Title: PSI 2016/2017 Quantum Field Theory III - Lecture 1

Date: Jan 30, 2017 11:30 AM

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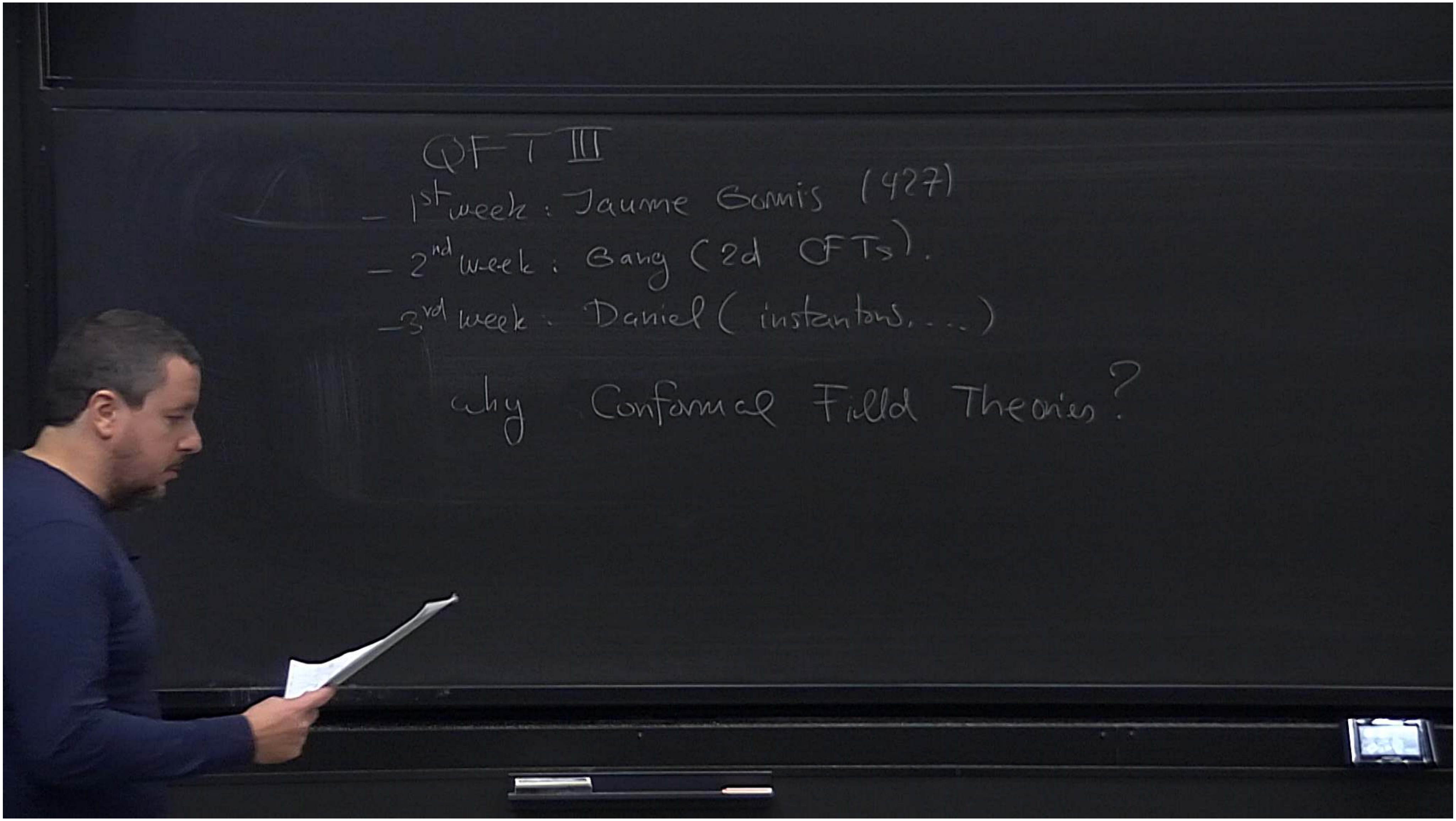
Abstract:

QFT III

– 1st week: Jaume Gomis (427)


QFT III
- 1st week: Jaume Gomis (427)
- 2nd week: Gang (2d CFTs).
- 3rd week: Daniel (instantons, ...)
why Conformal Field Theories?

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why Conformal Field Theories?

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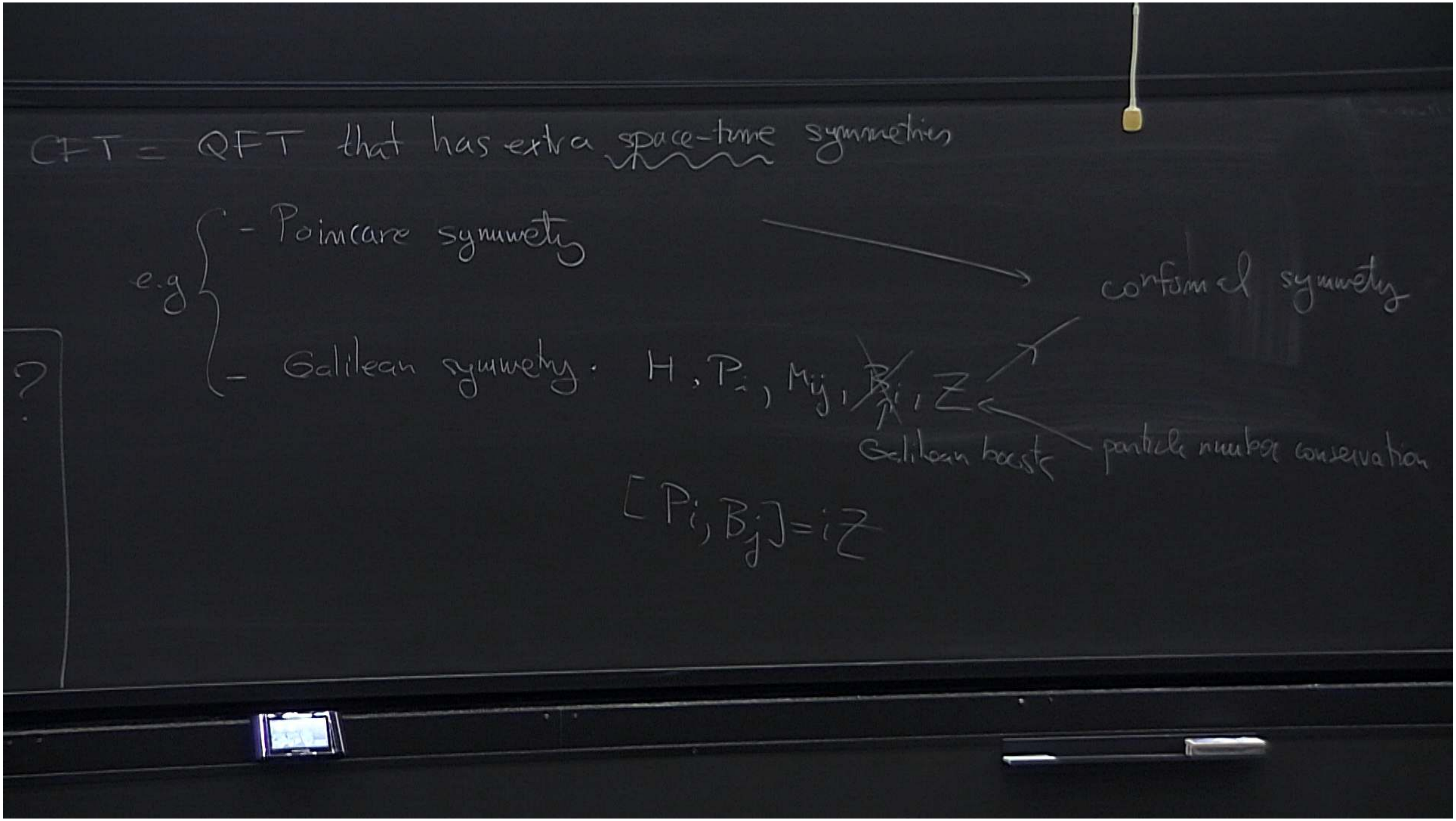
e.g.

- Poincare symmetry
- Galilean symmetry : $H, P_i, M_{ij}, \cancel{B_i}, Z$ particle number conservation

Galilean boosts

$$[P_i, B_j] = iZ$$


?
CFT = QFT that has extra space-time symmetries
e.g.
- Poincare symmetry
conformal symmetry
- Galilean symmetry: H, P_i, M_{ij}, $\cancel{B_i}$, Z
Galilean boosts
particle number conservation
$[P_i, B_j] = iZ$

…ories?

CFT = QFT that has extra space-time symmetries

e.g.
- Poincare symmetry → conformal symmetry
- Galilean symmetry. $H, P_i, M_{ij}, \cancel{B_i}, Z$

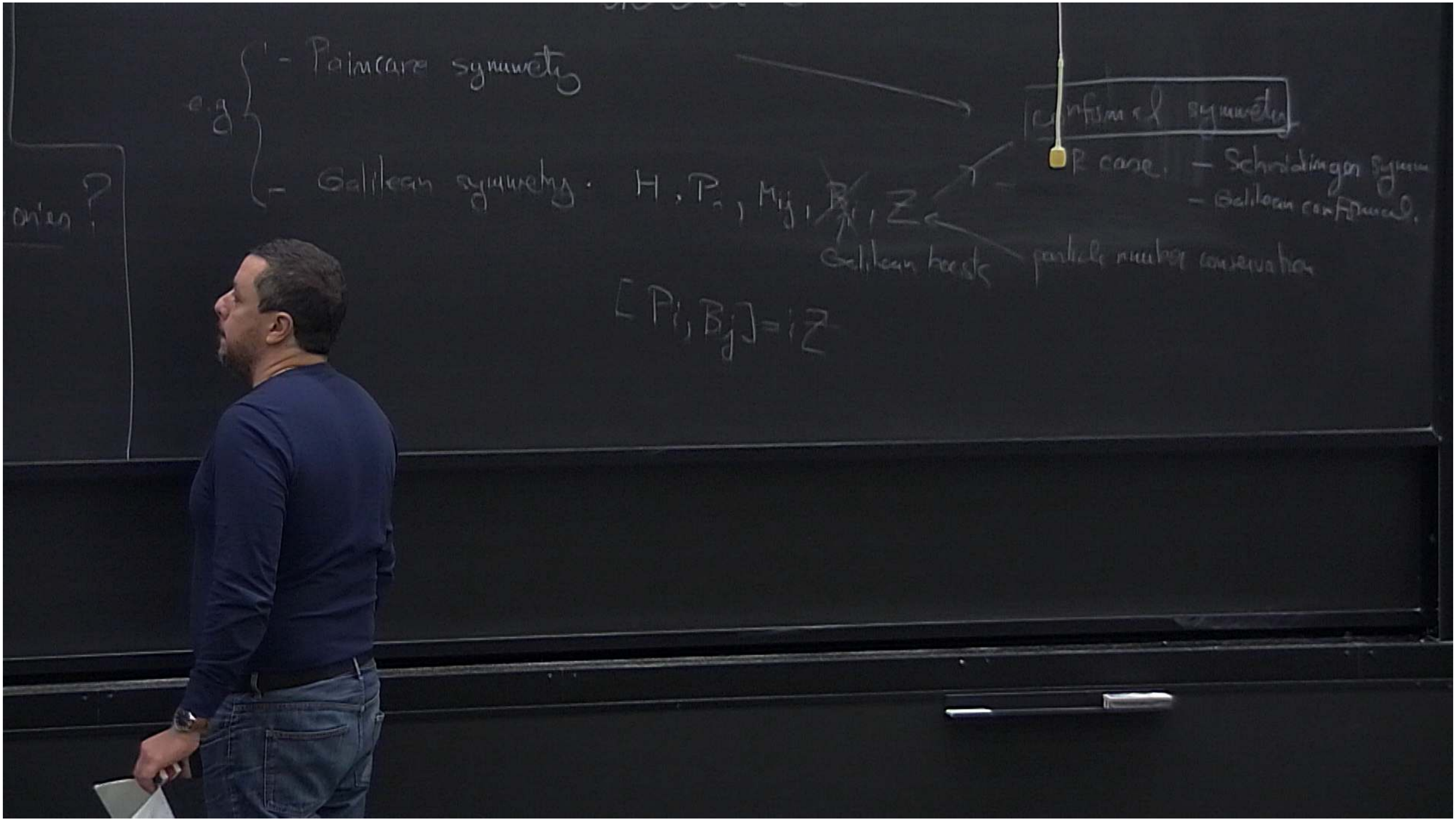
Galilean boosts

particle number conservation

NR case: – Schrödinger symm

– Galilean conformal.

$$[P_i, B_j] = iZ$$



e.g.
- Poincare symmetry
- Galilean symmetry. H, P, M, B, Z
Galilean boosts
particle number conservation
conformal symmetry
NR case. - Schrödinger symm
- Galilean conformal.
[P, B] = iZ

Scale transformations. (rotationally inv)

$$\vec{x} \to \lambda \vec{x} \qquad \lambda \in \mathbb{R}$$

$$t \to \lambda^{z} t \qquad z: \text{dynamical critical exponent}$$

- Relativistic: $z = 1$
- Non-relativistic
  - Schrödinger: $i \frac{\partial}{\partial t} \psi = -\nabla^2 \psi$ $\quad z=2$
  - $S = \int dx\, dt \ \frac{1}{2}(\partial_t \phi)^2 - (\nabla^2 \phi)^3$ $\quad z=3$

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A conf. transf.: $x^\mu \to \tilde{x}^\mu(x)$ that pre

$z=3.$

QFT's?

large distance / low energy behavior of any QFT

1. Massive theory (topological QFT's)

excitations in the boundary.

$z=3.$

FT's?

large distance / low energy behavior of any QFT

1. Massive theory (topological QFT's)

excitations in the boundary.

2. Scale invariant. Poincaré $\longrightarrow$ Poincaré $\rtimes R$.

$z=3$

why CFT's?

The large distance / low energy behavior of any QFT

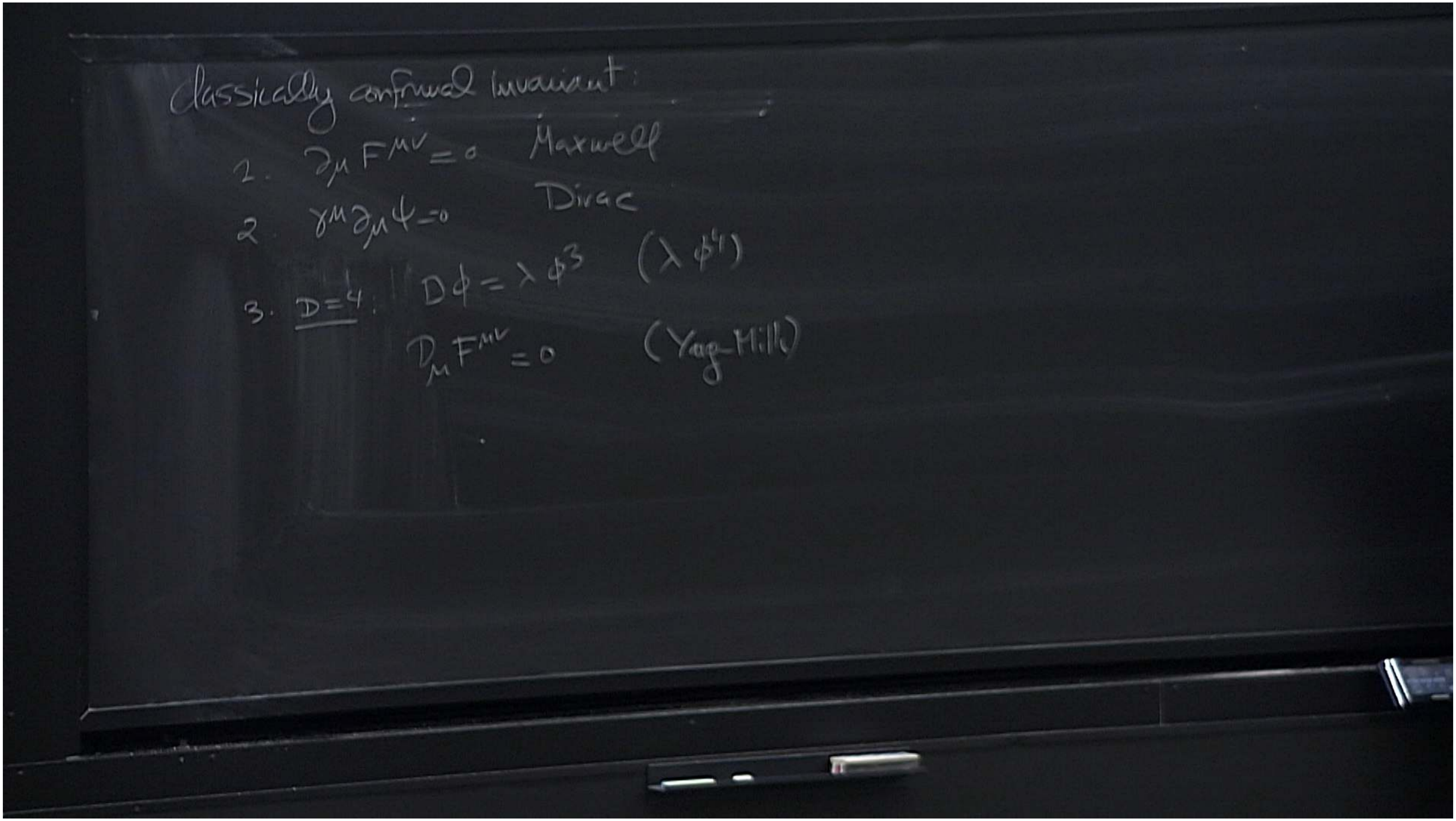
1. Massive theory (topological QFT's)

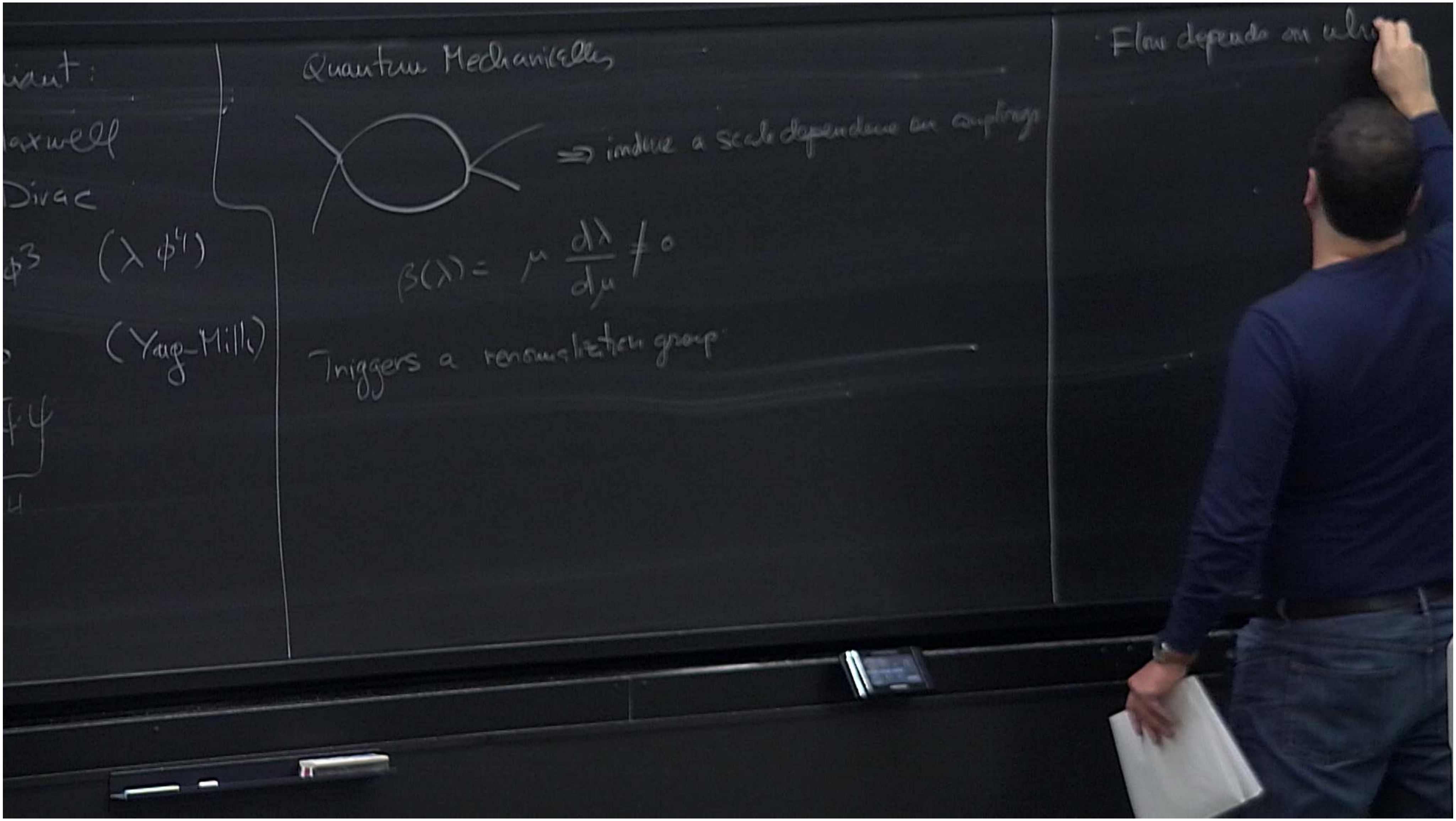
excitations in the boundary.

2. Scale invariant. Poincaré $\longrightarrow$ Poincaré $\rtimes R \xrightarrow{\text{enhanced}}$ conformal symmetry

Under reasonable conditions;

CFT's govern low E behavior of QFT's $\Longrightarrow$ Provide an ordering in spac


classically conformal invariant:
1. $\partial_\mu F^{\mu\nu}=0$ Maxwell
2. $\gamma^\mu\partial_\mu\psi=0$ Dirac
3. D=4: $\Box\phi=\lambda\phi^3$ $(\lambda\phi^4)$
$D_\mu F^{\mu\nu}=0$ (Yang-Mills)


axwell
Dirac
$(\lambda\phi^4)$
(Yang-Mills)
Quantum Mechanically
$\Rightarrow$ induce a scale dependence on couplings
$\beta(\lambda) = \mu \frac{d\lambda}{d\mu} \neq 0$
Triggers a renormalization group
Flow depends on

...pendence on couplings

· Flow depends on which operator triggers it

$\mathcal{O}$ dim Δ

$$\int d^Dx \cdot \lambda \, \mathcal{O} \qquad [\lambda] = D - \Delta$$

$$\bar{\lambda} = \lambda \mu^{\Delta - D}$$

$$\beta(\bar{\lambda}) = \mu \frac{d}{d\mu} \bar{\lambda} = (\Delta - D) \bar{\lambda}$$

- $\Delta > D$: irrelevant (dies towards IR)
- $\underline{\Delta < D}$: relevant (grows towards IR)
- $\Delta = D$: marginal

Universality

$\to$ irrelevant

scalar field in $D=4$. $\phi, \phi^2, \phi^3, \boxed{\phi^4}$

all relevant couplings.

Start w/ a CFT and perturb by a relevant operator

$$\beta_I(\lambda_I) = 0$$

CFT_1

CFT_2

Universality.

scalar field in $D=4$: $\underbrace{\phi,\ \phi^2,\ \phi^3,\ \boxed{\phi^4}}$ → irrelevant

all relevant couplings.

start w/ a CFT and perturb by a relevant operator

CFT_1 UV $\quad \beta_I(\lambda_*) = 0$

CFT_2 IR

CFT's induce an ordering in the space of QFT's.

$D = \text{even}$

$c \sim$ conformal anomalies

$$\begin{array}{ccc} \text{CFT}_1 \ \text{UV} & & \text{CFT}_2 \\ \Big\downarrow & \Longrightarrow \ \nexists & \Big\downarrow \\ \text{CFT}_2 \ \text{IR} & & \text{CFT}_1 \end{array}$$

$\beta_I(\lambda_*) = 0$

Assign quantity c to a CFT (c)

$c_{UV} > c_{IR}$

c: "entanglement entropy" of a spherical region in the CFT

CFT's induce an ordering in the space of QFT's.

D = even

$c \sim$ conformal anomalies

$\beta_I(\lambda_*) = 0$

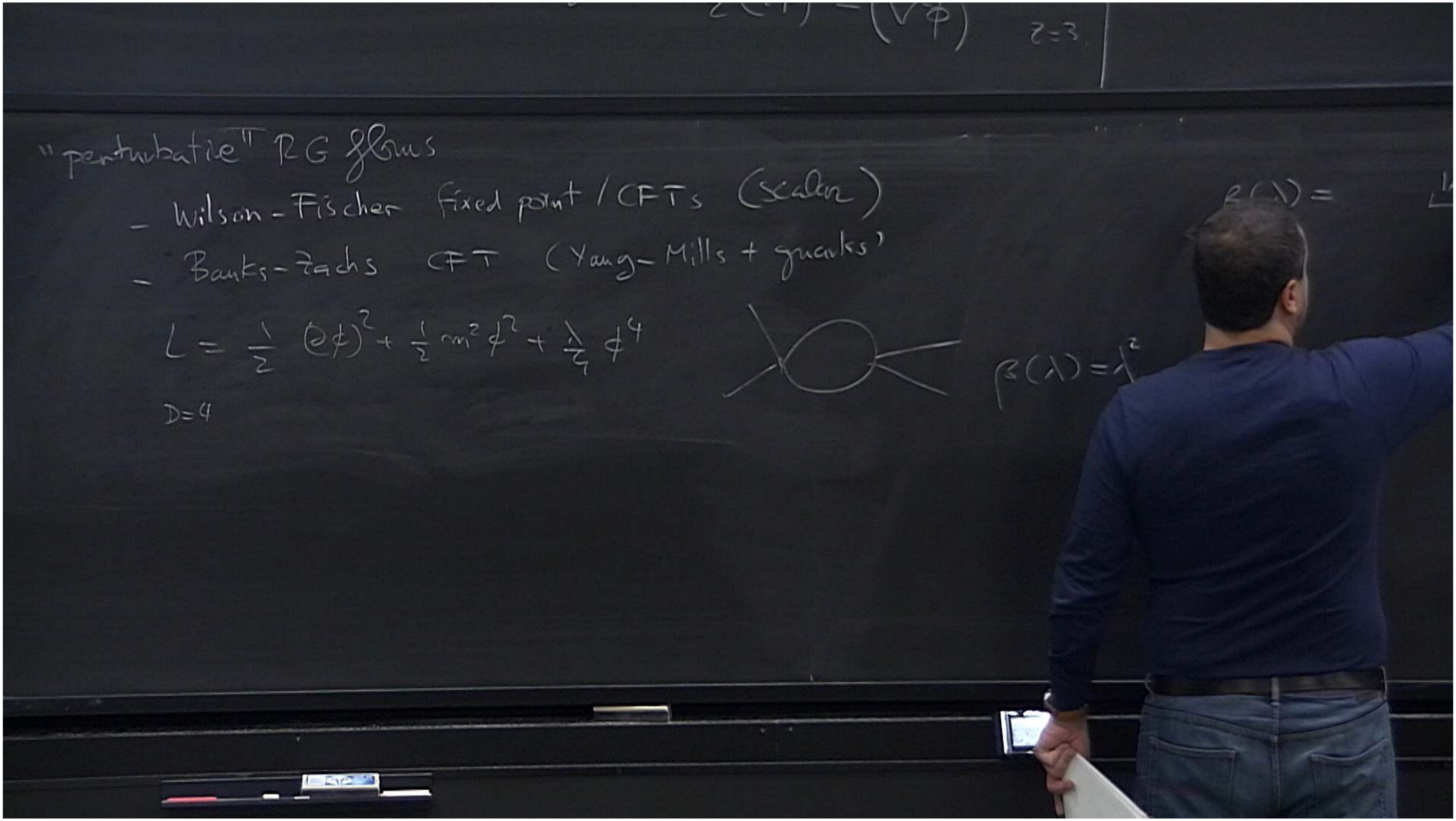
$$\mathrm{CFT}_1 \ \mathrm{UV} \quad \beta(\lambda^*) = 0 \ \text{s.t.} \ \lambda^* \ll 1 \quad \mathrm{CFT}_2 \ \mathrm{IR}$$

$$\Longrightarrow \quad \nexists \quad \mathrm{CFT}_2 \rightsquigarrow \mathrm{CFT}_1$$

Assign quantity c to a CFT (c)

$$c_{UV} > c_{IR}$$

c: "entanglement entropy" of a spherical region in the CFT



$z=3$
"perturbatie" RG flows
- Wilson-Fischer fixed point / CFTs (scalar)
- Banks-Zachs CFT (Yang-Mills + quarks)
$L = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$
$D=4$
$\beta(\lambda) =$
$\beta(\lambda) = \lambda^2$

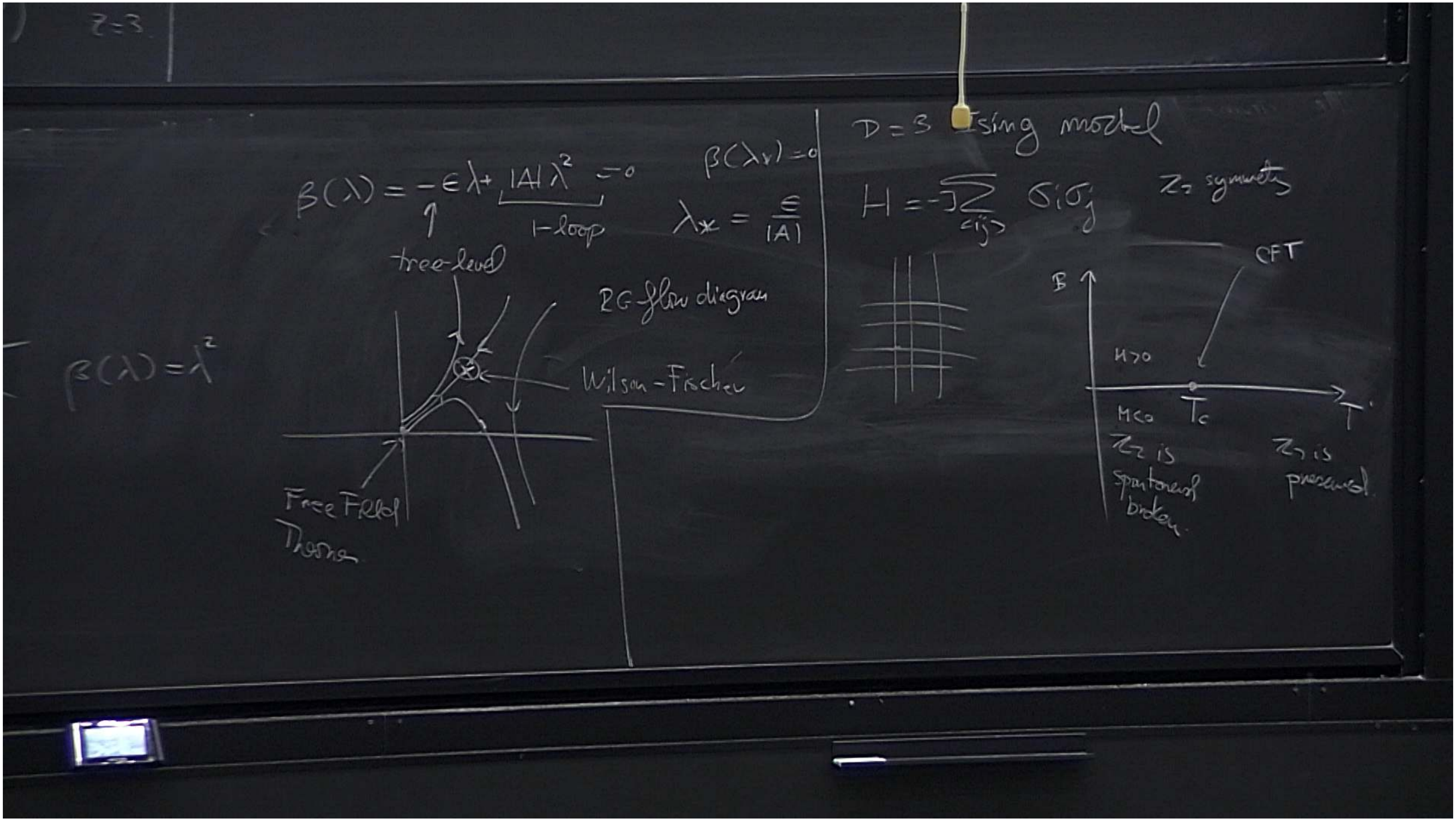
$(\nabla\phi)$ $z=3.$

FTs (scalar)

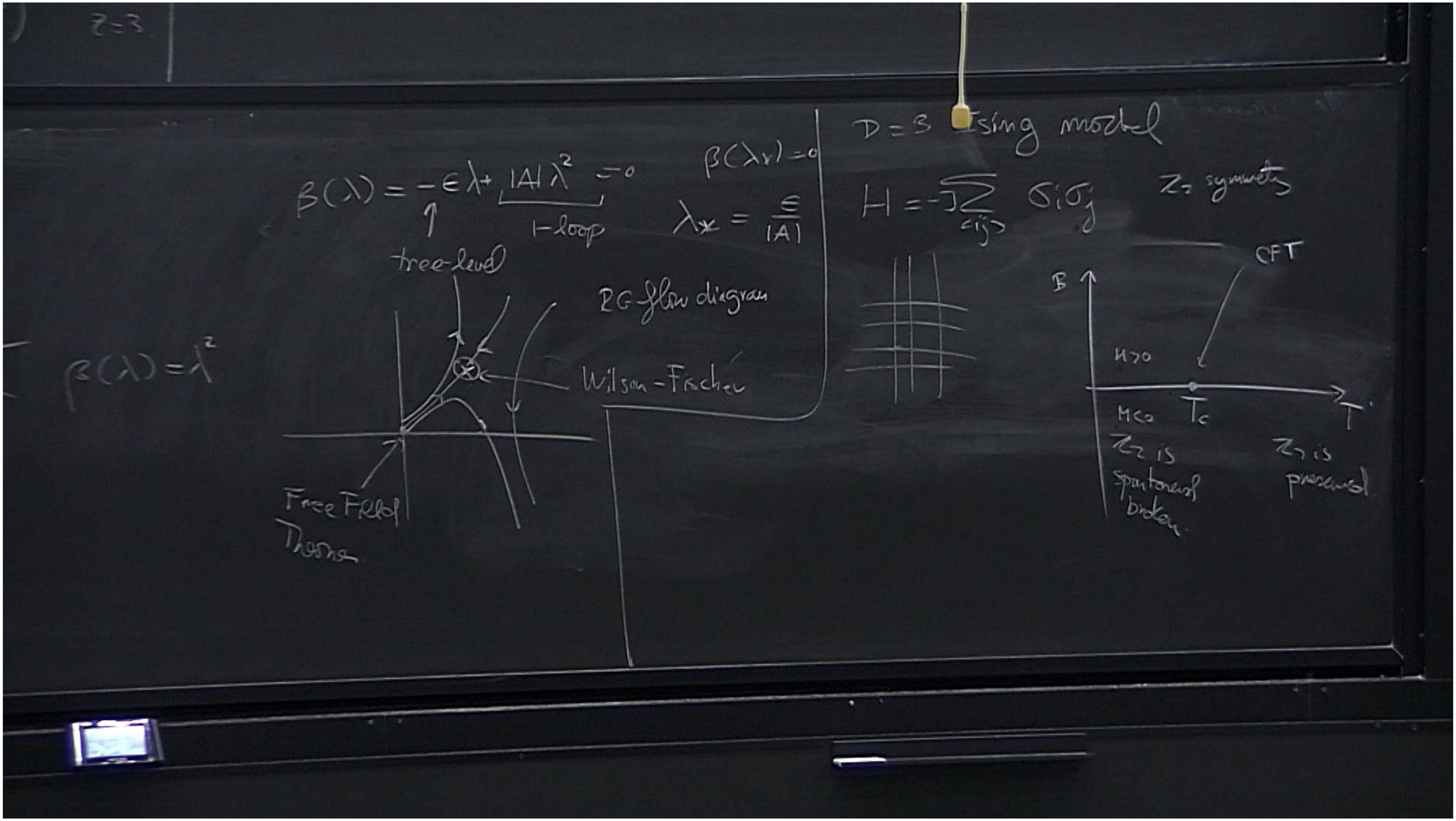
-Mills + quarks'

$$\beta(\lambda) = \underset{\substack{\uparrow \\ \text{tree-level}}}{} + \underbrace{|A|\lambda^2}_{\text{1-loop}}$$

$$\beta(\lambda) = \lambda^2$$



z=3
β(λ) = −ελ + |A|λ² = 0
tree-level
1-loop
β(λ*) = 0
λ* = ε/|A|
RG flow diagram
Wilson-Fischer
Free Field Theory
β(λ) = λ²
D = 3 Ising model
H = −J Σ<ij> σi σj
Z2 symmetry
CFT
B
M>0
M<0
Tc
T
Z2 is spontaneously broken
Z2 is preserved


z=3
β(λ)=λ²
β(λ) = −ελ + |A|λ² = 0
tree-level
1-loop
β(λ*) = 0
λ* = ε/|A|
RG flow diagram
Wilson-Fischer
Free Field Theory
D = 3 Ising model
H = −J Σ σᵢσⱼ
<ij>
Z₂ symmetry
CFT
B
M>0
M<0
T_c
T
Z₂ is spontaneously broken
Z₂ is preserved

CFT's are directly relevant for nature

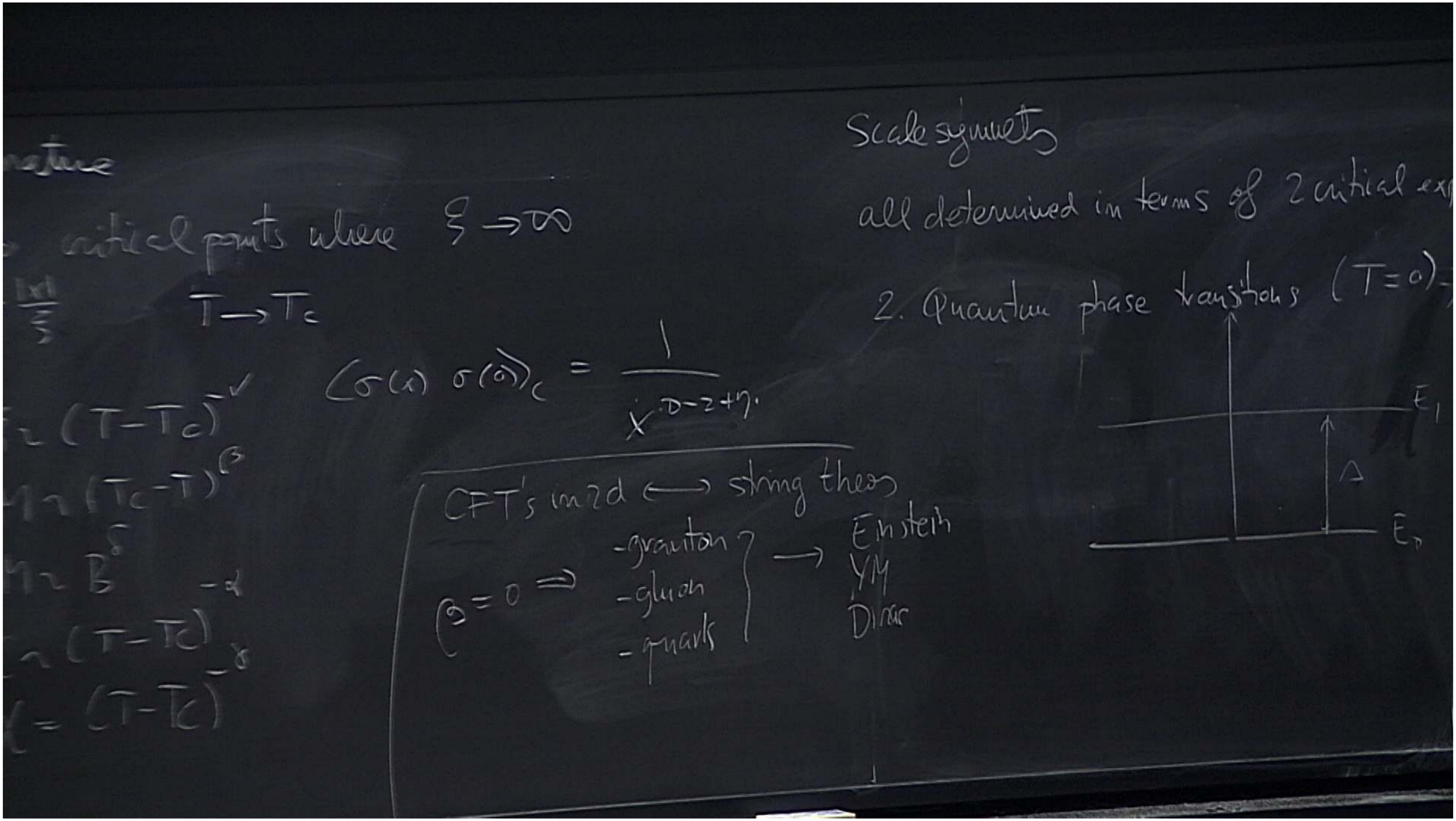
2. Critical phenomena   Go to critical points where $\xi \to \infty$

$$\langle \sigma(x)\, \sigma(0) \rangle_c \sim e^{-\frac{|x|}{\xi}} \qquad T \to T_c$$

$$\langle \sigma(x)\, \sigma(0) \rangle_c = \frac{1}{x^{D-2+\eta}}$$

Defines critical exponents:

$$\xi \sim (T - T_c)^{-\nu}$$
$$M \sim (T_c - T)^{\beta}$$
$$M \sim B^{\delta}$$
$$C \sim (T - T_c)^{-\alpha}$$
$$\chi \sim (T - T_c)^{-\gamma}$$



Scale symmetry
all determined in terms of 2 critical exp
critical points where ξ → ∞
T → Tc
2. Quantum phase transitions (T=0)
⟨σ(x) σ(0)⟩c = 1 / x^(D−2+η)
(T−Tc)^−ν
(Tc−T)^β
B^δ
(T−Tc)^−α
(T−Tc)^−γ
E1
Δ
E0
CFT's in 2d ⟷ string theory
β = 0 ⟹ −graviton −gluon −quarks → Einstein YM Dirac