Title: PSI 2016/2017 Condensed Matter (Review) - Lecture 1

Date: Jan 30, 2017 09:00 AM

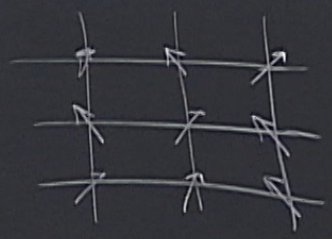
URL: http://pirsa.org/17010084

Abstract:

Many-body Entanglement and Tensor Networks

Condensed Matter Review

Motivation



$$H = -\sum_{\langle m,n \rangle}^{N} \sigma_m^x \sigma_n^x - h \sum_{m=1}^{N} \sigma_m^z$$

N quantum

$\sigma^x \equiv \begin{pmatrix} 0 & \\ 1 & \end{pmatrix}$

Hilbert space $V^{(N)} = \bigotimes_{m=1}^{N} V^{(1)}$ $V^{(1)}$

$\dim V^{(N)} = \left(\dim V^{(1)}\right)^N = \exp$

Networks

Condensed Matter Review

$$= -\sum_{\langle m,n \rangle}^{N} \sigma_m^x \sigma_n^x - h \sum_{m=1}^{N} \sigma_m^z$$

N quantum spins

$\sigma^x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Pauli matrices

st space $V^{(N)} = \bigotimes_{m=1}^{N} V^{(1)}$ $\quad V^{(1)} \cong \mathbb{C}_{12}$

$\dim V^{(N)} = \left(\dim V^{(1)}\right)^N = \exp aN \quad 2^N$

Task: find the ground state of H

$$H|\psi\rangle = E_0|\psi\rangle$$

↑ smallest eigenvalue

Task: find the ground state of H

$$H|\psi\rangle = E_0|\psi\rangle$$

E_0: smallest eigenvalue

1) minimal

2) Generic call

Possible approaches

a) Exact diagonalization $\longrightarrow$ cost $= \exp(N)$ in practice N

b) Perturbation theory

exact solution + small parameter $\rightarrow$ it fails for strong interactions

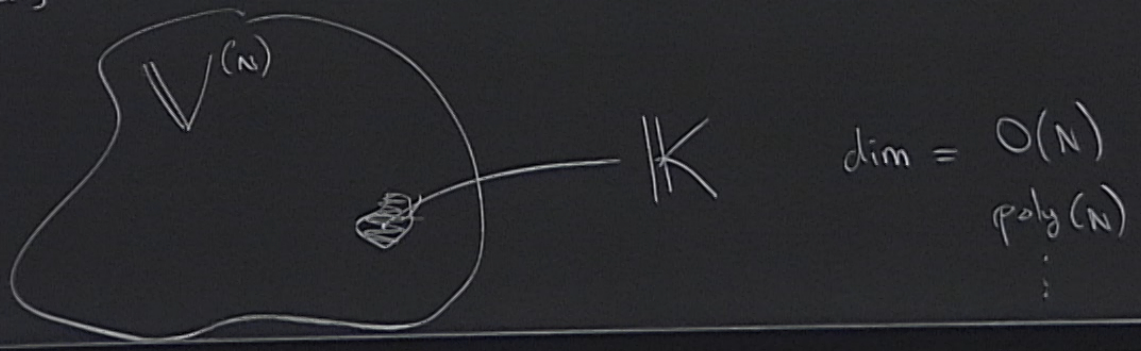
c) Quantum Monte Carlo (Sampling) $\longrightarrow$ Sign problem

1) minimal model Hamiltonian — crude approximation { - degrees of freedom; - interactions

2) Generically, we do not know how to solve H

N) in practice $N \lesssim 40$ spins

or strong interactions

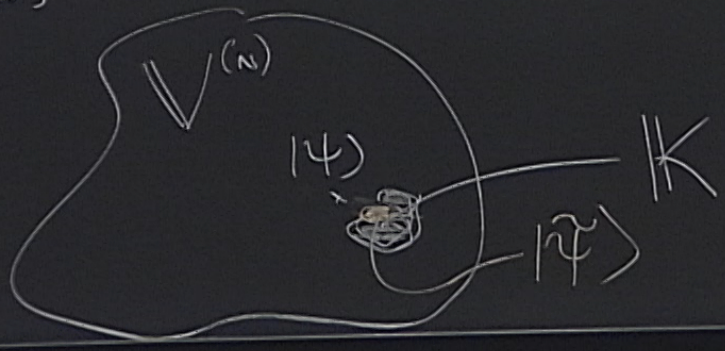


1) minimal model Hamiltonian — crude approximation { - degrees of freedom; - interactions

2) Generically, we do not know how to solve H

N) in practice $N \lesssim 40$ spins

for strong interactions



dim = $O(N)$
poly(N)
⋮

Task: find the ground state of H

$$H|\psi\rangle = E_0|\psi\rangle$$

E_0: smallest eigenvalue

1) minimal

2) Generically

Possible approaches

a) Exact diagonalization $\longrightarrow$ cost $= \exp(N)$ in practice N

b) Perturbation theory

exact solution + small parameter $\rightarrow$ it fails for strong interactions

c) Quantum Monte Carlo (Sampling) $\longrightarrow$ Sign problem

d) Variational approaches

$\dim = 2^N \rightarrow V^{(N)} \supseteq K$ submanifold

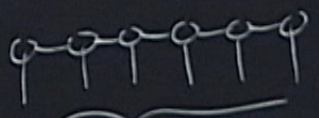
part I: The many-body Computational challenge

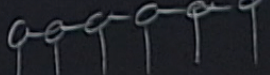
- exact diagonalization — $N \simeq 10$, $N \simeq 20$
- free fermions — $N \to 100$'s, $N = \infty$

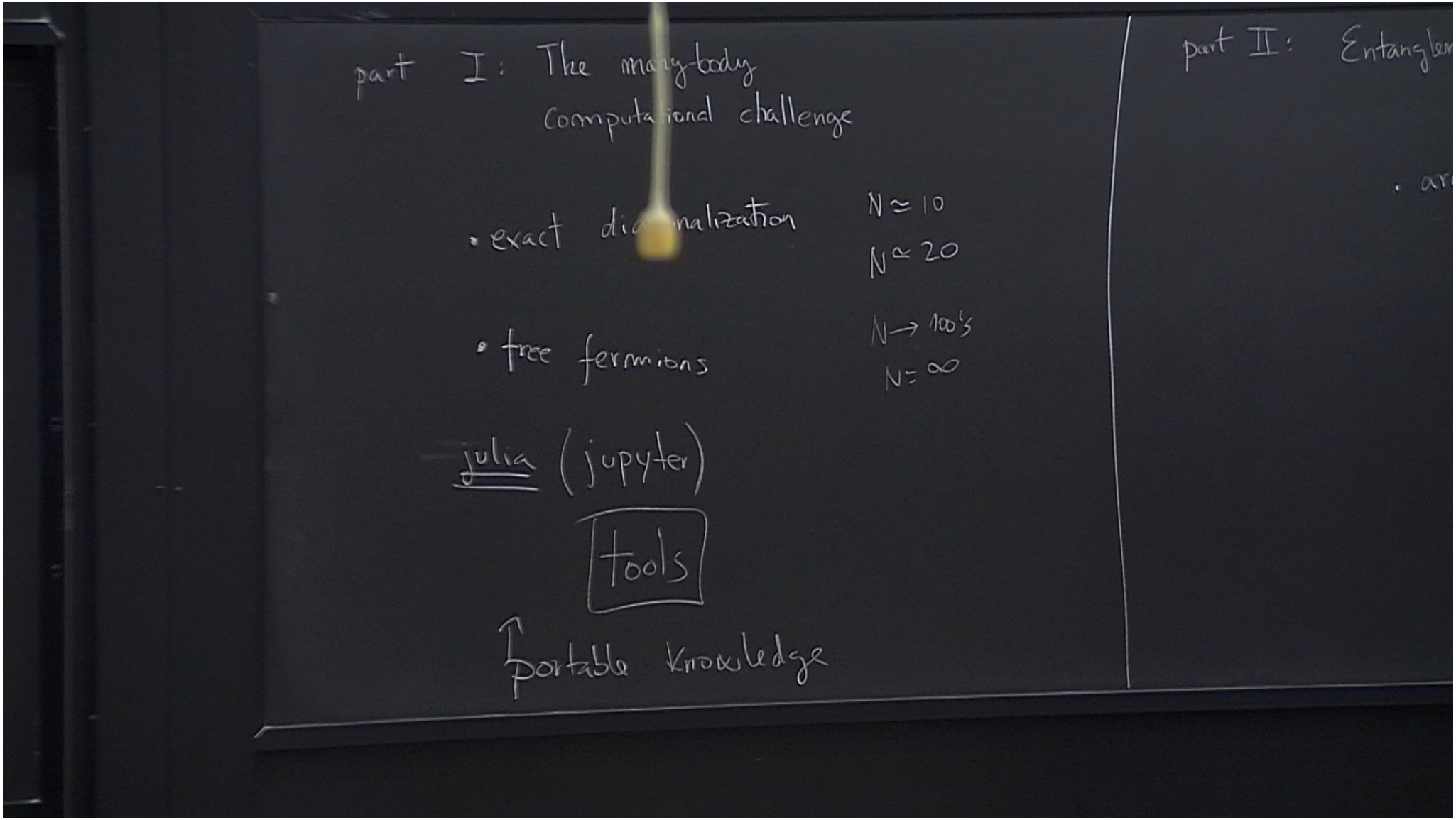
julia (jupyter)

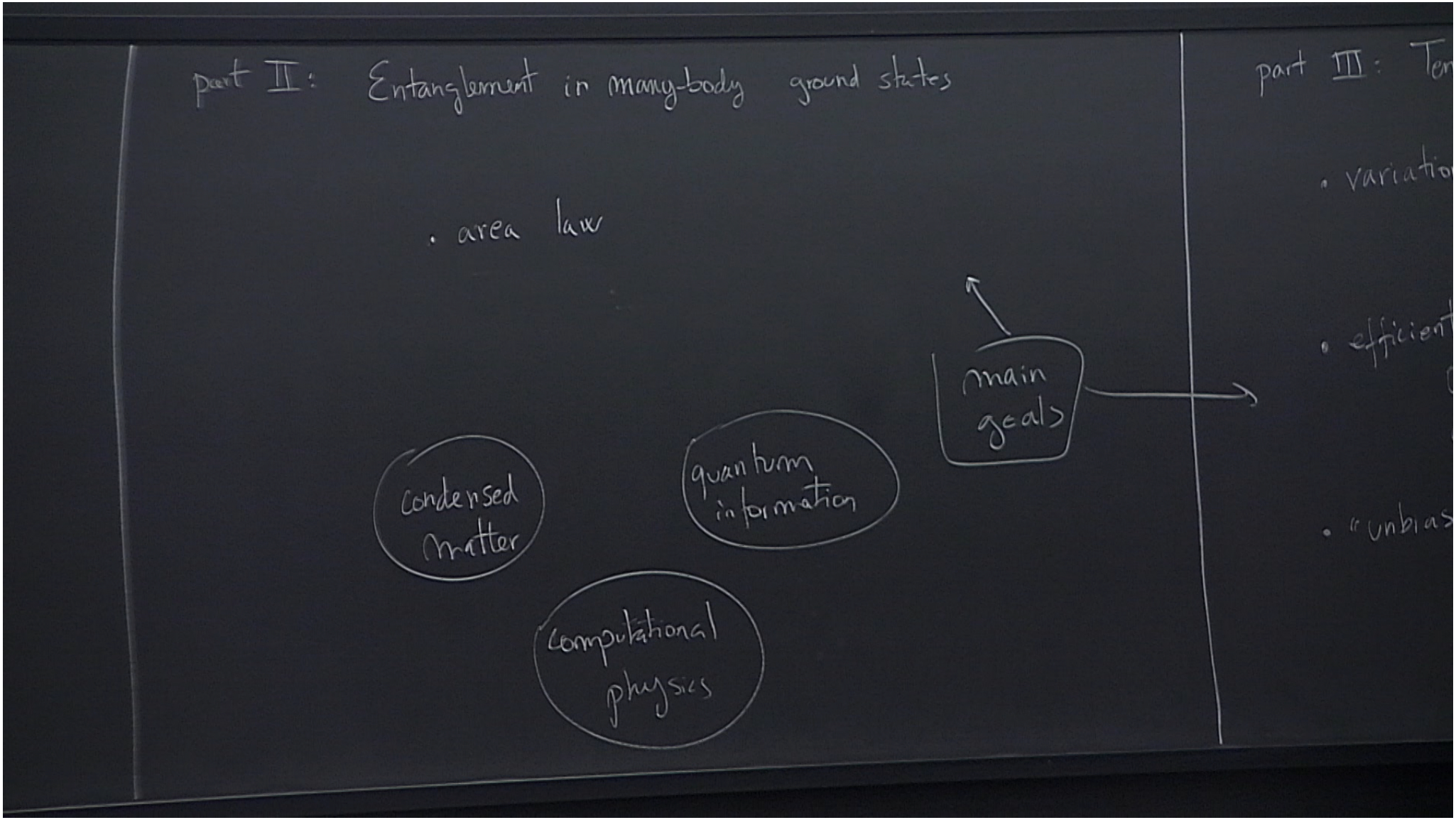
part III: Tensor network states

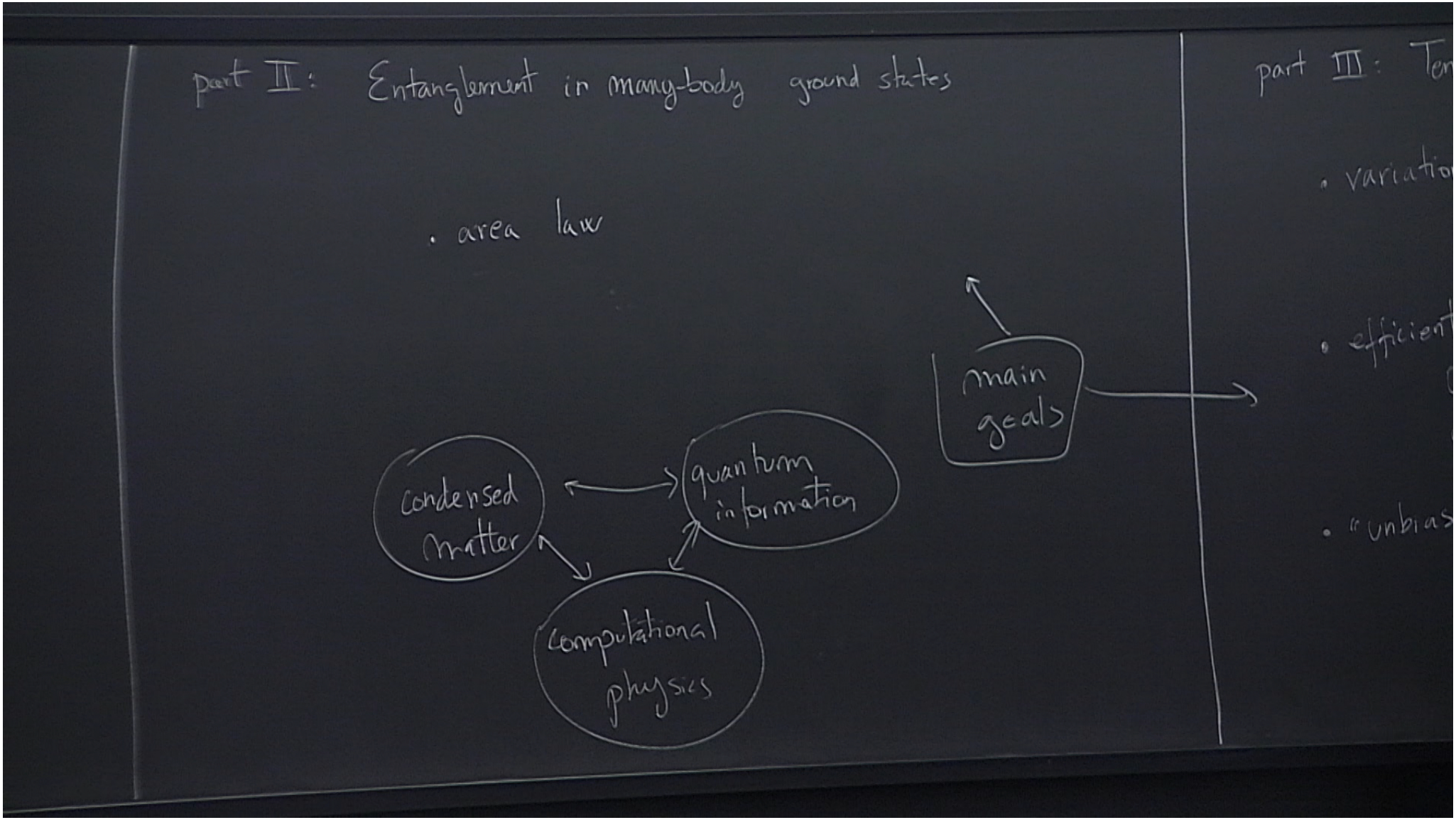
- variational approach

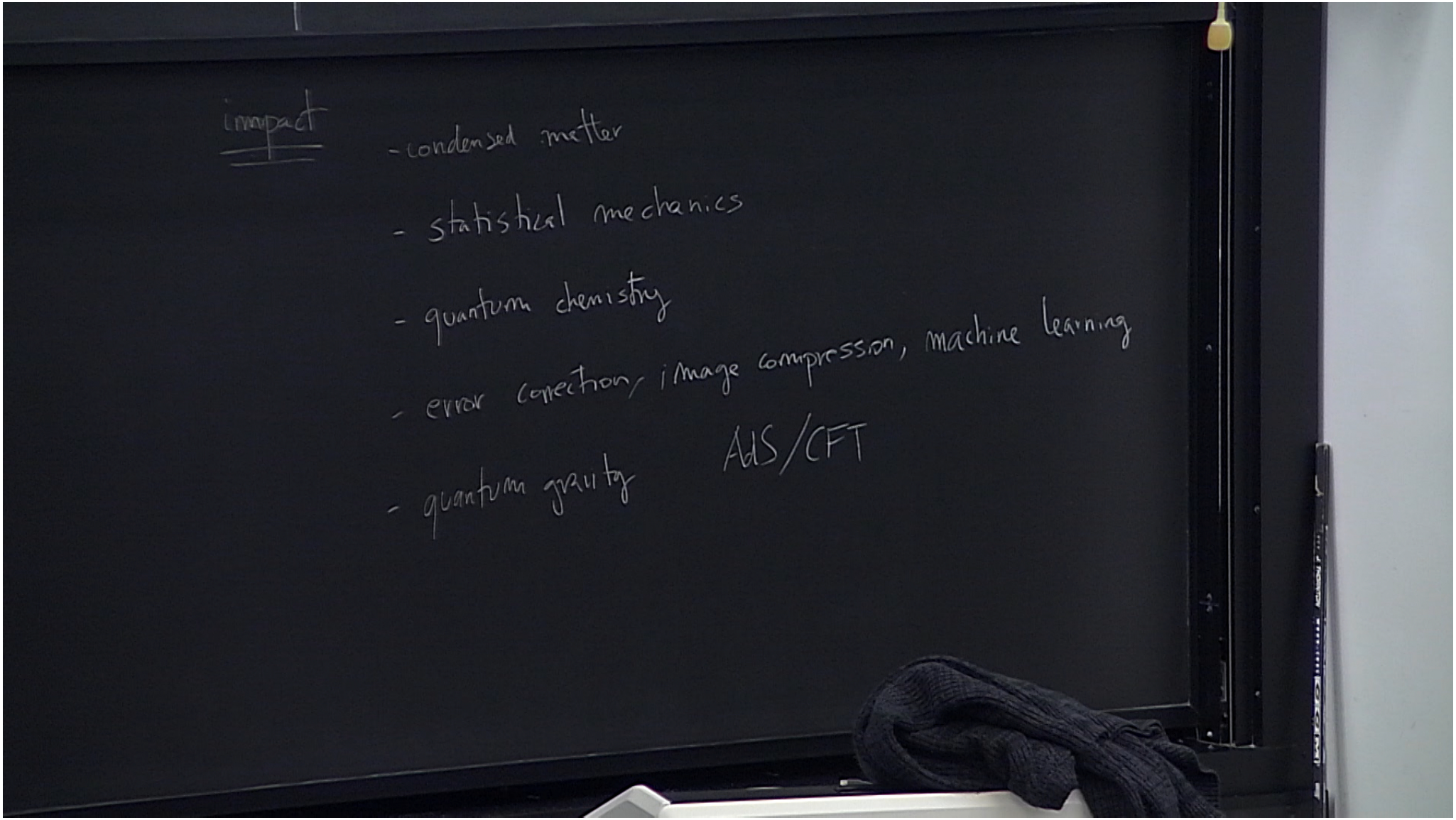
$|\Psi\rangle \sim$ 

- efficient
  - $O(N)$ tensors
  - $O(1)$ $N \to \infty$
- "unbiased" same 
  can represent "all" types of ground states.
  of <u>local</u> H


part I: The many-body
Computational challenge
• exact diagonalization
N ≃ 10
N ≃ 20
• free fermions
N → 100's
N = ∞
julia (jupyter)
tools
portable knowledge
part II: Entangle
. ar


part II: Entanglement in many-body ground states
• area law
main goals
condensed matter
quantum information
computational physics
part III: Ten
• variatio
• efficient
• "unbias


part II: Entanglement in many-body ground states
. area law
condensed matter
quantum information
computational physics
main goals
part III: Ten
. variatio
. efficient
. "unbias


impact
- condensed matter
- statistical mechanics
- quantum chemistry
- error correction, image compression, machine learning
- quantum gravity
AdS/CFT

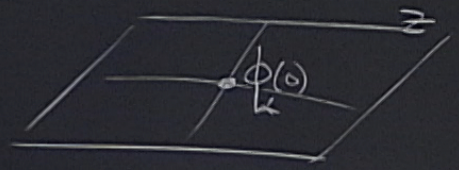
Lecture 1

Ising quantum spin chain $h=1$ critical magnetic field $\longrightarrow$ quantum

quantum spin chain $\xrightarrow[\text{long distance}]{\text{low energy}}$ quantum field theory QFT $\xrightarrow{h=1}$ conformal field theory CFT

CFT: ① scaling operators $\phi_\alpha(z)$

1+1 dimensions



Scale transformation

$z \longrightarrow \lambda z$

λ: scale factor

$\phi_\alpha(0) \longrightarrow \lambda^{\Delta_\alpha} \phi_\alpha(0)$

Δ_α: scaling dimens...

$h=1$ critical magnetic field $\longrightarrow$ quantum critical point $(T=0)$

quantum field theory QFT $\xrightarrow{h=1}$ conformal field theory CFT

$\phi_\alpha(0) \longrightarrow \lambda^{\Delta_\alpha} \phi_\alpha(0)$ ← scaling dimension of ϕ_α

rotations

$z \longrightarrow e^{i\theta} z$



$\phi_\alpha(0) \longrightarrow$

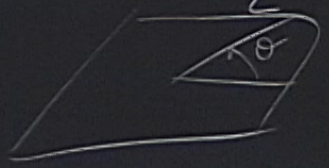
→ quantum critical point $(T=0)$

CFT

scaling dimension of ϕ_α

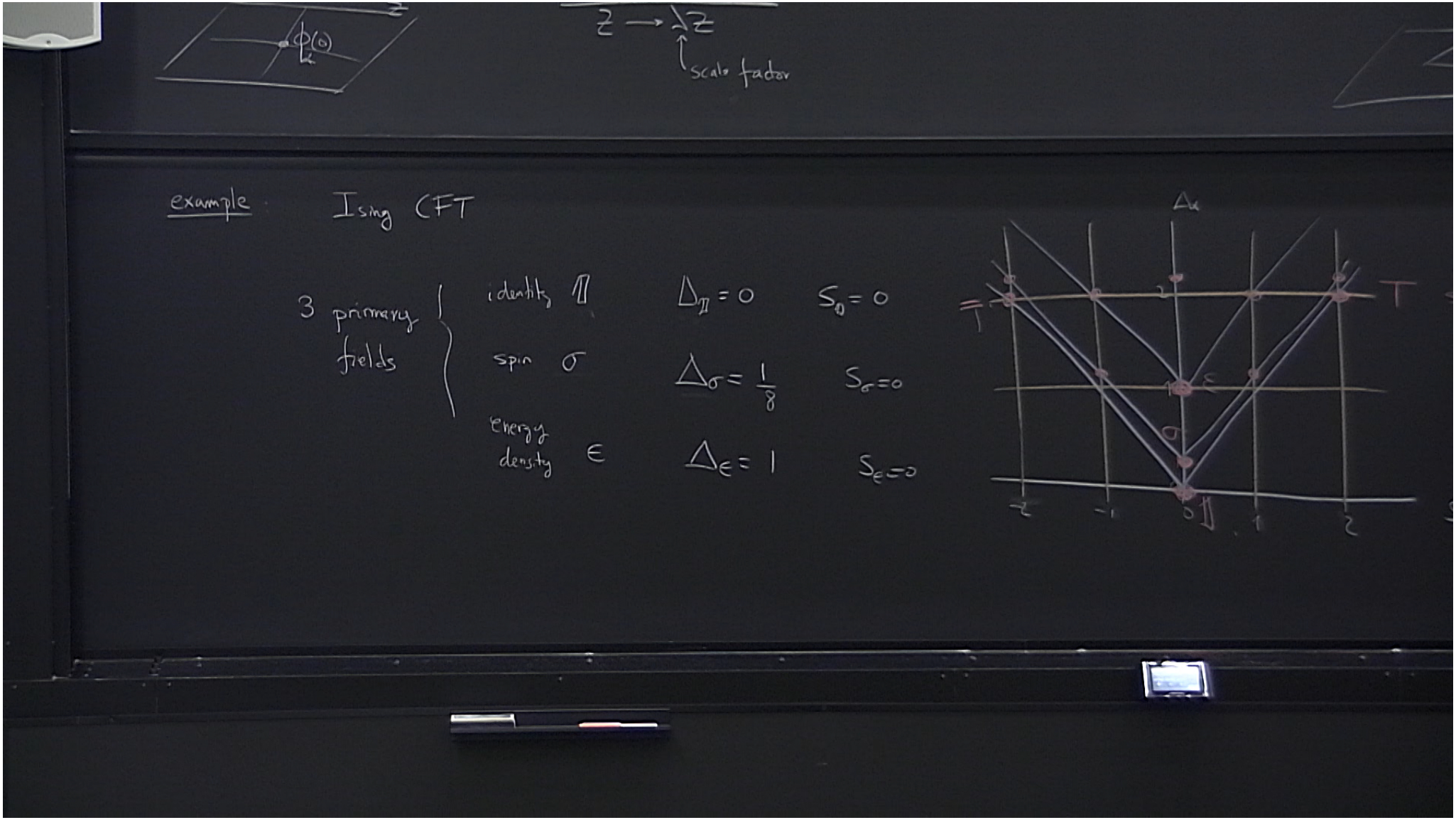
rotations

$z \to e^{i\theta} z$



$\phi_\alpha(0) \longrightarrow e^{i\theta\alpha} \phi_\alpha(0)$

conformal spin of $\phi_\alpha(0)$


$\phi(0)$
$z \to \lambda z$
scale factor
example
Ising CFT
3 primary fields
identity $\mathbb{1}$ $\Delta_{\mathbb{1}} = 0$ $S_{\mathbb{1}} = 0$
spin σ $\Delta_\sigma = \frac{1}{8}$ $S_\sigma = 0$
energy density ϵ $\Delta_\epsilon = 1$ $S_\epsilon = 0$
$\Delta_{\mathcal{O}}$
T
$\bar{T}$
ϵ
σ
$\mathbb{1}$
-2
-1
0
1
2

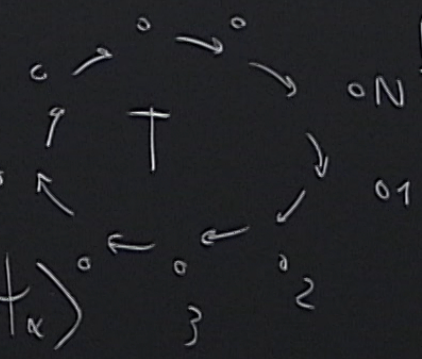
(2) - Operator / State correspondence

$\phi_\alpha \longleftrightarrow |\psi_\alpha\rangle$ '84

T stress-energy tensor - universal finite-size scaling ⟩ Cardy

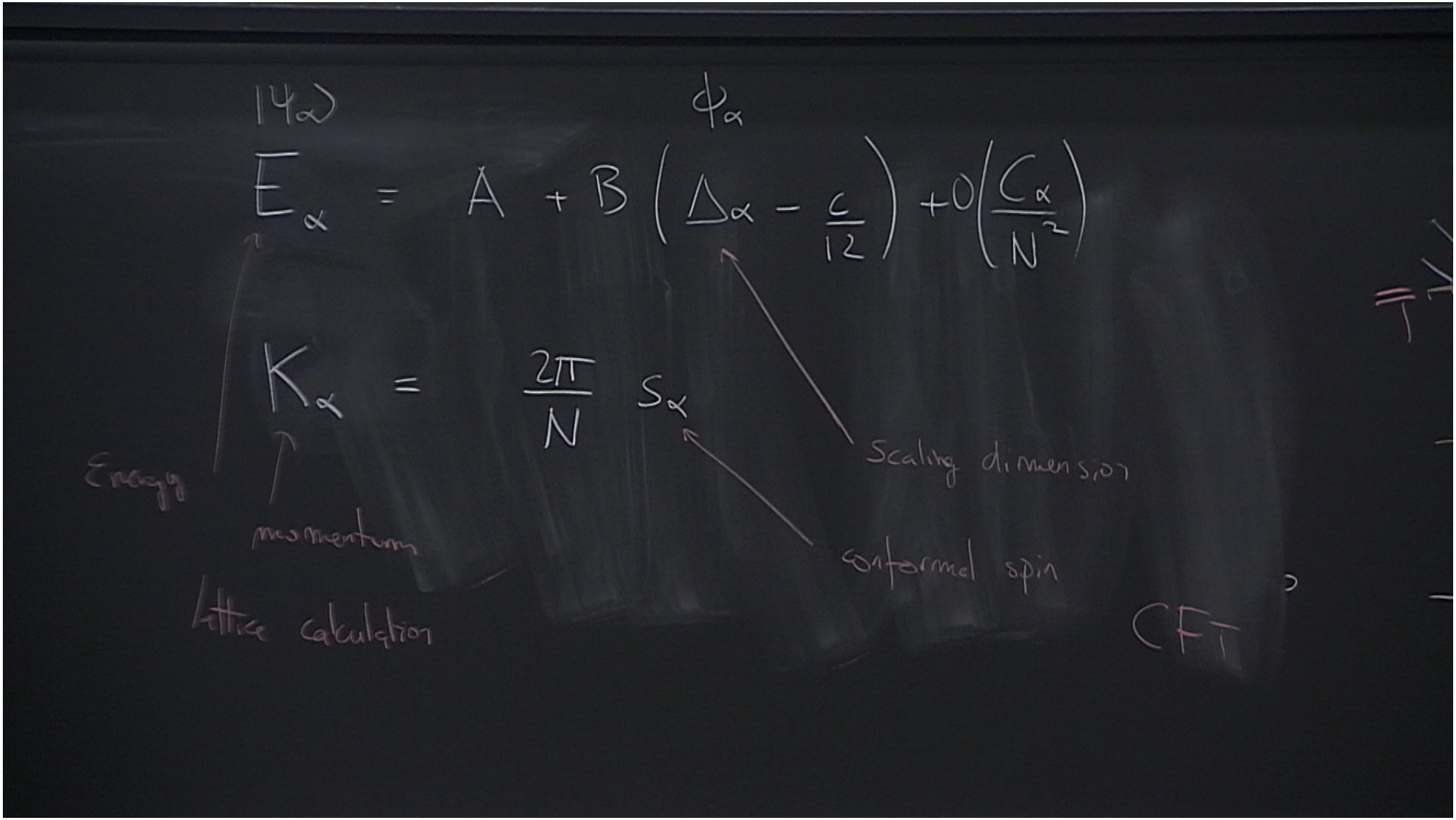
$[H, T] = 0$

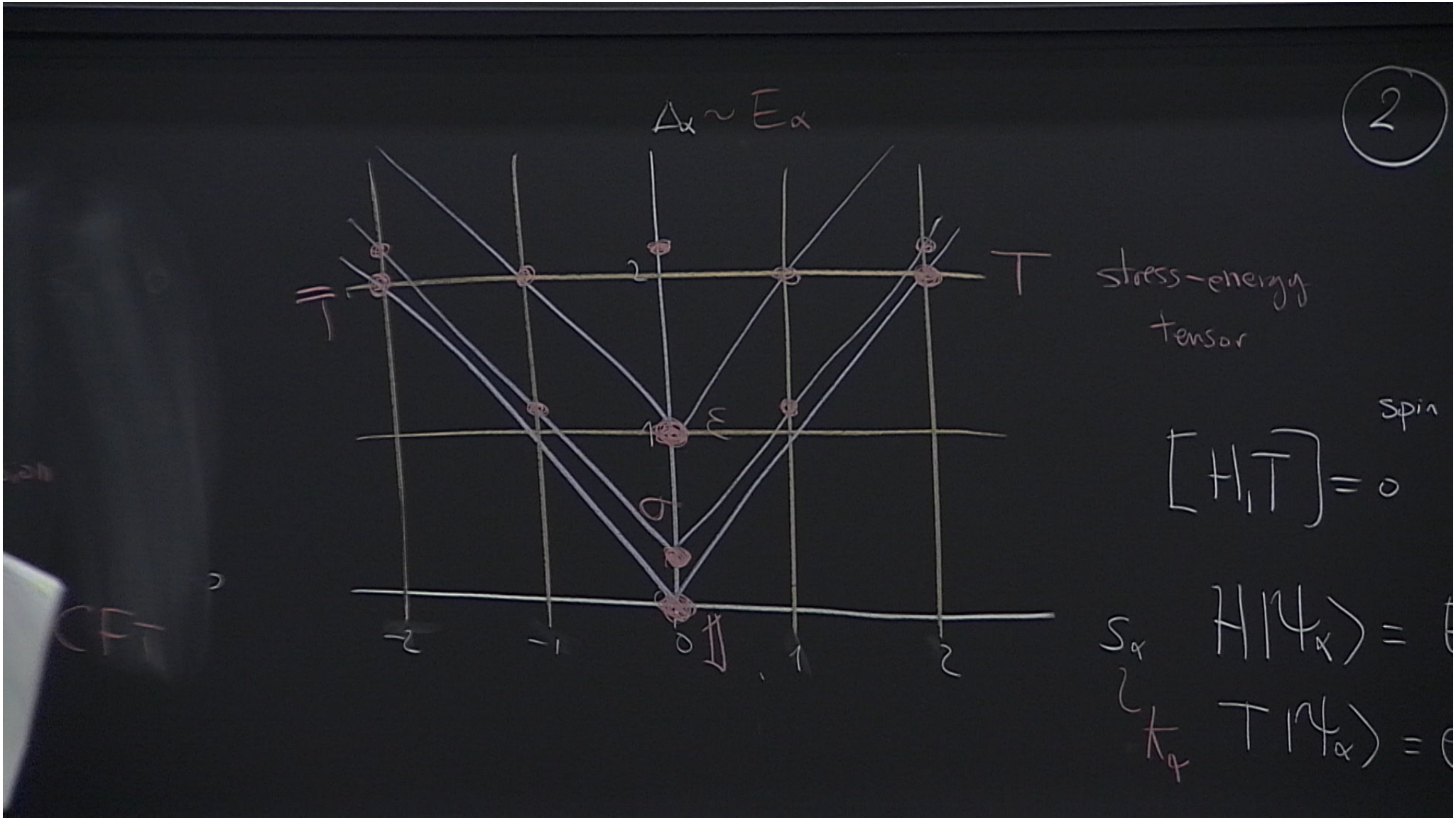
spin chain



$H|\psi_\alpha\rangle = E_\alpha|\psi_\alpha\rangle$ $T^N = \mathbb{1}$

$T|\psi_\alpha\rangle = e^{ik_\alpha}|\psi_\alpha\rangle$

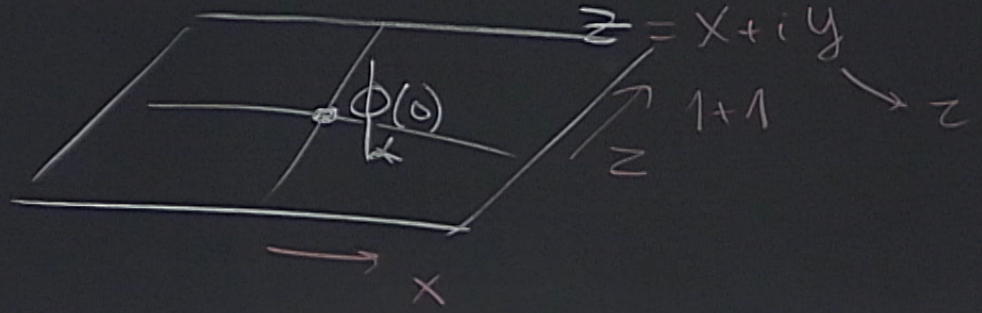

14a)
ψ_α
$E_\alpha = A + B\left(\Delta\alpha - \frac{c}{12}\right) + O\left(\frac{C_\alpha}{N^2}\right)$
$K_\alpha = \frac{2\pi}{N} S_\alpha$
Energy
momentum
Scaling dimension
conformal spin
lattice calculation
CFT


$\Delta_\alpha \sim E_\alpha$
2
T
stress-energy tensor
spin
$[H, T] = 0$
$H|\psi_\alpha\rangle =$
$T|\psi_\alpha\rangle =$
s_α
k_α
ε
σ
-2
-1
0
1
2

distance

CFT: (1) scaling operators $\phi_\alpha(z)$

1+1 dimensions



Scale transformation

$z \rightarrow \lambda z$

λ: scale factor

$$K_\alpha = \frac{2\pi}{N} S_\alpha$$

Energy

momentum

Scaling dimens