Title: The Conformal Bootstrap: From Magnets to Boiling Water

Date: Jan 25, 2017 02:00 PM

URL: http://pirsa.org/17010082

Abstract: <p>Conformal Field Theory (CFT) describes the long-distance
 dynamics of numerous quantum and statistical many-body systems. The
 long-distance limit of a many-body system is often so complicated that
 it is hard to do precise calculations. However, powerful new
 techniques for understanding CFTs have emerged in the last few years,
 based on the idea of the Conformal Bootstrap. I will explain how the
 Bootstrap lets us calculate critical exponents in the 3d Ising Model
 to world-record precision, how it explains striking relations between
 magnets and boiling water, and how it can be applied to questions
 across theoretical physics.</p>

The Conformal Bootstrap: From Magnets to Boiling Water

David Simmons-Duffin

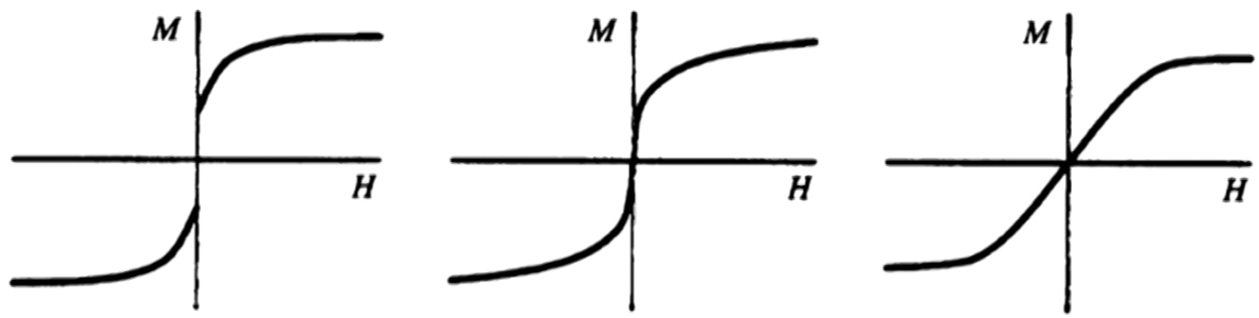
IAS

January 25, 2017

Outline

1. Critical Universality
2. Conformal Field Theory
3. Oracles
4. Prophecies

Critical Point of a Ferromagnet

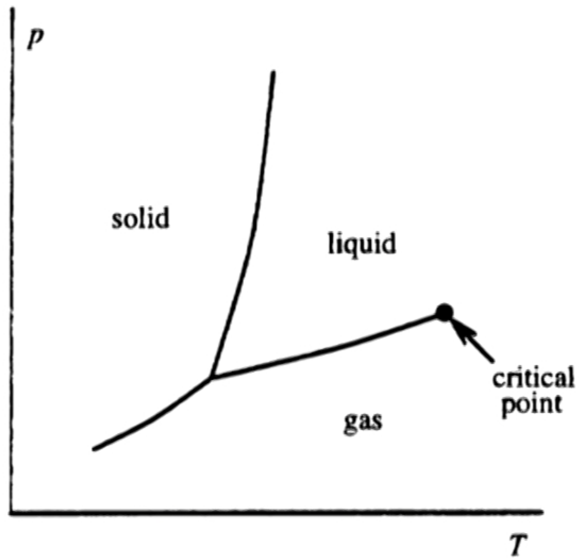


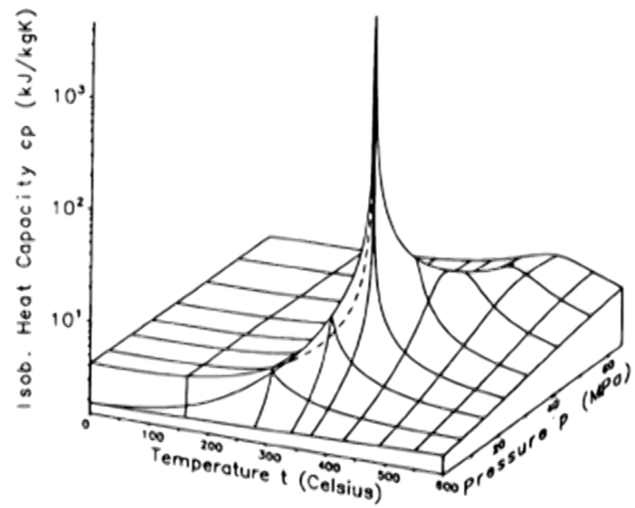
(a) $T < T_c$ (b) $T = T_c$ (c) $T > T_c$

Critical exponents:

- Specific heat $C \propto |T - T_c|^{-\alpha}$
- Susceptibility $\chi \propto |T - T_c|^{-\gamma}$

Critical Point of Water





Critical exponents:

- Specific heat $C \propto |T - T_c|^{-\alpha}$
- Compressibility $\chi_T \propto |T - T_c|^{-\gamma}$

Critical Universality

- magnet vs. liquid
- $\{T, H\}$ vs. $\{T, p\}$
- **Same** critical exponents!

$$\begin{aligned} \alpha_{\text{Magnet}} &= \alpha_{\text{Water}} &= 0.110\ldots \\ \gamma_{\text{Magnet}} &= \gamma_{\text{Water}} &= 1.237\ldots \\ &\vdots \end{aligned}$$

Universality class: 3d Ising model

$$H = -J \sum_{\langle ij \rangle} s_i s_j$$

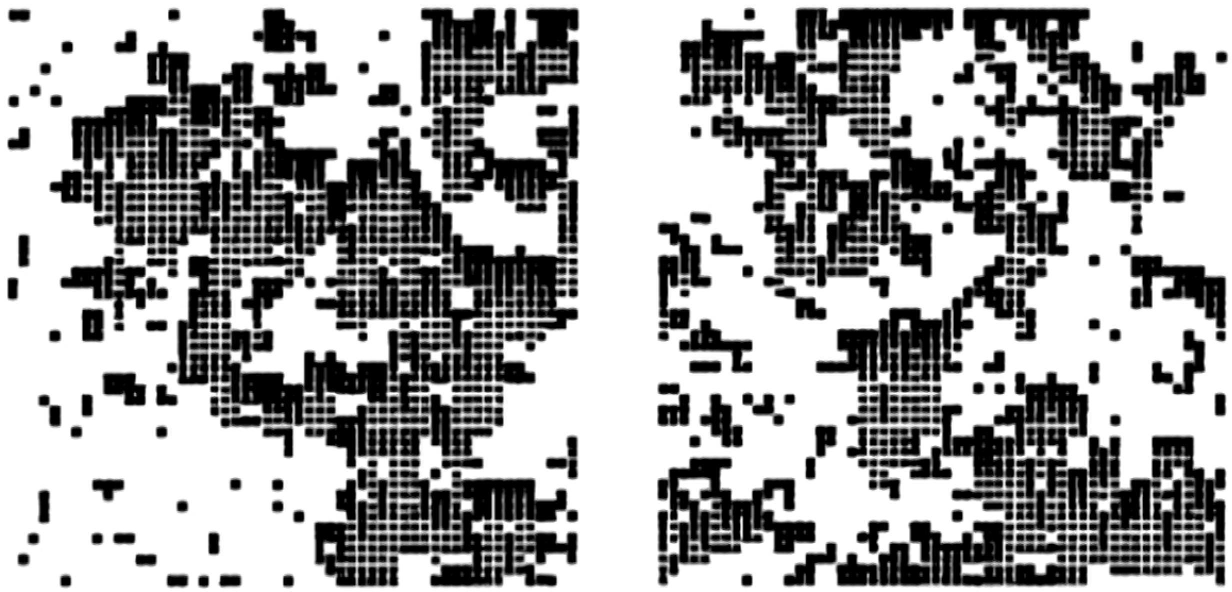
Critical Universality

- Why?
- What are $\alpha, \gamma, \ldots$?
- How do we calculate them?

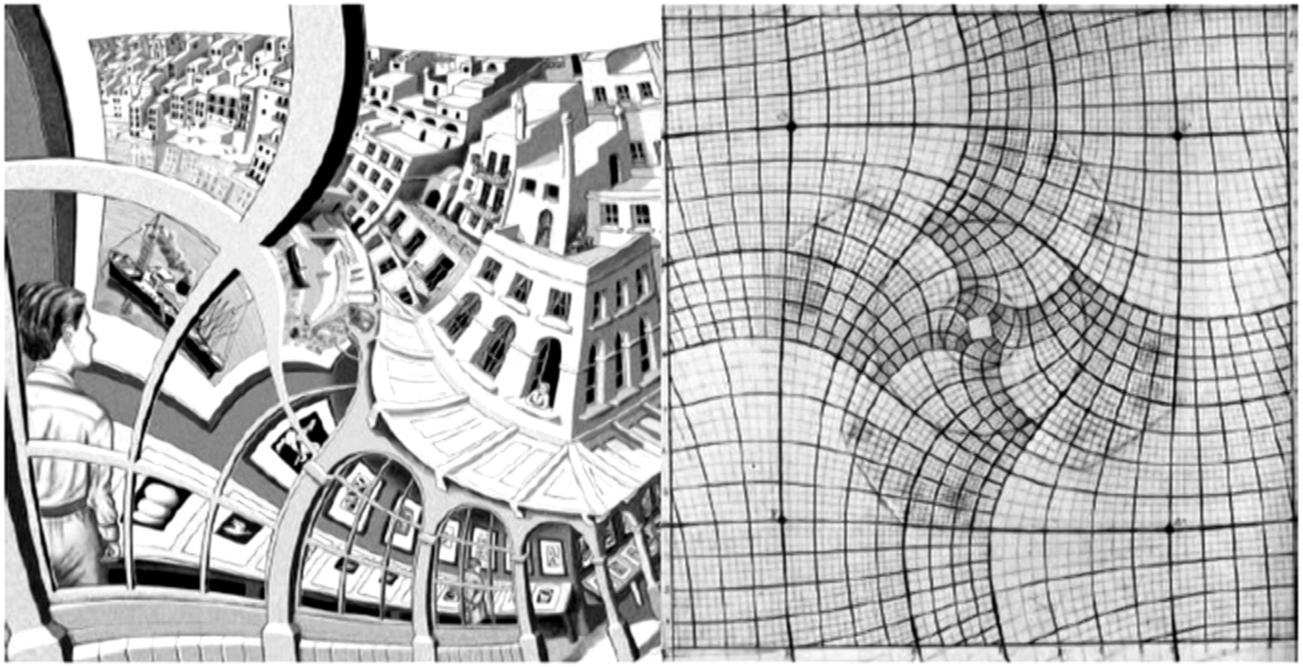
Outline

1. Critical Universality
2. Conformal Field Theory
3. Oracles
4. Prophecies

Scale Invariance



Scale $\Longrightarrow$ Conformal



$$\begin{matrix}\text{conformal}\\ \text{transformation}\end{matrix} = \begin{matrix}\text{rescaling + rotation}\\ \text{near each point}\end{matrix}$$

Quantum Field Theory

Thermal average = discrete path integral

$$Z = \sum_{\{s_i\}} e^{-J \sum_{\langle ij \rangle} s_i s_j}$$

$$s : \text{Lattice} \to \{\pm 1\}$$

Becomes regular path integral in continuum limit

$$\Longrightarrow Z = \int Ds(x)\, e^{-\int d^3x\, (\partial s)^2 + s^2 + s^4 + \dots}$$

$$s : \mathbb{R}^3 \to \mathbb{R}$$

Conformal Field Theory (CFT)

- Local operators $\sigma(x)$, $\epsilon(x)$, ...
  - Magnet: magnetization, energy, ...
  - Liquid: density, energy, ...
- Scaling dimensions $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = |x-y|^{-2\Delta_{\mathcal{O}}}$

$$\alpha = \frac{3-2\Delta_\epsilon}{3-\Delta_\epsilon}, \qquad \gamma = \frac{3-2\Delta_\sigma}{3-\Delta_\epsilon},$$

- Three-pt coefficients $\langle \mathcal{O}_i\mathcal{O}_j\mathcal{O}_k\rangle \propto f_{ijk}$
- Everything else determined by $\{\Delta_i, f_{ijk}\}$

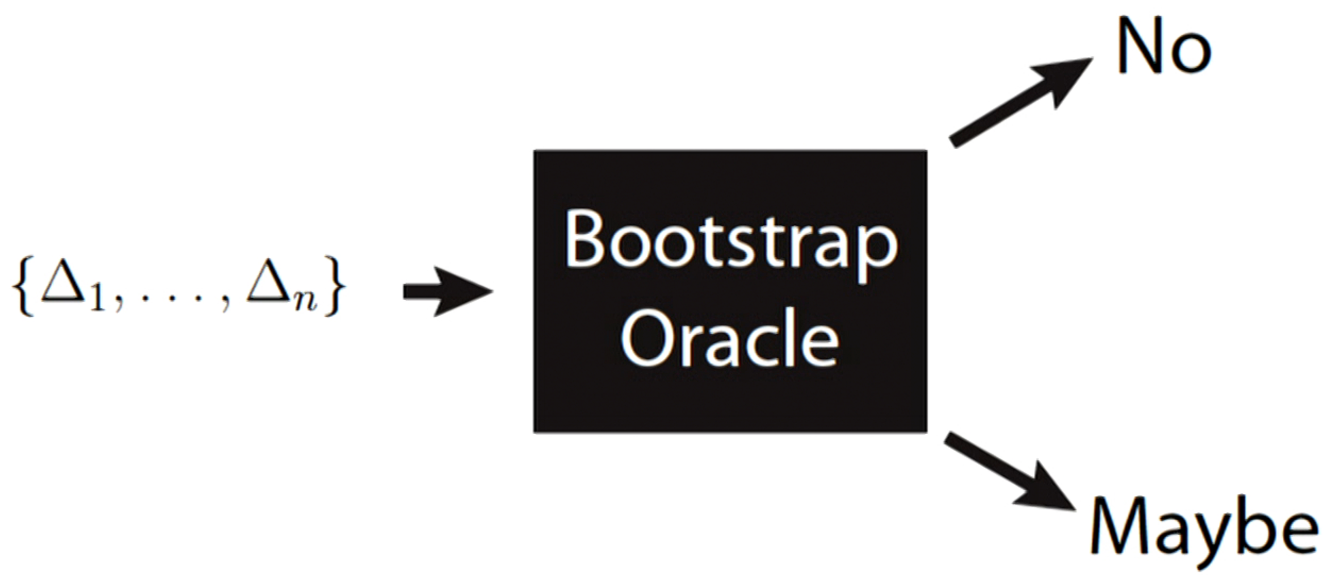
The Conformal Bootstrap

Find consistency conditions on $\{\Delta_i, f_{ijk}\}$ and solve them [Polyakov '74]

- *Nonperturbative* (no Feynman diagrams!)
- Progress in $d = 2$ throughout 80's and 90's.
    - ∞-dim conformal group [BPZ '83]
    - 2d Ising CFT exactly solved
    - Partial classification of 2d CFTs
- Huge revival for $d > 2$ a few years ago...

Bootstrap Revival [Rattazzi, Rychkov, Tonni, Vichi '08]

Is $\{\Delta_1, \ldots, \Delta_n\}$ part of some CFT spectrum?



Uses *conformal block decomposition* of $\langle\phi\phi\phi\phi\rangle$

Warmup: Spectral Decomposition

- Insert complete set of states (mass m)

$$\langle\phi(x)\phi(y)\rangle$$
$$= \langle 0|\phi(x)\left(\int dm^2 \int \frac{d^3\mathbf{p}}{2E_\mathbf{p}}|m,\mathbf{p}\rangle\langle m,\mathbf{p}|\right)\phi(y)|0\rangle$$

- Poincaré: $\langle 0|\phi(x)|m,\mathbf{p}\rangle = c(m^2)e^{-ip\cdot x}$,

$$= \int dm^2 \underbrace{\rho(m^2)}_{\text{unknown}} \times \underbrace{G_F(x-y,m^2)}_{\text{known (Feynman propagator)}}$$

- Unitarity:

$$\rho(m^2) = |c(m^2)|^2 \geq 0$$

Conformal Block Decomposition

- Insert complete set of states (dimension Δ, spin ℓ)

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

$$= \langle 0|\phi(x_1)\phi(x_2)\left(\sum_{\Delta,\ell}\sum_{\psi\in\text{rep}}|\psi\rangle\langle\psi|\right)\phi(x_3)\phi(x_4)|0\rangle$$

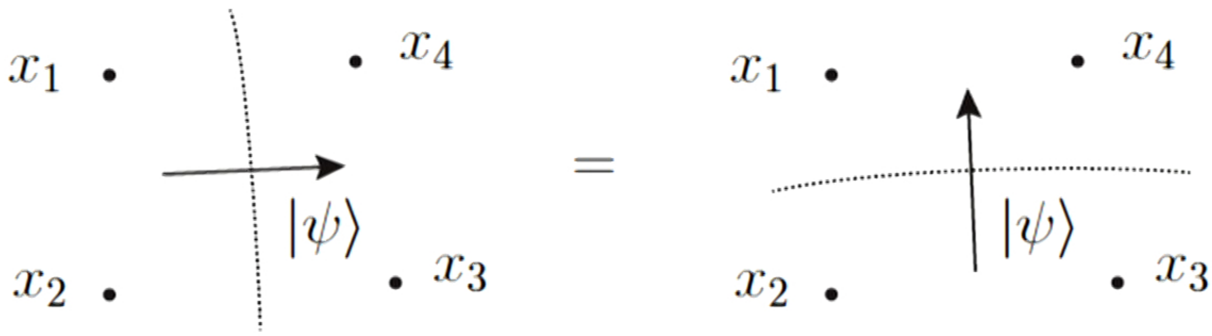
- Conformal symmetry fixes $\langle 0|\phi(x_1)\phi(x_2)|\psi\rangle$:

$$= \sum_{\Delta,\ell} \underbrace{p_{\Delta,\ell}}_{\text{unknown}} \times \underbrace{g_{\Delta,\ell}(x_1,x_2,x_3,x_4)}_{\text{known (conformal block)}}$$

- Unitarity:

$$p_{\Delta,\ell} \;\geq\; 0$$

Crossing Symmetry

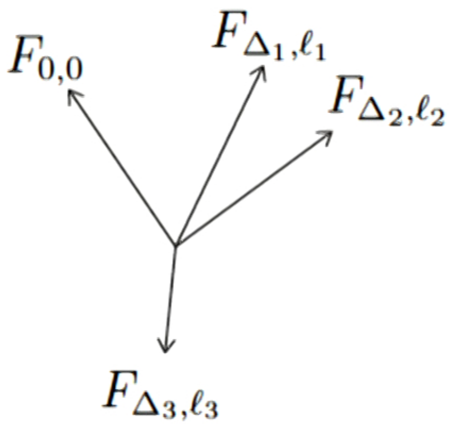


$$\sum_{\Delta,\ell} p_{\Delta,\ell} \underbrace{\left(g_{\Delta,\ell}(x_i) - g_{\Delta,\ell}(x_i)|_{1\leftrightarrow 3} \right)}_{F_{\Delta,\ell}(x_i)} = 0$$

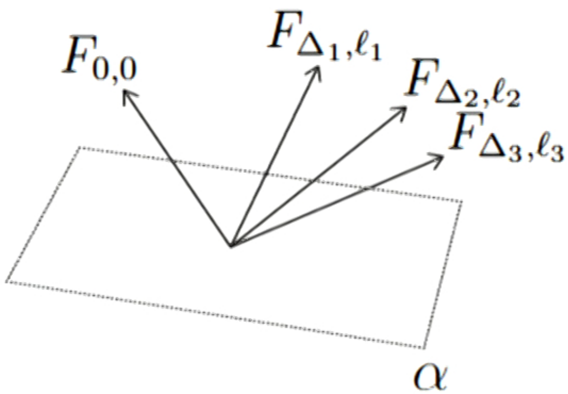
Crossing Symmetry vs. Unitarity

$$\sum_{\Delta,\ell} p_{\Delta,\ell} F_{\Delta,\ell}(x_i) = 0, \qquad p_{\Delta,\ell} \geq 0 \text{ (unitarity)}$$

Maybe



No

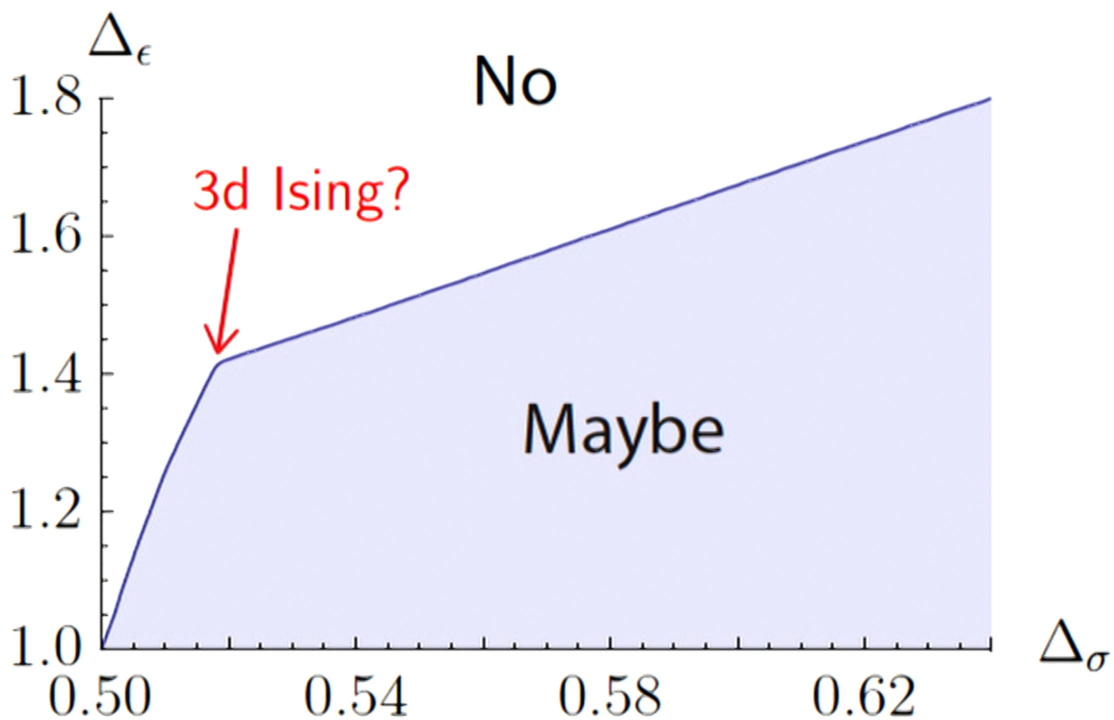


Finding $\alpha \implies$ Linear/Semidefinite Programming

Outline

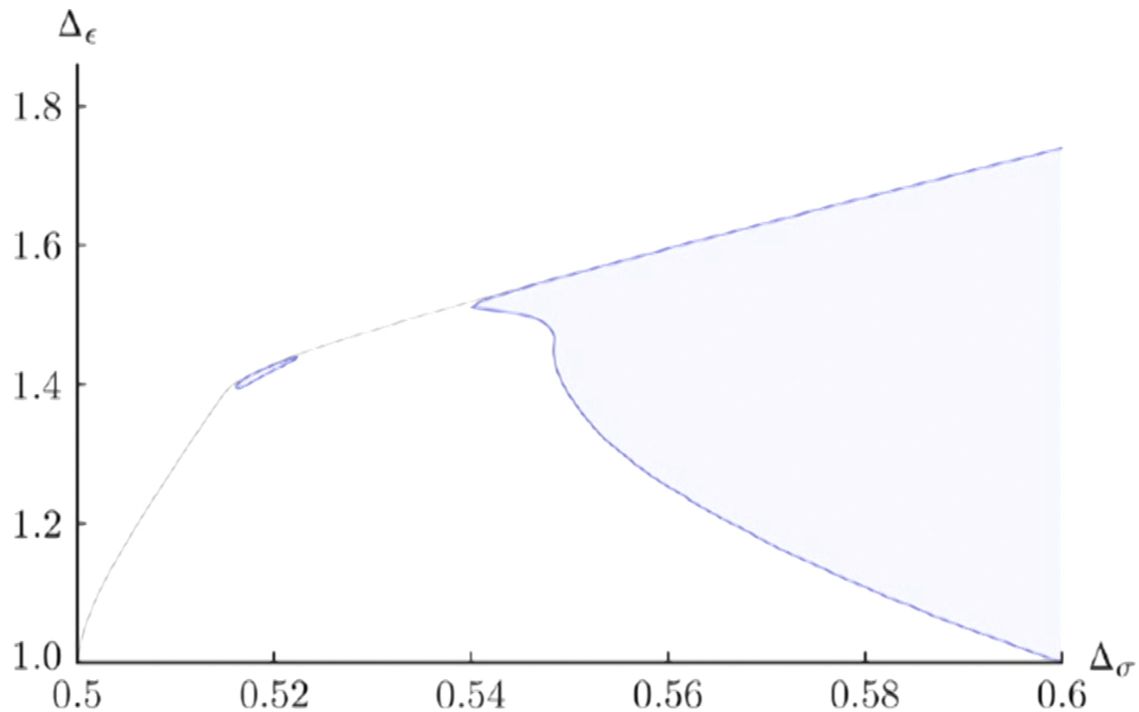
1. Critical Universality
2. Conformal Field Theory
3. Oracles
4. Prophecies

A Surprise [El-Showk, Paulos, Poland, Rychkov, DSD, Vichi '12]



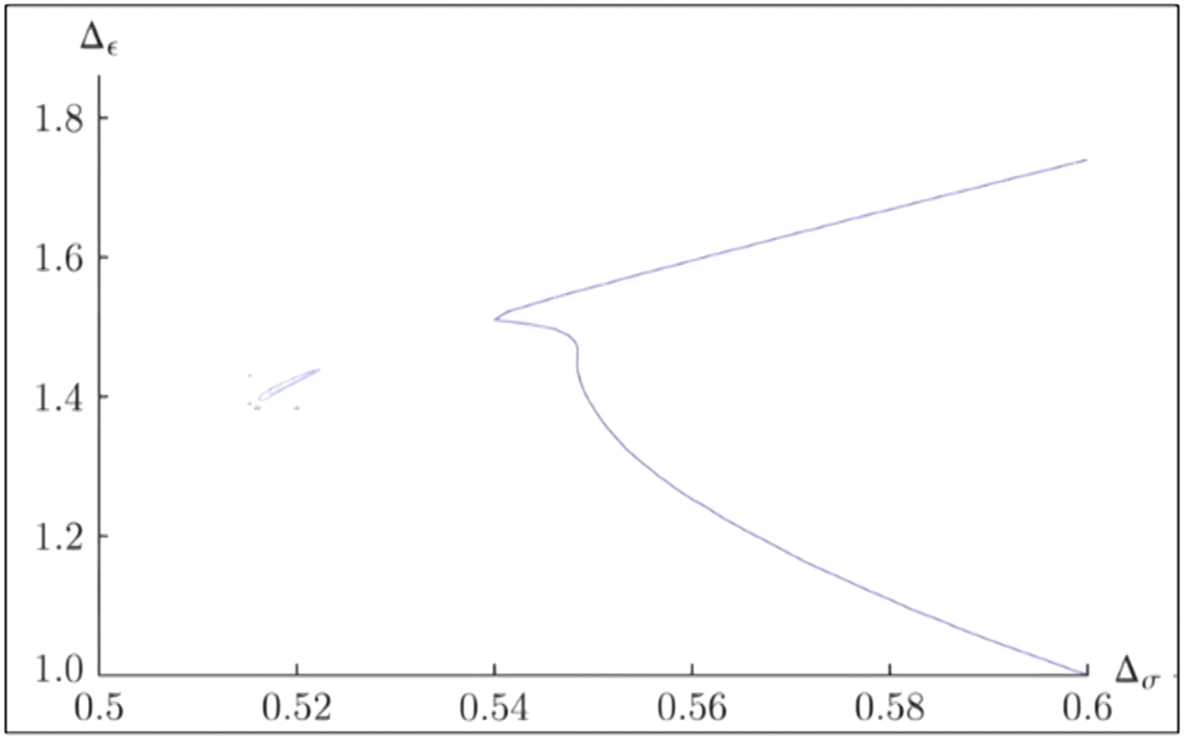
- Using $\langle \sigma\sigma\sigma\sigma \rangle$

Multiple Correlators [Kos, Poland, DSD '14], [DSD '15]



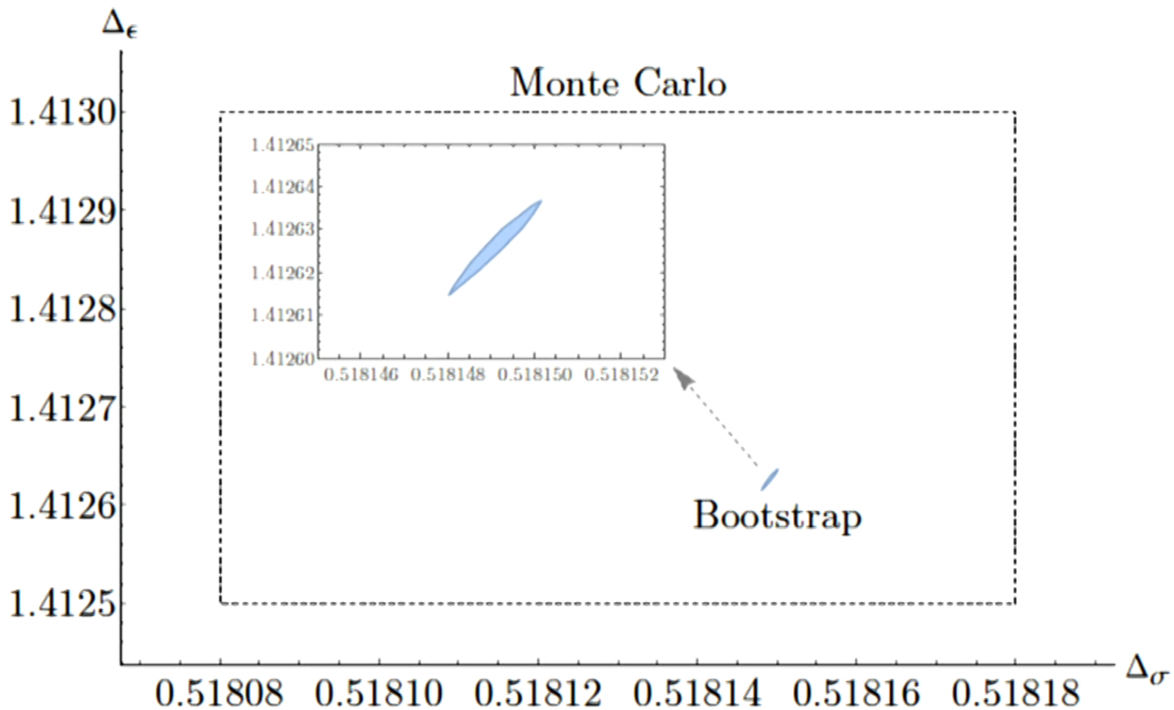
- Using $\langle\sigma\sigma\sigma\sigma\rangle, \langle\sigma\sigma\epsilon\epsilon\rangle, \langle\epsilon\epsilon\epsilon\epsilon\rangle$
- Assuming σ, ϵ are only relevant scalars

Multiple Correlators [Kos, Poland, DSD '14], [DSD '15]



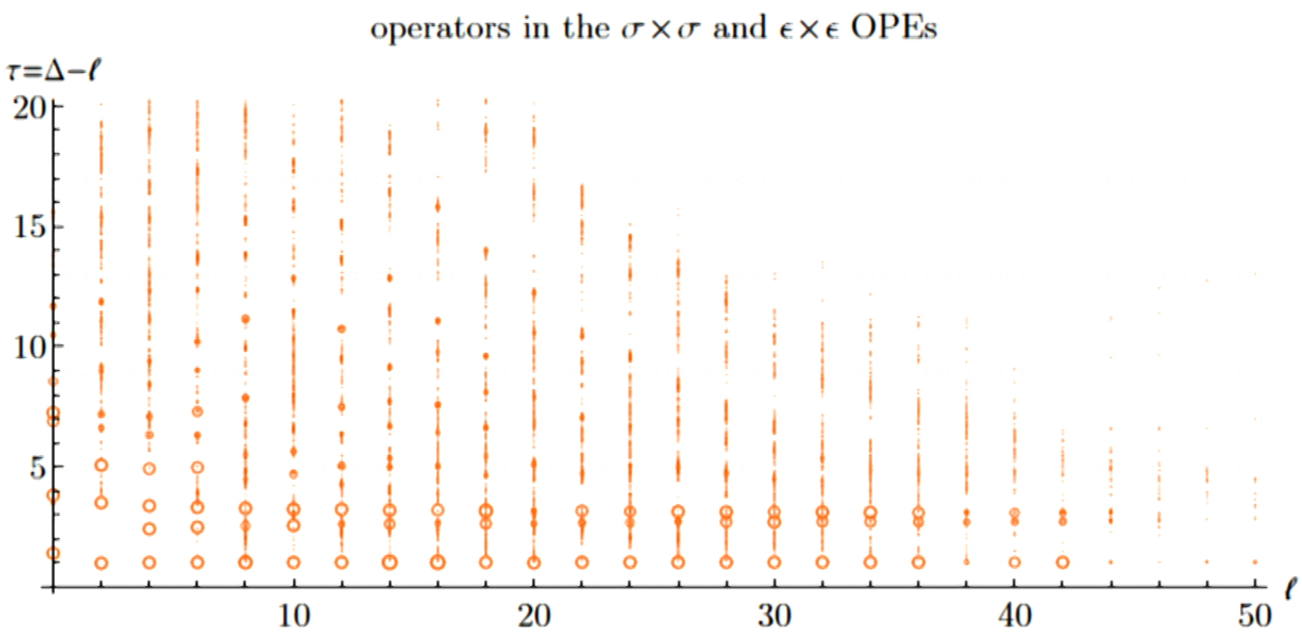
- Using $\langle\sigma\sigma\sigma\sigma\rangle, \langle\sigma\sigma\epsilon\epsilon\rangle, \langle\epsilon\epsilon\epsilon\epsilon\rangle$
- Assuming σ, ϵ are only relevant scalars

New Oracle [DSD '15], [Kos, Poland, DSD, Vichi '16]

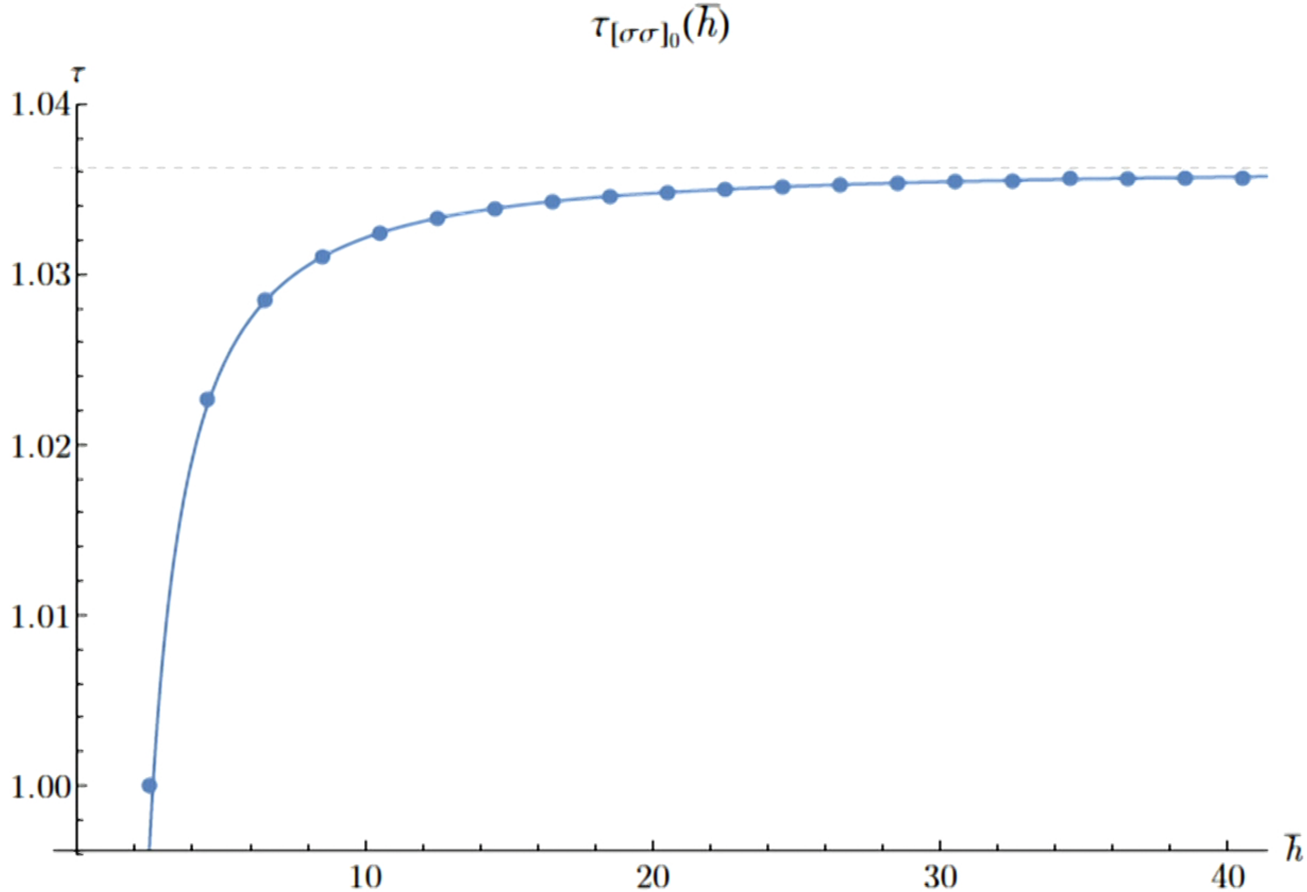


- $\Delta_\sigma = 0.5181489(10)$, $\Delta_\epsilon = 1.412625(10)$.
- "Explains" critical universality!

New Numerics: 100 Operators [DSD '16]



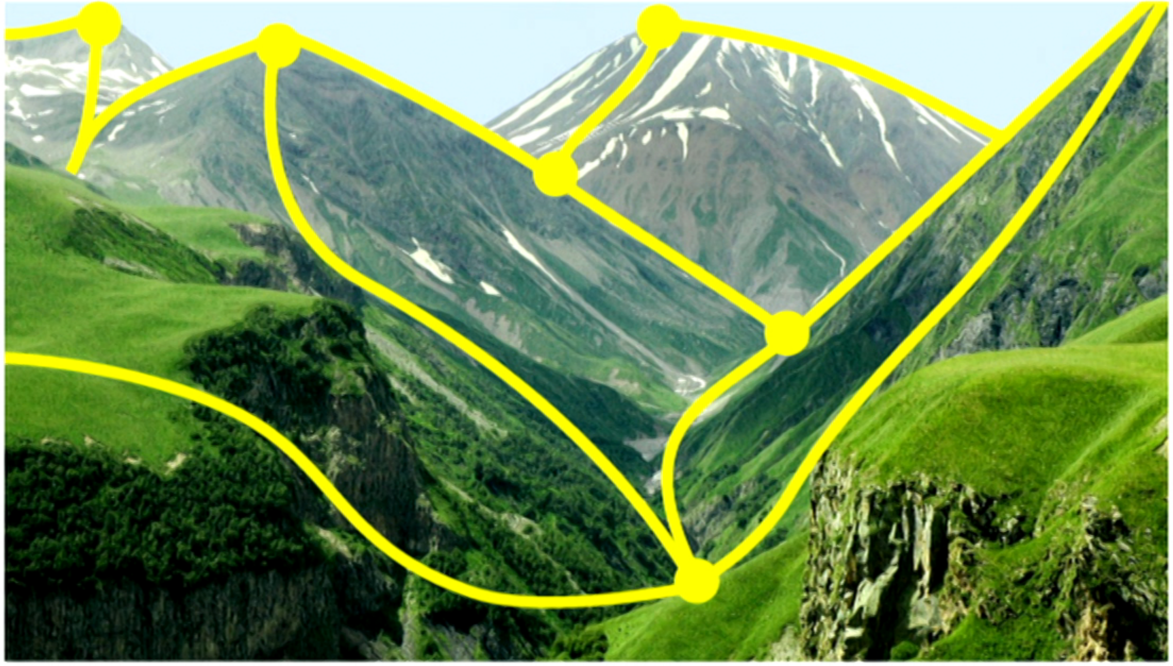
Matches analytics! [Kaplan, Fitzpatrick, Poland, DSD '12; Komargodski, Zhiboedov '12; Alday, Zhiboedov '15; DSD '16]



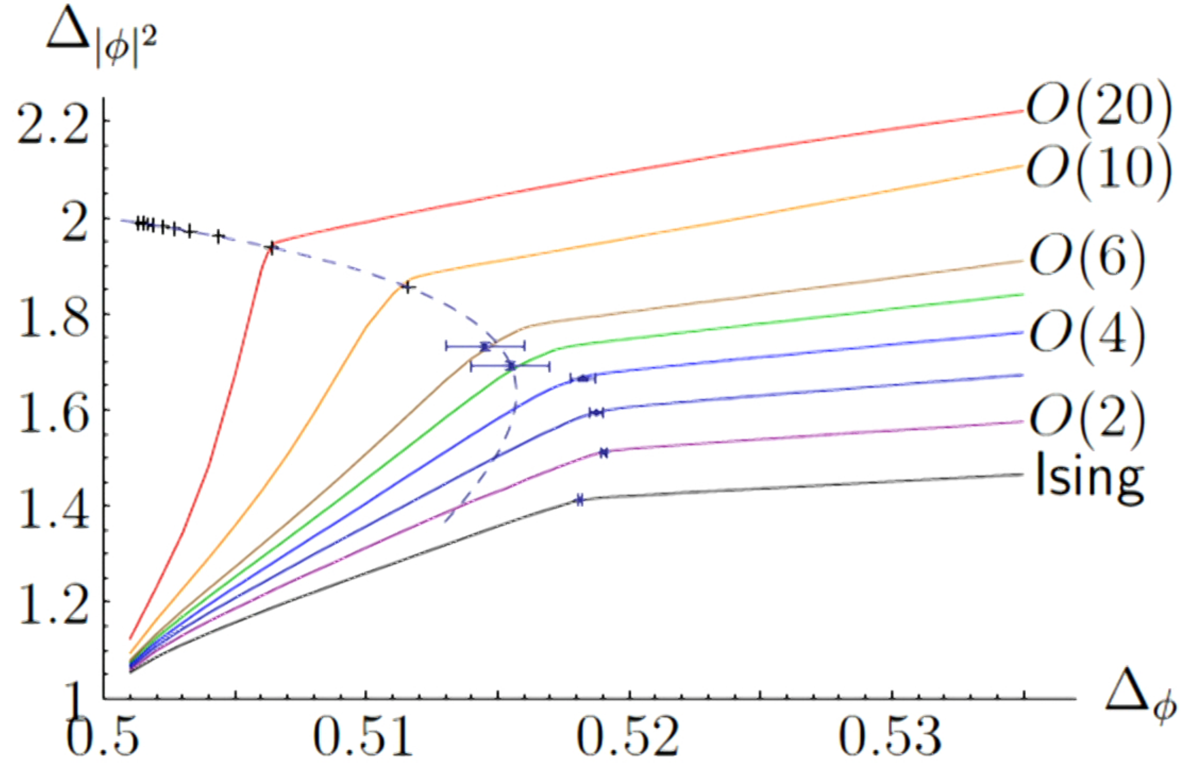
CFTs are Ubiquitous

- They describe 2nd order phase transitions in condensed matter systems
- They may appear in Beyond the Standard Model physics
- They encode theories of quantum gravity via AdS/CFT

Landmarks in the Space of Physical Theories

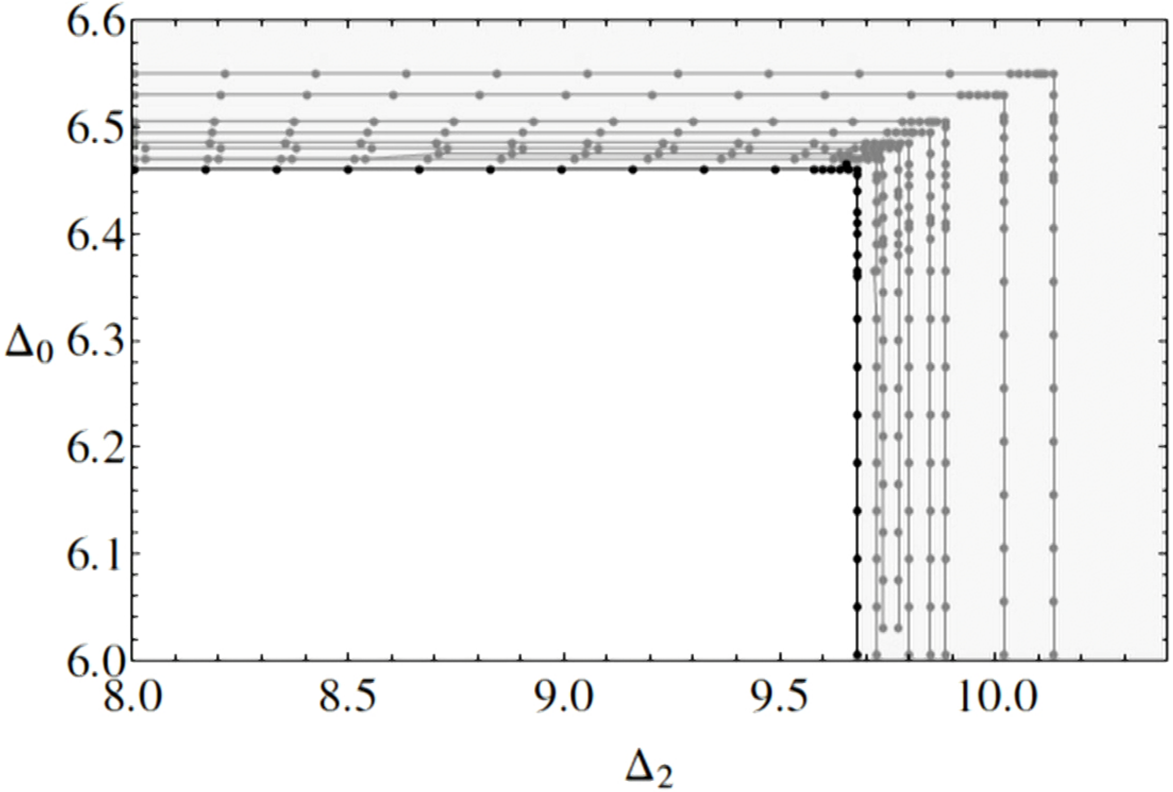


3d $O(N)$ Vector Models [Kos, Poland, DSD '13]



Lots of new results to compare with experiment!

$6d\ \mathcal{N} = (2,0)$ Theory (stack of M5 branes) [Beem, Lemos, Rastelli, van Rees '15]



The Conformal Bootstrap

- One of the few *nonperturbative* tools in QFT (others include integrability and supersymmetric localization)
- Can explore many interesting strongly-coupled systems in condensed matter physics, particle physics, and string theory!