

Title: PSI 2016/2017 Quantum Field Theory III - Lecture 1

Date: Jan 30, 2017 11:30 AM

URL: <http://pirsa.org/17010088>

Abstract:

QFT III

- 1st week: Jaume Gomis (427)

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- 2nd week: Gang (2d QFTs).
- 3rd week: Daniel (instantons, ...)

why Conformal Field Theories?

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e.g. { - Poincare symmetry

- Galilean symmetry.

$H, P_i, M_{ij}, \cancel{B_i}, Z$

Galilean boosts

particle number conservation

$$[P_i, B_j] = iZ$$

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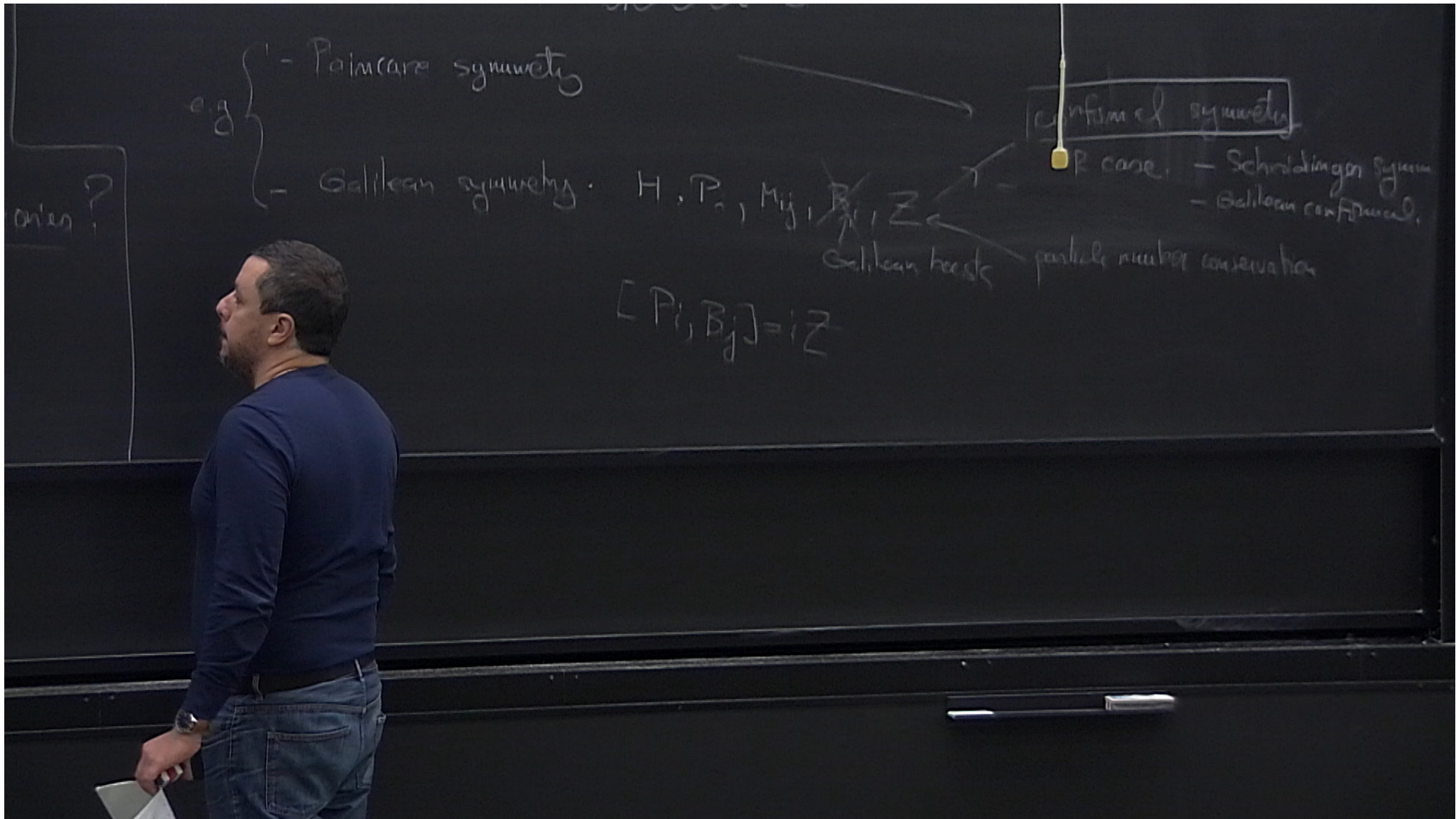
ories?

$H, P_i, M_{ij}, \cancel{B_i}, Z$
Galilean boosts

conformal symmetry
NR case: - Schrödinger Symm.
- Galilean conformal.

particle number conservation

$$[P_i, B_j] = iZ$$



e.g. {
 - Poincare symmetry
 - Galilean symmetry

H, P_i, M_{ij}, B_i, Z
 Galilean boosts

confirm of symmetry

R case:
 - Schrödinger system
 - Galilean invariance

particle number conservation

$$[P_i, B_j] = iZ$$

Scale transformations. (rotationally inv)

$$\vec{x} \rightarrow \lambda \vec{x} \quad \lambda \in \mathbb{R}$$

$$t \rightarrow \lambda^z t \quad z: \text{dynamical critical exponent}$$

- Relativistic: $z=1$

- Non-relativistic

- Schrödinger: $i \frac{\partial \psi}{\partial t} = -\nabla^2 \psi \quad z=2$

- $S = \int dx dt \frac{1}{2} (\partial_t \phi)^2 - (\nabla^2 \phi)^3 \quad z=3$

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A conf. transf.

$x^\mu \rightarrow \tilde{x}^\mu(x)$ that pre

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$$z(\gamma) = (v\phi) \quad z=3$$

QFT's?

large distance / low energy behavior of any QFT

1. Massive theory (topological QFT's)



excitations in the boundary.

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QFT's?

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1. Massive theory (topological QFT's)



← excitations in the boundary.

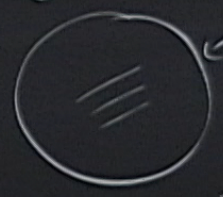
2. Scale invariant. Poincaré' \rightarrow Poincaré' \times R.

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why CFT's?

The large distance / low energy behavior of any QFT

1. Massive theory (topological QFT's)



excitations in the boundary.

2. Scale invariant. Poincaré' \rightarrow Poincaré' \times R

enhanced

conformal symmetry

Under reasonable conditions,

CFT's govern low E behavior of QFT's \Rightarrow Provide an ordering in spacetime

classically conformal invariant:

2. $\partial_\mu F^{\mu\nu} = 0$ Maxwell

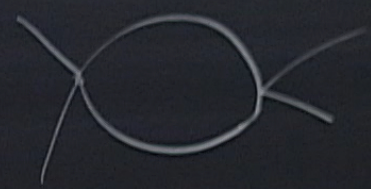
2. $\gamma^{\mu\nu} \partial_\mu \psi = 0$ Dirac

3. $\underline{D=4}$: $D\phi = \lambda \phi^3$ ($\lambda \phi^4$)

$D_\mu F^{\mu\nu} = 0$ (Yang-Mills)

Maxwell
Dirac
 ϕ^3 ($\lambda \phi^4$)
(Yang-Mills)
 ψ
4

Quantum Mechanics



\Rightarrow induce a scale dependence on couplings

$$\beta(\lambda) = \mu \frac{d\lambda}{d\mu} \neq 0$$

Triggers a renormalization group

Flow depends on ultraviolet



Flow depends on which operator triggers it

θ dim Δ

$$\int d^D x \cdot \lambda \theta$$

$$\bar{\lambda} = \lambda \mu^{\Delta-D}$$

$$[\bar{\lambda}] = D - \Delta$$

$\Delta > D$: irrelevant (dies towards IR)

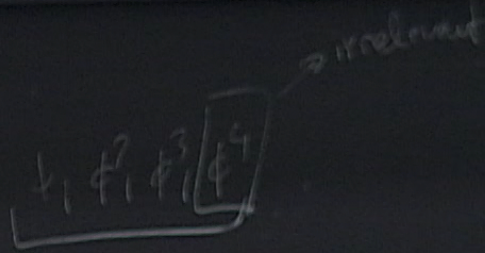
$\Delta < D$: relevant (grows towards IR)

$$\beta(\bar{\lambda}) = \mu \frac{d\bar{\lambda}}{d\mu} = (\Delta - D) \bar{\lambda}$$

$\Delta = D$: marginal

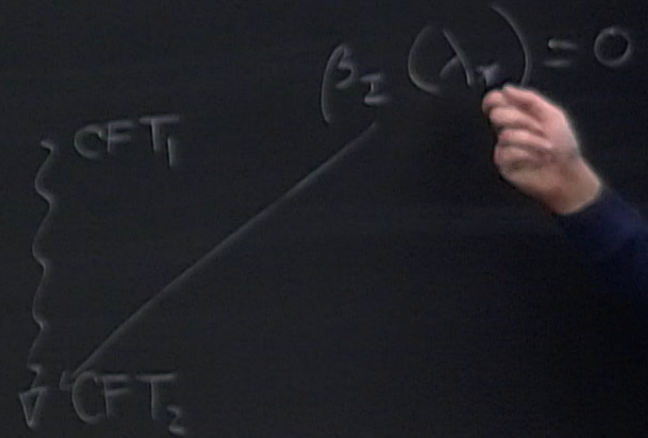
Universality

scalar field in $D=4$.



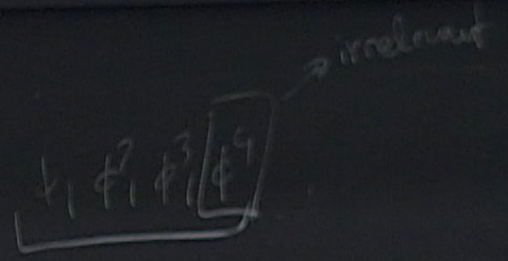
all relevant couplings.

Start w a CFT and perturb by a relevant operator



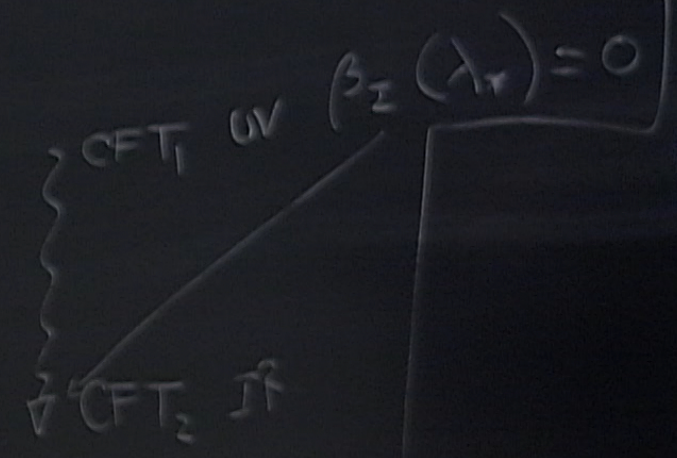
Universality

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CFT's induce an ordering in the space of QFT's.

CFT₁ UV

CFT₂

$D = \text{even}$

$c \sim \text{conformal anomalies}$

\Rightarrow

CFT₂ IR

CFT₁

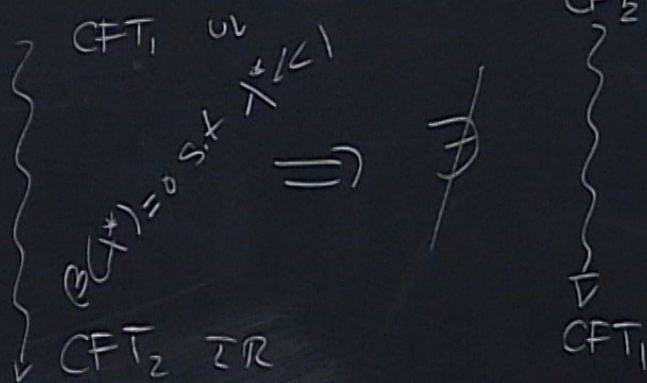
$$\beta_{\mathbb{I}}(\lambda_{\mathbb{I}}) = 0$$

Assign quantity c to a CFT (c)

$$c_{UV} > c_{IR}$$

c : "entanglement entropy" of a spherical region in the CFT

CFT's induce an ordering in the space of QFT's.



$D = \text{even}$

$c \sim \text{conformal anomalies}$

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c : "entanglement entropy" of a spherical region in the CFT

$$z(\lambda) = (\sqrt{\lambda}) \quad z=3$$

"perturbative" RG flows

- Wilson-Fischer fixed point / CFTs (scalars)
- Banks-Zaks CFT (Yang-Mills + quarks)

$$L = \frac{1}{2} (\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4$$

D=4



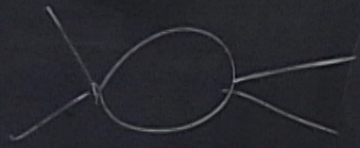
$$\beta(\lambda) = -\lambda^2$$

$$\beta(\lambda) =$$

$\mathcal{L}(\psi) = (\psi^\dagger \not{\partial} \psi) - \frac{1}{2} (\partial_\mu \phi)^2 - \lambda \phi^4$

FTs (scalars)
- Mills + quarks

$$\beta(\lambda) = \underbrace{\quad}_{\text{tree-level}} + \underbrace{+ |\lambda|^2}_{\text{1-loop}}$$



$$\beta(\lambda) = \lambda^2$$

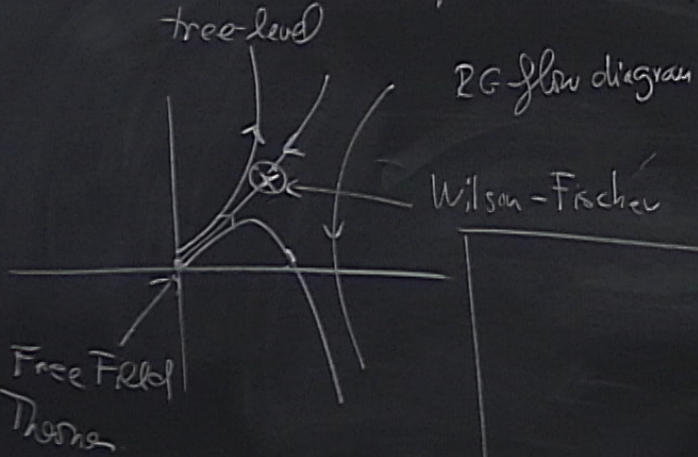
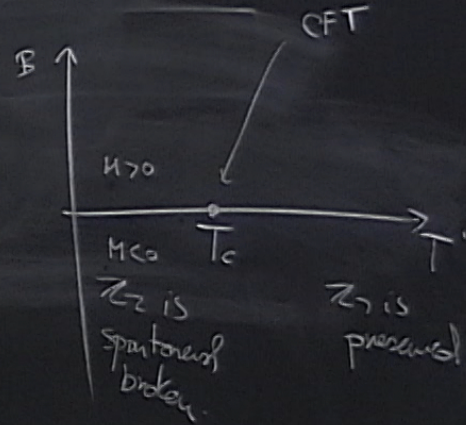
$z=3$

$$\beta(\lambda) = \underbrace{-\epsilon\lambda}_{\text{tree-level}} + \underbrace{|\Lambda|\lambda^2}_{\text{1-loop}} = 0 \quad \beta(\lambda^*) = 0$$

$$\lambda^* = \frac{\epsilon}{|\Lambda|}$$

$D=3$ Ising model

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \quad Z_2 \text{ symmetry}$$



$$\beta(\lambda) = \lambda^2$$

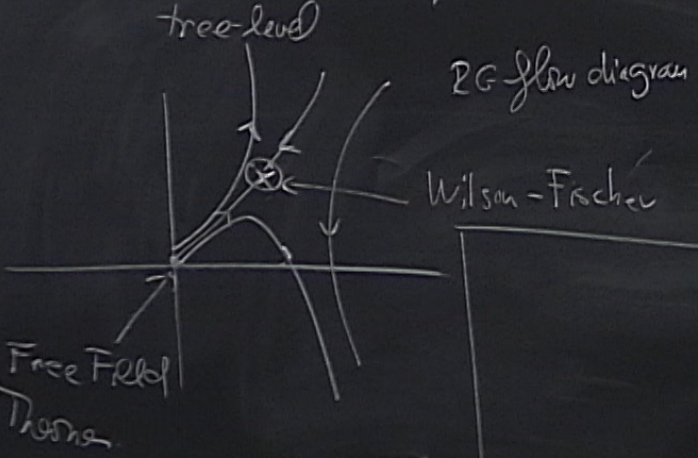
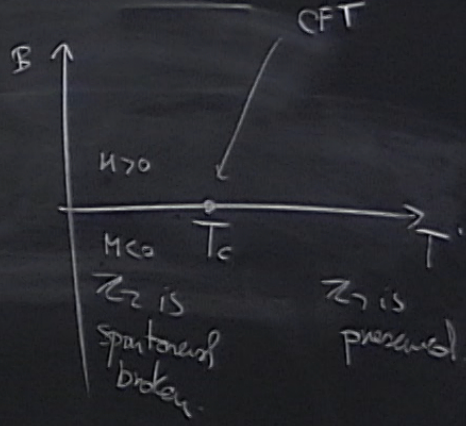
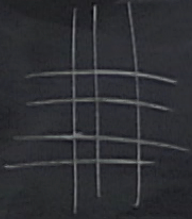
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$$\beta(\lambda) = \lambda^2$$

CFT's are directly relevant for nature

2. Critical phenomena $G_c \rightarrow$ critical points where $\xi \rightarrow \infty$

$$\langle \sigma(x) \sigma(0) \rangle_c \sim e^{-\frac{|x|}{\xi}} \quad T \rightarrow T_c$$

Defines critical exponents:

$$\xi \sim (T - T_c)^{-\nu}$$

$$M \sim (T_c - T)^{\beta}$$

$$M \sim B^{\frac{\beta}{\delta}}$$

$$C \sim (T - T_c)^{-\gamma}$$

$$\chi \sim (T - T_c)^{-\alpha}$$

$$\langle \sigma(x) \sigma(0) \rangle_c = \frac{1}{|x|^{2-\alpha}}$$

nature

critical points where $\xi \rightarrow \infty$
 $T \rightarrow T_c$

$\sim (T-T_c)^{-\nu}$
 $\sim (T_c-T)^{\beta}$
 $\sim B^{-\alpha}$
 $\sim (T-T_c)^{-\delta}$
 $\sim (T-T_c)$

$$\langle \sigma(x) \sigma(y) \rangle_c = \frac{1}{x^{\Delta-2+\eta}}$$

CFT's in 2d \leftrightarrow string theory
 $\beta = 0 \Rightarrow$

- graviton	}	→ Einstein
- gluon		
- quark		

YM
Dirac

Scale symmetry

all determined in terms of 2 critical exponents
2. Quantum phase transitions ($T=0$)

