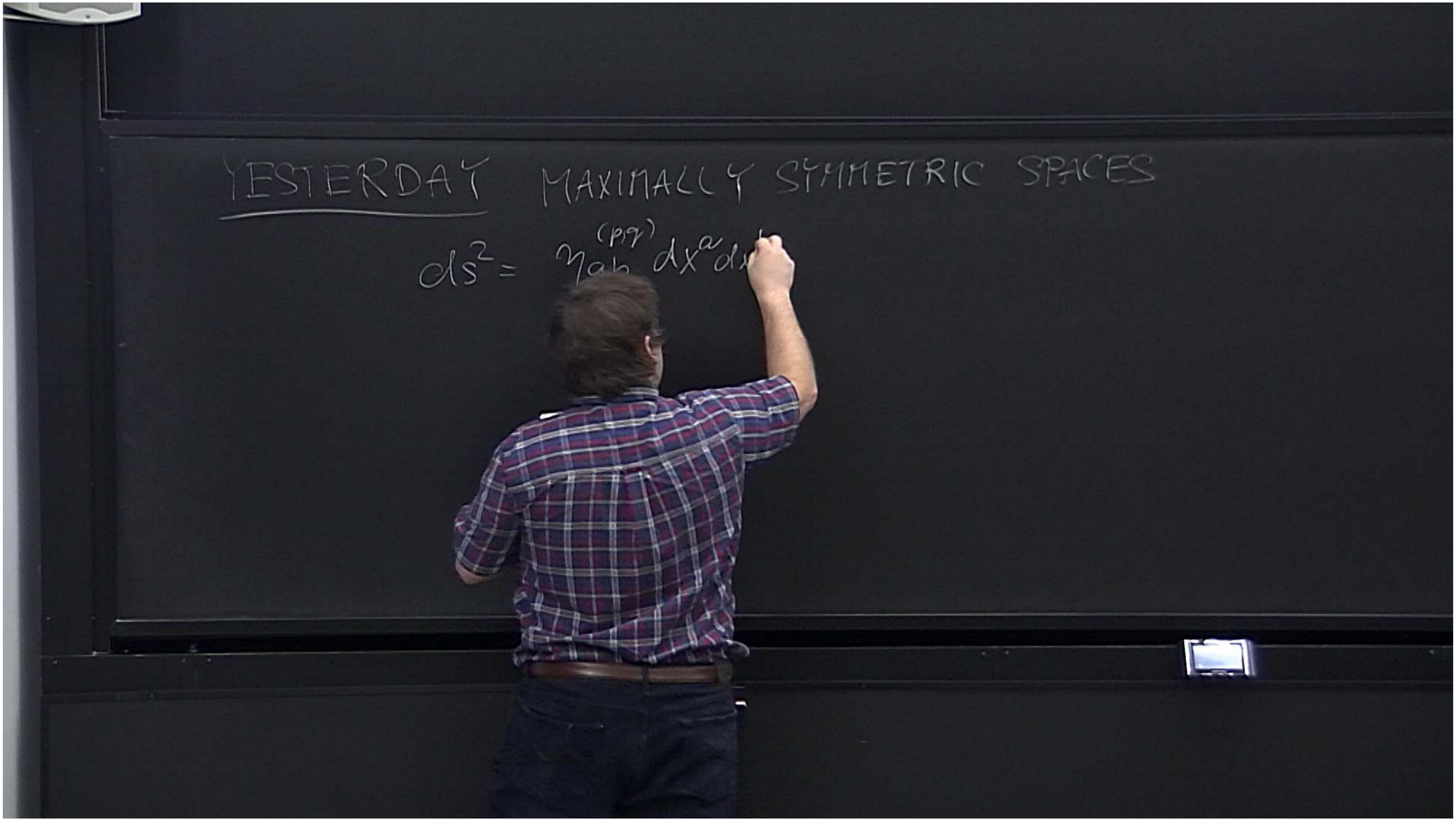
Title: PSI 2016/2017 Cosmology (Review) - Lecture 2

Date: Jan 31, 2017 10:15 AM

URL: http://pirsa.org/17010087

Abstract:


YESTERDAY MAXIMALLY SYMMETRIC SPACES
$ds^2 = \eta^{(p,q)}_{ab}\, dx^a\, dx$

YESTERDAY MAXIMALLY SYMMETRIC SPACES

$$\boxed{\begin{aligned} ds^2 &= \eta^{(p,q)}_{ab}\, dx^a dx^b + K\,(dx^{n+1})^2 = \eta^{(p,q|K}_{AB}\, dx^A dx^B \\ K\rho^2 &= \eta^{(p,q)K}_{AB}\, X^A X^B \end{aligned}}$$

YESTERDAY MAXIMALLY SYMMETRIC SPACES

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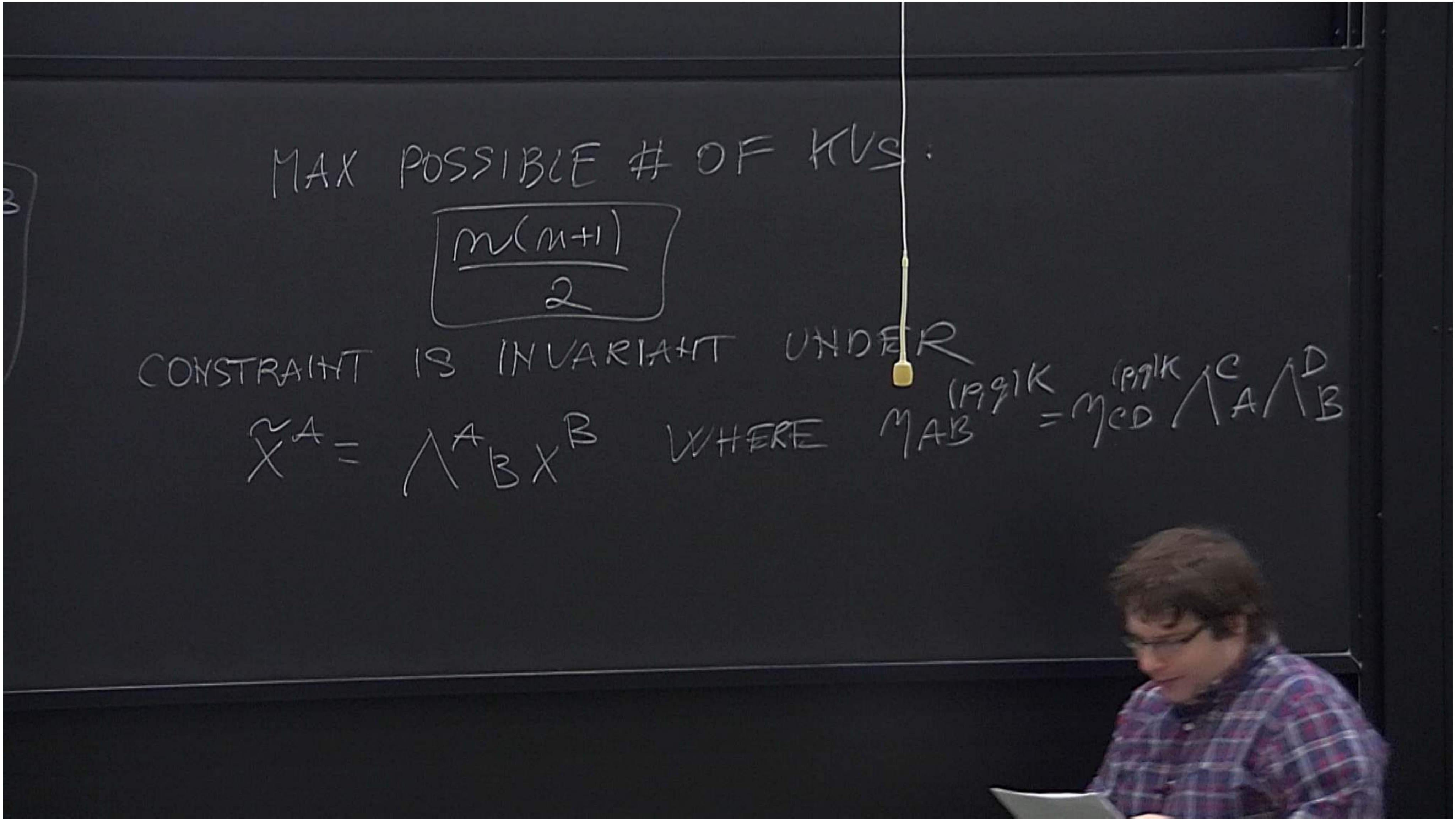
$$\Longrightarrow \quad \boxed{g^{(p,q|K)}_{ab} = \eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2}}$$



MAX POSSIBLE # OF KVs:
m(m+1)/2
CONSTRAINT IS INVARIANT UNDER
X̃^A = Λ^A_B X^B WHERE η_AB^(p,q)K = η_CD^(p,q)K Λ^C_A Λ^D_B

CES

$^{(p,q)|k}$
$_{AB}\, dx^A dx^B$

INT

MAX POSSIBLE # OF KVs:

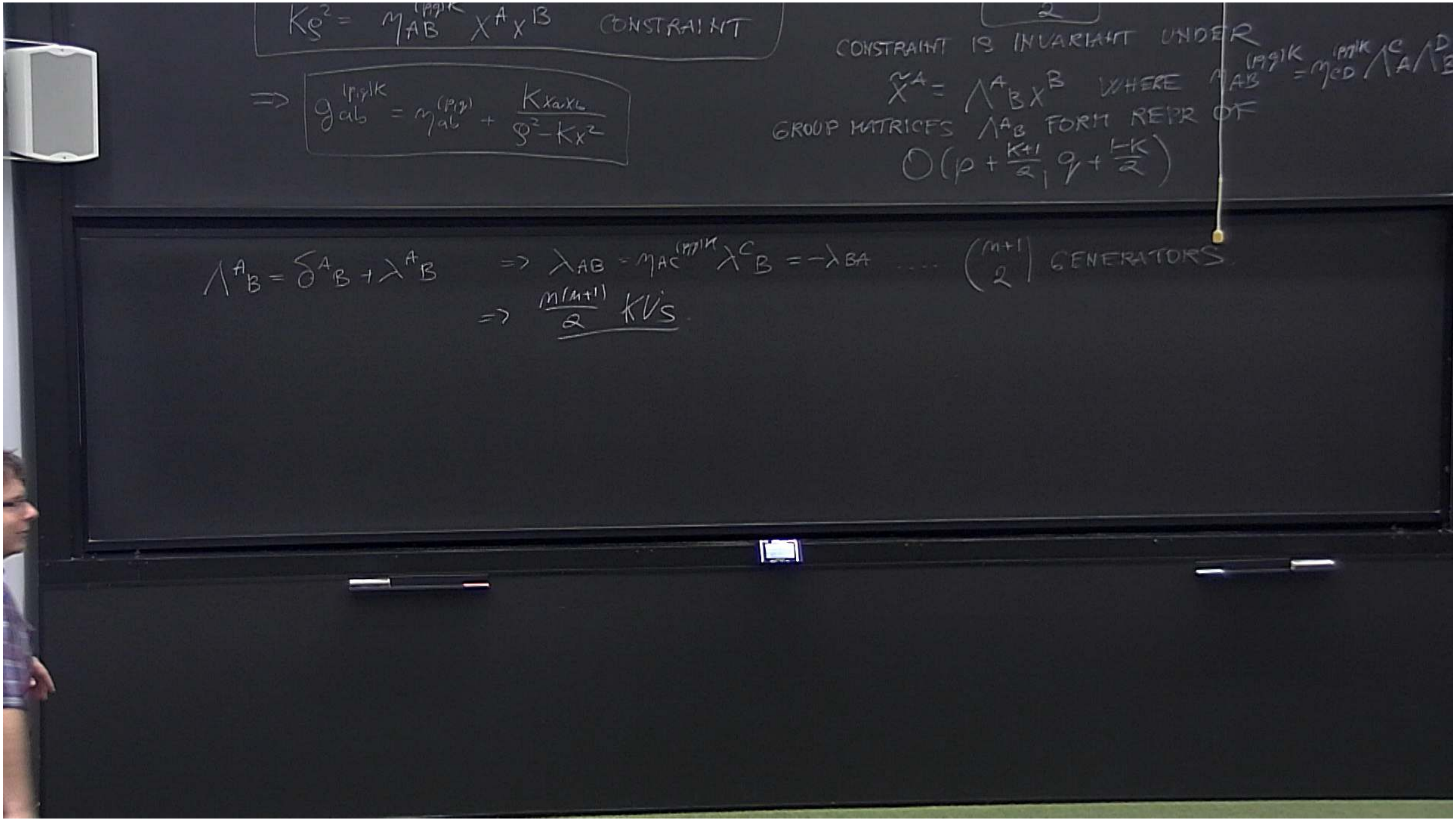
$$\boxed{\frac{n(n+1)}{2}}$$

CONSTRAINT IS INVARIANT UNDER

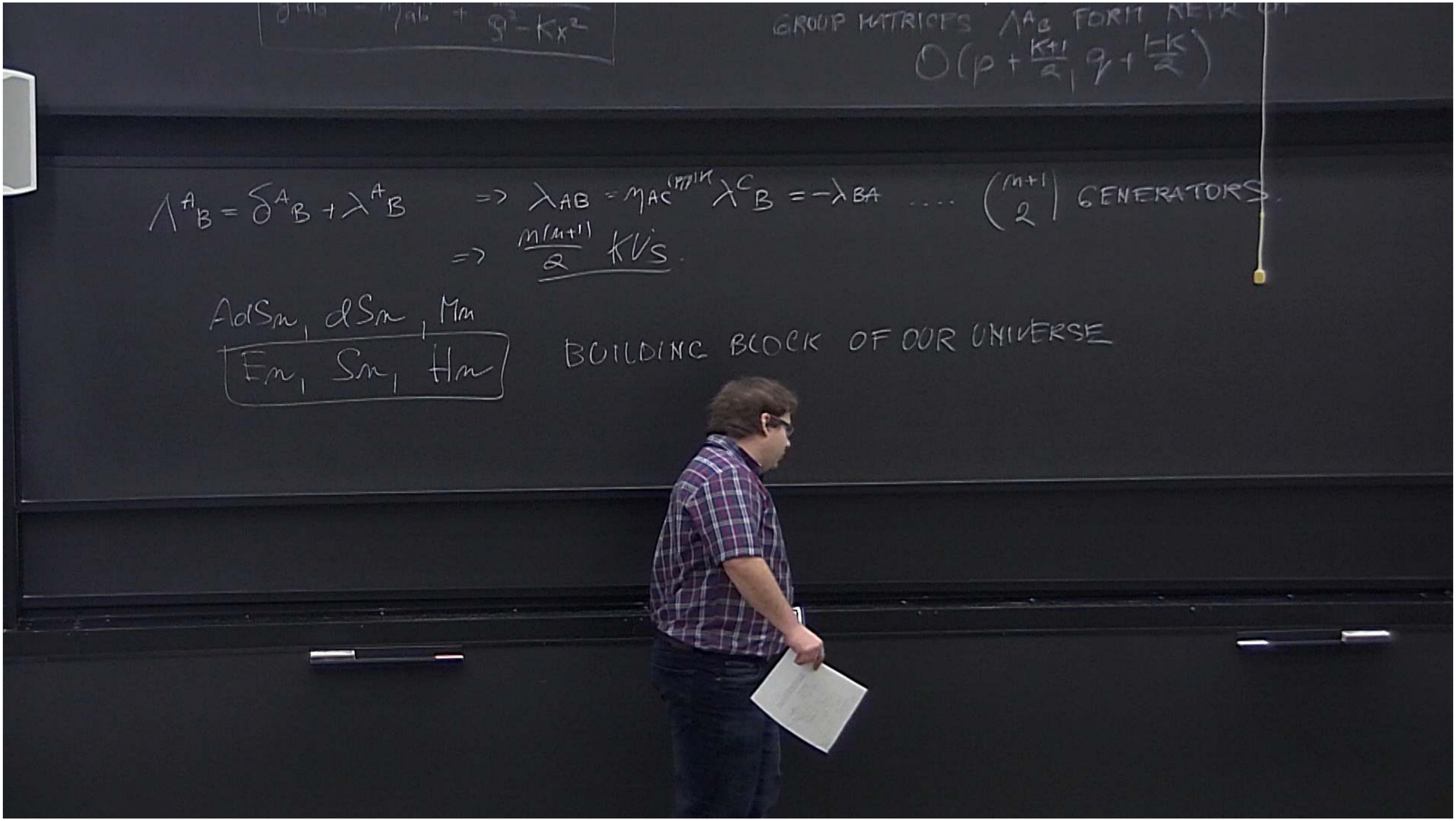
$\tilde{X}^A = \Lambda^A{}_B X^B$ WHERE $\eta_{AB}^{(p,q)|k} = \eta_{CD}^{(p,q)|k} \Lambda^C{}_A \Lambda^D{}_B$

GROUP MATRICES $\Lambda^A{}_B$ FORM REPR OF

$$O\left(p + \frac{k+1}{2},\, q + \frac{1-k}{2}\right)$$



$K\rho^2 = \eta^{(p,q)K}_{AB} X^A X^B$ CONSTRAINT
$\Rightarrow g^{(p,q)K}_{ab} = \eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2}$
CONSTRAINT IS INVARIANT UNDER
$\tilde{X}^A = \Lambda^A{}_B X^B$ WHERE $\eta^{(p,q)K}_{AB} = \eta^{(p,q)K}_{CD} \Lambda^C{}_A \Lambda^D{}_B$
GROUP MATRICES $\Lambda^A{}_B$ FORM REPR OF
$O(p + \frac{K+1}{2}, q + \frac{1-K}{2})$
$\Lambda^A{}_B = \delta^A{}_B + \lambda^A{}_B$
$\Rightarrow \lambda_{AB} = \eta^{(p,q)K}_{AC} \lambda^C{}_B = -\lambda_{BA}$ .... $\binom{n+1}{2}$ GENERATORS
$\Rightarrow \frac{n(n+1)}{2}$ KVs


GROUP MATRICES $\Lambda^A{}_B$ FORM
$O(p + \frac{K+1}{2}, q + \frac{1-K}{2})$
$\Lambda^A{}_B = \delta^A{}_B + \lambda^A{}_B \Rightarrow \lambda_{AB} = \eta_{AC}{}^{(m)(K)} \lambda^C{}_B = -\lambda_{BA} \ldots \binom{m+1}{2}$ GENERATORS.
$\Rightarrow \frac{m(m+1)}{2}$ KVs.
AdSm, dSm, Mm
Em, Sm, Hm
BUILDING BLOCK OF OUR UNIVERSE

$\boxed{E_m,\ S_m,\ H_m}$ BUILDING BLOCK OF OUR ...

b) FRW SPACETIME

OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE
BUT SPATIAL GEOMETRY CAN "ARBITRARY" STRETCH/SHRINK
WITH TIME. $a(t)$. SCALE FACTOR

· CAN SET $\varrho = 1$

$$\boxed{\begin{aligned} ds^2 &= -dt^2 + a^2(t)\, g^{(3,0)K}_{ij}\, dx^i dx^j \\ &= -dt^2 + a^2(t)\left(\delta_{ij} + \frac{K x_i x_j}{\varrho^2 - K x^2}\right) dx^i dx^j \end{aligned}}$$

Hm

BUILDING BLOCK OF OUR UNIVERSE

E IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE

GEOMETRY CAN "ARBITRARY" STRETCH/SHRINK

$a(t)$ .. SCALE FACTOR

$$+a^2(t)\, g_{ij}^{(3,0)K}\, dx^i dx^j$$

$$+a^2(t)\left(\delta_{ij} + \frac{K x_i x_j}{\rho^2 - Kx^2}\right) dx^i dx^j$$

- CAN SET $\rho = 1$: $x^i = \rho \tilde{x}^i$, $a = \tilde{a}/\rho$
- $K=0$ $\quad E_3$ $\quad$ "FLAT UNIVERSE"
- $K=+1$ $\quad S_3$ $\quad$ "CLOSED"
- $K=-1$ $\quad H_3$ $\quad$ "OPEN"

• SYMMETRY: 3 SPATIAL TRANSL & 3 ROTATIONS. = 6 KVs.

NO BOOST INVARIANCE.

$\Rightarrow \exists$ PREFERRED FRAME .. COMOOVING OBSERVER

t ... PROPER TIME, x^i = CONST.

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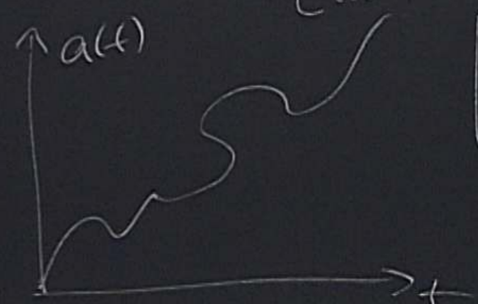

• $\boxed{H = \frac{\dot{a}}{a}}$ ... HUBBLE'S EXPANSION RATE

$\boxed{q = -\frac{\ddot{a}}{a H^2}}$ DECELERATION PARAMETER.

- OTHER COORDINATE SYSTEMS

a) SPHERICAL

$$\boxed{\begin{aligned} x^1 &= r\sin\theta\cos\varphi, \\ x^2 &= r\sin\theta\sin\varphi \\ x^3 &= r\cos\theta \end{aligned}}$$

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$$

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b) NEW RADIAL COORDINATE

$$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$$

$$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$$

$\ldots \theta \cos\varphi,$

$\ldots \theta \sin\varphi$

$\cos\theta$

$\ldots r^2 d\Omega^2)$

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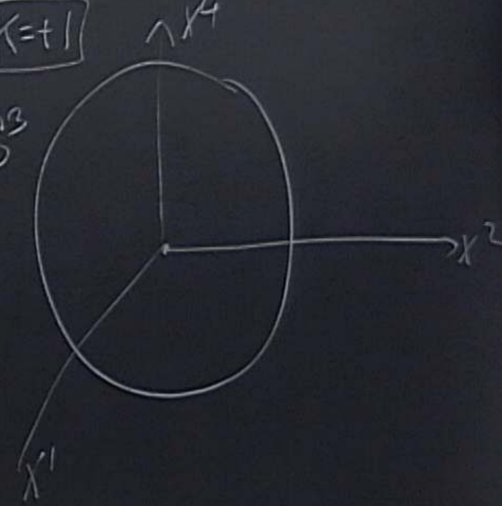
$\boxed{K=+1}$

S^3



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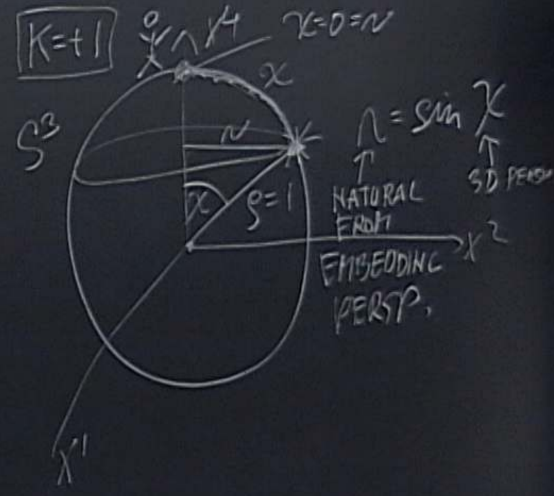
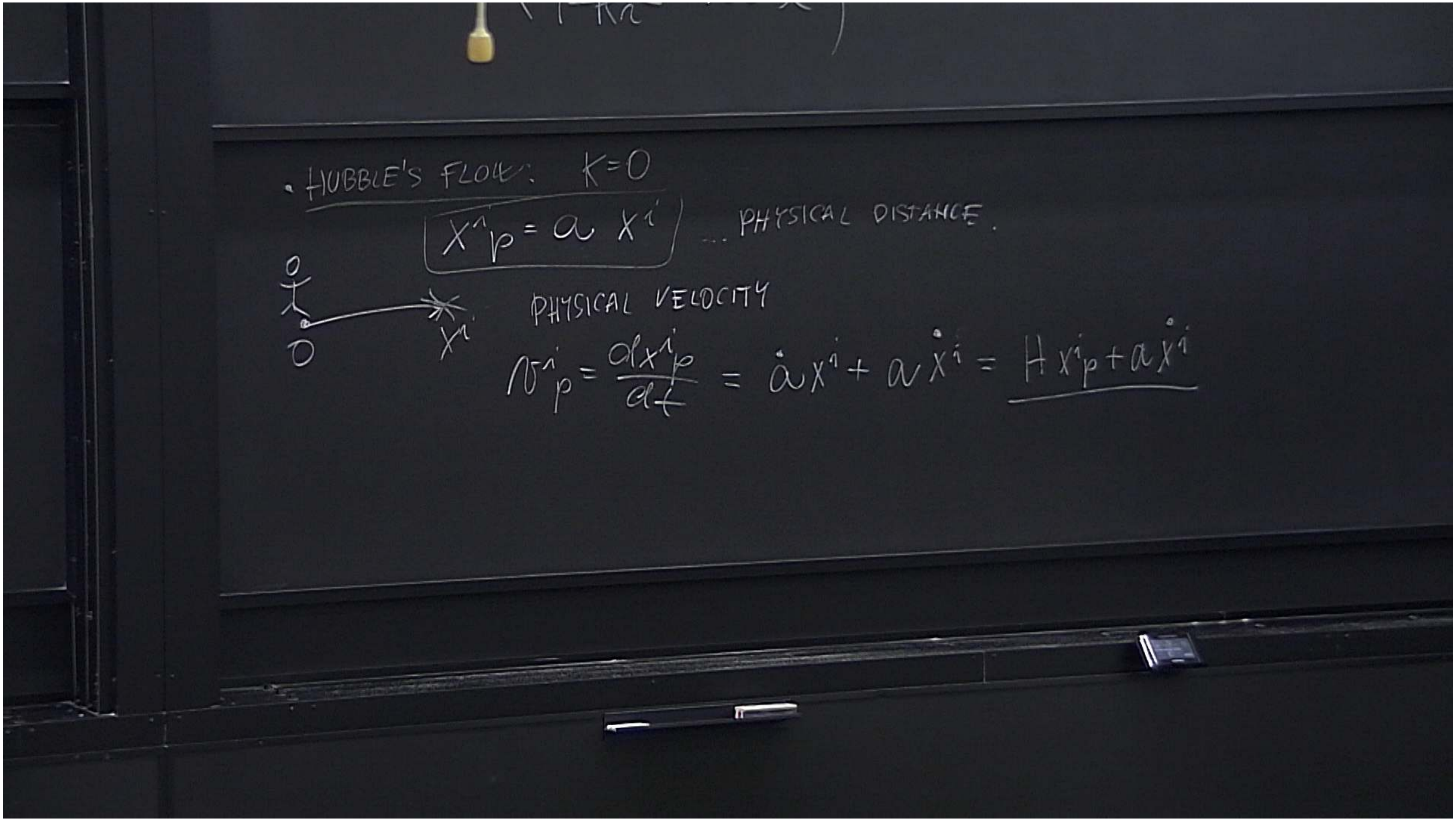
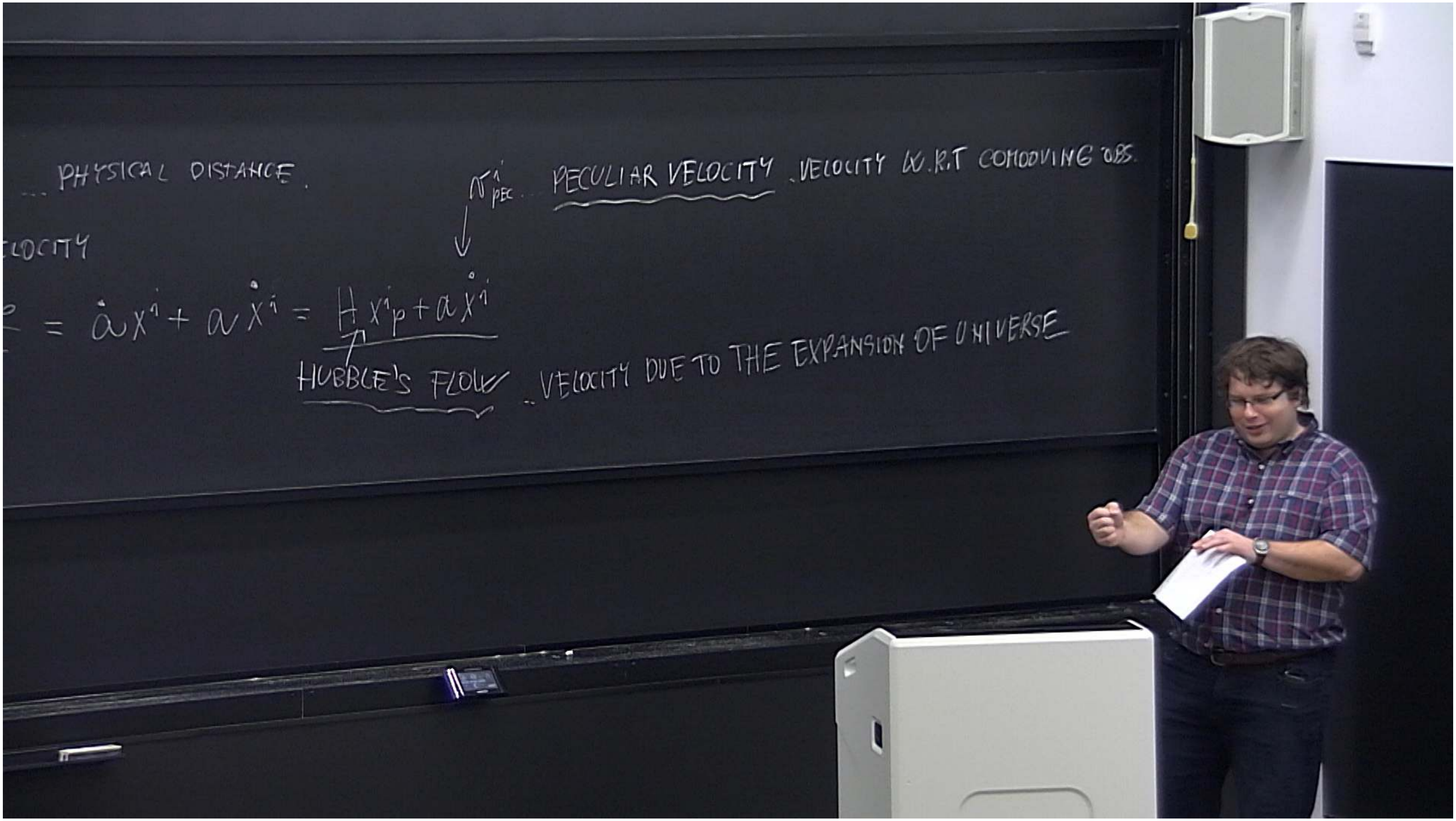
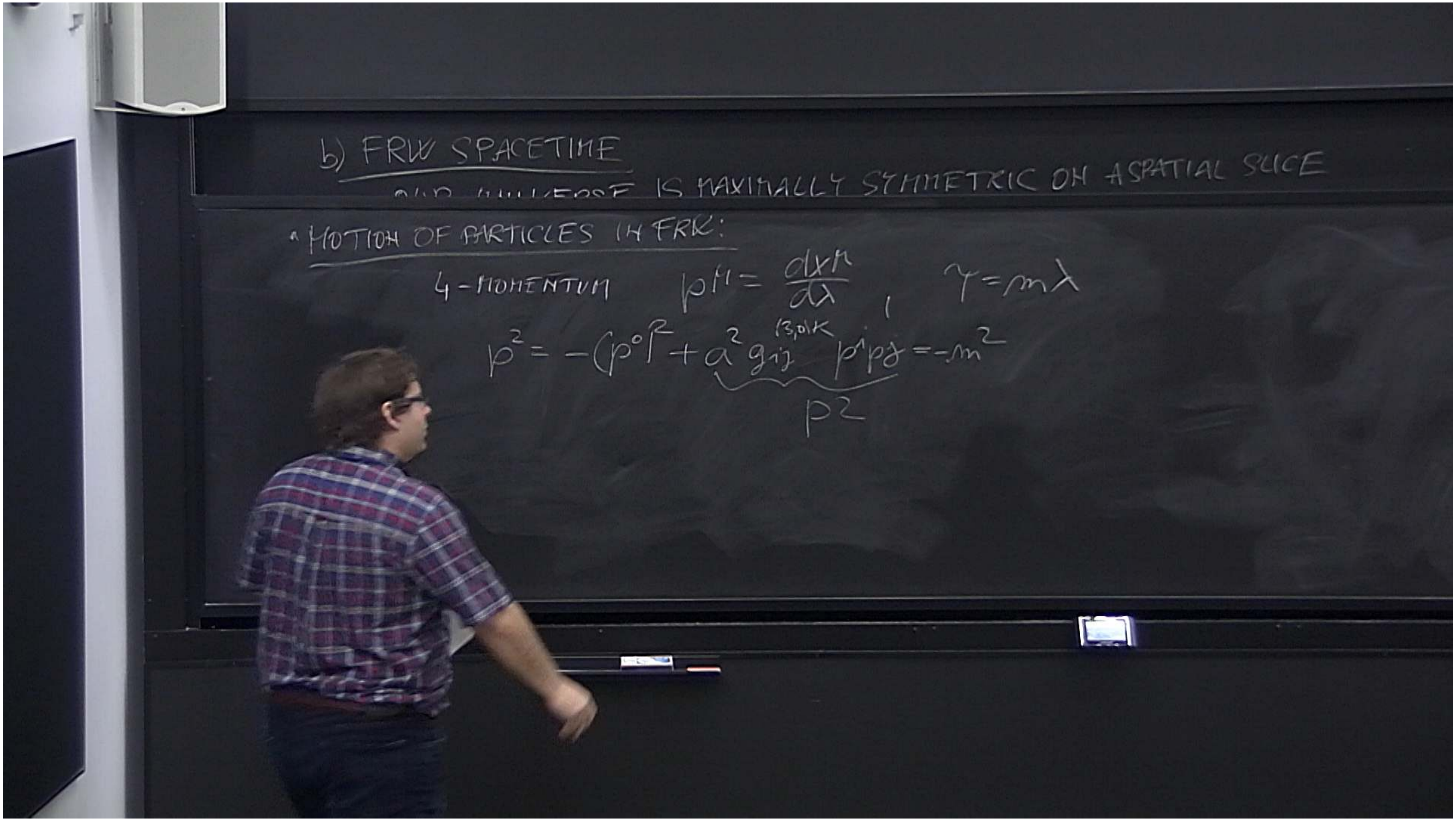
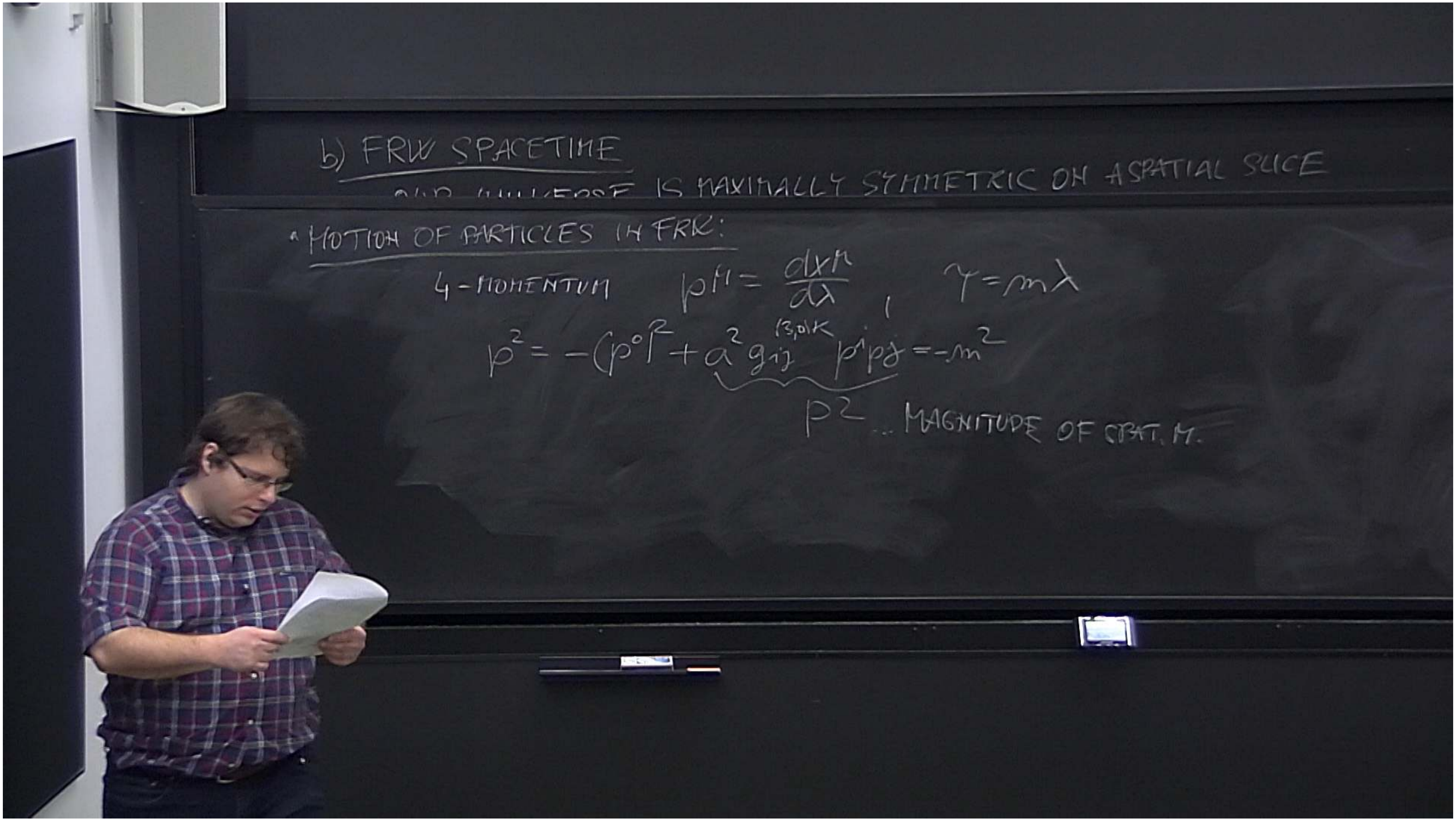



• HUBBLE'S FLOW: $K=0$
$\boxed{x^i_p = a\, x^i}$ ... PHYSICAL DISTANCE.
O
x^i
PHYSICAL VELOCITY
$v^i_p = \frac{dx^i_p}{dt} = \dot{a}\, x^i + a\, \dot{x}^i = \underline{H x^i_p + a\, \dot{x}^i}$


PHYSICAL DISTANCE
PECULIAR VELOCITY
VELOCITY W.R.T COMOOVING OBS.
HUBBLE'S FLOW
VELOCITY DUE TO THE EXPANSION OF UNIVERSE


b) FRW SPACETIME
IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE
MOTION OF PARTICLES IN FRW:
4-MOMENTUM $p^\mu = \frac{dx^\mu}{d\lambda}$, $\tau = m\lambda$
$p^2 = -(p^0)^2 + \underbrace{a^2 g_{ij}^{(3),0|K} p^i p^j}_{p^2} = -m^2$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• MOTION OF PARTICLES IN FRW:
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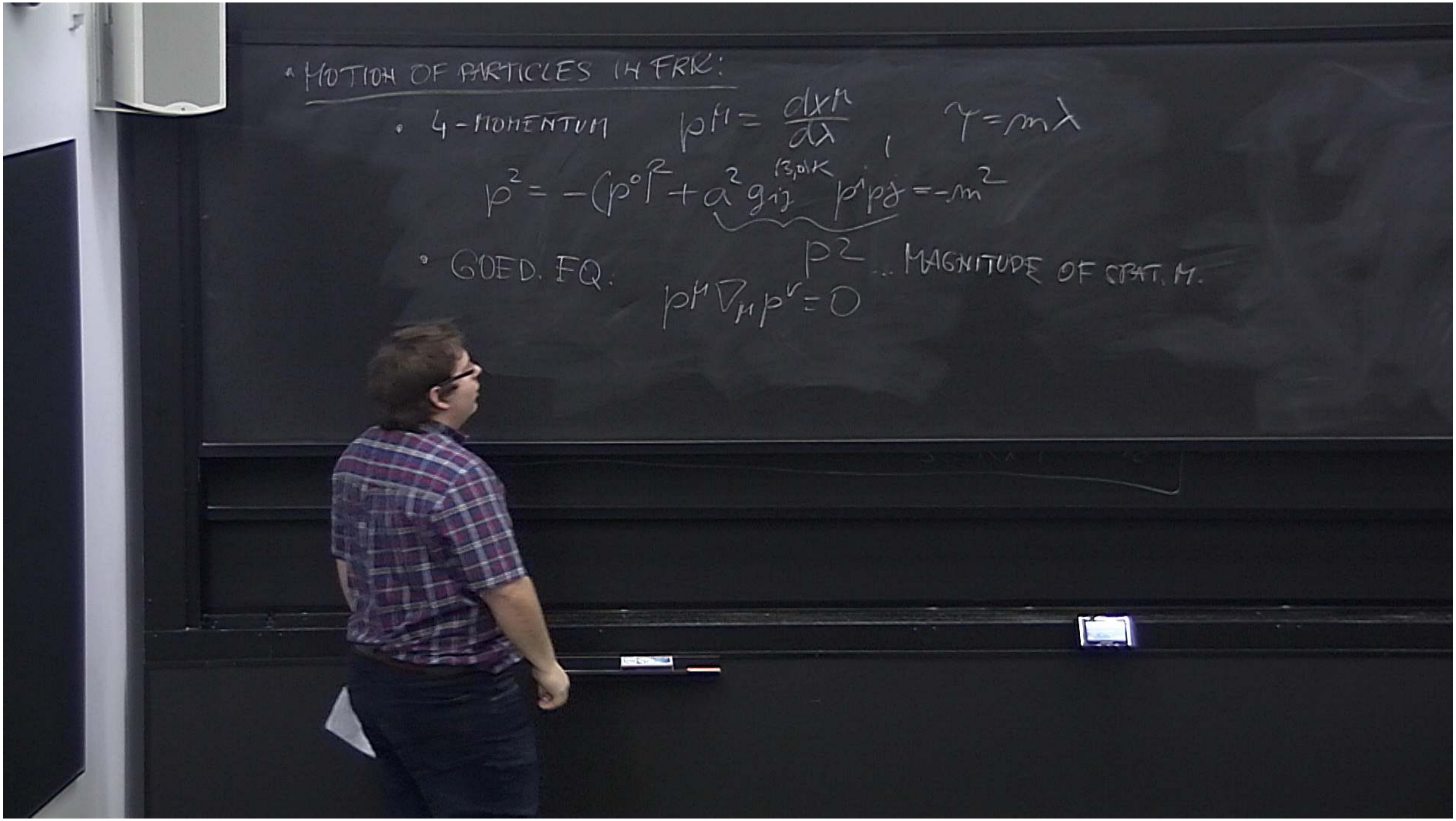
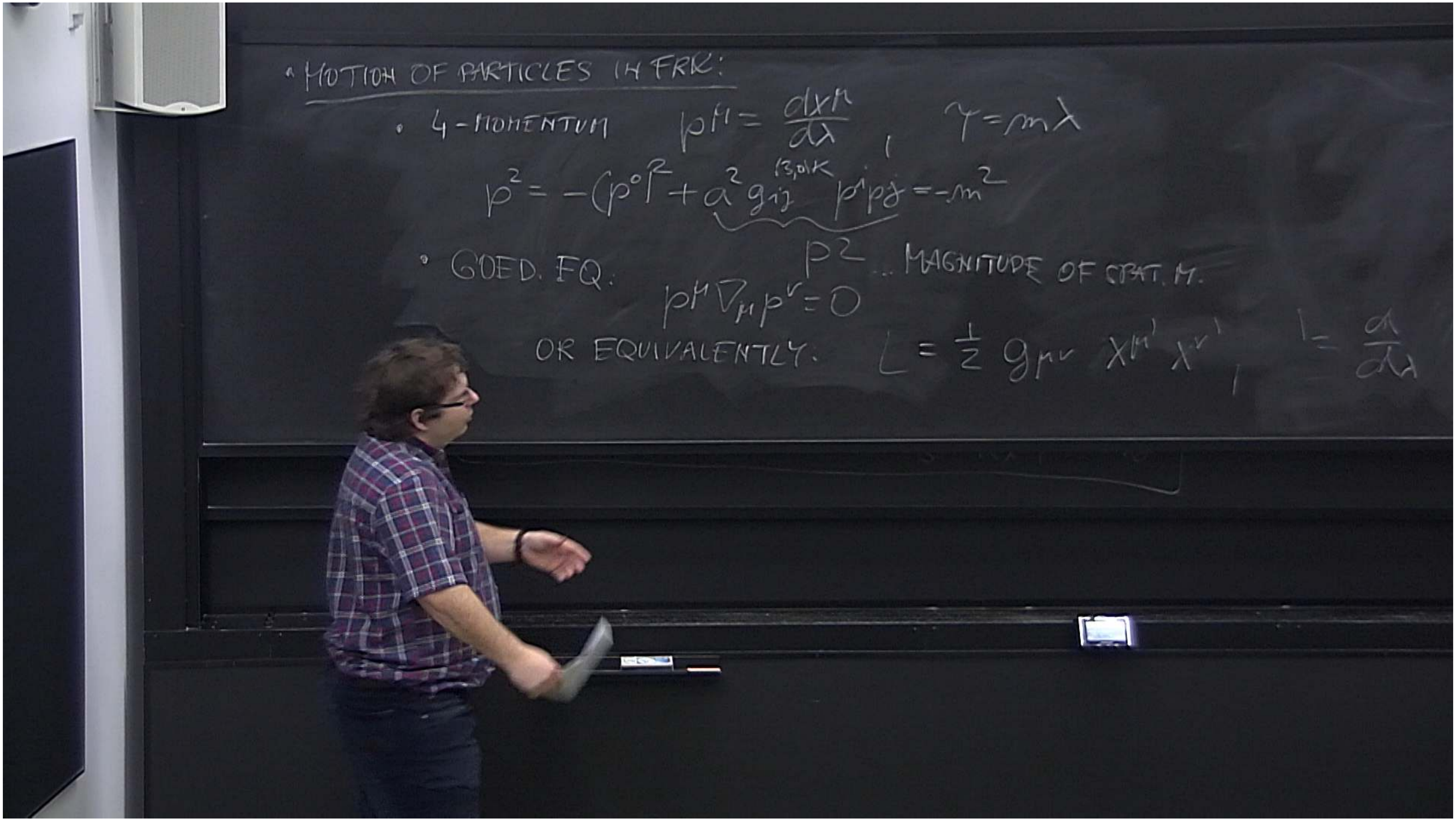
p^2 ... MAGNITUDE OF SPAT. M.


MOTION OF PARTICLES IN FRW:
4-MOMENTUM $p^\mu = \frac{dx^\mu}{d\lambda}$, $\tau = m\lambda$
$p^2 = -(p^0)^2 + a^2 g_{ij}^{(3),FRW} p^i p^j = -m^2$
p^2 ... MAGNITUDE OF SPAT. M.
GEOD. EQ.:
$p^\mu \nabla_\mu p^\nu = 0$


MOTION OF PARTICLES IN FRW:
4-MOMENTUM $p^\mu = \frac{dx^\mu}{d\lambda}$, $\tau = m\lambda$
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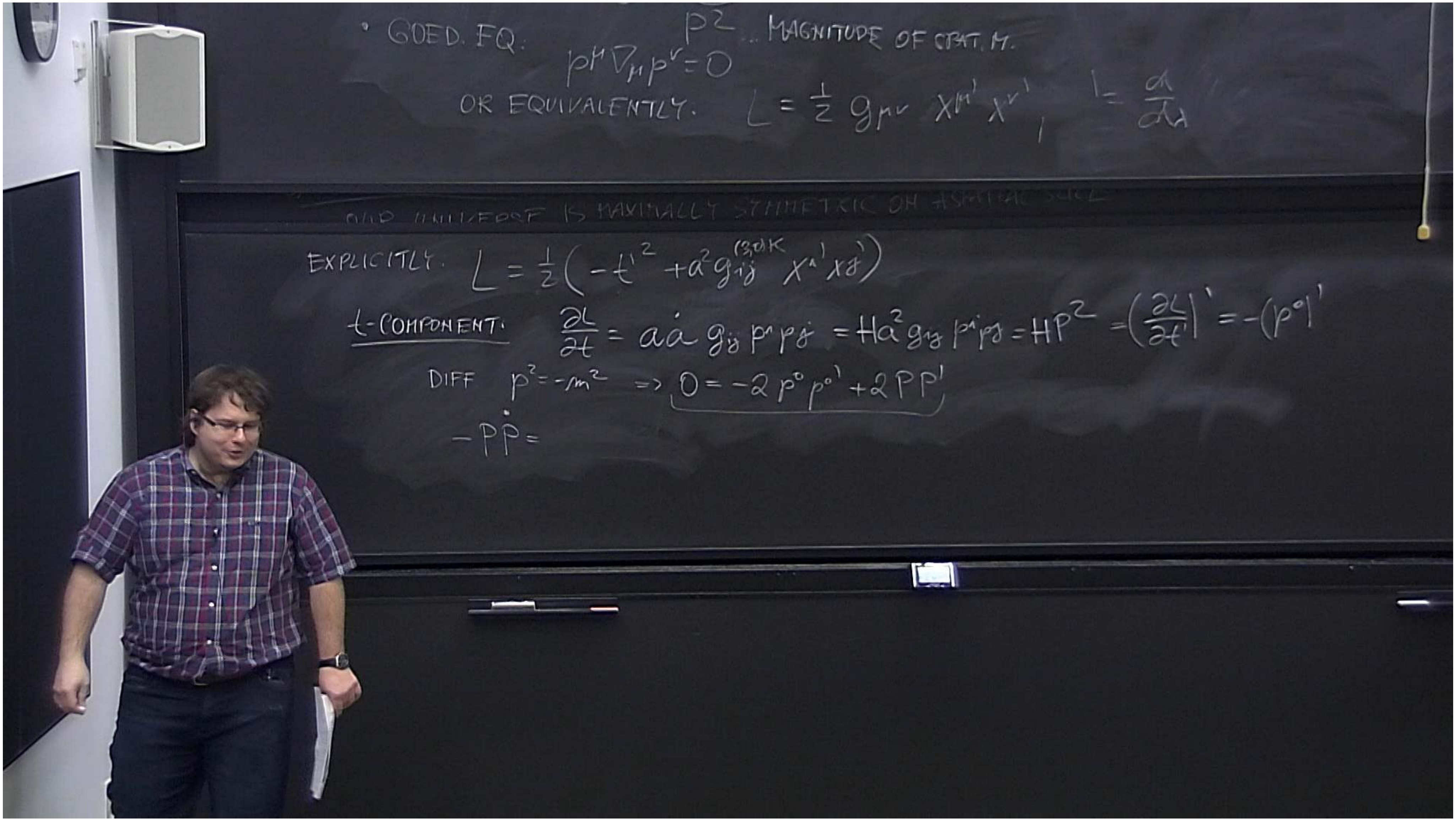
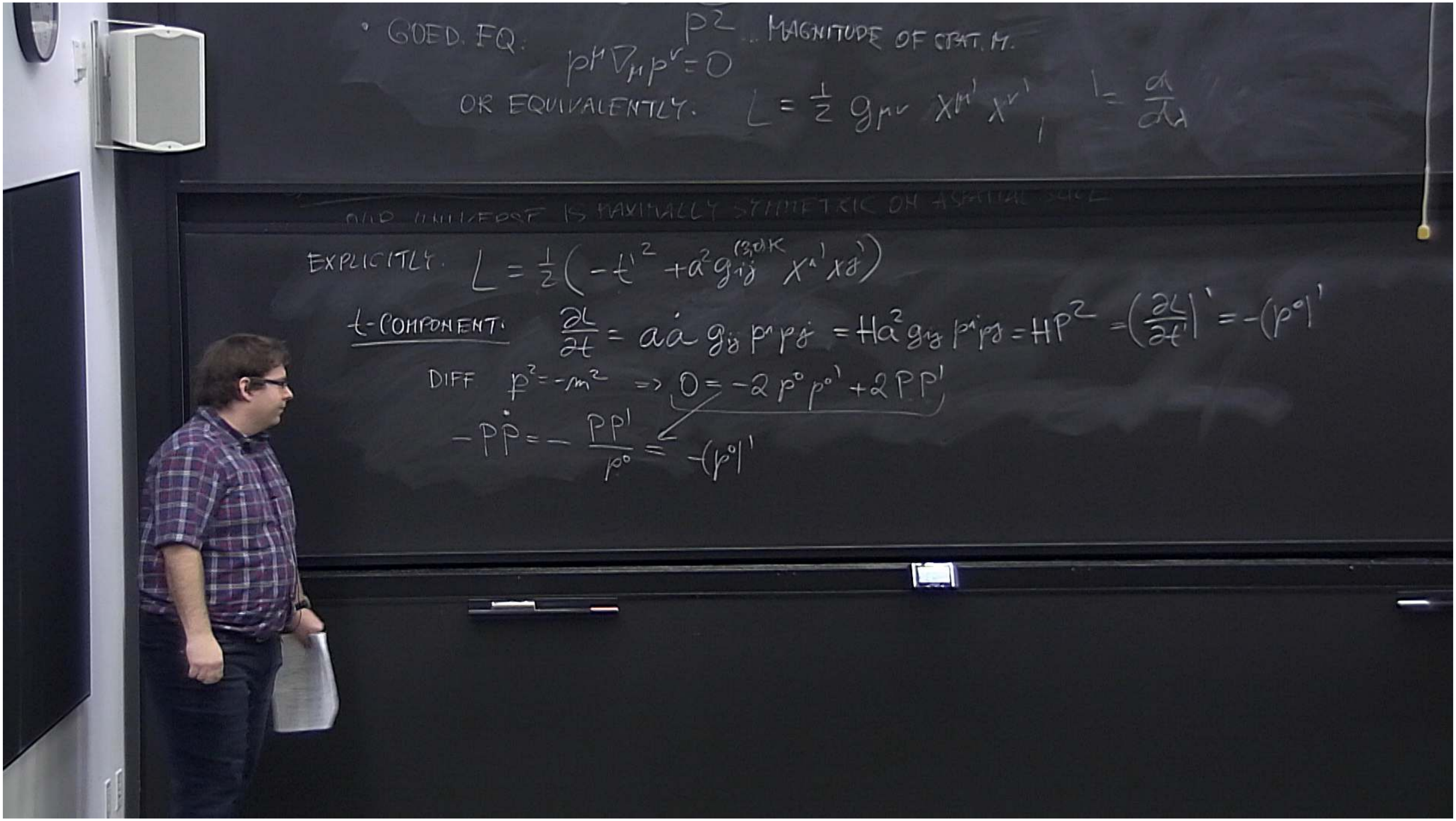
p^2 ... MAGNITUDE OF SPAT. M.
GEOD. EQ.: $p^\mu \nabla_\mu p^\nu = 0$
OR EQUIVALENTLY: $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$, $' = \frac{d}{d\lambda}$



GOED. EQ.
P2 ... MAGNITUDE OF SPAT. M.
$p^\mu \nabla_\mu p^\nu = 0$
OR EQUIVALENTLY. $L = \frac{1}{2} g_{\mu\nu} x^{\mu'} x^{\nu'}$
EXPLICITLY. $L = \frac{1}{2}\left(-t'^2 + a^2 g_{ij} x^{i'} x^{j'}\right)$
t-COMPONENT.
$\frac{\partial L}{\partial t} = a\dot{a}\, g_{ij} p^i p^j = Ha^2 g_{ij} p^i p^j = HP^2 = \left(\frac{\partial L}{\partial t'}\right)' = -(p^0)'$
DIFF $p^2 = -m^2 \Rightarrow 0 = -2p^0 p^{0'} + 2PP'$
$-P\dot{P} =$


GEOD. EQ.
p^2 .. MAGNITUDE OF SPAT. M.
$p^\mu \nabla_\mu p^\nu = 0$
OR EQUIVALENTLY. $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$, $\prime = \frac{d}{d\lambda}$
EXPLICITLY. $L = \frac{1}{2}\left(-t'^2 + a^2 g_{ij}^{(3),K} x^{i\prime} x^{j\prime}\right)$
t-COMPONENT. $\frac{\partial L}{\partial t} = a\dot{a}\, g_{ij} p^i p^j = H a^2 g_{ij} p^i p^j = H P^2 = \left(\frac{\partial L}{\partial t'}\right)' = -(p^0)'$
DIFF $p^2 = -m^2 \Rightarrow 0 = -2p^0 p^{0\prime} + 2PP'$
$-P\dot{P} = -\frac{PP'}{p^0} = -(p^0)'$

$$p^\mu \nabla_\mu p^\nu = 0$$

OR EQUIVALENTLY: $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}, \quad {}' = \frac{d}{d\lambda}$

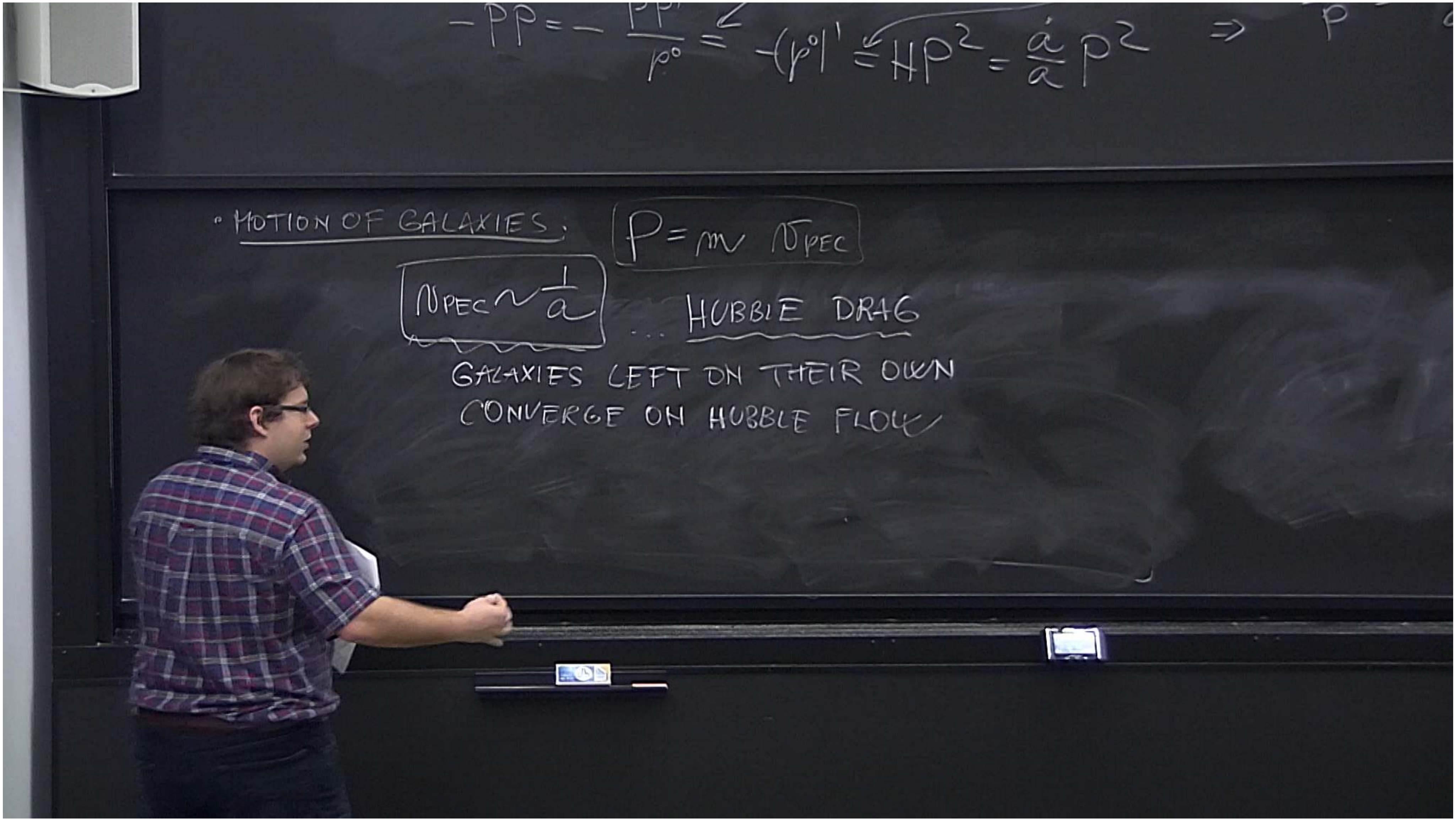
OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE

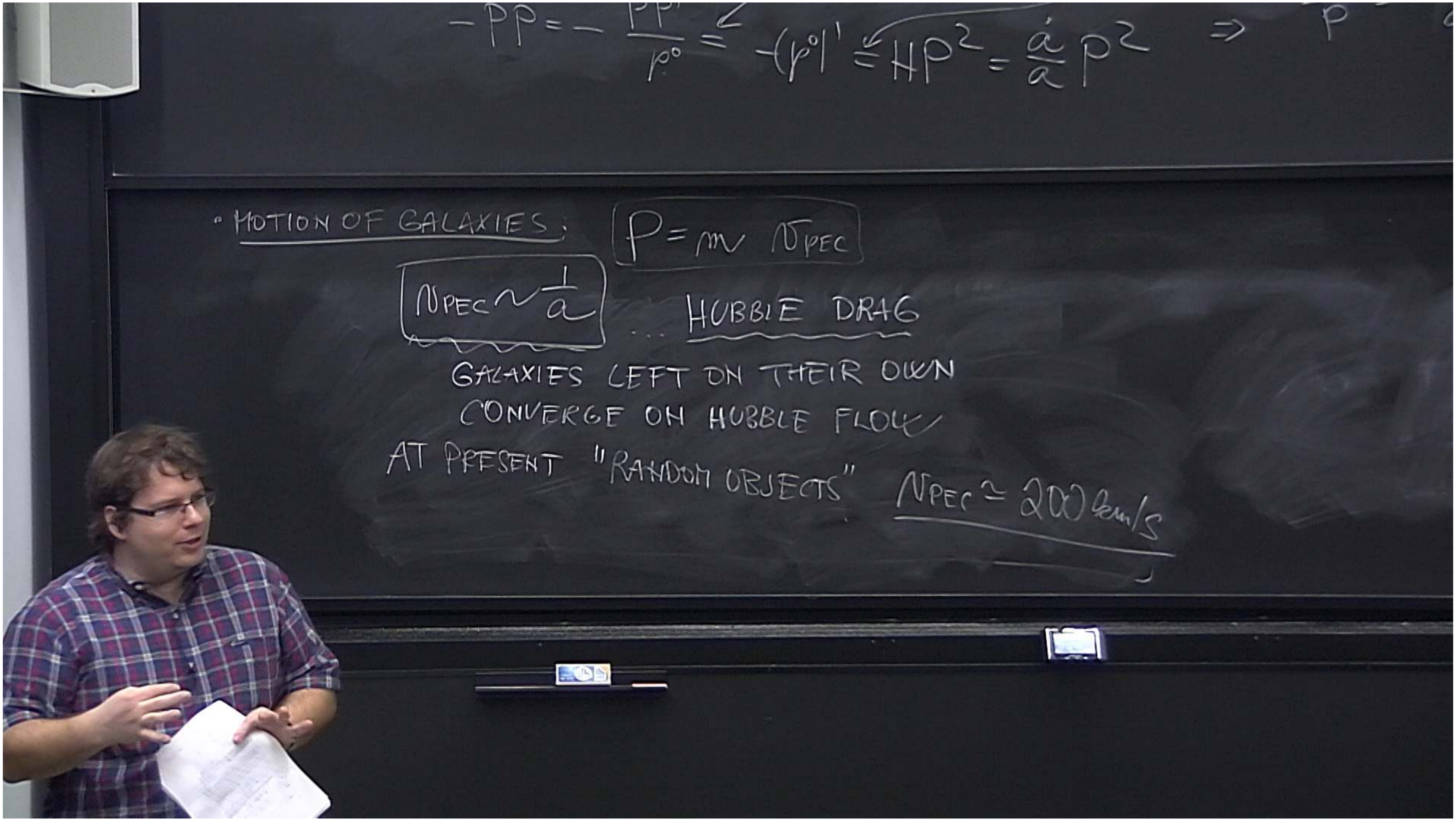
EXPLICITLY: $L = \frac{1}{2}\left(-t'^2 + a^2 g^{(3,0)K}_{ij} x^{i\prime} x^{j\prime}\right)$

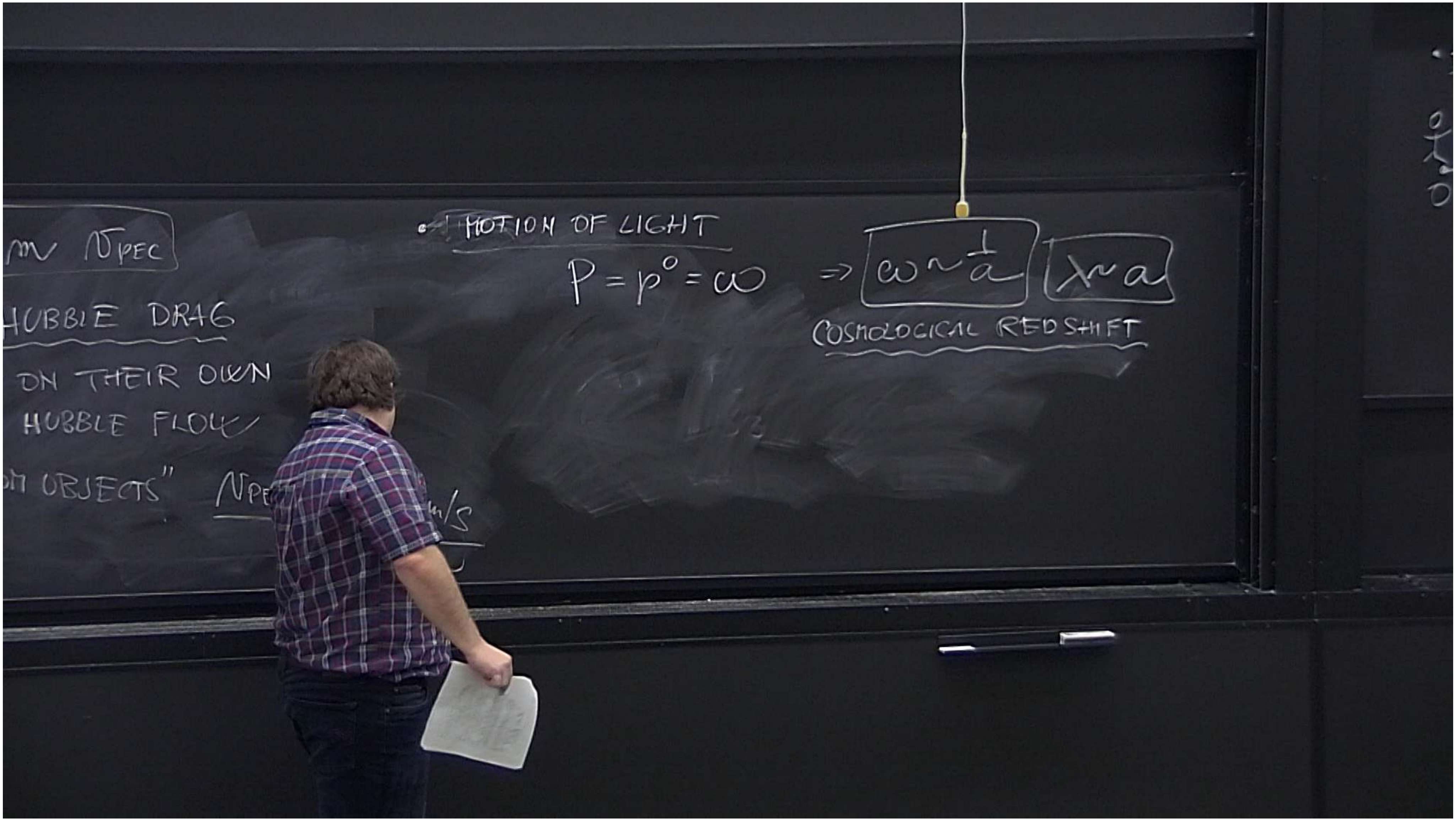
t-COMPONENT: $\frac{\partial L}{\partial t} = a\dot{a}\, g_{ij} p^i p^j = H a^2 g_{ij} p^i p^j = \underline{H P^2} = \left(\frac{\partial L}{\partial t'}\right)' = \underline{-(p^0)'}$

DIFF $p^2 = -m^2 \Rightarrow 0 = -2p^0 p^{0\prime} + 2PP'$

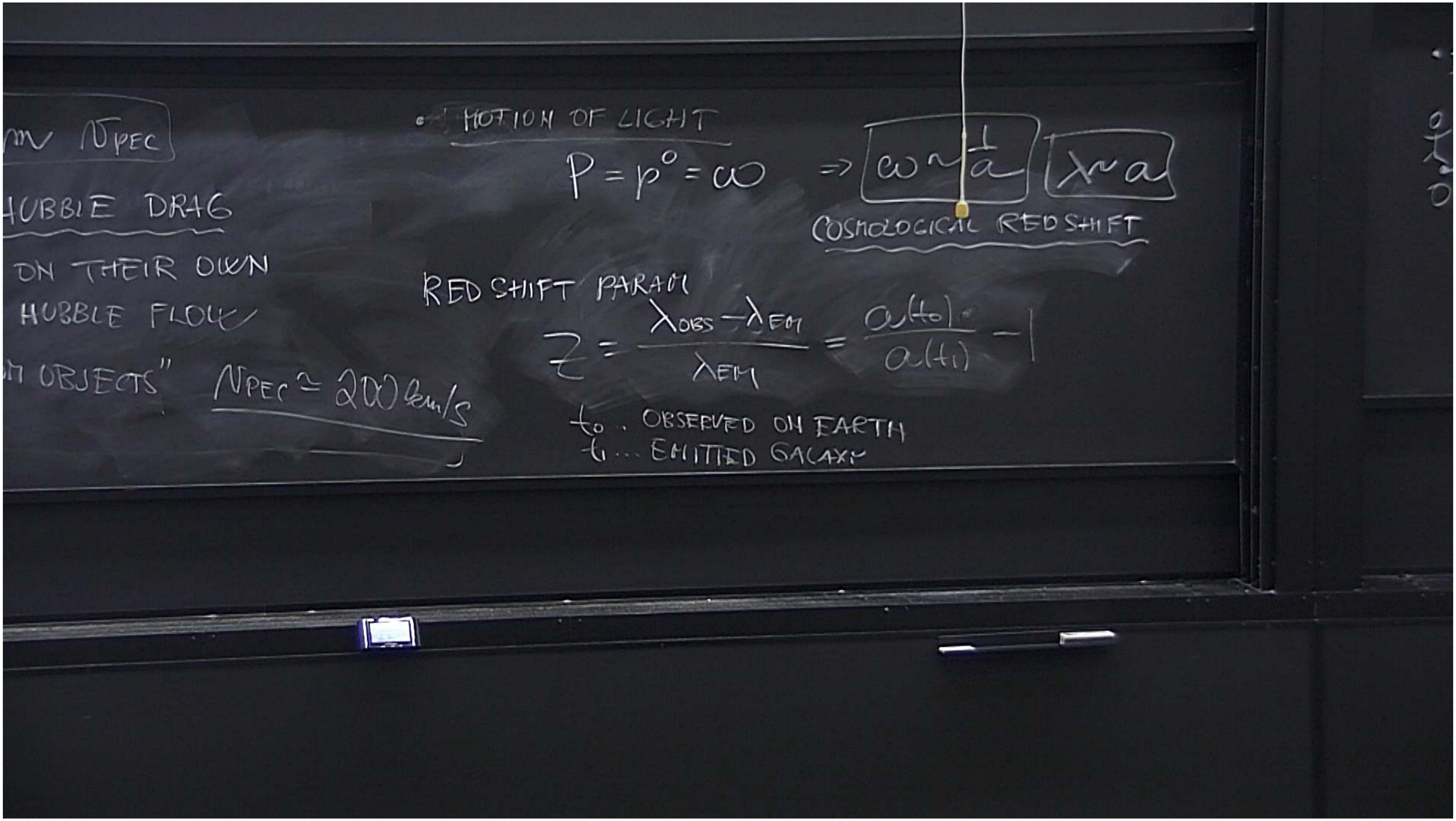
$$-P\dot{P} = -\frac{PP'}{p^0} = -(p^0)' = HP^2 = \frac{\dot{a}}{a}P^2 \Rightarrow \frac{dP}{P} = -\frac{da}{a} \Rightarrow \boxed{P \sim \frac{1}{a}}$$


MOTION OF GALAXIES:
P = m v_PEC
v_PEC ~ 1/a
... HUBBLE DRAG
GALAXIES LEFT ON THEIR OWN
CONVERGE ON HUBBLE FLOW


• MOTION OF GALAXIES:
P = m v_PEC
v_PEC ~ 1/a
... HUBBLE DRAG
GALAXIES LEFT ON THEIR OWN
CONVERGE ON HUBBLE FLOW
AT PRESENT "RANDOM OBJECTS" v_PEC ≃ 200 km/s



HUBBLE DRAG
ON THEIR OWN
HUBBLE FLOW
OBJECTS"
MOTION OF LIGHT
P = p° = ω
⇒ ω ~ 1/a
λ ~ a
COSMOLOGICAL REDSHIFT



MOTION OF LIGHT
$P = p^0 = \omega \Rightarrow \boxed{\omega \sim \frac{1}{a}}\ \boxed{\lambda \sim a}$
COSMOLOGICAL REDSHIFT
HUBBLE DRAG
ON THEIR OWN
HUBBLE FLOW
OBJECTS"
$v_{PEC} \simeq 200\ km/s$
REDSHIFT PARAM
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}} = \frac{a(t_0)}{a(t_1)} - 1$
t_0 .. OBSERVED ON EARTH
t_1 ... EMITTED GALAXY

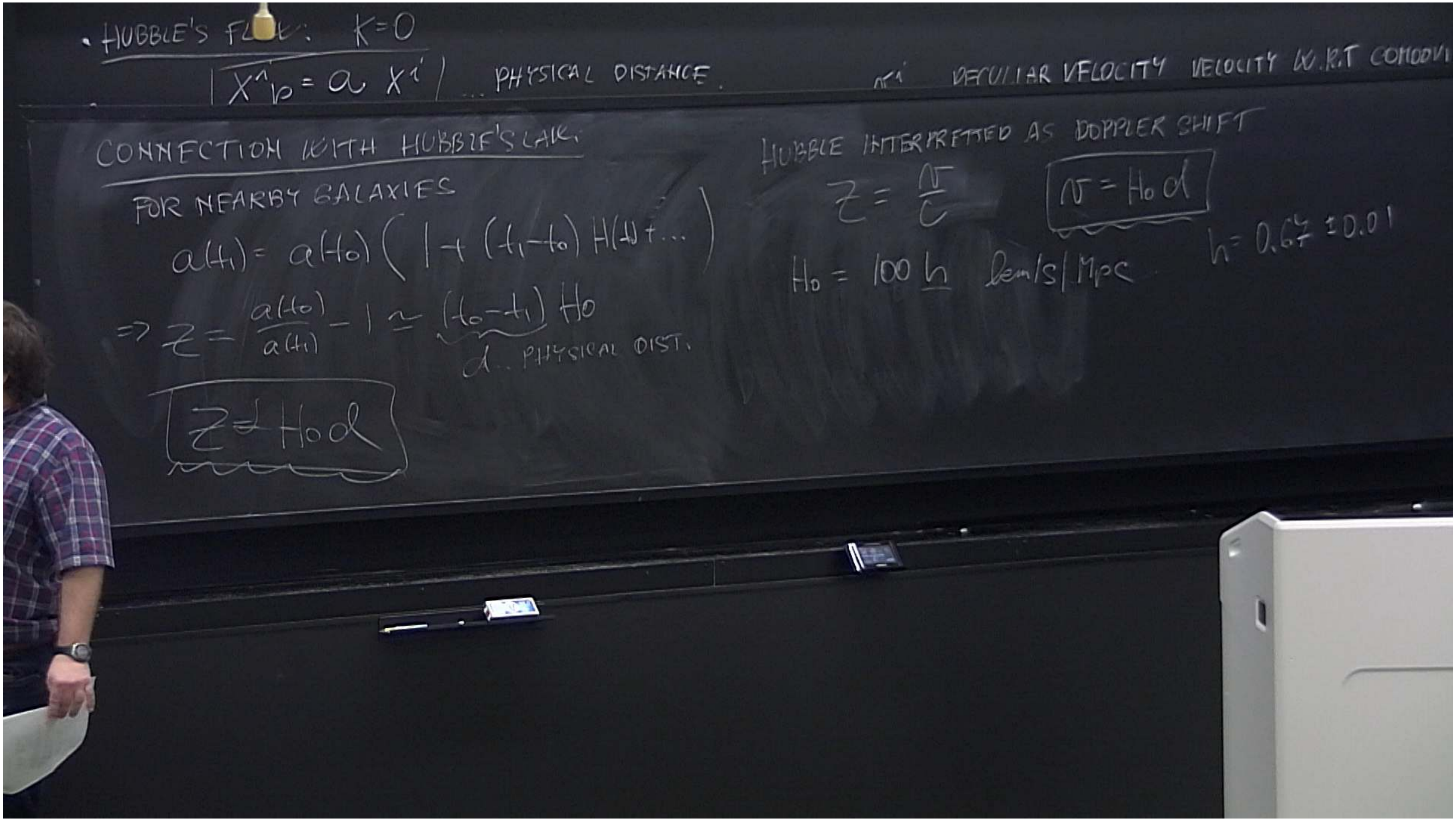
CONNECTION WITH HUBBLE'S LAW:

FOR NEARBY GALAXIES

$$a(t_1) = a(t_0)\left(1 + (t_1 - t_0) H(t_0) + \dots\right)$$

$$\Rightarrow z = \frac{a(t_0)}{a(t_1)} - 1 \simeq \underbrace{(t_0 - t_1)}_{d\text{: PHYSICAL DIST.}} H_0$$

$$\boxed{z = H_0 d}$$



• HUBBLE'S FLOW: K=0
PHYSICAL DISTANCE
PECULIAR VELOCITY VELOCITY W.R.T COMOVI
CONNECTION WITH HUBBLE'S LAW:
FOR NEARBY GALAXIES
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0) H(t_0) + \ldots\right)$
$\Rightarrow z = \frac{a(t_0)}{a(t_1)} - 1 \simeq (t_0 - t_1) H_0$
d .. PHYSICAL DIST.
$z \simeq H_0 d$
HUBBLE INTERPRETED AS DOPPLER SHIFT
$z = \frac{v}{c}$
$v = H_0 d$
$H_0 = 100\, h$ km/s/Mpc
$h = 0.67 \pm 0.01$