Title: Orders and disorder in high-Tc superconductors

Date: Jan 13, 2017 03:30 PM

URL: http://pirsa.org/17010065

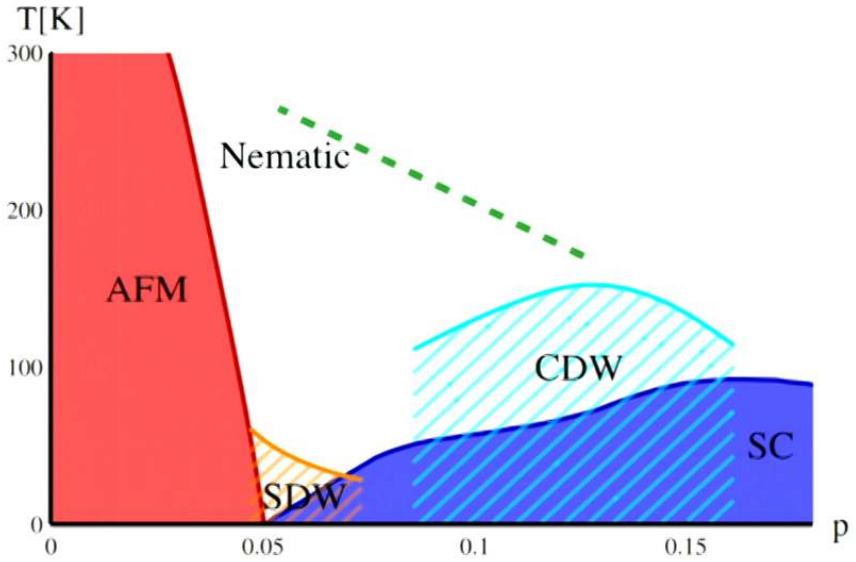
Abstract: Since its first discovery in 1986, high-Tc superconductors have been attracting constant interests and meticulous efforts from both theorists and experimentalists, not merely due to its large transition temperature, but also because it offers a well characterized laboratory for the study of exotic phenomena such as quantum criticality, non-Fermi liquid behavior, and intertwined orders. One pressing question in the field is the role played by disorder:

inevitable in real materials, disorder is able to fundamentally alter the properties of the system under certain circumstances. In this talk, we will discuss from a theoretical point of view, how different types of disorders affect various electronic orders in copper-based high-Tc superconductors. We also discuss the implications in experiments, and investigate the possibility of generalizing our conclusions to a variety of other physical systems.

Orders and disorder in high-T_c superconductors

Laimei Nie

Stanford University



Sketch of cuprates phase diagram, adapted from Schütt and Fernandes, PRL 2015

Perimeter Institute, Jan 13 2017

Orders and disorder in high-T_c superconductors

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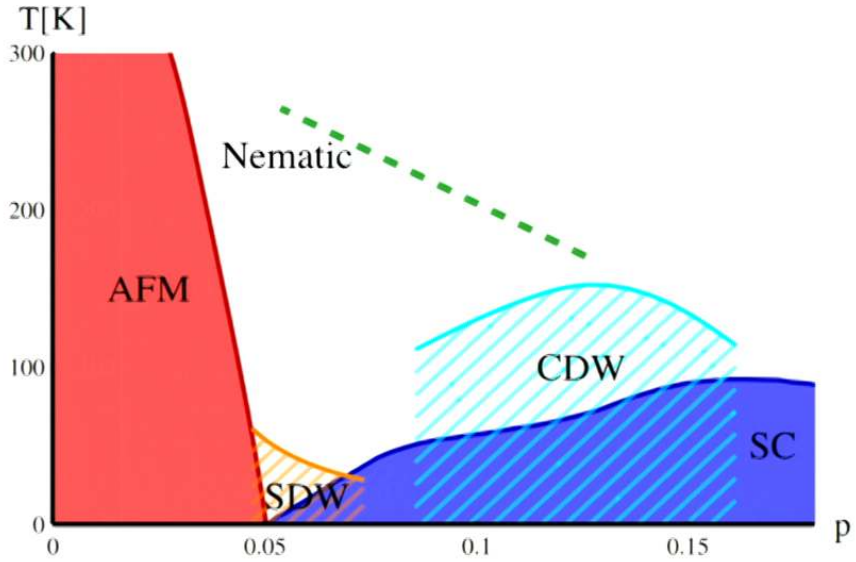
Stanford University

Experiments
Theories and numerics



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Perimeter Institute, Jan 13 2017

Orders and disorder in high-T_c superconductors

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Stanford University

Collaborators:

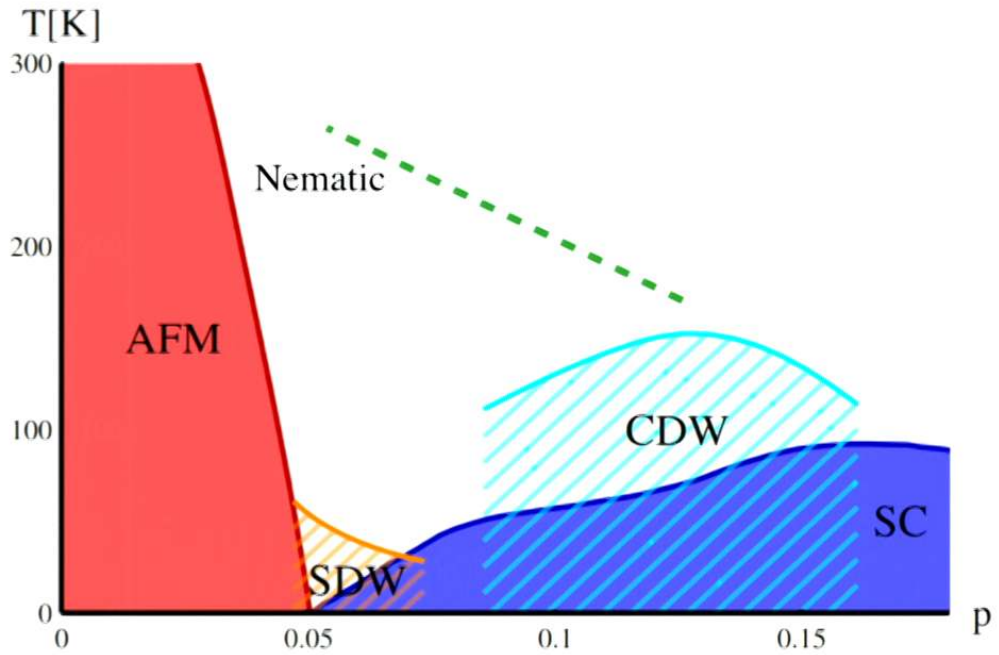
Steven Kivelson
Akash Maharaj
Stanford

Roger Melko
Lauren Hayward Sierens
Perimeter Institute

Eduardo Fradkin
UIUC

Subir Sachdev
Harvard

Orders and disorder in high-T_c superconductors



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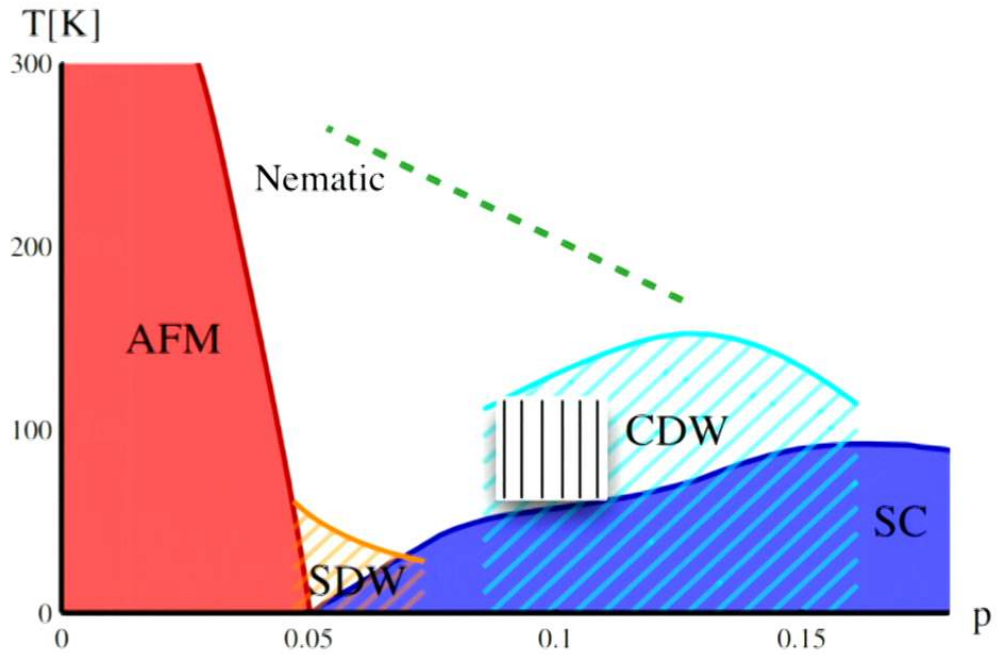

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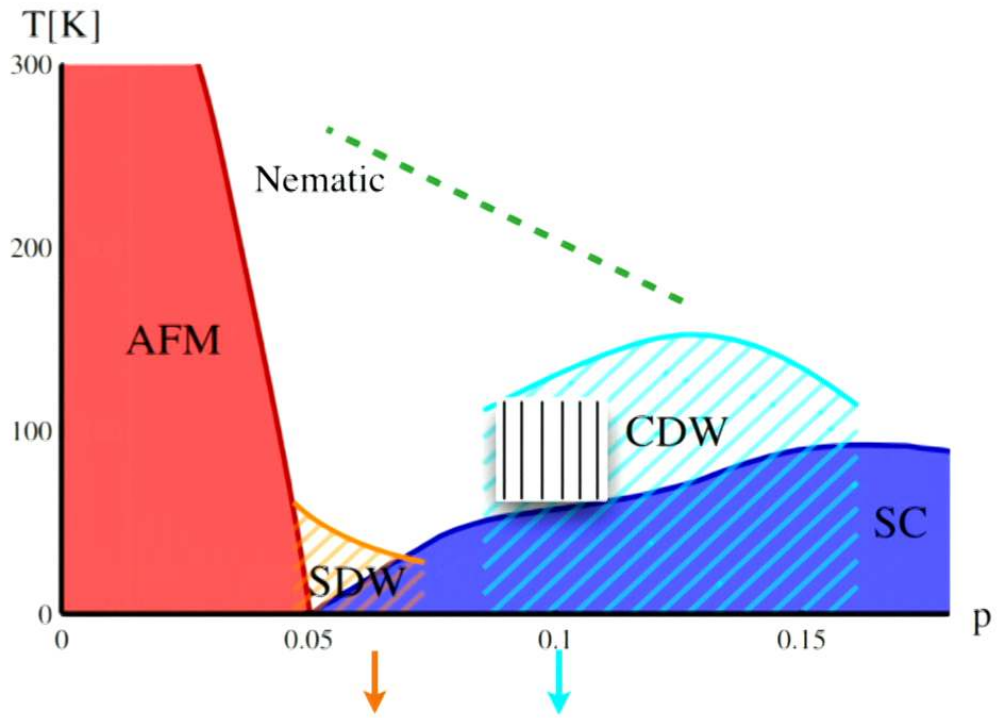
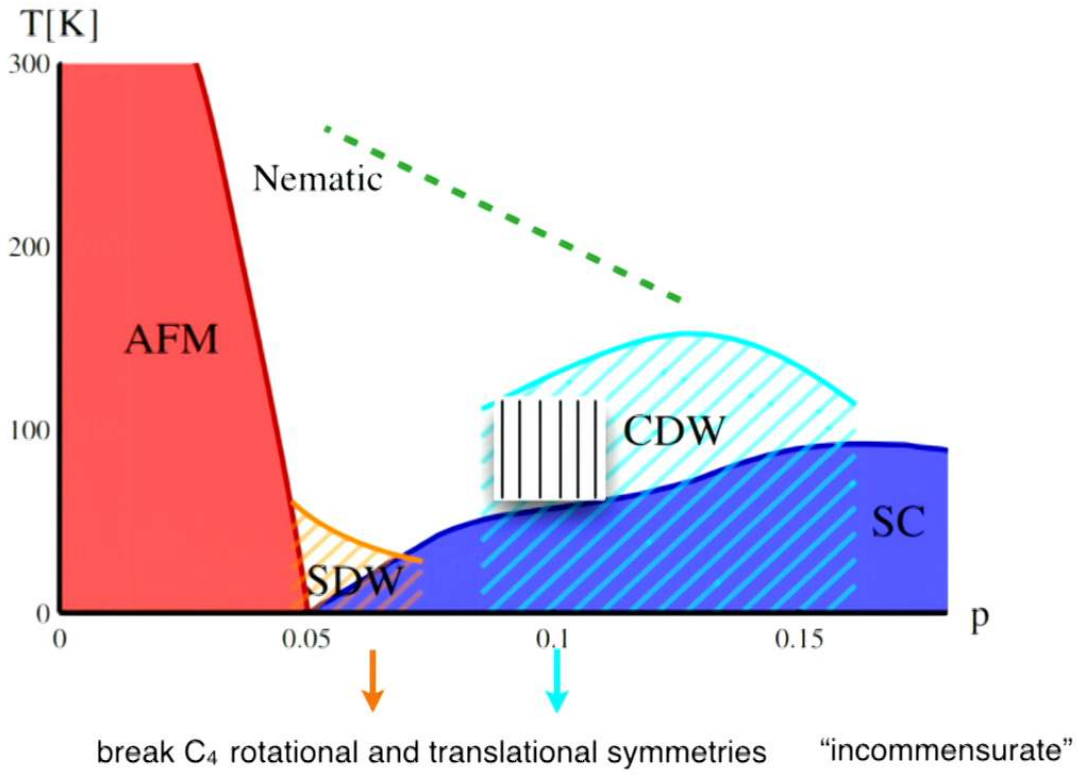


break C_4 rotational and translational symmetries

Orders and disorder in high-T_c superconductors



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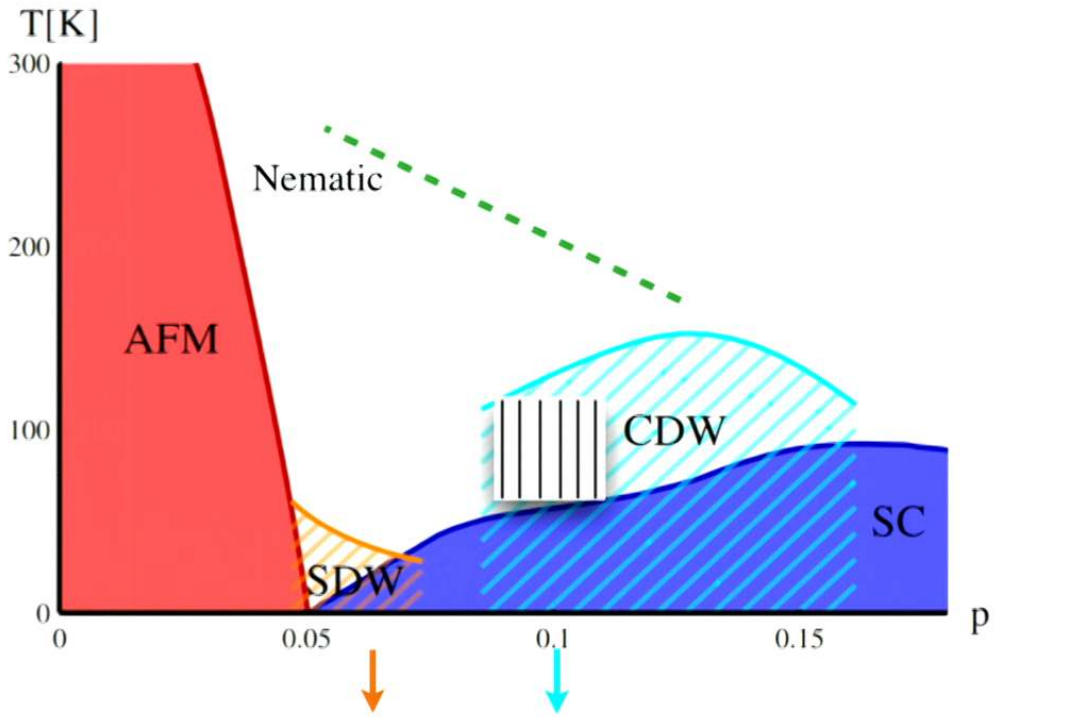

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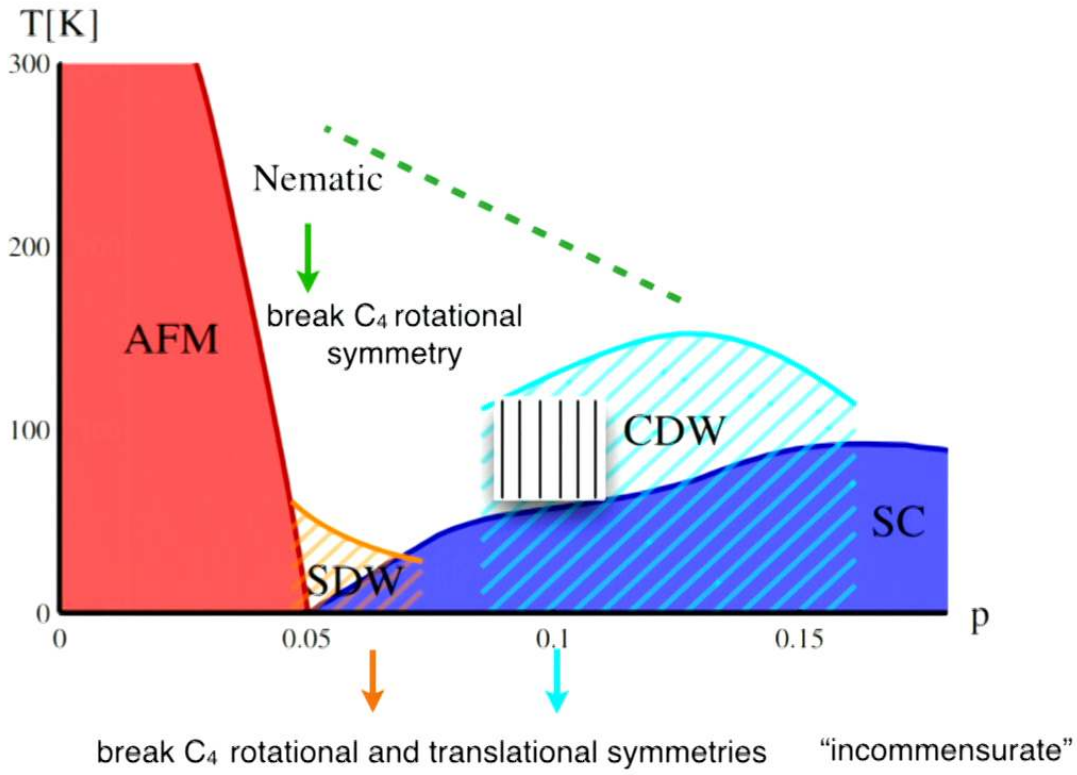

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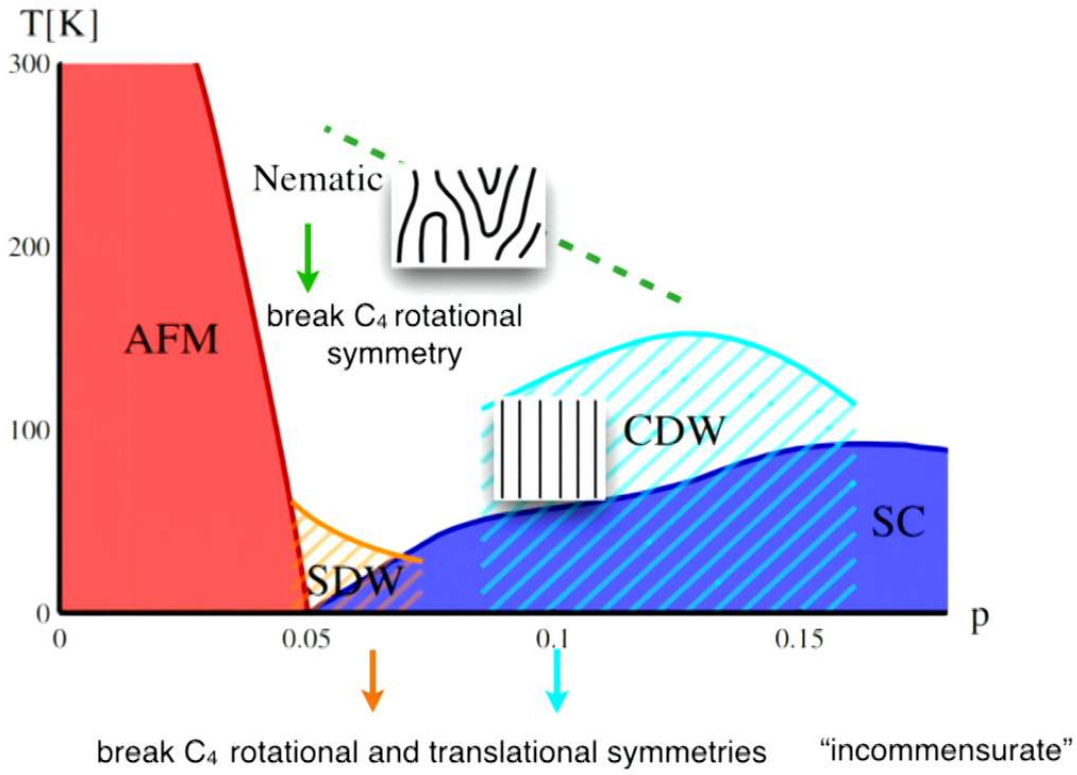
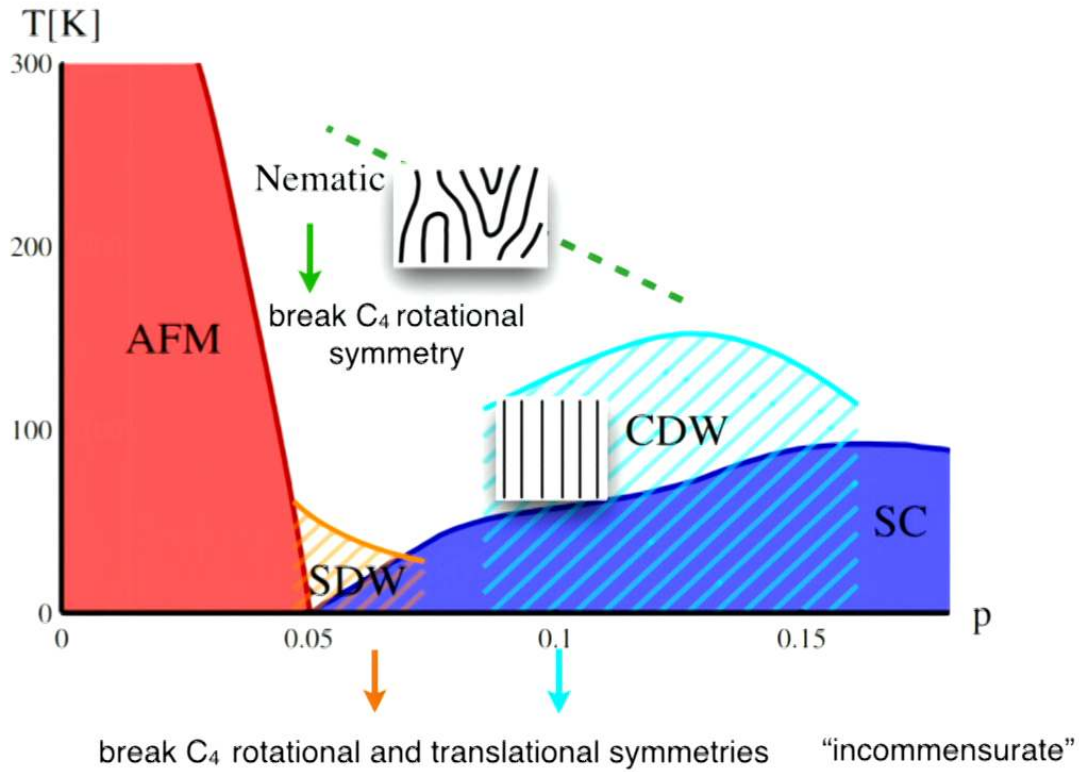


CDW SDW SC

Orders and disorder in high-T_c superconductors



CDW SDW SC

Orders and disorder in high-T_c superconductors

1. CDW + disorder PNAS 111, 7980 (2014)

   Continuous symmetry (CDW) *vs* Ising symmetry (nematic)

2. CDW + SC + disorder PRB 92, 174505 (2015)

   CDW structure factor (X-ray experiments)

3. CDW + SDW + disorder arXiv: 1701.02751

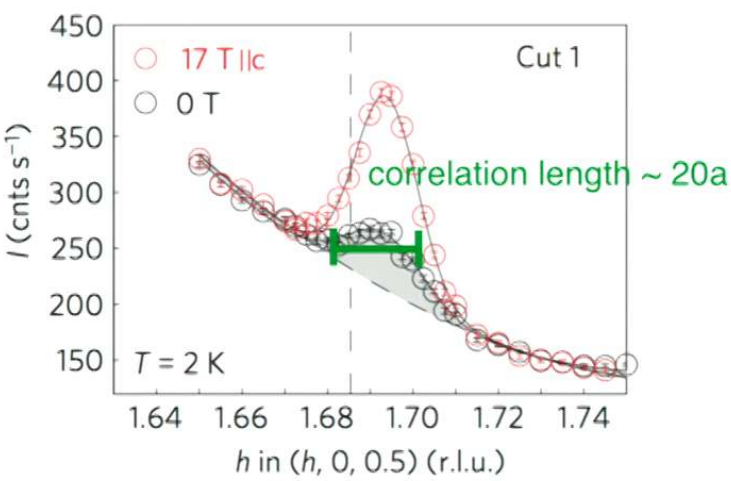
   Two nematics?

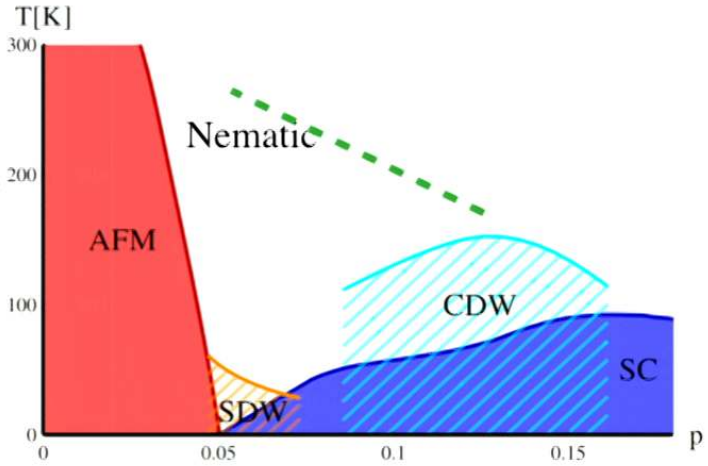
CDW + disorder:

Short-ranged CDW & long-ranged nematicity

CDW + disorder:
Short-ranged CDW & long-ranged nematicity

- X-ray in YBCO, Bi2201, LBCO, etc.

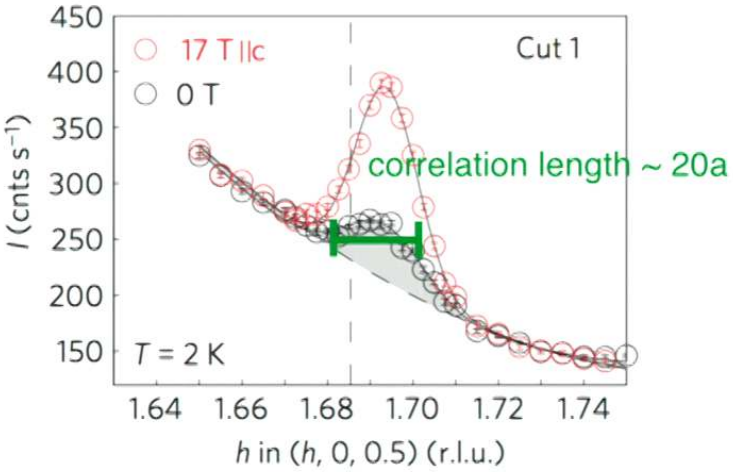




Chang *et al*, Nature Physics 8, 871 (2012)

CDW + disorder:
Short-ranged CDW & long-ranged nematicity

- X-ray in YBCO, Bi2201, LBCO, etc.



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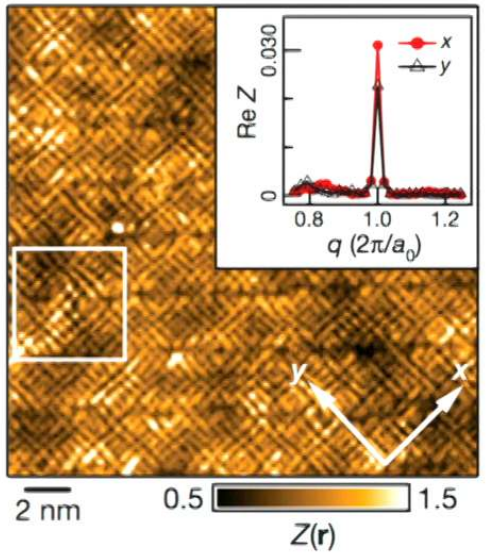
- STM in Bi2212



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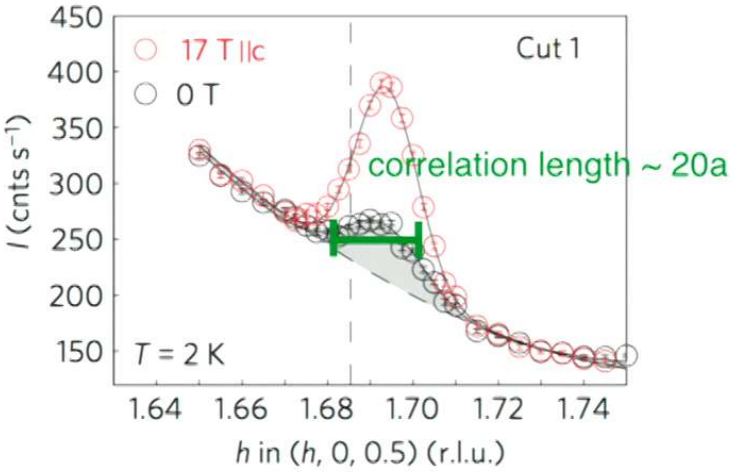

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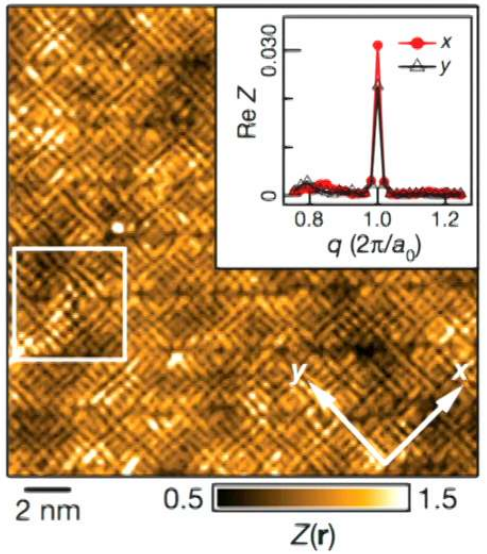
correlation length > 90a



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CDW + disorder:
Short-ranged CDW & long-ranged nematicity

A general argument: Imry-Ma Imry and Ma, PRL 35, 1399 (1975)

No continuous symmetry breaking phase in 3D,
if there is quenched random field

→ **No** truly long-ranged incommensurate CDW order
in real materials

Discrete symmetry breaking phase is allowed in 3D
(weak disorder)

→ Nematic order (Ising) can be long-ranged

CDW + disorder:
Short-ranged CDW & long-ranged nematicity

An explicit model: Landau-Ginzburg theory Nie, Tarjus, Kivelson PNAS 111, 7980 (2014)

$$\rho(\mathbf{r}) = \overline{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x\cdot\mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y\cdot\mathbf{r}}\right] + c.c. + ...$$

$\Phi \equiv (\Phi_x, \Phi_y)$ SO(2) x SO(2) x Z_2

$$H = \alpha\left[|\Phi_x|^2 + |\Phi_y|^2\right] + J\left[|\nabla\Phi_x|^2 + |\nabla\Phi_y|^2\right]$$

$$+U\left[|\Phi_x|^2 + |\Phi_y|^2 - 1\right]^2 - \Delta\left[|\Phi_x|^2 - |\Phi_y|^2\right]^2 + h^*(\Phi_x + \Phi_y) + c.c.$$

$U \to +\infty$

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$U \to +\infty$

$\Delta > 0$ stripe

$\Delta < 0$ checkerboard

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$\Delta > 0$ stripe

$\overline{h(\mathbf{r})} = 0$

$\overline{h^*(\mathbf{r})h(\mathbf{r}')} = 2\sigma^2\delta(\mathbf{r} - \mathbf{r}')$

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$U \to +\infty$

$\Delta > 0$ stripe

Replica trick

$$\overline{h(\mathbf{r})} = 0$$

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$U \to +\infty$

Hubbard-Stratonovich
large *N*, saddle point

$\Delta > 0$ stripe

Replica trick

$$\overline{h(\mathbf{r})} = 0$$
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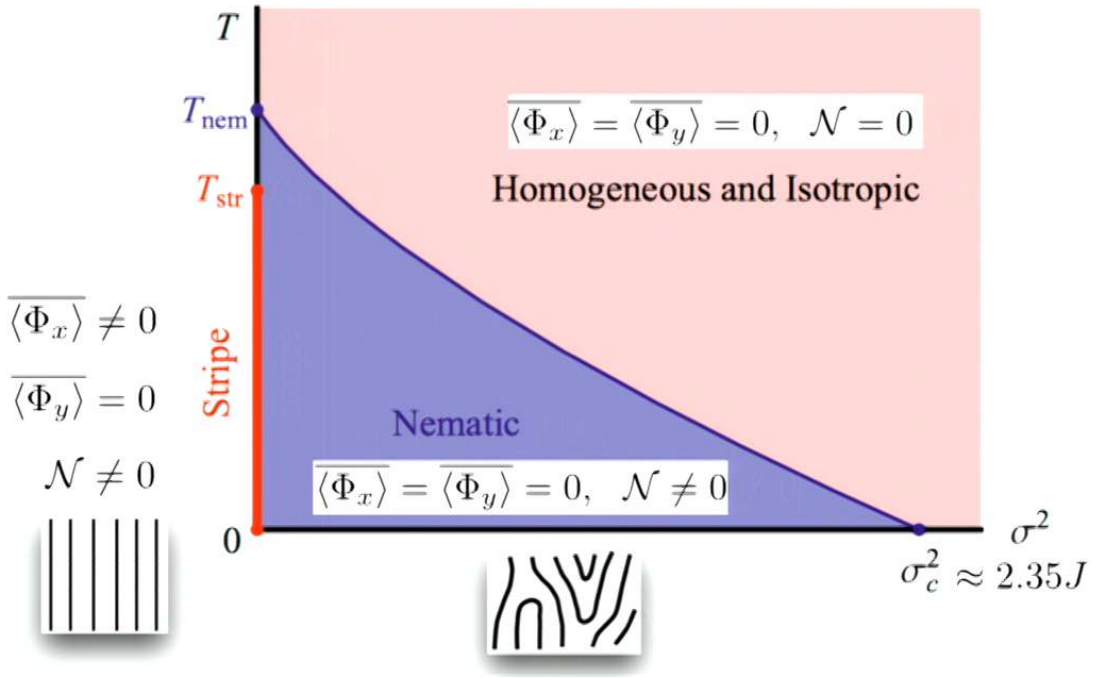
CDW + disorder:
Short-ranged CDW & long-ranged nematicity

Phase diagram

Nie, Tarjus, Kivelson PNAS 111, 7980 (2014)

$$\overline{\langle \Phi_x \rangle} \ , \ \overline{\langle \Phi_y \rangle} \ , \ \mathcal{N} \equiv \overline{\langle |\Phi_x|^2 \rangle} - \overline{\langle |\Phi_y|^2 \rangle}$$



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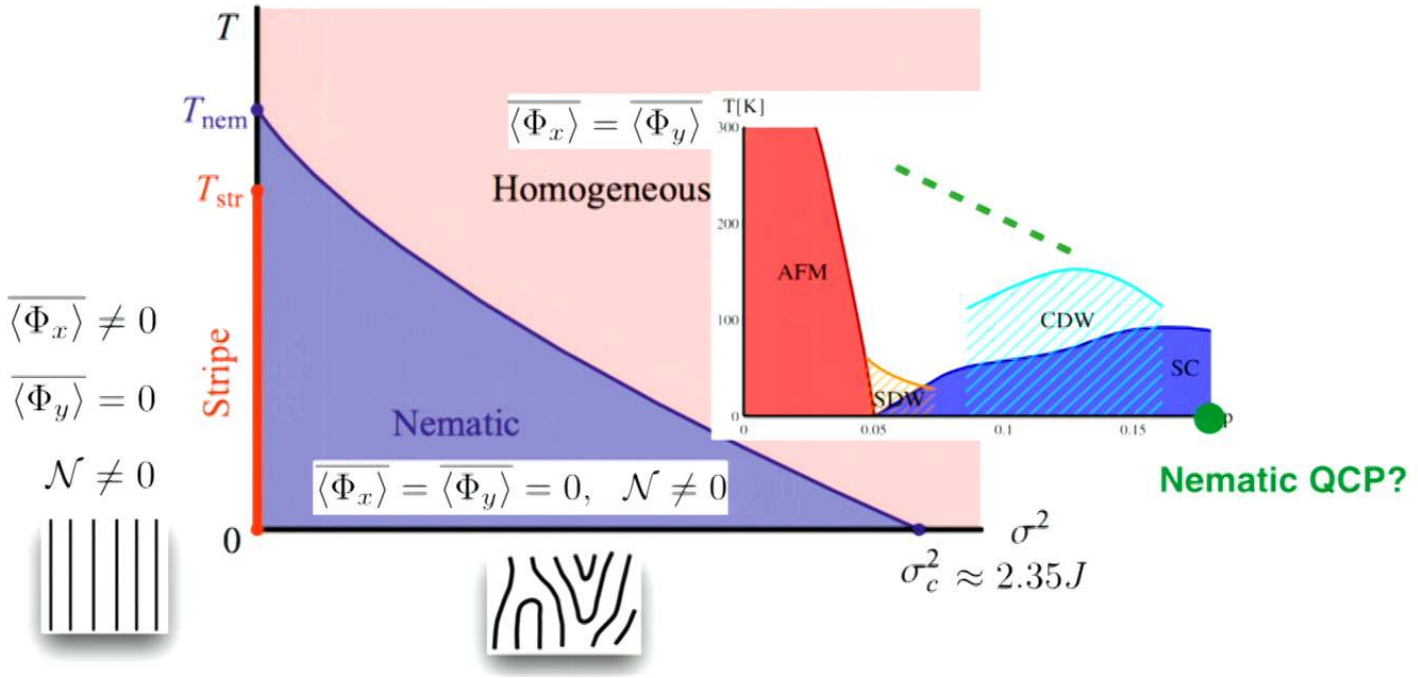

Nematic QCP?

CDW + disorder:
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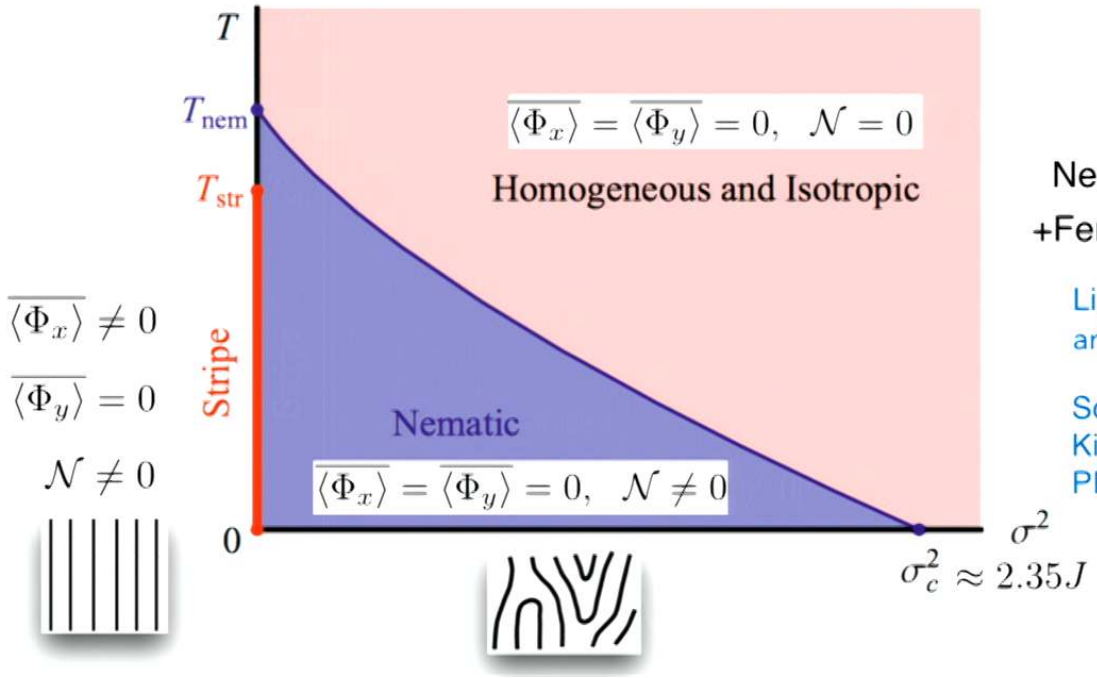

$\overline{\langle \Phi_x \rangle} \neq 0$

$\overline{\langle \Phi_y \rangle} = 0$

$\mathcal{N} \neq 0$

Nematic fluctuations
+Fermi surface: QMC

Li, Wang, Yao, Lee
arXiv:1512.04541

Schattner, Lederer,
Kivelson, Berg
PRX 6, 031028 (2016)

1. CDW + disorder

- An explicit model of Imry-Ma's argument
- Short-ranged CDW and long-ranged nematicity

2. CDW + SC + disorder

CDW + SC + disorder: The model

Nie, Sierens, Melko, Sachdev, Kivelson, PRB 92, 174505 (2015)

$$\rho(\mathbf{r}) = \overline{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x\cdot\mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y\cdot\mathbf{r}}\right] + c.c. + \ldots$$

CDW $\left(\Phi_x(\mathbf{r}), \Phi_y(\mathbf{r})\right) \equiv \Phi(\mathbf{r})$

SC $\Psi(\mathbf{r})$

SO(2) x SO(2) x SO(2) x Z_2

$$H = \alpha\left[|\Phi_x|^2 + |\Phi_y|^2 + |\Psi|^2\right] + J\left[|\nabla\Phi_x|^2 + |\nabla\Phi_y|^2 + |\nabla\Psi|^2\right] + J_z\left[|\nabla_\perp\Phi_x|^2 + |\nabla_\perp\Phi_y|^2 + |\nabla_\perp\Psi|^2\right]$$

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$h^*\Psi$

$U \to +\infty$

$\Delta > 0$ stripe

CDW + SC + disorder: The model

Nie, Sierens, Melko, Sachdev, Kivelson, PRB 92, 174505 (2015)

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~~$h^*\Psi$~~ gauge invariance

$U \to +\infty$

$\Delta > 0$ stripe

CDW + SC + disorder: The model

Nie, Sierens, Melko, Sachdev, Kivelson, PRB 92, 174505 (2015)

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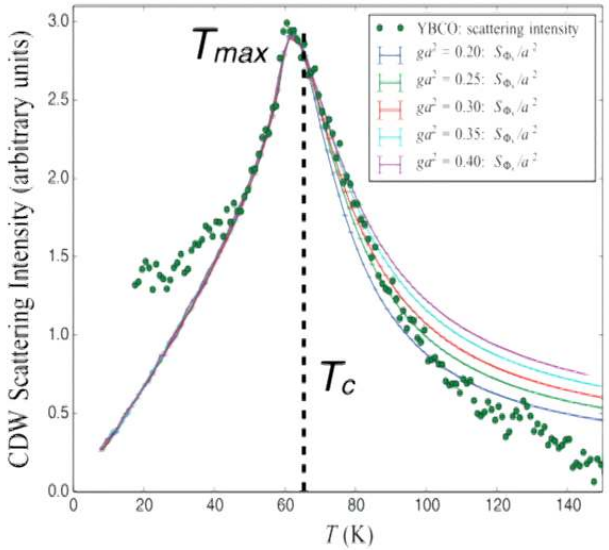
Replica trick + saddle point $(N = \infty)$ & Classical Monte Carlo $(N = 2)$

CDW + SC + disorder: CDW structure factor

$$\rho(\mathbf{r}) = \overline{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x\cdot\mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y\cdot\mathbf{r}}\right] + c.c. + \ldots$$

$$S_{\Phi_x}(T) \equiv \overline{\langle\rho(\mathbf{Q}_x)\rho(\mathbf{Q}_x)\rangle} = \overline{\langle\Phi_x^\dagger(\mathbf{k})\Phi_x(\mathbf{k})\rangle}\Big|_{\mathbf{k}=0}$$



Monte Carlo: similar model 2D, no disorder

- $S_{\Phi_x}(T=0)$?
- T_{max} vs. T_c ?

Hayward *et al.*, Science 343, 1336 (2014)
Achkar *et al.*, PRL 113, 107002 (2014)

CDW + SC + disorder: CDW structure factor

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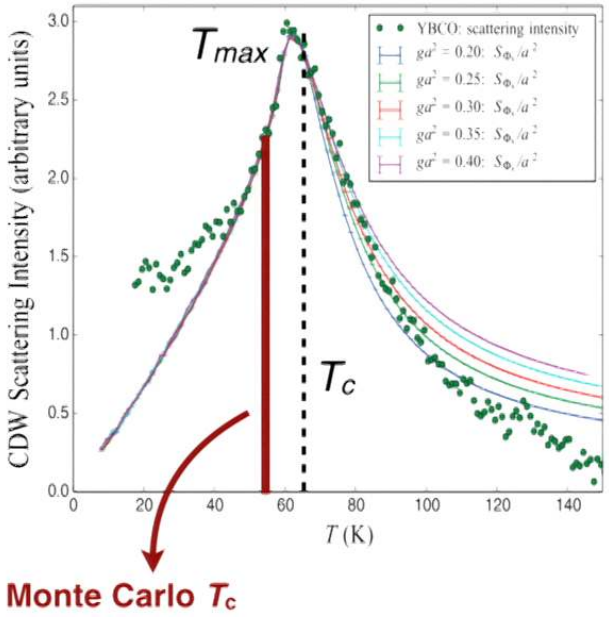
$$S_{\Phi_x}(T) \equiv \overline{\langle\rho(\mathbf{Q}_x)\rho(\mathbf{Q}_x)\rangle} = \overline{\langle\Phi_x^\dagger(\mathbf{k})\Phi_x(\mathbf{k})\rangle}\Big|_{\mathbf{k}=0}$$

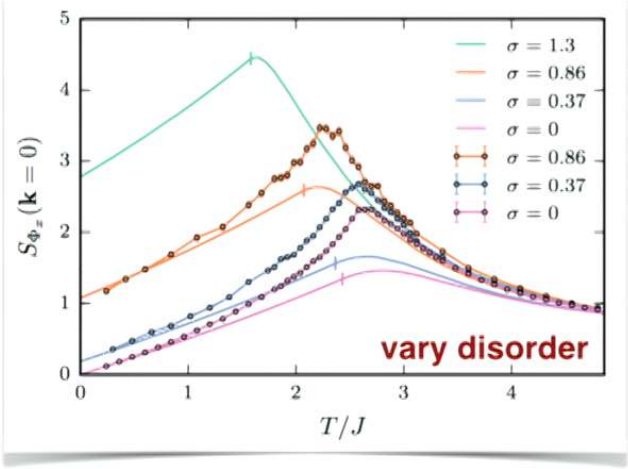


Monte Carlo: similar model 2D, no disorder

- $S_{\Phi_x}(T=0)$?
- T_{max} vs. T_c ?

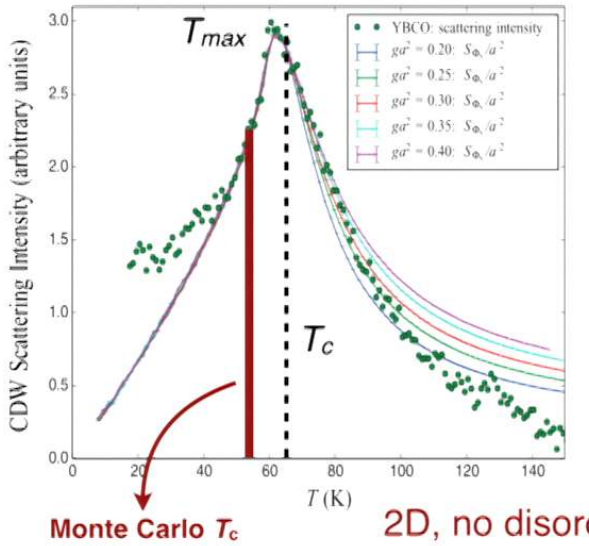
Hayward *et al.*, Science 343, 1336 (2014)
Achkar *et al.*, PRL 113, 107002 (2014)

CDW structure factor



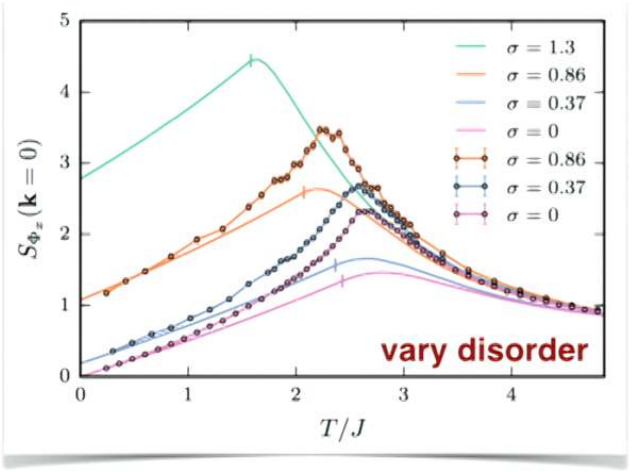
increase σ ⟶ $S_{\Phi_x}(T=0) \neq 0$

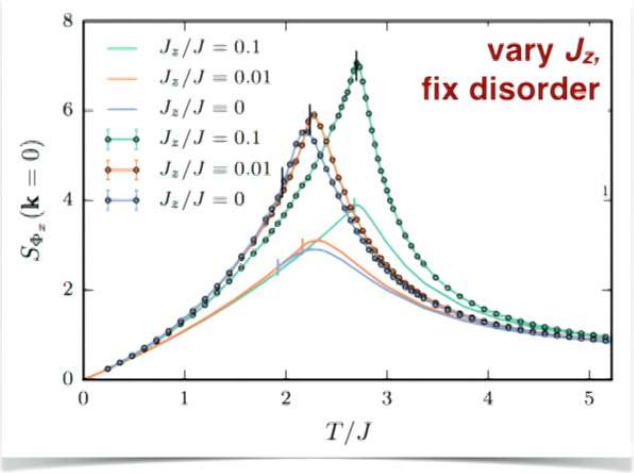
⟶ T_{max} and T_c closer

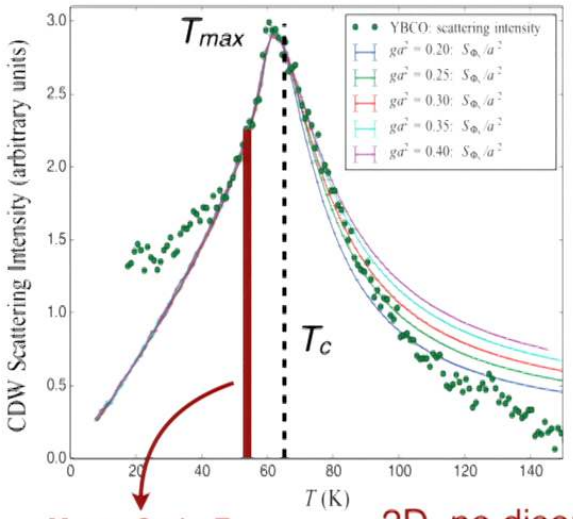


2D, no disorder

CDW structure factor



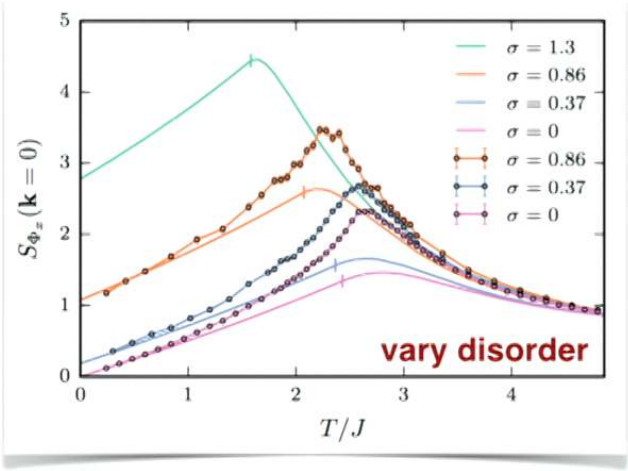


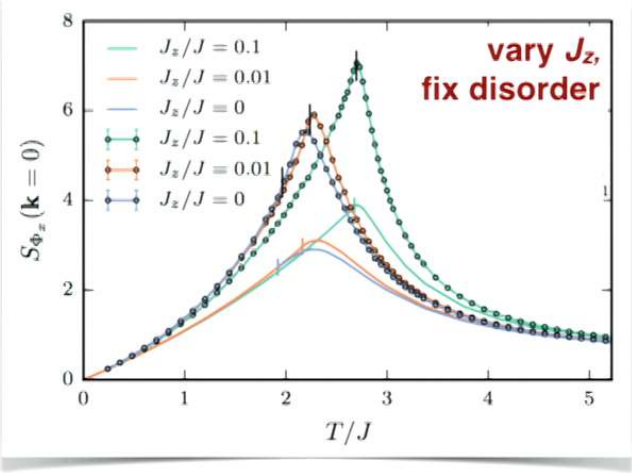


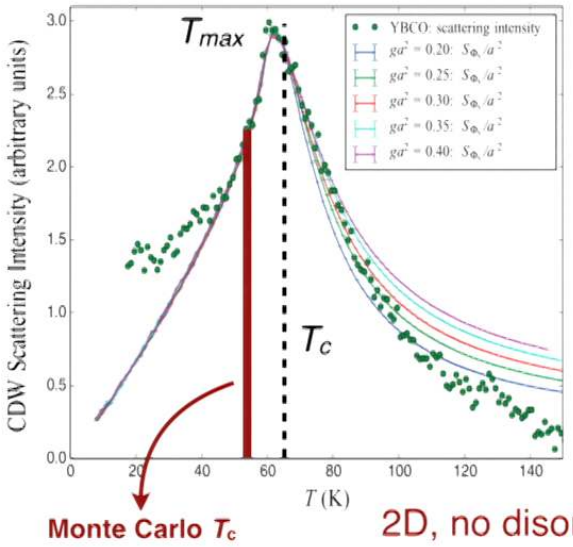
increase σ → $S_{\Phi_x}(T=0) \neq 0$

→ T_{max} and T_c closer

CDW structure factor







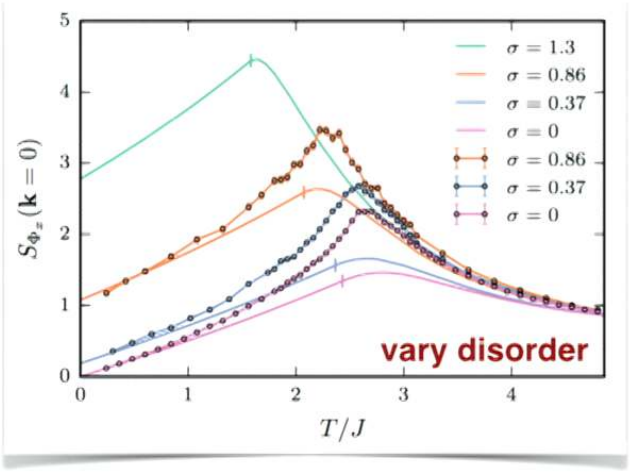
2D, no disorder

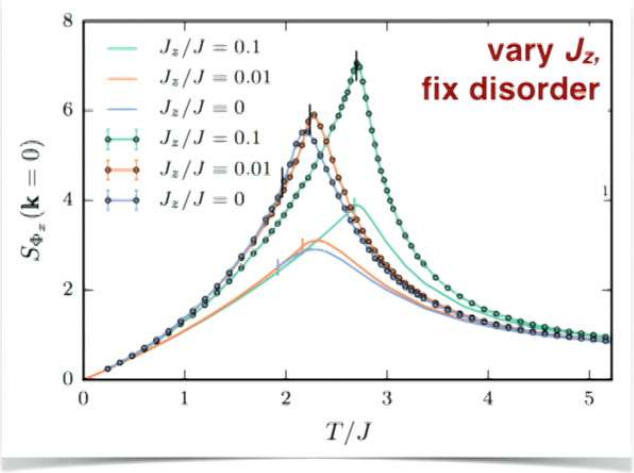
increase σ → $S_{\Phi_x}(T=0) \neq 0$

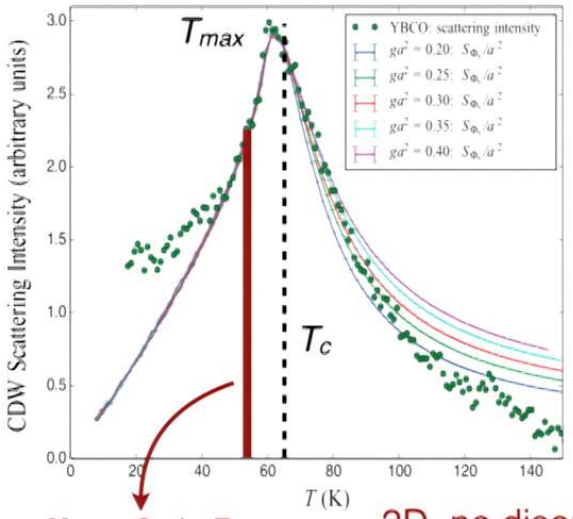
increase J_z → T_{max} and T_c closer

Agreement between large N and Monte Carlo

CDW structure factor







increase σ → $S_{\Phi_x}(T=0) \neq 0$

increase J_z → T_{max} and T_c closer

Agreement between large N and Monte Carlo

2. CDW + SC + disorder

- Thermal evolution of CDW structure factor:

  disorder and interlayer coupling are important

CDW + SDW + disorder: The model

Nie, Maharaj, Fradkin, Kivelson
arXiv: 1701.02751

CDW $$\rho(\mathbf{r}) = \overline{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x\cdot\mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y\cdot\mathbf{r}}\right] + c.c. + \ldots$$

SDW $$\mathbf{S}(\mathbf{r}) = \mathbf{S}_x(\mathbf{r})e^{i\mathbf{K}_x\cdot\mathbf{r}} + \mathbf{S}_y(\mathbf{r})e^{i\mathbf{K}_y\cdot\mathbf{r}} + c.c. + \ldots$$

$$H = H_{\mathbf{S}}[\mathbf{S}_x, \mathbf{S}_y] + H_{\Phi}[\Phi_x, \Phi_y] + H_{\rm int}[\mathbf{S}_x, \mathbf{S}_y, \Phi_x, \Phi_y]$$

$$H_{\Phi} = \alpha_{\Phi}\left[|\Phi_x|^2 + |\Phi_y|^2\right] + J_{\Phi}\left[|\nabla\Phi_x|^2 + |\nabla\Phi_y|^2\right] + U_{\Phi}\left[|\Phi_x|^2 + |\Phi_y|^2\right]^2 + \gamma_{\Phi}|\Phi_x|^2|\Phi_y|^2$$

$$+\left[h^*(\Phi_x + \Phi_y) + c.c.\right] \qquad h|\mathbf{S}|^2$$

$$H_{\mathbf{S}} = \alpha_s\left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2\right] + J_s\left[|\nabla\mathbf{S}_x|^2 + |\nabla\mathbf{S}_y|^2\right] + U_s\left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2\right]^2 + \gamma_s|\mathbf{S}_x|^2|\mathbf{S}_y|^2$$

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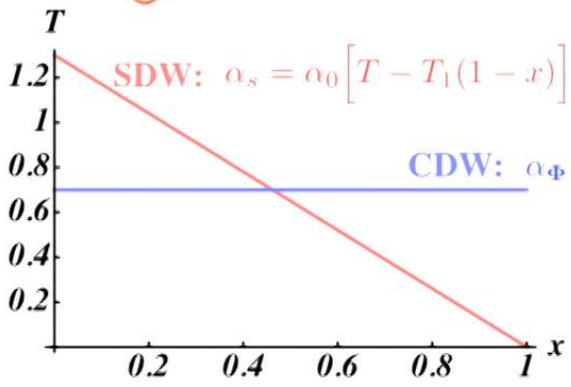

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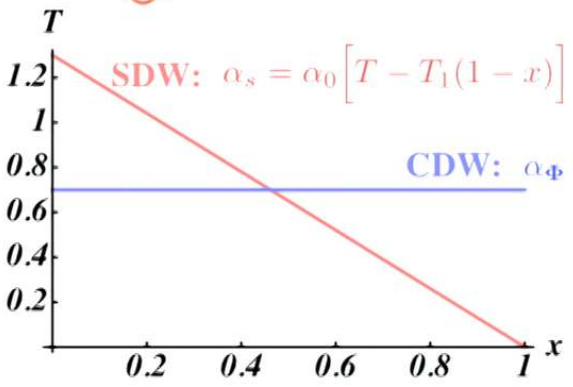

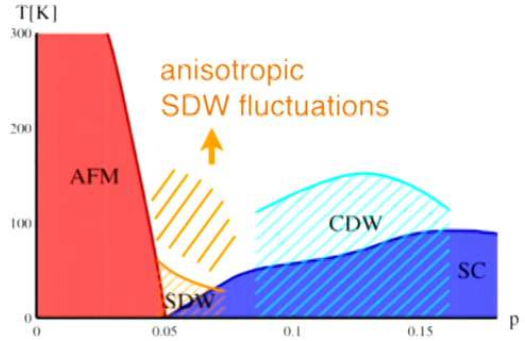

CDW + SDW + disorder: The model

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CDW $\rho(\mathbf{r}) = \overline{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x\cdot\mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y\cdot\mathbf{r}}\right] + c.c. + \ldots$

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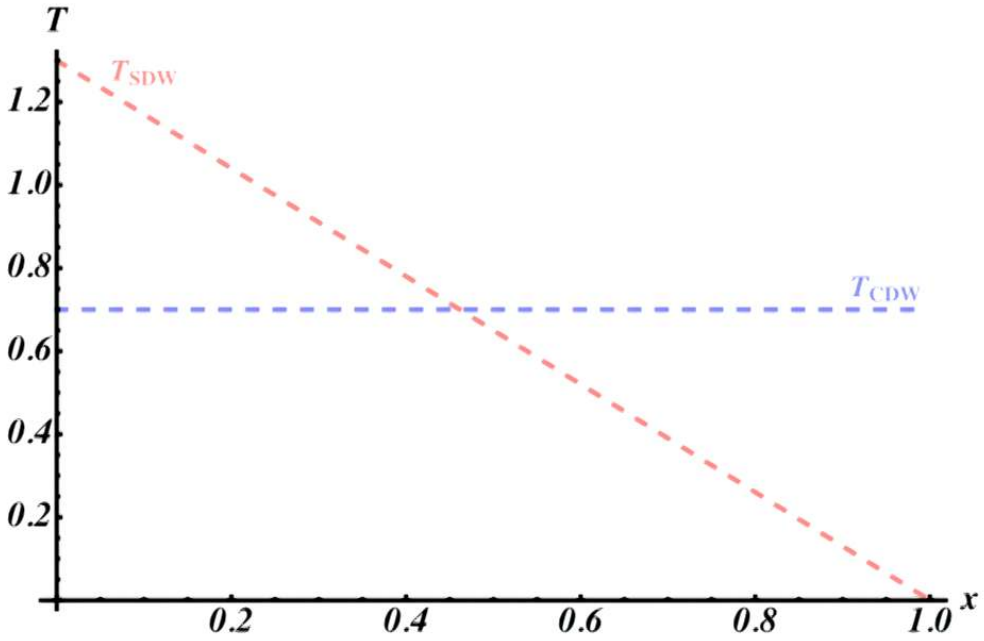
$$H_{\mathbf{S}} = \alpha_s\left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2\right] + J_s\left[|\nabla\mathbf{S}_x|^2 + |\nabla\mathbf{S}_y|^2\right] + U_s\left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2\right]^2 + \gamma_s|\mathbf{S}_x|^2|\mathbf{S}_y|^2$$

$$H_{\rm int} = v\left[|\Phi_x|^2|\mathbf{S}_x|^2 + |\Phi_y|^2|\mathbf{S}_y|^2\right] + w\left[|\Phi_x|^2|\mathbf{S}_y|^2 + |\Phi_y|^2|\mathbf{S}_x|^2\right]$$

$v > w$: SDW $\perp$ CDW

Result: zero disorder $(\sigma = 0)$



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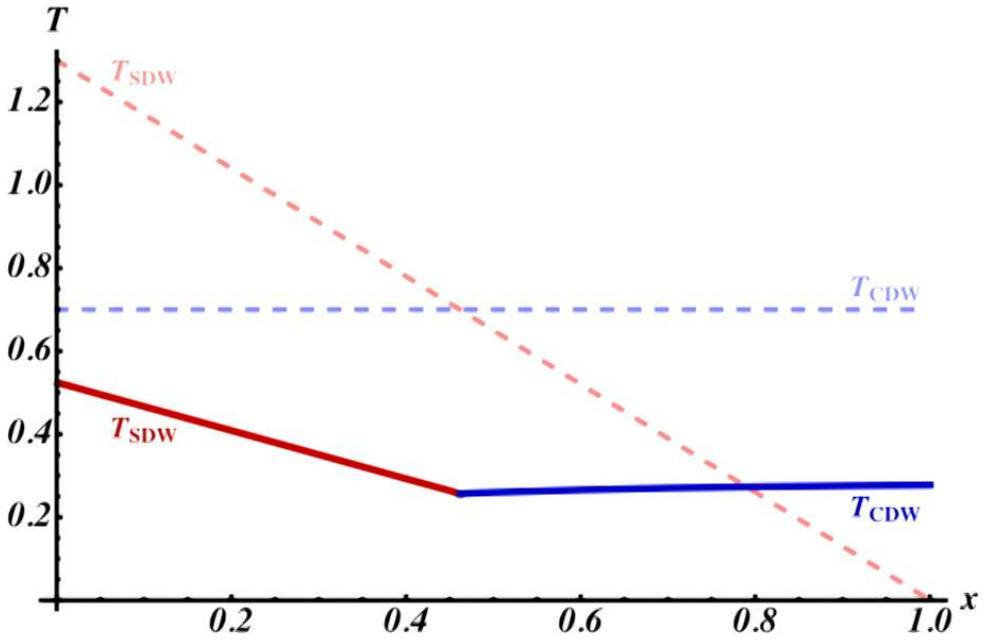

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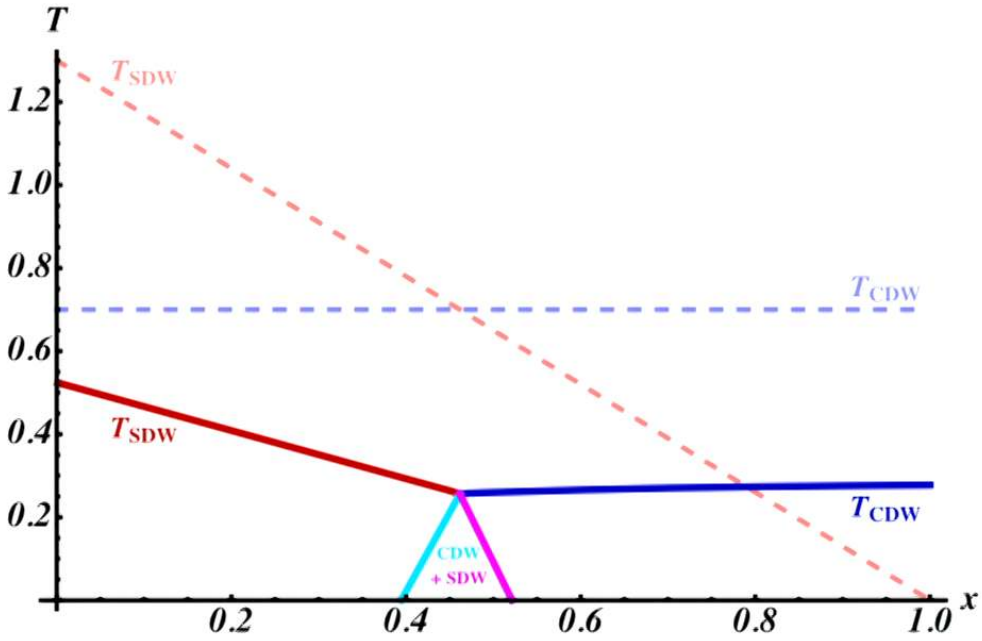

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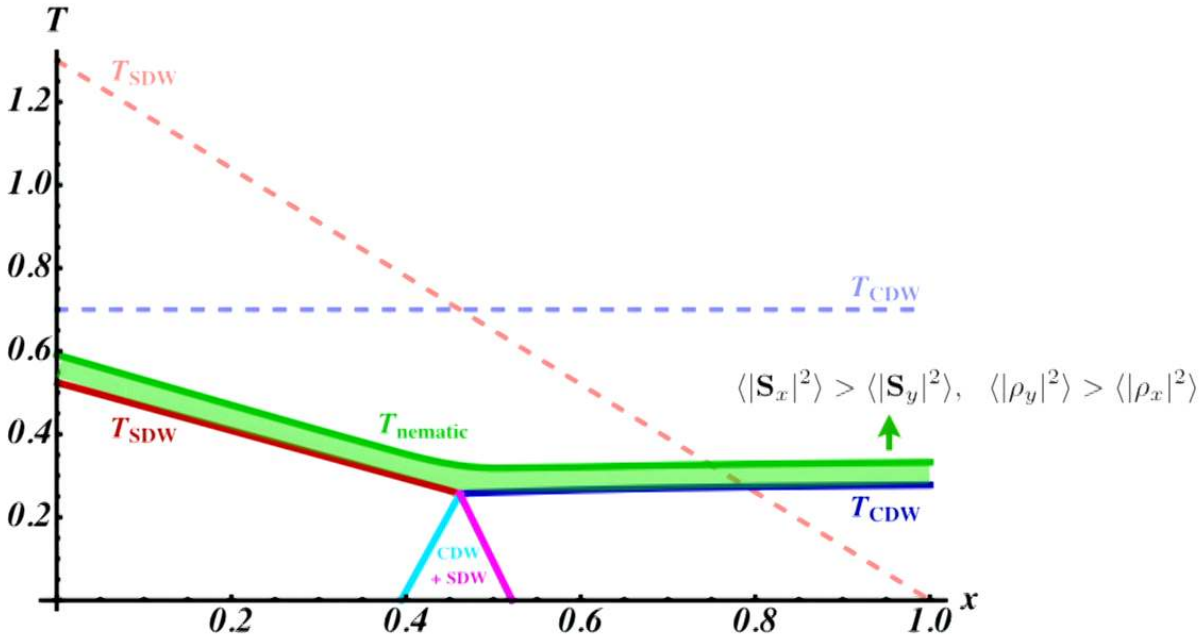
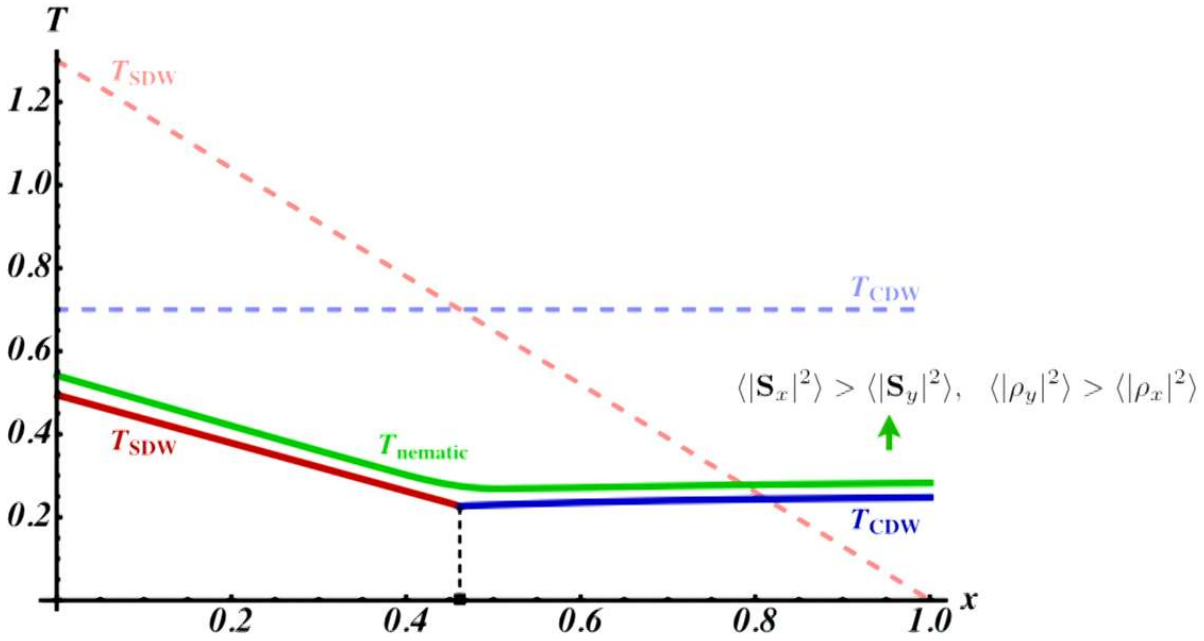
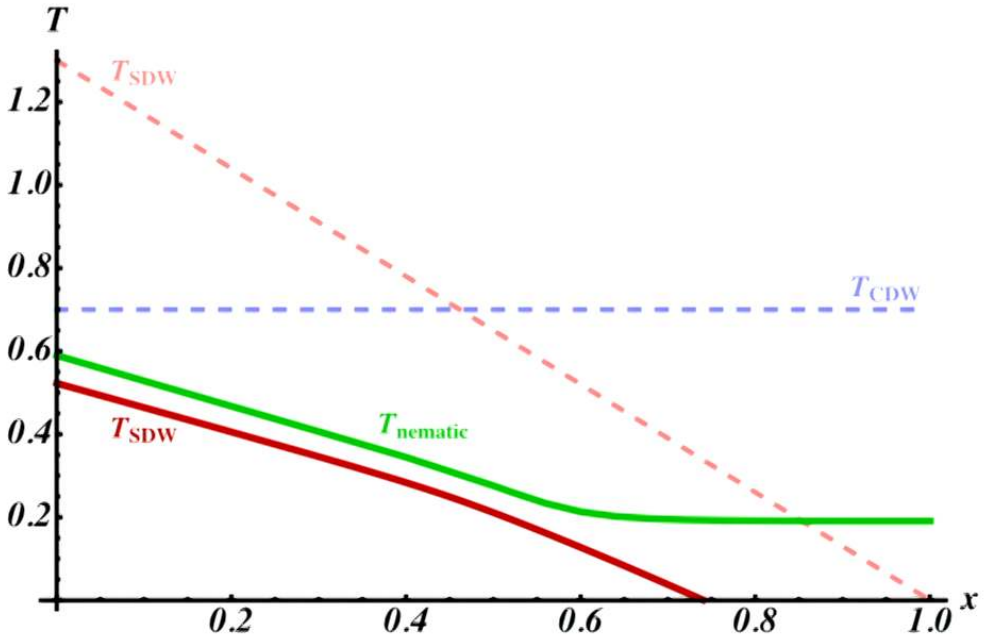


Result: zero disorder $(\sigma = 0)$

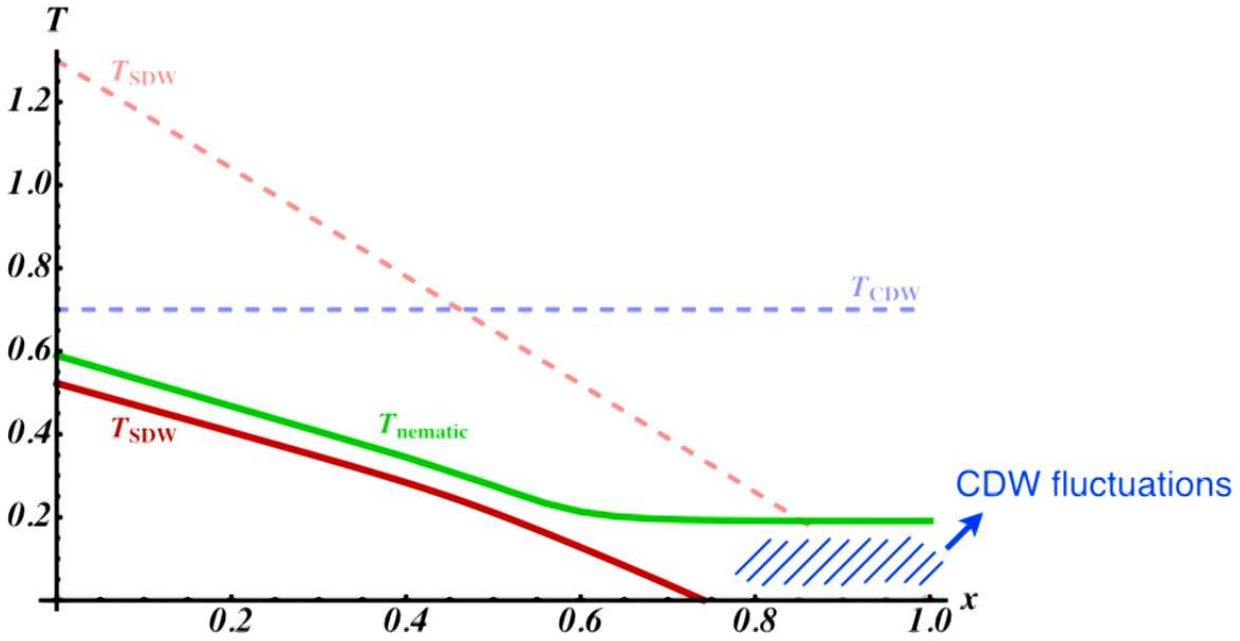


Result: finite disorder $(\sigma \neq 0)$



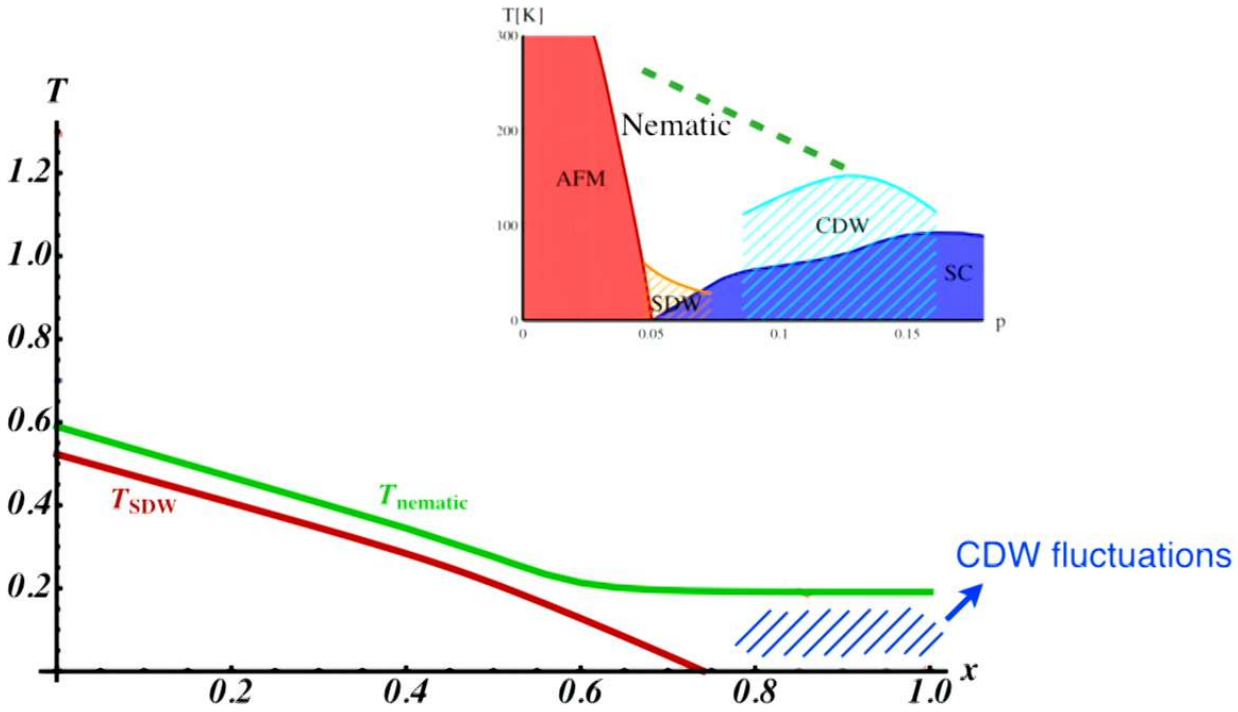
$\sigma = 0.1$

Result: finite disorder $(\sigma \neq 0)$



$\sigma = 0.1$

Result: finite disorder $(\sigma \neq 0)$



$\sigma = 0.1$

3. CDW + SDW + disorder

- A universal nematic transition across the entire doping range,

  from partial melting of density waves

  (thermal & random-field disorder for CDW)

Summary

1. CDW + disorder

   Short-ranged CDW and long-ranged nematicity

2. CDW + SC + disorder

   Disorder and interlayer coupling (dimensionality)

3. CDW + SDW + disorder

   One nematic phase, driven by CDW and SDW