Title: Quantum supremacy of fault-tolerant quantum computation in a pre-threshold region

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Abstract: <p>Demonstrating quantum supremacy, a complexity-guaranteed quantum advantage against over the best classical algorithms by using less universal quantum devices, is an important near-term milestone for quantum information processing. Here we develop a threshold theorem for quantum supremacy with noisy quantum circuits in the pre-threshold region, where quantum error correction does not work directly. By using the postselection argument, we show that the output sampled from the noisy quantum circuits cannot be simulated efficiently by classical computers based on a stable complexity theoretical conjecture, i.e., non-collapse of the polynomial hierarchy. By applying this to fault-tolerant quantum computation with the surface codes, we obtain the threshold value 2.84% for quantum supremacy, which is much higher than the standard threshold 0.75% for universal fault-tolerant quantum computation with the same circuit-level noise model. Moreover, contrast to the standard noise threshold, the origin of quantum supremacy in noisy quantum circuits is quite clear; the threshold is determined purely by the threshold of magic state distillation, which is essential to gain a quantum advantage.</p>

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Quantum supremacy of fault-tolerant quantum computation in a pre-threshold region

arXiv:1610.03632

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Outline

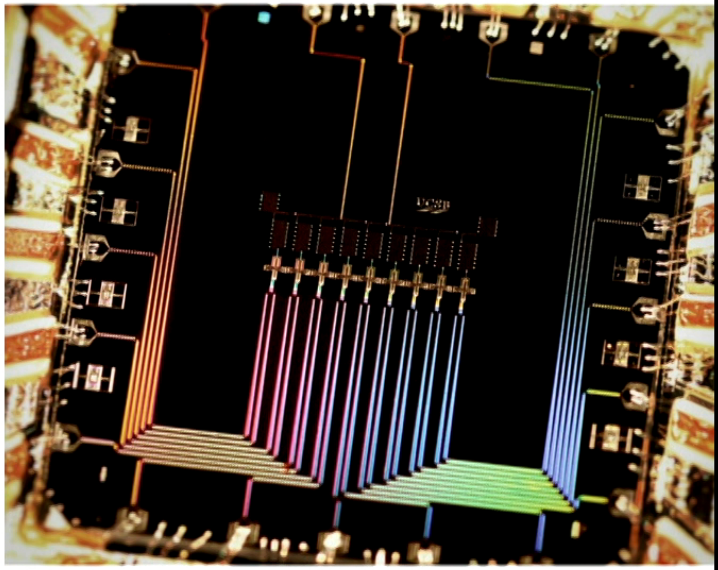
- Motivations
- Hardness proof by postselection (IQP, DQC1)
- Threshold theorem for quantum supremacy
- Applications: 3D topological cluster computation & 2D surface code
- Summary

Quantum supremacy with near-term quantum devices

"QUANTUM COMPUTING AND THE ENTANGLEMENT FRONTIER" by J Preskill

The 25th Solvay Conference on Physics
19-22 October 2011; arXiv:1203.5813

How can we best achieve quantum supremacy with the *relatively small systems that may be experimentally accessible fairly soon*, systems with of order 100 qubits?



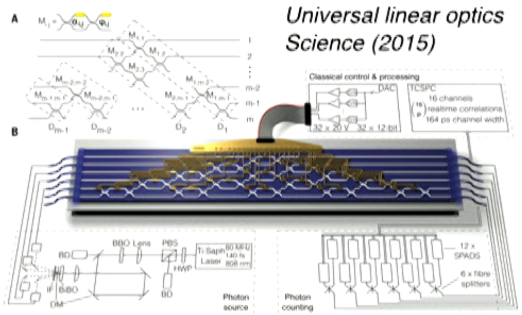
https://www.technologyreview.com/s/601668/google-reports-progress-on-a-shortcut-to-quantum-supremacy/

Intermediate models for non-universal quantum computation

Boson Sampling

Aaronson-Arkhipov '13

Universal linear optics
Science (2015)



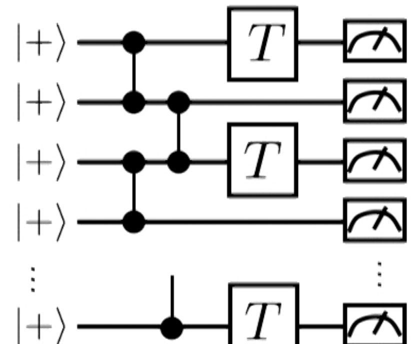
Linear optical quantum computation

Experimental demonstrations

J. B. Spring *et al*. Science **339**, 798 (2013)
M. A. Broome, Science **339**, 794 (2013)
M. Tillmann *et al*., Nature Photo. **7**, 540 (2013)
A. Crespi *et al*., Nature Photo. **7**, 545 (2013)
N. Spagnolo *et al*., Nature Photo. **8**, 615 (2014)
J. Carolan *et al*., Science **349**, 711 (2015)

IQP (commuting circuits)

Bremner-Jozsa-Shepherd '11

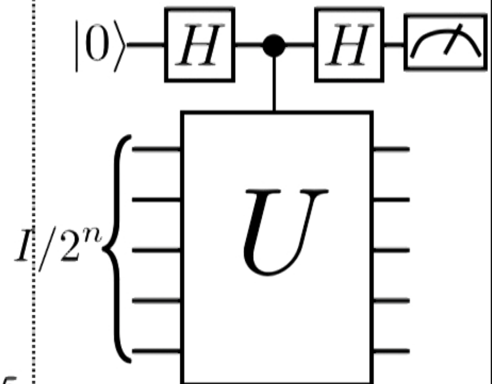


Ising type interaction

KF-Morimae '13
Bremner-Montanaro-Shepherd '15
Gao-Wang-Duan '15
Farhi-Harrow '16

DQC1 (one-clean qubit model)

Knill-Laflamme '98
Morimae-KF-Fitzsimons '14
KF et al, '16



NMR spin ensemble

The role of this study:
→ universal but (very) noisy quantum circuits

Noisy quantum circuits approaching fault-tolerance threshold

SUPERCONDUCTING QUBITS

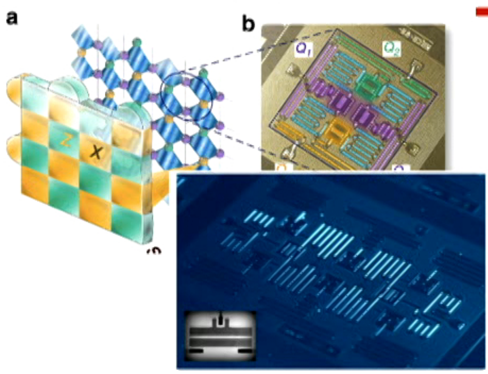
Solving a wonderful problem

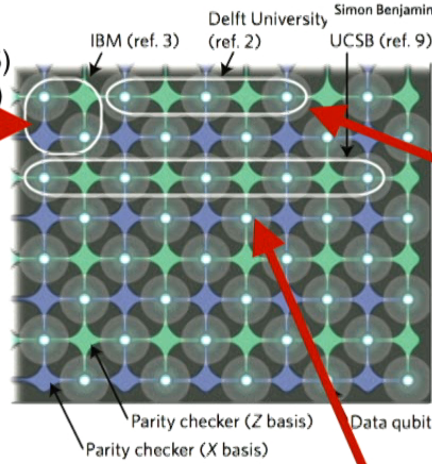
Superconducting qubits are used to demonstrate features of quantum fault tolerance, making an important step towards the realization of a practical quantum machine.

Simon Benjamin and Julian Kelly

IBM:

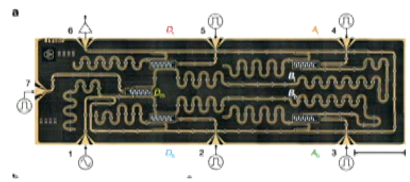
Chow et al., Nat. Comm. **5** 4015 (2015)
C´orcoles et al., Nat. Comm. **6**, 6979 (2015)
Gambetta et al., npj quant. info. **3**, 2 (2017)

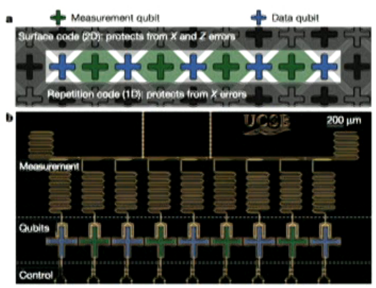


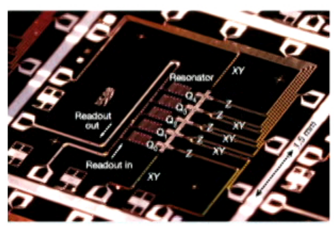


Delft (QuTech):

Riste et al., Nat. Comm. **6** 6983 (2015







UCSB(Martinis)+Google:

Kelly et al., Nature **519,** 66 (2015)
Barends et al., Nature **508**, 500 (2014)

[fidelities]
single-qubit gate: 99.92%
two-qubit gate: 99.4%
measurement: 99%

Noisy quantum circuits above standard noise threshold

Threshold theorem: if the noise strength is smaller than a certain constant threshold value, quantum computation can be performed with an arbitrary accuracy.

(standard) noise threshold

universal QC

classically simulatable?

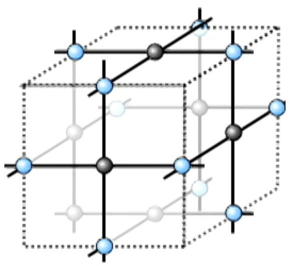
distillability of of magic state

classically simulatable by GK theorem

clean → **noisy**

phenomenological noise **2.9-3.3%**
circuit-based noise **0.75%**

phenomenological noise **14.6%**

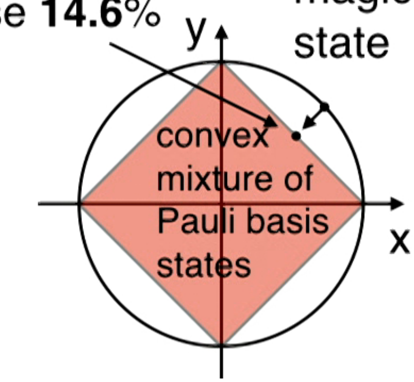


Topological fault-tolerance in cluster state quantum computation

R Raussendorf, J Harrington and K Goyal

New Journal of Physics **9** (2007) 199

Ann. Phys. ***321*** *2242* (2006)



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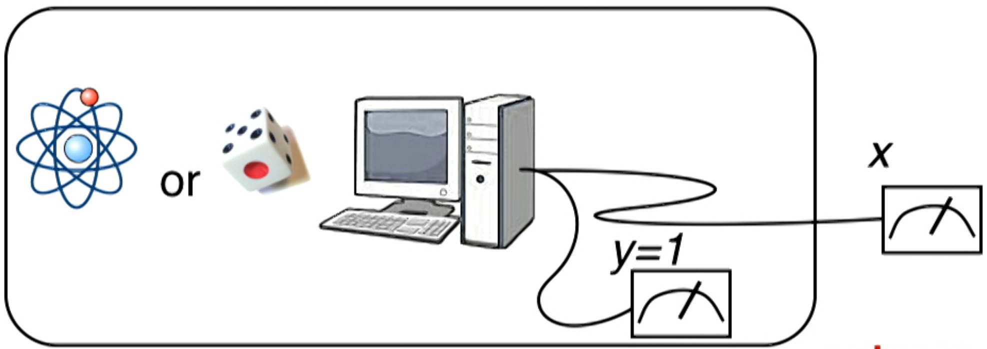
Postselected computation

= solving a decision problem by using conditional probability distribution.

while(y==1)



$$p(x|y=1) = p(x,y)/\boxed{p(y=1)}$$

not zero but can be exponentially small

yes: $p(x=1|y=1) \geq 2/3$

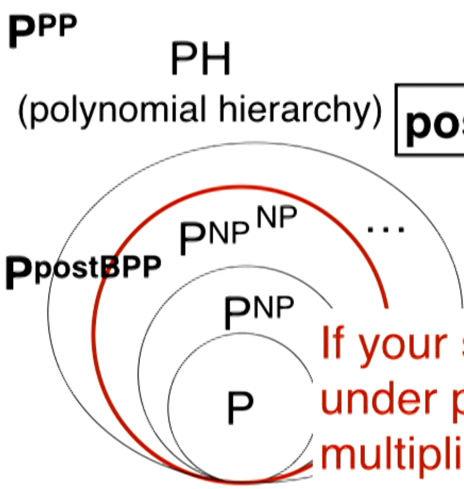
no: $p(x=0|y=1) \geq 2/3$

Aaronson, Proc. of the Royal Society A: Math., Phys. and Eng. Sci. **461**, 3473 (2005).

Hardness proof via postBQP = PP

A (fictitious) ability to postselect a possibly exponentially rare events allows us to distinguish quantum and classical tasks!

classical+ postselection → **postBPP** ≠ quantum+ postselection → **PP=postBQP** [Aaronson05]



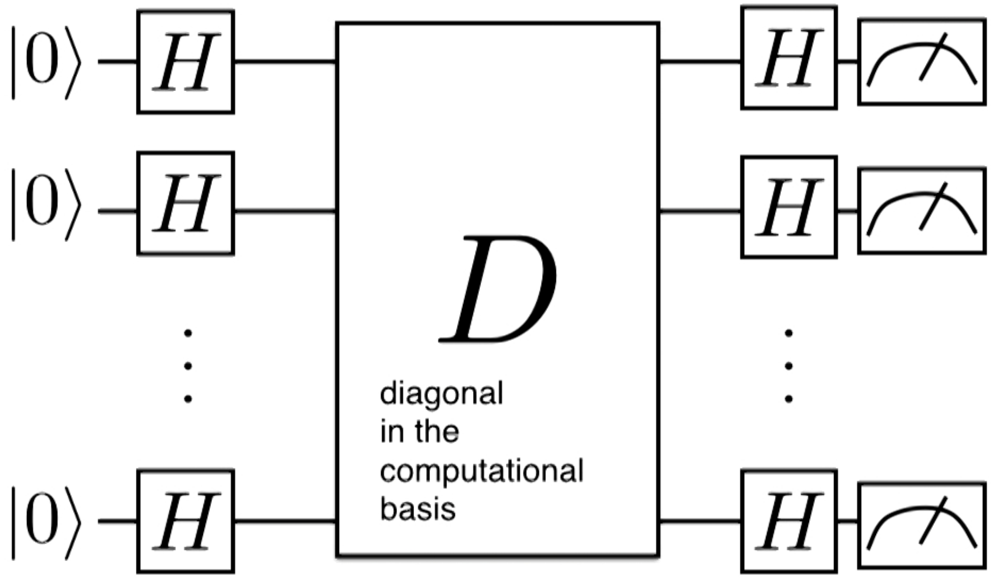
unless the PH collapses to the 3rd level.

If your system is potentially as powerful as postBQP under postselection, then its classical simulation (with multiplicative error) is hard unless the PH collapses!

Bremner-Jozsa-Shepherd, Proc. Royal Soc. A: Math. Phys. and Eng. Sic. 467, 2126 (2011)
Aaronson, Proc. of the Royal Society A: Math., Phys. and Eng. Sci. **461**, 3473 (2005).

IQP

instantaneous quantum polynomial-time computation



Bremner-Jozsa-Shepherd, Proc. Royal Soc. A: Math. Phys. and Eng. Sic. 467, 2126 (2011)

IQP

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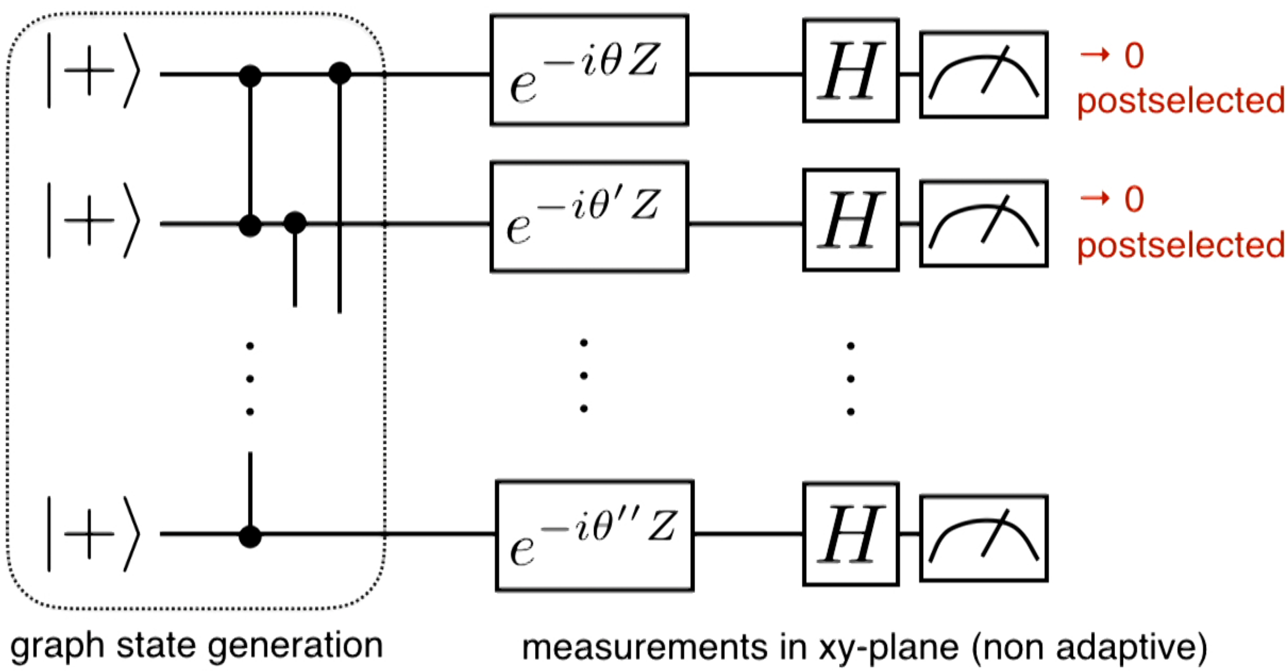


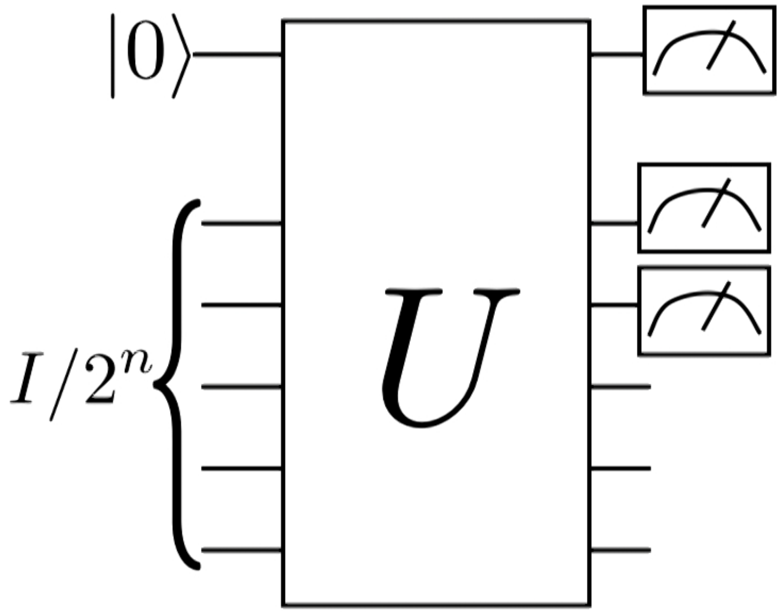
postIQP = postBQP

Bremner-Jozsa-Shepherd, Proc. Royal Soc. A: Math. Phys. and Eng. Sic. 467, 2126 (2011)

DQC1$_3$

of measurement ports

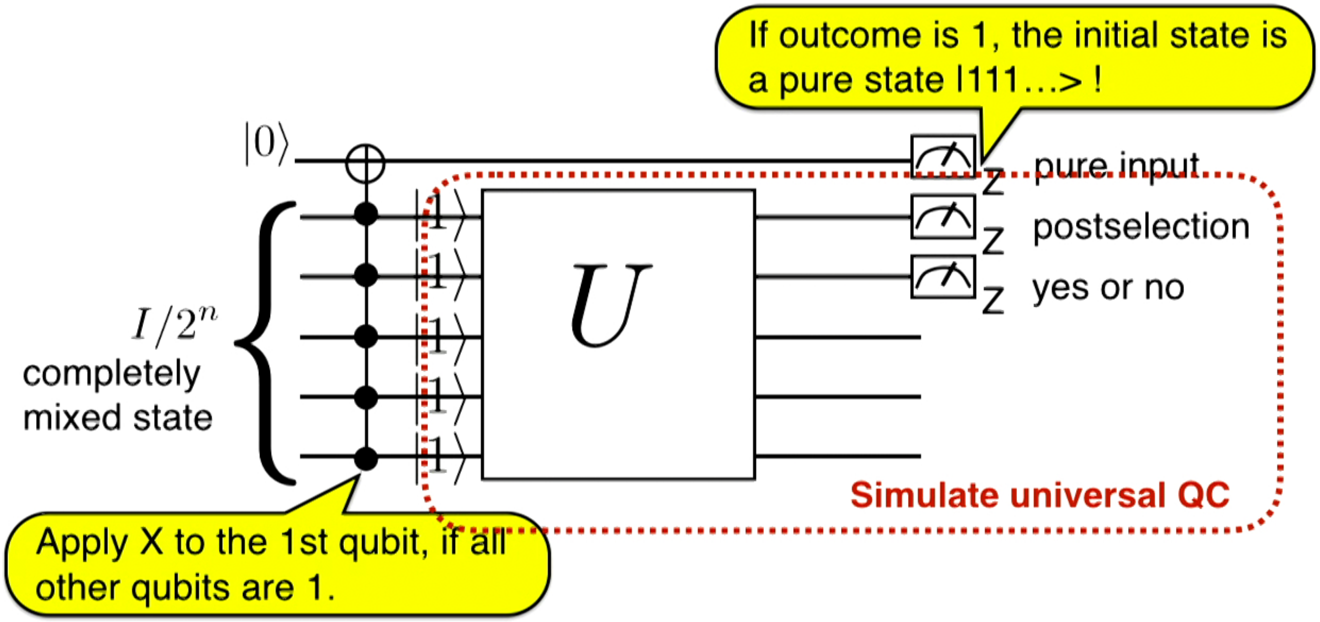
(one clean qubit with three-qubit measurement)



DQC1: Knill-Laflamme, PRL 81, 5672 (1998)

DQC1$_3$

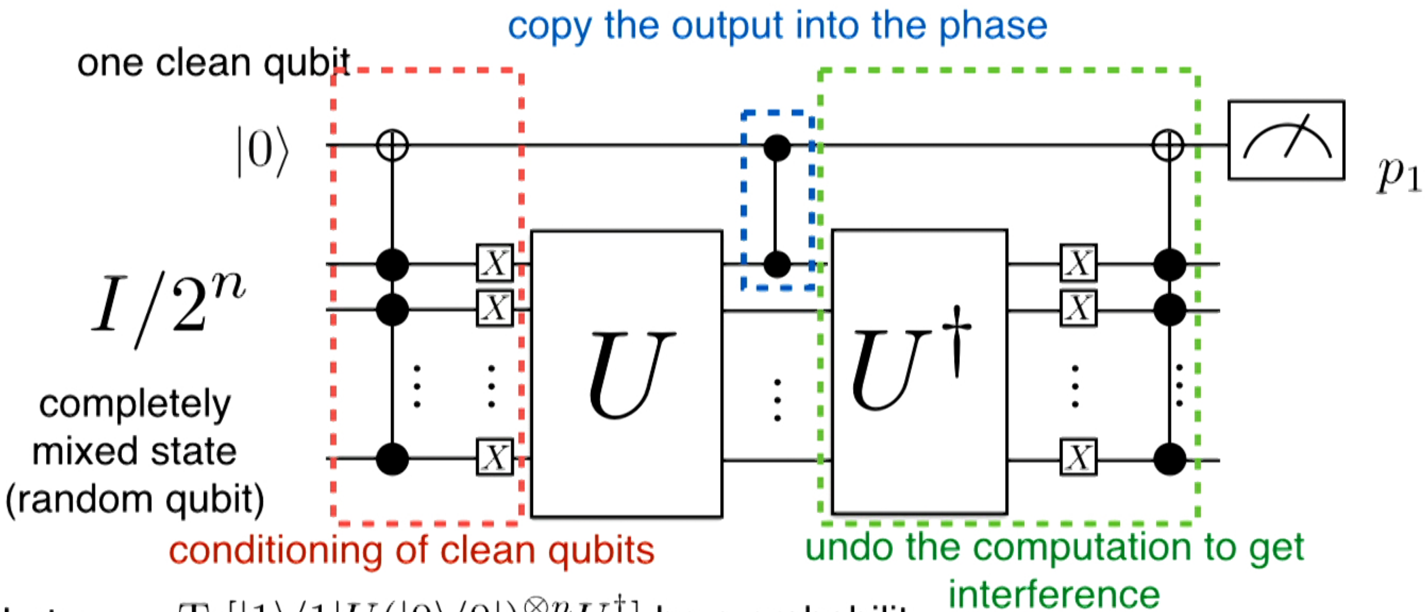
(one clean qubit with three-qubit measurement)



Morimae-KF-Fitzsimons, PRL **112**, 130502 (2014)

DQC1$_1$

(one clean qubit with three-qubit measurement)



Let $q_1 = \mathrm{Tr}[|1\rangle\langle 1|U(|0\rangle\langle 0|)^{\otimes n}U^\dagger]$ be a probability of pure quantum computation.

$$\rightarrow \quad p_1 = \frac{4}{2^n} q_1(1-q_1)$$

SBQP = SBQ$_1$P

(An efficient classical simulation implies PH=AM, collapse of PH to 2nd level)

KF-Kobayashi-Morimae-Nishimura-Tamate-Tani, ICALP2016 (arXiv:1509.07276)

Subtlety of quantum supremacy with noisy sampling

- multiplicative error (or exponentially small additive error)

$$\frac{1}{c} p^{\mathrm{ideal}}(x) < p^{\mathrm{samp}}(x) < c p^{\mathrm{ideal}}(x) \qquad (c > 1)$$

[Bremner-Jozsa-Shepherd, '11]

- constant additive error with l_1-norm (total variation distance)

$$\| p^{\mathrm{samp}}(x) - p^{\mathrm{ideal}}(x) \|_1 = \sum_x | p^{\mathrm{samp}}(x) - p^{\mathrm{ideal}}(x) | < c$$

(using Stockmeyer's counting algorithm)

[Aaronson-Arkhipov, '11, Bremner-Montanaro-Shepherd '16]

Small amount of noise can easily break these conditions.

Can noisy quantum circuits exhibit quantum speedup as a sampling problem?

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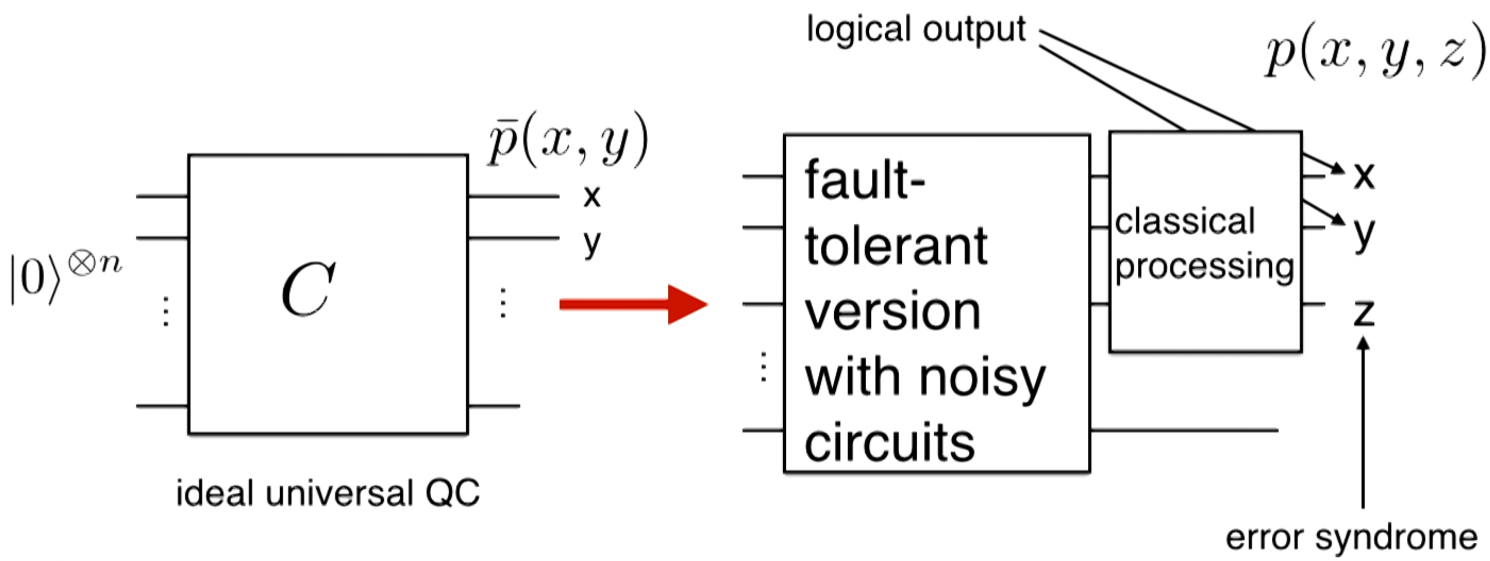
Main idea: simulation of fault-tolerant quantum computation under postselection



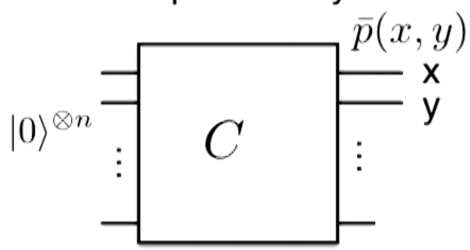
Detect any erroneous event and postselect more reliable quantum computation!

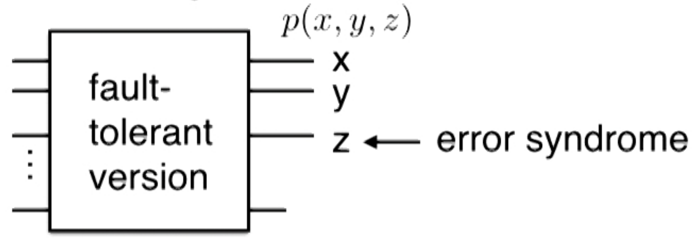
$p(x, y, |z = 0)$
postselect the events where no error syndrome is activated

arXiv:1610.03632

Threshold theorem for quantum supremacy

- Part1: An exponentially small additive error is enough.





$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

where $\kappa = \mathrm{poly}(n)$ the overhead is polynomial in *n*. Then, classical simulation of $p(x,y,z)$ with a multiplicative error $1 < c < \sqrt{2}$ is hard.

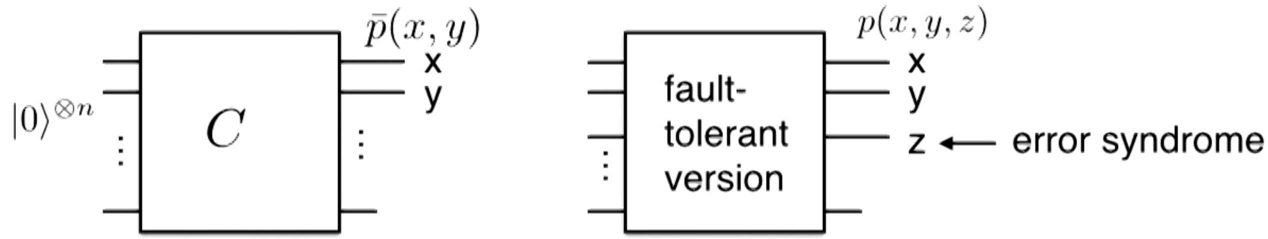
- Part2: The exponentially small additive error

$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

is achievable by quantum error correction under postselection.

arXiv:1610.03632

Part1: an exponential small additive error is enough



Solve a PP-complete problem (**MAJSAT**) using $\bar{p}(x|y)$ as in [Aaronson05]

→ probability for postselection: $\bar{p}(y=0) > 2^{-6n-4}$

Therefore, if $|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$ with $\kappa = \mathrm{poly}(n)$

then we have

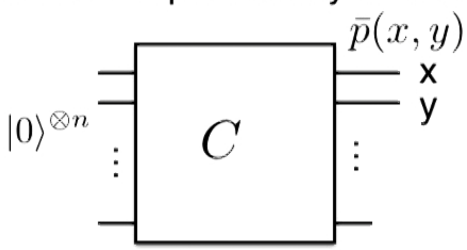
$$|\bar{p}(x|y=0) - p(x|y=0, z=0)| < 1/2$$

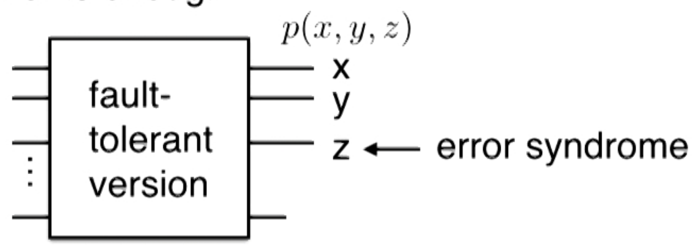
→ $p(x,y,z)$ can solve the PP-complete problem under postselection.

arXiv:1610.03632

Threshold theorem for quantum supremacy

- Part1: An exponentially small additive error is enough.





$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

where $\kappa = \mathrm{poly}(n)$ the overhead is polynomial in *n*. Then, classical simulation of $p(x,y,z)$ with a multiplicative error $1 < c < \sqrt{2}$ is hard.

- Part2: The exponentially small additive error

$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

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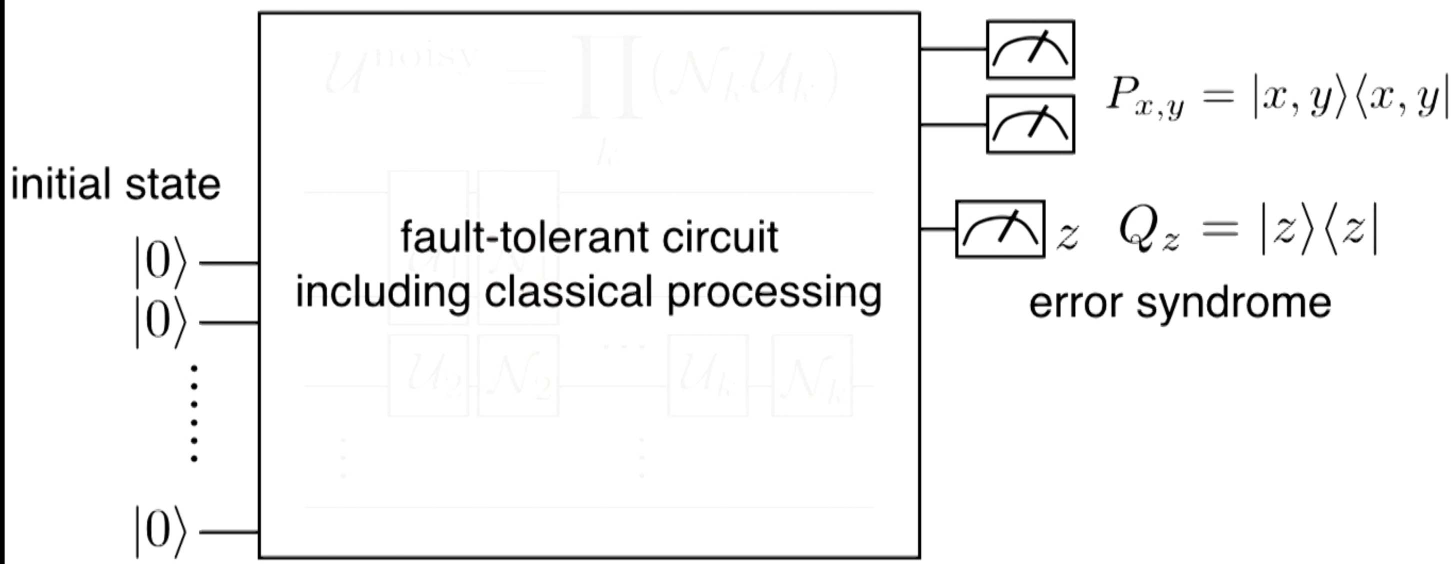
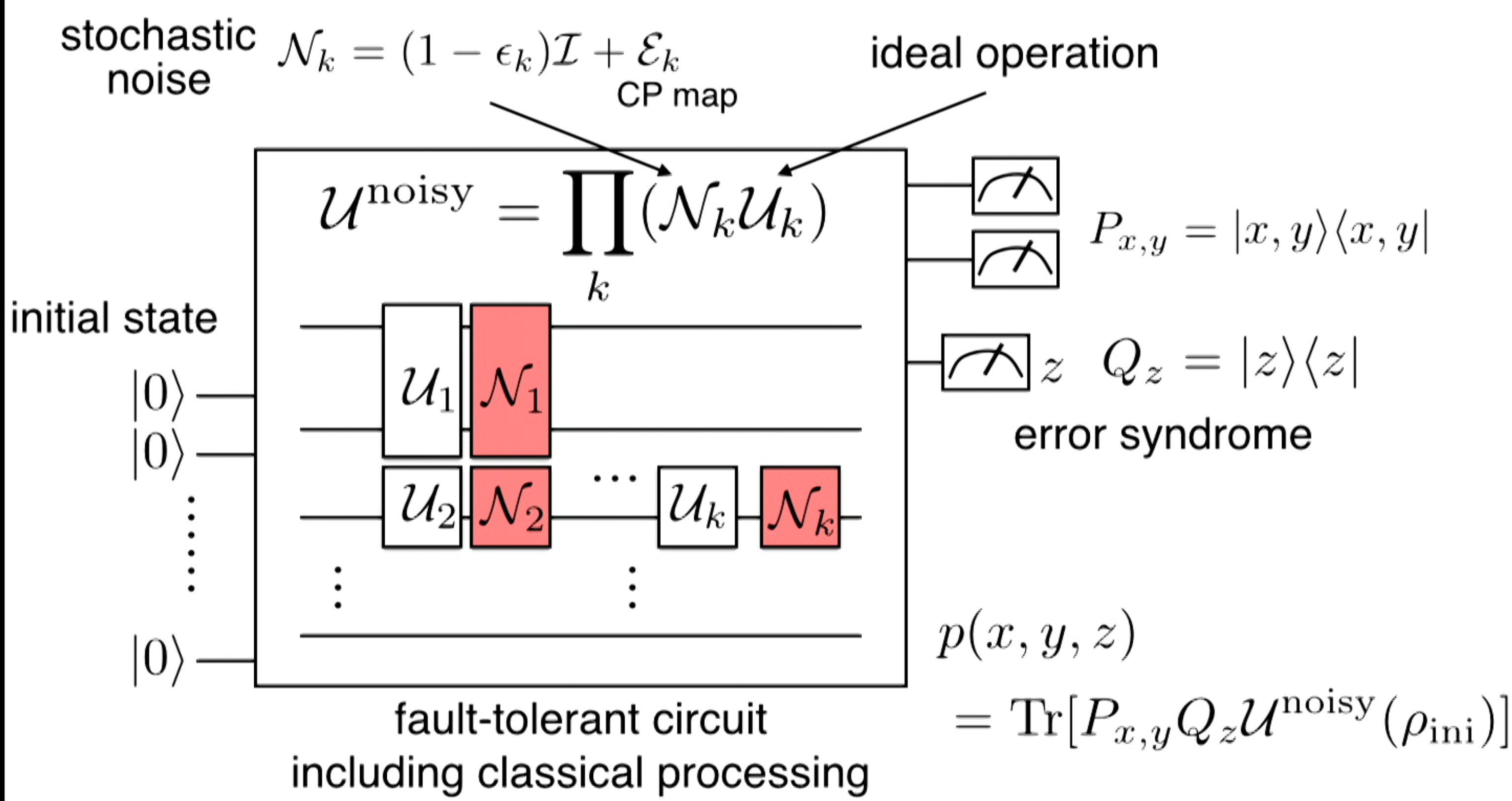
Part2: error reduction under postselection (sketch)



arXiv:1610.03632

Part2: error reduction under postselection (sketch)



arXiv:1610.03632

Part2: error reduction under postselection (sketch)

Using $\mathcal{N}_k = (1-\epsilon_k)\mathcal{I} + \mathcal{E}_k$, we decompose $\mathcal{U}^{\text{noisy}}$ into

$$\mathcal{U}^{\text{noisy}}(\rho_{\text{ini}}) = \rho_{\text{sparse}} + \rho_{\text{faulty}}$$

such that $\bar{p}(x,y) \propto \mathrm{Tr}[P_{x,y} Q_{z=0} \rho_{\text{sparse}}]$.

Then we can show that

$$\|\bar{p}(x,y) - p(x,y|z=0)\|_1 < 2\|\rho_{\text{faulty}}\|_1 / q_{z=0}$$

where $q_{z=0} \equiv \mathrm{Tr}[Q_{z=0}\mathcal{U}^{\text{noisy}}(\rho_{\text{ini}})]$. (prob. of null syndrome measurement)

$$< 2\sum_{r\geq d} C(r)\left(\frac{\epsilon}{1-\epsilon}\right)^r \quad (\epsilon \equiv \max_k \epsilon_k)$$

arXiv:1610.03632

There is a constant threshold ε_{th} below which the output $p(x, y, z) = \mathrm{Tr}[P_{x,y} Q_z \mathcal{U}^{\mathrm{noisy}}(\rho_{\mathrm{ini}})]$ from the noisy quantum circuits cannot be simulated efficiently on a classical computer unless the PH collapses to the 3rd level.

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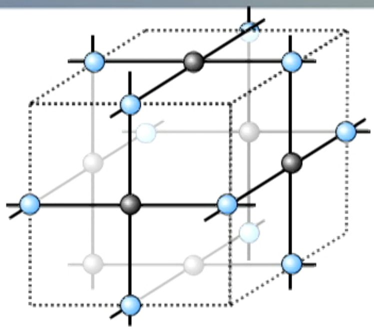
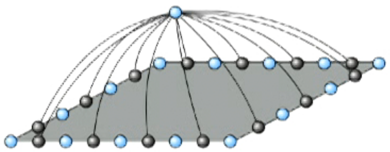
Topological MBQC on a 3D cluster state



- MBQC on a graph state of degree log(n)
  (corresponds to commuting circuits of depth log(n))
- Noise: independent single-qubit dephasing w prob. ϵ
  (phenomenological noise model)



- Faulty part comes from either clifford operations or magic state distillation $\rho_{\text{faulty}} = \rho_{\text{cl}} + \rho_{\text{magic}}$

Clifford operations
(counting # of self-avoiding walks: Dennis et al '02)

$$\frac{12}{5}\text{poly}(n)\left(\frac{5\epsilon}{1-\epsilon}\right)^d \longrightarrow \epsilon_{\text{cl}} = 0.167$$

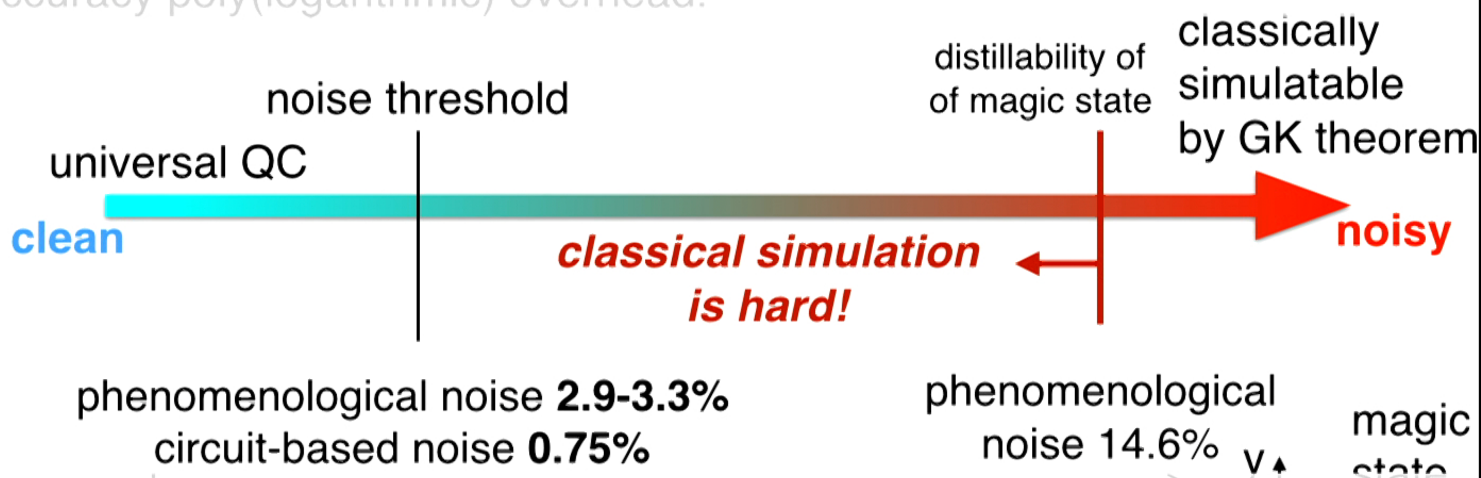
magic state distillation $\longrightarrow$ $\epsilon_{\text{magic}} = 0.146$

(Bravyi-Kitaev '05; Reichardt '06)

see also KF-Tamate '16

Noisy quantum circuits above standard noise threshold

Threshold theorem: if the noise strength is smaller than a certain constant threshold value, quantum computation can be done with an arbitrary accuracy poly(logarithmic) overhead.



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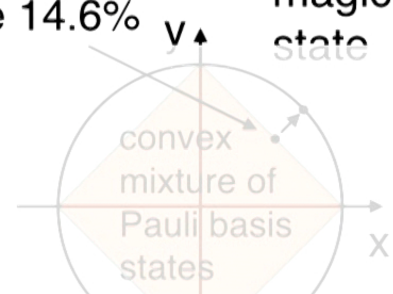
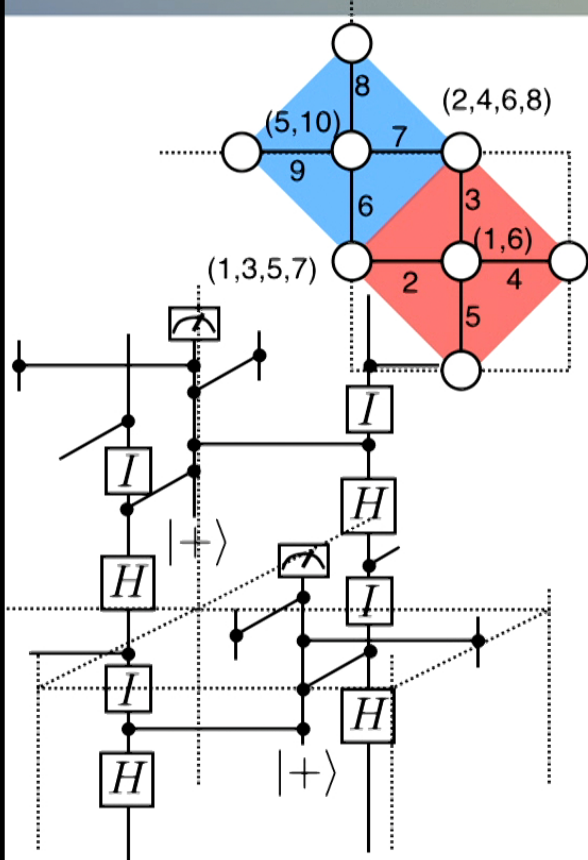
New Journal of Physics **9** (2007) 199

Ann. Phys. **321** 2242 (2006)



Circuit-based noise model with 2D surface code



arXiv:1610.03632

- 2D nearest-neighbor gates on a square grid
- circuit-based depolarizing noise model: prep., meas., 1- and 2-qubit gates with probability *p*.

$$\epsilon(\nu,\mu) \equiv \left(\frac{\nu}{1-\nu}\right)^{1/2}\left[\left(\frac{\nu}{1-\nu}\right)+\left(\frac{2\mu}{1-\mu}\right)\right]^{1/2}$$

ν: independent error rate
μ: correlated error rate
$\nu = 54p/15, \mu = 6p/5$

- threshold value: p=2.84% (distillability of magic state)
- higher than the standard threshold 0.75%

Summary

- Sampling with noisy quantum circuits can exhibit quantum supremacy.

- The threshold for supremacy is much higher than that for universal fault-tolerant quantum computation.

- The threshold is determined purely by distillability of the magic state (in a phenomenological model it sharply separate classically simulatable and not-simulatable regions).

- Can we directly verify or identify quantum supremacy of the near-term noisy quantum devices in a pre-fault-tolerant region?

Thank you for your attention!

arXiv:1610.03632