Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 2

Date: Feb 01, 2017 09:00 AM

URL: http://pirsa.org/17020038

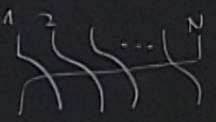
Abstract:

Lecture 4 (th)

4.1 Summary of previous lectures (2 & 3 → I Julia)

a) build $2^N \times 2^N$ matrices for

Hamiltonian $\quad H = -\sum_{m=1}^{N} \sigma_m^x \sigma_{m+1}^x - h \sum_{m=1}^{N} \sigma_m^z$

Translation operator $\quad T =$ 

Spin flip operator $\quad S = \sigma_1^z \sigma_2^z \cdots \sigma_N^z$

b) Simultaneous diagonalization of H, T, S $\longrightarrow \{|\psi_\alpha\rangle\}$

$[H,T] = [H,S] = [T,S] = 0$

c) for $h=1$ (critical) numer

extract conformal data

Δ_α scaling dimensions $\left(\leftrightarrow \text{critical exponents}\right)$

S_α conformal spins

$$H|\psi_\alpha\rangle = E_\alpha|\psi_\alpha\rangle \qquad \text{energy } E_\alpha \in \mathbb{R}$$

$$T|\psi_\alpha\rangle = e^{iK_\alpha}|\psi_\alpha\rangle \qquad \text{momentum } K_\alpha \in (-\pi, \pi]$$

$$S|\psi_\alpha\rangle = (-1)^{P_\alpha}|\psi_\alpha\rangle \qquad \text{parity } P_\alpha = 0, 1$$

$N \simeq 10$ cost $\sim \exp(N)$

$N \sim 100$

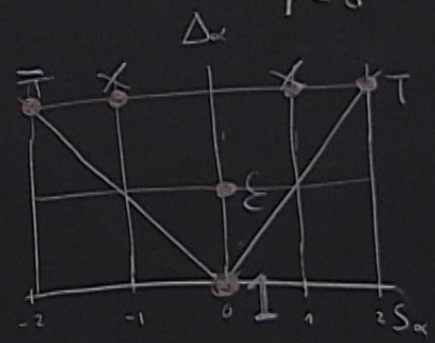
numerical

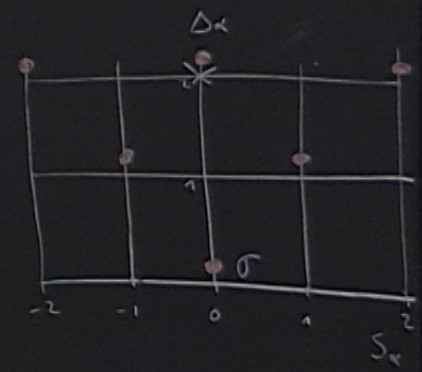
critical exponents

$$E_\alpha = A + \frac{B}{N}\left(\Delta_\alpha - \frac{c}{12}\right) + O\left(\frac{1}{N^2}\right)$$

$$K_\alpha = \frac{2\pi}{N} S_\alpha$$

Universal





	time	space
Examples a) vector-vector multiplication		
$$v^\dagger \cdot w = \sum_{\alpha=1}^{m} v_\alpha^* w_\alpha$$ $\alpha = 1, \dots, m$	$O(m)$	$O(m)$
b) matrix-vector multiplication	$O(m^2)$	$O(m^2)$
$$w_\alpha = \sum_{\beta=1}^{m} M_{\alpha\beta} v_\beta$$ $\alpha, \beta = 1, \dots m$		

$$\sum_\beta M_{1\beta} v_\beta$$

Examples

	time	space
a) vector-vector multiplication $v^\dagger \cdot w = \sum_{\alpha=1}^{m} v_\alpha^* w_\alpha$, $\alpha = 1, \dots, m$	$O(m)$	$O(m)$
b) matrix-vector multiplication $w_\alpha = \sum_{\beta=1}^{m} M_{\alpha\beta} v_\beta$, $\alpha, \beta = 1, \dots m$	$O(m^2)$	$O(m^2)$
c) matrix-matrix multiplication $C_{\alpha\gamma} = \sum_\beta A_{\alpha\beta} B_{\beta\gamma}$, $\alpha, \beta, \gamma = 1, \dots, m$	$O(m^3)$	$O(m^2)$

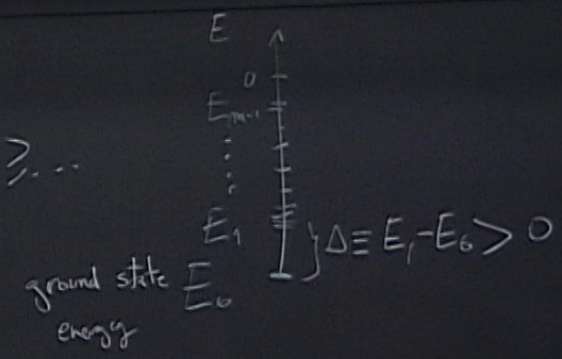
4.2 Computational cost

given problem of size m

- how much space (= memory) ? $O(m)$? $O(m^2)$?
- how much time? $\exp m$?

43 Power method

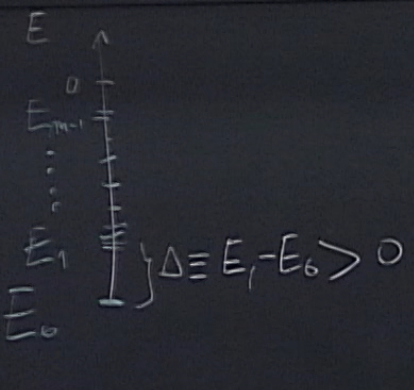
$$H = \sum_{\alpha=0}^{m-1} E_\alpha |E_\alpha\rangle\langle E_\alpha| \quad \text{with} \quad |E_0| > |E_1| \geq |E_2| \geq \dots$$



$$|\psi\rangle = \sum c_\alpha |E_\alpha\rangle \qquad H|\psi\rangle = \sum E_\alpha c_\alpha |E_\alpha\rangle$$

$$H^p|\psi\rangle = \sum (E_\alpha)^p c_\alpha |E_\alpha\rangle = E_0^p \left[c_0 |E_0\rangle + \left(\frac{E_1}{E_0}\right)^p c_1 |E_1\rangle + \right.$$

$$|E_1| \geq |E_2| \geq \dots$$



$$= E_0^p \left[c_0 |E_0\rangle + \left(\frac{E_1}{E_0}\right)^p c_1 |E_1\rangle + \left(\frac{E_2}{E_0}\right)^p c_2 |E_2\rangle + \dots \right] = E_0^p \left[c_0 |E_0\rangle + O(\exp -p) \right]$$

$$\left| \frac{E_1}{E_0} \right| < 1 \qquad \left(\frac{E_1}{E_0}\right)^p \ll 1$$

4.4 Exact diagonalization of H

method	function	based on
→ full diagonalization		
$H \to D, U$ $\quad H = UDU^{\dagger}$	$\mathrm{eig}(H)$	matrix-matrix multiplication
→ sparse diagonalization		matrix-vector multiplication
$H^{p}\lvert\psi\rangle \to \lvert E_0\rangle$	$\mathrm{eigs}(H, n)$	
power method		
→ Lanczos method	$E_0, E_1, E_2 \ldots E_n$ $\quad \lvert E_0\rangle, \lvert E_1\rangle \ldots$	

n: # of eigenstates we want

computational cost $\quad m = 2^N \quad$ dim of $\mathbb{V}^{(N)}$

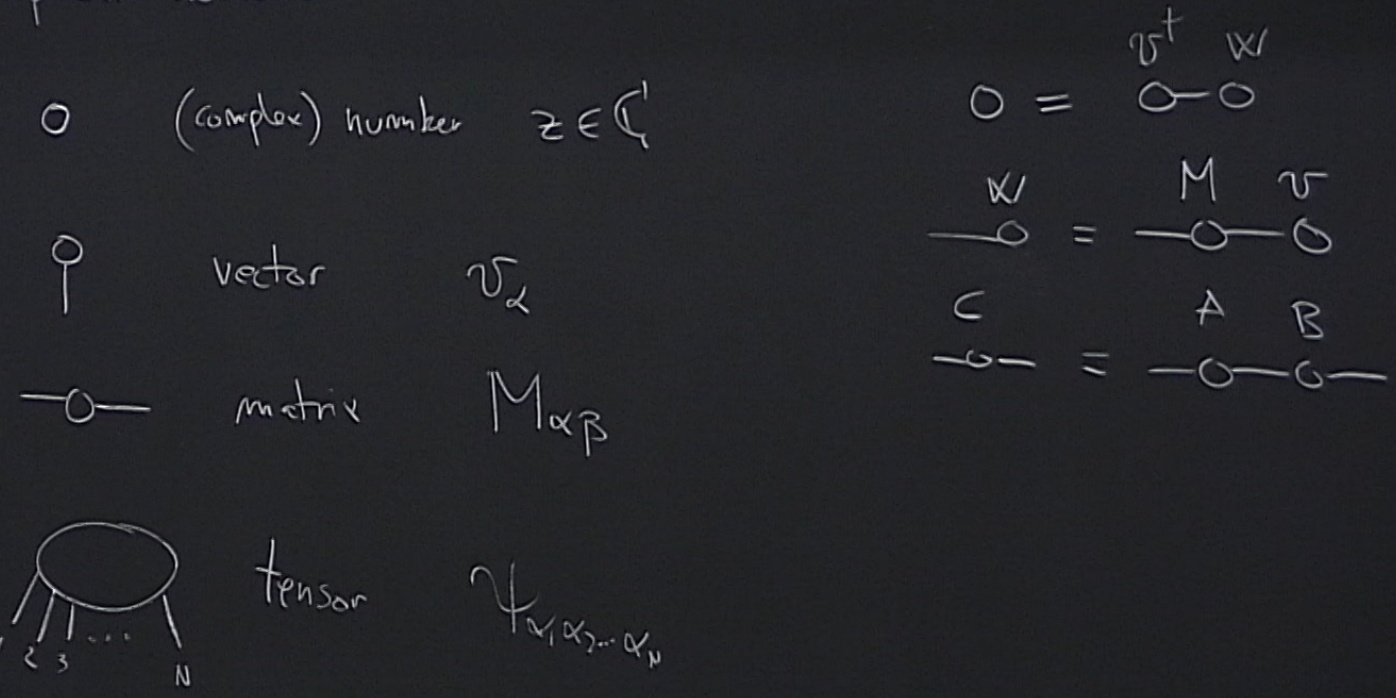
based on	time	space	
matrix-matrix multiplication	$m^3 = 2^{3N}$	$m^2 = 2^{2N}$	$\longrightarrow$ N = 10 spin
matrix-vector multiplication	$m^2 = 2^{2N}$	$m^2 = 2^{2N}$	$\longrightarrow$ N = 20 spin
sparse vector update	$m = 2^N$	$m = 2^N$	$\longrightarrow$ N = 30 spin

ground state $|E_0\rangle, |E_1\rangle$

$$H = \sum_{\ell=1}^{N} h_{\ell,\ell+1}$$

(We never need to build H $2^N \times 2^N$ matrix)

4.5 Graphical notation



4.6 Sparse vector update

$$|\psi\rangle = \sum_{\alpha_1,\alpha_2 \ldots \alpha_N = 0}^{1} \psi_{\alpha_1\alpha_2 \ldots \alpha_N} |\alpha_1 \alpha_2 \ldots \alpha_N\rangle$$

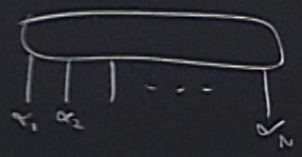
$\alpha_i = 0, 1$

$|100000\rangle$
$|100001\rangle$
$|100010\rangle$
$|100011\rangle$
$\vdots$
$|111111\rangle$

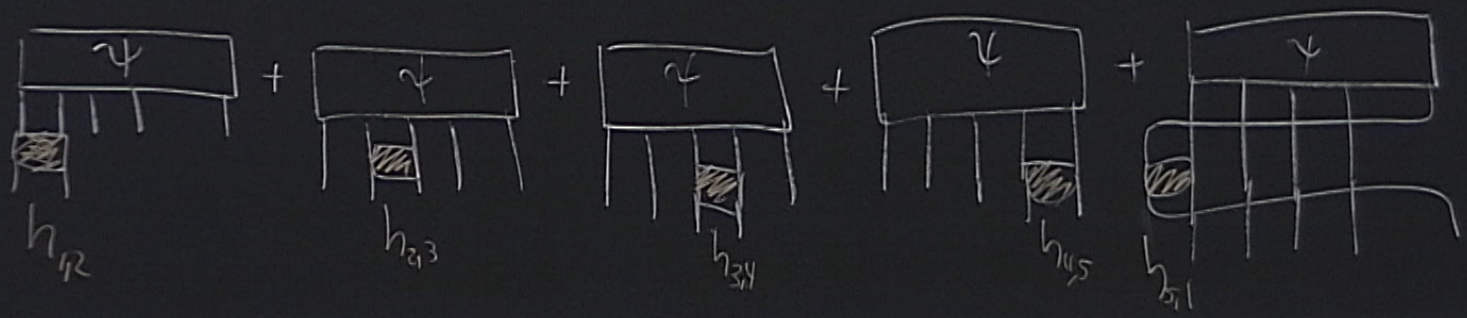
2^N

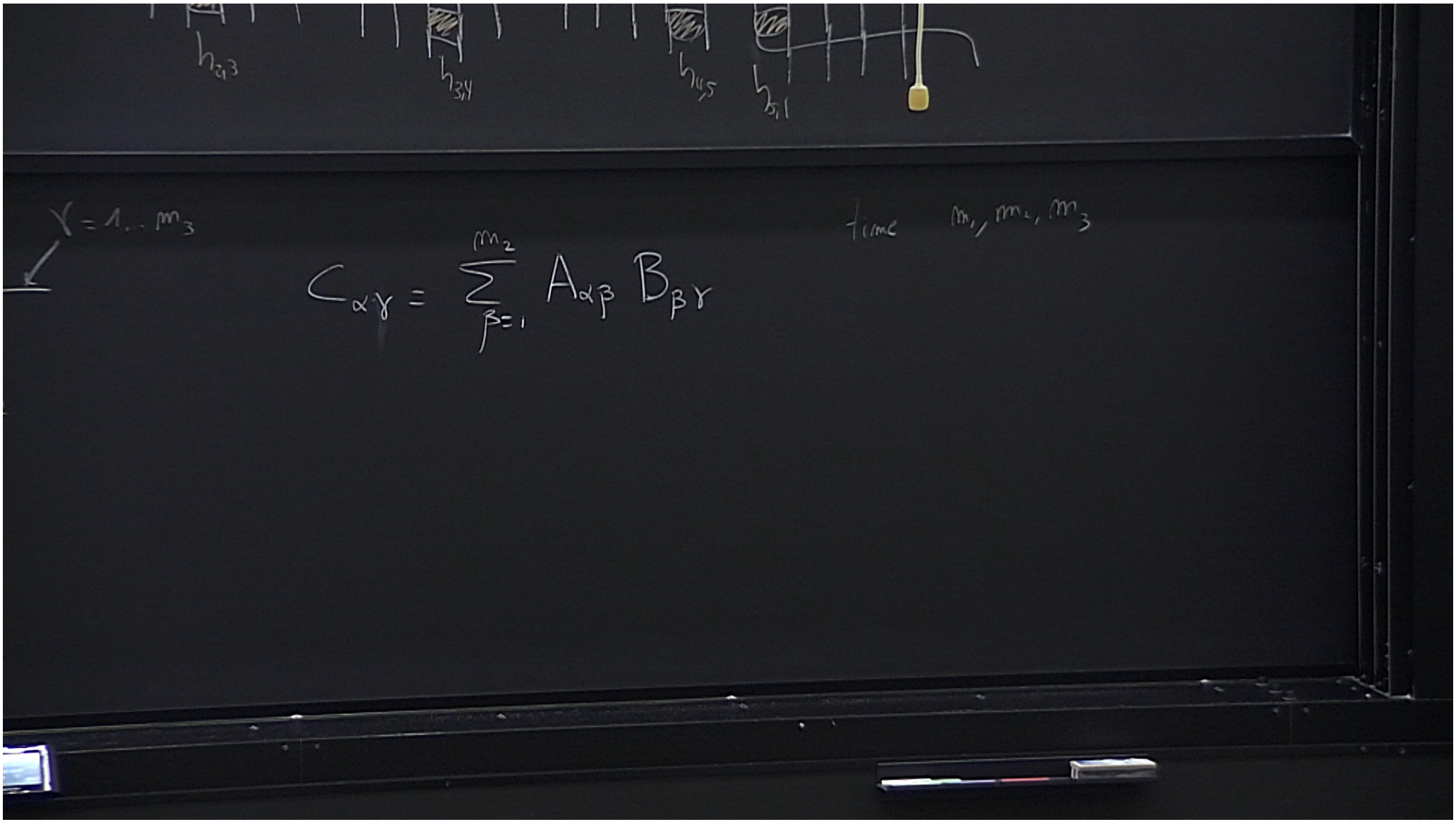
$|\psi\rangle \to |\psi'\rangle = H|\psi\rangle$

$|\psi\rangle \leftarrow$

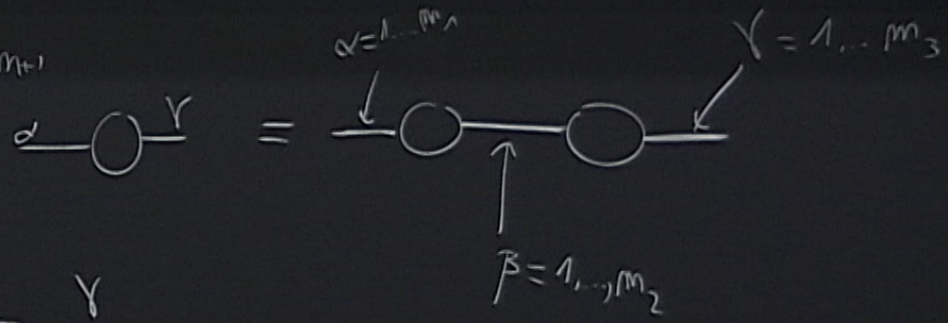


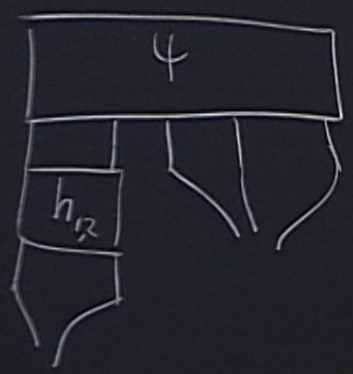
$|\psi'\rangle =$

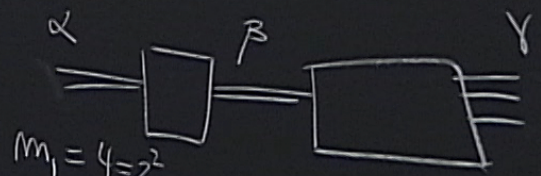



$h_{2,3}$
$h_{3,4}$
$h_{4,5}$
$h_{5,1}$
$\gamma = 1 \ldots m_3$
time m_1, m_2, m_3
$C_{\alpha\gamma} = \sum_{\beta=1}^{m_2} A_{\alpha\beta} B_{\beta\gamma}$

$$H = \sum_m \left[-\sigma_m^x \sigma_{m+1}^x - \frac{\sigma_m^z + \sigma_{m+1}^z}{2} h \right] = \sum_m h_{m,m+1}$$



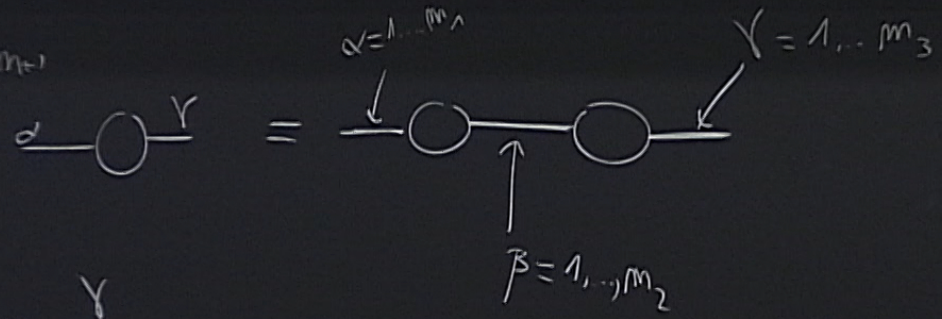




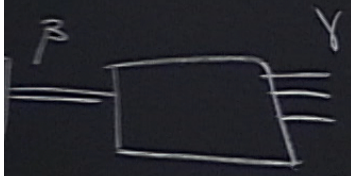
$m_1 = 4 = 2^2$ $\quad m_2 = 4 = 2^2$ $\quad m_3 = 8 = 2^3 = 2^{N-2}$

$h_{1,2}$ $h_{2,3}$ $h_{3,4}$ $h_{4,5}$ $h_{5,1}$

$\sum_m h_{m,m+1}$



$$C_{\alpha\gamma} = \sum_{\beta=1}^{m_2} A_{\alpha\beta} B_{\beta\gamma}$$

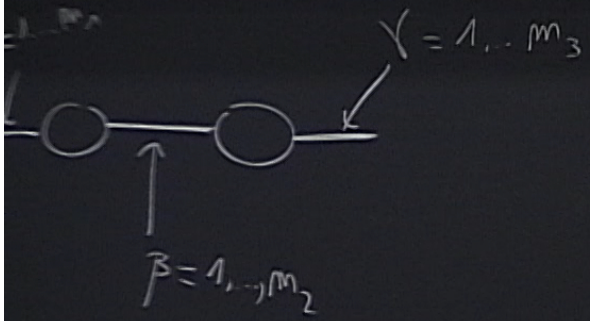


$m_1 \, m_2 \, m_3$

$m_2 = 4 = 2^2$ $\quad m_3 = 8 = 2^3 = 2^{N-2}$

cost $2^2 \, 2^2 \, 2^{N-2} = 2^{N+2}$

$h_{1,2}$ $h_{2,3}$ $h_{3,4}$ $h_{4,5}$ $h_{5,1}$



$$C_{\alpha\gamma} = \sum_{\beta=1}^{m_2} A_{\alpha\beta} B_{\beta\gamma}$$

time m_1, m_2, m_3

$O(m_1 \cdot m_2 \cdot m_3)$

$m_1 m_2 m_3$

2^{N-2}

cost $2^2 2^2 2^{N-2} = 2^{N+2}$ $O(N 2^N)$

$$H = \sum_m \left[-\sigma_m^x \sigma_{m+1}^x - \frac{\sigma_m^z + \sigma_{m+1}^z}{2} h \right] = \sum_m h_{m,m+1}$$