Title: Emergent Chiral Spin Liquids in Frustrated Magnetism

Date: Jan 20, 2017 04:30 PM

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Abstract: <p>Topological states of matter are of of fundamental interest in contemporate condensed matter physics. Today, the fractional Quantum Hall effect remains the only known experimental system expected to exhibit intrinsic topological order. The question remains whether also different systems might stabilize this kind of ordering. Chiral spin liquids are an analogue of Fractional Quantum Hall Effect wave functions for spin systems. These wavefunctions have been envisioned in 1987 but only very recently several simple frustrated quantum spin models have been proposed realizing this physics. In this talk we will introduce chiral spin liquids, discuss their relation to the Fractional Quantum Hall effect and present our recent numerical studies that provide conclusive evidence for the emergence of this exotic state of matter in extended frustrated Heisenberg models. In the course of these projects novel algorithms and techniques for large scale Exact Diagonalization were developed and applied. We briefly discuss these techniques and show that they allow for sparse diagonalizations of Heisenberg systems up to 48-50 spin-1/2 particles.</p>

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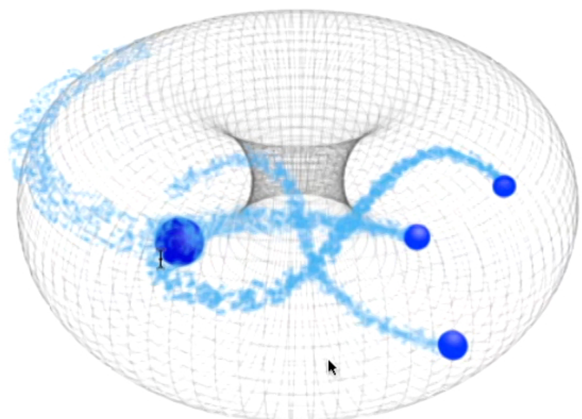
Emergent Chiral Spin Liquids in Frustrated Magnetism



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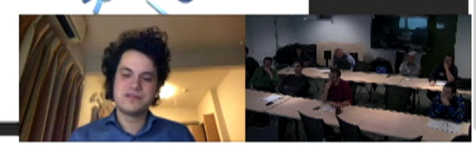

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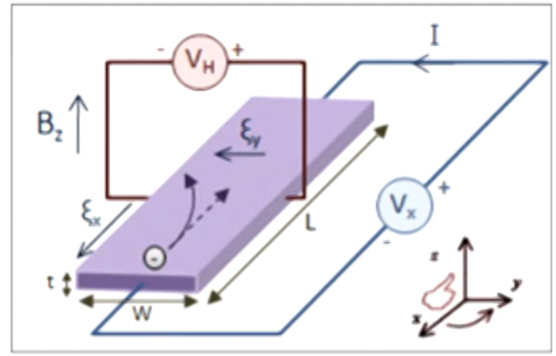
The Hall effects

Classical Hall effect:

- the Hall resistivity

$$R_H = \frac{V_H}{I} = \frac{B}{ne}$$

is proportional to the applied magnetic field.



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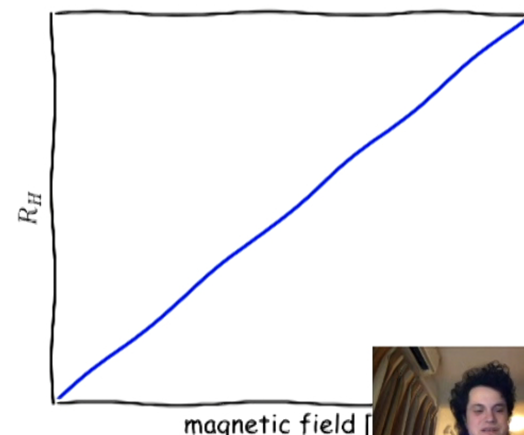
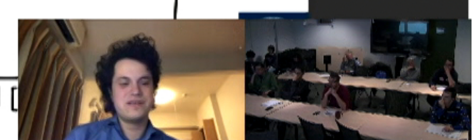




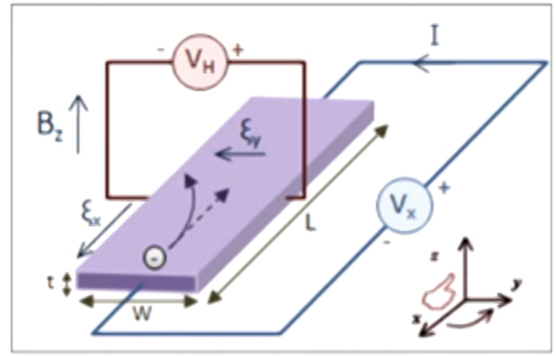
The Hall effects

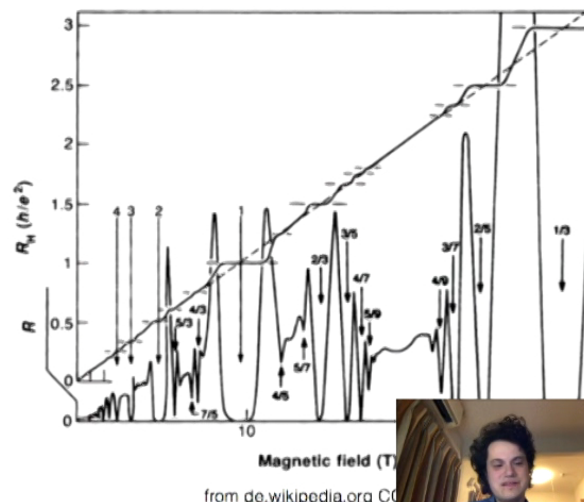
Quantum Hall effect:

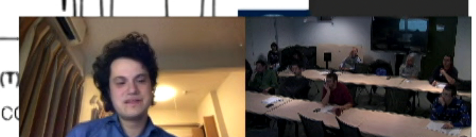
- several plateaux appear in the graph of the Hall resistivity:

$$R_H = \frac{1}{\nu}\frac{h}{e^2}$$

- $\nu = 1, 2, \ldots$ -> Integer QHE
- $\nu = \frac{1}{3}, \frac{2}{5} \ldots$ -> Fractional QHE
- pure quantum mechanical effect







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Fractional QHE

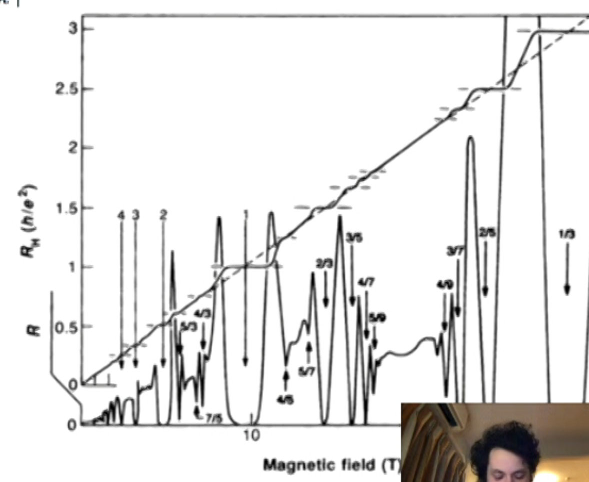
- Strongly interacting electrons in 2D limit at low temperatures, high magnetic fields and pure samples

- groundstate at filling fractions $\nu = 1/p$ be described by Laughlin wave function

$$\psi_p(z_1, \ldots, z_n) = \prod_{i<j}(z_i - z_j)^p \prod_k e^{-|z_k|^2}$$

- phase transitions between different filling fractions without breaking a symmetry

- beyond Landau theory, no local order parameter, quantum entanglement key ingredient



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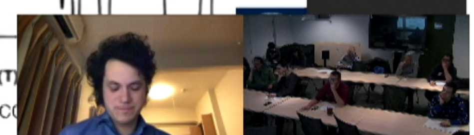

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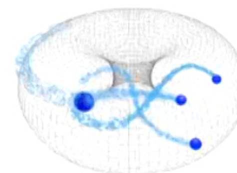
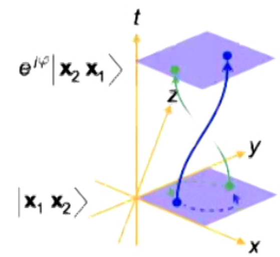
Topological Order (in FQHE)

- groundstate degeneracy depends on topology of the surface the system lives on



- quasiparticle excitations have exotic anyonic braiding statistics (neither fermions nor bosons)



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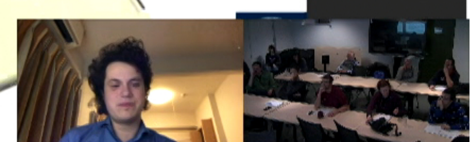
- system is long-range entangled

- (gapless) edge excitations on open boundary conditions

Are there different physical systems exhibiting these phenomena?



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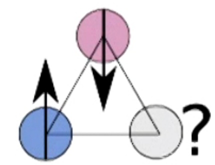
Frustrated Magnetism

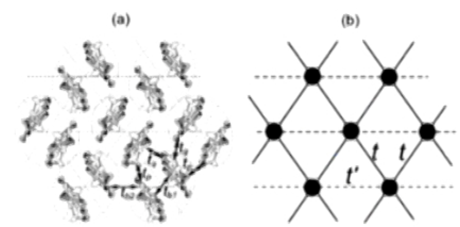
- Antiferromagnetic materials where local energy constraints cannot be minimized simultaneously



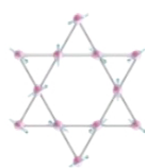
- Theoretically described by local frustrated spin models, e.g. Heisenberg model

$$H = \sum_{(i,j)\in \text{bonds}} J_{ij}(\vec{S}_i \cdot \vec{S}_j)$$

- simple to write down, very hard to solve



$\kappa - (\mathrm{BEDT-TTF})_2\mathrm{Cu}_2(\mathrm{CN})_3$
[Shimizu et al, Phys. Rev. Lett. 93 (2003)]



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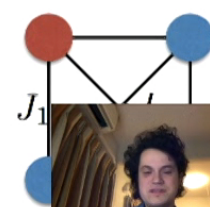

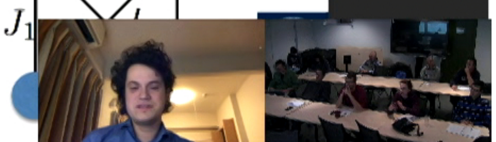

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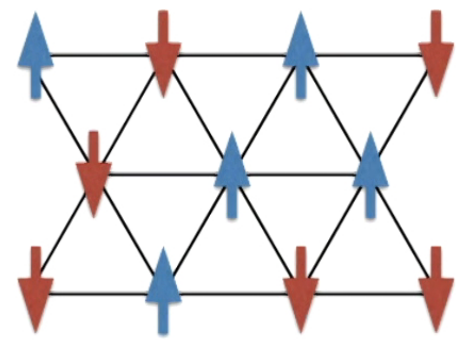
Chiral Spin Liquids

- translate fractional QHE physics for spins

  [V. Kalmeyer, R.B. Laughlin, Phys. Rev. Lett. 59 (1987)]

  [X.G. Wen, F. Wilczek, A. Zee Phys. Rev. B 39 (1989)]

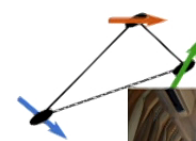


- mapping continuum wavefunction
  for bosonic $\nu = 1/2$ Laughlin state
  to hard-core bosons (i.e. spins) on a lattice

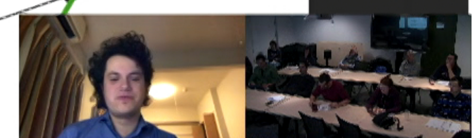
$$\psi_2(z_1, \ldots, z_n) = \prod_{i<j} (z_i - z_j)^2 \prod_k e^{-|z_k|^2}$$

- lattice symmetry and spin rotational SU(2) symmetry unbroken

- time-reversal and parity symmetry broken
  scalar chirality as an order parameter



$$\vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$$



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Chiral Spin Liquids

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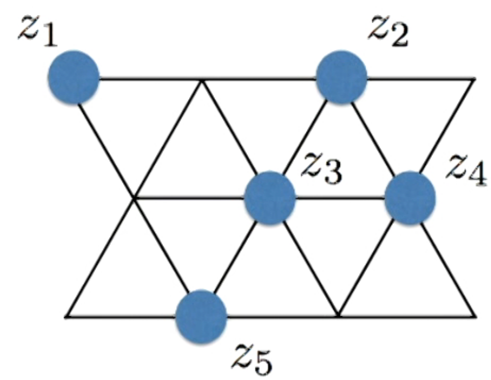

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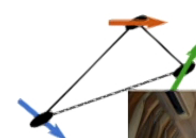

$\vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$

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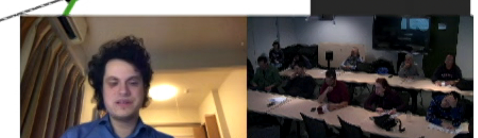

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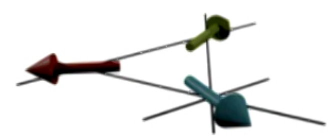
Chiral Spin Liquids

- topological order for Laughlin $\nu = 1/2$ CSL:
  - Spin 1/2 excitations (spinons)
  - spinons have semionic statistics
  - twofold degenerate ground state on torus
  - gapless chiral edge modes

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

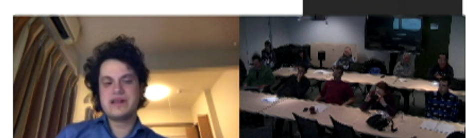
$$T = \mathrm{e}^{-i\frac{2\pi}{24}} \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{i\frac{\pi}{2}} \end{pmatrix}$$

- proposed as ground state of triangular lattice Heisenberg antiferromagnet
- turned out to be incorrect: ground state is Néel ordered



[Bernu et al., Phys Rev B (1994)]

Is there any model realizing this physics?



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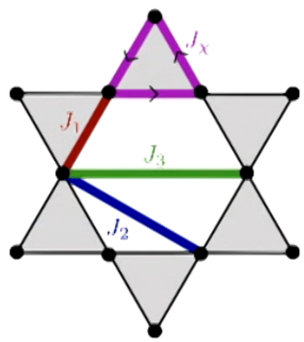
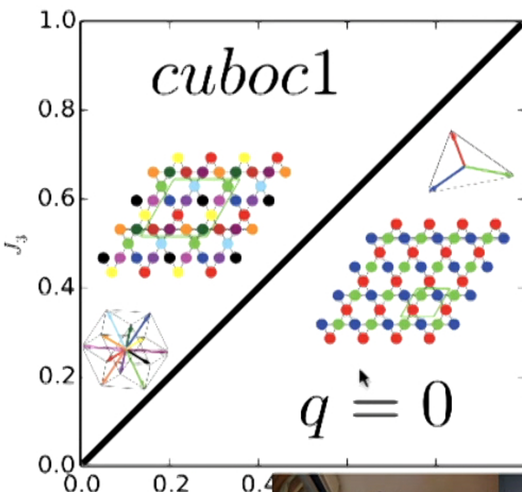
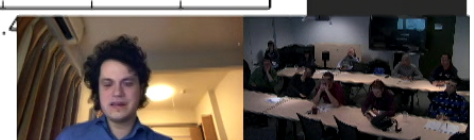
CSL on Kagome

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

- Classical phase diagram was investigated by
  [Messio et al., *Phys. Rev. Lett.* 108 (2012)]
  [Messio et al., *Phys. Rev. B* 83 (2011)]
- proposed CSL is on the critical line between the classical $cuboc1$ and order $q = 0$
- $cuboc1$ non-coplanar finite value of scalar chirality $\vec{S}_1 \cdot \left(\vec{S}_2 \times \vec{S}_3\right)$







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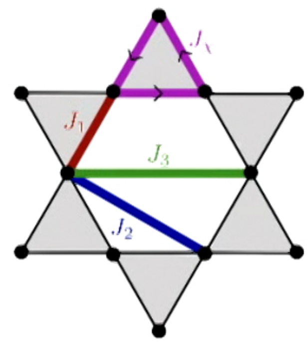
CSL on Kagome

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

- DMRG studies showed conclusive evidence that indeed a CSL is realized here

[S. Gong, W. Zhu & D. N. Sheng, Nature *Sci. Rep.* **4**, 6317 (2014)]

[Yin-Chen He, D. N. Sheng, and Yan Chen, Phys. Rev. Lett. **112**, (2014)]



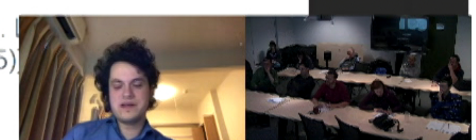
- Further kagome lattice model realizing CSL found by DMRG

$$H = J_\chi \sum_{(i,j,k) \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

[B. Bauer et al., Nature *Comm.* **5**, 5137 (2014)]

- Results confirmed with Exact Diagonalization, variational wave function approach

[A. Wietek, A. Sterdyniak, A. M.
Phys. Rev. B 92, 125122 (2015)]



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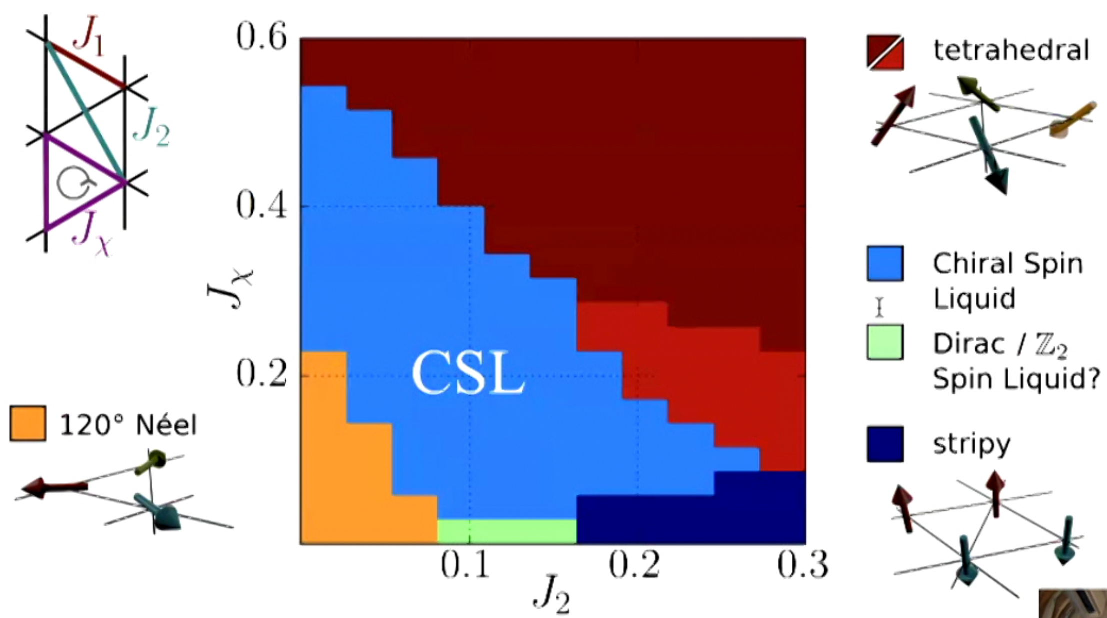
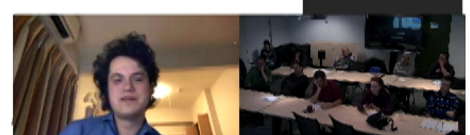
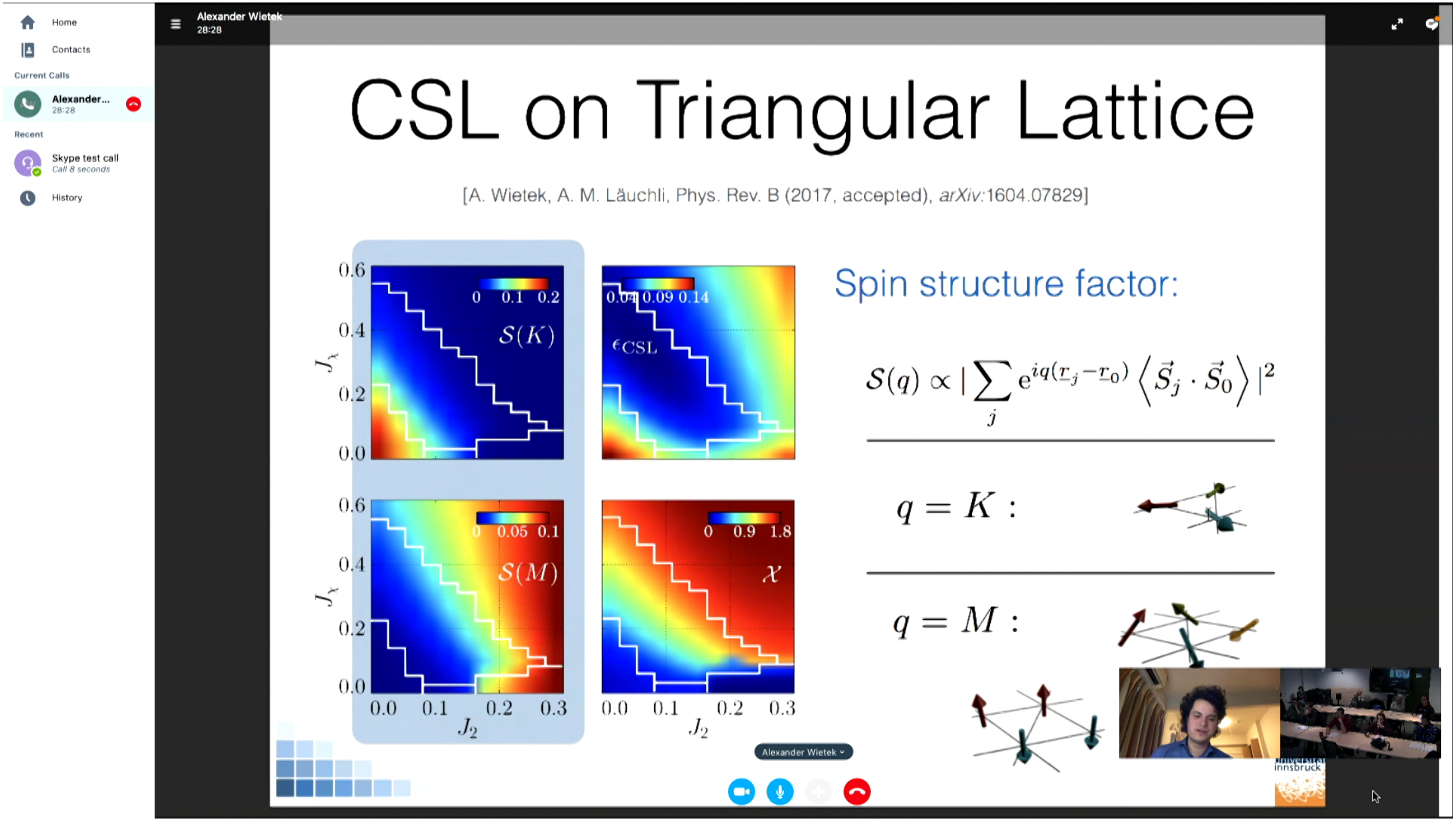
CSL on Triangular Lattice

[A. Wietek, A. M. Läuchli, Phys. Rev. B (2017, accepted), *arXiv*:1604.07829]



$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times$$

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CSL on Triangular Lattice
[A. Wietek, A. M. Läuchli, Phys. Rev. B (2017, accepted), arXiv:1604.07829]
Spin structure factor:
$\mathcal{S}(q) \propto |\sum_j e^{iq(r_j - r_0)} \langle \vec{S}_j \cdot \vec{S}_0 \rangle|^2$
$q = K$:
$q = M$:
$\mathcal{S}(K)$
ϵ_{CSL}
$\mathcal{S}(M)$
χ
J_χ
J_2
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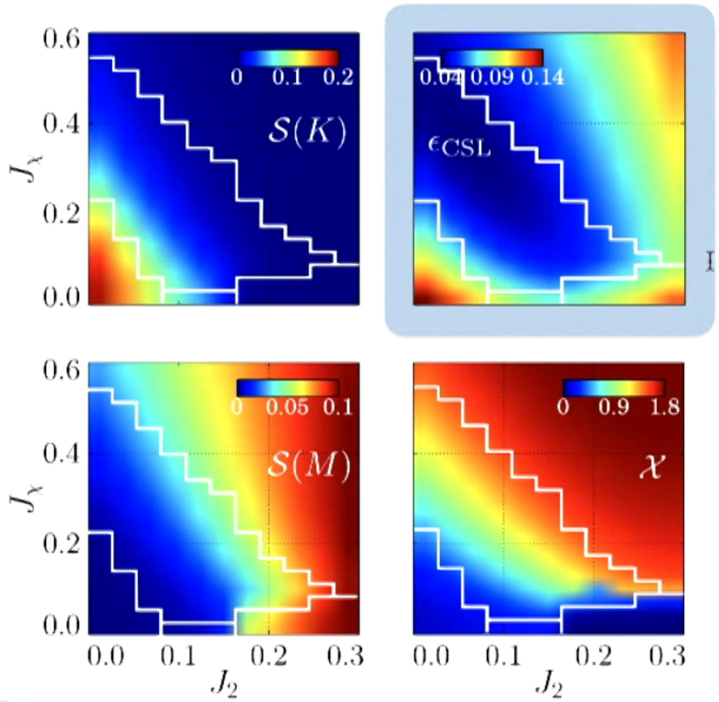
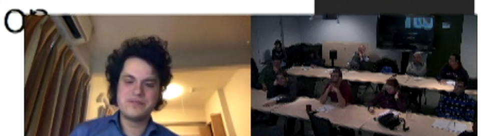
[A. Wietek, A. M. Läuchli, Phys. Rev. B (2017, accepted), *arXiv*:1604.07829]



Energy of model CSL:

$$\epsilon_{\mathrm{CSL}} = (E_{\mathrm{CSL}} - E_0)/E_0$$

- variational energy of model CSL wave function
- Gutzwiller projected w.f. similar to Laughlin w.f.
- low energy in the spin disordered regio...



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CSL on Triangular Lattice

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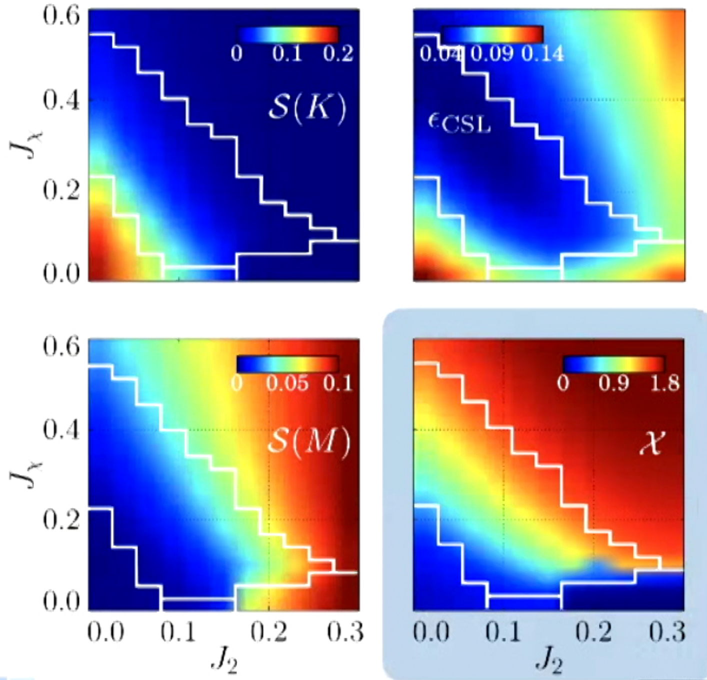
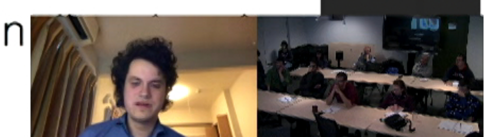


Chirality correlation:

$$\mathcal{X} = \sum_{(i,j,k)\in\triangle} \langle \chi_{(0,1,2)} \cdot \chi_{(i,j,k)} \rangle$$

- measure of non-coplanarity of the spins
- indicative for the CSL phase as well as the tetrahedral phase
- large also in spin ... region



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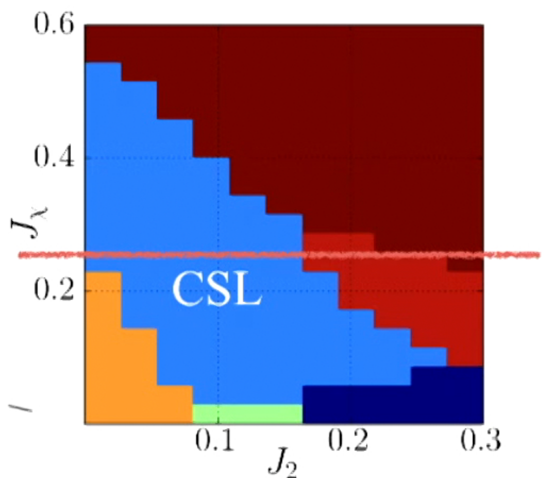
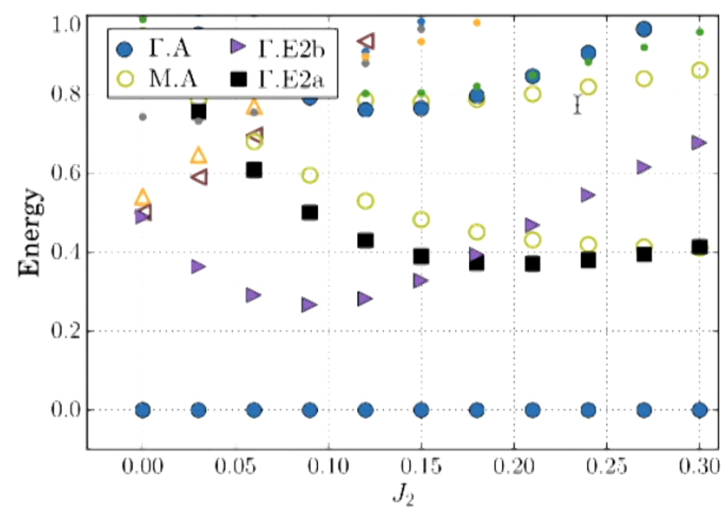
CSL on Triangular Lattice

[A. Wietek, A. M. Läuchli, Phys. Rev. B (2017, accepted), *arXiv*:1604.07829]

- many-body spectrum computed with Exact Diagonalization on 36 sites



$J_\chi = 0.24$



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CSL on Triangular Lattice

[A. Wietek, A. M. Läuchli, Phys. Rev. B (2017, accepted), *arXiv*:1604.07829]

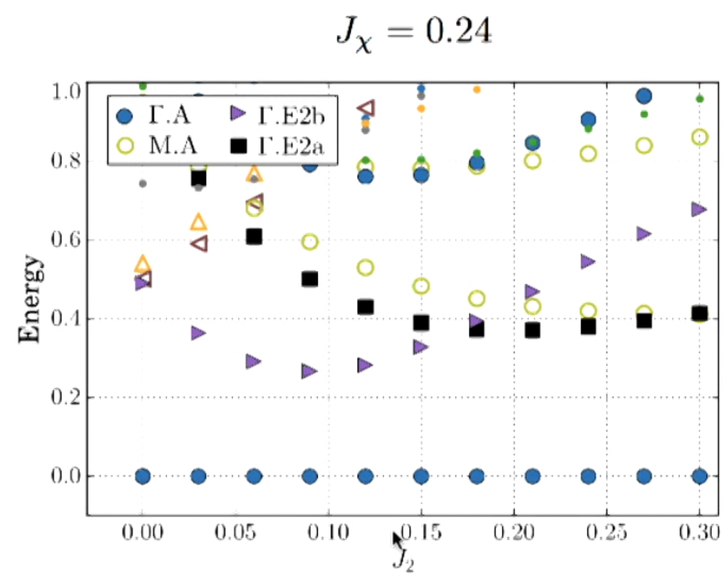
- construct two CSL wave functions on torus from Gutzwiller projection

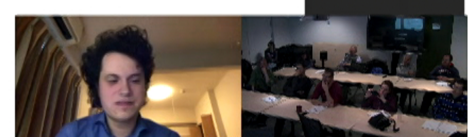
  $|\psi_{\text{CSL-I}}\rangle$ $|\psi_{\text{CSL-II}}\rangle$

- linearly independent with comparable low variational energy

- compute overlap with exact numerical eigenvalues

$$\mathcal{O}_{\text{GW-ED}} \equiv \left|\langle \psi^0_{\text{ED}} | \psi_{\text{CSL}} \rangle\right|^2 + \left|\langle \psi^1_{\text{ED}} | \psi_{\text{CSL}} \rangle\right|^2$$





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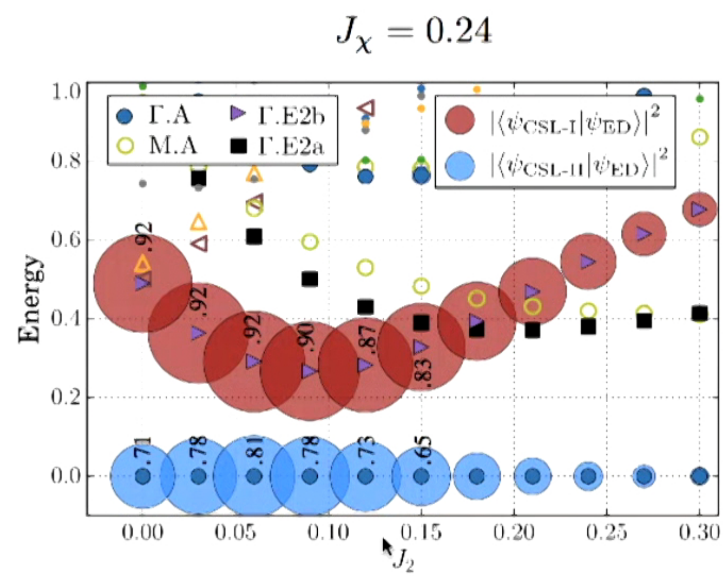
CSL on Triangular Lattice

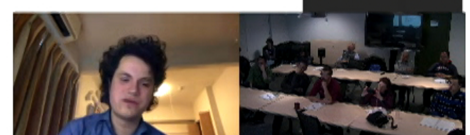
[A. Wietek, A. M. Läuchli, Phys. Rev. B (2017, accepted), *arXiv*:1604.07829]

- overlaps of up to 0.92
- dimension of Hilbertspace

  $$(\mathcal{H}) = 2^{36} = 68 \text{ billion}$$

- orthogonality catastrophe: overlaps expected to converge to zero exponentially





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CSL on Kagome

[A. Wietek, A. Sterdyniak, A. M. Läuchli, Phys. Rev. B 92, 125122 (2015)]

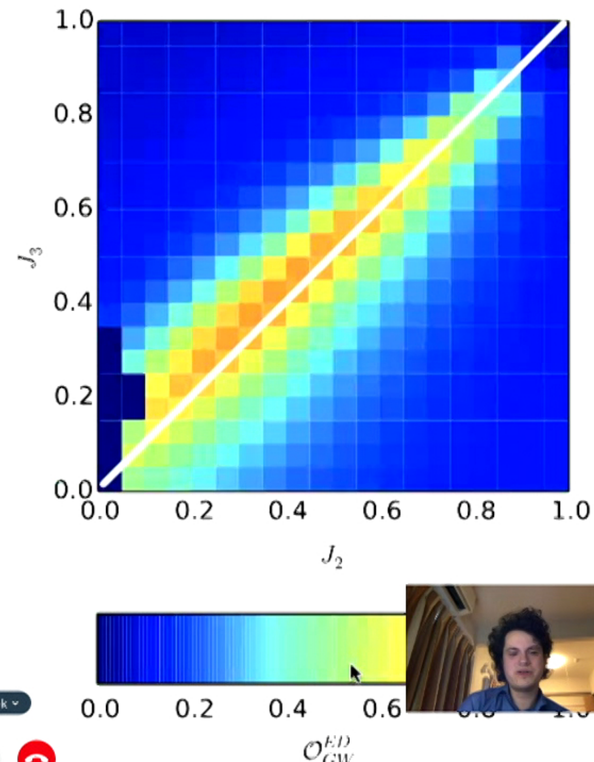
- Gutzwiller construction performed for Kagome CSL

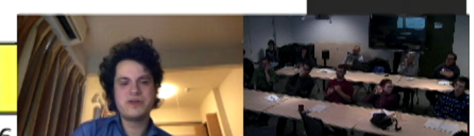
$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

- overlaps of up to 0.8
- adding a scalar chirality term

$$J_\chi \sum_{(i,j,k) \in \triangle} \vec{S}_i (\vec{S}_j \times \vec{S}_k)$$

yields overlaps of up to 0.95





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CSL in SU(N) fermionic Mott insulators

[P. Nataf, M. Lajkó, A. Wietek, K. Penc, F. Mila, A. M. Läuchli, Phys. Rev. Lett. 117, 167202]

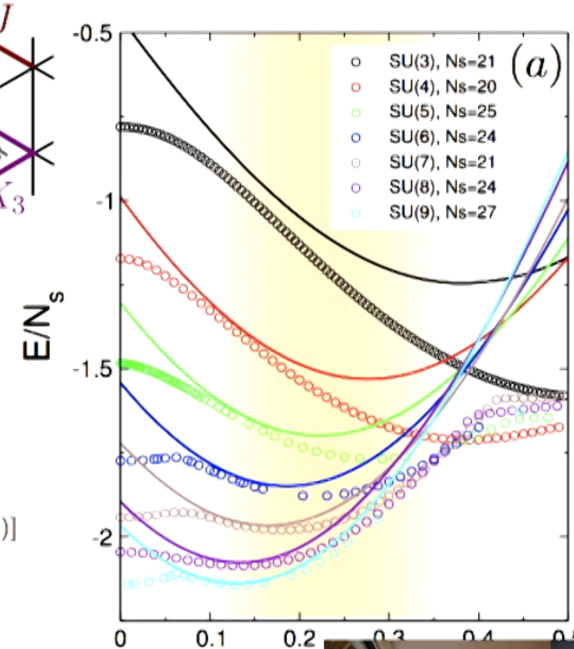
- Triangular lattice SU(N) Heisenberg model

$$H = J \sum_{\langle i,j \rangle} P_{ij} + K_3 \sum_{(i,j,k)} (iP_{ijk} + \text{h.c.})$$

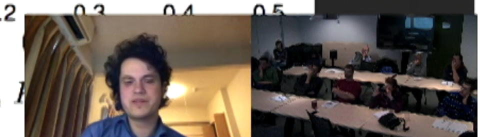
- numerical investigation using novel techniques for diagonalizing SU(N) Hamiltonians

[Pierre Nataf and Frédéric Mila, Phys. Rev. Lett. **113**, 127204 (2014)]

- Emergence of CSL phase for $N = 3, \ldots, 9$



$J = \cos(\theta)$,



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Exact Diagonalization

- Solution of Schrödinger equation

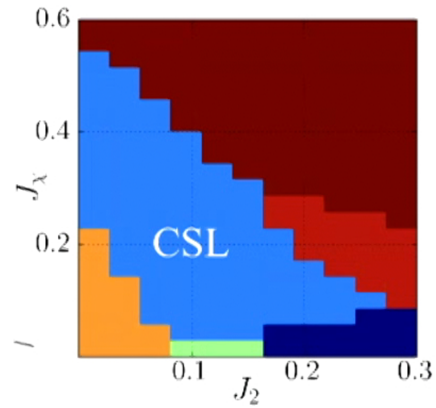
$$H\,|\psi\rangle = E\,|\psi\rangle$$

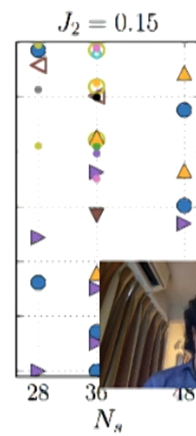
- Straightforward numerical method:
  a) Choose a Basis for the Hilbertspace
  b) Build up Hamiltonian Matrix
  c) Diagonalize it

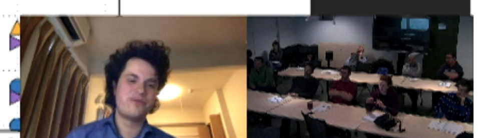
- exponential scaling in system size

- still ~48 Spin 1/2 particles can now be simulated

$$\dim(\mathcal{H}) = 2^{48} = 2.8 \cdot 10^{14}$$







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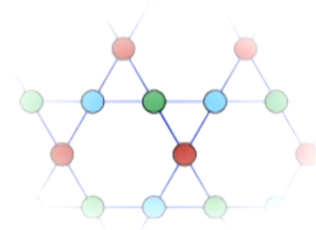
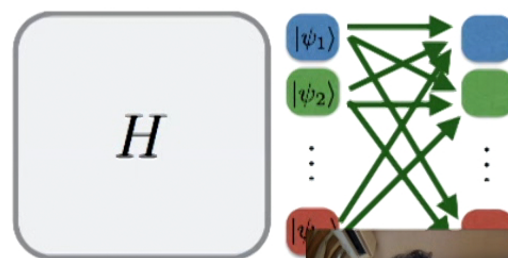
Exact Diagonalization

- Use of symmetries:
  - developed algorithms for efficiently working in a symmetrized basis
  - efficient reduction of Hilbertspace dimension



- MPI Parallelization:
  - solved several load balancing problems by randomly distributing the Hilbertspace to MPI processes
  - good scaling behavior up to several 1000 cores



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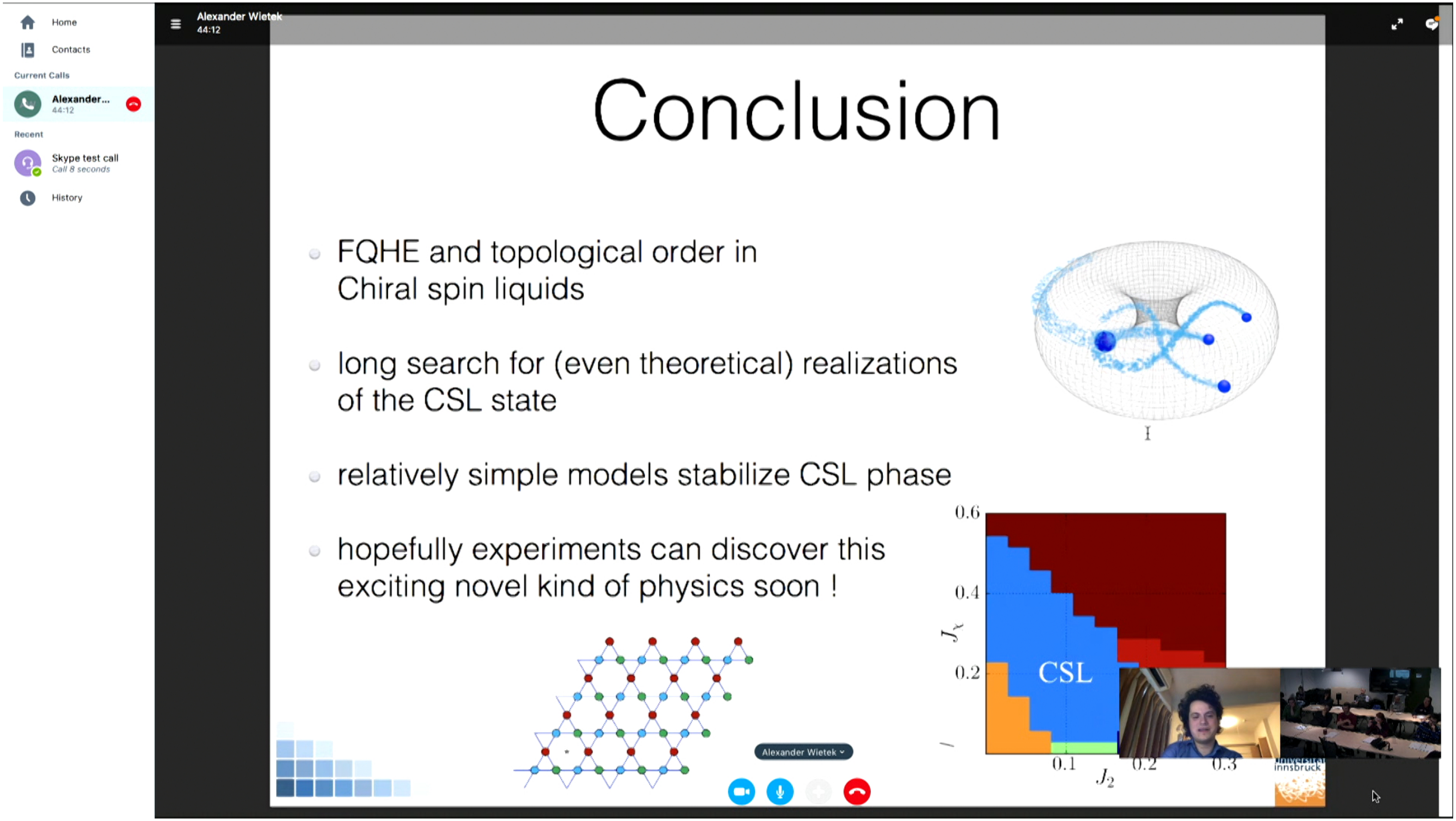
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Conclusion
FQHE and topological order in Chiral spin liquids
long search for (even theoretical) realizations of the CSL state
relatively simple models stabilize CSL phase
hopefully experiments can discover this exciting novel kind of physics soon !
0.6
0.4
0.2
CSL
0.1
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J_χ
J_2
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