Title: PSI 2016/2017 Quantum Field Theory III - Lecture 2

Date: Jan 31, 2017 11:30 AM

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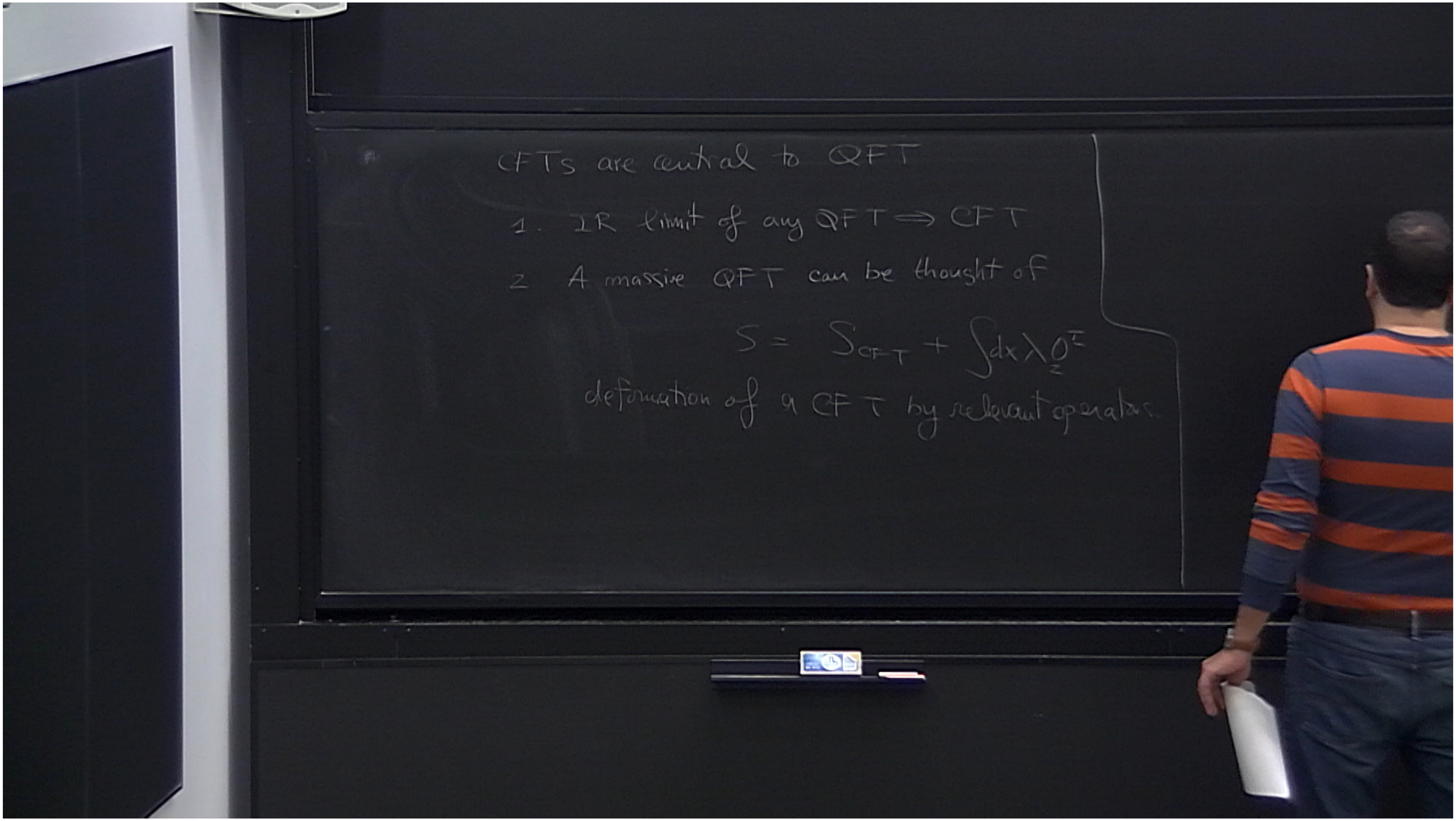
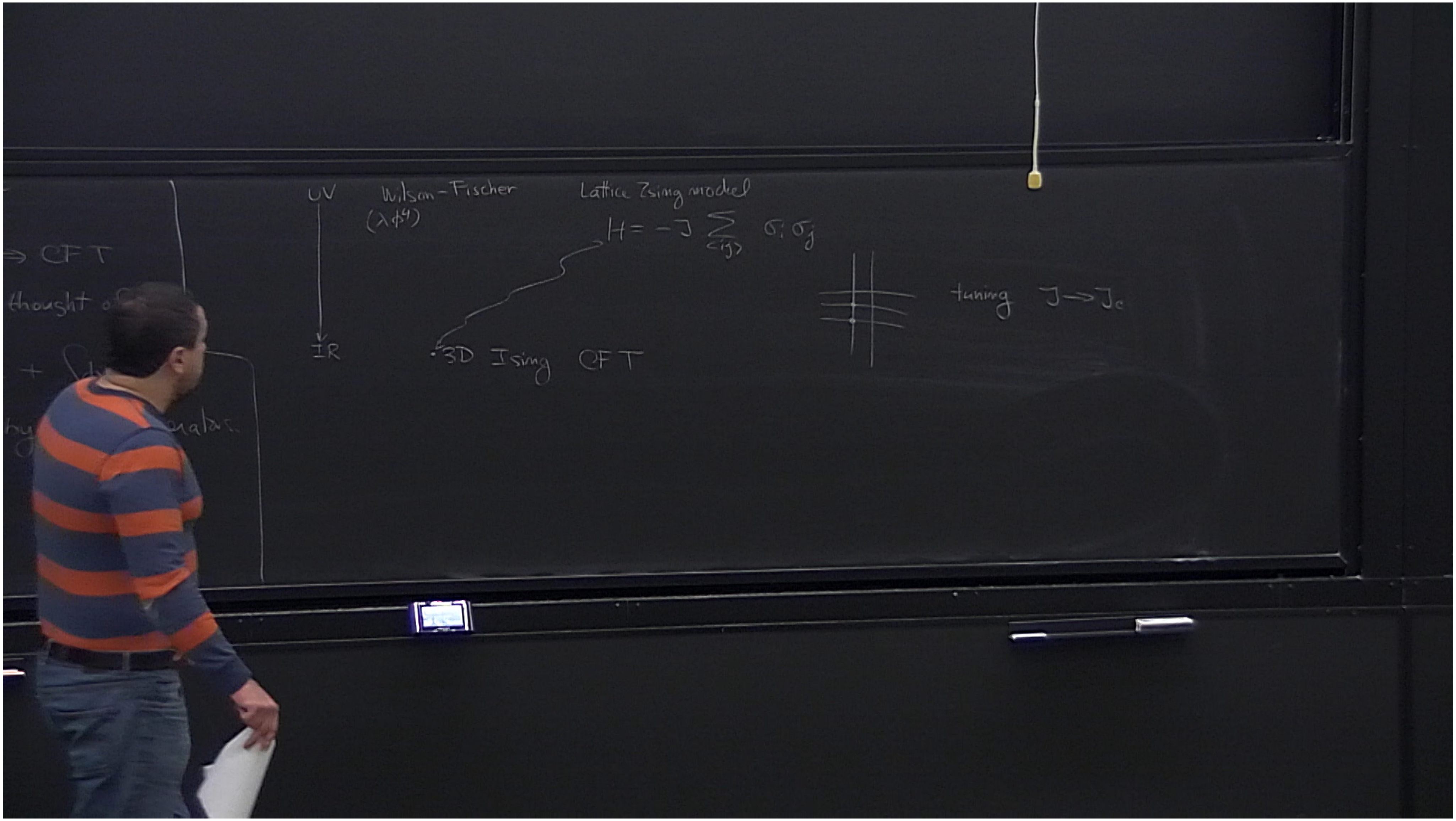
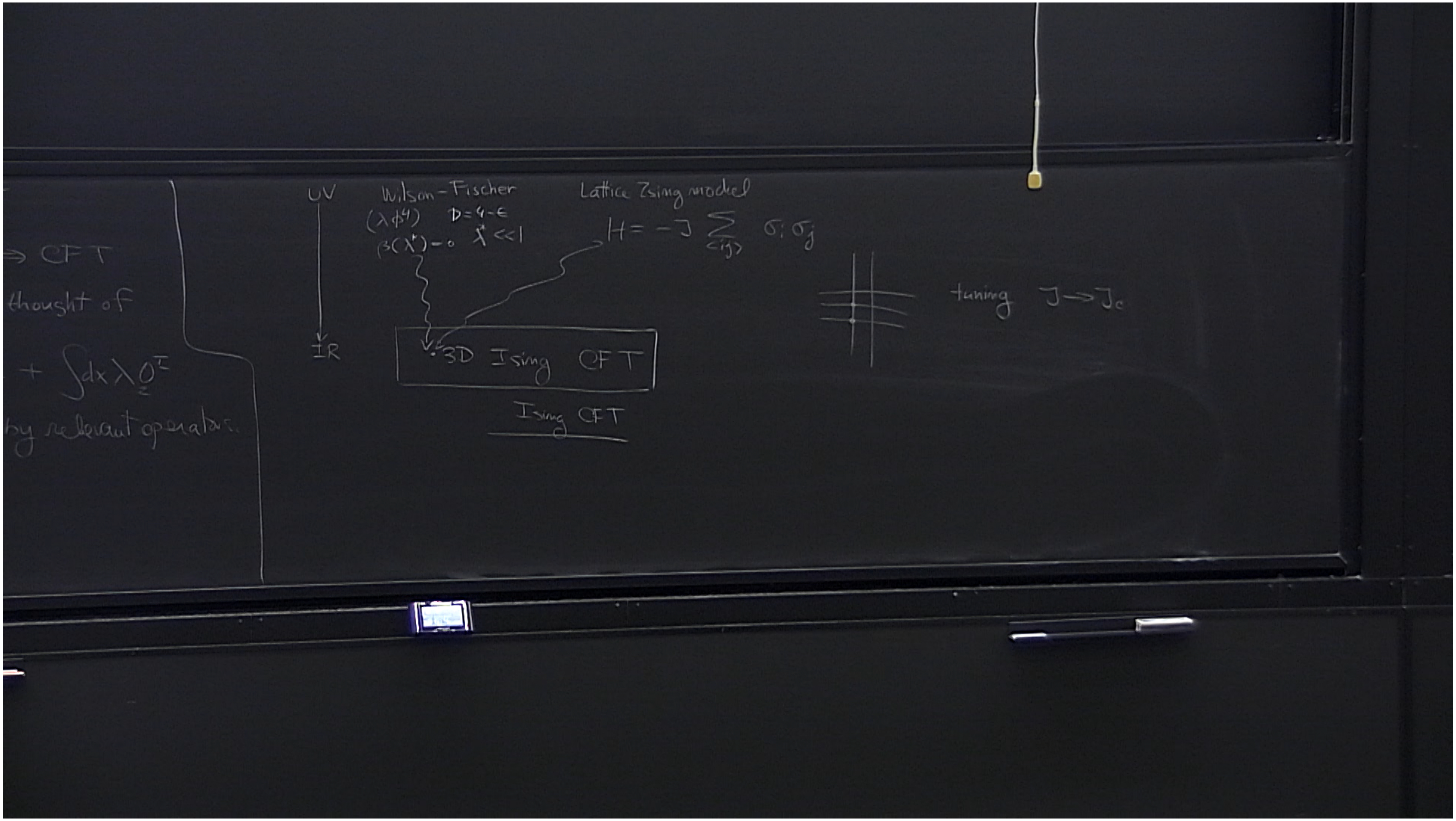
Abstract:


CFTs are central to QFT
1. IR limit of any QFT ⟹ CFT
2. A massive QFT can be thought of
$S = S_{CFT} + \int dx\, \lambda\, O$
deformation of a CFT by relevant operators.


⇒ CFT
thought
UV
Wilson-Fischer
$(\lambda\phi^4)$
Lattice Ising model
$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$
IR
3D Ising CFT
tuning $J \to J_c$


$\Rightarrow$ CFT
thought of
$+ \int dx\, \lambda_I O^I$
by relevant operators
UV
IR
Wilson-Fischer
$(\lambda\phi^4)$ $D=4-\epsilon$
$\beta(\lambda^*)=0$ $\lambda^* \ll 1$
Lattice Ising model
$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$
3D Ising CFT
Ising CFT
tuning $J \to J_c$

$\Rightarrow$ CFT

thought of

$+ \int dx\, \lambda\, \underline{O}^{I}$

by relevant operators.

UV

$\downarrow$

IR

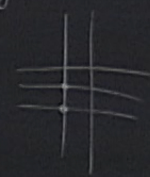
Wilson-Fischer

$(\lambda \phi^4)$ $\quad D = 4-\epsilon$

$\beta(\lambda^*) = 0 \quad \lambda^* \ll 1$

Lattice Ising model

$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$



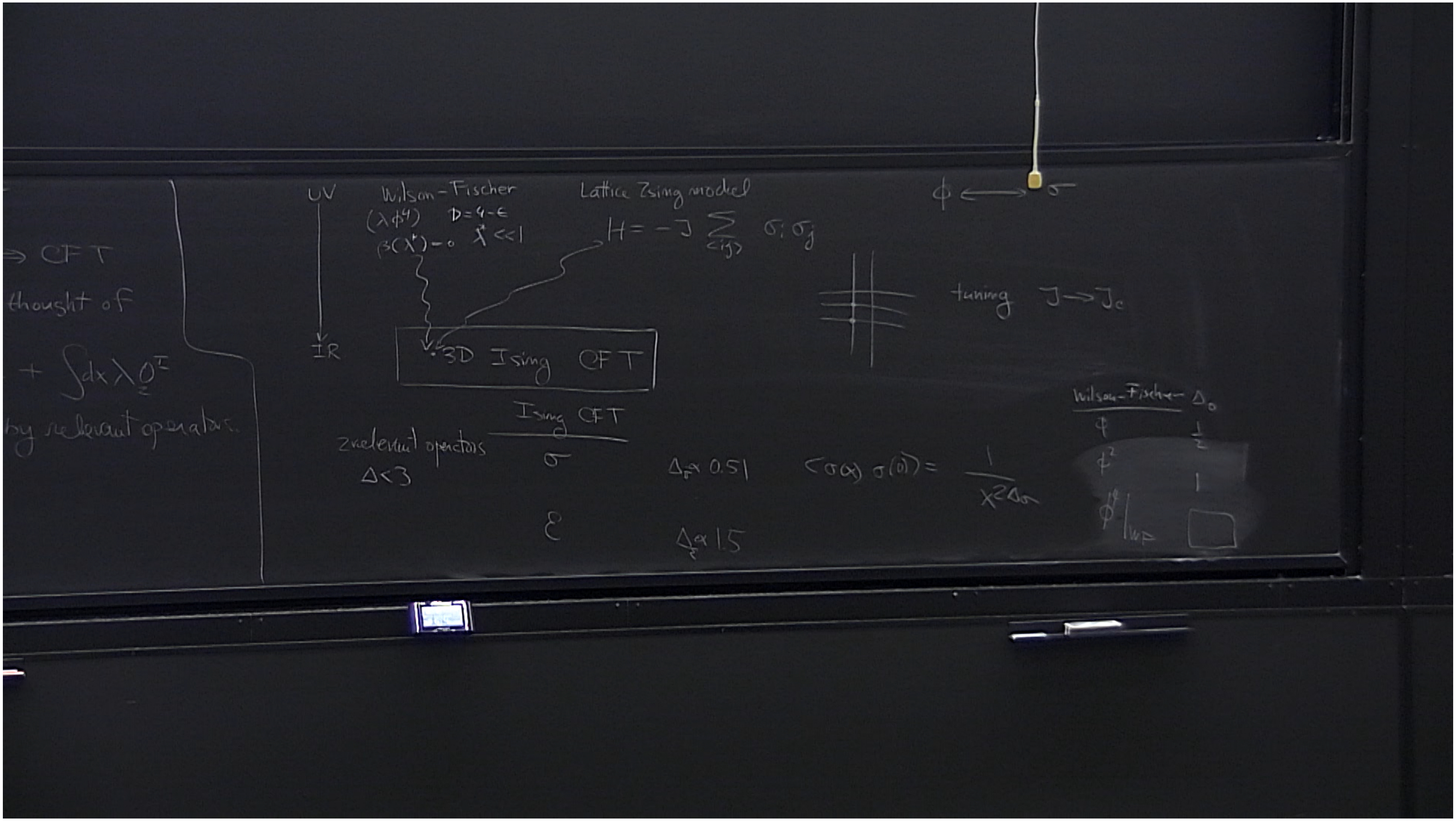
tuning $J \to J_c$

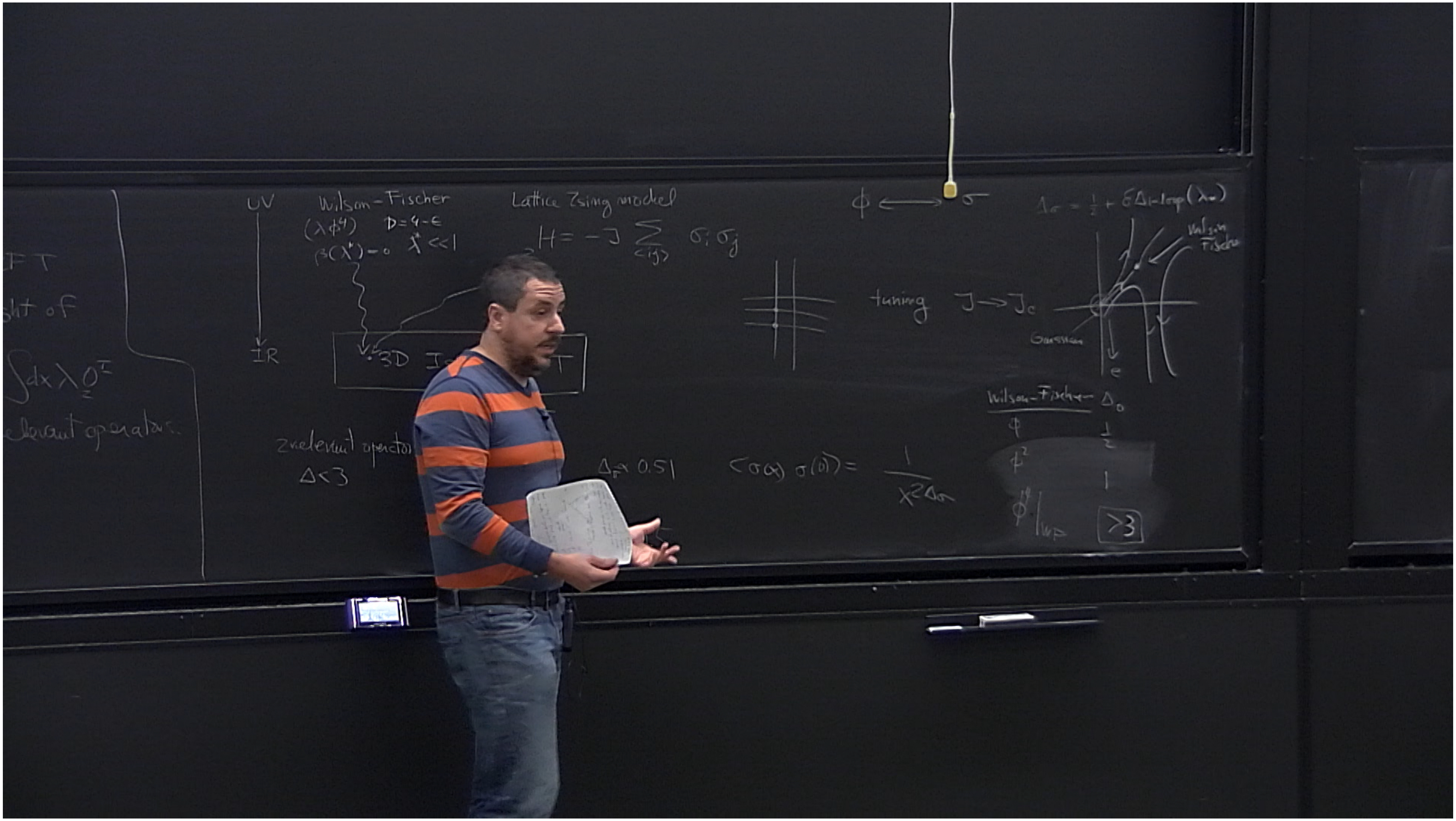
$\boxed{\text{3D Ising CFT}}$

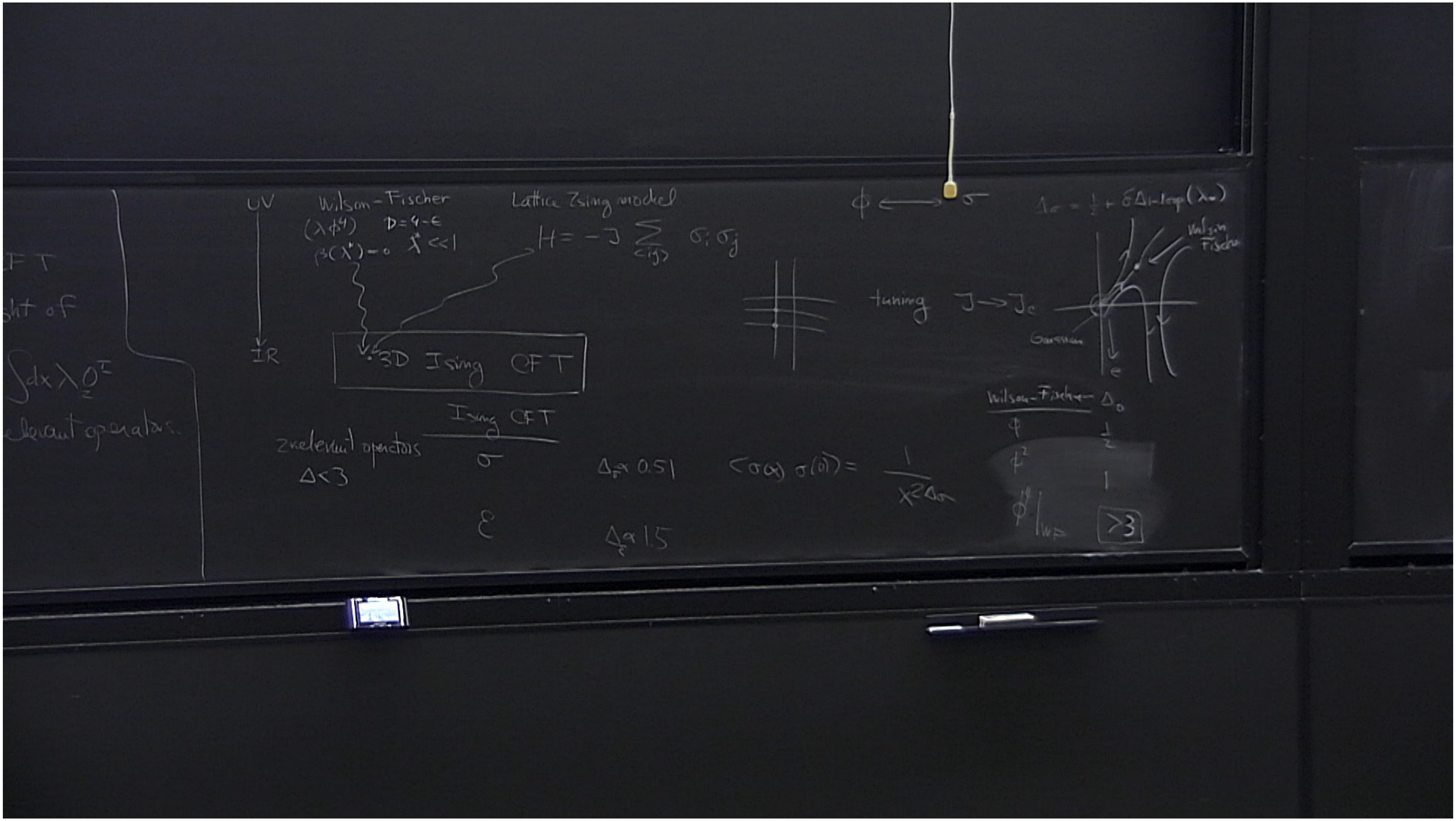
2 relevant operators

$\Delta < 3$

Ising CFT		
σ	$\Delta_\sigma \approx 0.51$	$\langle \sigma(x)\, \sigma(0) \rangle = \dfrac{1}{x^{2\Delta_\sigma}}$
ε	$\Delta_\varepsilon \approx 1.5$	


⇒ CFT
thought of
+ ∫dx λ O
by relevant operators
UV
IR
Wilson-Fischer
(λφ⁴) D=4-ε
β(λ*)=0 λ* << 1
Lattice Ising model
H = -J Σ σᵢ σⱼ
3D Ising CFT
Ising CFT
relevant operators
Δ<3
σ
ε
Δσ ≈ 0.51
Δε ≈ 1.5
<σ(x) σ(0)> = 1/x^{2Δσ}
φ ⟷ σ
tuning J → Jc
Wilson-Fischer Δ₀
φ 1/2
φ² 1
φ⁴|WF


UV
IR
Wilson-Fischer
Lattice Ising model
$\phi \longleftrightarrow \sigma$
tuning $J \to J_c$



FT
ght of
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relevant operators
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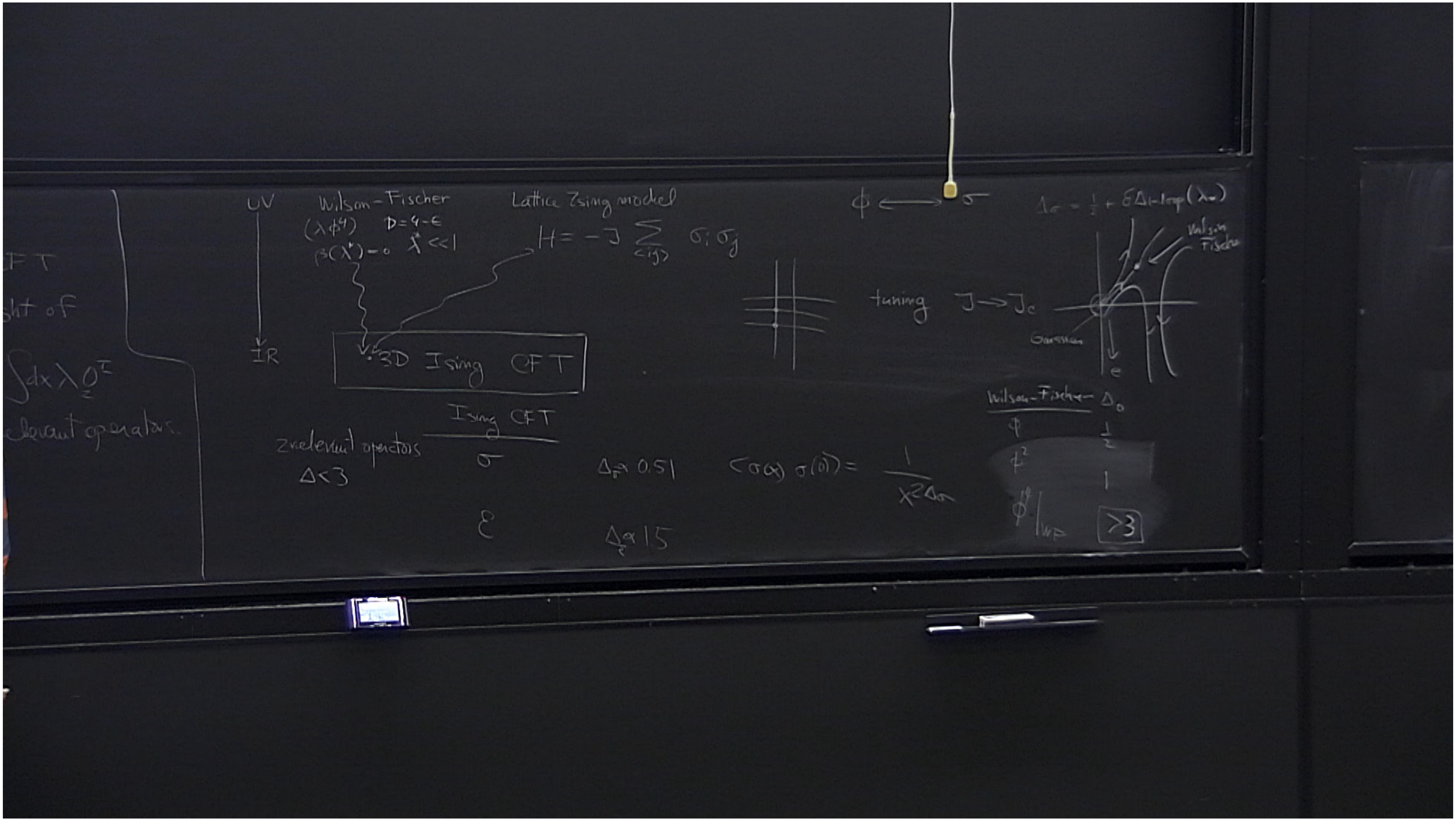
σ $\Delta_\sigma \approx 0.51$
ϵ $\Delta_\epsilon \approx 1.5$
$\langle \sigma(x)\, \sigma(0) \rangle = \frac{1}{x^{2\Delta_\sigma}}$
$\phi \longleftrightarrow \sigma$
tuning $J \to J_c$
$\Delta_\sigma = \frac{1}{2} + \delta\Delta_{1\text{-loop}}(\lambda_*)$
Wilson-Fischer
Gaussian
Wilson-Fischer Δ_0
ϕ $\frac{1}{2}$
ϕ^2 1
$\phi^4|_{WF}$ >3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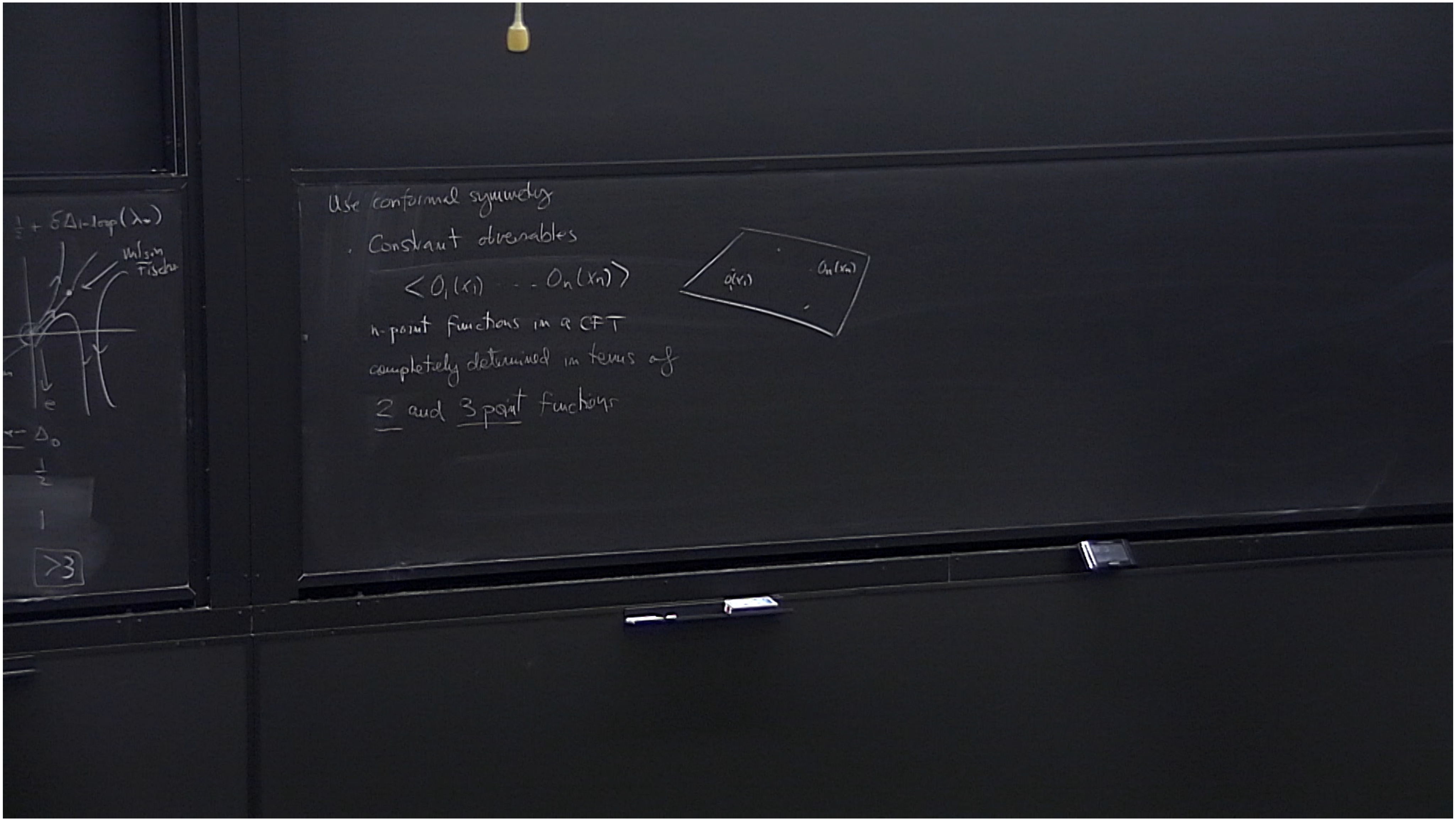
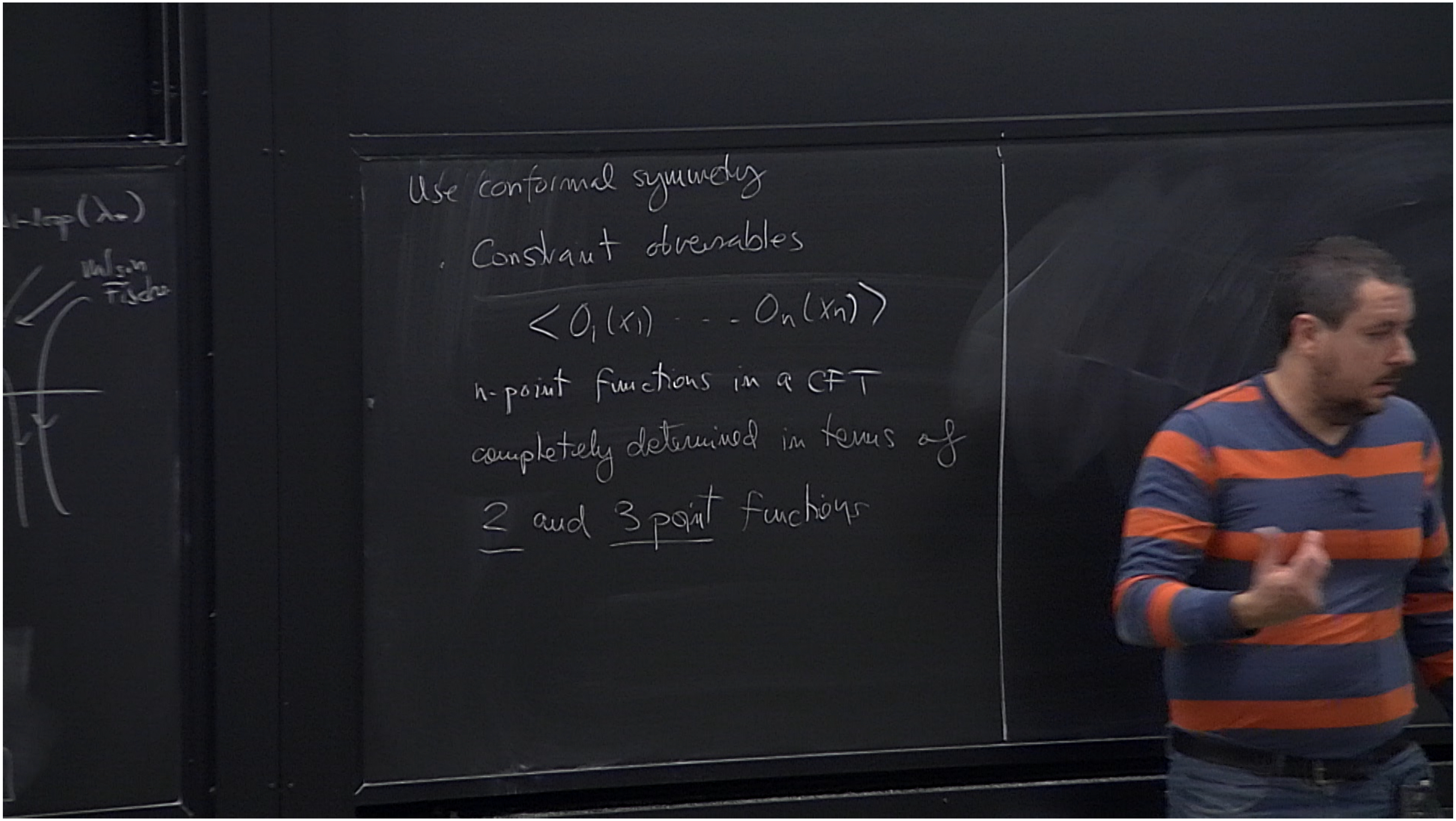
Δσ = 1/2 + εΔ1-loop(λ*)
Wilson-Fischer
Gaussian
Wilson-Fischer Δ0
φ 1/2
φ² 1
φ⁴|WF ≥3


Use conformal symmetry
. Constraint observables
< O1(x1) . . . On(xn) >
n-point functions in a CFT
completely determined in terms of
2 and 3 point functions
O1(x1)
On(xn)
Wilson Fischer
>3


Use conformal symmetry
Constraint observables
$\langle O_1(x_1) \dots O_n(x_n) \rangle$
n-point functions in a CFT
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2 and 3 point functions

Use conformal symmetry

- Constraint observables

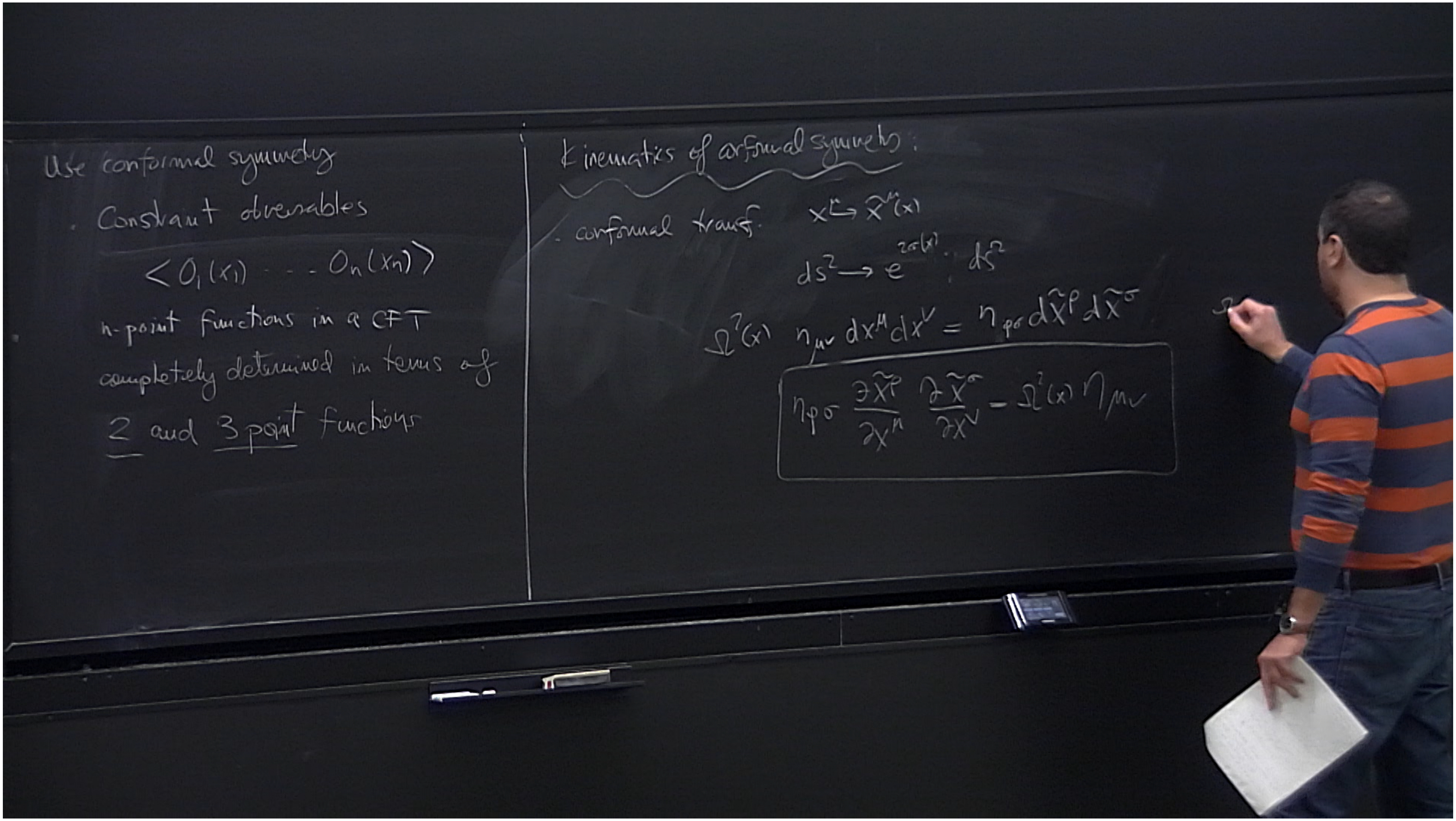
$$\langle O_1(x_1) \cdots O_n(x_n) \rangle$$

n-point functions in a CFT completely determined in terms of <u>2</u> and <u>3 point</u> functions

Kinematics of conformal symmetry:

- conformal transf. $x^\mu \mapsto \tilde{x}^\mu(x)$

$$ds^2 \to e^{2\sigma(x)} \, ds^2$$


Use conformal symmetry
. Constraint observables
$\langle O_1(x_1) \cdots O_n(x_n) \rangle$
n-point functions in a CFT
completely determined in terms of
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Kinematics of conformal symmetry:
. conformal transf. $x^\mu \mapsto \tilde{x}^\mu(x)$
$ds^2 \to e^{2\sigma(x)} ds^2$
$\Omega^2(x)\, \eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma$
$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\, \eta_{\mu\nu}$

$\langle O_1(x_1) \cdots O_n(x_n) \rangle$

n-point functions in a CFT completely determined in terms of 2 and 3 point functions

· conformal transf $x^\mu \to \tilde{x}^\mu(x)$

$$ds^2 \to e^{2\sigma(x)} ds^2$$

$$\Omega^2(x)\, \eta_{\mu\nu}\, dx^\mu dx^\nu = \eta_{\rho\sigma}\, d\tilde{x}^\rho d\tilde{x}^\sigma$$

$$\boxed{\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\, \eta_{\mu\nu}}$$

$$\Lambda^\nu_\mu(x) = \boxed{\frac{\partial x^\nu}{\partial \tilde{x}^\mu}}\, \Omega \qquad \Lambda^T \eta \Lambda = \eta$$

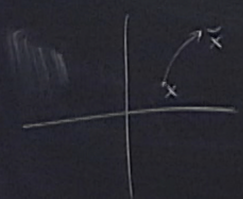
translation, Lorentz

$\Omega = 1$ Poincaré transformations $\tilde{x}^\mu = a^\mu + \Lambda^\mu_\nu x^\nu$

$$\Omega(x) = \left| \frac{\partial \tilde{x}}{\partial x} \right|^{1/D} = 1 + \frac{1}{D}(\partial \cdot \xi)$$

For transformations connected to identity

$$\tilde{x}^\mu = x^\mu + \xi^\mu(x)$$



$$\boxed{\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D}(\partial \cdot \xi)\, \eta_{\mu\nu}}$$

conformal Killing vector

conformal transformation is a local Lorentz transformation w/ a dilation

$= 0$ Killing vector

$$\xi^\mu = a^\mu + \omega^\mu_\nu x^\nu$$

· Isometries: $\partial_\mu \partial_\nu \xi_\rho = 0$

· Conformal transf: $\partial_\mu \partial_\nu \partial_\lambda \xi_\rho = 0$

$$\xi^\mu = \underbrace{a^\mu}_{\text{translations}} + \underbrace{\omega^\mu{}_\nu x^\nu}_{\text{Lorentz}} + \underbrace{\lambda x^\mu}_{\text{scale}} + \underbrace{b^\mu x^2 - 2x^\mu\, b\cdot x}_{\text{special conformal.}}$$

Conformal algebra

$$\xi_{(1)} = \xi^\mu_{(1)}\,\partial_\mu$$

$$\xi_{(2)} = \xi^\mu_{(2)}\,\partial_\mu$$

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)}$$

$$\xi^\mu_{(3)} = \xi_1^\nu \partial_\nu \xi_2^\mu - \xi_2^\nu \partial_\nu \xi_1^\mu$$

$a^\mu \longleftrightarrow P^\mu$

$\omega^{\mu\nu} \longrightarrow M^{\mu\nu}$

$\lambda \longrightarrow D$ (dilatation)

$b^\mu \longrightarrow K^\mu$ (special conf.)

Poincaré

$\xi_P = a^\mu \partial_\mu$

$\xi_M = \omega^\lambda{}_\nu x^\nu \partial_\lambda$

$\omega_{\mu\nu} = -\omega_{\nu\mu}$.

$[\xi_P^{(1)}, \xi_P^{(2)}] = 0 \iff [P^\mu, P^\nu] = 0$

$[\xi_{M}^{(1)}, \xi_P^{(2)}] = \underbrace{a^\mu \omega^\lambda_\mu}\, \partial_\lambda = \xi_P^{(3)}$

$\tilde{a}^\lambda = a^\mu \omega_\mu^\lambda$

$[M_{\mu\nu}, P_\rho] = \eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu$

$[\xi_M^{(1)}, \xi_M^{(2)}] = \xi_M^{(3)}$.

Poinca

$$[\xi_P^{(1)}, \xi_P^{(2)}] = 0 \iff [P^\mu, P^\nu] = 0$$

$$[\xi_M^{(1)}, \xi_P^{(2)}] = \underbrace{a^\mu \omega^\lambda_\mu} \qquad \partial_\lambda = \xi_P^{(3)}$$

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$$[M_{\mu\nu}, P_\rho] = \eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu$$

$$[\xi_M^{(1)}, \xi_M^{(2)}] = \xi_M^{(3)}$$

Poincaré

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu)$$

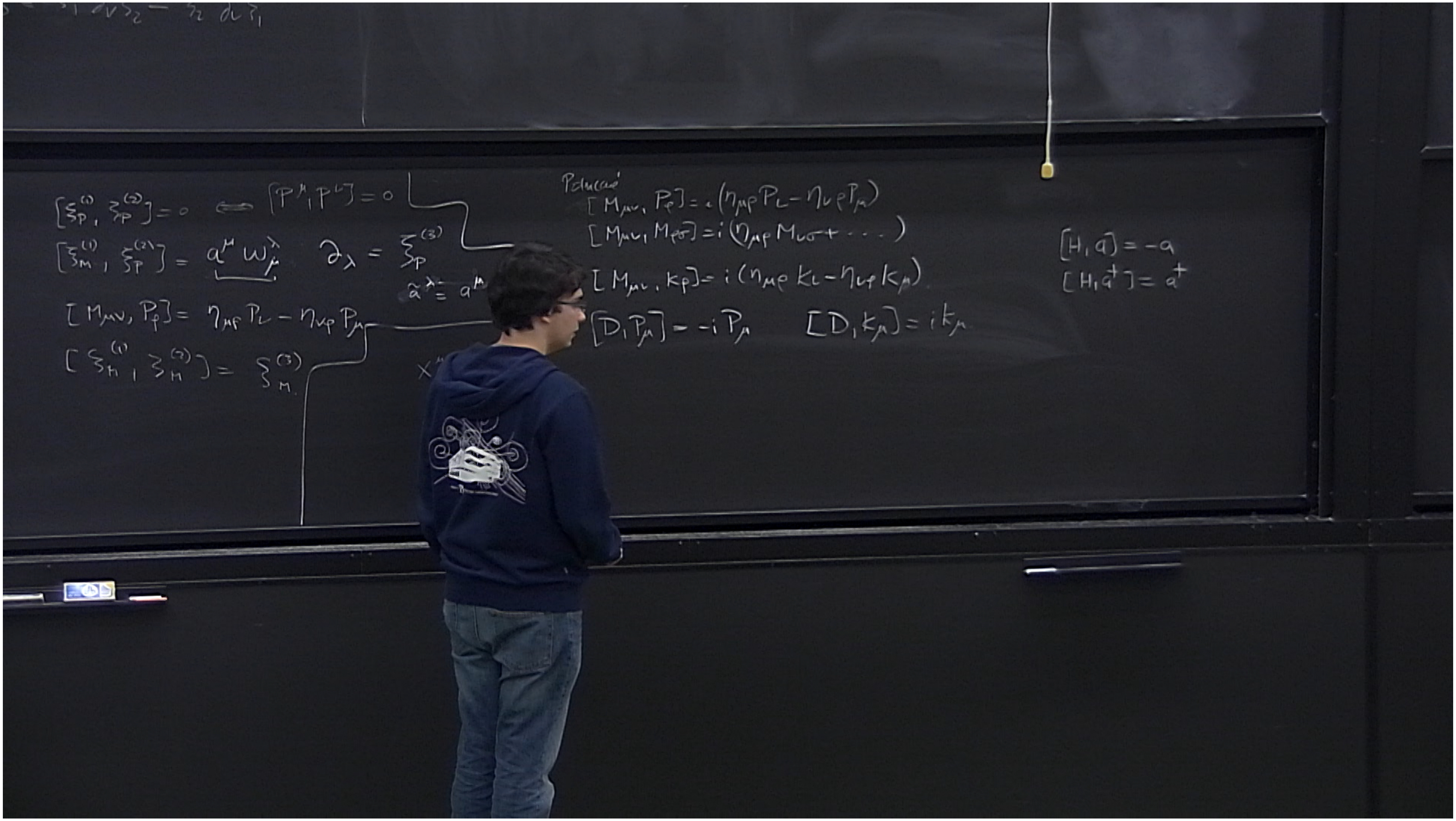
$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\mu\rho} M_{\nu\sigma} + \dots)$$

$$[M_{\mu\nu}, K_\rho] = i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu)$$

$$[D, P_\mu] = -i P_\mu \qquad [D, K_\mu] = i K_\mu$$

$$[H, a] = -a$$

$$[H, a^\dagger] = a^\dagger$$


$[\xi_P^{(1)}, \xi_P^{(2)}] = 0 \Longleftrightarrow [P^\mu, P^\nu] = 0$
$[\xi_M^{(1)}, \xi_P^{(2)}] = a^\mu \omega_\mu^\lambda \quad \partial_\lambda = \xi_P^{(3)}$
$\tilde{a}^\lambda = a^\mu$
$[M_{\mu\nu}, P_\rho] = \eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu$
$[\xi_M^{(1)}, \xi_M^{(2)}] = \xi_M^{(3)}$
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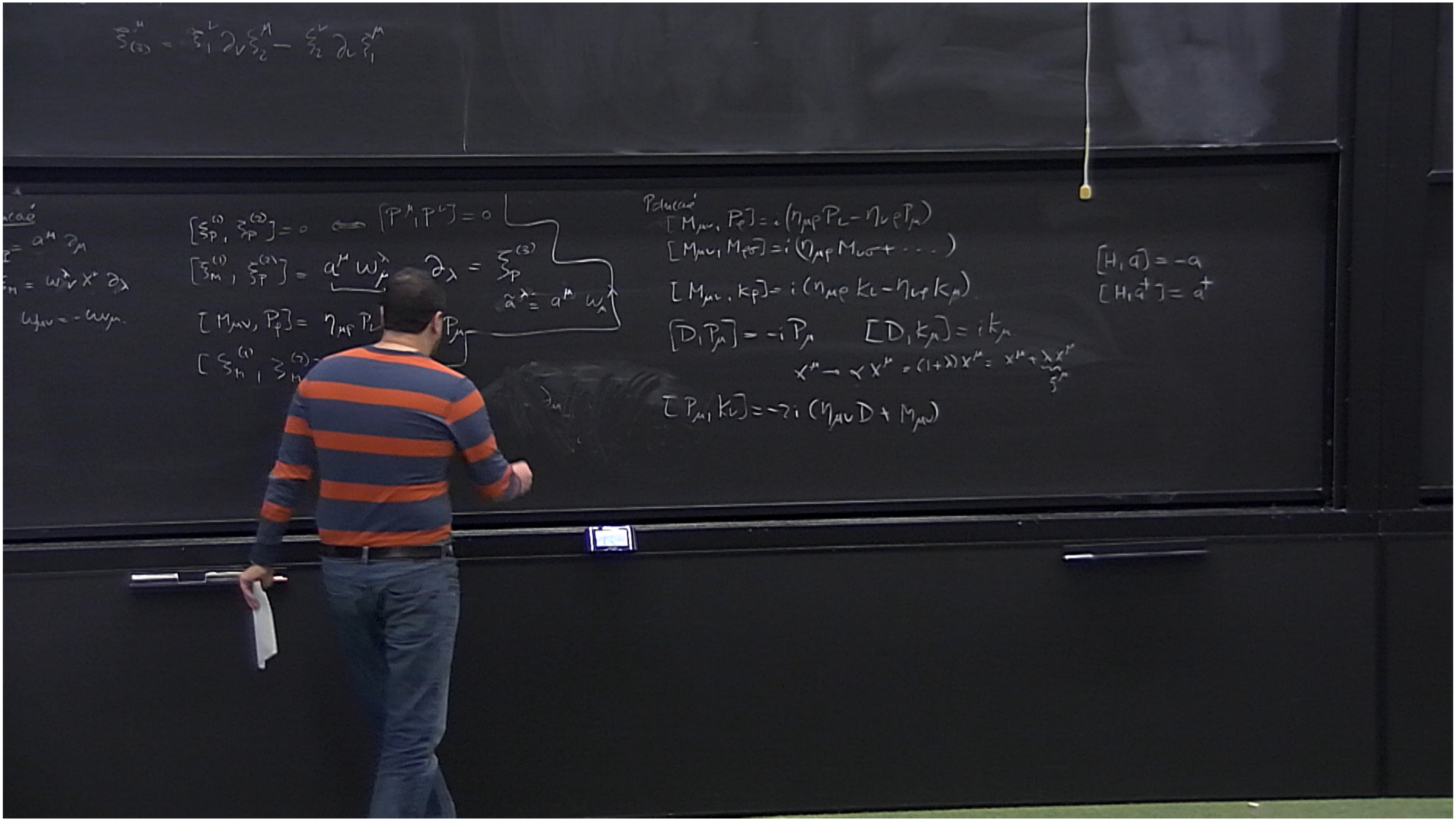
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$[M_{\mu\nu}, K_\rho] = i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu)$
$[D, P_\mu] = -i P_\mu$
$[D, K_\mu] = i K_\mu$
$[P_\mu, K_\nu] = -2i(\eta_{\mu\nu} D + M_{\mu\nu})$
$[H, a] = -a$
$[H, a^\dagger] = a^\dagger$
$[P^\mu, P^\nu] = 0$

$$\xi_{(3)}^{\mu} = \xi_1^{\nu}\partial_\nu \xi_2^{\mu} - \xi_2^{\nu}\partial_\nu \xi_1^{\mu}$$

...racaé

$\xi_P = a^{\mu}\partial_\mu$

$\xi_M = \omega^{\lambda}{}_{\nu} x^{\nu} \partial_\lambda$

$\omega_{\mu\nu} = -\omega_{\nu\mu}$.

$$[\xi_P^{(1)}, \xi_P^{(2)}] = 0 \iff [P^{\mu}, P^{\nu}] = 0$$

$$[\xi_M^{(1)}, \xi_P^{(2)}] = \underbrace{a^{\mu}\omega^{\lambda}_{\mu}}\,\partial_\lambda = \xi_P^{(3)}$$

$$\tilde{a}^{\lambda} = a^{\mu}\omega_{\mu}^{\lambda}$$

$$[M_{\mu\nu}, P_\rho] = \eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu$$

$$[\xi_M^{(1)}, \xi_M^{(2)}] = \xi_M^{(3)}$$

∂_m

$\Longrightarrow$ Lie algebra

Poincaré

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\mu\rho}M_{\nu\sigma} + \dots)$$

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$$[D, P_\mu] = -iP_\mu \qquad [D, K_\mu] = iK_\mu$$

$$x^{\mu} \to \alpha x^{\mu} = (1+\lambda)x^{\mu} = x^{\mu} + \underbrace{\lambda x^{\mu}}_{\xi^{\mu}}$$

$$[P_\mu, K_\nu] = -2i(\eta_{\mu\nu}D + M_{\mu\nu})$$

$$[H, a] = -a$$

$$[H, a^{\dagger}] = a^{\dagger}$$

conformal algebra

$SO(2,D)$ $\quad$ Minkowski $\mathbb{R}^{1,D-1}$

$SO(1,D+1)$ $\quad$ Euclid $\mathbb{R}^{D}$

L_{MN}, $\quad M = \underbrace{0, \ldots D-1}_{\mu}, D, D+1$

$$[L_{MN}, L_{PQ}] = i\eta_{MP} L_{NQ} + \cdots$$

$$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{D\,D+1} = D \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu\,D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$$

$\Rightarrow$ Lie algebra $\quad [P_\mu, K_\nu] = -2i(\eta_{\mu\nu} D + M_{\mu\nu})$

$SO(1, D+1)$

Euclid

$\mathbb{R}^D$

$$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{D\,D+1} = D \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$$

CFT is a theory that has this symmetry

Poincaré: $\partial^\mu T_{\mu\nu} = 0 \qquad T_{\mu\nu} = T_{\nu\mu}$

what properties does $T_{\mu\nu}$ must have to be

- Dilation inv

- Special conf. inv

For tran

Use conformal

. Constant

$< 0 | ($

n-point fun

completely

2 and 3

$$\delta S = \frac{1}{2}\int dx\, T_{\mu\nu}\, \delta h^{\mu\nu} \sim \int dx\, T_{\mu\nu}\, (\partial\cdot\xi)\, \eta^{\mu\nu} = \int dx\, T^{\mu}_{\mu}\, (\partial\cdot\xi)$$

(1) $\delta h^{\mu\nu} = -\partial^\mu \xi^\nu - \partial^\nu \xi^\mu$

(2) conformal $\partial^\mu \xi^\nu + \partial^\nu \xi^\mu = \frac{1}{2}(\partial\cdot\xi)\,\eta_{\mu\nu}$

(a) Scale invariant: $\boxed{T^{\mu}_{\mu} = \partial_\mu L^\mu}$ $\partial\cdot\xi_S = \text{constant}$

(b) conformal

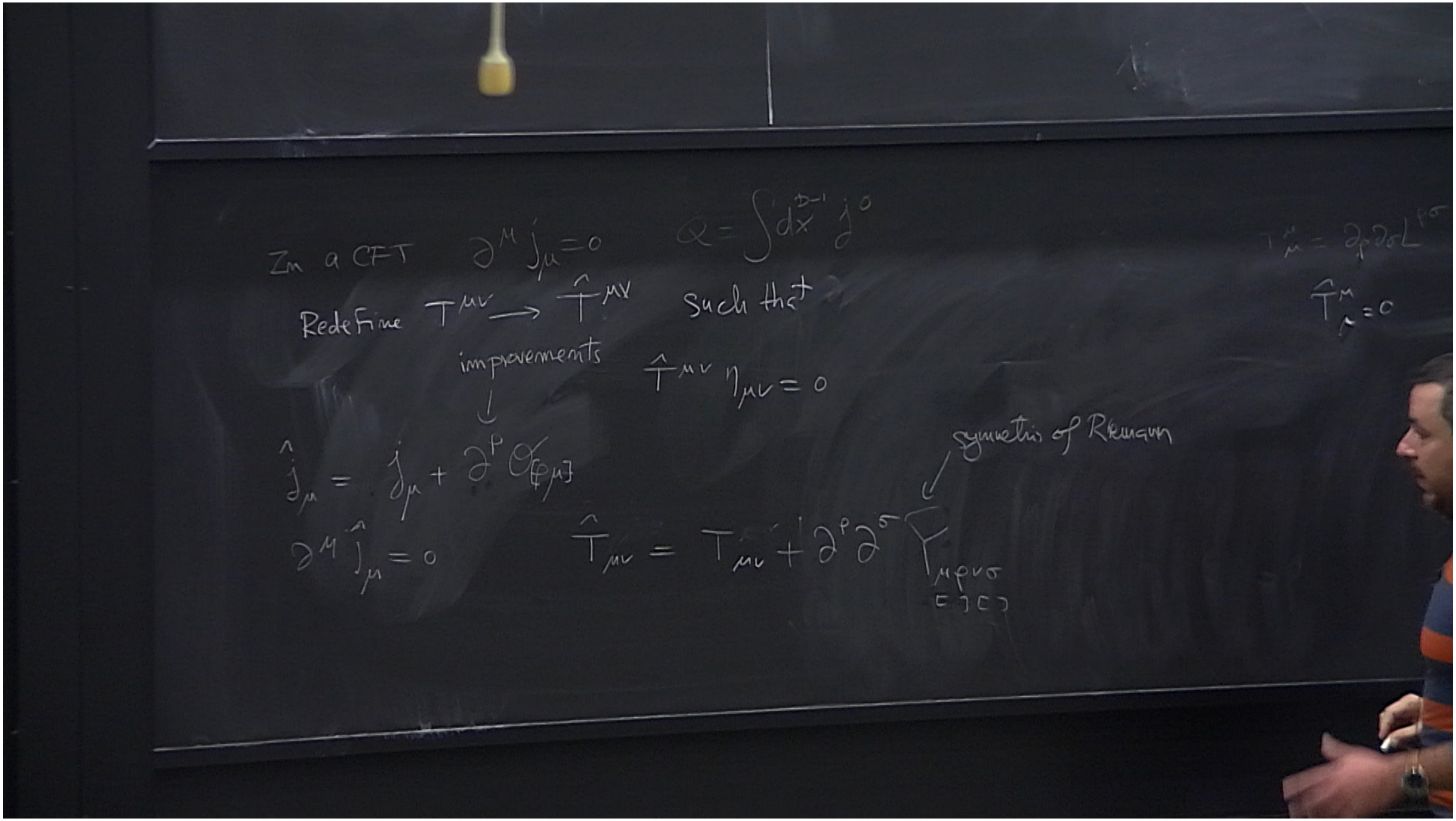
special

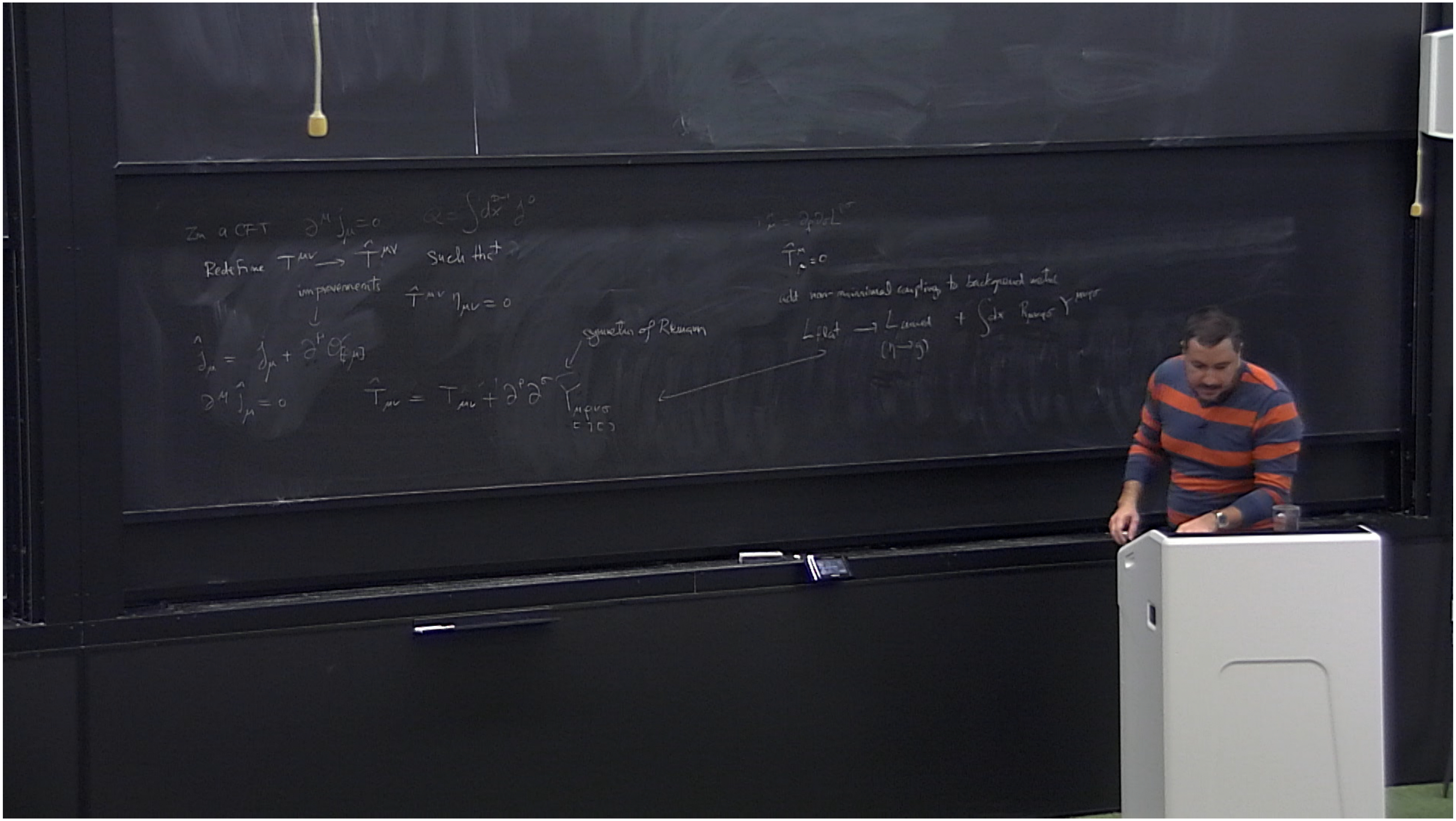
$\boxed{T^{\mu}_{\mu} = \partial_\mu \partial_\nu L^{\mu\nu}}$ $\partial\cdot\xi_K \sim x$

$\partial_\mu = \frac{2}{D}(\partial\cdot\xi)\,\eta_{\mu\nu}$ conformal Killing vector

conformal transformation is a local Lorentz

vector


In a CFT $\partial^\mu j_\mu = 0$ $Q = \int d^{D-1}x\, j^0$
$T^\mu_\mu = \partial_\rho \partial_\sigma L^{\rho\sigma}$
Redefine $T^{\mu\nu} \longrightarrow \hat{T}^{\mu\nu}$ such that
$\hat{T}^\mu_\mu = 0$
improvements
$\hat{T}^{\mu\nu} \eta_{\mu\nu} = 0$
symmetries of Riemann
$\hat{j}_\mu = j_\mu + \partial^\rho \sigma_{[\rho\mu]}$
$\partial^\mu \hat{j}_\mu = 0$
$\hat{T}_{\mu\nu} = T_{\mu\nu} + \partial^\rho \partial^\sigma Y_{\mu\rho\nu\sigma}$
$[\;][\;]$


In a CFT
Redefine
improvements
such that
symmetries of Riemann
add non-minimal coupling to background metric