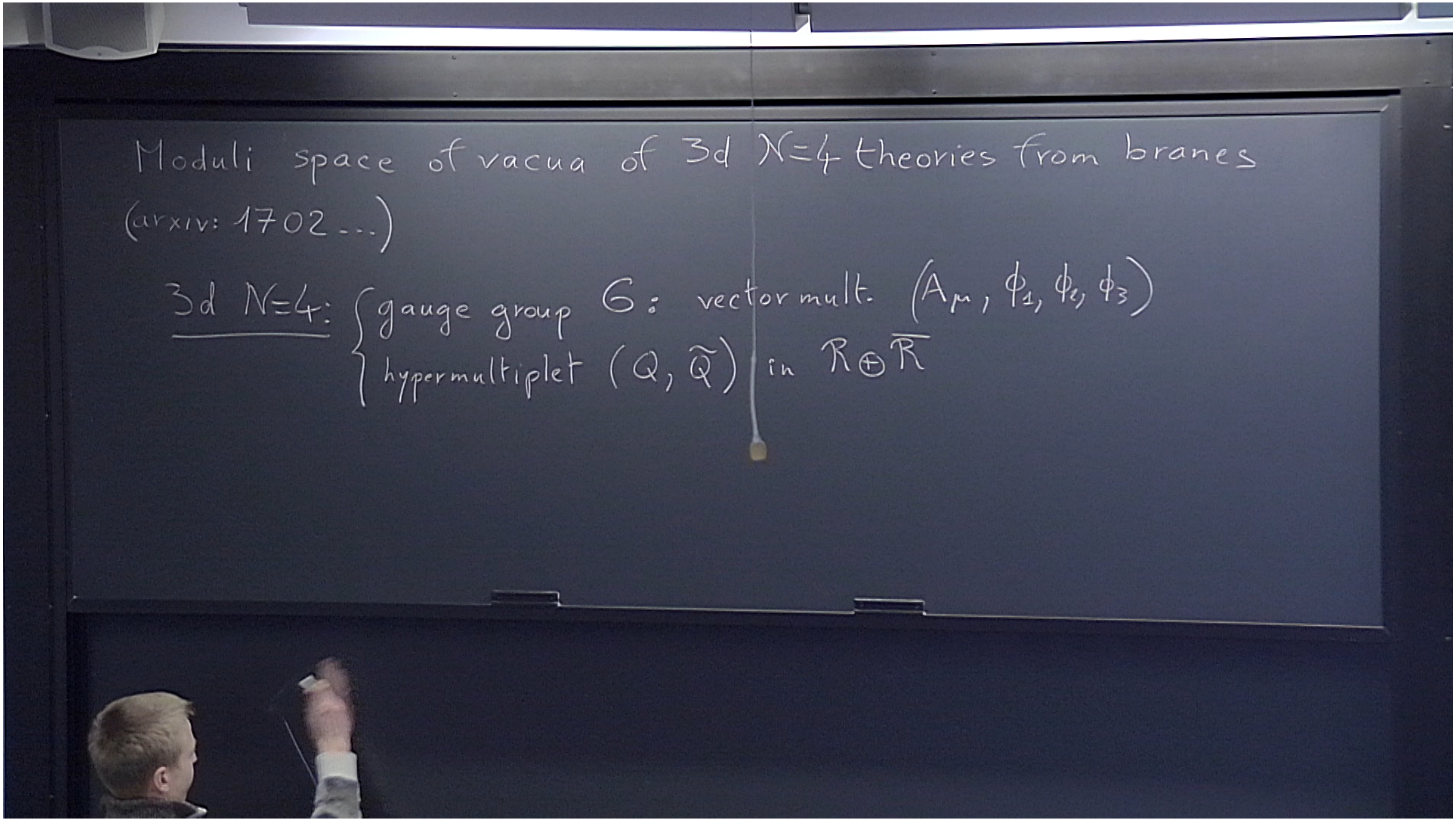
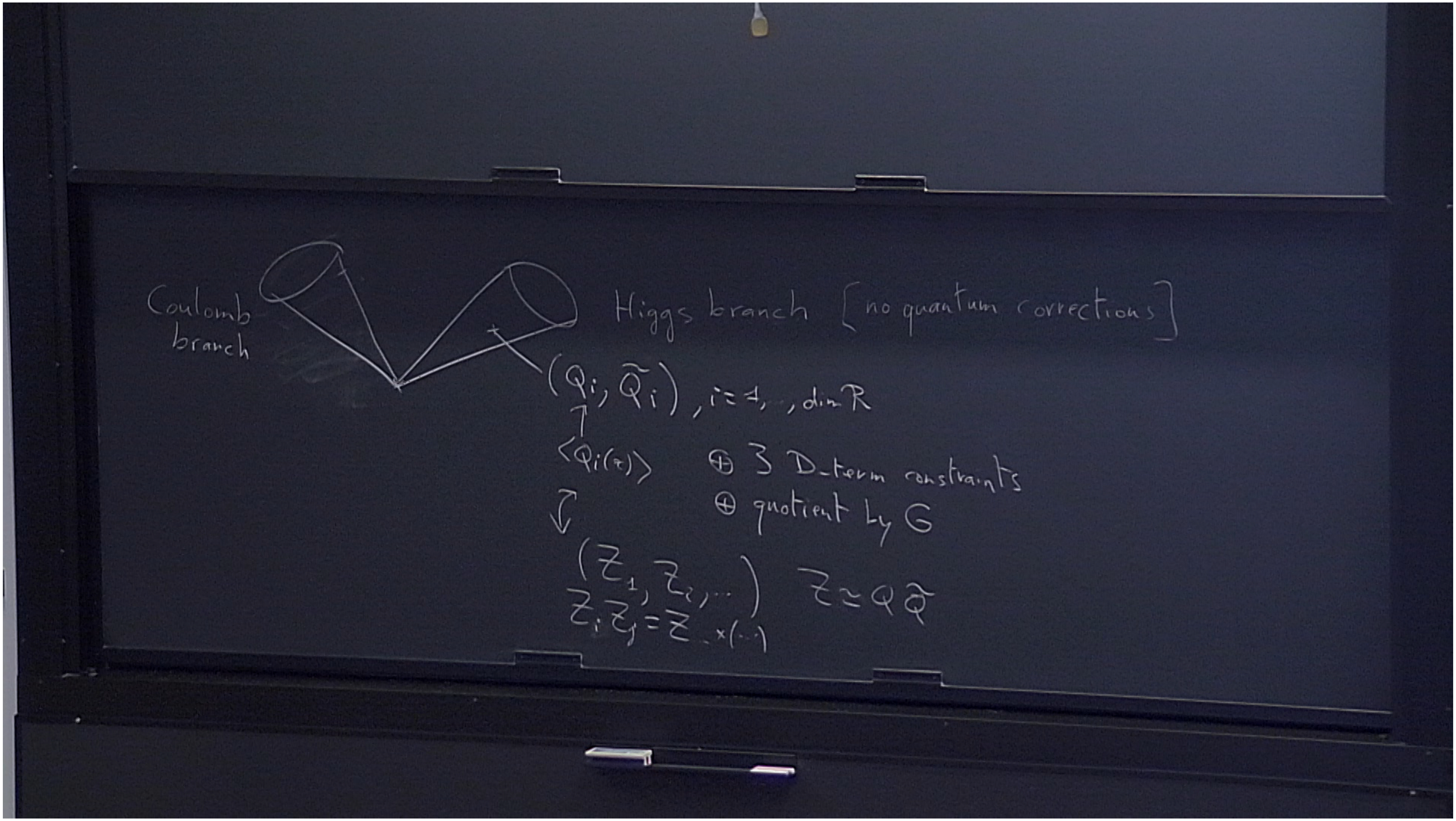
Title: The Higgs and Coulomb branches of 3d N=4 theories from branes

Date: Jan 24, 2017 02:30 PM

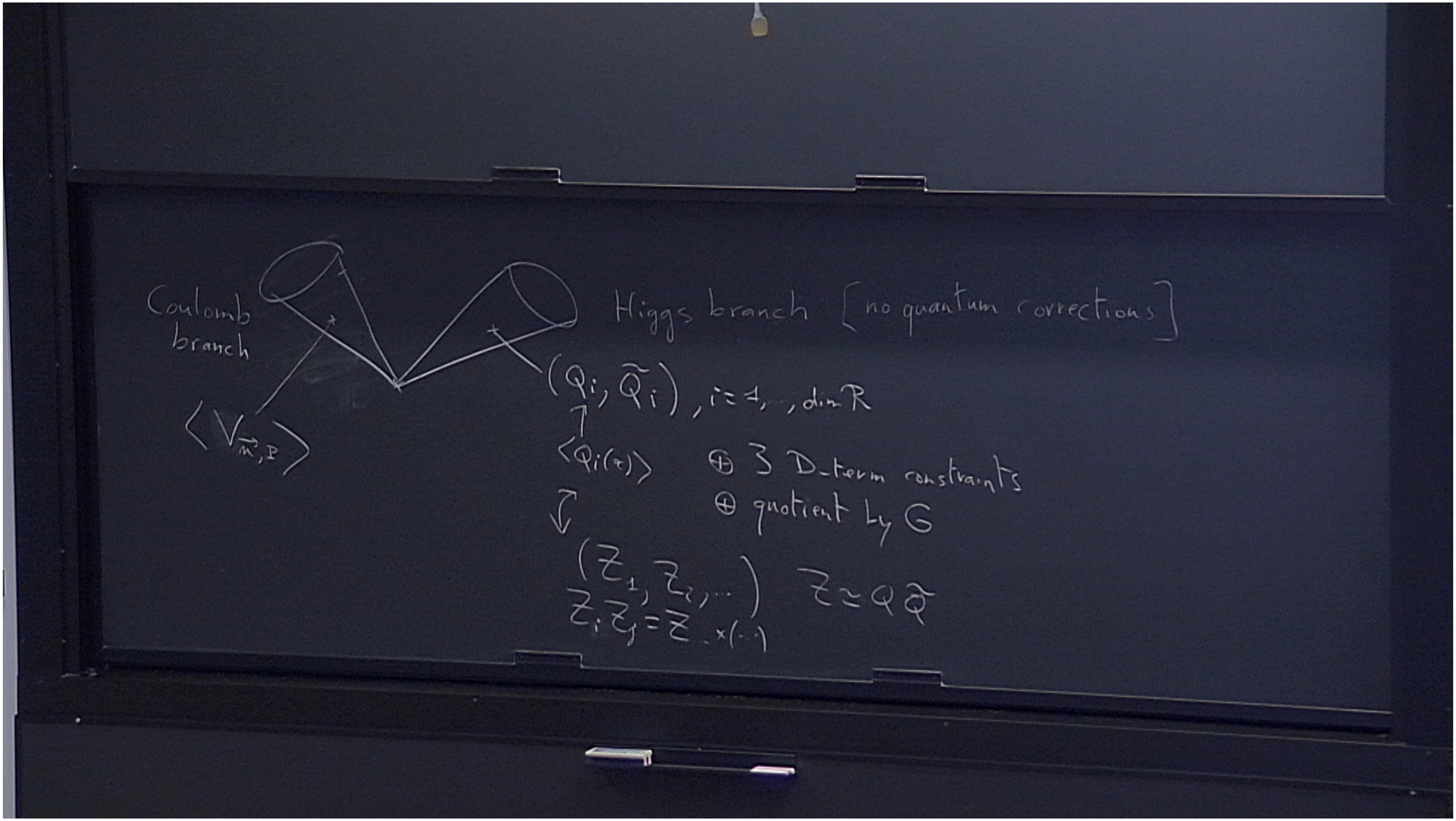
URL: http://pirsa.org/17010078

Abstract: <p>The moduli space of vacua of 3d N=4 gauge theories splits essentially into two ``branchesâ€•: the Higgs branch parametrised by the vevs of hypermultiplet scalars and the Coulomb branch parametrised by the vevs of dressed monopole operators. They can be described as complex algebraic varieties. I will present a new approach to the analysis of the Higgs and Coulomb branches based on the type IIB brane realisation of the gauge theory. Focusing on abelian theories, I will show how the addition of new ingredients in the brane setup can be linked to the path integral insertions of Higgs or Coulomb operators. I will explain how this construction can be used to derive the ring relations and study mirror symmetry.</p>


Moduli space of vacua of 3d $\mathcal{N}=4$ theories from branes
(arxiv: 1702 ...)
3d $\mathcal{N}=4$: gauge group G: vector mult. $(A_\mu, \phi_1, \phi_2, \phi_3)$
hypermultiplet $(Q, \tilde{Q})$ in $R \oplus \overline{R}$



Coulomb branch
Higgs branch [no quantum corrections]
(Q_i, Q̃_i) , i = 1, .., dim R
⟨Q_i(τ)⟩
⊕ 3 D-term constraints
⊕ quotient by G
(Z_1, Z_2, ...)
Z_i Z_j = Z ... × (...)
Z ≃ Q Q̃


Coulomb branch
Higgs branch [no quantum corrections]
⟨V_{n,P}⟩
(Q_i, Q̃_i), i=1,…,dim R
⟨Q_i(τ)⟩
⊕ ∃ D-term constraints
⊕ quotient by G
(Z_1, Z_2, …)
Z ≃ Q Q̃
Z_i Z_j = Z × (…)

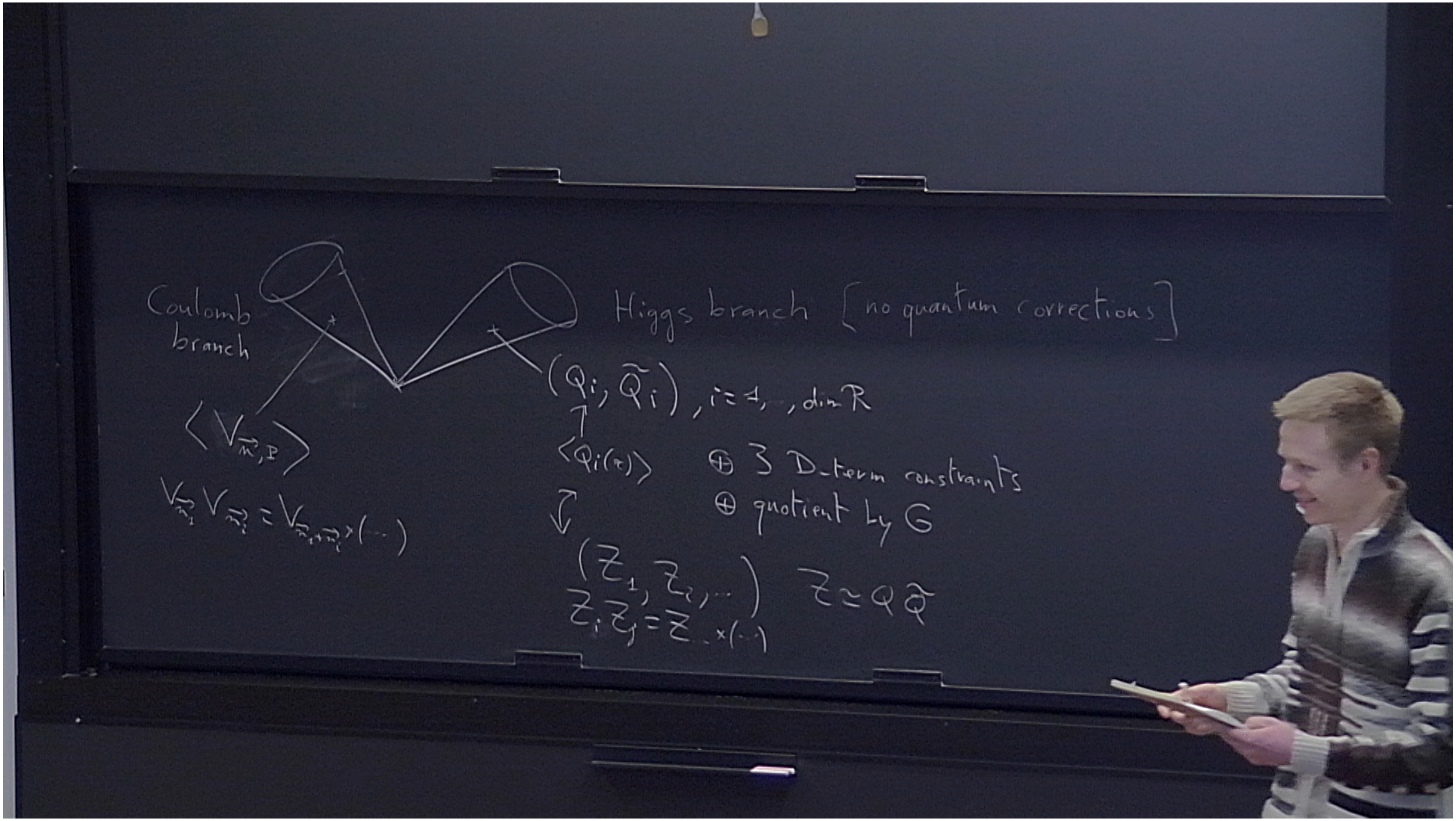
path integral

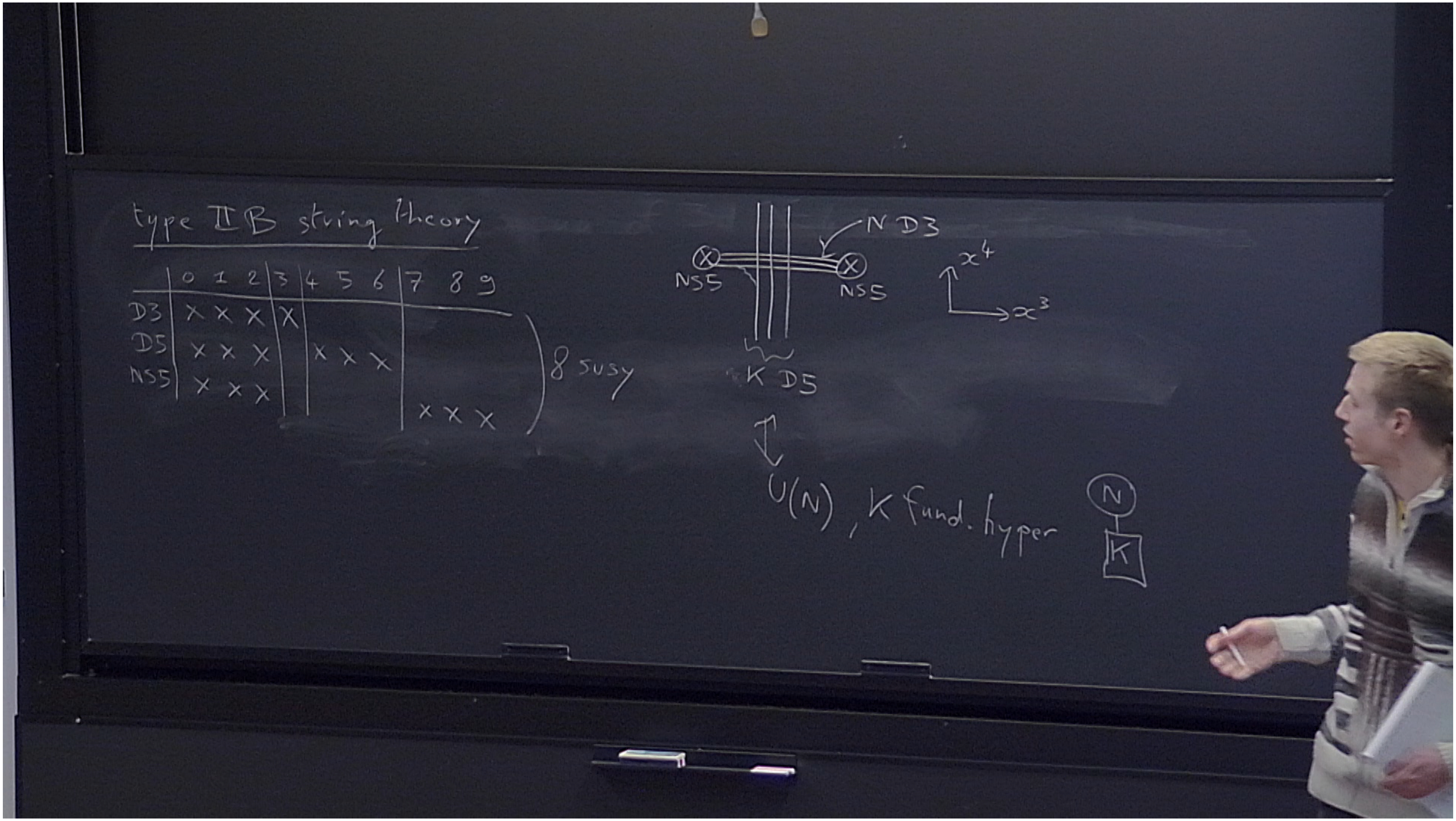
$$A_\mu = \begin{pmatrix} A_{1\mu} & & & \\ & A_{2\mu} & & \\ & & \ddots & \\ & & & A_{N,\mu} \end{pmatrix}$$

- $\int_x dA_i = n_i$, singularity for ϕ_1

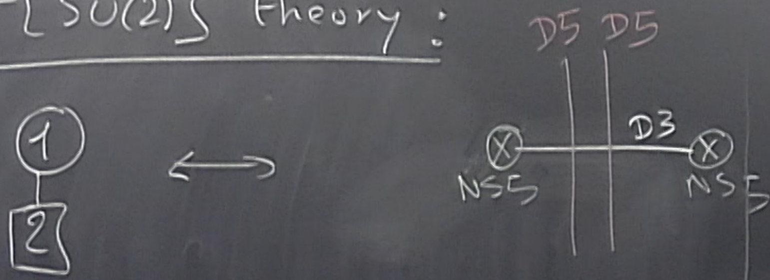
⊕ "dressed" with $\mathbb{P}(\varphi_i(x))$, $\varphi = \phi_2 + i\phi_3$

⊕ average over Weyl group


Coulomb branch
Higgs branch [no quantum corrections]
⟨V_{m,B}⟩
V_{m1} V_{m2} = V_{m1+m2} × (...)
(Q_i, Q̃_i), i = 1, ..., dim R
⟨Q_i(x)⟩
⊕ ∃ D-term constraints
⊕ quotient by G
(Z_1, Z_2, ...)
Z ≃ Q Q̃
Z_i Z_j = Z ×(...)

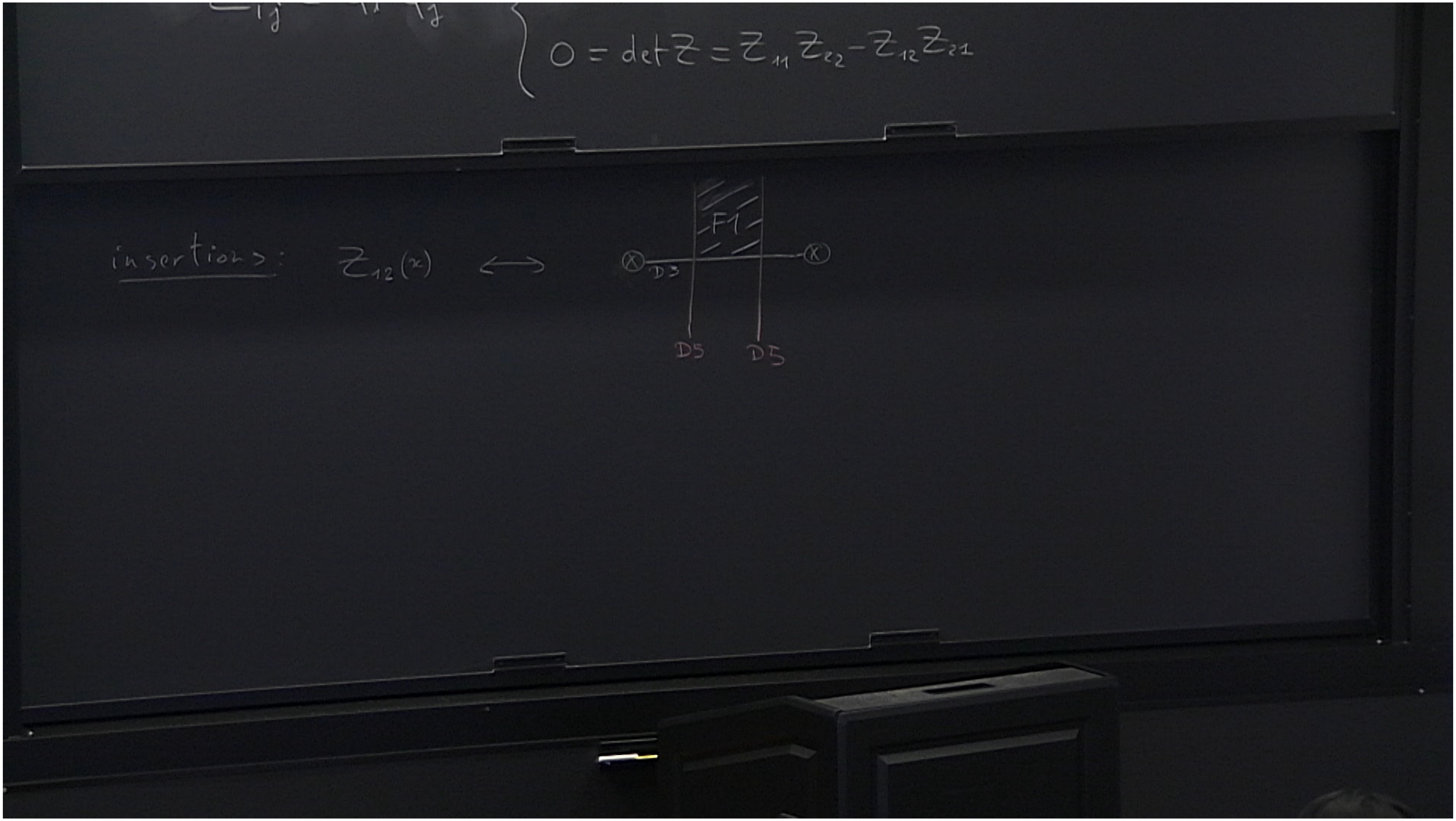

type IIB string theory
0 1 2 3 4 5 6 7 8 9
D3 x x x x
D5 x x x x x x
NS5 x x x x x x
8 susy
N D3
NS5
NS5
K D5
x^4
x^3
U(N), K fund. hyper
N
K

T[SU(2)] theory:



Higgs branch: $(Q_1, \widetilde{Q}_1), (Q_2, \widetilde{Q}_2)$

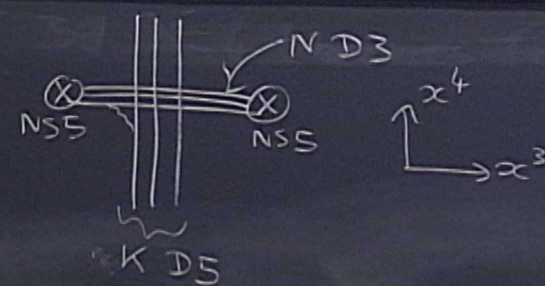
$$Z_{ij} = \widetilde{Q}_i Q_j \qquad \begin{cases} 0 = \mathrm{Tr}\, Z = Z_{11} + Z_{22} \\ 0 = \det Z = Z_{11} Z_{22} - Z_{12} Z_{21} \end{cases}$$


0 = det Z = Z11 Z22 − Z12 Z21
insertions:
Z12(x) ⟷
F1
D3
D5
D5

type IIB string theory

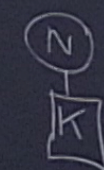
	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D5	x	x	x		x	x	x			
NS5	x	x	x					x	x	x
F1				x	x					

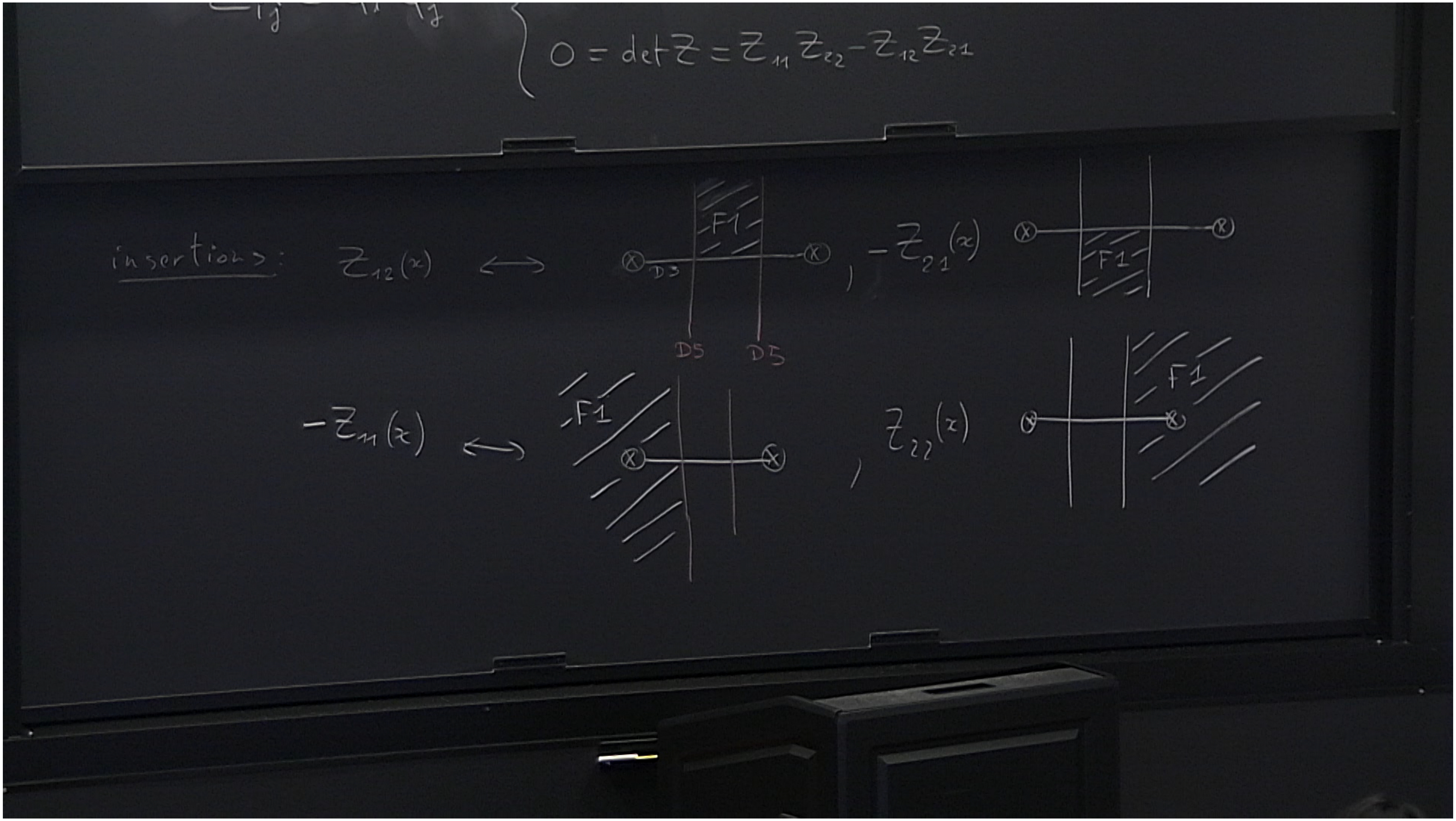
8 susy

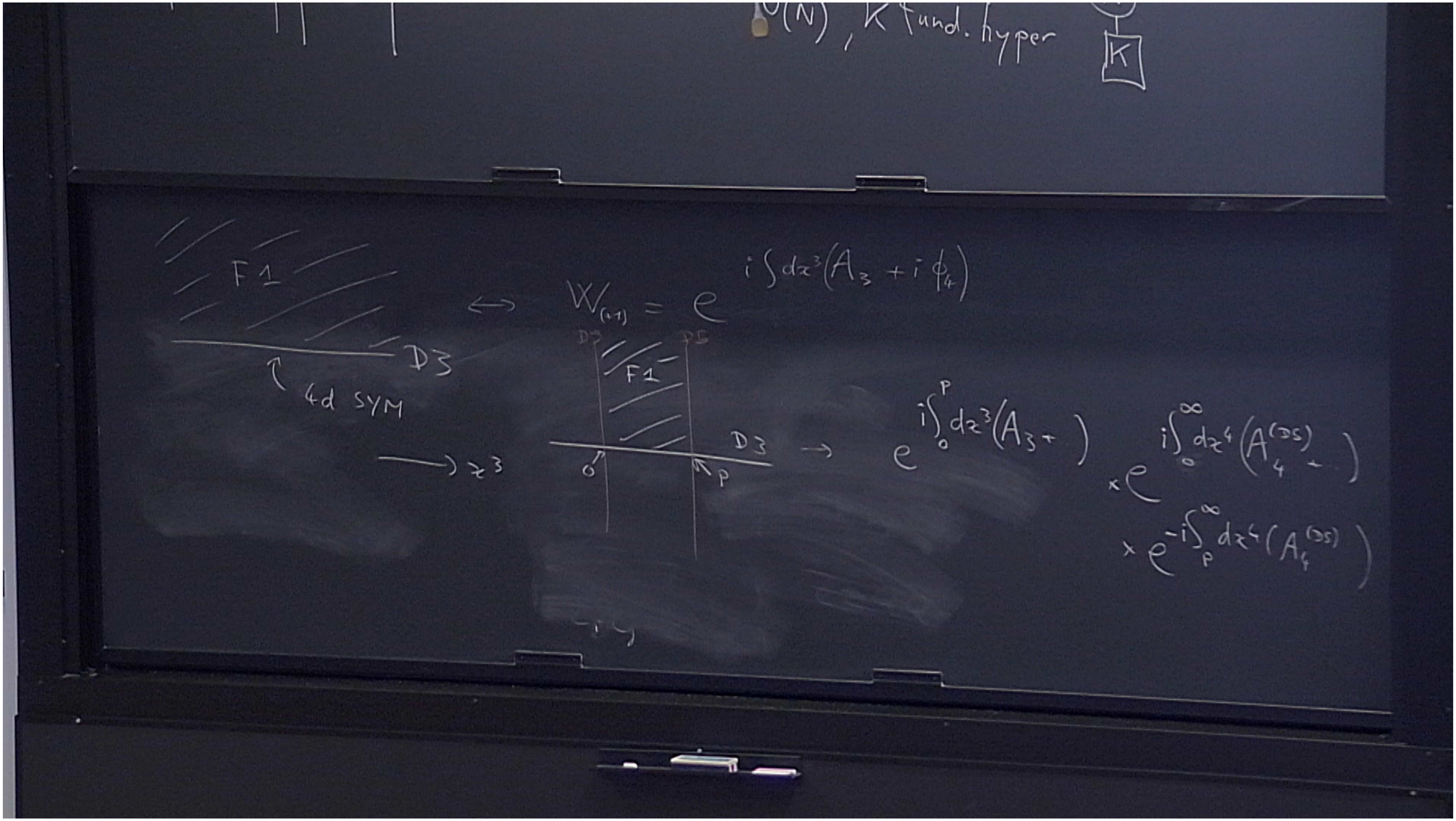


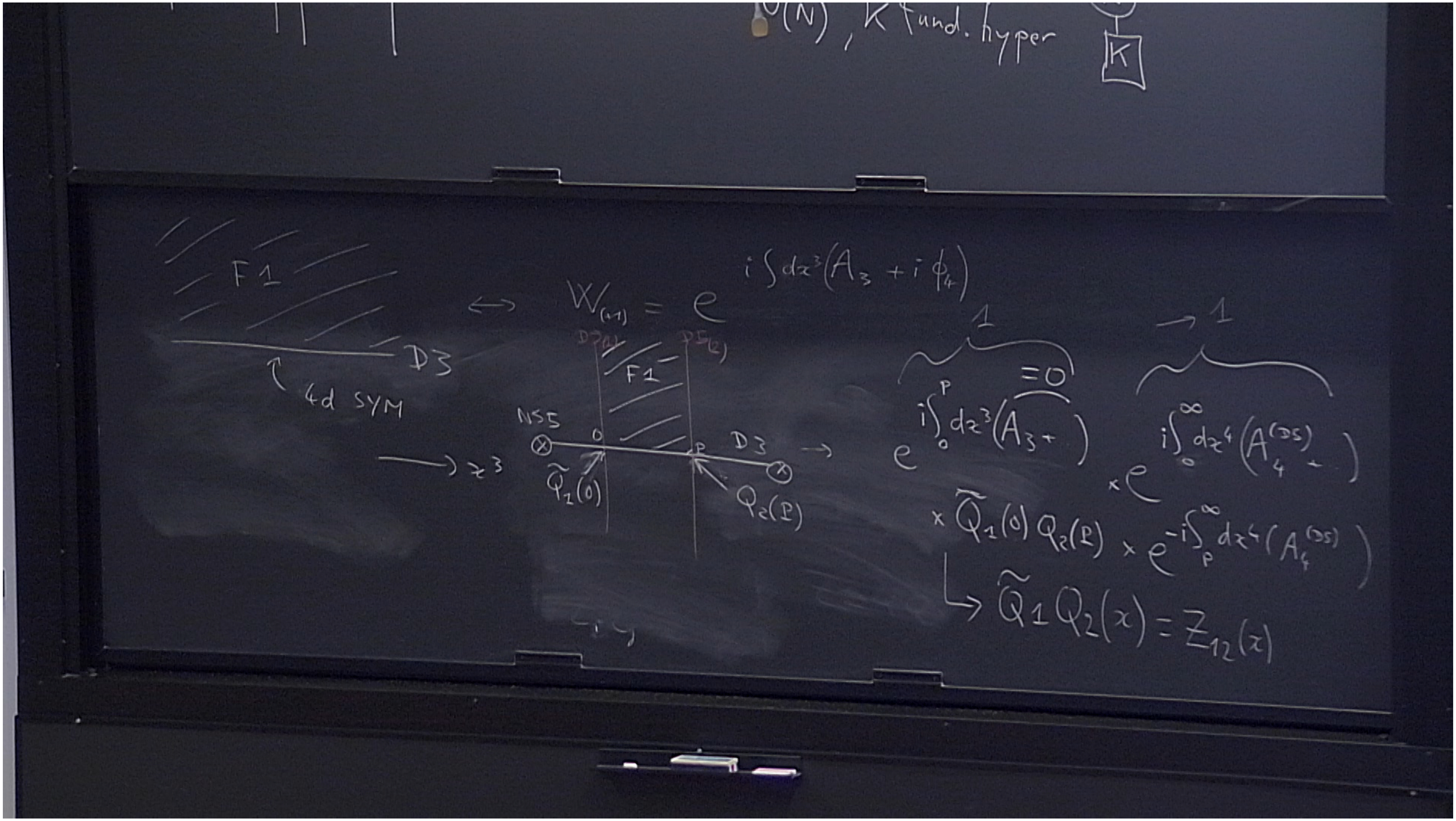
$\Updownarrow$

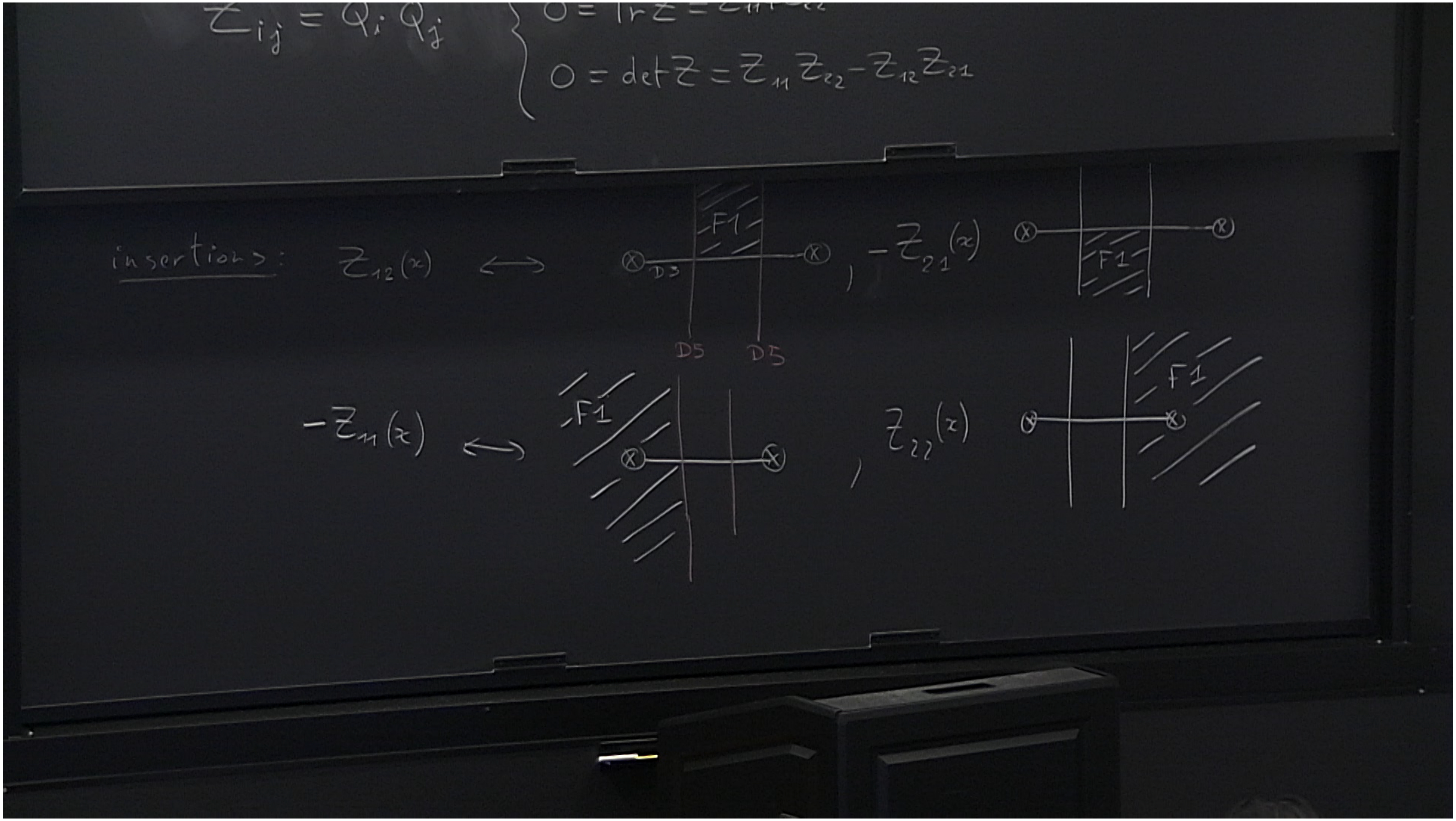
U(N), K fund. hyper




0 = det Z = Z11 Z22 − Z12 Z21
insertions:
Z12(x) ⟷
F1
D3
D5
D5
, −Z21(x)
F1
−Z11(x) ⟷
F1
, Z22(x)
F1

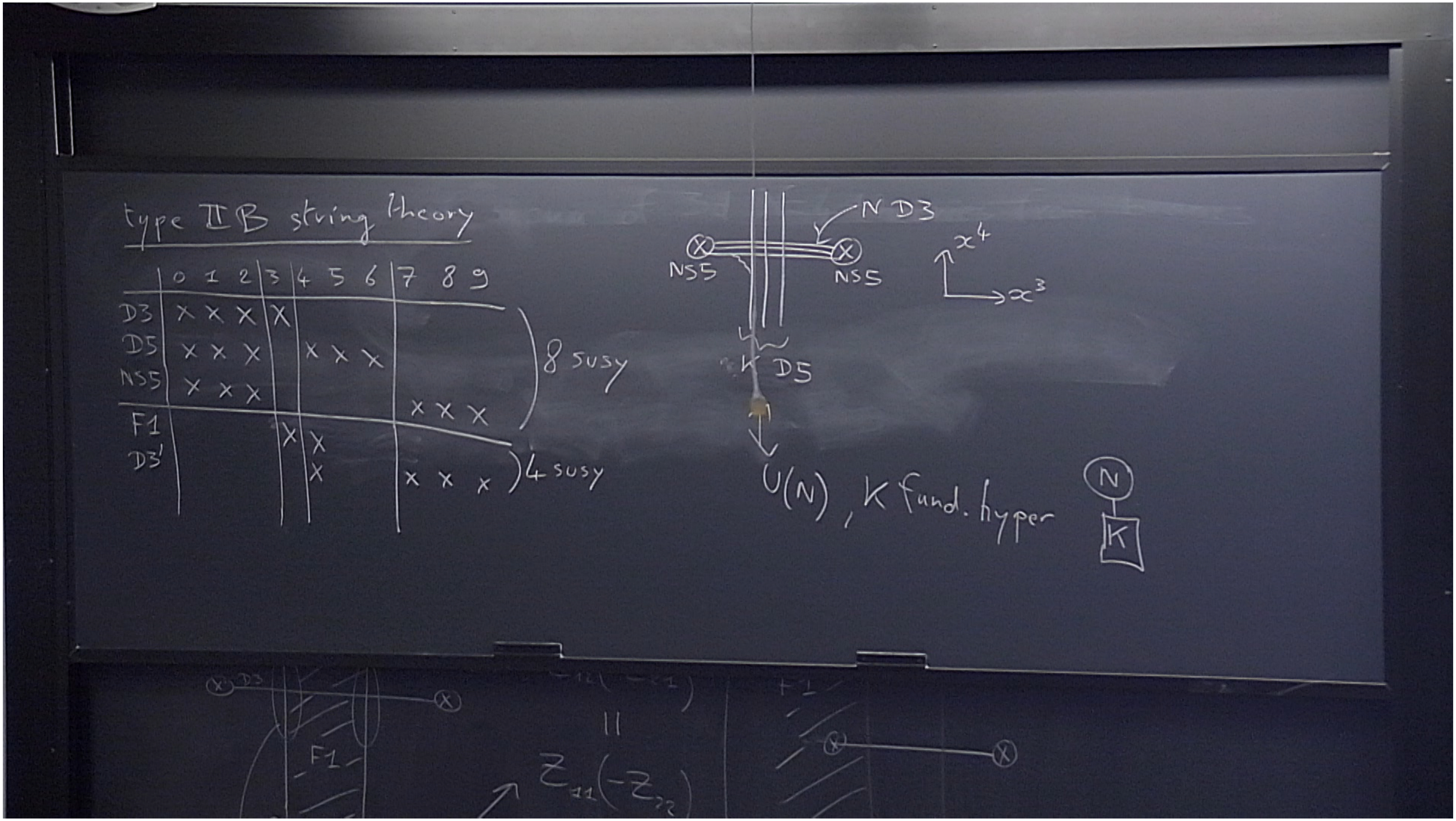

U(N), K fund. hyper
K
F1
D3
4d SYM
z^3
W_(1,1) = e^{i∫dz^3(A_3 + iφ_4)}
D5
D5
F1
D3
0
p
e^{i∫_0^p dz^3(A_3 + …)}
× e^{i∫_0^∞ dz^4(A_4^(D5) + …)}
× e^{-i∫_p^∞ dz^4(A_4^(D5))}


U(N), K fund. hyper
K
F1
D3
4d SYM
NS5
D3
F1
D5(1)
D5(2)
z^3
Q̃_1(0)
Q_2(p)
W(+1) = e^{i∫dz^3(A_3 + iφ_4)}
→ e^{i∫_0^p dz^3(A_3 +)} × e^{i∫_0^∞ dz^4(A_4^(D5) +)}
= 0
1
→ 1
× Q̃_1(0) Q_2(p) × e^{-i∫_p^∞ dz^4(A_4^(D5))}
→ Q̃_1 Q_2(x) = Z_12(x)

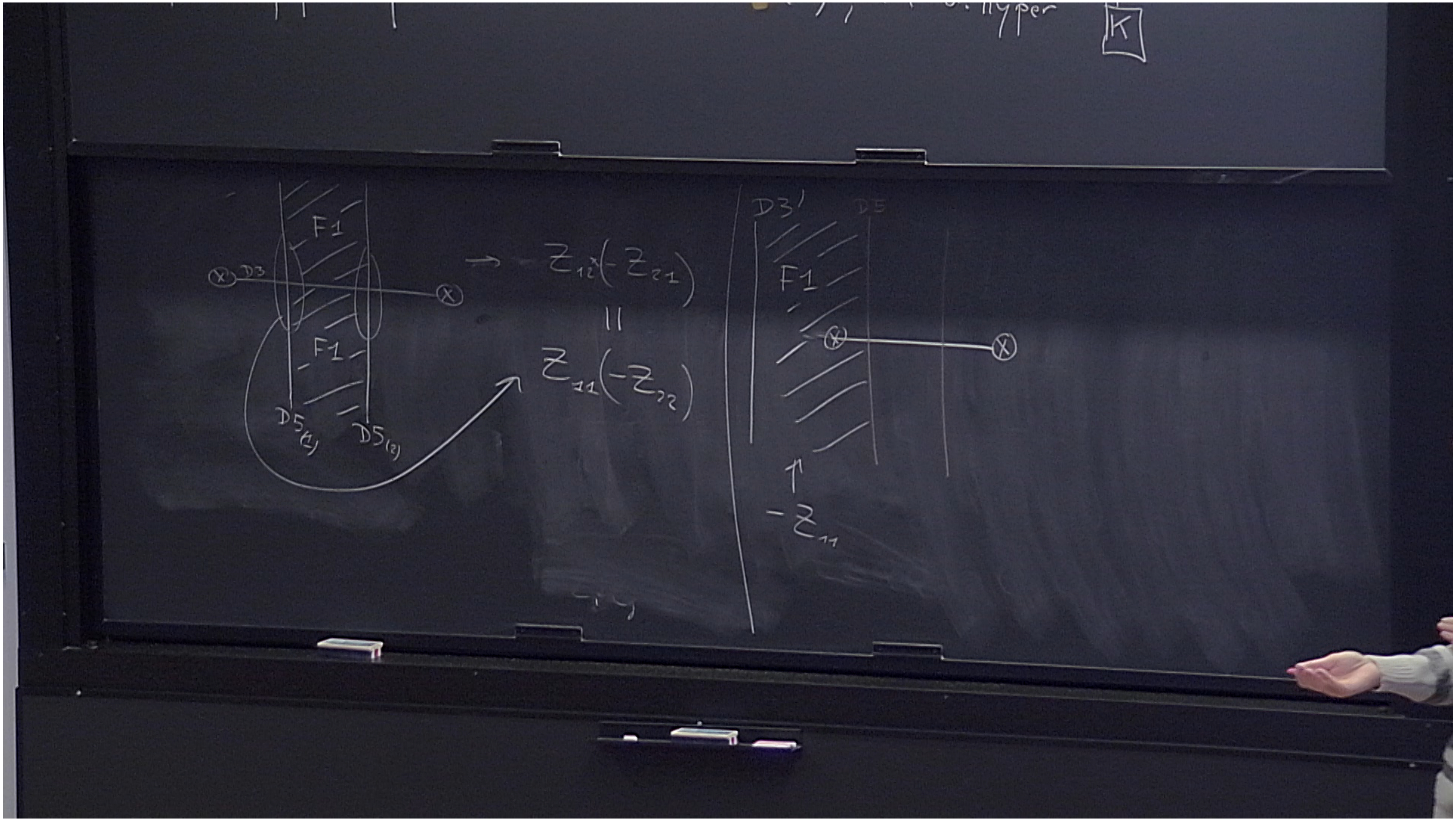


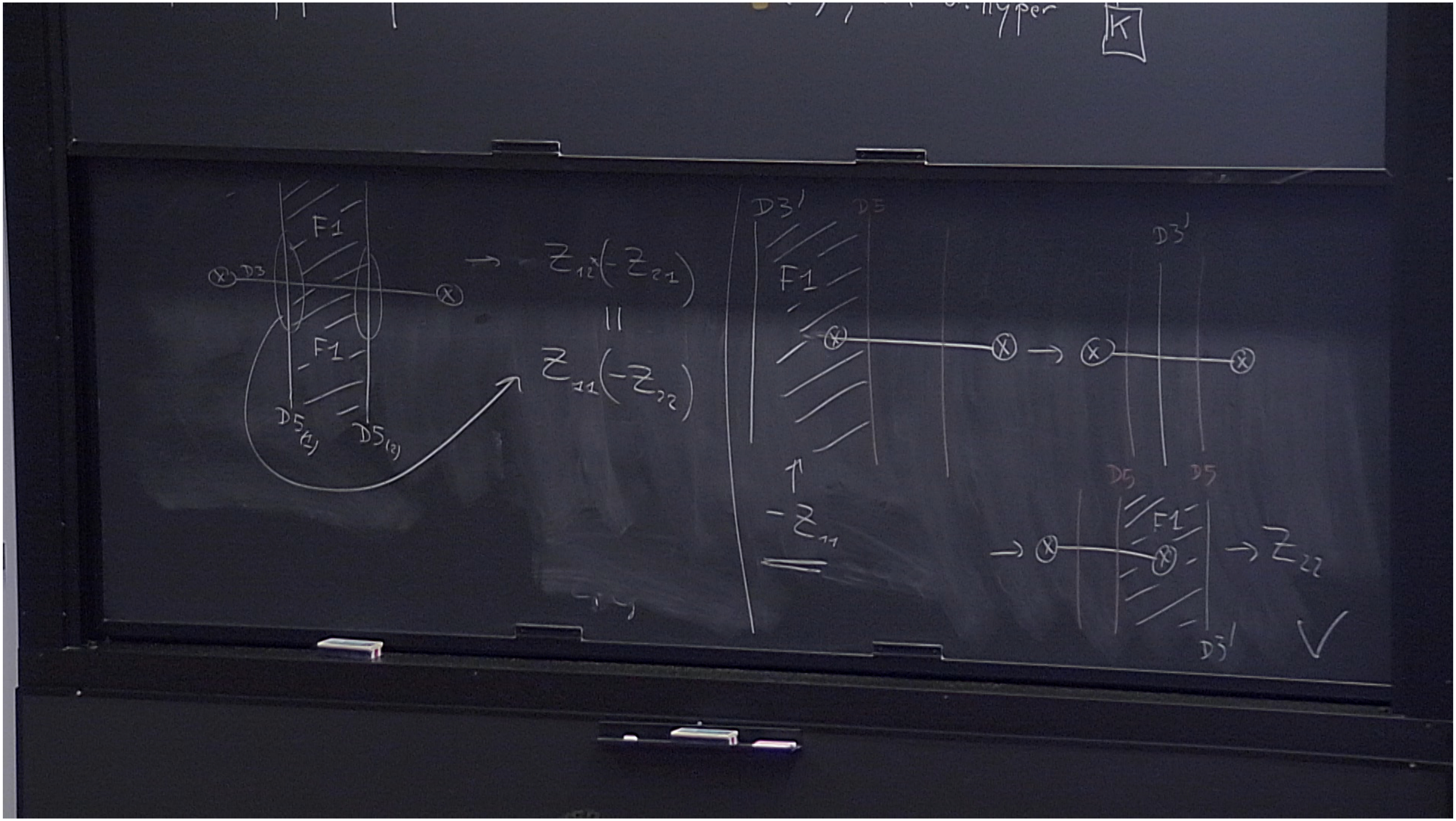
Z_ij = Q_i Q_j
0 = det Z = Z_11 Z_22 − Z_12 Z_21
insertions:
Z_12(x)
F1
D3
D5
D5
−Z_21(x)
F1
−Z_11(x)
F1
Z_22(x)
F1

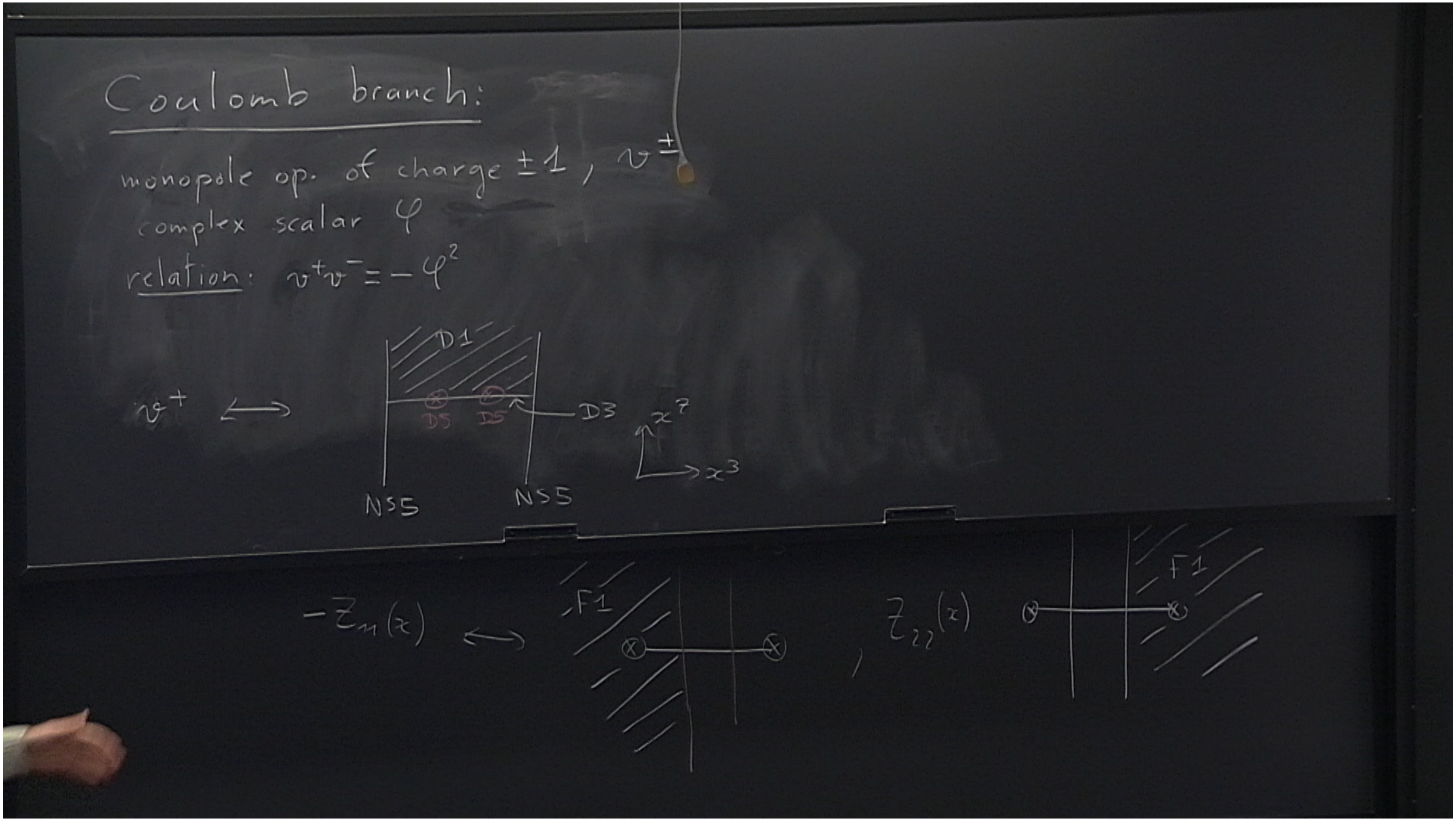

U(N), K fund. hyper
K
F1
D3
F1
D5(1)
D5(2)
Z12(-Z21)
=
Z11(-Z22)

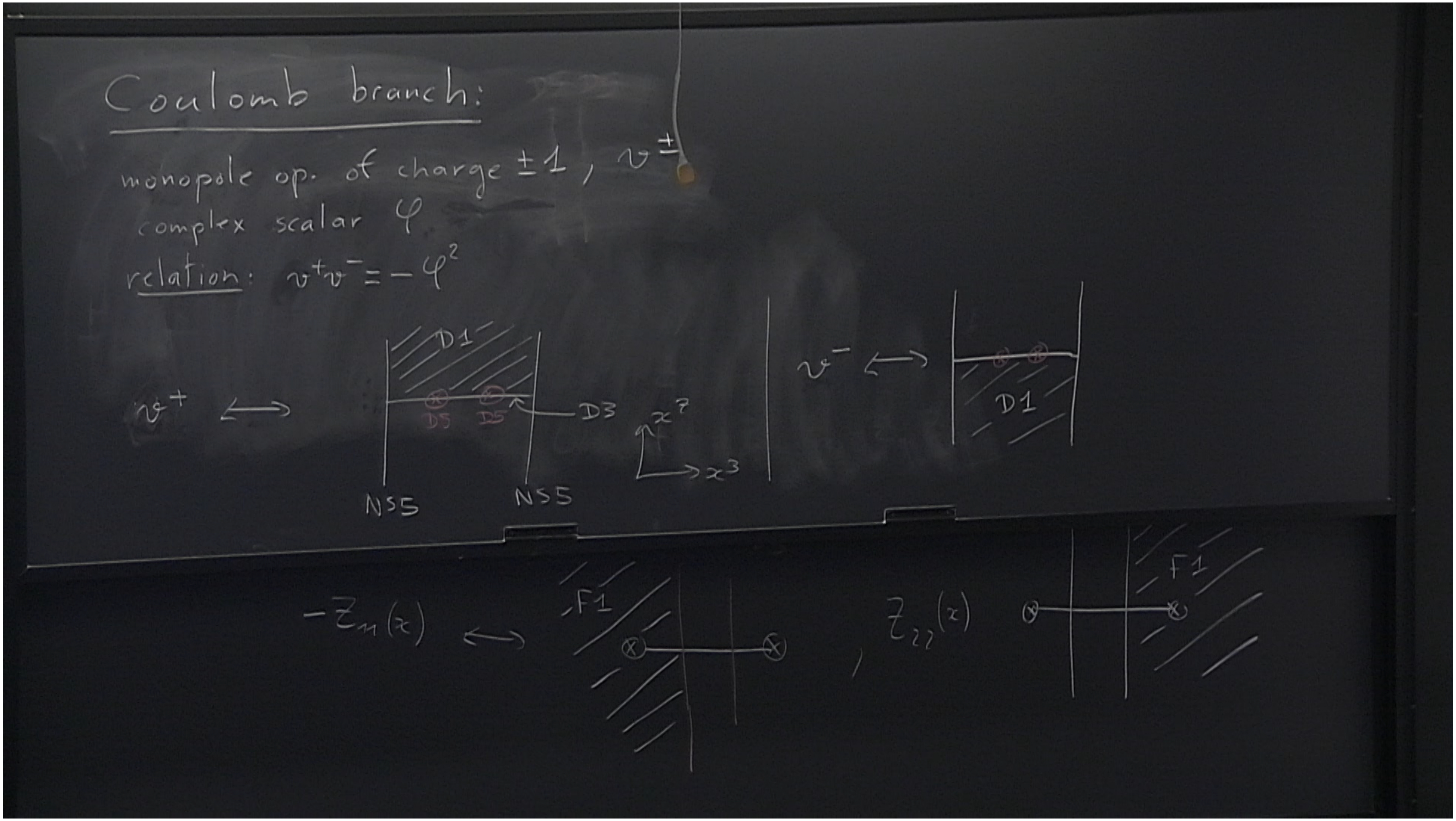


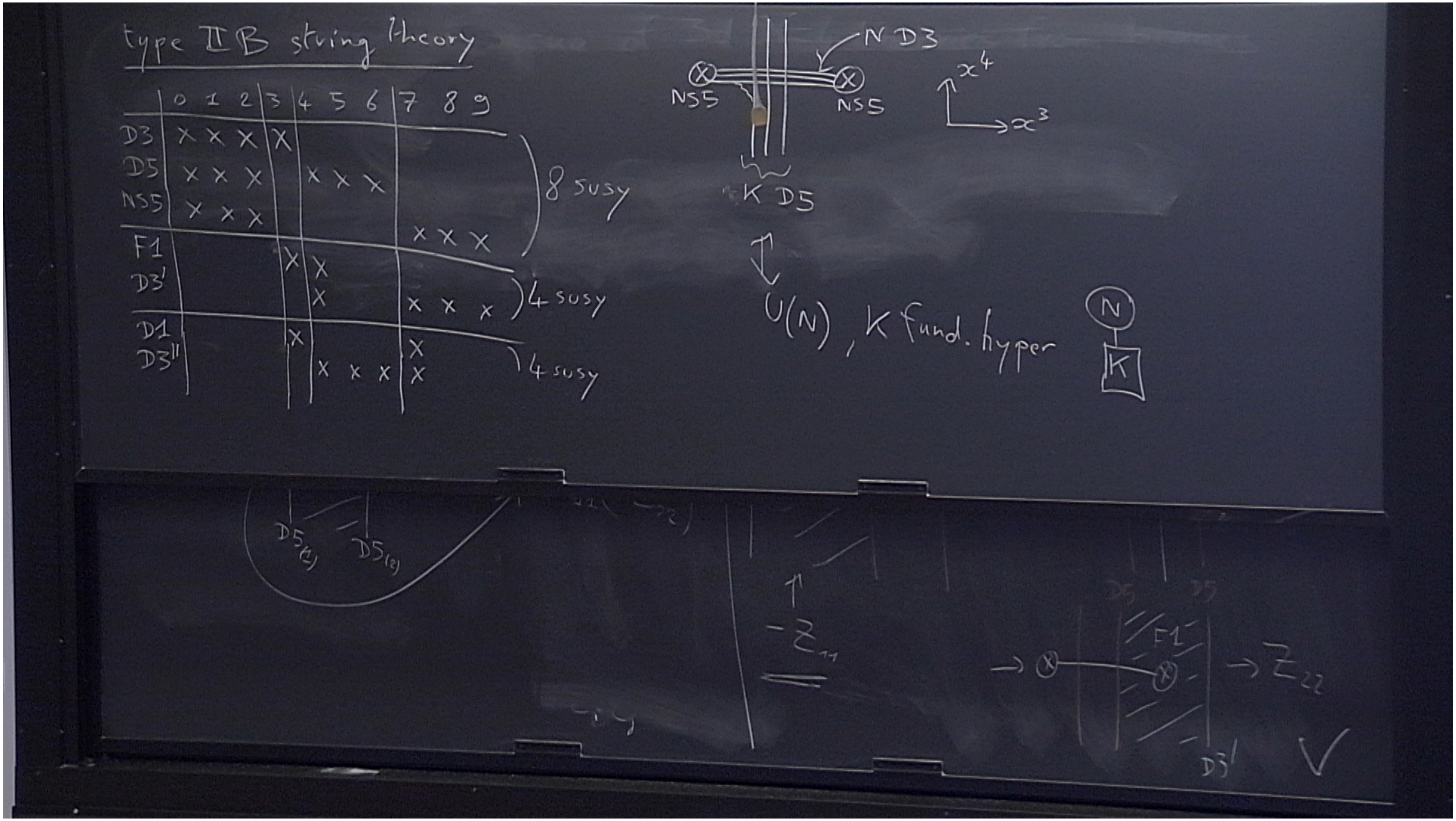
type IIB string theory
0 1 2 3 4 5 6 7 8 9
D3
D5
NS5
F1
D3'
8 susy
4 susy
N D3
NS5
NS5
x^4
x^3
K D5
U(N), K fund. hyper
N
K
F1

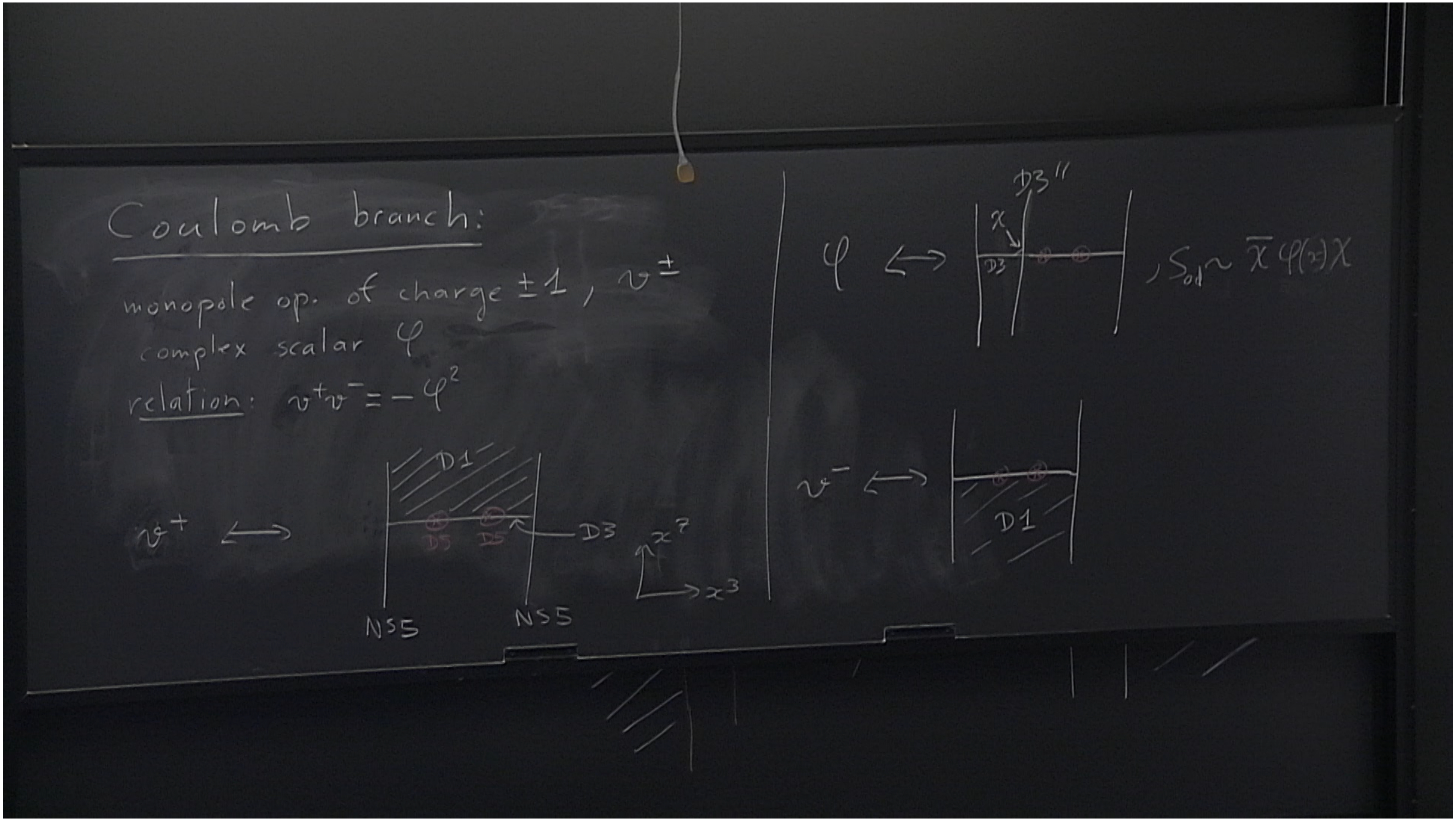

K
F1
D3
F1
D5(1)
D5(2)
Z12(-Z21)
=
Z11(-Z22)
D3'
D5
F1
-Z11

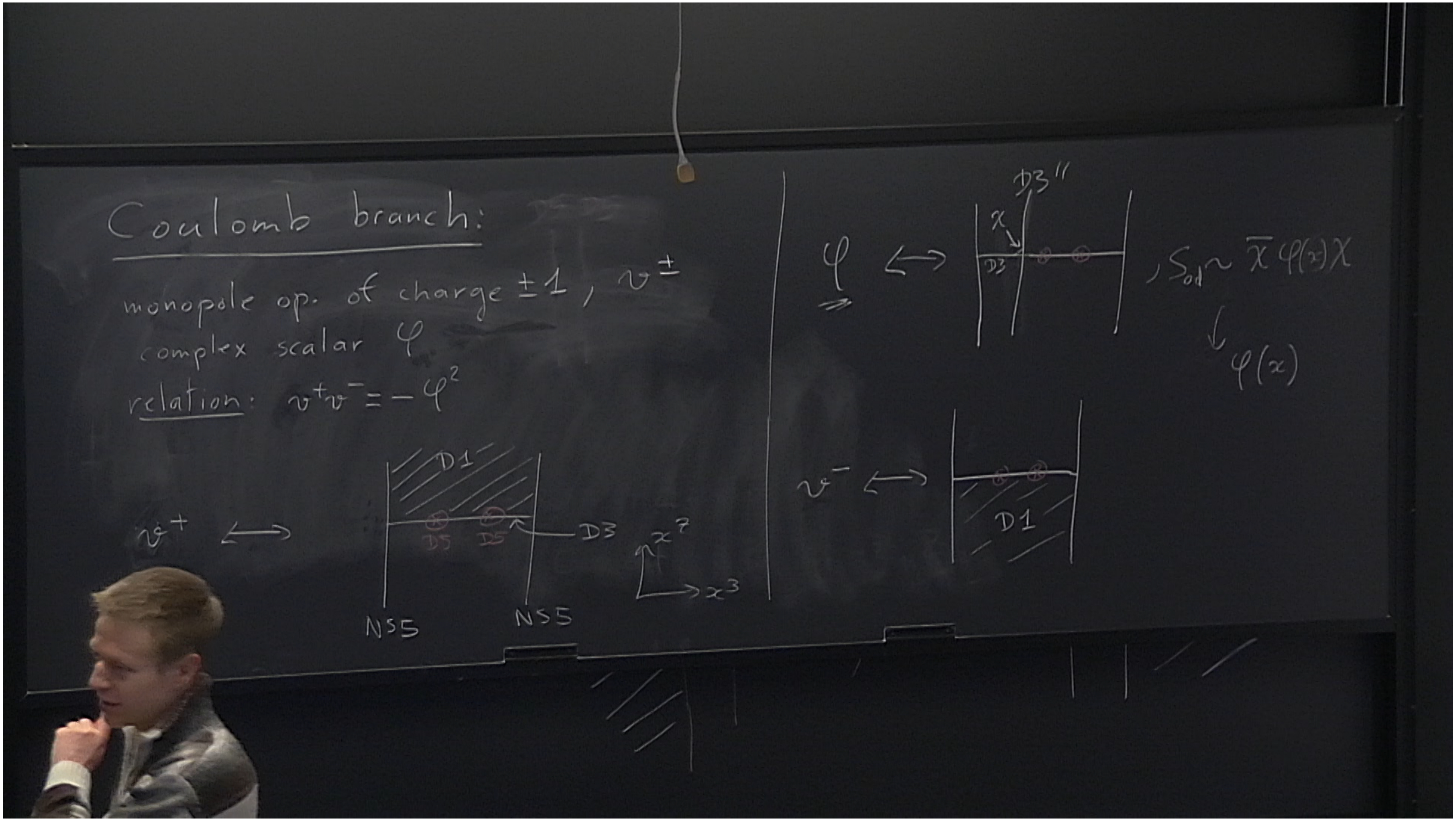

D3
F1
F1
D5(1)
D5(2)
Z12(-Z21)
=
Z11(-Z22)
D3'
D5
F1
-Z11
D3'
D5
D5
F1
Z22
D3'
K

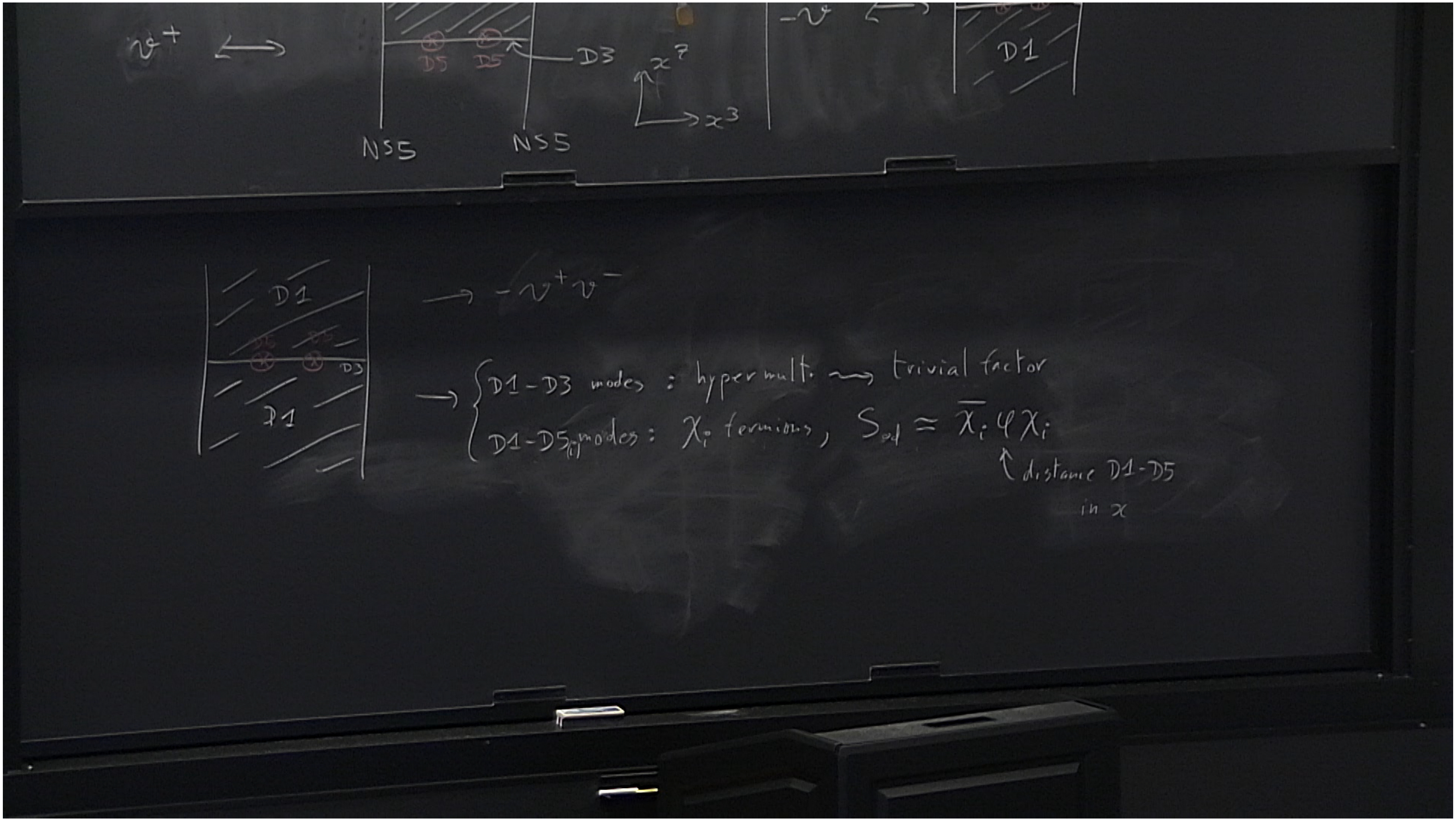

Coulomb branch:
monopole op. of charge ±1 , v^±
complex scalar φ
relation: v^+ v^- = - φ^2
v^+ ⟷
D1
D5
D5
D3
x^7
x^3
NS5
NS5
- Z_11(x) ⟷
F1
, Z_22(x)
F1


Coulomb branch:
monopole op. of charge ±1, v±
complex scalar φ
relation: v⁺v⁻ = −φ²
v⁺ ⟷
D1
D5 D5
D3
NS5 NS5
x⁷
x³
v⁻ ⟷
D1
−Z₁₁(x) ⟷
F1
Z₂₂(x)
F1

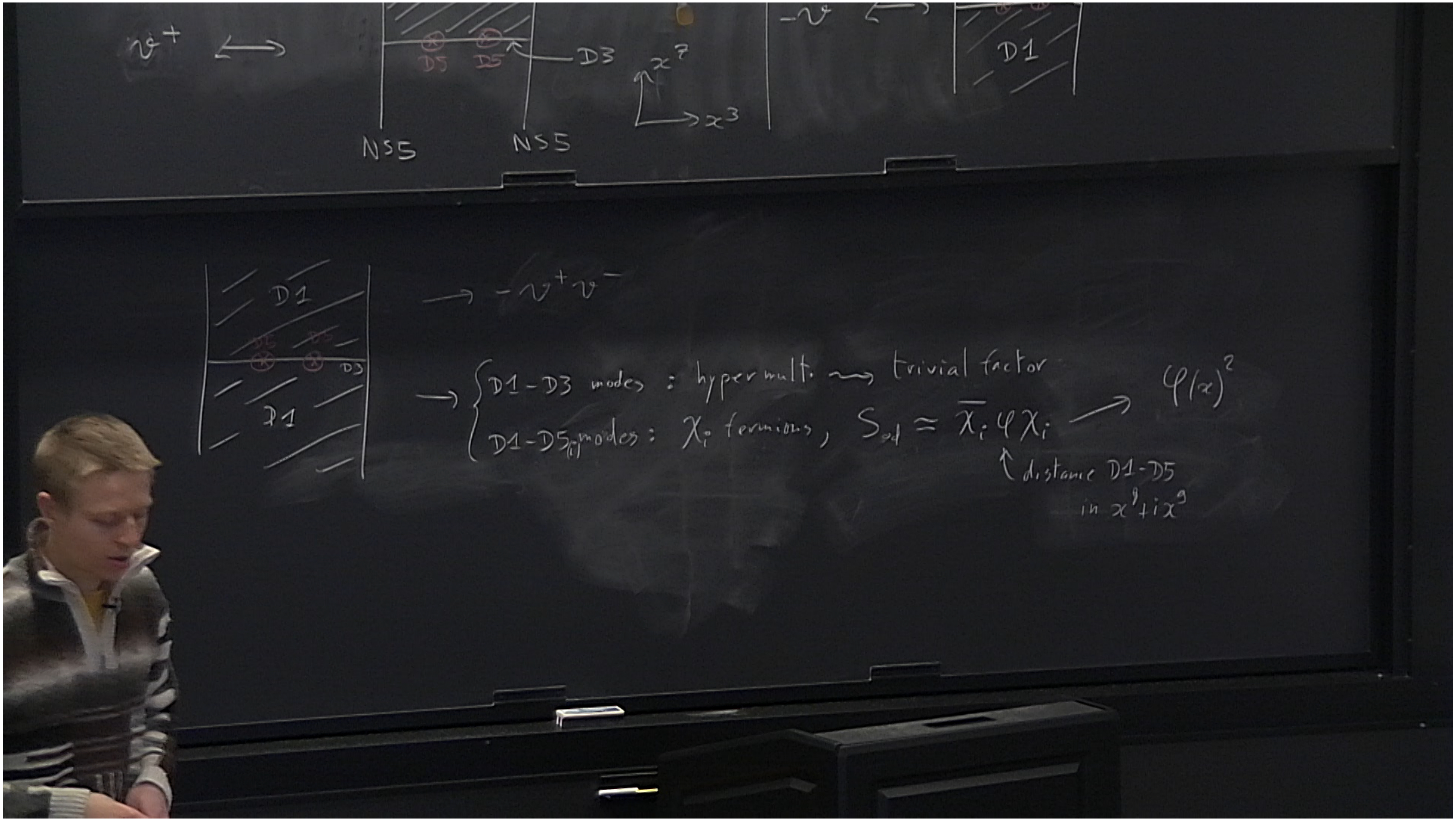

type IIB string theory
0 1 2 3 4 5 6 7 8 9
D3
D5
NS5
F1
D3'
D1
D3''
8 susy
4 susy
4 susy
N D3
NS5
NS5
K D5
U(N), K fund. hyper
N
K
D5
D5
F1
D3'

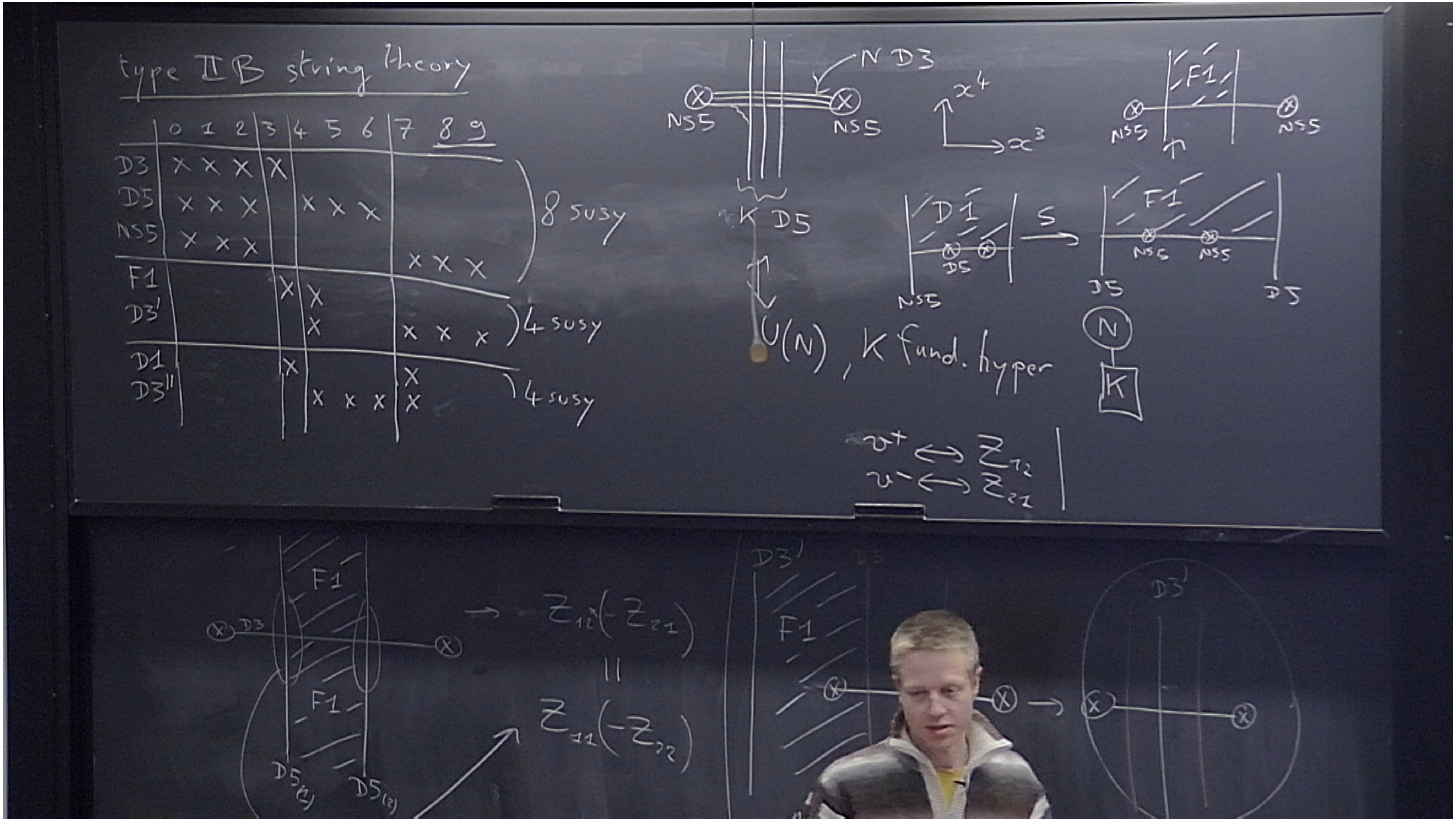

Coulomb branch:
monopole op. of charge ±1, $v^\pm$
complex scalar φ
relation: $v^+ v^- = -\varphi^2$
$v^+ \longleftrightarrow$
D1
D5
D5
D3
NS5
NS5
x^7
x^3
$\varphi \longleftrightarrow$
D3''
χ
D3
, $S_{0d} \sim \bar{\chi}\,\varphi(x)\,\chi$
$v^- \longleftrightarrow$
D1


Coulomb branch:
monopole op. of charge ±1, $v^{\pm}$
complex scalar φ
relation: $v^+ v^- = -\varphi^2$
$v^+ \longleftrightarrow$
D1
D5
D5
D3
NS5
NS5
x^7
x^3
$\varphi \longleftrightarrow$
"D3"
χ
D3
$S_{od} \sim \bar{\chi}\,\varphi(x)\,\chi$
$\varphi(x)$
$v^- \longleftrightarrow$
D1



D3
NS5
NS5
D1
D1
D1
D1-D3 modes : hypermult. ⟶ trivial factor
D1-D5 modes : χ fermions
distance D1-D5
in x


D5 D5
D3
NS5 NS5
x^7
x^3
D1
D1
D3
D1
$\rightarrow -v^+ v^-$
D1 - D3 modes : hypermult. ⇝ trivial factor
D1 - D5 modes : χ_i fermions, $S \simeq \bar{\chi}_i \varphi \chi_i$
$\varphi(x)^2$
distance D1-D5
in $x^{?} + i x^9$


type IIB string theory
0 1 2 3 4 5 6 7 8 9
D3
D5
NS5
F1
D3'
D1
D3''
8 susy
4 susy
4 susy
N D3
NS5
NS5
K D5
U(N), K fund. hyper
F1
D1
S
D5
N
K
F1
D3
D5(1)
D5(2)
D3'
F1
D3'