Title: Recovery maps in quantum thermodynamics

Date: Jan 11, 2017 04:00 PM

URL: http://pirsa.org/17010066

Abstract: <p>A research line that has been very active recently in quantum information is that of recoverability theorems. These, roughly speaking, quantify how well can quantum information be restored after some general CPTP map, through particular 'recovery maps'. In this talk, I will outline what this line of work can teach us about quantum thermodynamics.</p>

<p>On one hand, dynamical semigroups describing thermalization, namely Davies maps, have the curious property of being their own recovery map, as a consequence of a condition named quantum detailed balance. For these maps, we derive a tight bound relating the entropy production at time t with the state of the system at time 2t, which puts a strong constraint on how systems reach thermal equilibrium.
On the other hand, we also show how the Petz recovery map appears in the derivation of quantum fluctuation theorems, as the reversed work-extraction process. From this fact alone, we show how a number of useful expressions follow. These include a generalization of the majorization conditions that includes fluctuating work, Crooks and Jarzynski's theorems, and an integral fluctuation theorem that can be thought of as the second law as an equality.</p>

Recovery maps in quantum thermodynamics

Informational vs. physical (ir)reversibility

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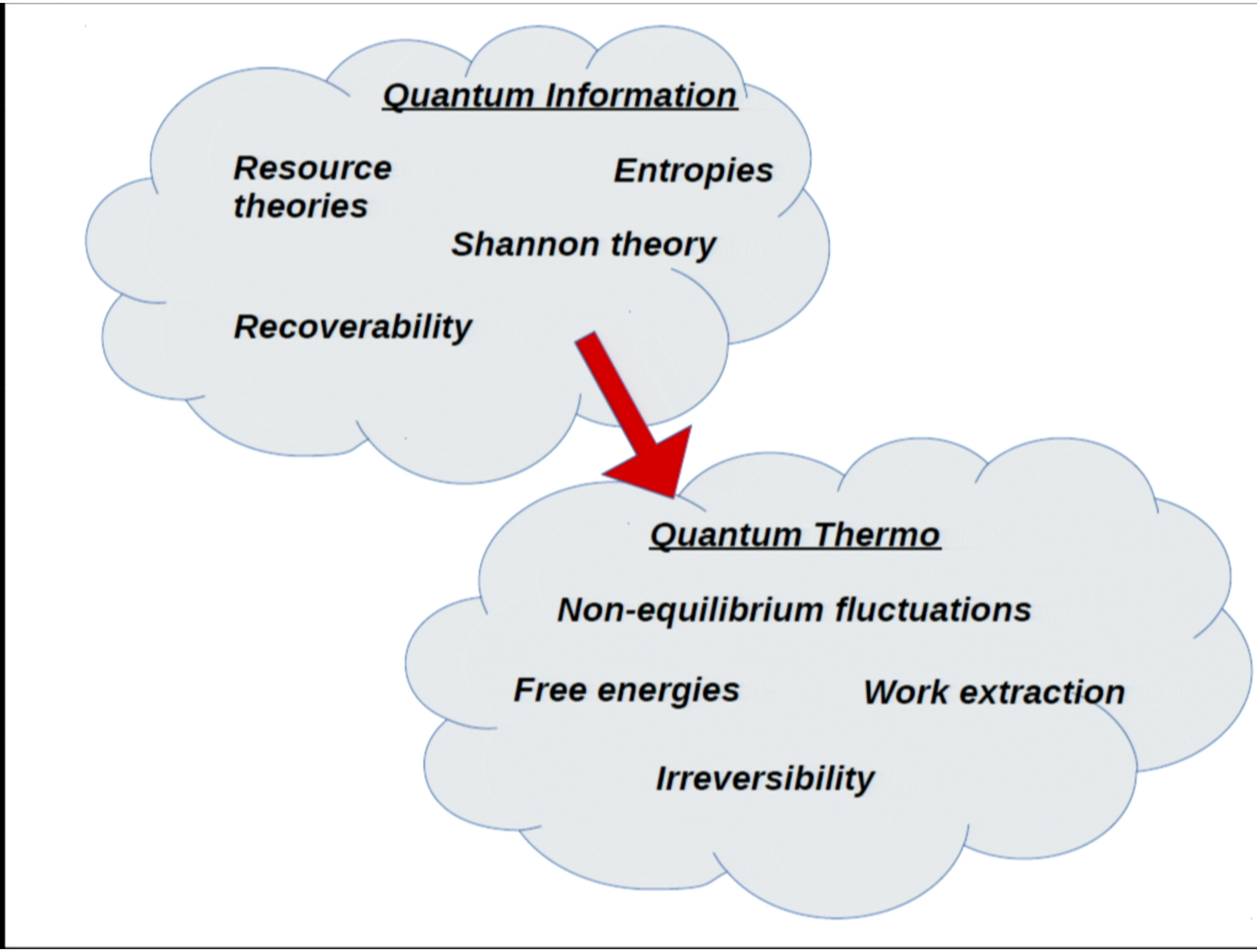
1601.05799 ***w/ Jonathan Oppenheim, Lluis Masanes, Chris Perry***


Quantum Information
Resource theories
Entropies
Shannon theory
Recoverability
Quantum Thermo
Non-equilibrium fluctuations
Free energies
Work extraction
Irreversibility

‘Reversed’ map

$$\sum_i p(j|i)p(i) = q(j)$$

$$\tilde{p}(i|j) = \frac{p(i)}{q(j)} p(j|i) \;\; \rightarrow \qquad \sum_j \tilde{p}(i|j) q(j) = p(i)$$

Quantum ‘reversed’ map

$$T(\cdot) \text{ CPTP}, \sigma \text{ quantum state}$$

$$\tilde{T}(\cdot) = \sigma^{1/2} T^\dagger (T(\sigma)^{-1/2} \cdot T(\sigma)^{-1/2}) \sigma^{1/2}$$

$$\rightarrow \tilde{T}(T(\sigma)) = \sigma$$

Petz recovery map

$$\tilde{T}(\cdot) = \sigma^{1/2} T^{\dagger}(T(\sigma)^{-1/2} \cdot T(\sigma)^{-1/2}) \sigma^{1/2}$$

- Relative entropy

$$D(\rho||\sigma) = \mathrm{Tr}[\rho(\log \rho - \log \sigma)]$$

$$D(\rho||\sigma) - D(T(\rho)||T(\sigma)) \geq 0$$

- Petz theorem '86

$$D(\rho||\sigma) = D(T(\rho)||T(\sigma)) \iff \tilde{T}(T(\rho)) = \rho$$

- Approximate version?

$$D(\rho||\sigma) - D(T(\rho)||T(\sigma)) \sim 0 \quad \overset{??}{\to} \quad \tilde{T}(T(\rho)) \sim \rho$$

Approximate recoverability

- Recoverability theorem: Tightened monotonicity
  (Junge et al. '16 + others)

$$D(\rho||\sigma) - D(T(\rho)||T(\sigma)) \geq -2\log F(\rho, \tilde{T}'(T(\rho)))$$

$$F(\rho,\sigma) = \mathrm{Tr}[\sqrt{\sqrt{\sigma}\rho\sqrt{\sigma}}]$$

- Useful particular cases (e.g. Fawzi-Renner '14)

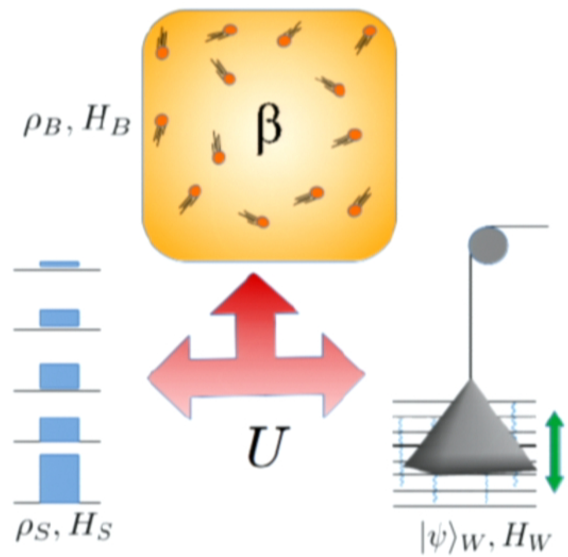
$$I(A:C|B) \geq -2\log F(\rho_{ABC}, \tilde{T}'_{B\to BC}(\rho_{AB}))$$

Quantum thermodynamics

Study irreversibility of quantum processes, so what can we learn from recoverability of QI?

Two kinds of processes:

- Equilibration with environment.
- Work gain/expenditure.



Equilibration with environment

- Weak coupling between system-heat bath (Davies maps '76)
- Dynamical semigroup (Markovian, RWA,...)

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = -i[H_S, \rho_S] + \mathcal{L}_D(\rho_S)$$

- Stationary state $\tau_S = \frac{e^{-\beta H_S}}{Z_S}$
- Classical detailed balance $p(j|i) = \frac{p(j)}{p(i)} p(i|j)$
- Quantum detailed balance

$$\mathcal{L}_D(\cdot) = \tau_S^{1/2} \mathcal{L}_D^\dagger(\tau_S^{-1/2} \cdot \tau_S^{-1/2}) \tau_S^{1/2}$$

QDB implies recovery map equals original map!

Bound on entropy production

- Entropy production: measure of irreversibility

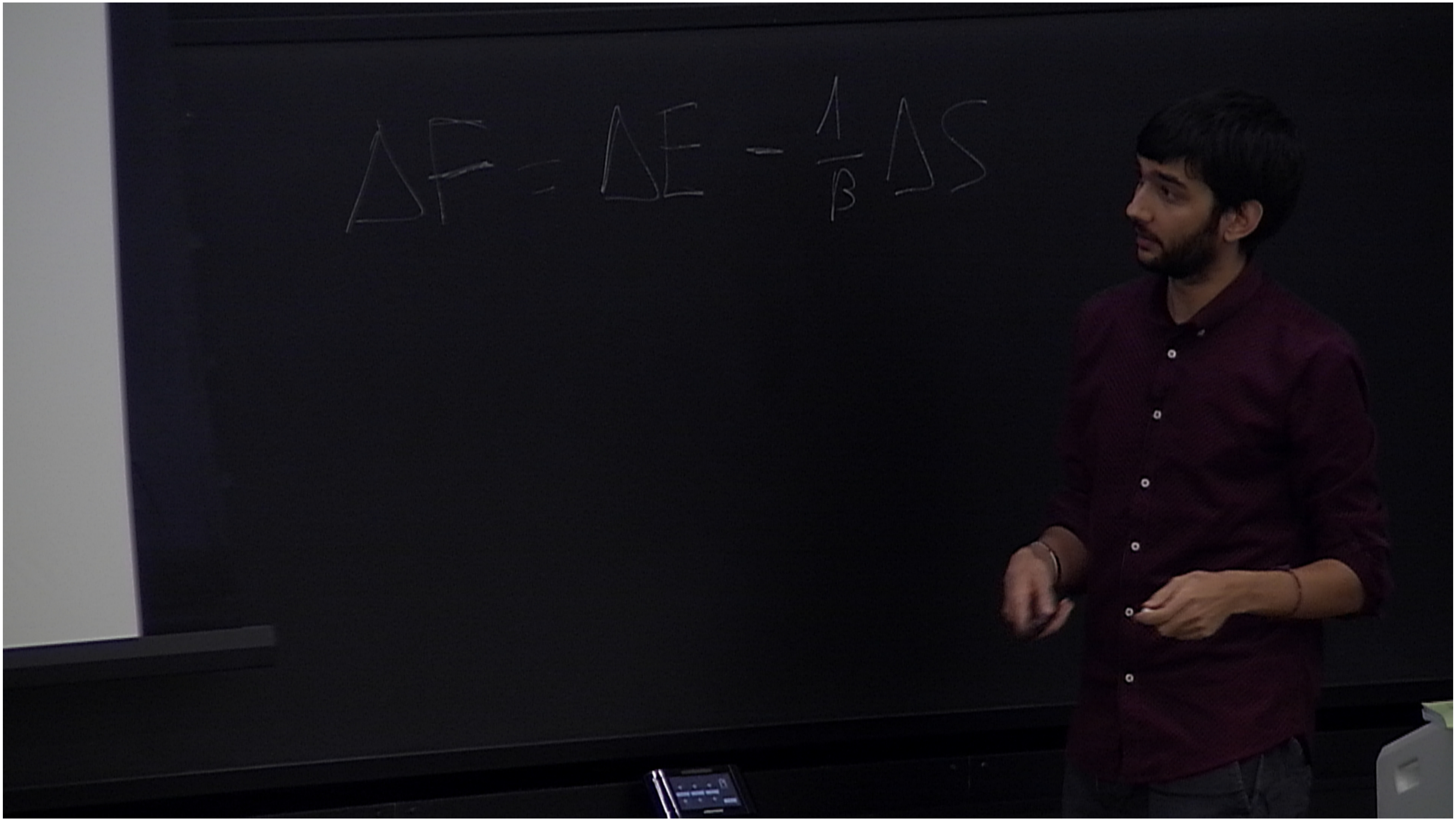
$$D(\rho_S(0)||\tau_S) - D(\rho_S(t)||\tau_S) \equiv \frac{1}{\beta}F_\beta(\rho_S(0)) - \frac{1}{\beta}F_\beta(\rho_S(t))$$

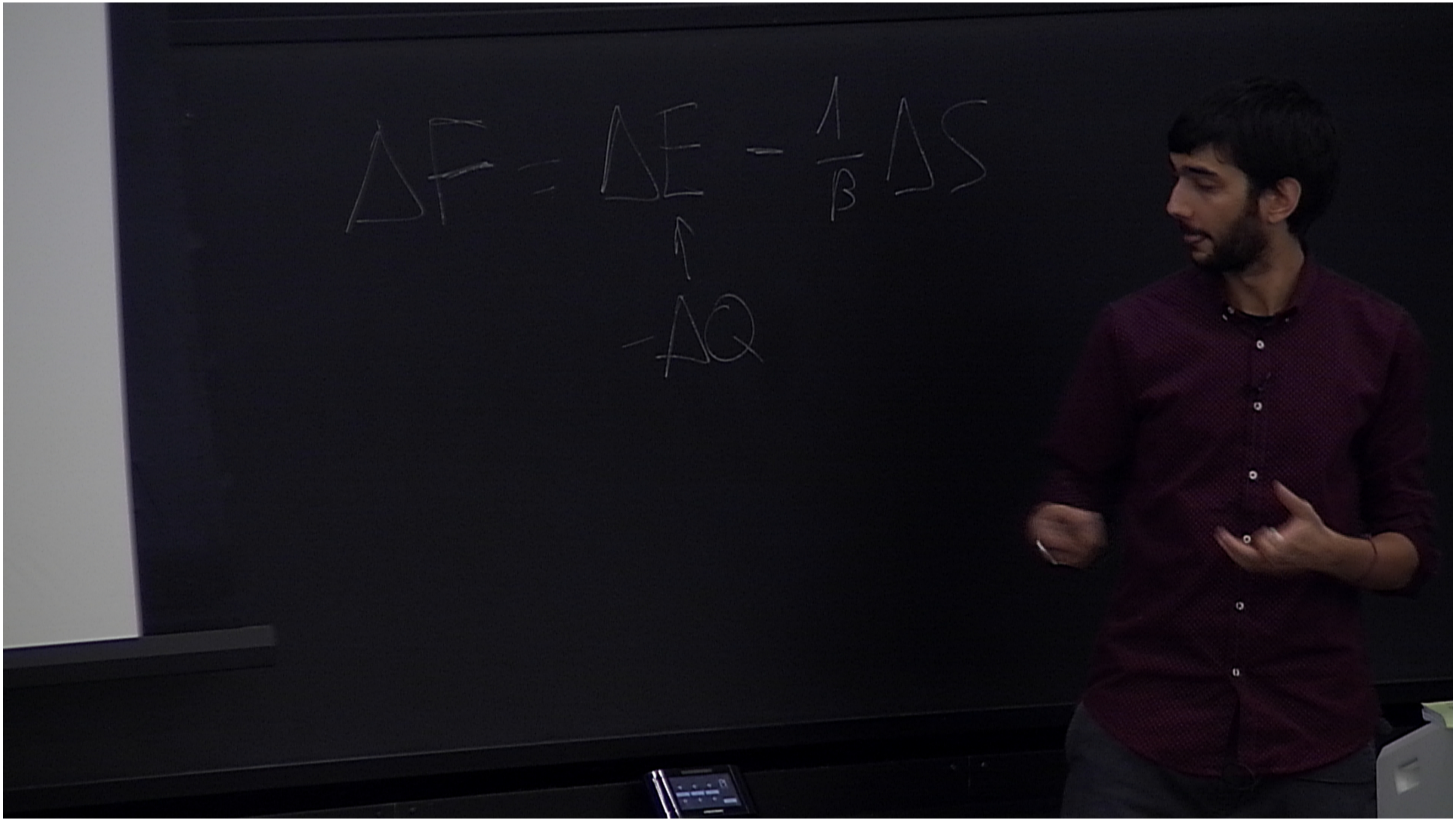
- RESULT #1: Strongest version of recoverability theorem holds + QDB

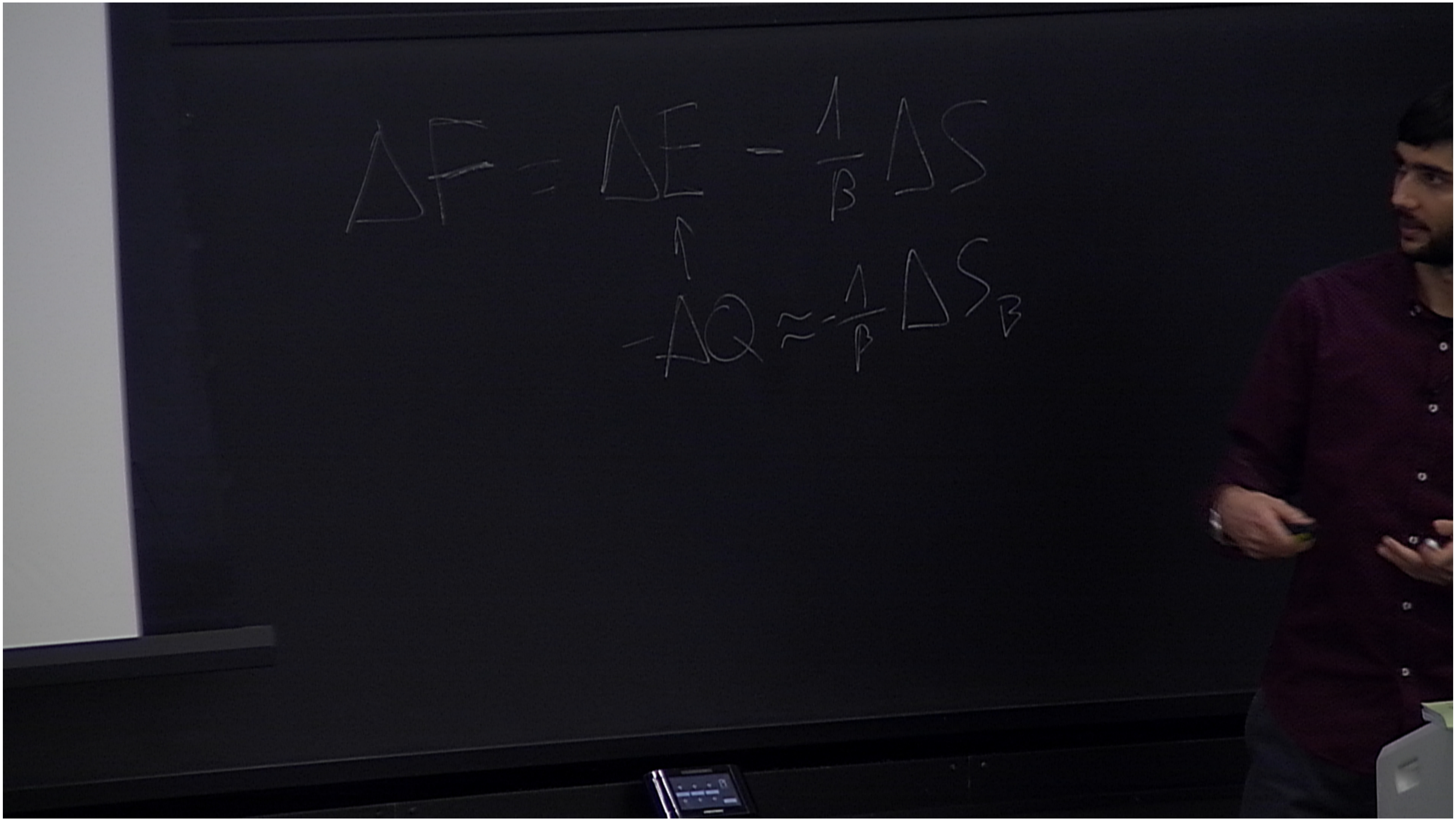
$$F_\beta(\rho_S(0)) - F_\beta(\rho_S(t)) \geq \beta D(\rho_S(0)||\rho_S(2t))$$

(As opposed to $D(\rho||\sigma) - D(T(\rho)||T(\sigma)) \geq -2\log F(\rho, \tilde{T}'(T(\rho)))$ ***)***

- Entropy production related to change of state (as expected)


ΔF = ΔE - 1/β ΔS


ΔF = ΔE − 1/β ΔS
−ΔQ


ΔF = ΔE − 1/β ΔS
−ΔQ ≈ −1/β ΔS_B

Bound on entropy production

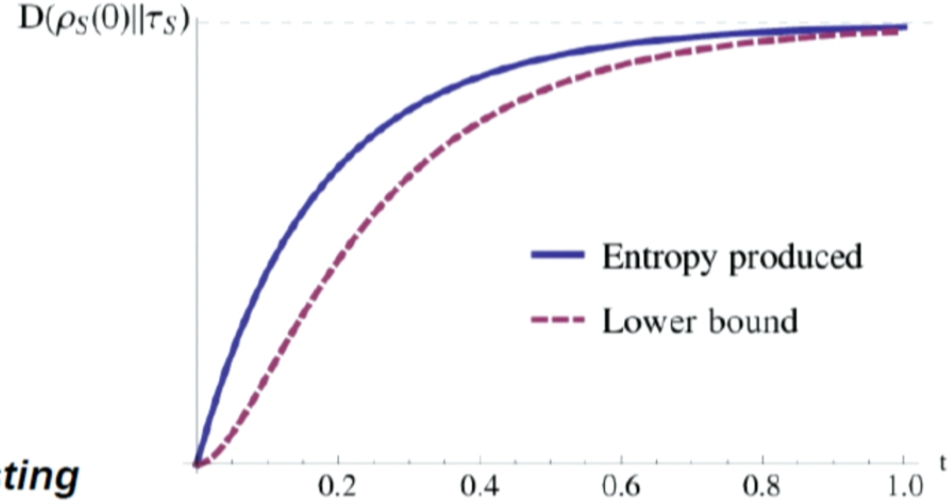
$$F_\beta(\rho_S(0)) - F_\beta(\rho_S(t)) \geq \beta D(\rho_S(0)||\rho_S(2t))$$

-Tight lower bound (relative entropy!)

-Equality at equilibrium

-Relates state at 2 different times

-Condition on reaching equilibrium



Claim: physically interesting instance of recoverability theorem

Microscopic thermodynamics w/ work

- Resource theory framework: tracking energy, entropy, coherence
- System+bath+weight
- Work may fluctuate $P(W)$

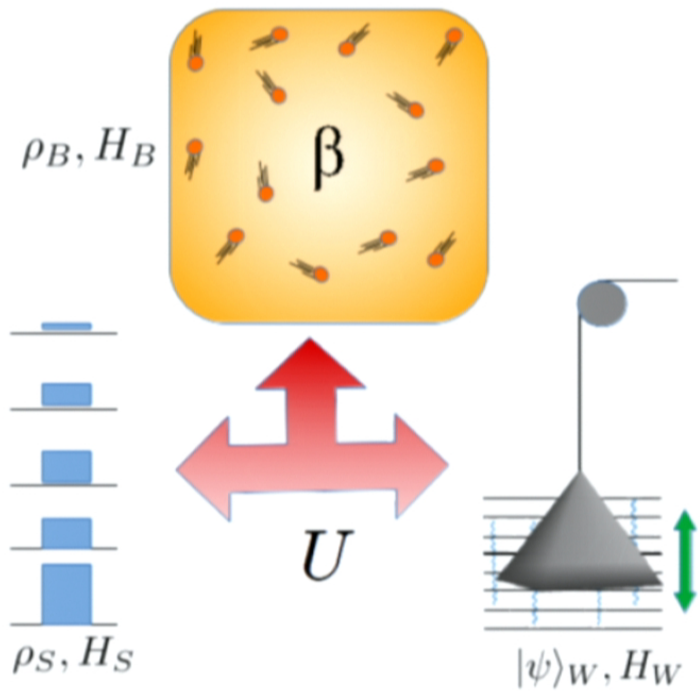
Global unitary: U

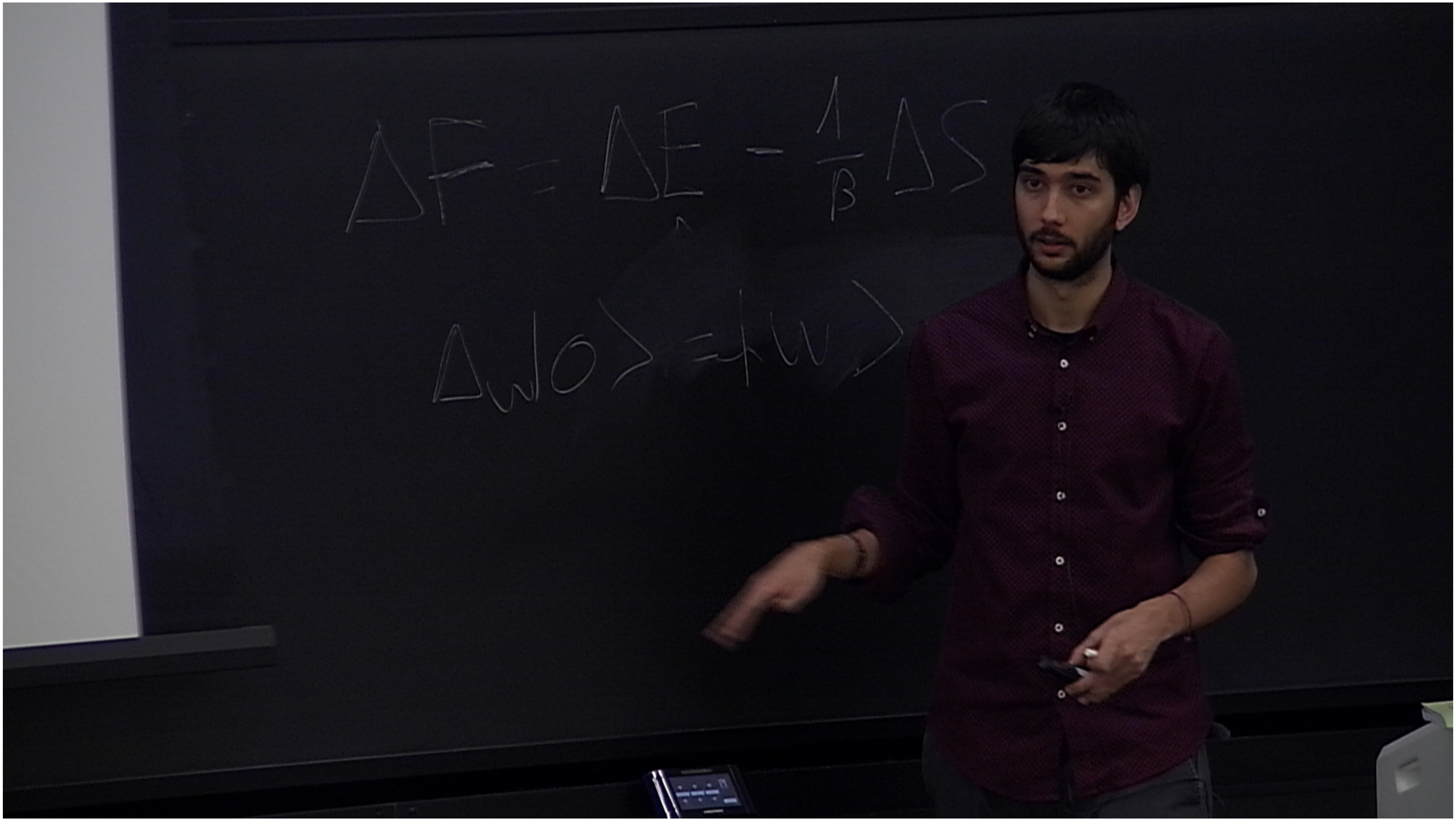
Energy conservation:

$$[U, H_S + H_B + H_W] = 0$$

Independent on state of weight:

$$[U, \Delta_W] = 0$$

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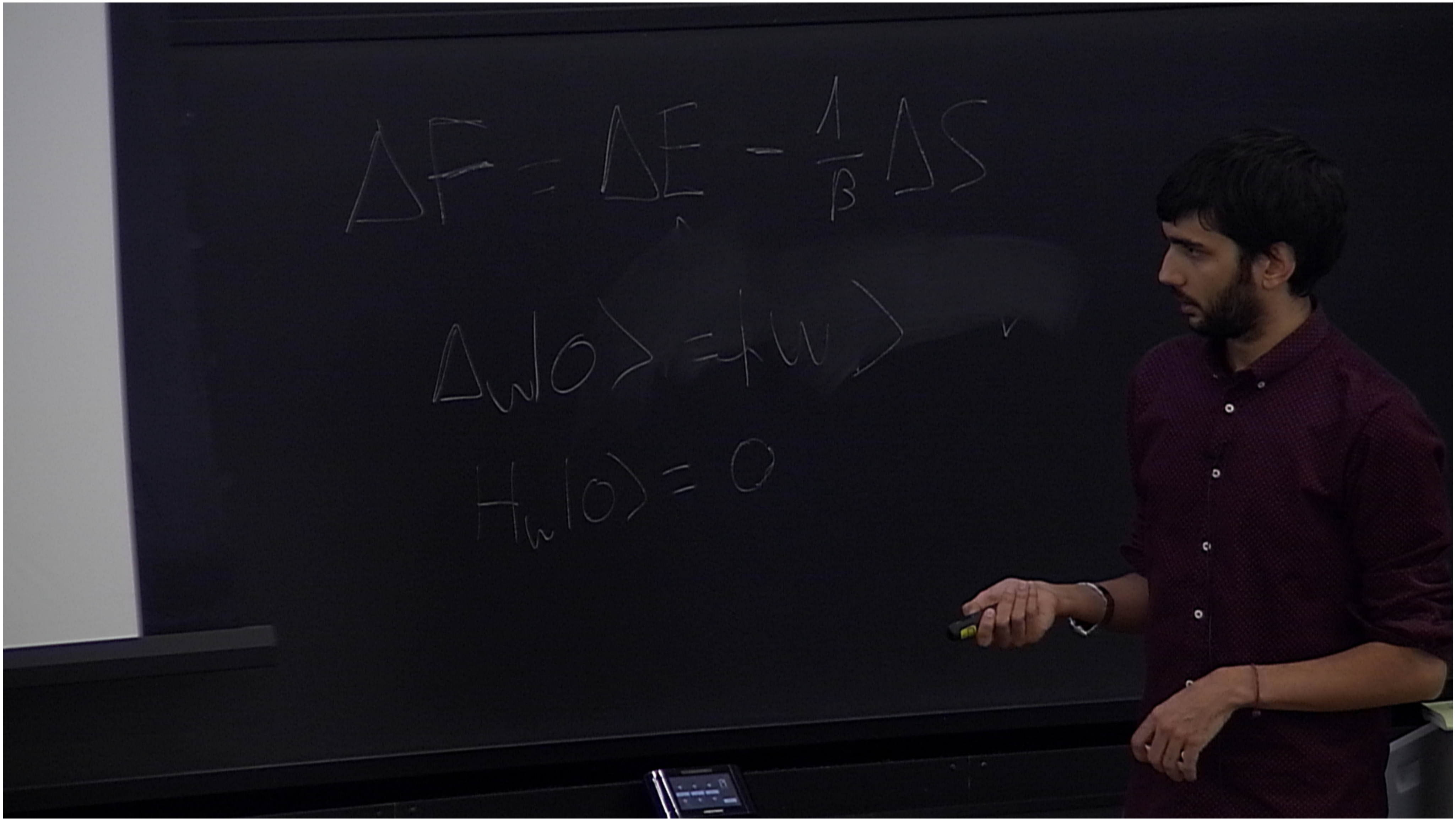
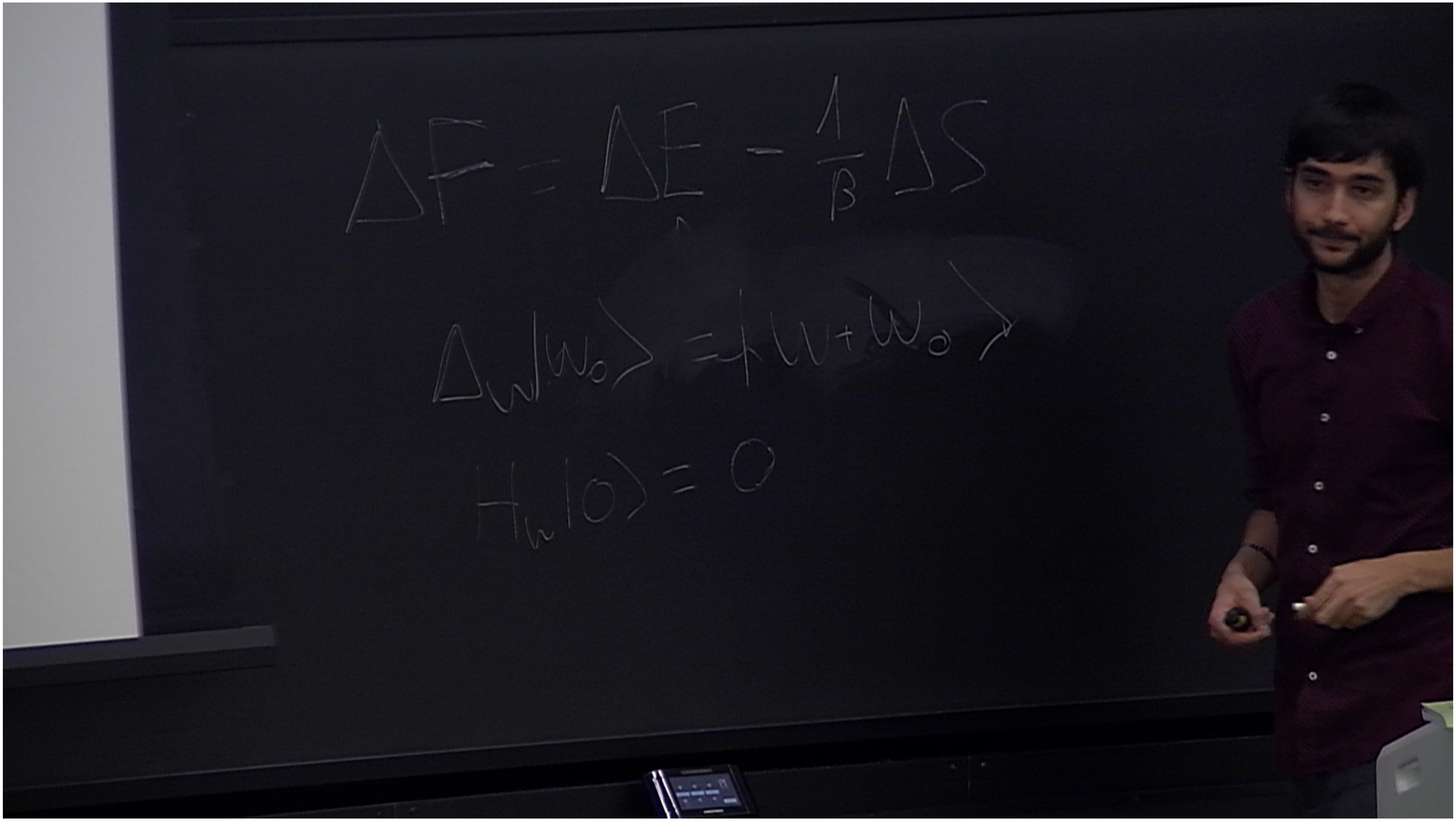

ΔF = ΔE − 1/β ΔS
H_w|0⟩ = 0


ΔF = ΔE - 1/β ΔS
Δw|w0> = |w+w0>
Hw|0> = 0

Quantum fluctuation theorems

Relation between forwards and backwards process.

$$T_{SW}^{\text{forw}}(\cdot) = \mathrm{Tr}_B[U(\cdot) \otimes \tau_B \otimes \rho_W U^\dagger]$$

$$T_{SW}^{\text{back}}(\cdot) = \mathrm{Tr}_B[U^\dagger(\cdot) \otimes \tau_B \otimes \rho_W U]$$

Result #2: One is the recovery map of the other!

(quantum Crooks theorem)

$$T_{SW}^{\text{back}}(\cdot) =$$

$$e^{\frac{\beta}{2}(H_S+H_W)} T_{SW}^{\text{forw}\,\dagger}(e^{-\frac{\beta}{2}(H_S+H_W)} \cdot e^{-\frac{\beta}{2}(H_S+H_W)}) e^{\frac{\beta}{2}(H_S+H_W)}$$

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Quantum fluctuation theorems

- Can we understand their 'quantumness'? (coherence in energy)

(see Aberg '16)

- If we look at energy eigenbasis, we can derive important general statements:

$$p_{\text{back}}(i, -W|j) = e^{\beta(E_j - E_i + W)} p_{\text{forw}}(j, W|i)$$

Crooks theorem $\dfrac{p^{forw}(W)}{p^{back}(-W)} = e^{-\beta W} \dfrac{Z'_S}{Z_S}$

Jarzynski's theorem $\langle e^{\beta W} \rangle = \dfrac{Z'_S}{Z_S} \quad \rightarrow \log Z'_S / Z_S \geq \beta \langle W \rangle$

Second-law equality $\langle e^{\beta(f'_j - f_i + W)} \rangle = 1$

$f_i = E_i + \log p(i)$ $\quad \rightarrow \ \Delta F \geq \langle W \rangle$

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Resource theories

	Model of Thermodynamics	**Constraint on maps**	**Conditions for transitions**
Noisy operations $\mathrm{Tr}_B[U\rho\otimes\frac{\mathbb{I}_B}{d}U^\dagger]$	$H = I$ (purity)	$\sum_i p(j\|i) = 1$	Majorization
Thermal operations $\mathrm{Tr}_B[U\rho\otimes\tau_B U^\dagger]$	any H W fixed (deterministic)	$\sum_i p(j\|i)e^{\beta(E_j-E_i)} = 1$	Thermo-majorization
Thermal operations + fluctuating work $\mathrm{Tr}_B[U\rho\otimes\tau_B\otimes\rho_W U^\dagger]$	any H $p(W)$ (fluctuating)	$\sum_{i,W} p(j,W\|i)e^{\beta(E_j-E_i+W)} = 1$	????

Conclusions

- How well can we recover quantum information?
- The notions of informational and thermodynamical reversibility seem to coincide (as expected?).
- Useful for: Equilibration with environment, work extraction (not unrelated).
- Consequences for the dissipative character of stochastic & quantum processes (2^{nd} law).
- Tools of information theory become useful in thermodynamics (following Landauer).

1609.07496 (Dynamical semigroups)

Thanks!!

1601.05799 (Work fluctuations)