Title: PSI 2016/2017 Cosmology (Review) - Lecture 1

Date: Jan 30, 2017 10:15 AM

URL: http://pirsa.org/17010086

Abstract:

COSMOLOGY REVIEW

WHY COSMOLOGY?

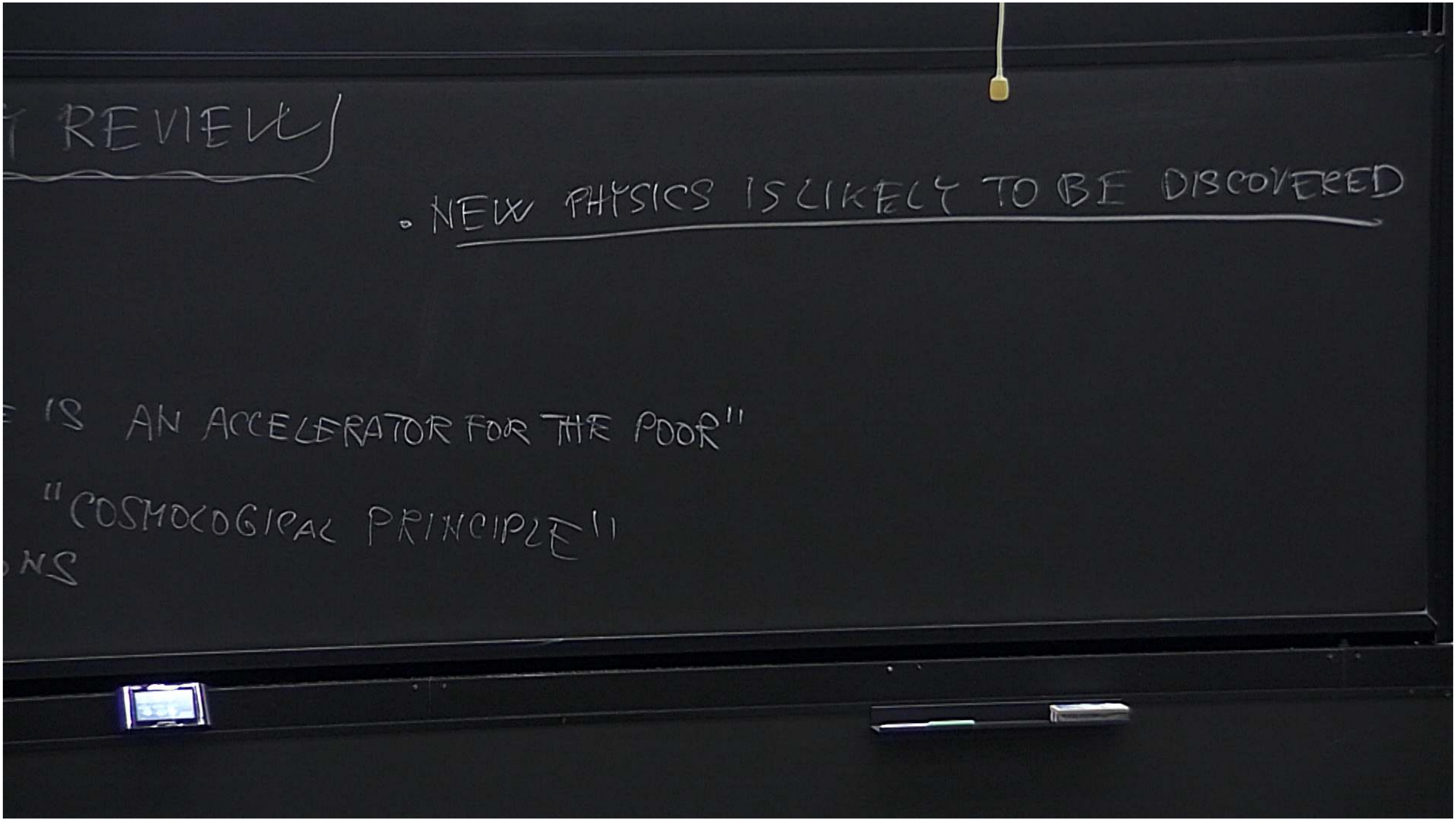
- "RICH PHYSICS"

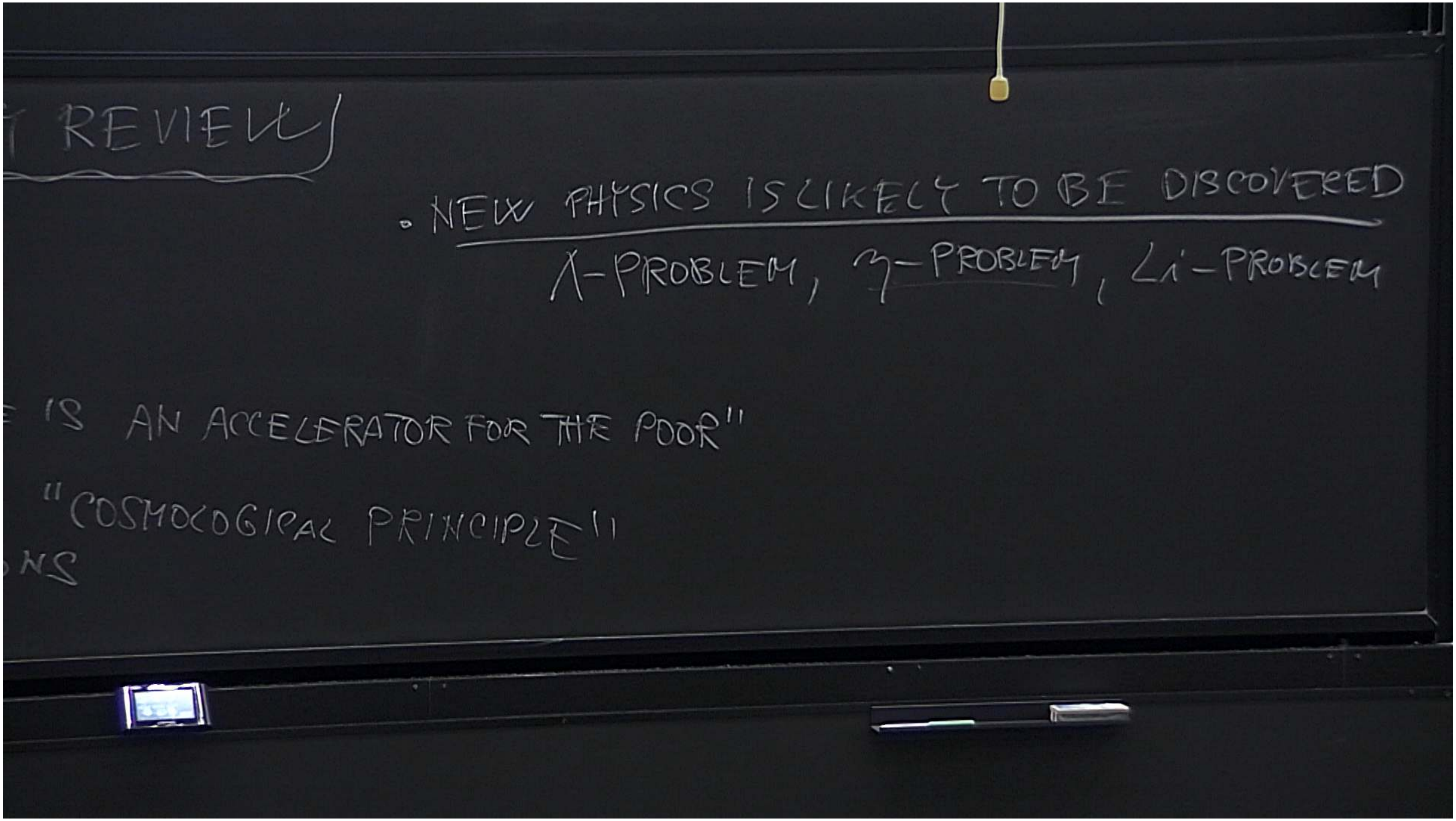
  ZELDOVICH – "EARLY UNIVERSE IS AN ACCELERATOR

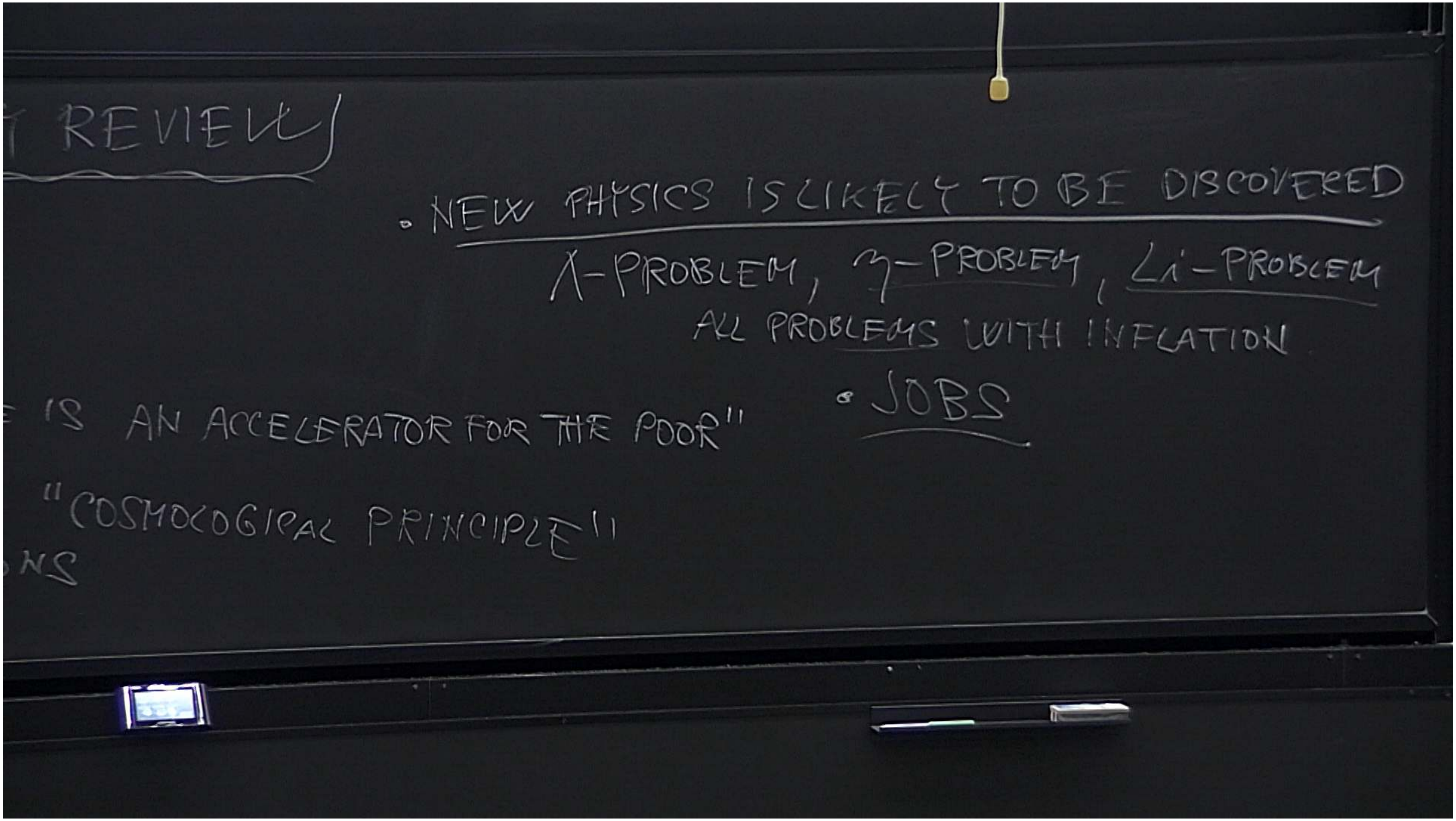
- "OBSERVATION BASED" NO LONGER "COSMOLOGICAL PRI

  BASED ON SOLID OBSERVATIONS

  GW's ?


REVIEW
• NEW PHYSICS IS LIKELY TO BE DISCOVERED
IS AN ACCELERATOR FOR THE POOR"
"COSMOLOGICAL PRINCIPLE"
NS


REVIEW
• NEW PHYSICS IS LIKELY TO BE DISCOVERED
Λ-PROBLEM, η-PROBLEM, Li-PROBLEM
IS AN ACCELERATOR FOR THE POOR"
"COSMOLOGICAL PRINCIPLE"


REVIEW
• NEW PHYSICS IS LIKELY TO BE DISCOVERED
λ-PROBLEM, η-PROBLEM, Li-PROBLEM
ALL PROBLEMS WITH INFLATION
• JOBS
IS AN ACCELERATOR FOR THE POOR"
"COSMOLOGICAL PRINCIPLE"
NS

ESTABLISHED FACTS

- HOMOGENEOUS & ISOTROPIC
- EXPANDS ACCORDING TO HUBBLE'S LAW ✓
- CMB $T \simeq 2,73K$
- BARYONIC MATTER: 75% H, 25% He
- DM, DE (95% OF UNIVERSE)
- SMALL FLUCTUATIONS 10^{-5} IN DENSITY WHEN UNIVERSE 1000× SMALLER

ISOTROPIC

NS TO HUBBLE'S LAW

73K

R: 75% H, 25% He

5% OF UNIVERSE)

IONS 10^{-5} IN DENSITY

UNIVERSE 1000 × SMALLER → STRUCTURE FORMATION

TROPIC

TO HUBBLE'S LAW

HOMOGENEOUS U.

K

75% H, 25% He

MATTER

OF UNIVERSE)

10^{-5} IN DENSITY

NIVERSE 1000× SMALLER → STRUCTURE FORMATION

FLUCTUATIONS

WHEN UNIVERSE 1000× SMALLER → STRUCTURE FO

I) HOMOGENEOUS UNIVERSE

- OBSERVABLE U 3×10^4 MPc
- HOM. ON LARGE SCALES 100 Mpc
  OBSERVED BY "TYPICAL FREE FALING OBSERVER"
- WE SEE ISOTROPY, ASSUME COPERNICUS PRINCIPLE
  ⇒ ISOTROPIC EVERYWHERE ⇒ HOMOGENEOUS

WHEN UNIVERSE 1000x SMALLER → STRUCTURE FO

I) HOMOGENEOUS UNIVERSE

- OBSERVABLE U 3×10^4 MPc
- HOM. ON LARGE SCALES 100 Mpc

  OBSERVED BY "TYPICAL FREE FALING OBSERVER"
- WE SEE ISOTROPY, ASSUME COPERNICUS PRINCIPLE

  ⟹ ISOTROPIC EVERYWHERE ⟹ HOMOGENEOUS
- FIRST APROXIMATION: U IS HOM & ISOTROPIC

...RUCTURE FORMATION }

...RVER"

...NCIPLE

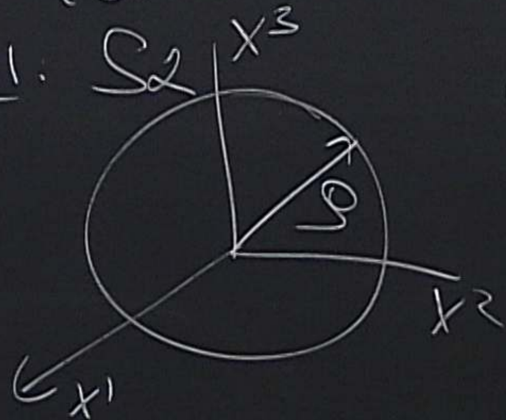
...OS

a) MAXIMALLY SYMMETRIC SPACES

"ALL HOM & ISOTROPIC SPACES"

CONSTRUCTION .. EMBEDDING IN (n+1) DIM SPACE

EX1: S_2



$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$\rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$$

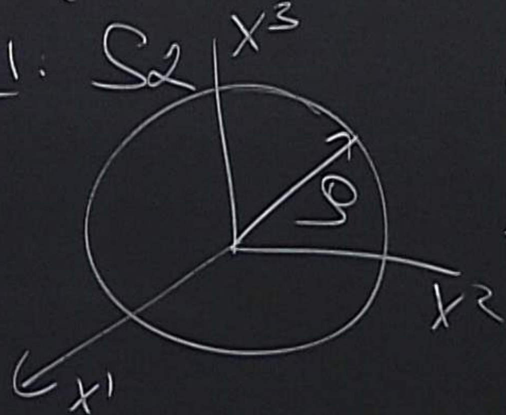
...RUCTURE FORMATION }

a) MAXIMALLY SYMMETRIC SPACES

"ALL HOM & ISOTROPIC SPACES"

CONSTRUCTION .. EMBEDDING IN $(n+1)$ DIM SPACE

EX 1: S_2



$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$\rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2 \quad \text{CONSTRAINT}$$

$$0 = x^1 dx^1 + x^2 dx^2 + x^3 dx^3$$

...RVER"

...NCIPLE

...OS

...RUCTURE FORMATION

...RVER"

...NCIPLE

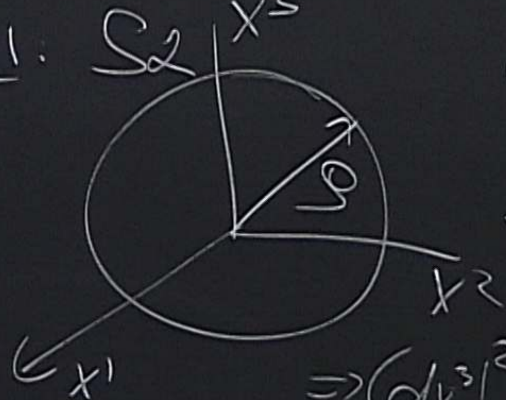
...OS

a) MAXIMALLY SYMMETRIC SPACES

"ALL HOM & ISOTROPIC SPACES"

CONSTRUCTION .. EMBEDDING IN $(n+1)$ DIM SPACE

EX 1: S_2



$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$\rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$$

CONSTRAINT

$$0 = x^1 dx^1 + x^2 dx^2 + x^3 dx^3$$

$$\Rightarrow (dx^3)^2 = (x^1 dx^1 + x_2 dx^2)^2$$

...RUCTURE FORMATION }

...RVER"

...NCIPLE

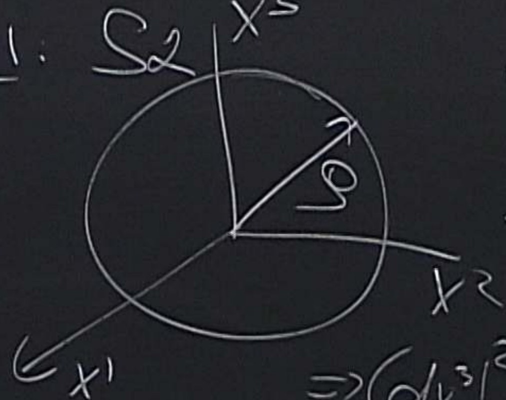
...OS

a) MAXIMALLY SYMMETRIC SPACES

"ALL HOM & ISOTROPIC SPACES"

CONSTRUCTION .. EMBEDDING IN $(n+1)$ DIM SPACE

EX 1: S_2



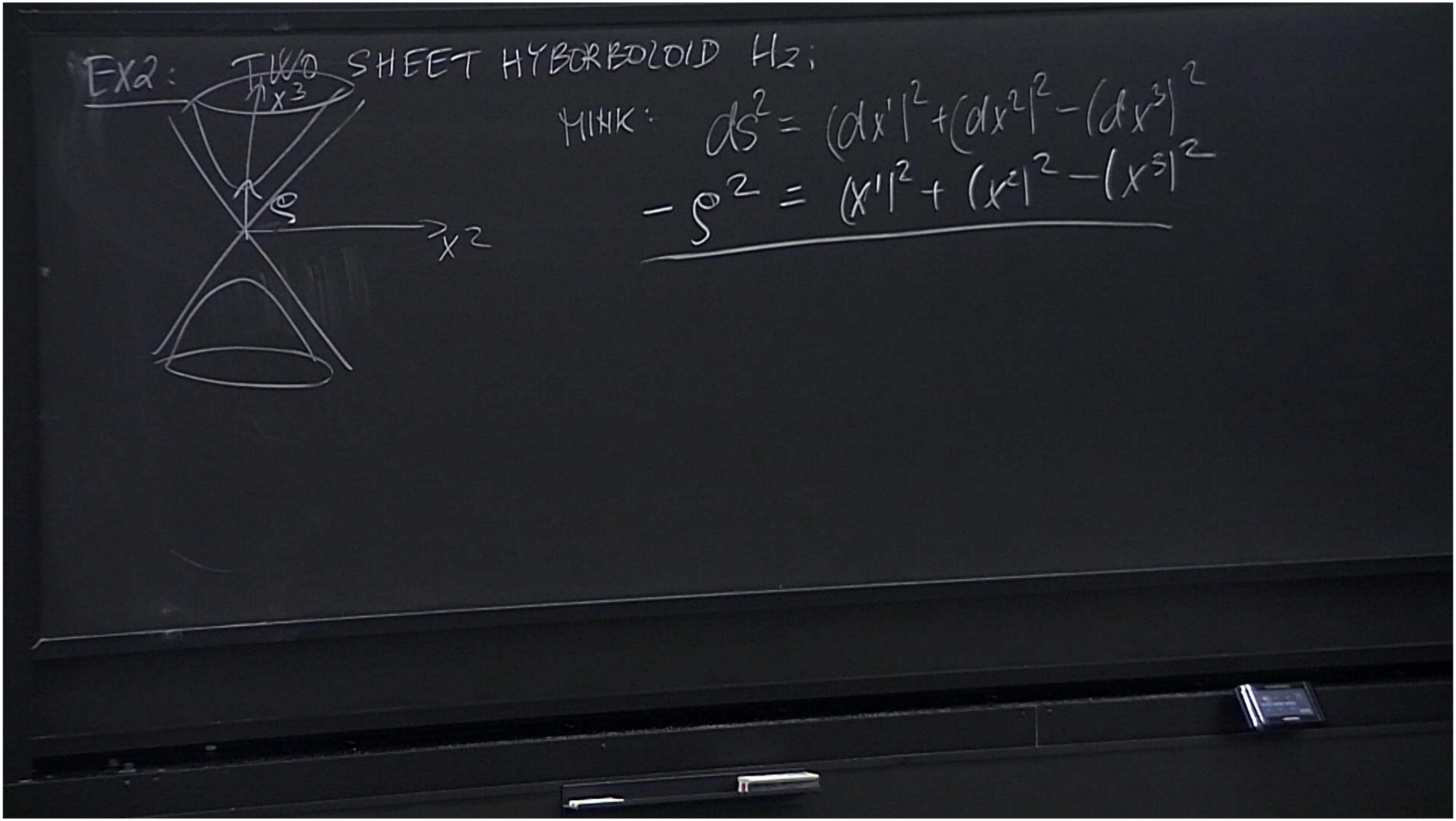
$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$\rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$$

CONSTRAINT

$$0 = x^1 dx^1 + x^2 dx^2 + x^3 dx^3$$

$$\Rightarrow (dx^3)^2 = \frac{(x^1 dx^1 + x^2 dx^2)^2}{\rho^2 - (x^1)^2 - (x^2)^2}$$


EX2: TWO SHEET HYBORBOLOID H_2:
MINK: $ds^2 = (dx^1)^2 + (dx^2)^2 - (dx^3)^2$
$-\rho^2 = (x^1)^2 + (x^2)^2 - (x^3)^2$
x^3
ρ
x^2

GENERALLY n-DIM MAXIMALLY SYMMETRIC SPACE OF SIGNATURE (p,q) AND CURVATURE $K = 0, \pm 1$ IS GIVEN BY

$$ds^2 = \eta_{ab}^{(p,q)} + K\,(dx^{n+1})^2$$

GENERALLY n-DIM MAXIMALLY SYMMETRIC SPACE OF SIGNATURE (p,q) AND CURVATURE $K = 0, \pm 1$ IS GIVEN BY

$$\boxed{\begin{aligned} ds^2 &= \eta^{(p,q)}_{ab}\, dx^a dx^b + K\,(dx^{n+1})^2 = \eta^{(p,q)K}_{AB}\, dx^A dx^B \\ K\rho^2 &= \eta^{(p,q)K}_{AB}\, x^A x^B \end{aligned}}$$

GENERALLY n-DIM MAXIMALLY SYMMETRIC SPACE OF SIGNATURE (p,q) AND CURVATURE $K=0,\pm 1$ IS GIVEN BY

$$\boxed{\begin{aligned} ds^2 &= \eta^{(p,q)}_{ab}\, dx^a dx^b + K\,(dx^{n+1})^2 = \eta^{(p,q)K}_{AB}\, dx^A dx^B \\ K\rho^2 &= \eta^{(p,q)K}_{AB}\, x^A x^B \end{aligned}}$$

$\eta^{(p,q)}_{ab} =$

ETRIC SPACE

AND CURVATURE

BY

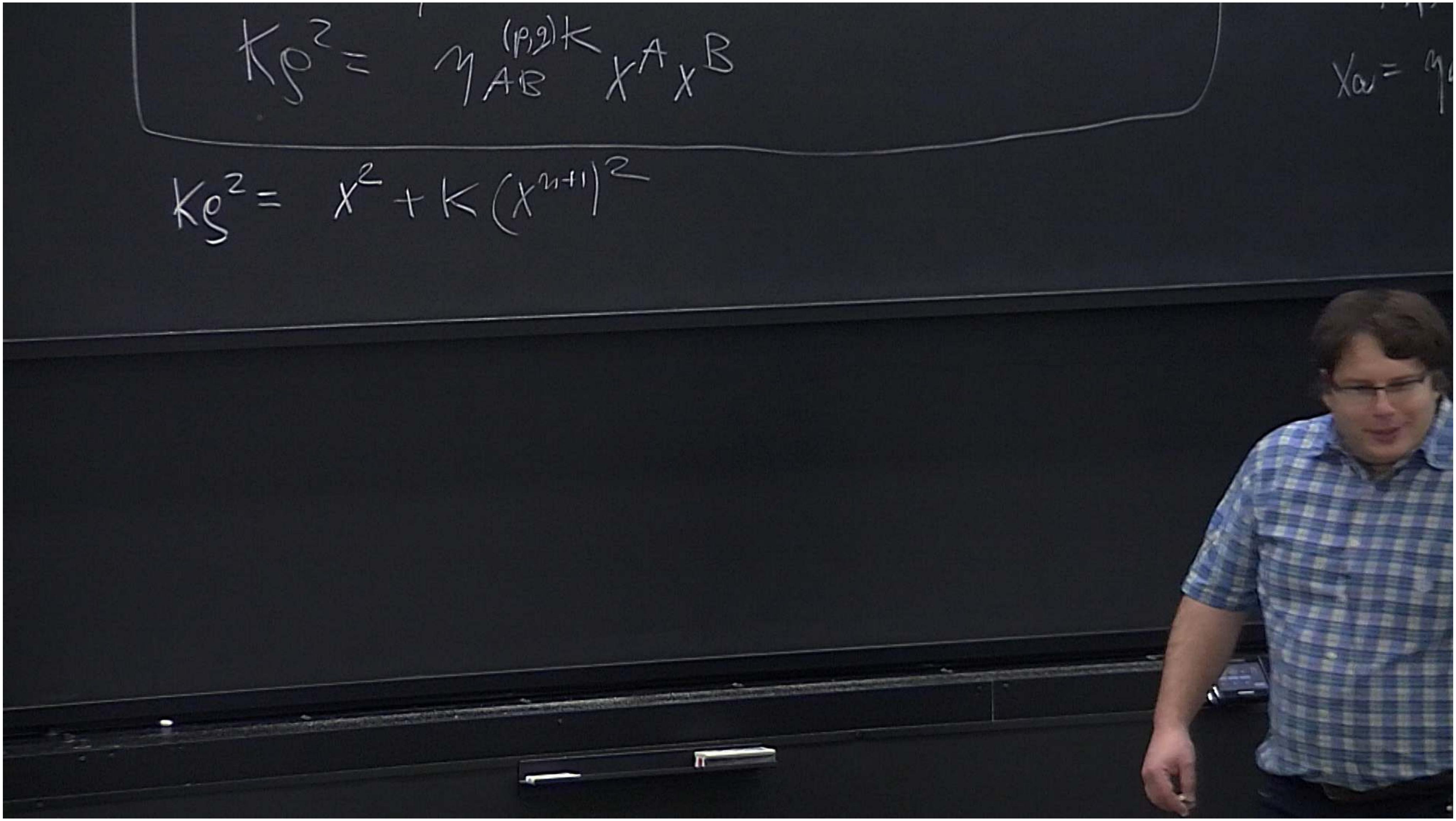
$$\boxed{(x^{n+1})^2 = \eta^{(p,q)k}_{AB}\, dx^A dx^B}$$

$$\eta^{(p,q)}_{ab} = (\underbrace{+,+,\dots,+}_{p},\underbrace{-,-,\dots,-}_{q})$$

$a,b = 1,\dots,n$

$A,B = 1,\dots,n+1$

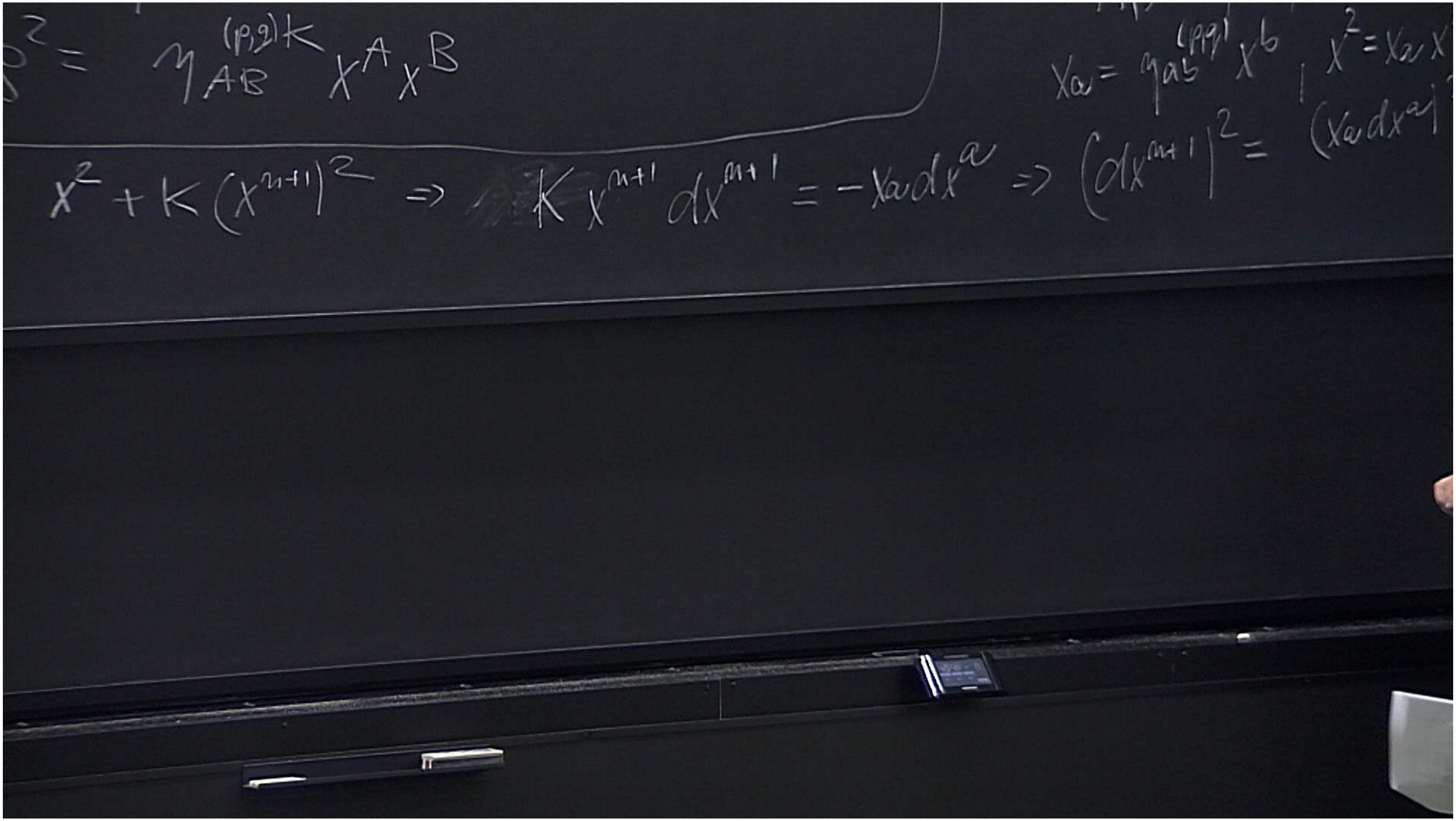
$x_a = \eta^{(p,q)}_{ab} x^b \,,\; x^2 = x_a x^a$

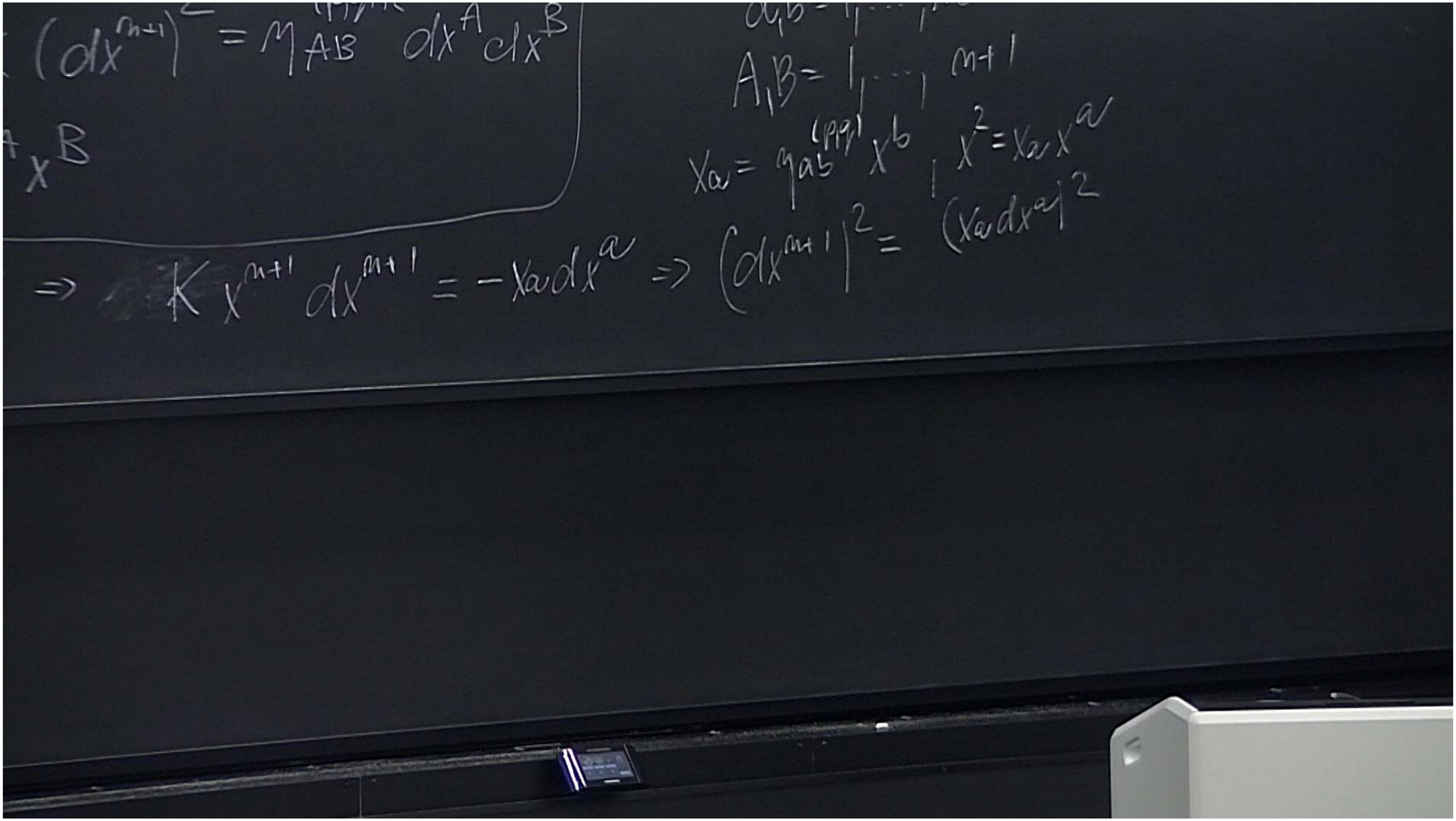

$K\rho^2 = \eta^{(p,q)K}_{AB} X^A X^B$
$K\rho^2 = X^2 + K(X^{n+1})^2$

$$\boxed{K\rho^2 = \eta^{(p,q)K}_{AB}\, X^A X^B}$$

$$X_a = \eta$$

$$K\rho^2 = x^2 + K\left(X^{n+1}\right)^2 \Rightarrow K X^{n+1} dX^{n+1} = -X_a dX^a$$


$g^2 = \eta^{(p,q)K}_{AB} X^A X^B$
$X_a = \eta^{(p,q)}_{ab} X^b \; ; \; X^2 = X_a X^a$
$X^2 + K(X^{n+1})^2 \Rightarrow K X^{n+1} dX^{n+1} = -X_a dX^a \Rightarrow (dX^{n+1})^2 = (X_a dX^a)$


$(dx^{n+1})^2 = \eta_{AB}\, dx^A dx^B$
$A, B = 1, \dots, n+1$
$x_a = \eta_{ab}^{(p,q)} x^b, \quad x^2 = x_a x^a$
$\Rightarrow K x^{n+1} dx^{n+1} = -x_a dx^a \Rightarrow (dx^{n+1})^2 = (x_a dx^a)^2$

$$\boxed{g_{ab}^{(p,q|K)} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\rho^2 - K x^2}}$$

$$\boxed{g_{ab}^{(p,q)|K} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\rho^2 - K x^2}}$$

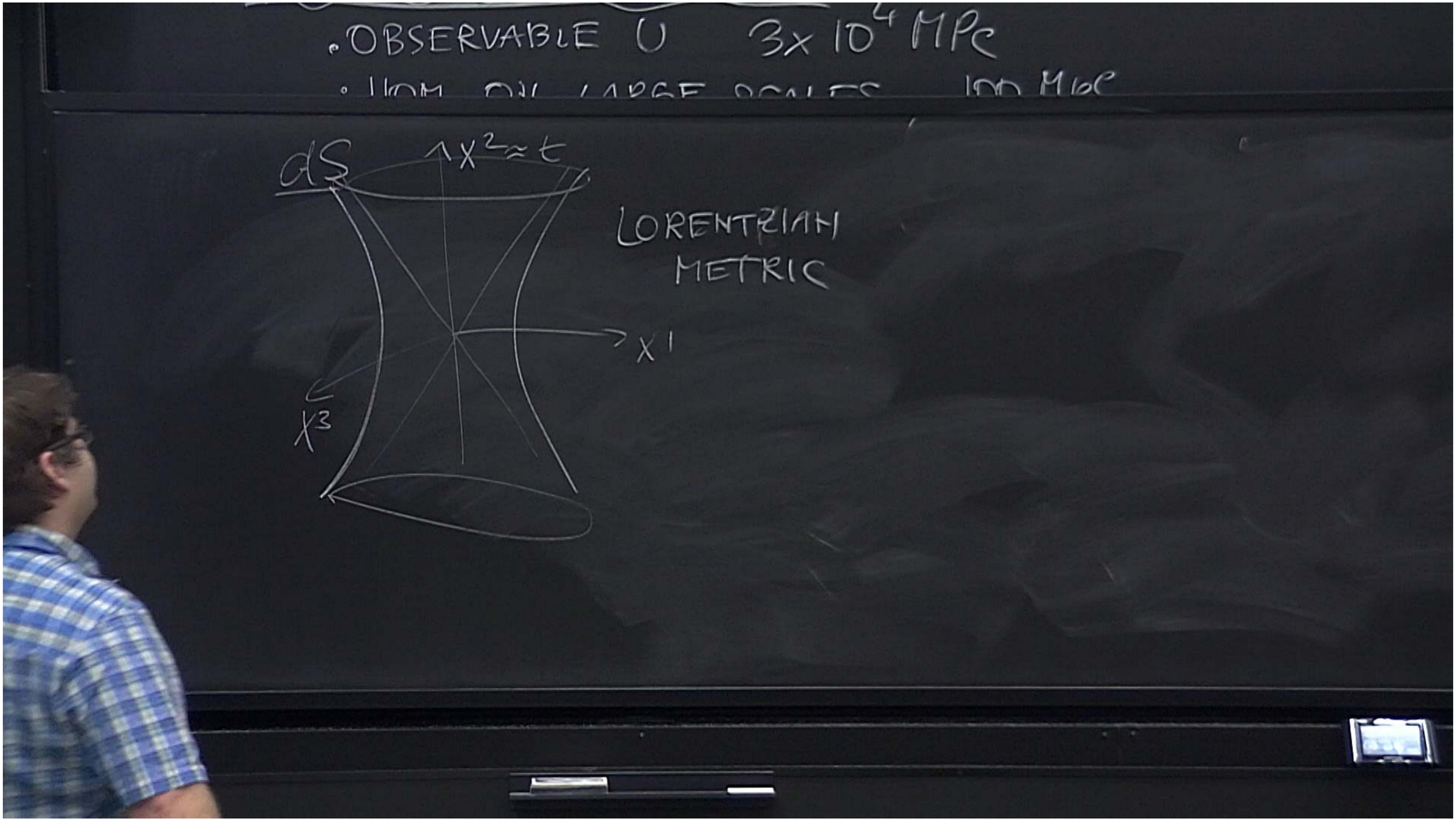
$$\Rightarrow \quad R_{abcd} = \frac{K}{\rho^2}\left(g_{ac} g_{bd} - g_{ad} g_{bc}\right)$$

EE: $\quad G_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$

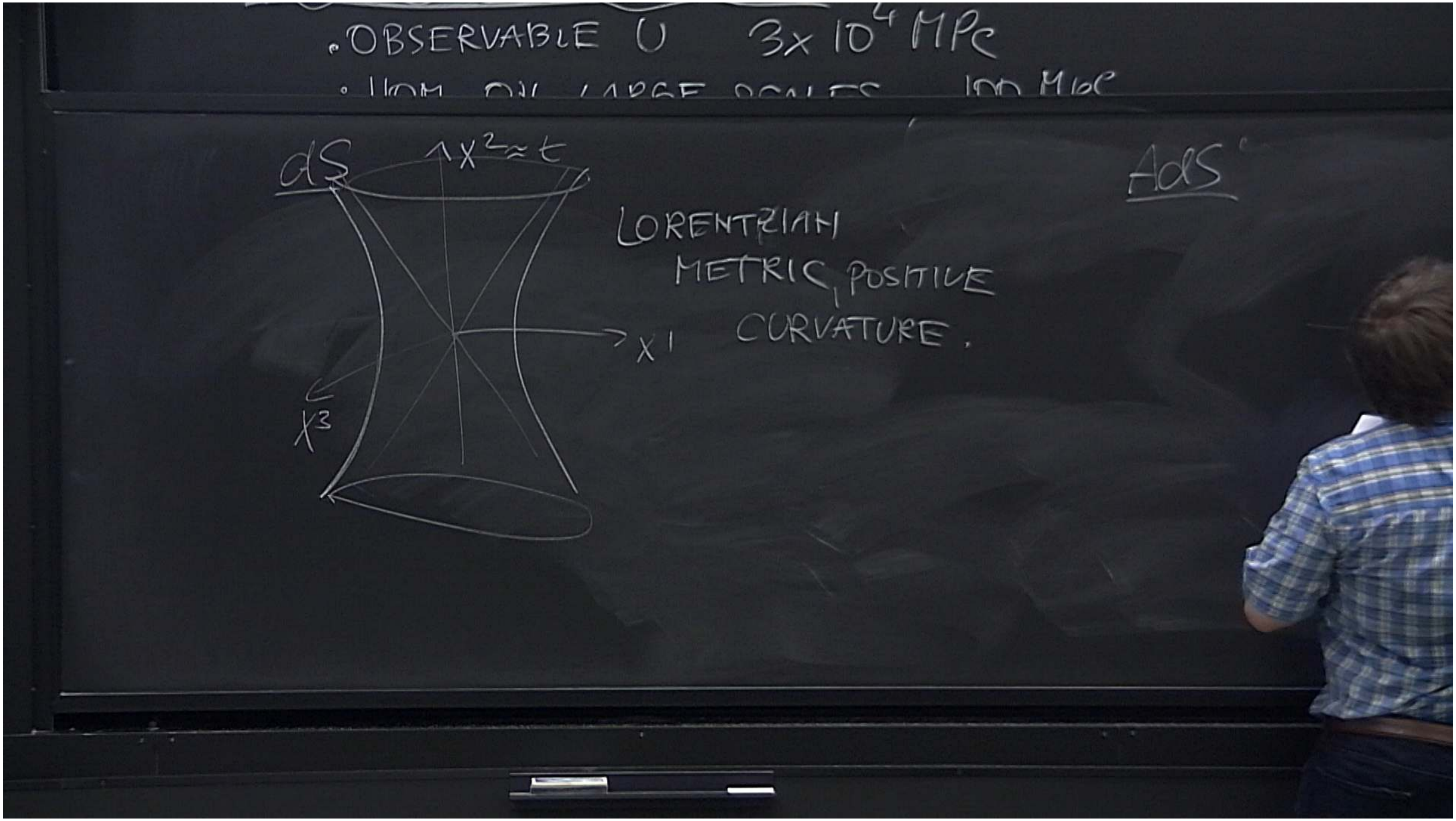
SOLUTIONS OF $T_{ab} = 0$, $\Lambda = \frac{K(n-1)(n-2)}{2\rho^2}$ EE.

(p,q)	$K=0$	$K=+1$	$K=-1$
$(m,0)$	E_m	S_m	H_m
$(m-1,1)$	M_m	dS_m	AdS_m
$(m-2,2)$	END OF PHYSICS STORY.		

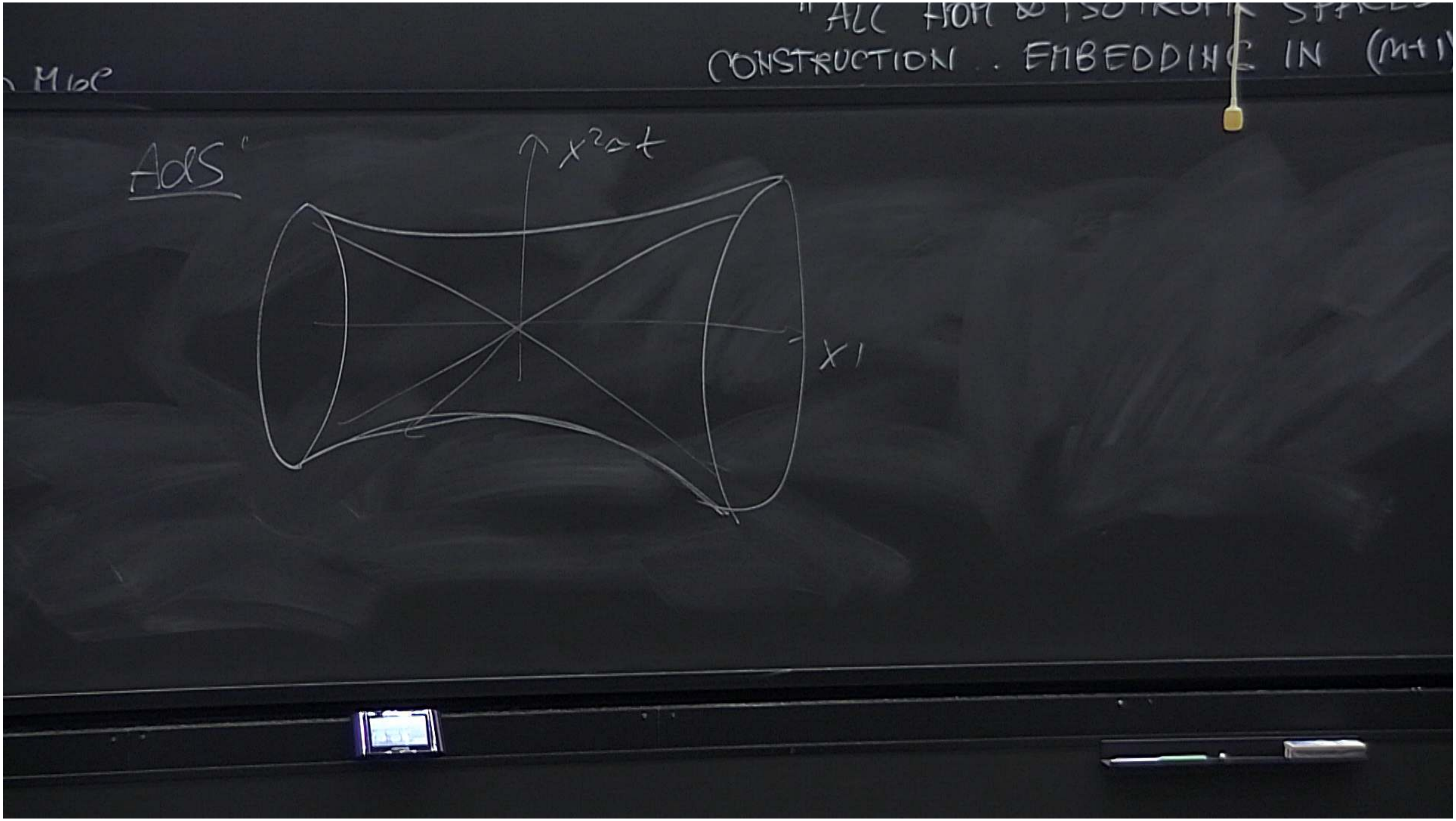
E:

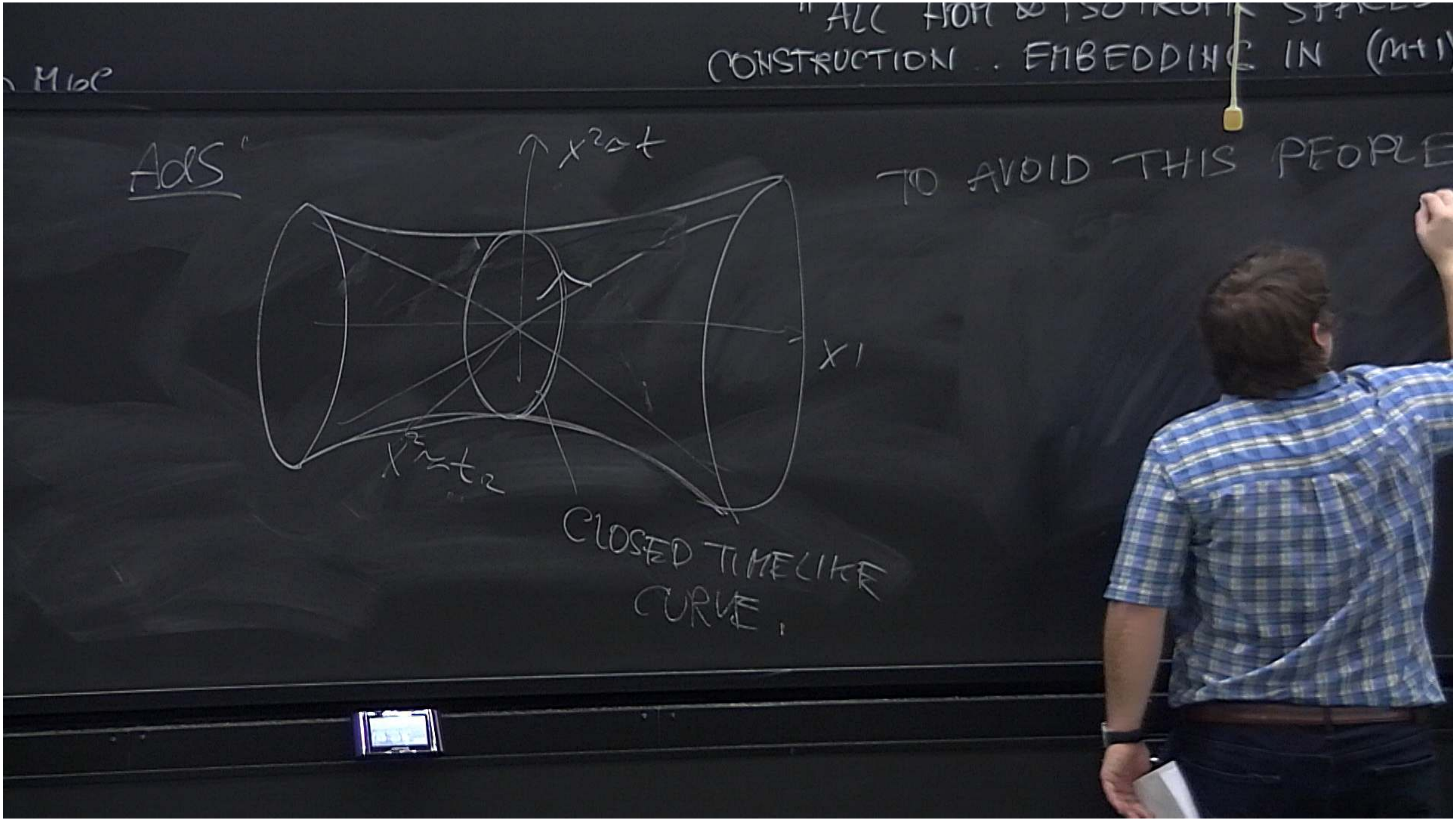


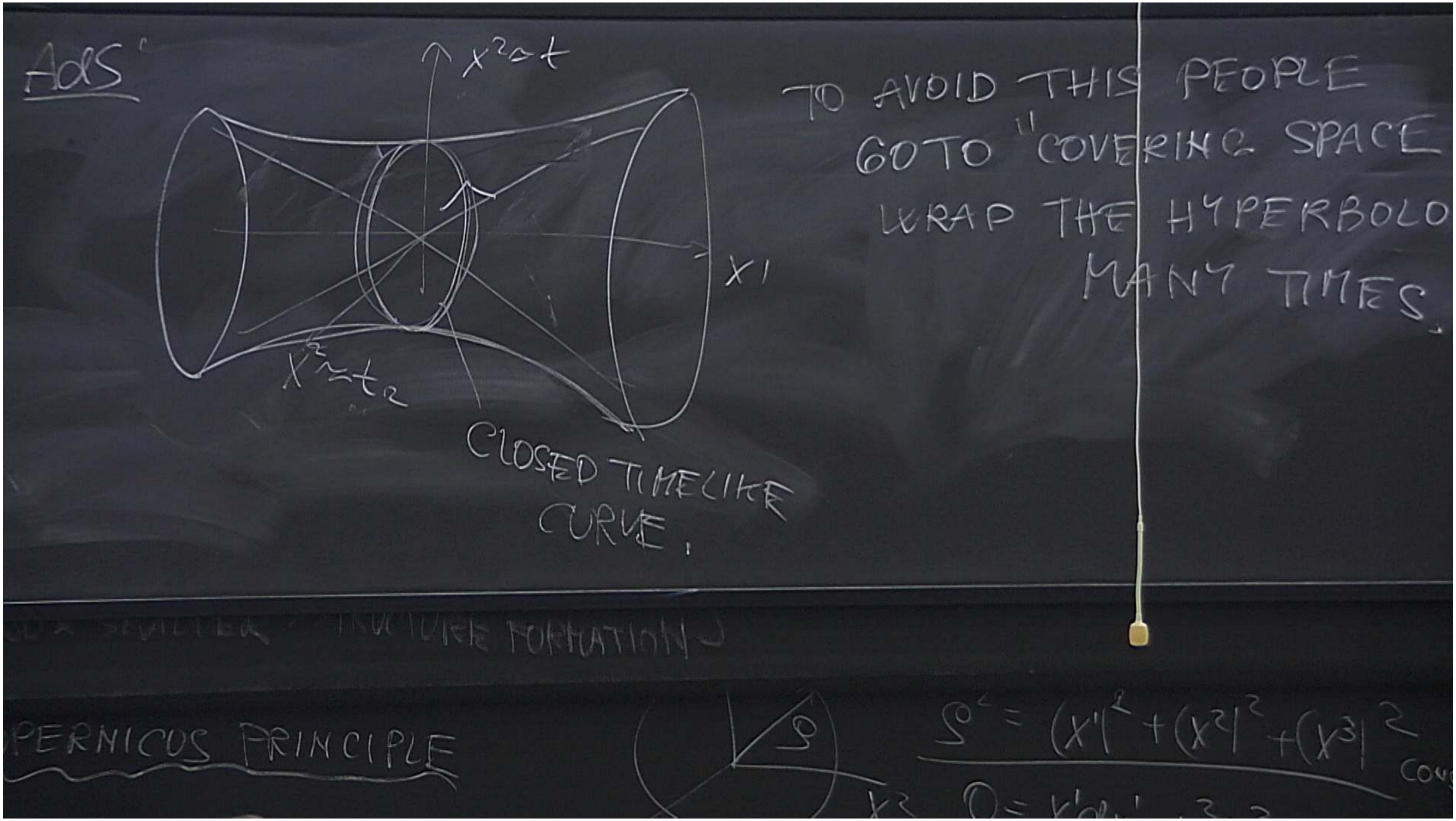
.OBSERVABLE U 3x 10^4 MPc
dS
x^2 ≈ t
LORENTZIAN
METRIC
x^1
x^3



• OBSERVABLE U 3x 10^4 MPc
dS
X² ≈ t
LORENTZIAN
METRIC, POSITIVE
CURVATURE.
X¹
X³
AdS


CONSTRUCTION .. EMBEDDING IN (n+1)
AdS
$x^2 = t$
x^1


CONSTRUCTION . . EMBEDDING IN (n+1)
AdS
x2~t
X1
CLOSED TIMELIKE CURVE.
TO AVOID THIS PEOPLE


AdS
$x^2 \simeq t$
x^1
$x^2 \simeq t_2$
CLOSED TIMELIKE CURVE.
TO AVOID THIS PEOPLE GOTO "COVERING SPACE WRAP THE HYPERBOLO MANY TIMES.
…RUCTURE FORMATION
…PERNICUS PRINCIPLE
$\rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$
CONS

WHY MAXIMALLY SYMMETRIC?

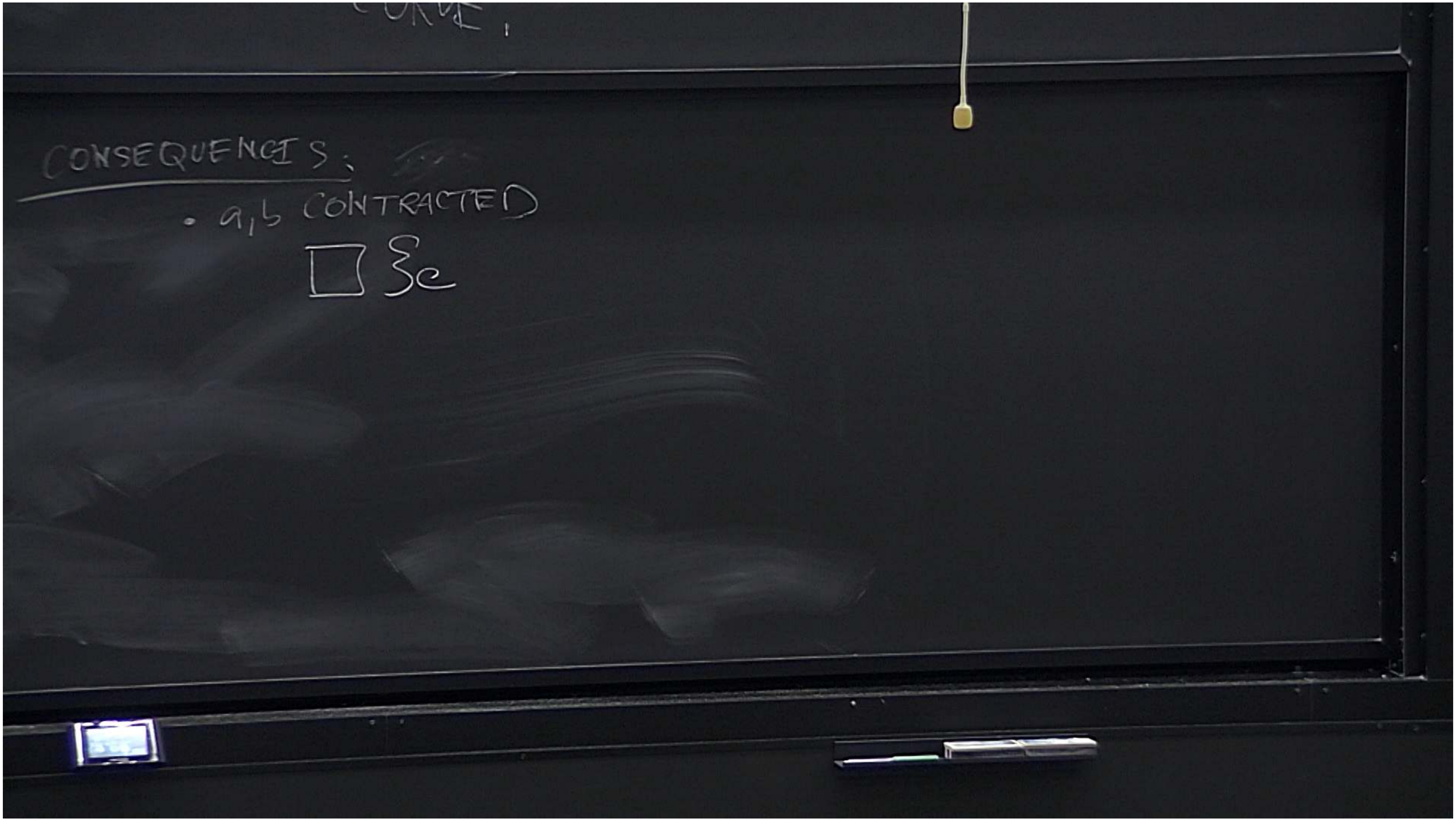
CONSEC

SYMMETRIES ≏ KILLING VECTORS

$$\nabla_a \xi_b + \nabla_b \xi_a = 0$$

STUDY INTEGRABILITY COND.

$$\boxed{\nabla_a \nabla_b \xi_c = - R_{bca}{}^{d} \xi_d}$$


CONSEQUENCES:
• a,b CONTRACTED
□ ξc

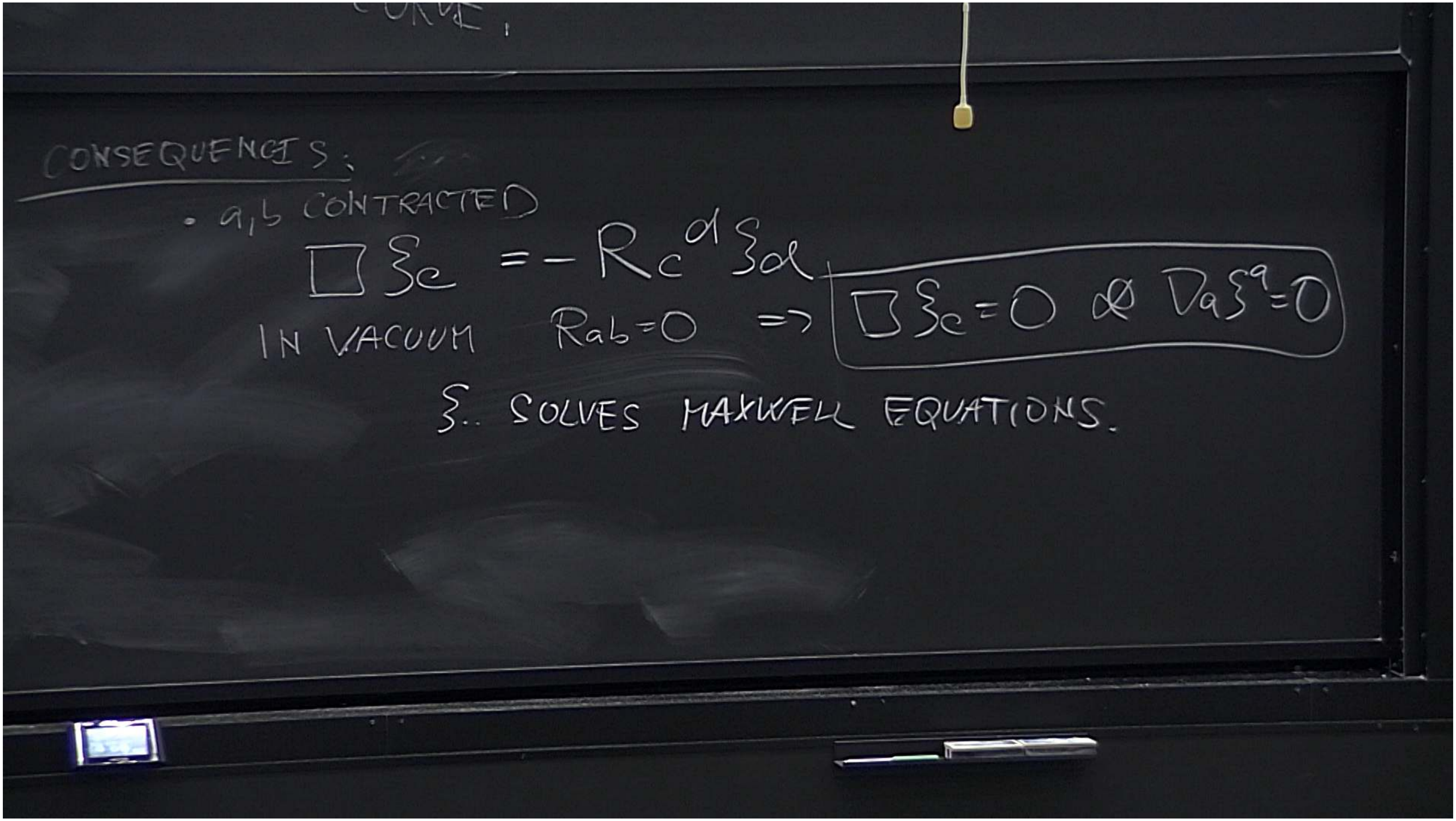
CURVE.

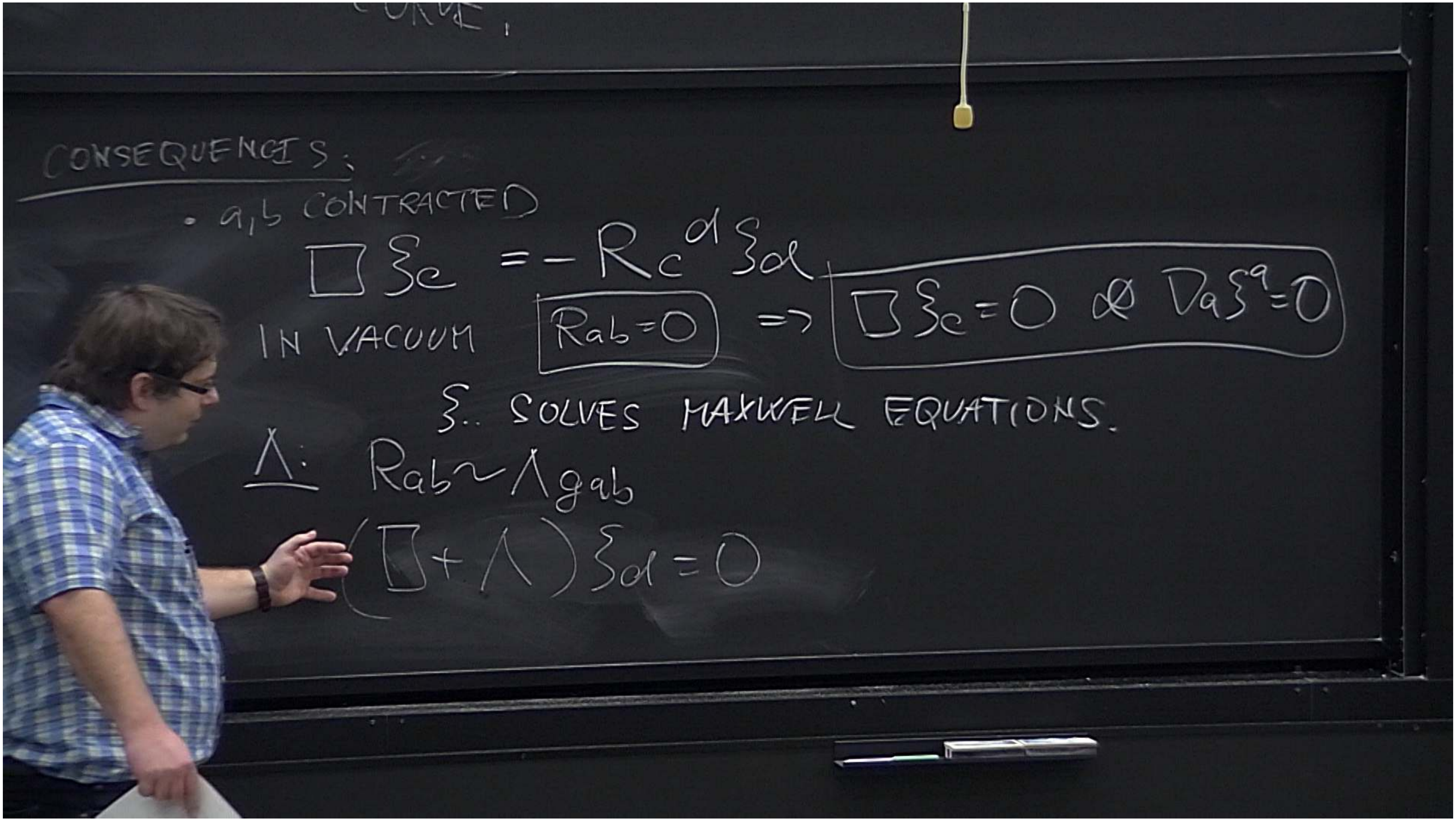
CONSEQUENCES:

• a, b CONTRACTED

$$\Box \xi_c = -R_c{}^d \xi_d$$

IN VACUUM $R_{ab}=0$ $\Rightarrow$ $\boxed{\Box \xi_c = 0 \;\&\; \nabla_a \xi^a = 0}$

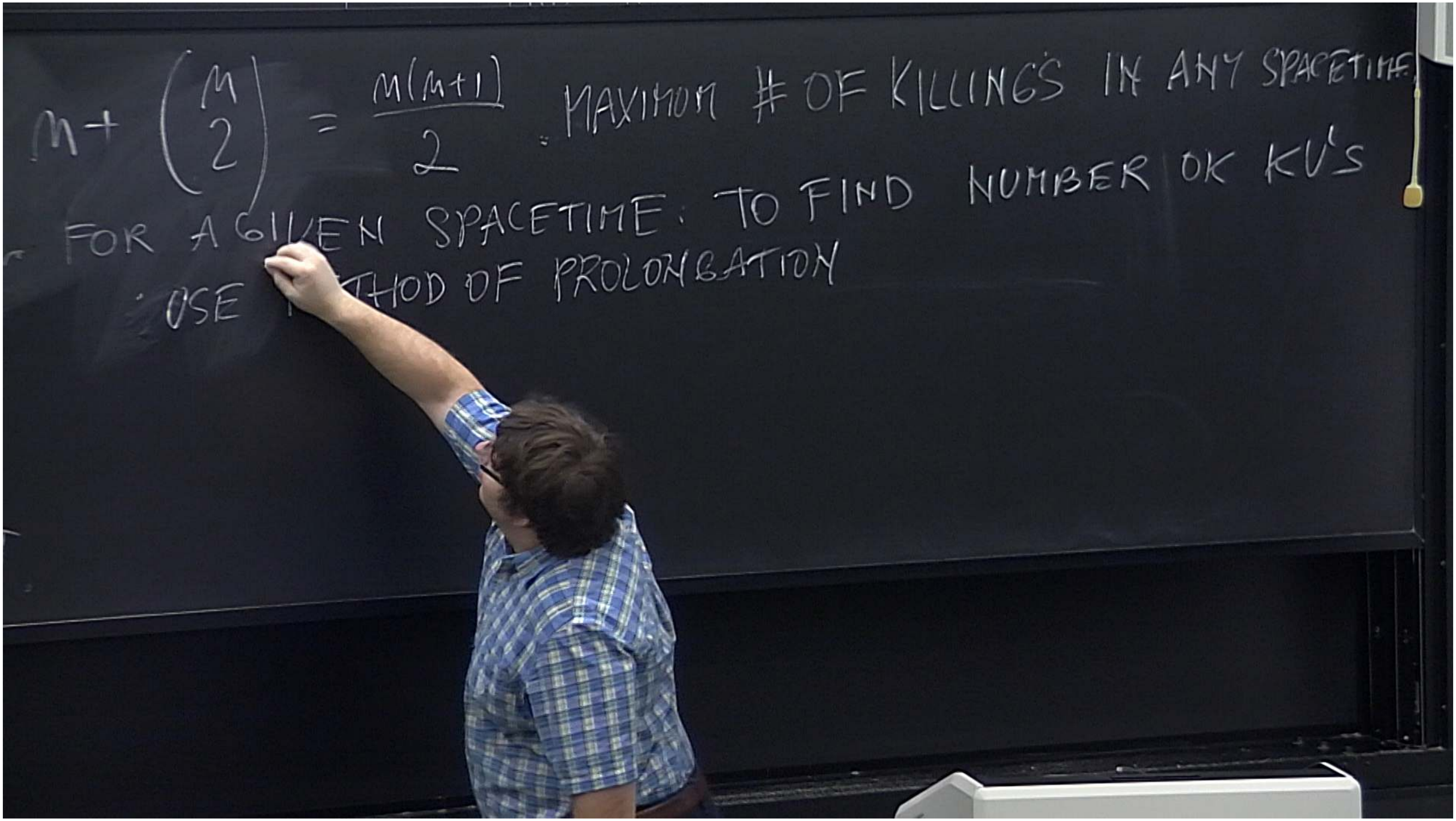

CONSEQUENCES:
• a,b CONTRACTED
$\Box \xi_c = -R_c{}^d \xi_d$
IN VACUUM $R_{ab}=0 \Rightarrow \boxed{\Box \xi_c = 0 \ \& \ \nabla_a \xi^a = 0}$
ξ.. SOLVES MAXWELL EQUATIONS.


CONSEQUENCES:
• a, b CONTRACTED
$\Box \xi_c = -R_c{}^d \xi_d$
IN VACUUM $\boxed{R_{ab}=0}$ $\Rightarrow$ $\boxed{\Box \xi_c = 0 \ \& \ \nabla_a \xi^a = 0}$
ξ.. SOLVES MAXWELL EQUATIONS.
Λ: $R_{ab} \sim \Lambda g_{ab}$
$(\Box + \Lambda)\xi_d = 0$

- $\nabla_a \xi_b = \nabla_{[a}\xi_{b]} = L_{ab}$
- $\nabla_a L_{bc} = -R_{bca}{}^{d}\xi_d$

CLOSED SYSTEM OF EQS. FOR L_{ab}, ξ_c

$\Rightarrow$ TO SOLVE THIS, ENOUGH TO SPECIFY L_{ab}, ξ_c AT ONE POINT.


$m + \binom{m}{2} = \frac{m(m+1)}{2}$: MAXIMUM # OF KILLINGS IN ANY SPACETIME
FOR A GIVEN SPACETIME: TO FIND NUMBER OF KV's
USE
THOD OF PROLONGATION

$n + \binom{n}{2} = \frac{n(n+1)}{2}$: MAXIMUM # OF KILLINGS IN ANY SPACETIME

- FOR A GIVEN SPACETIME: TO FIND NUMBER OF KV'S

: USE "METHOD OF PROLONGATION"

$$R \llcorner L = (DR) \llcorner \xi + R \llcorner L$$

$\frac{\ }{2}$

FOR A GIVEN SPACETIME: TO FIND NUMBER OF KV's

USE "METHOD OF PROLONGATION"

$$R \llcorner L = (DR) \llcorner \xi + R \llcorner L$$

↑ KNOW ... ALGEBRAIC EQ FOR ξ, L.