Title: Algebraic EE and holography

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Abstract: <p>In the first part of the talk, I will explain how the lengths of non-minimal geodesics in AdS3 conical defect backgrounds can be interpreted as the entanglement entropy of certain subalgebras in the dual CFT. This part will be based on 1608.02040. In the second part of the talk, I will discuss how the Ryu-Takayanagi area term seems to be the analog of a certain edge term for EE in a gauge theory, and how one might try to test this. This part is more speculative.</p>

Algebraic EE and holography

Jennifer Lin
IAS

1608.02040 "A Toy Model of Entwinement" + ongoing work

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Entanglement and spacetime

In recent years, many people have suggested that "spacetime emerges from quantum entanglement."

This can be made most precise in AdS/CFT where we have **the Ryu-Takayanagi formula**

$$S_{EE}(B) = \frac{A}{4G_N} + \mathcal{O}(G_N^0)$$

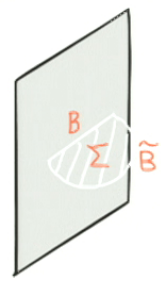
for B a region of the CFT, and A the area of the minimal-area bulk surface homologous to it.

The RT formula has led to many nice applications:

- Einstein equations from entanglement dynamics around vacuum AdS
- Entanglement wedge reconstruction

Einstein equations from entanglement

Around the AdS vacuum, the linearized Einstein equations are dual to the entanglement first law for ball shaped regions in the CFT. (van Raamsdonk et al., 2013)



$$\begin{array}{ccc} \delta S_{EE} & = & \delta\langle H_{mod}\rangle \\ \updownarrow & & \updownarrow \\ \int_{\tilde{B}} F_0(\delta g_{ab}) & = & \int_B F_1(\delta g_{ab}) \end{array}$$

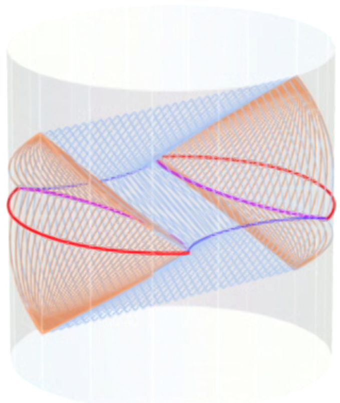
where $S_{EE} = -\mathrm{Tr}\rho\log\rho$, $\langle H_{mod}\rangle = -\langle\log\rho\rangle$, and δ means that we take the difference in these quantities on states that are perturbatively close by in the CFT. For a ball shaped region in the vacuum state of a CFT, H_{mod} is a weighted integral of the CFT stress tensor over the ball.

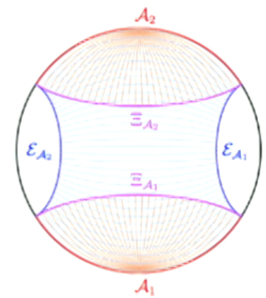
For each ball, we map both sides to the bulk, giving one nonlocal bulk equation for each bulk point. These can be inverted to get the linearized EFE's.

Entanglement wedge reconstruction (2016)

In a CFT state with a geometric dual description, suppose that you want to reconstruct a local bulk operator. How nonlocal is the operator in the CFT?

On a constant time slice, partition the sphere to split the CFT into two regions $\mathcal{A}$ and $\bar{A}$. Label the Ryu-Takayanagi surface of $\mathcal{A}$ as $\mathcal{E}_{\mathcal{A}}$. Then the **entanglement wedge** of $\mathcal{A}$ is the bulk domain of dependence of the region bounded by A and $\mathcal{E}_{\mathcal{A}}$.





Any local bulk operator in the entanglement wedge can be reconstructed as a CFT operator supported on region $\mathcal{A}$!

The progress so far raises two natural questions.

Question 1: Decoding the Hologram

Given a CFT with $R_{AdS} \gg \ell_s, \ell_P$ and a state $|\psi\rangle$ in the CFT that has a geometric dual, we should be able to read off from it:

- What the bulk metric is;
- What CFT physics is dual to the Einstein equations point-by-point in the bulk

As discussed above, the Ryu-Takayanagi formula allows us to answer these questions around the CFT vacuum, which is empty AdS. But what about around a general state?

Question 2: What is the organizing principle?

Even more ambitiously, we would like to understand the microscopic meaning of the RT formula, and its closely related cousin, the Bekenstein-Hawking entropy. More concretely:

- Is it true beyond AdS/CFT that the leading contribution to EE for any region in a theory of quantum gravity scales with area?
- What degrees of freedom is this counting (the entanglement of)?
- Can we derive an α'-exact version of the RT formula in string theory?
- Can we understand the origin of "1/4" in the RT/B-H formulas, in string theory?

Outline

These will be the subjects of the two parts of my talk.

In part 1, which is the main part, I'll show how to generalize the RT formula to the areas of non-minimal extremal surfaces in a specific set of holographic states.

In part 2 I'll briefly explain how the RT area term seems to be the stringy analog of a certain boundary term that one finds when defining EE in gauge theories. I'll then describe how we might take steps towards deriving the area term in string theory.

Part 1: Bulk Reconstruction

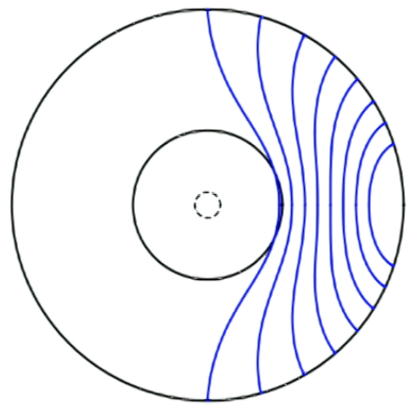
To recapitulate, the problem is the following. Given an excited state ψ with a geometric dual, we want to read off the complete bulk metric from ψ, or generalize Mark's argument to find the CFT description of the Einstein equations around ψ.

Our goal is to explore whether entanglement in the CFT is the organizing principle for the emergent bulk geometry(e.g., concretely: whether the EE first law is dual to the linearized Einstein equations around general states).

Reading off the bulk metric requires an entry in the holographic dictionary, that is a functional of the bulk metric, that has access to every point in the bulk.

Entanglement shadow

The RT formula is a nice functional of the bulk metric. However, position space EE in the CFT is **not enough** to solve this problem of bulk reconstruction, because of the entanglement shadow...



We have to find a shadowless class of covariant bulk probes.

At the same time, our discussion of EE is incomplete. The RT formula geometrizes the bulk dual to *position-space* EE's in the CFT. But we can also assign an EE quantifying the ignorance of an observer with access to any operator subalgebra in the CFT, that need not be organized by position space.

It is natural to ask whether the EE's of any such subalgebras geometrize to areas of codimension-2 surfaces in the bulk.

I'll now define EE for operator subalgebras and present one (very specific) example where the CFT dual of a non-minimal, boundary-anchored extremal bulk surface is the EE of a subalgebra in the boundary CFT.

Algebraic definition of EE

EE quantifies the ignorance of an observer who can only access a subalgebra of observables in a quantum system. Typically, one takes the subalgebra to all the operators in a spatial region, but one can also define EE for any subalgebra that need not be organized by position space.

The definition is the following. A theory is specificed by the Hilbert space $\mathcal{H}$ and an algebra of observables, $\mathcal{A}$. Given a state $\psi \in \mathcal{H}$ and a subalgebra $\mathcal{A}_0 \in \mathcal{A}$, let ρ be the unique element in $\mathcal{A}_0$ s.t.

$$\mathrm{Tr}_{\mathcal{H}}(\rho\mathcal{O}) = \langle\psi|\mathcal{O}|\psi\rangle \,.$$

Then the EE of the subalgebra is its von Neumann entropy,

$$S_{EE}(\mathcal{A}_0, \psi) = -\mathrm{Tr}_{\mathcal{H}}\rho \log \rho \,.$$

Comments:

1. Since ρ is an element of $\mathcal{A}_0$, we can expand it as

$$\rho = \sum_{\mathcal{O}_i \in \mathcal{A}_0} \rho_i \mathcal{O}_i \, .$$

   The condition $\mathrm{Tr}_{\mathcal{H}}(\rho\mathcal{O}) = \langle \mathcal{O} \rangle$ then gives one equation for each unknown ρ_i, so there is in general a unique solution.

2. The EE of a subregion in a local QFT is the EE of the maximal subalgebra supported on the subregion.

3. This algebraic definition of EE is intractable in interacting QFT's. One problem is that we have to enumerate all the operators and their expectation values. The other, which appears already in generic QM systems, is that $\mathrm{Tr}_{\mathcal{H}}$ needs to be regulated for infinite-dimensional Hilbert spaces.

Our approach will be to start from the simplest possible quantum mechanics system and build toy models of more complicated systems out of it.

Algebraic EE for a qubit

Consider a single spin.

- Hilbert space: $|\uparrow\rangle, |\downarrow\rangle$.
- Operator algebra: $\mathbf{1}, \sigma_x, \sigma_y, \sigma_z$.
- Subalgebras: $\{\{\mathbf{1}\}, \{\mathbf{1}, \sigma_\mathbf{x}\}, \{\mathbf{1}, \sigma_\mathbf{y}\}, \{\mathbf{1}, \sigma_\mathbf{z}\}\}$.

Pauli matrices are orthogonal: $Tr_{\mathcal{H}}(\sigma^a \sigma^b) = 2\delta^{ab}$.

It's easy to see that

- For any subalgebra $\mathcal{A}_0$, in a global state $\rho = \sum_{\mathcal{A}} \rho_i \mathcal{O}_i$, the reduced density matrix $\rho_{\mathcal{A}_0}$ that reproduces their expectation values is the projection onto the subalgebra, $\rho_{\mathcal{A}_0} = \sum_{\mathcal{A}_0} \rho_i \mathcal{O}_i$.
- The coefficients are given by

$$\rho_i = \frac{\langle \mathcal{O}_i \rangle}{\dim \mathcal{H}} .$$

These properties obviously generalize to chains of d spins.

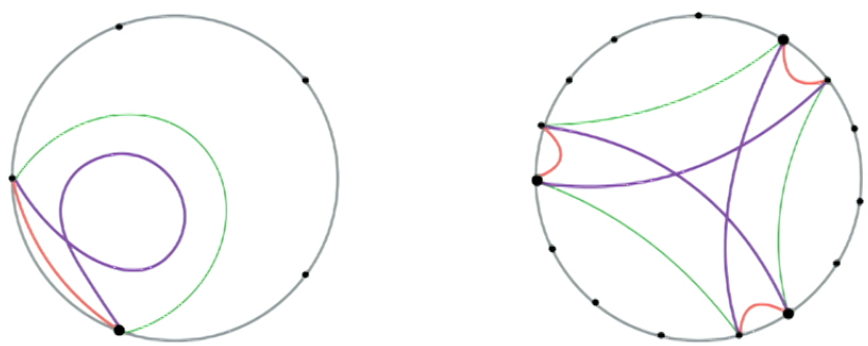
I'll now describe a holographic example where I claim that features of bulk geometry can be understood using EE's of non-maximal subalgebras.

Conical defect in AdS_3

This situation goes by the name of entwinement. In AdS_3/CFT_2, consider the geometry $AdS_3/\mathbb{Z}_n$ for integer n.

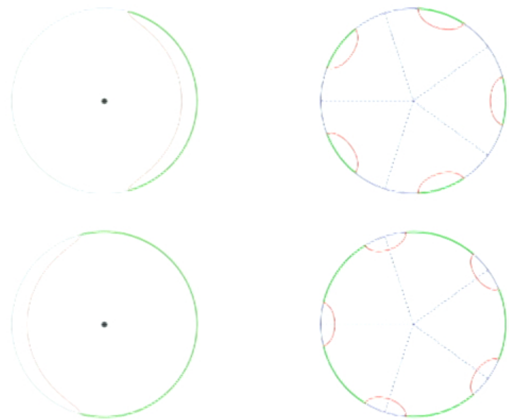
This geometry has a covering space, which is an empty AdS with a n-times-longer circumference. We can compute any geometric feature in the defect background by doing the same computation over its n copies in the covering space.

For example, we can find the lengths of geodesics in this way.

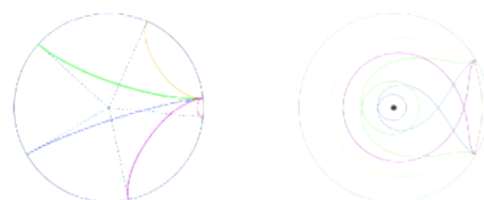


It's easy to see that the conical defect geometry of strength $\mathbb{Z}_n$:

1. Has an entanglement shadow.



2. Contains $n-1$ non-minimal geodesics homologous to each interval. The union of minimal and non-minimal geodesics have no shadow.



If we think of the defect geometry as an excited state for quantum gravity in AdS_3: what is its CFT dual?

Naively, this question is intractable since the dual of a state we've described geometrically belongs to a strongly coupled CFT. But we can make it well-defined by embedding the defect into the D1-D5 system and interpolating to the free orbifold point, where the dual CFT is a free field theory with the target space $(\mathbf{T}^4)^N/Sym(N)$.

Which state in the free orbifold CFT does the state dual to the $AdS_3/\mathbb{Z}_n$ defect interpolate to as we move along the moduli space?

For n a divisor of $N = N_1 N_5$, the answer turns out to be

$$\sigma^{N/n}|0\rangle$$

where $\sigma^{N/n}$ is the twist operator that sets up boundary conditions on the fields, s.t.

$$\begin{array}{ccccccccc} \phi_1 & \rightarrow & \phi_2 & \rightarrow & \dots & \rightarrow & \phi_n & \rightarrow & \phi_1 \\ \phi_{n+1} & \rightarrow & \phi_{n+2} & \rightarrow & \dots & \rightarrow & \phi_{2n} & \rightarrow & \phi_n \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ \phi_{N-n+1} & \rightarrow & \phi_{N-n+2} & \rightarrow & \dots & \rightarrow & \phi_N & \rightarrow & \phi_{N-n+1} \end{array}$$

as one goes around the spatial $\mathbf{S}^1$ of the CFT.

Long string picture and CFT covering space

A convenient way to visualize twisted sectors in orbifold CFT's is to use the long string picture.

A field configuration on a target space $M^N/Sym(N)$ can be thought of as N copies of noninteracting fields valued in M up to symmetrization. In the untwisted sector, fields on a spatial slice describes N maps from $\mathbf{S}^1 \to M$. So we can picture states in the untwisted sector as N loops inside M.

In a twisted sector, fields rotate into one another as we go around $\mathbf{S}^1$. States now describe maps from a fewer number of longer $\mathbf{S}^1$'s into M. We can picture twisted states as $< N$ longer loops ("long strings") in M.

Looking back at the twisted state dual to the $AdS_3/\mathbb{Z}_N$ defect, the same "n-times-longer $\mathbf{S}^1$" covering space appears in both the AdS$_3$ description of the conical defect geometry and the CFT description of its dual state.

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Entwinement is the CFT dual of the long geodesic

Identifying the covering space construtions, we find that

Non-minimal surface	$\rightarrow_{\text{covering space}}$	minimal surface
		$\downarrow_{RT}$
EE over long string segment	$\leftarrow$	pos. space EE in CFT cover

The CFT dual of the non-minimal geodesic in the defect background is the EE over a segment of the long string, that goes more than once around the spatial $\mathbf{S}^1$. It's not a position space EE in the CFT, but rather, an EE for internal degrees of freedom, that we defined by un-gauging part of the gauge group

This quantity has been called **entwinement**.

My goal for the next few slides will be to define entwinement in a manifestly gauge-invariant manner. In particular, I will argue that it's equal to the algebraic EE of a gauge-invariant subalgebra.

A toy model of entwinement

To compute algebraic EE, we'll replace the orbifold CFT by a spin chain that captures some of its essential properties.

We put the CFT on a lattice, discretize the spatial $\mathbf{S}^1$ into k cells, and replace the N scalar fields by Nk quantum variables. Also, we will take the variables to be $\mathbb{Z}_2$- instead of $\mathbf{T}^4$-valued, so they are now quantum spins.

The simplest case is $k = 1, N = 2$: two spins with a $Sym(2) = \mathbb{Z}_2$ gauge symmetry.

Two spins with a $\mathbb{Z}_2$ gauge symmetry

From the 4d Hilbert space of two spins, we project out the singlet state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$.

Of the 16 linearly independent operators $\sigma \otimes \sigma$ that constitute the algebra in the theory of 2 spins, 9 combinations of them are gauge-invariant:

$$\mathcal{A} = \{\mathbf{1} \otimes \mathbf{1}, \sigma_i \otimes \sigma_i, \frac{1}{2}(\mathbf{1} \otimes \sigma_i + \sigma_i \otimes \mathbf{1}), \frac{1}{2}\sigma_{(i} \otimes \sigma_{j)}\}$$

where $i \in x, y, z$. (This list contains ten operators but one combination $\frac{3}{4}\mathbf{1} \otimes \mathbf{1} + \frac{1}{4}(\sigma_i \otimes \sigma_i)$ acts as the identity on our Hilbert space).

In short, this is a fancy way to label the 9 linearly independent 3×3 matrices that act on 3d Hilbert space.

Comments:

(*) This basis of $\mathcal{A}$ has the property $\mathcal{O}^3 = \mathcal{O}$, hence, subalgebras include $\{1, \mathcal{O}, \mathcal{O}^2\}$ for each $\mathcal{O} \in \mathcal{A}$.

(*) The analog of entwinement in this simple system is the EE between the two spins when that state is lifted to the 4d Hilbert space, with the $\mathbb{Z}_2$ constraint removed.

A simple example

E.g. consider the family of states $|\psi\rangle = a|\uparrow\uparrow\rangle + \sqrt{1-a^2}|\downarrow\downarrow\rangle$ with a real.

Pretending that this is a state in 4d Hilbert space and tracing out the secnod spin in the usual way, we get the reduced density matrix $\rho = a^2|\uparrow\rangle\langle\uparrow| + (1-a^2)|\downarrow\rangle\langle\downarrow|$, whose von Neumann entropy is the entwinement:

$$S_{ent} = -[a^2 \log a^2 + (1-a^2)\log(1-a^2)].$$

The density matrix for the state ψ in the physical, 3d Hilbert space with basis in the basis $(|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, \frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ is

$$\begin{aligned} \rho &= \begin{pmatrix} a^2 & a\sqrt{1-a^2} & 0 \\ a\sqrt{1-a^2} & 1-a^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ &= \frac{1}{3}\mathbf{1}\otimes\mathbf{1} + \frac{1}{6}(\sigma_z\otimes\sigma_z - \frac{1}{2}[\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y]) \\ &\quad + \frac{1}{2}\left[a^2 - \frac{1}{2}\right](\mathbf{1}\otimes\sigma_z + \sigma_z\otimes\mathbf{1}) + \frac{1}{2}a\sqrt{1-a^2}(\sigma_x\otimes\sigma_x - \sigma_y\otimes\sigma_y) \end{aligned}$$

where I've written out the expansion of $\rho = \sum_i \rho_i \mathcal{O}_i$.

$\{\mathbf{1}\otimes\mathbf{1}, \frac{1}{2}(\mathbf{1}\otimes\sigma_z+\sigma_z\otimes\mathbf{1}), \sigma_z\otimes\sigma_z\}$ form a subalgebra. The projection of ρ onto it is

$$\rho_{\mathcal{A}} = \mathrm{diag}(a^2, 1-a^2, 0)$$

whose EE clearly equals

$$S_{ent} = -[a^2 \log a^2 + (1-a^2)\log(1-a^2)].$$

So for this subset of states, the entwinement is the algebraic EE of the subalgebra generated by $\frac{1}{2}(\mathbf{1}\otimes\sigma_z+\sigma_z\otimes\mathbf{1})$.

If we do the same thing for the subset of states $a\left[\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)\right] + \sqrt{1-a^2}\left[\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right]$ with a real, we find that the entwinement is equal to the algebraic EE generated by $\frac{1}{2}(\mathbf{1}\otimes\sigma_x + \sigma_x\otimes\mathbf{1})$.

Entwinement as algebraic EE in the two-spin system

So now there's basically a unique prescription for how to define the entwinement of any state in the two-spin system as an algebraic EE. For any state of this gauged two-spin system, the entwinement is the algebraic entanglement entropy of the subalgebra generated by the projection of the global density matrix onto operators of the form

$$\mathbf{1} \otimes \sigma_i + \sigma_i \otimes \mathbf{1} \,.$$

In my paper, I show that this is true. Here's a handwavy and quick argument for it.

- When we lift a state from the gauged (3d) Hilbert space to the extended (4d) one, its expansion $\rho = \sum_i \rho_i \mathcal{O}_i$ is unchanged.
- Entwinement is a position-space EE in the extended Hilbert space. Algebraically, a position space EE is the projection of the global density matrix onto the maximal subalgebra supported in that region.
- So if we wanted to algebraically define entwinement in the extended Hilbert space, we would project the density matrix onto operators of the form $\sigma \otimes \mathbf{1}$. The coefficients of these guys in the expansion of ρ are the relevant data.
- In the physical theory, such operators always come with $\mathbf{1} \otimes \sigma$ to get a gauge-invariant combination.
- The only real check to do is that various constants work out.

Entwinement as algebraic EE for larger N and k

The two-spin example with $\mathbb{Z}_2$ gauge symmetry was the simplest possible case for our discretization of the CFT into a spin chain. What about a longer spin chain with a larger discrete gauge group?

The argument in the last slide generalizes immediately.

Originally, we were interested in a twisted state in an orbifold CFT. There we wanted to algebraically define "EE for part of the long string". The claim is that it's the algebraic EE of a state-dependent subalgebra, generated by the operator one gets from projecting the global state onto those operators that are nontrivial on the segment of the long string and trivial elsewhere, summed over the minimal discrete permutations that turn them into gauge-invariant operators.

Summary

To summarize, we've shown in the very specific example of the conical defect backround in AdS_3, the entanglement entropy of a subalgebra which is not the maximal one in position space is dual to the area of a non-minimal codimension 2 surface in the bulk.

But the example is naive, with everything trivializing in a covering space. Also, the boundary subalgebra is not very nice.

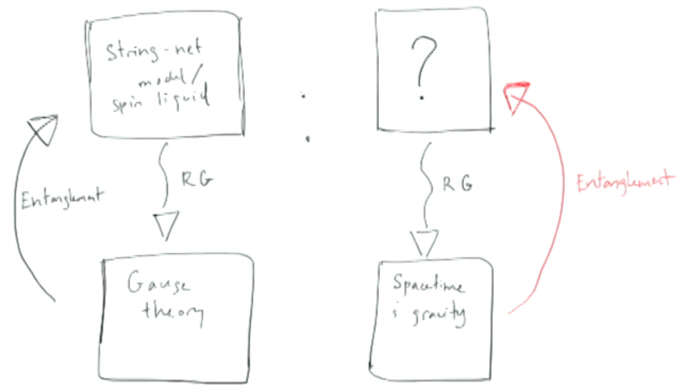
This is a **proof of principle** that entanglement entropies can translate to geometric features in AdS/CFT beyond the Ryu-Takayanagi formula, which is insufficient for bulk reconstruction.

But it's unclear that organizing by subalgebras of the dual CFT is the most efficient way to extract this information.

If there are no questions, I'd now like to briefly discuss what bulk principle might underlie the area formula, which will also shed light on why we might expect EE's of certain subalgebras in holographic CFT's to geometrize on general grounds.

There are no new results in this part.

The idea



...is that we're going to make an analogy between emergent gauge theories and the **bulk** in AdS/CFT. As I'll review, when one defines entanglement carefully in an emergent gauge theory, there is an "edge term" measuring classical correlations between UV variables that the gauge theory emerges from.

I'll argue that "$A/4G_N$" of the Ryu-Takayanagi formula is such an edge term, measuring classical correlations of stringy DOF's that gravity emerges from. As such, it should be extractable from a worldsheet calculation.

EE in gauge theory

In the last few years, people have tried to define EE in (lattice) gauge theory. Gauge theories don't factorize so we need something beyond the usual partial trace.

One idea that agrees with the replica trick is the "extended Hilbert space" prescription. Given a subregion in a state of the gauge theory, we embed our Hilbert space into the minimal larger Hilbert space that factorizes across the region, and then take the EE to be the usual thing in the extended Hilbert space.

$$\mathcal{H} \subset \mathcal{H}_{ext} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$$
$$\rho_A = \mathrm{Tr}_A \rho \text{ in } \mathcal{H}_{ext}$$
$$S_{EE} = -\mathrm{Tr} \rho_A \log \rho_A \,.$$

Upon doing this, one finds that EE contains edge terms...

Ex. 1: EE in electrodynamics on $\mathbf{S}^1$

Consider $U(1)$ gauge theory on $\mathbf{S}^1$. The gauge-invariant algebra contains just one canonically conjugate pair of stuff, $\oint \mathbf{A}$ and $E(x)$ which is constant by Gauss's law. A good basis for the Hilbert space are eigenstates of E: $E|n\rangle = n|n\rangle, n \in \mathbb{Z}$.

Now to define EE across an interval for a state ψ, we follow the recipe. Defining $\mathcal{H} \subset \mathcal{H}_{ext} = \mathcal{H}_{phys} \otimes \mathcal{H}_{phys}$ with $|n\rangle \to |n\rangle \otimes |n\rangle$,

$$|\psi\rangle = \sum_n \psi_n |n\rangle \in \mathcal{H} = \sum_n \psi_n |n\rangle \otimes |n\rangle \in \mathcal{H}_{ext}$$

$$\rho_A = \sum_n p_n |n\rangle\langle n|, \qquad p_n = |\psi_n|^2$$

$$S_{EE} = -\sum_n p_n \log p_n \qquad (\text{"Shannon edge term"}).$$

What we're measuring here is the kinematic correlation of the electric field operator in regions A and $\bar{A}$ due to Gauss's law.

Ex. 2: EE in Yang-Mills on $\mathbf{S}^1$

Now consider Yang-Mills with gauge group G on $\mathbf{S}^1$. The gauge-invariant algebra now contains Wilson loops $\mathrm{Tr}_R \exp(i \oint \mathbf{A})$ and Casimirs made out of the electric field. A convenient basis for the Hilbert space is labeled by representations of G: $\mathcal{H} = \{|R\rangle\}$.

To define EE, we embed $\mathcal{H} \subset \mathcal{H}_{ext} = \oplus_R \{|R, i, j\rangle\} \otimes \{|R, i, j\rangle\}$, now assigning a subspace of dimension $(\dim R)^2$ to each state $|R\rangle$. Now for

$$|\psi\rangle = \sum_R \psi_R |R\rangle \in \mathcal{H} = \sum_R \psi_R |R, i, j\rangle \otimes |R, j, i\rangle \in \mathcal{H}_{ext},$$

$$\rho_A = \sum_R p_R \sum_{i,j} |R, i, j\rangle\langle R, i, j|, \qquad p_R = |\psi_R|^2,$$

$$S_{EE} = -\sum_R p_R \log p_R + \sum_R p_R \log \dim R. \qquad (\text{"Shannon + } \log d_R \text{ edge terms"}).$$

Interpretation of the edge terms

- In a more general, (lattice) gauge theory in $d > 2$ dimensions, the extended Hilbert space yields the result

$$S_{EE} = \text{Shannon edge term} + \log d_R \text{ edge term} + \text{distillable EE}\,.$$

- What do the edge terms mean? They measure correlations due to constraint equations that are visible classically:
  - (*) The Shannon edge term measures correlations between IR operators due to the Gauss law.
  - (*) The $\log d_R$ edge term measures correlations between UV variables due to the IR gauge symmetry.

Algebraic EE vs. extended Hilbert space on the lattice

Earlier in this talk, I described the algebraic definition of EE.

It turns out that the algebraic EE for the maximal gauge-invariant subalgebra supported on a region of the lattice, differs from the extended Hilbert space definition by precisely the $\log d_R$ term:

$$S_{EE}(\rho_A) = S_{alg,ginv}(A) + \log\, d_R \text{ edge term}\,.$$

This is not surprising since the $\log d_R$ term measures correlations between gauge-*variant* UV operators in an emergent gauge theory.

"$A/4G_N$" is a $\log d_R$ edge term

Now I'd like to make an analogy between lattice gauge theory and the **bulk** side of AdS/CFT.

In AdS/CFT, Harlow has recently proved an algebraic version of the statement

RT + $1/N$ (FLM) correction $\leftrightarrow$ entanglement wedge reconstruction

$$\left(\begin{array}{l} \text{EE of region A in CFT} = \\ \text{RT area} + \\ \text{algebraic EE of g-inv. operators in } \mathcal{E}_A \end{array} \right) \leftrightarrow \left(\text{Region A in CFT} = \text{Region } \mathcal{E}_A. \right)$$

$\Downarrow$ (Take EE of both sides)

$$\left(\begin{array}{l} \text{EE of region A in CFT} = \\ \text{EE of region } \mathcal{E}_A \text{ in bulk EFT} \\ = \text{algebraic EE} + \log d_R \text{ edge term} \end{array} \right).$$

(NB: One can indirectly test this by repeating Lewkowycz-Maldacena in the presence of a bulk nonabelian gauge field.)

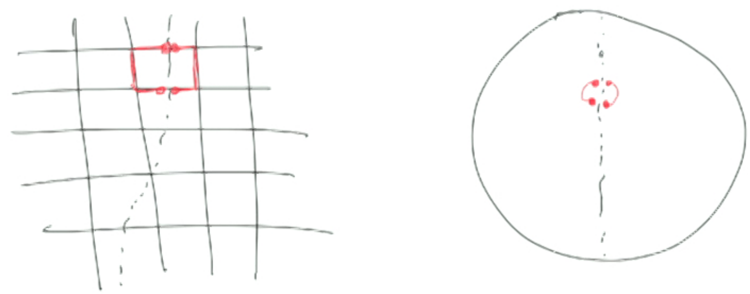
The $\log d_R$ edge term counts classical correlations between UV degrees of freedom that the IR gauge theory emerges from.

As long as the analogy doesn't qualitatively break down, this suggests that "$A/4G_N$" canonically counts classical string degrees of freedom, and should be extractable from a string worldsheet calculation, without having to use the replica trick!

What is it counting?

Here is a cartoon.

1. To define EE along a bulk extremal surface, we are partitioning a string background into two halves.
2. String theory contains extended objects (e.g. closed strings) that can naively lie across this bulk partition.
3. By a naive analogy between a Wilson loop in an emergent gauge theory and a closed string,



a resolution is that the Hilbert space of string theory contains open strings ending on the entangling surface and "$A/4G_N$" counts their Chan-Paton factors.

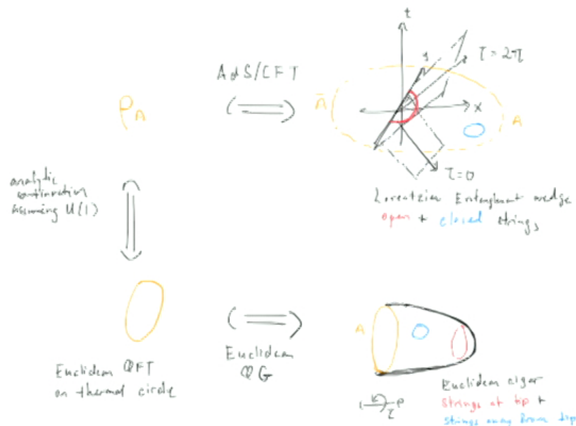
How can we test this?

The real test is whether we can extract "$1/4G_N$" from a worldsheet calculation.

I suggested that in the Lorentzian entanglement wedge, it comes from Chan-Paton factors of a new open string sector that we don't know anything about.

But if the Hamiltonian generates a geometric modular flow (i.e. the AdS-Rindler wedge), we can analytically continue the boundary state and extract α'-exact data from the Euclidean worldsheet in the dominant Euclidean saddle. Naively, the dominant saddle is Euclidean Rindler (flat space) as $R_{AdS} \to \infty$.

The bulk picture suggests that the dominant saddle is actually a modified Euclidean Rindler background with a nonvanishing sphere partition function for closed strings at the tip of the Euclidean cigar.



Question: How to efficiently compute the $\mathbf{S}^2$ string partition function?

Summary

To summarize, it would be very interesting to pursue this idea that the Ryu-Takayanagi area term is a "$\log d_R$" edge term in string theory.

Also, we can now return to the question of what is needed to derive the Einstein equations around general asymptotically AdS backgrounds in AdS/CFT, from the point of view that area terms are edge terms for bulk entanglement. Clearly what is needed is to find which boundary subalgebras are needed to reconstruct all the operators in arbitrary bulk subregions, generalizing subregion-subregion duality.

Thank you!