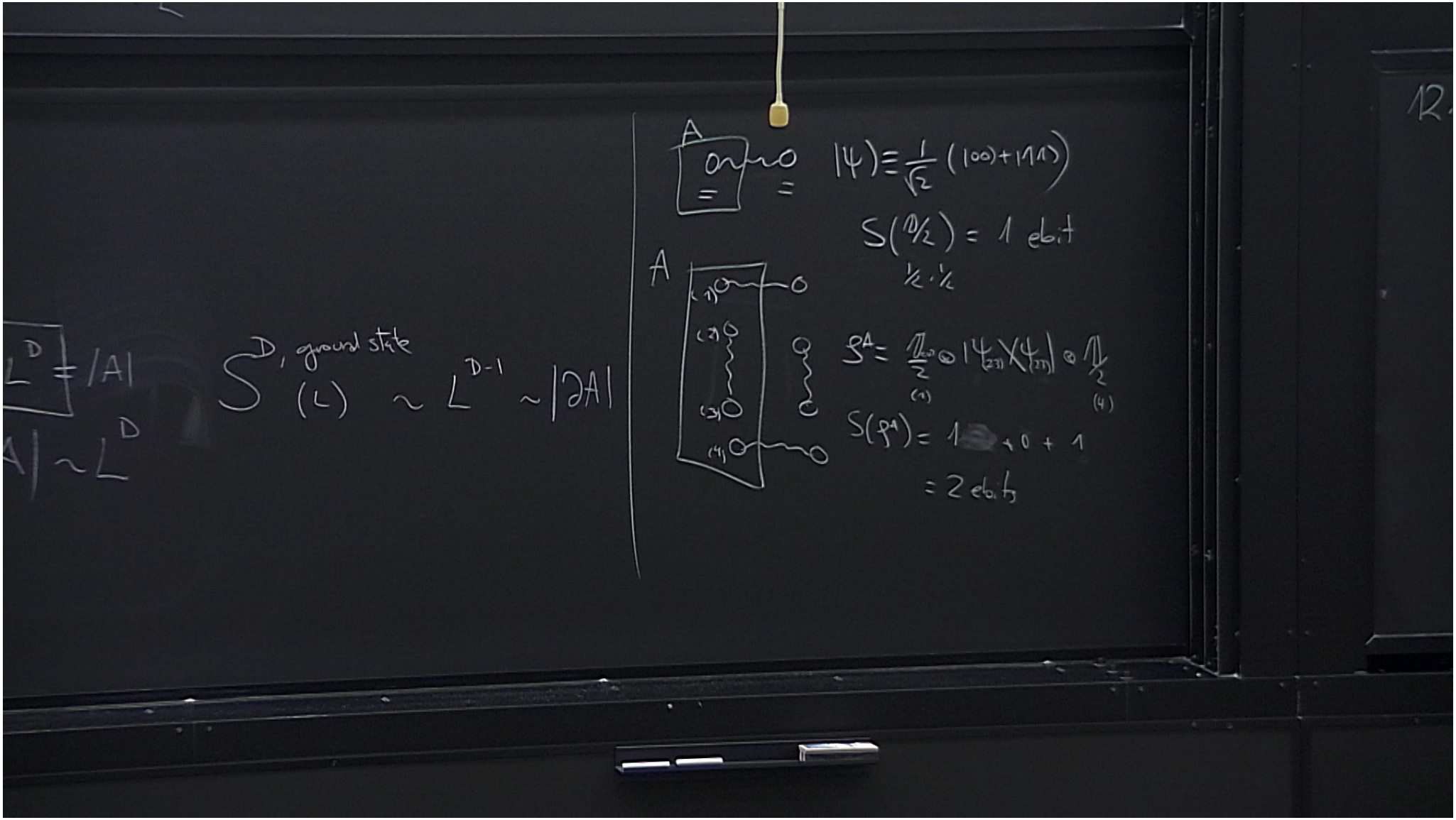

B 2+1 spacetime dimensions
gapped
E_k
$\Delta > 0$
k_x
k_y
$S(L)$
$S(L) \sim L$
L
gapless I
E_k
k_x
k_y
$S(L)$
$S(L) \sim L$
L
gapless II
E_k
k_x
k_y
Ferm. Surface
$S(L)$
$S(L) \sim L \log L$
L
L

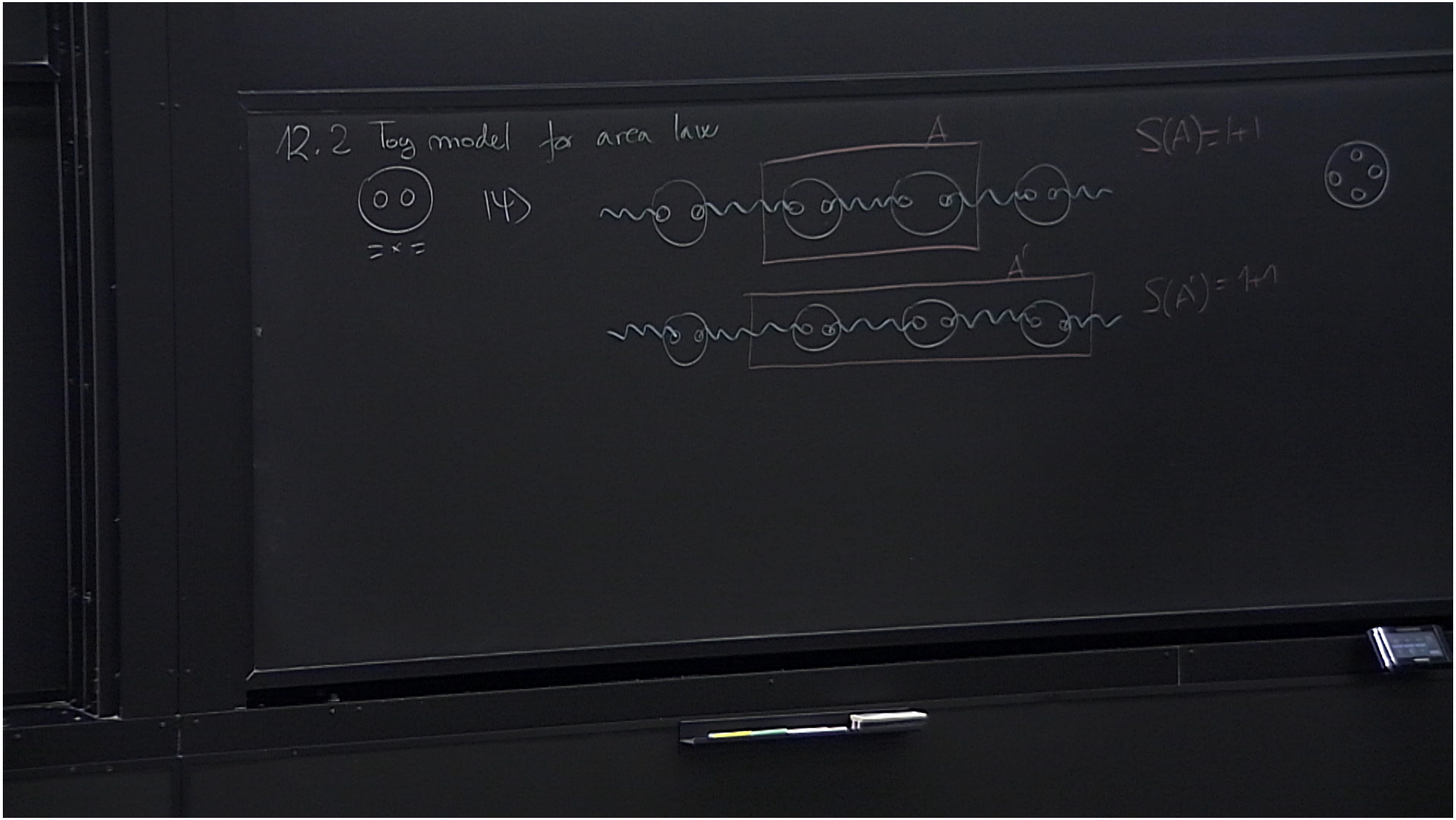

gapless II
E_2
k_x
k_y
Fermi surface
region A
L
L
$S(L)$
L^2 sites
$S(L) \sim L$
L
$S(L)$
$S(L) \sim L \log L$
L
$S^{Rand}(L) = L^2$

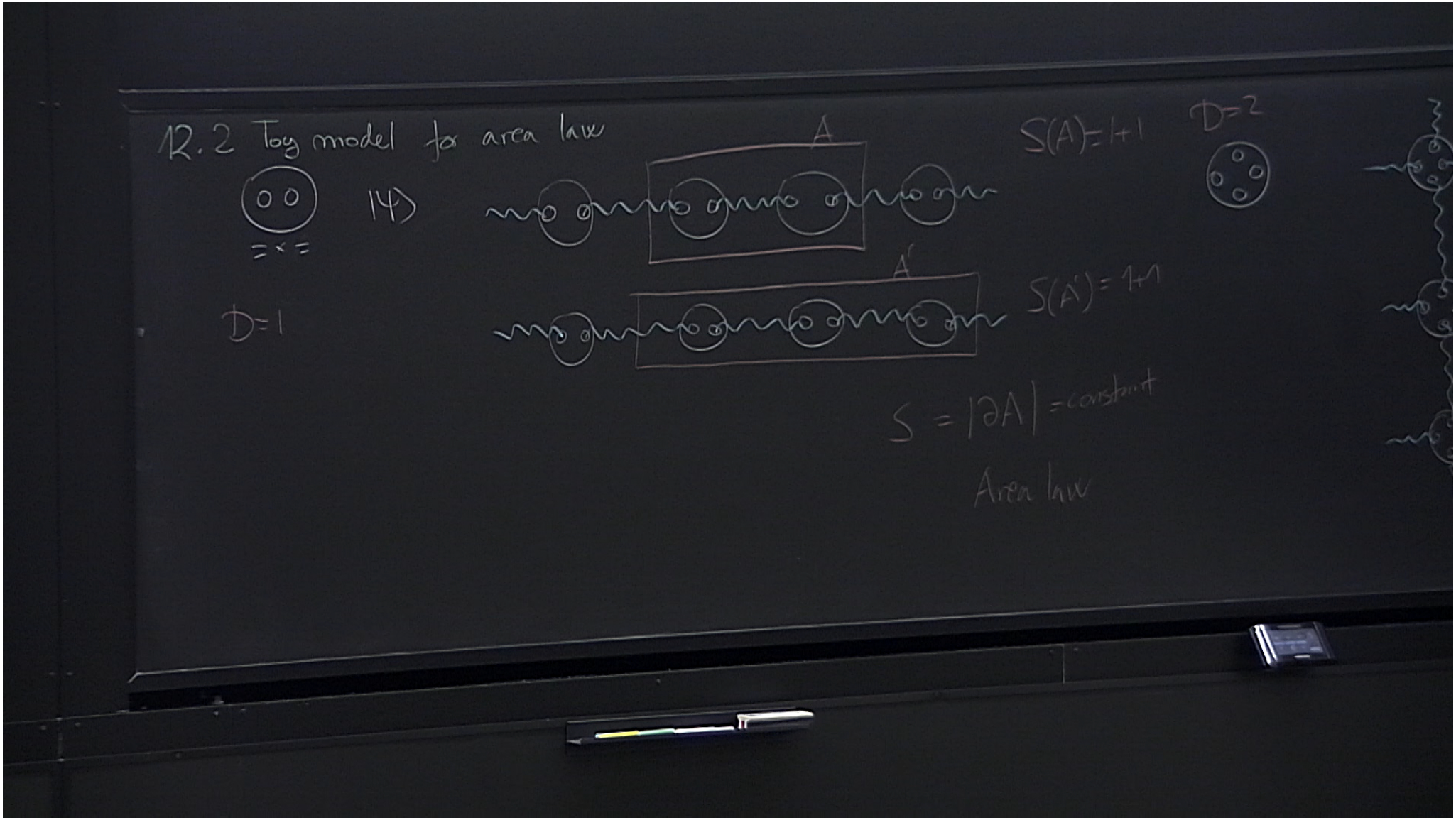

S(L)~L
S(L)~L
S(L)~L log L
S^Rand(L) = L^2
L
L
L
A
L^3
L^D
S^{D,Rand}(L) ~ L^D = |A|
|A| ~ L^D
S^{D, ground state}(L) ~ L^{D-1} ~ |∂A|
boundary of A
∂A
D+1 dimensions
gapped
gapless
D = 1
S_L ~ const
log L
type I
type II
D = 2
S_L ~ L
L
L log L
D = 3
S_L ~ L^2
L^2
L^2 log L
Area law
S_L ~ L^{D-1}

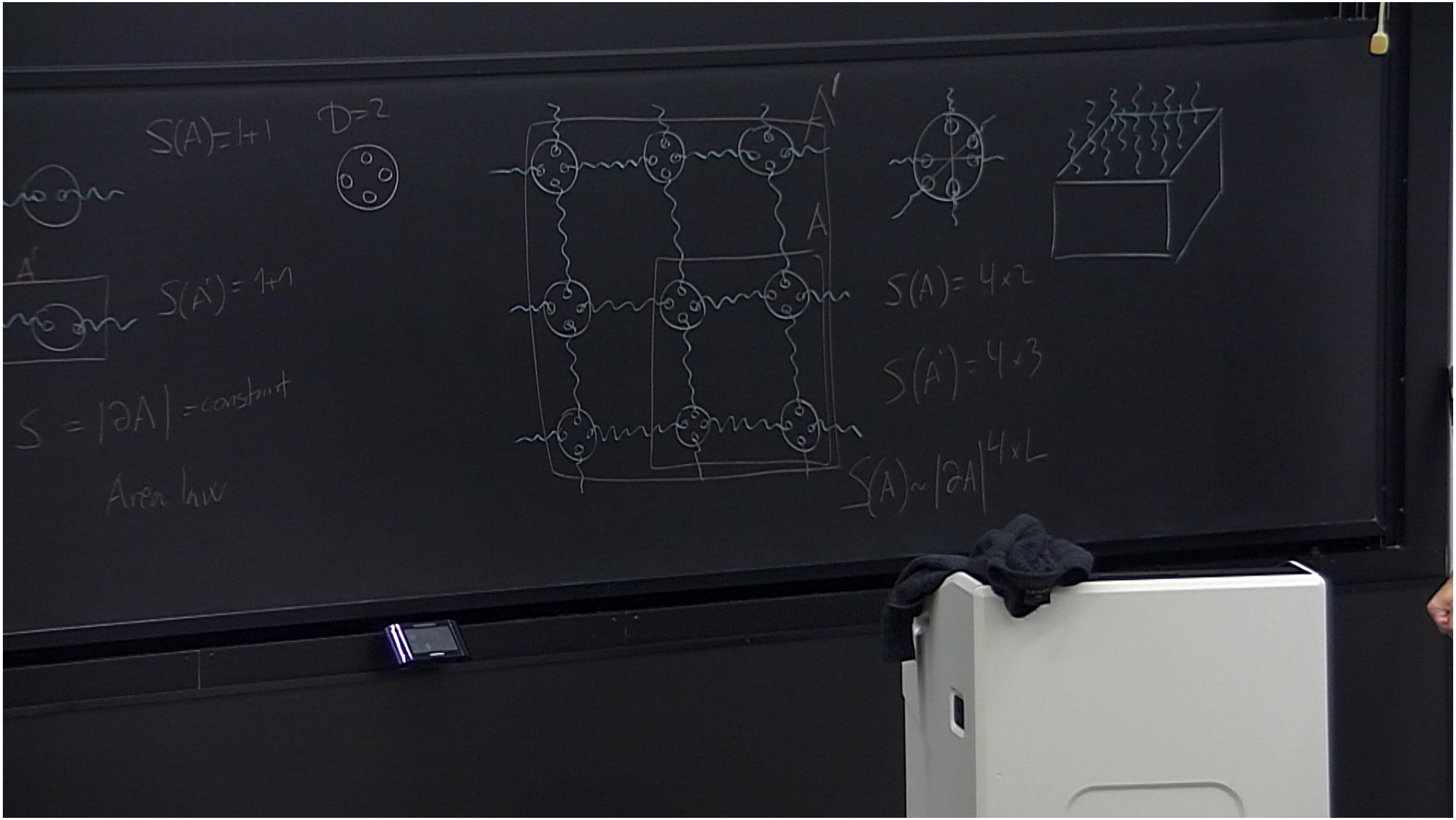

L
L
gapless
log L
type I
type II
L
L log L
L^2
L^2 log L
L
A
L
L
L
boundary of A
∂A
L^3
L^D
S^{D, Rand}(L) ~ L^D = |A|
|A| ~ L^D
S^{D, ground state}(L) ~ L^{D-1} ~ |∂A|

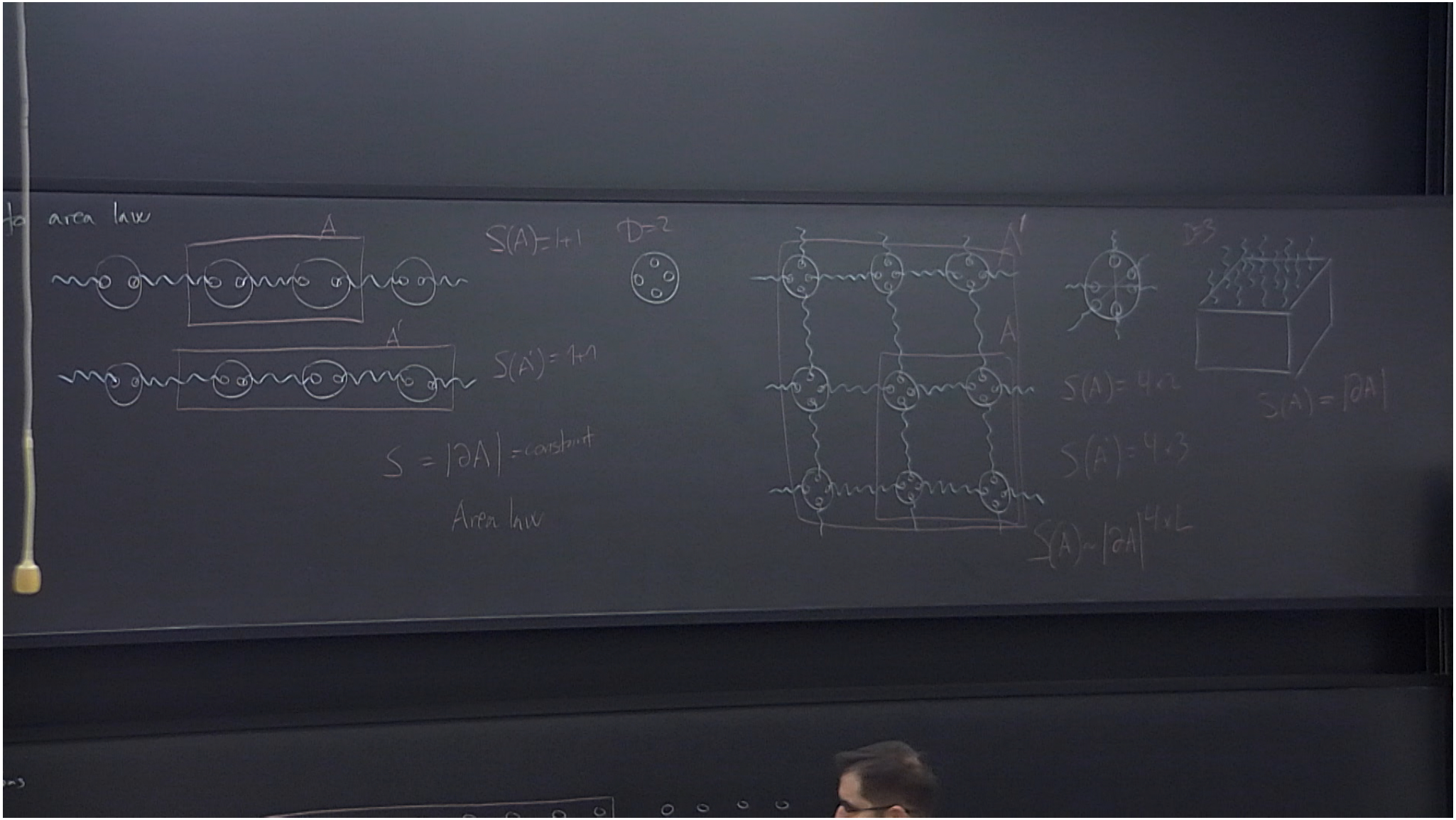

L
L
L
D+1 dimensions
gapped
gapless
Ferm. surface of dimension D-1
D = 1
$S_L \sim$ const
log L
type I
type II
D = 2
$S_L \sim L$
L
L log L
D = 3
$S_L \sim L^2$
L^2
$L^2 \log L$
Area law
$S_L \sim L^{D-1}$
$L^{D-1} \log L$
L
L
L
A
boundary of A
∂A
L^3
L^D
$S^{D,Rand}(L) \sim L^D = |A|$
$|A| \sim L^D$

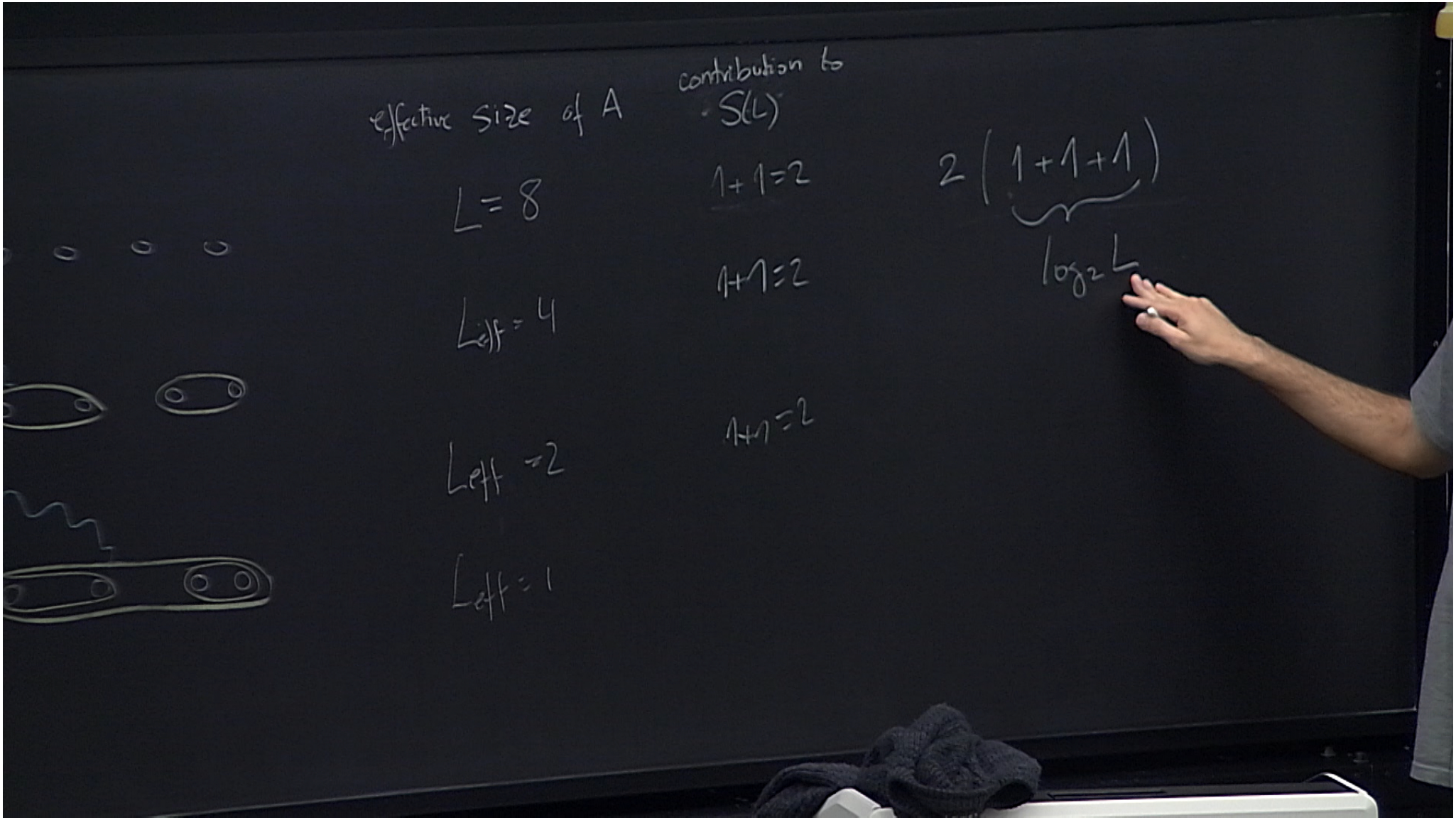

A
$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
$S(\rho_{1/2}) = 1$ ebit
$\frac{1}{2}, \frac{1}{2}$
A
$\rho^A = \frac{\mathbb{1}}{2} \otimes |\psi_{(23)}\rangle\langle\psi_{(23)}| \otimes \frac{\mathbb{1}}{2}$
$S(\rho^A) = 1 + 0 + 1$
$= 2$ ebits
$L^D = |A|$
$|A| \sim L^D$
S^D, ground state
$S(L) \sim L^{D-1} \sim |\partial A|$
12.

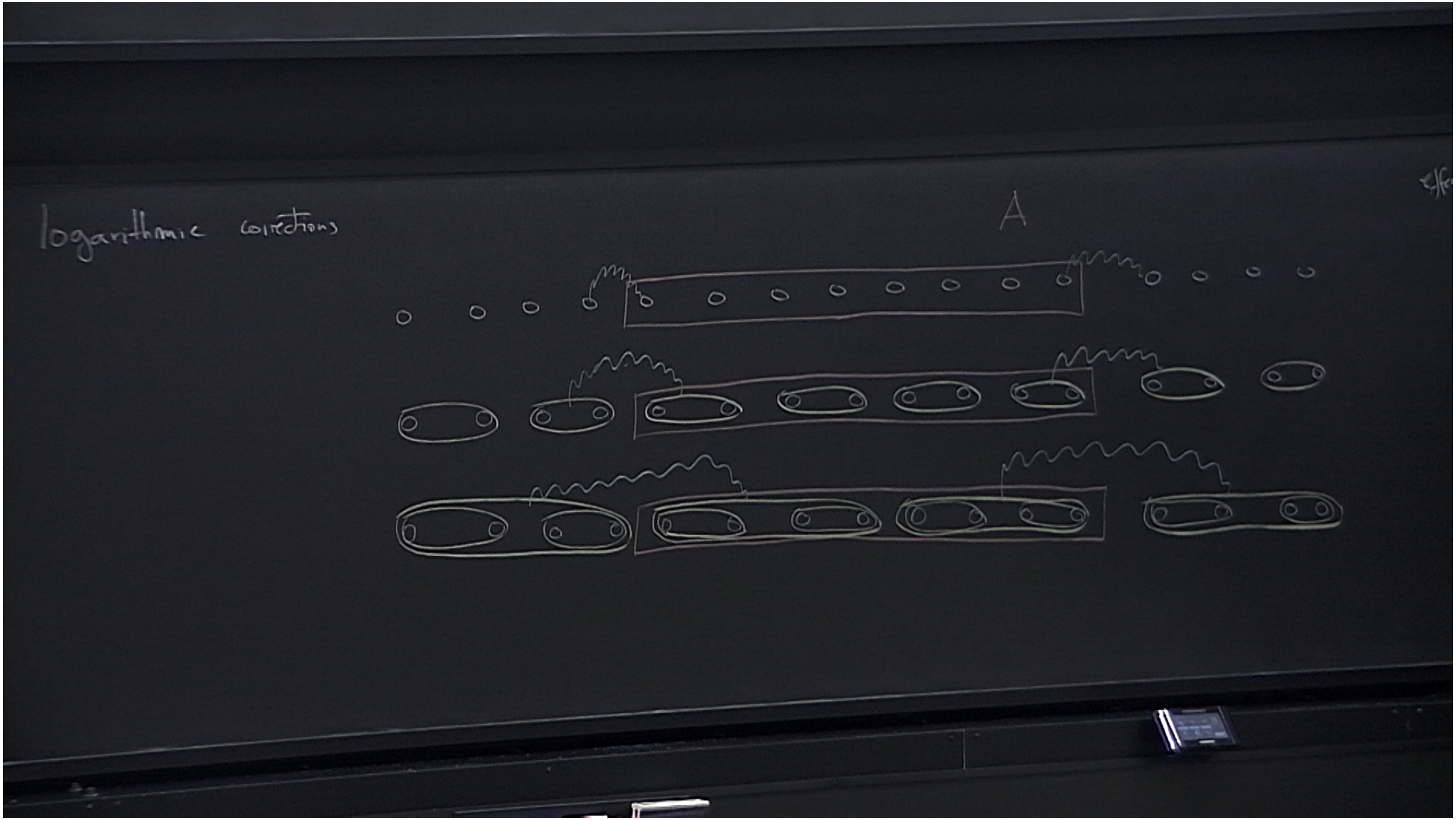

12.2 Toy model for area law
A
S(A)=1+1
A'
S(A')=1+1

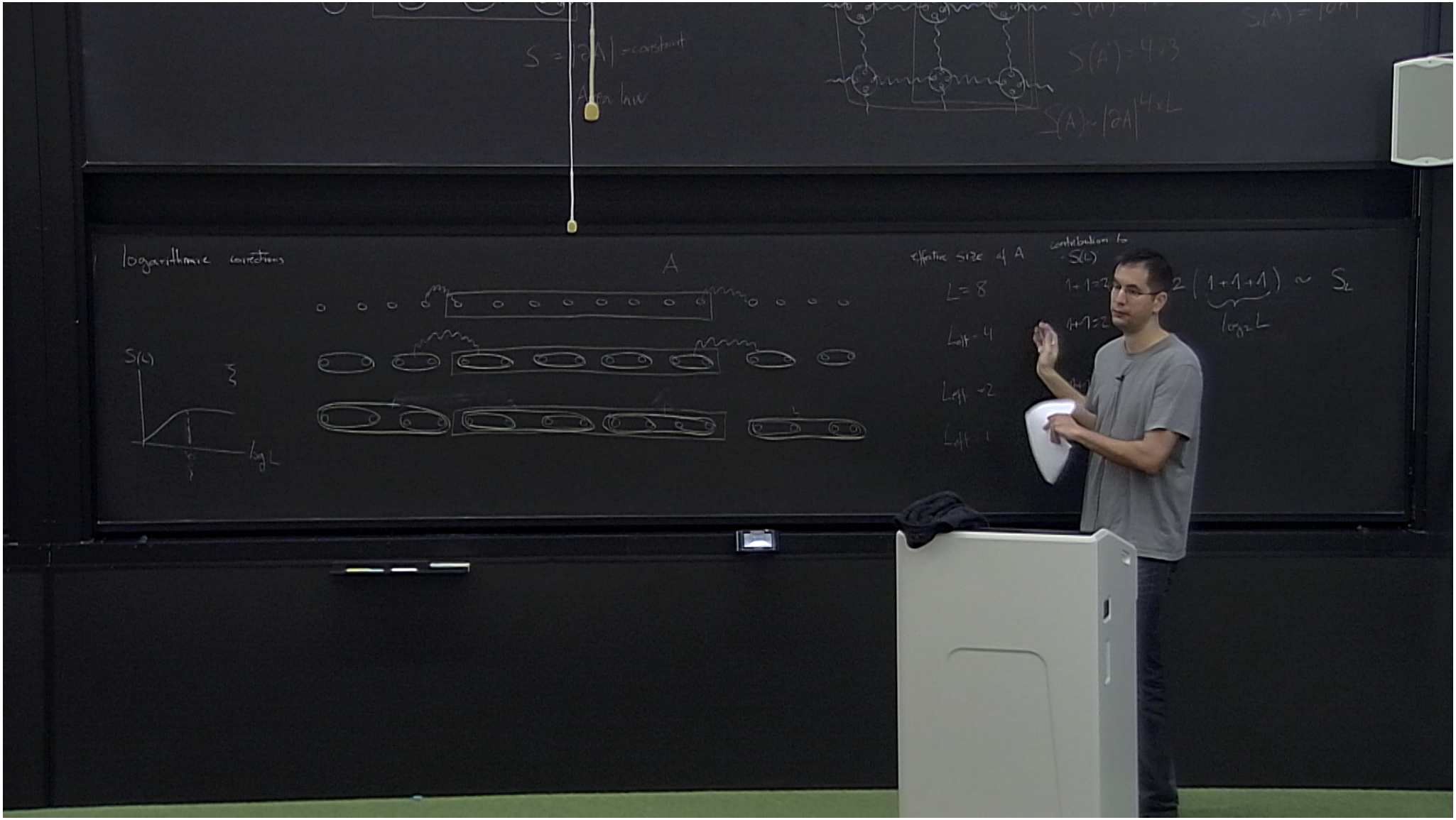

12.2 Toy model for area law
A
S(A)=1+1
D=2
|4⟩
D=1
A'
S(A')=1+1
S = |∂A| = constant
Area law

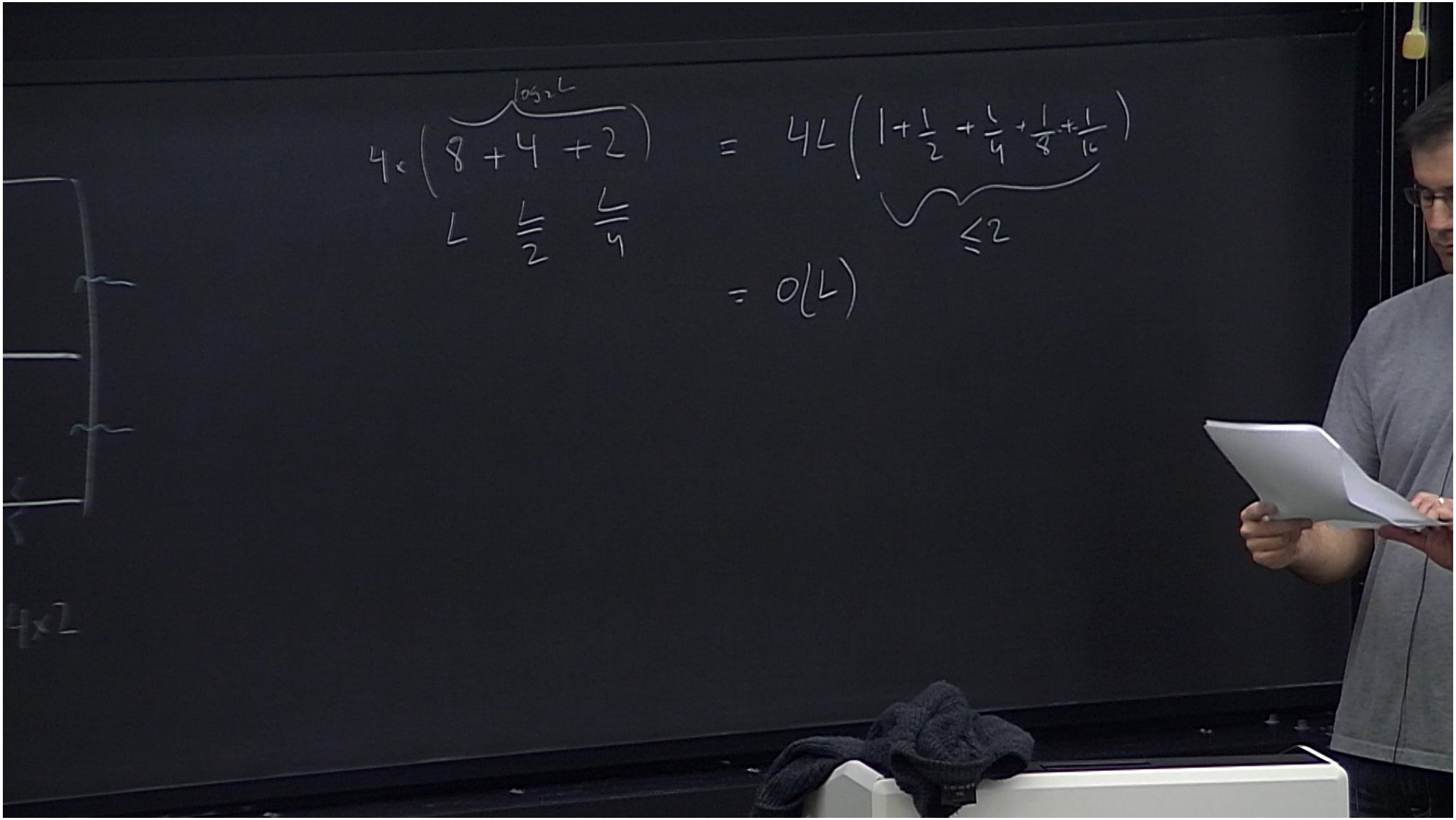

S(A)=1+1
D=2
A'
A
S(A')=1+1
S=|∂A|=constant
Area law
S(A)=4×2
S(A')=4×3
S(A)~|∂A|^{4×L}

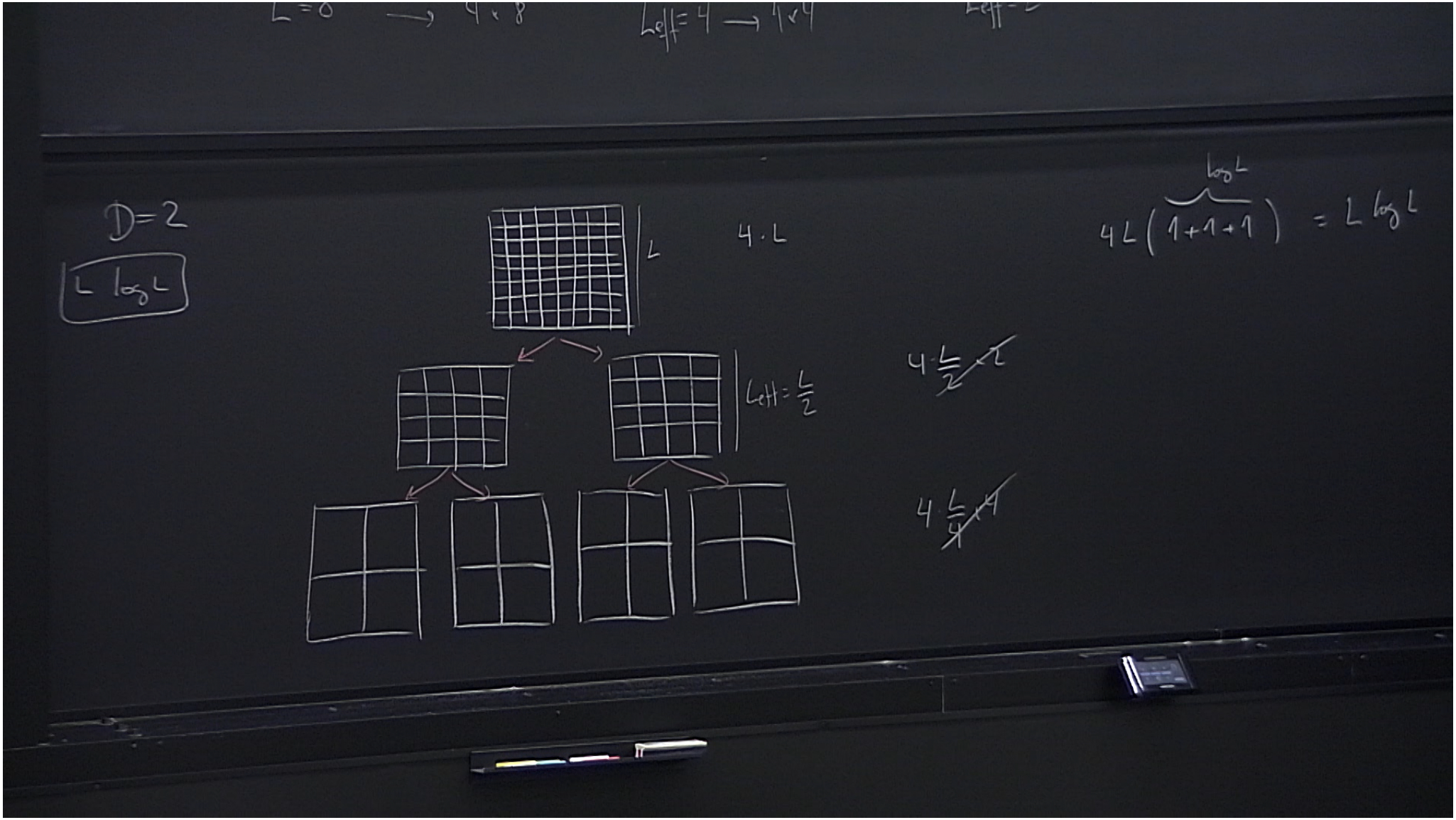

area law
A
A'
S(A)=1+1
S(A')=1+1
S = |∂A| = constant
Area law
D=2
A'
A
S(A)=4+2
S(A')=4+3
S(A)~|∂A|^{4×L}
D=3
S(A)=|∂A|


effective size of A
contribution to S(L)
L = 8
1+1=2
2(1+1+1)
log2 L
Left = 4
1+1=2
Left = 2
1+1=2
Left = 1


logarithmic corrections
A


S = |∂A| = constant
S(A) = 4·3
S(A) ~ |∂A|^{4×L}
logarithmic corrections
A
S(L)
log L
effective size of A
contribution to S(L)
L = 8
Leff = 4
Leff = 2
Leff = 1
1+1=2
1+1=2
2 (1+1+1) ~ S_L
log_2 L


$4 \times \overbrace{(8 + 4 + 2)}^{\log_2 L}$
$L \quad \frac{L}{2} \quad \frac{L}{4}$
$= 4L\left(\underbrace{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}}_{\leq 2}\right)$
$= O(L)$
4×2


D=2
L log L
L
4·L
Left = L/2
4 · L/2 × L
4 · L/4 × L
log L
4L(1+1+1) = L log L