

Title: Orders and disorder in high-Tc superconductors

Date: Jan 13, 2017 03:30 PM

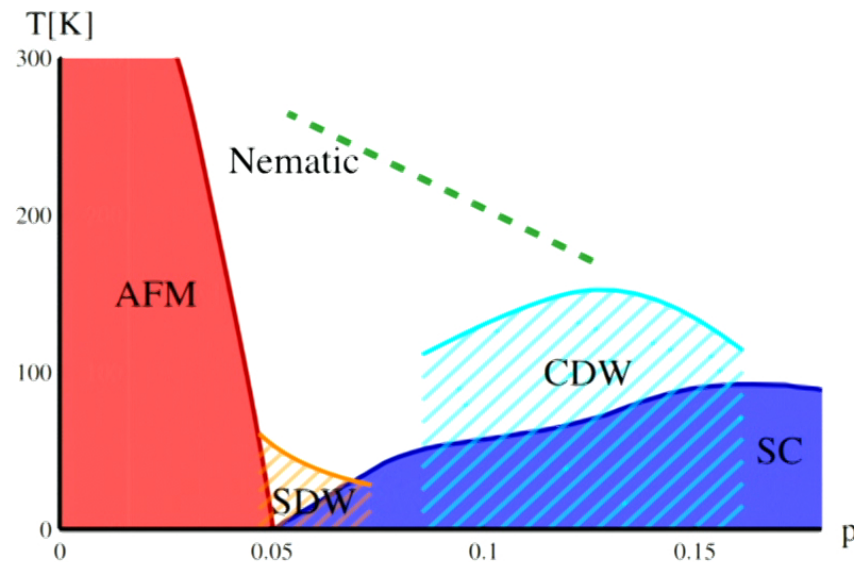
URL: <http://pirsa.org/17010065>

Abstract: <p>Since its first discovery in 1986, high-Tc superconductors have been attracting constant interests and meticulous efforts from both theorists and experimentalists, not merely due to its large transition temperature, but also because it offers a well characterized laboratory for the study of exotic phenomena such as quantum criticality, non-Fermi liquid behavior, and intertwined orders. One pressing question in the field is the role played by disorder:</p>

<p>inevitable in real materials, disorder is able to fundamentally alter the properties of the system under certain circumstances. In this talk, we will discuss from a theoretical point of view, how different types of disorders affect various electronic orders in copper-based high-Tc superconductors. We also discuss the implications in experiments, and investigate the possibility of generalizing our conclusions to a variety of other physical systems.</p>

Orders and disorder in high- T_c superconductors

Laimei Nie
Stanford University



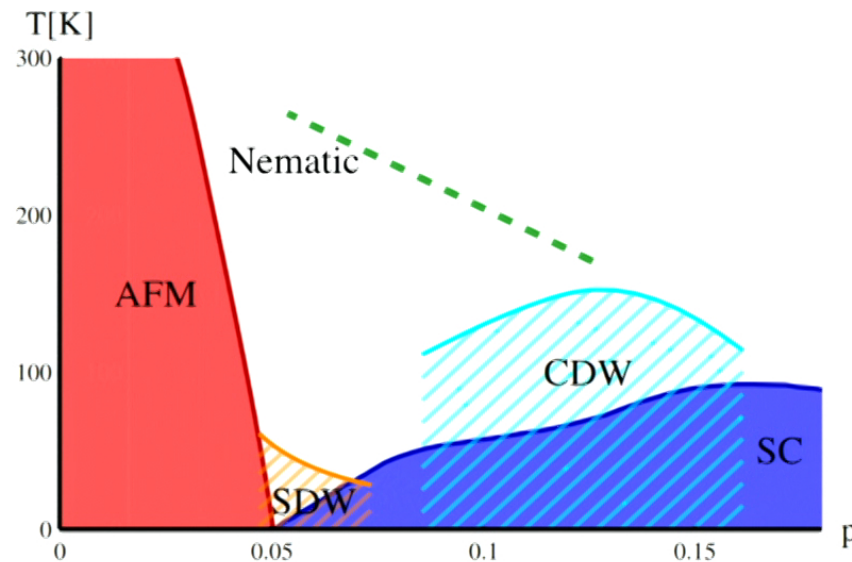
Sketch of cuprates phase diagram, adapted from Schütt and Fernandes, PRL 2015

Perimeter Institute, Jan 13 2017

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Experiments
Theories and numerics



Sketch of cuprates phase diagram, adapted from Schütt and Fernandes, PRL 2015

Perimeter Institute, Jan 13 2017

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Collaborators:

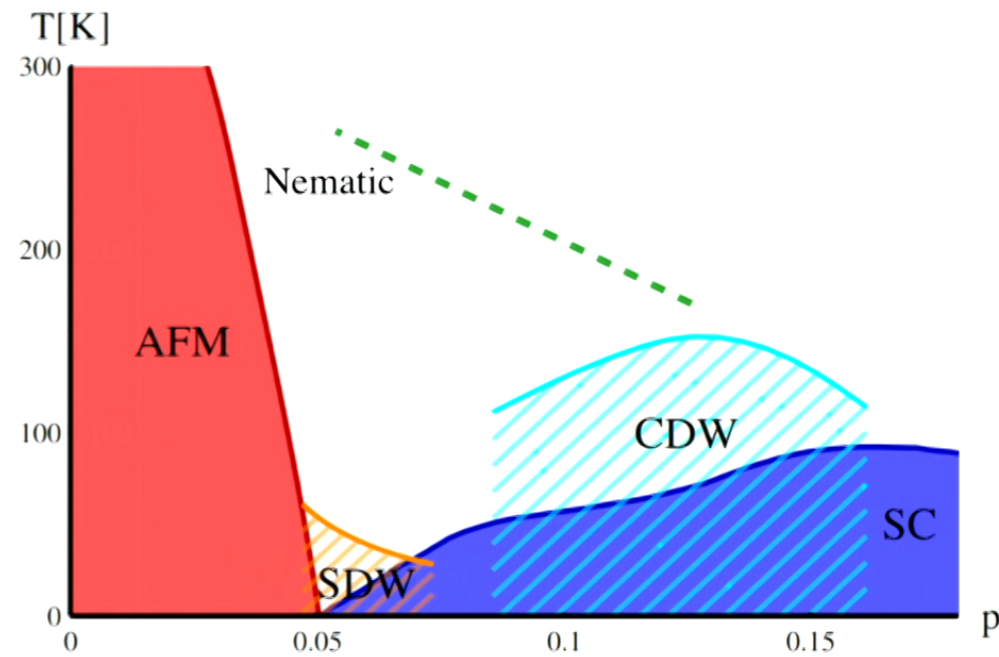
Steven Kivelson
Akash Maharaj
Stanford

Roger Melko
Lauren Hayward Sierens
Perimeter Institute

Eduardo Fradkin
UIUC

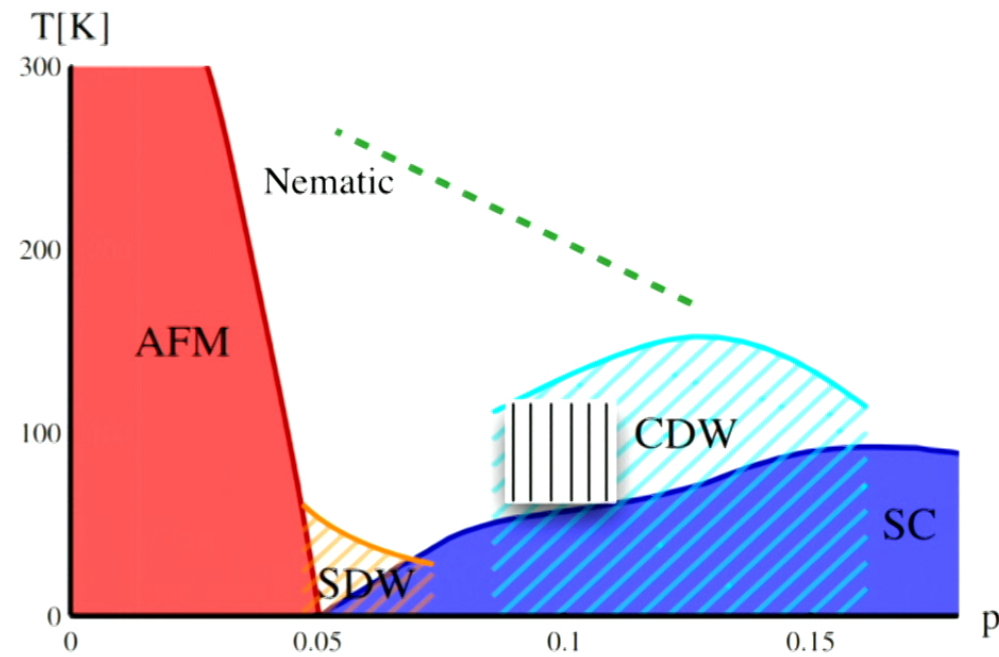
Subir Sachdev
Harvard

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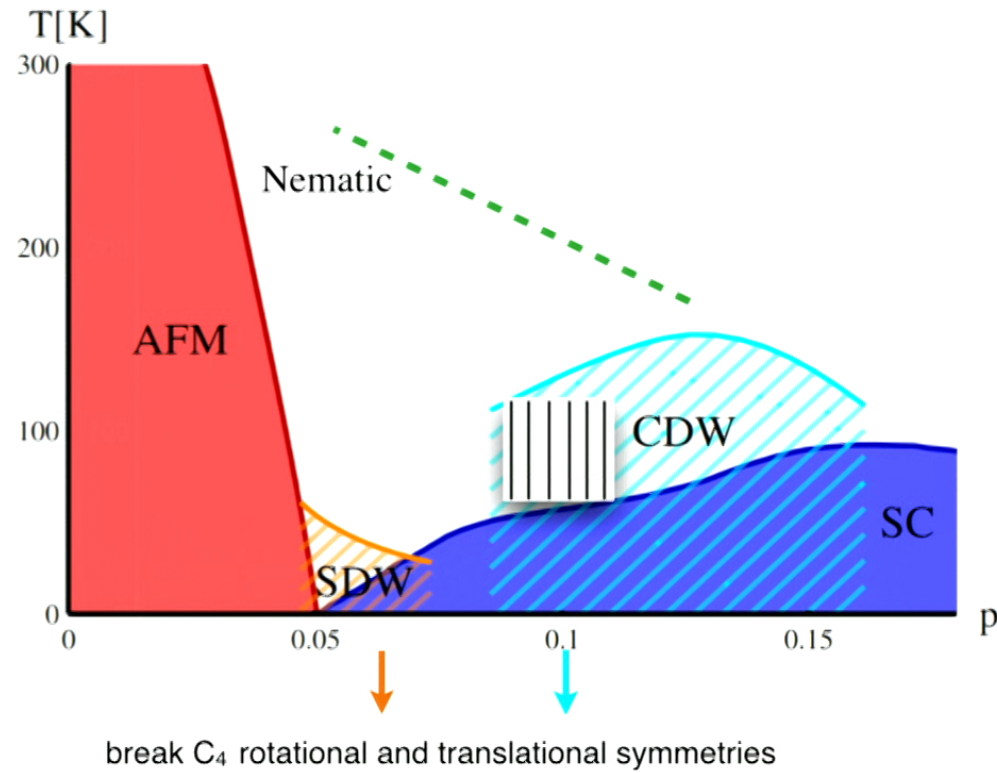


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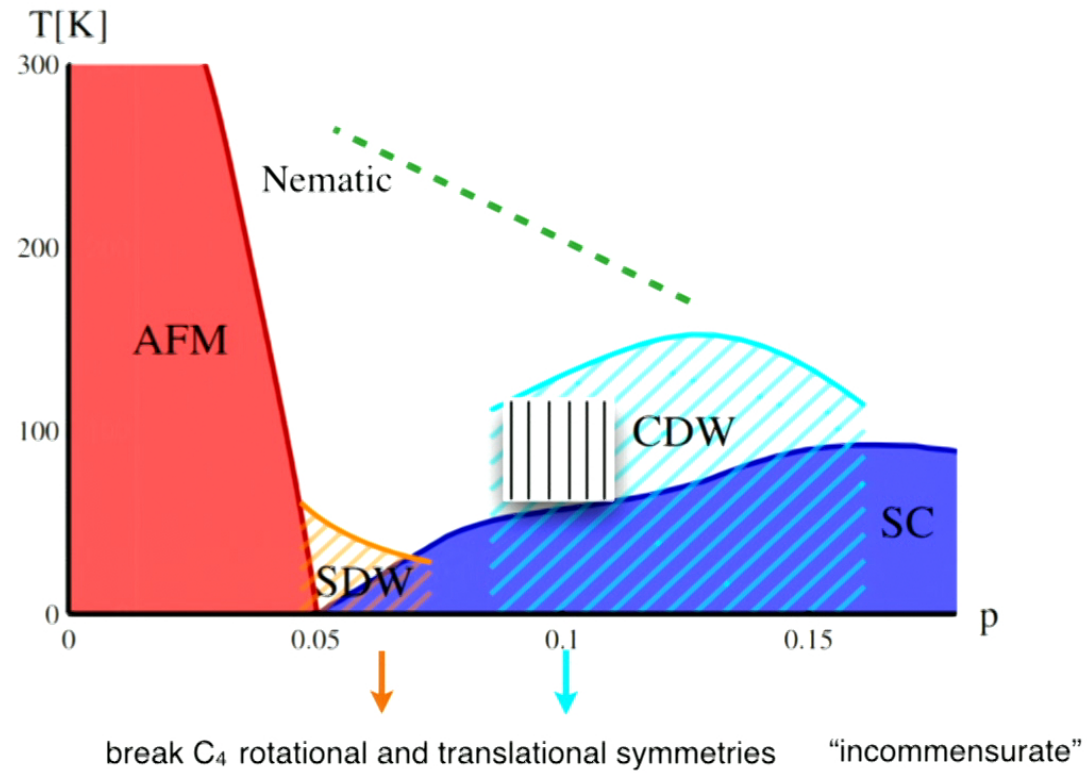
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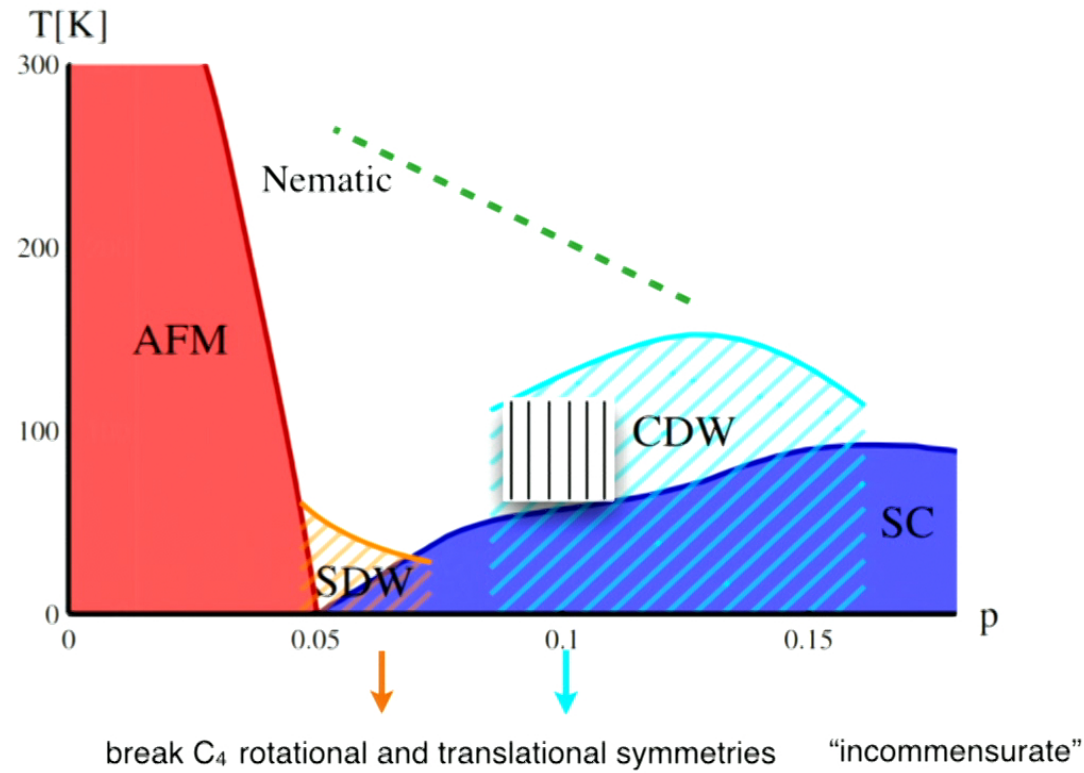
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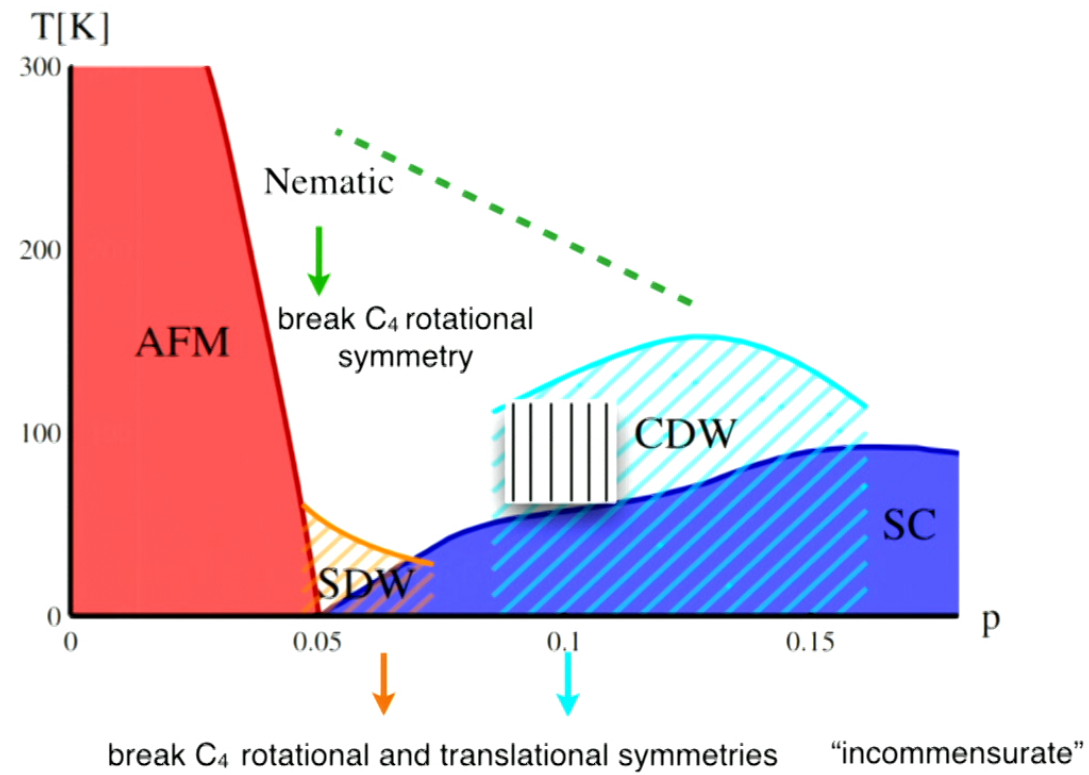
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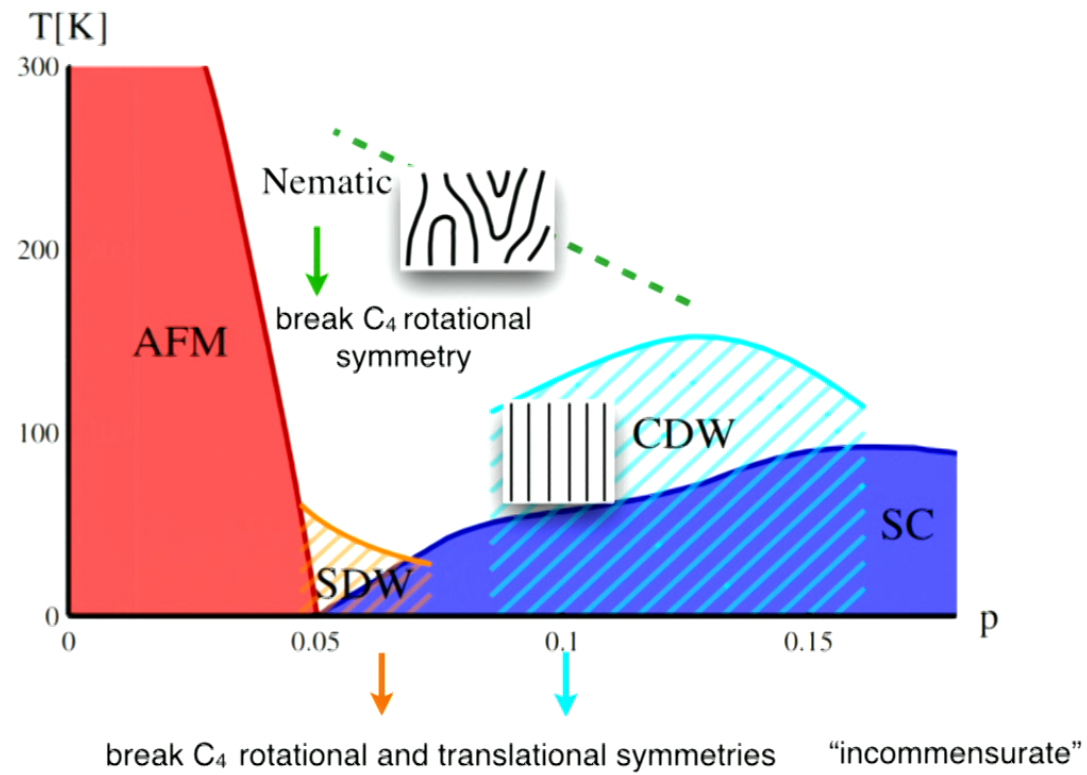
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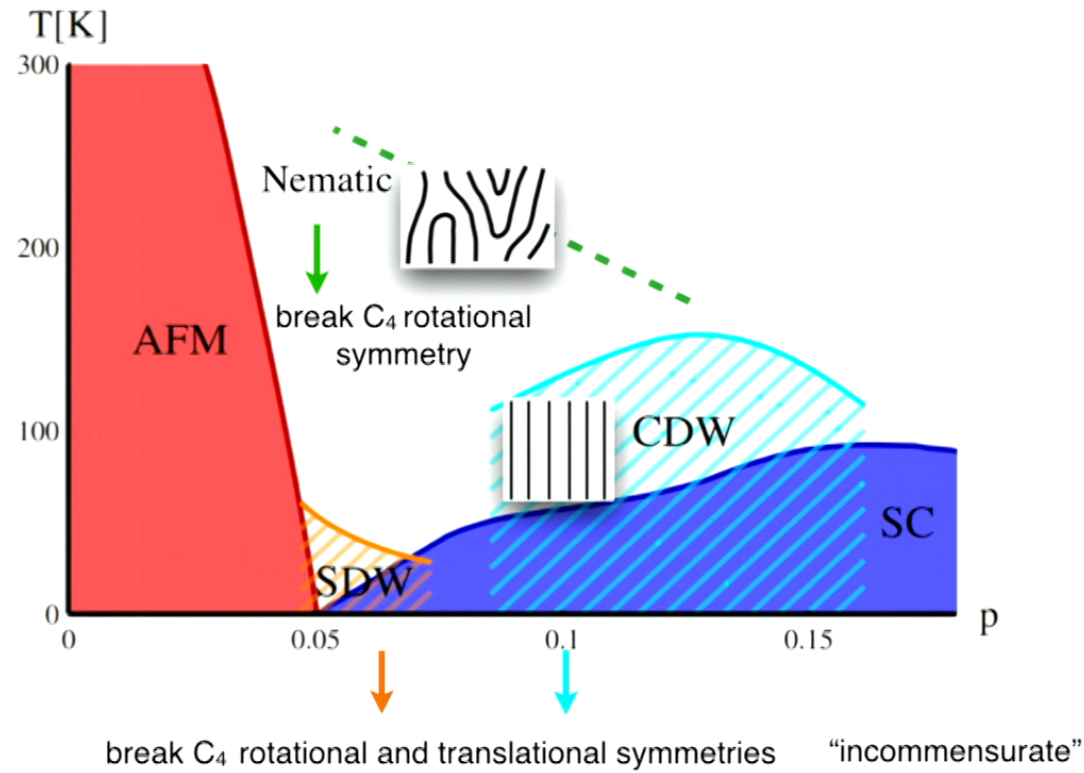


Orders and disorder in high- T_c superconductors



↗ CDW SDW SC

Orders and disorder in high- T_c superconductors



↗ CDW SDW SC

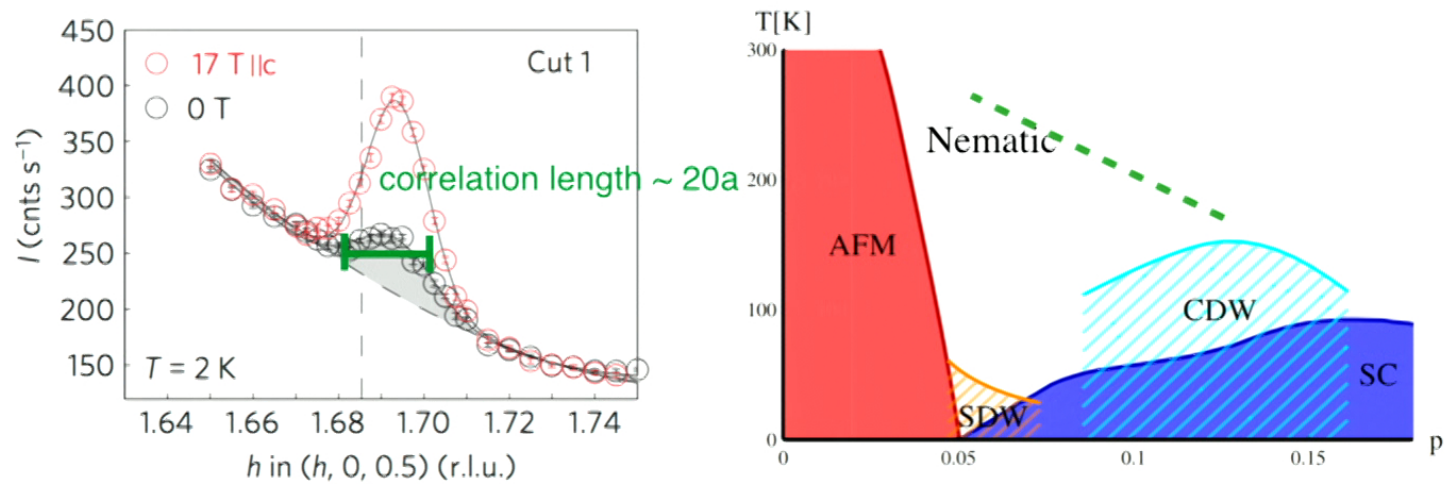
Orders and disorder in high- T_c superconductors

1. CDW + disorder [PNAS 111, 7980 \(2014\)](#)
Continuous symmetry (CDW) vs Ising symmetry (nematic)
2. CDW + SC + disorder [PRB 92, 174505 \(2015\)](#)
CDW structure factor (X-ray experiments)
3. CDW + SDW + disorder [arXiv: 1701.02751](#)
Two nematics?

CDW + disorder:
Short-ranged CDW & long-ranged nematicity

CDW + disorder: Short-ranged CDW & long-ranged nematicity

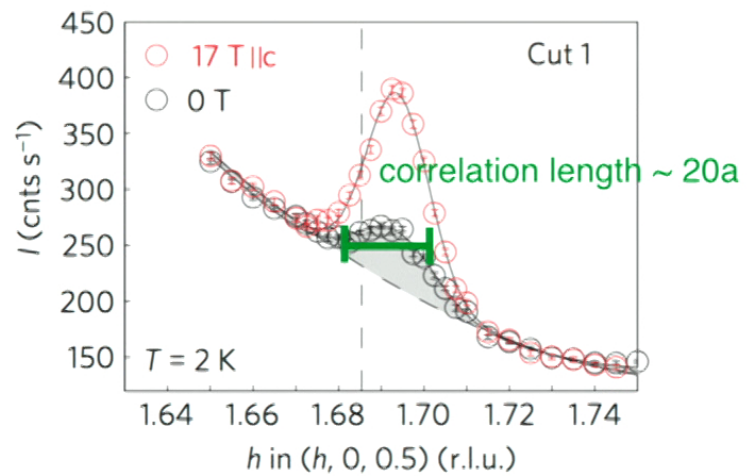
- X-ray in YBCO, Bi2201, LBCO, etc.



Chang *et al*, Nature Physics 8, 871 (2012)

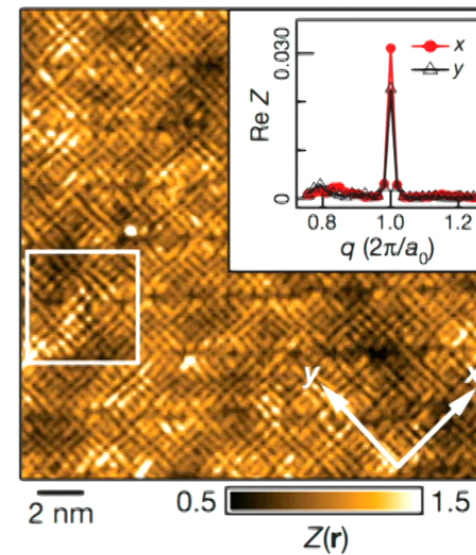
CDW + disorder: Short-ranged CDW & long-ranged nematicity

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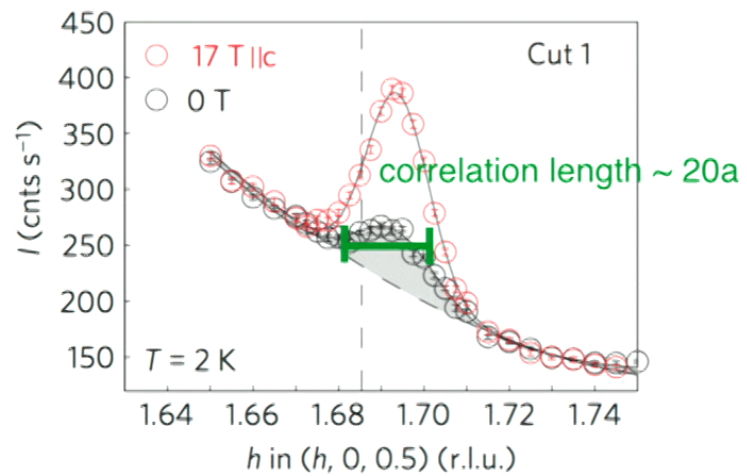
- STM in Bi2212



Lawler *et al*, Nature 466, 347 (2010)

CDW + disorder: Short-ranged CDW & long-ranged nematicity

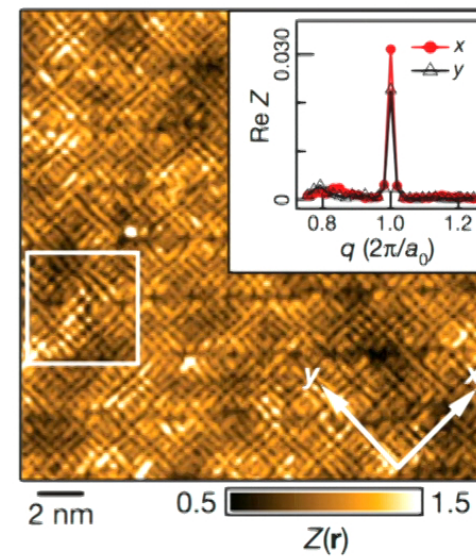
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- STM in Bi2212

correlation length $> 90a$



Lawler *et al*, Nature 466, 347 (2010)

CDW + disorder: Short-ranged CDW & long-ranged nematicity

A general argument: Imry-Ma

Imry and Ma, PRL 35, 1399 (1975)

No continuous symmetry breaking phase in 3D,
if there is **quenched random field**

→ **No** truly long-ranged incommensurate CDW order
in real materials

Discrete symmetry breaking phase is allowed in 3D
(weak disorder)

→ Nematic order (Ising) can be long-ranged

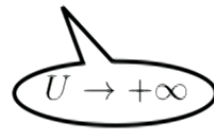
CDW + disorder: Short-ranged CDW & long-ranged nematicity

An explicit model: Landau-Ginzburg theory [Nie, Tarjus, Kivelson PNAS 111, 7980 \(2014\)](#)

$$\rho(\mathbf{r}) = \bar{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x \cdot \mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y \cdot \mathbf{r}} \right] + c.c. + \dots$$

$$\Phi \equiv (\Phi_x, \Phi_y) \quad \text{SO}(2) \times \text{SO}(2) \times \mathbb{Z}_2$$

$$H = \alpha \left[|\Phi_x|^2 + |\Phi_y|^2 \right] + J \left[|\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 \right] \\ + U \left[|\Phi_x|^2 + |\Phi_y|^2 - 1 \right]^2 - \Delta \left[|\Phi_x|^2 - |\Phi_y|^2 \right]^2 + h^* (\Phi_x + \Phi_y) + c.c.$$


$$U \rightarrow +\infty$$

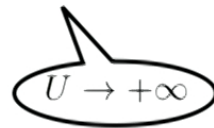
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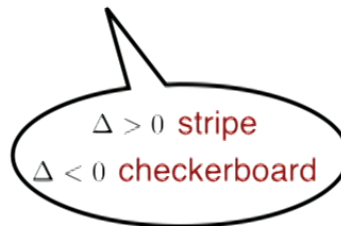
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$$U \rightarrow +\infty$$


$$\Delta > 0 \text{ stripe} \\ \Delta < 0 \text{ checkerboard}$$

CDW + disorder: Short-ranged CDW & long-ranged nematicity

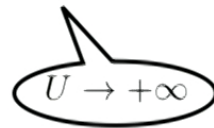
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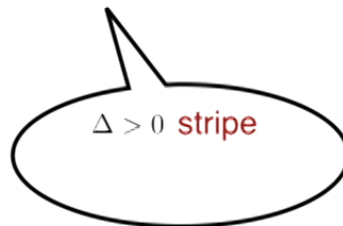
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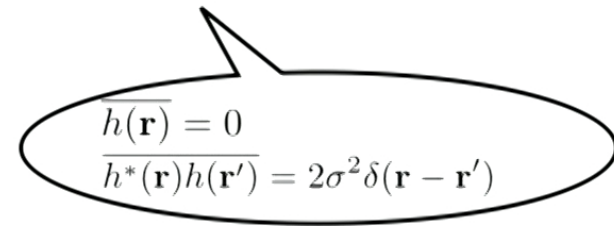
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$U \rightarrow +\infty$



$\Delta > 0$ stripe



$\overline{h(\mathbf{r})} = 0$
 $\overline{h^*(\mathbf{r})h(\mathbf{r}')} = 2\sigma^2\delta(\mathbf{r} - \mathbf{r}')$

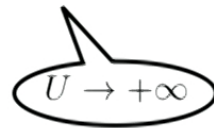
CDW + disorder: Short-ranged CDW & long-ranged nematicity

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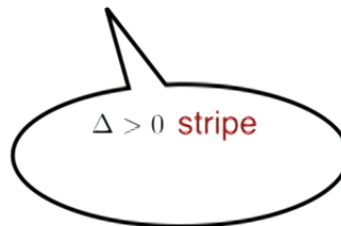
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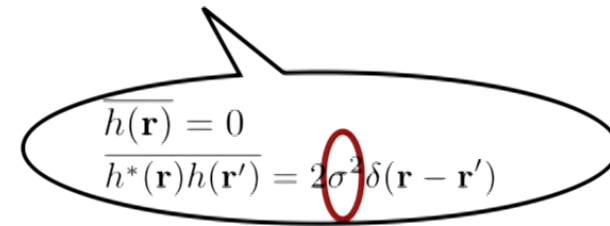
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$U \rightarrow +\infty$

$\Delta > 0$ stripe

Replica trick

$$\overline{h(\mathbf{r})} = 0$$

$$\overline{h^*(\mathbf{r})h(\mathbf{r}')} = 2\sigma^2 \delta(\mathbf{r} - \mathbf{r}')$$

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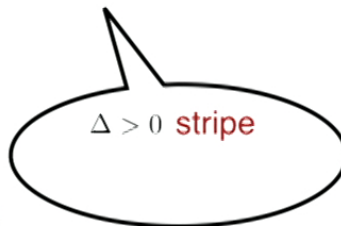
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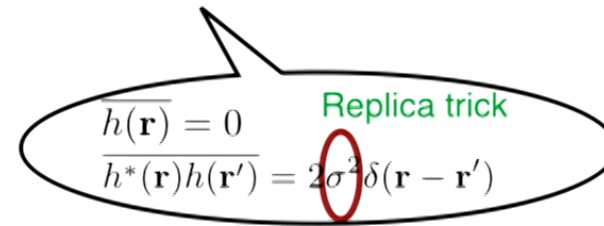


$$U \rightarrow +\infty$$

Hubbard-Stratonovich
large N , saddle point



$$\Delta > 0 \text{ stripe}$$



$$\overline{h(\mathbf{r})} = 0$$

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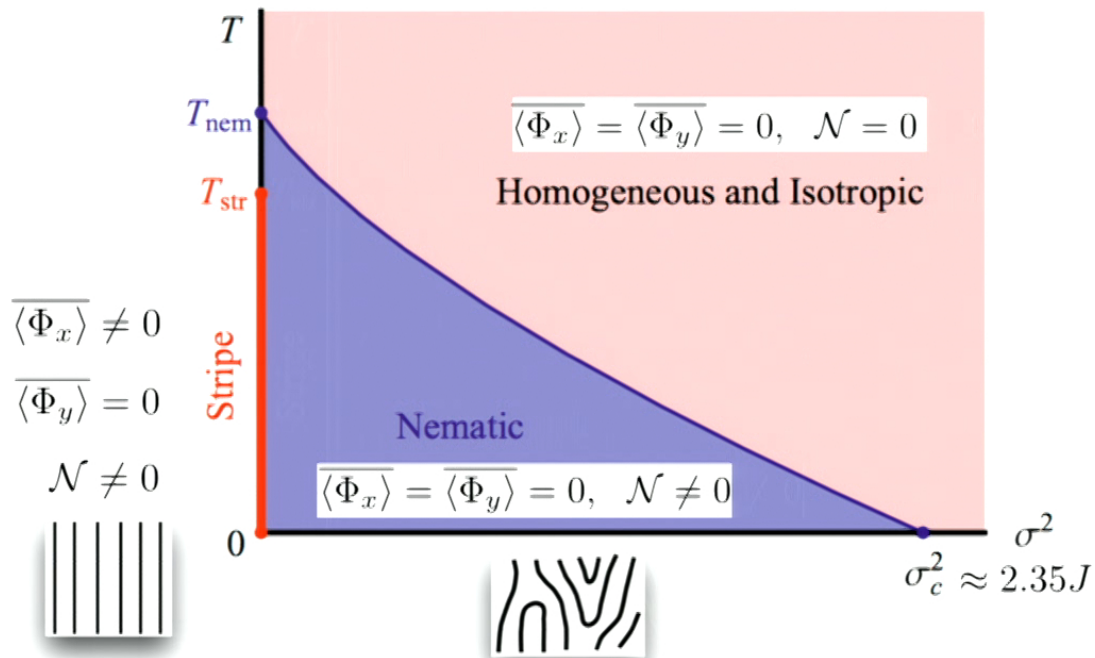
Replica trick

CDW + disorder: Short-ranged CDW & long-ranged nematicity

Phase diagram

Nie, Tarjus, Kivelson PNAS 111, 7980 (2014)

$$\langle \overline{\Phi_x} \rangle, \langle \overline{\Phi_y} \rangle, \mathcal{N} \equiv \langle |\overline{\Phi_x}|^2 \rangle - \langle |\overline{\Phi_y}|^2 \rangle$$

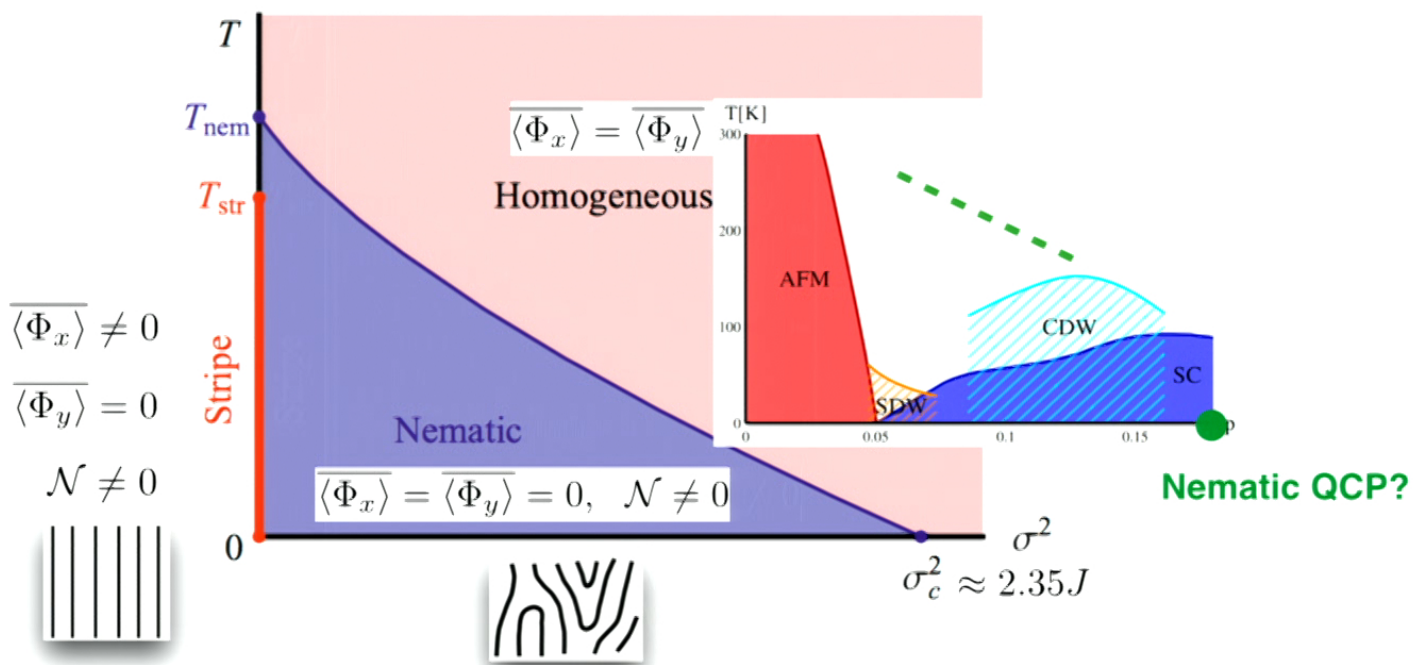


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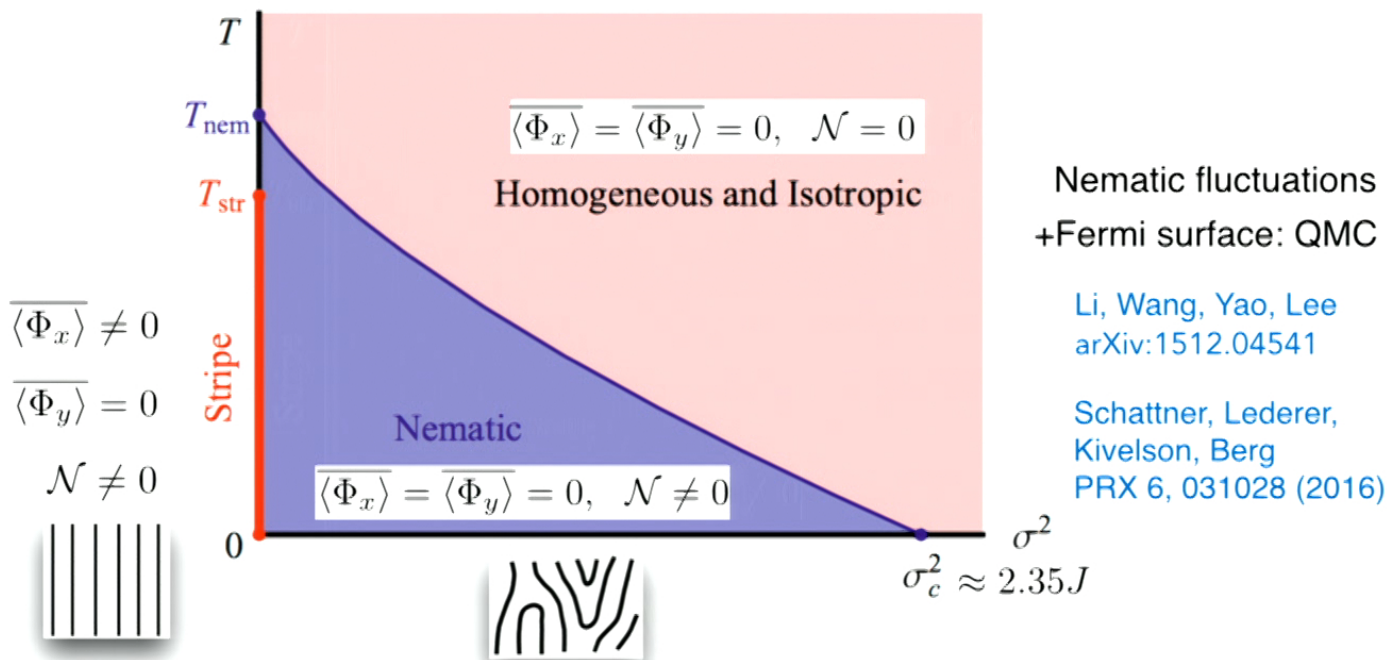


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1. CDW + disorder

- An explicit model of Imry-Ma's argument
- Short-ranged CDW and long-ranged nematicity

2. CDW + SC + disorder

CDW + SC + disorder: The model

Nie, Sierens, Melko, Sachdev, Kivelson, PRB 92, 174505 (2015)

$$\rho(\mathbf{r}) = \bar{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x \cdot \mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y \cdot \mathbf{r}} \right] + c.c. + \dots$$

$$\begin{array}{l} \text{CDW} \quad \left(\Phi_x(\mathbf{r}), \Phi_y(\mathbf{r}) \right) \equiv \Phi(\mathbf{r}) \\ \text{SC} \quad \Psi(\mathbf{r}) \end{array} \quad \text{SO}(2) \times \text{SO}(2) \times \text{SO}(2) \times \text{Z}_2$$

$$H = \alpha \left[|\Phi_x|^2 + |\Phi_y|^2 + |\Psi|^2 \right] + J \left[|\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + |\nabla \Psi|^2 \right] + J_z \left[|\nabla_{\perp} \Phi_x|^2 + |\nabla_{\perp} \Phi_y|^2 + |\nabla_{\perp} \Psi|^2 \right]$$

$$+ U \left[|\Phi_x|^2 + |\Phi_y|^2 + |\Psi|^2 - 1 \right]^2 - \Delta \left[|\Phi_x|^2 - |\Phi_y|^2 \right]^2 + g' |\Psi|^4 + \underline{h^* (\Phi_x + \Phi_y)} + c.c.$$

$$h^* \Psi$$

$$U \rightarrow +\infty$$

$$\Delta > 0 \text{ stripe}$$

CDW + SC + disorder: The model

Nie, Sierens, Melko, Sachdev, Kivelson, PRB 92, 174505 (2015)

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$$U \rightarrow +\infty$$

$$\Delta > 0 \text{ stripe}$$

~~$h^* \Psi$~~ gauge invariance

CDW + SC + disorder: The model

Nie, Sierens, Melko, Sachdev, Kivelson, PRB 92, 174505 (2015)

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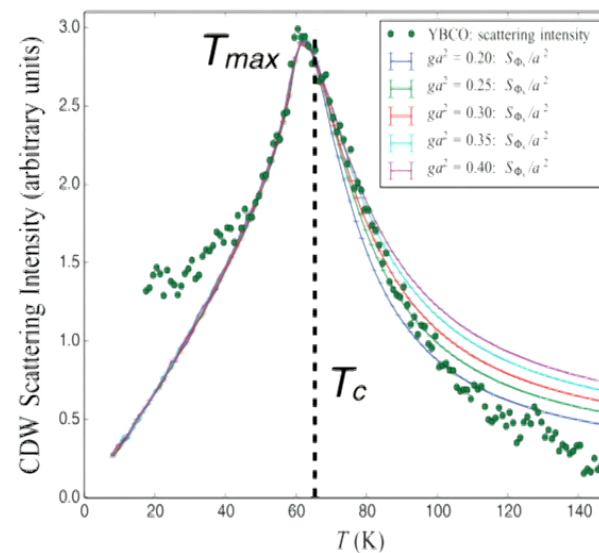
$$\begin{aligned} H = & \alpha \left[|\Phi_x|^2 + |\Phi_y|^2 + |\Psi|^2 \right] + J \left[|\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + |\nabla \Psi|^2 \right] + J_z \left[|\nabla_{\perp} \Phi_x|^2 + |\nabla_{\perp} \Phi_y|^2 + |\nabla_{\perp} \Psi|^2 \right] \\ & + U \left[|\Phi_x|^2 + |\Phi_y|^2 + |\Psi|^2 - 1 \right]^2 - \Delta \left[|\Phi_x|^2 - |\Phi_y|^2 \right]^2 + g' |\Psi|^4 + h^* (\Phi_x + \Phi_y) + c.c. \end{aligned}$$

Replica trick + saddle point ($N = \infty$) & Classical Monte Carlo ($N = 2$)

CDW + SC + disorder: CDW structure factor

$$\rho(\mathbf{r}) = \bar{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x \cdot \mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y \cdot \mathbf{r}} \right] + c.c. + \dots$$

$$S_{\Phi_x}(T) \equiv \overline{\langle \rho(\mathbf{Q}_x) \rho(\mathbf{Q}_x) \rangle} = \overline{\langle \Phi_x^\dagger(\mathbf{k}) \Phi_x(\mathbf{k}) \rangle} \Big|_{\mathbf{k}=0}$$



Monte Carlo:
similar model
2D, no disorder

- ▶ $S_{\Phi_x}(T = 0)$?
- ▶ T_{\max} vs. T_c ?

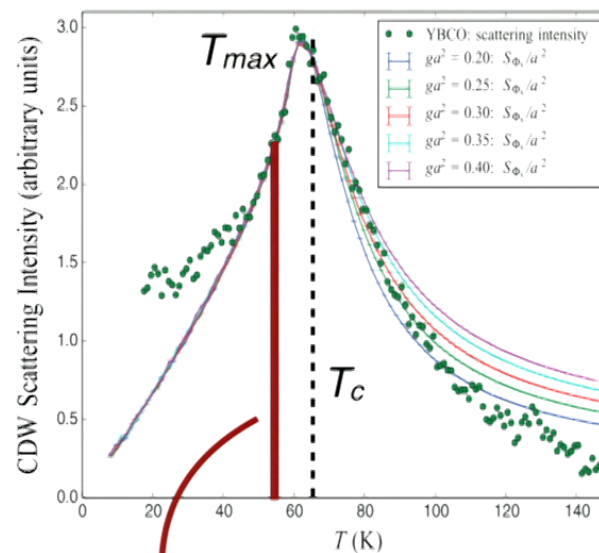
Hayward *et al.*, Science 343, 1336 (2014)

Achkar *et al.*, PRL 113, 107002 (2014)

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Monte Carlo T_c

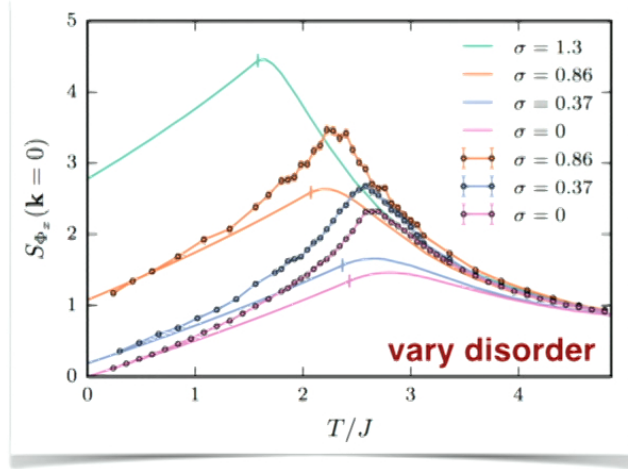
Hayward *et al.*, Science 343, 1336 (2014)

Achkar *et al.*, PRL 113, 107002 (2014)

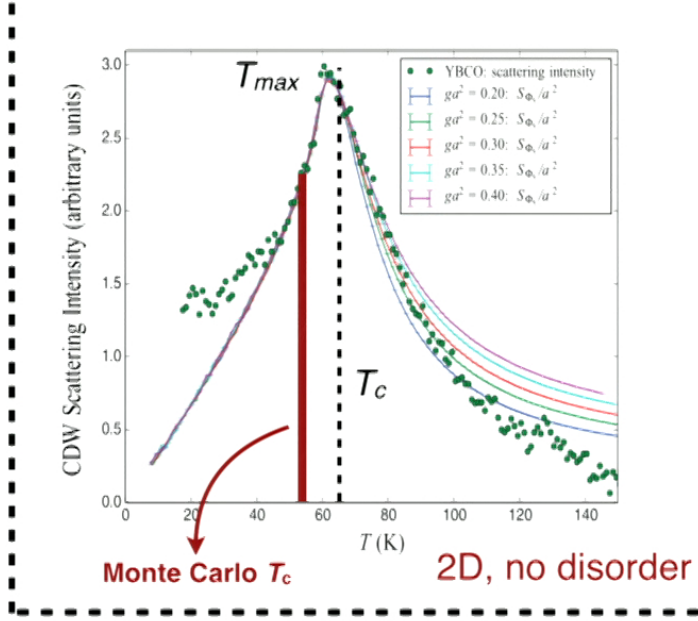
Monte Carlo:
similar model
2D, no disorder

- ▶ $S_{\Phi_x}(T = 0)$?
- ▶ T_{\max} VS. T_c ?

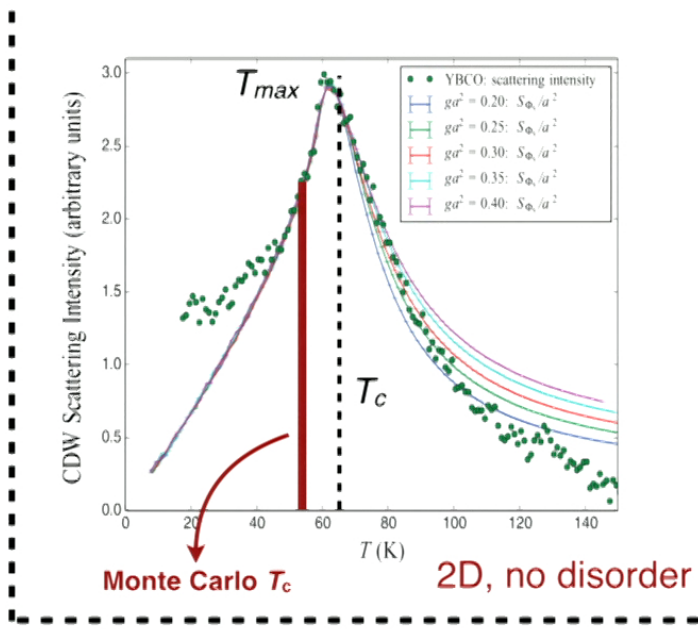
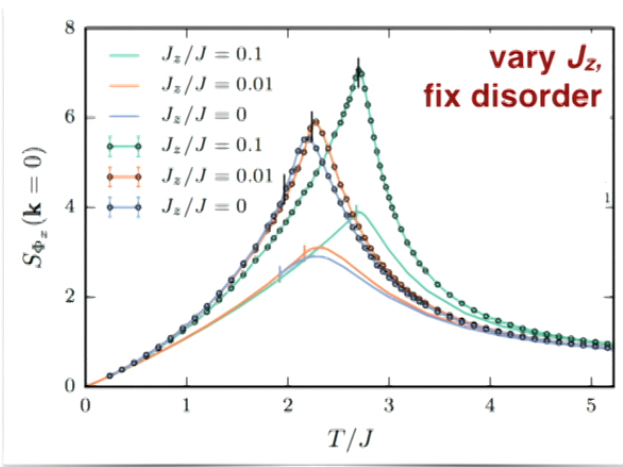
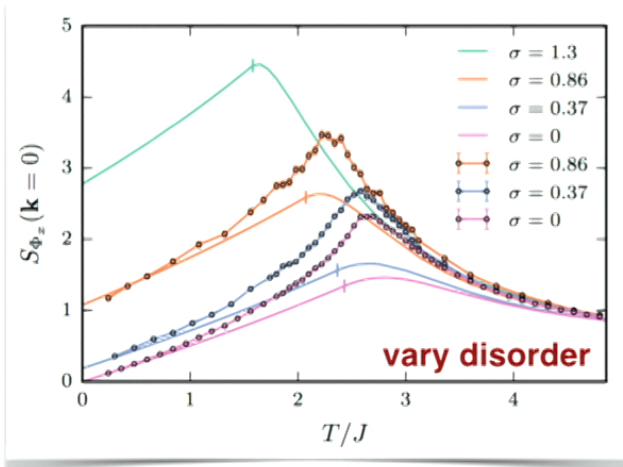
CDW structure factor



increase $\sigma \rightarrow S_{\Phi_x}(T=0) \neq 0$
 $\rightarrow T_{max}$ and T_c closer

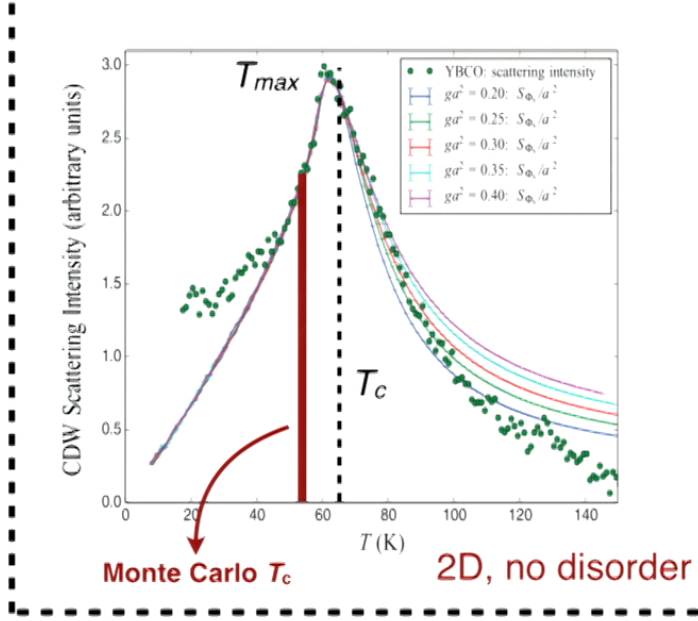
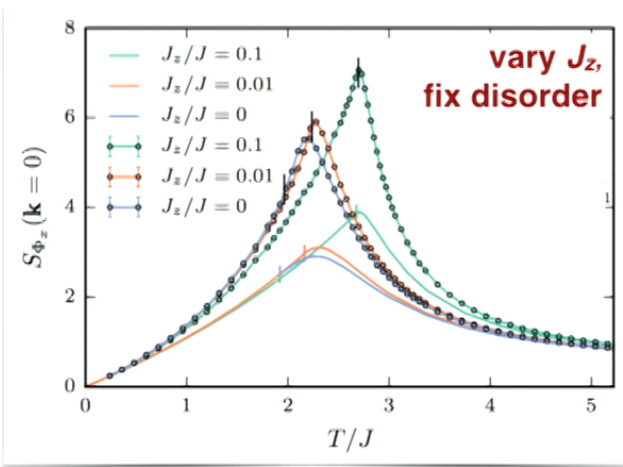
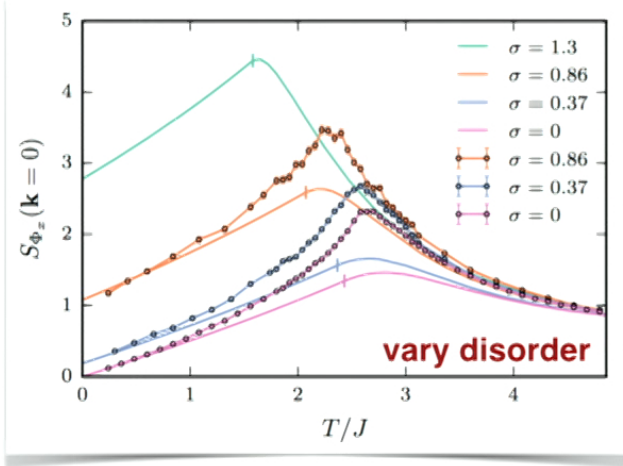


CDW structure factor



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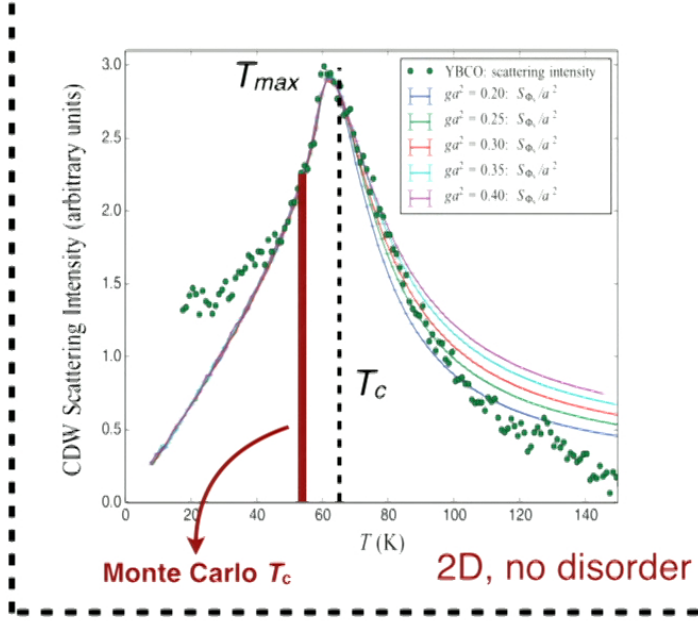
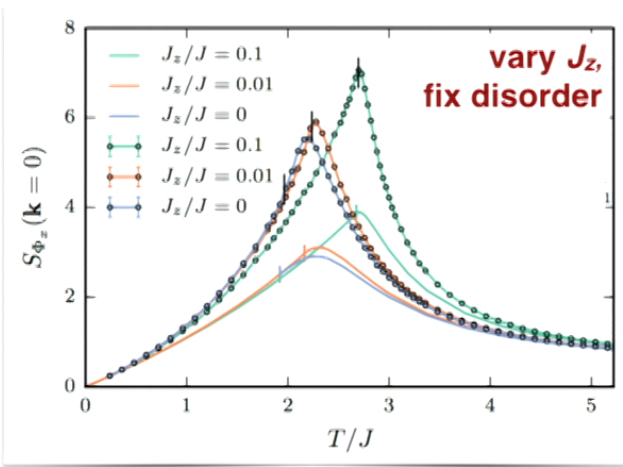
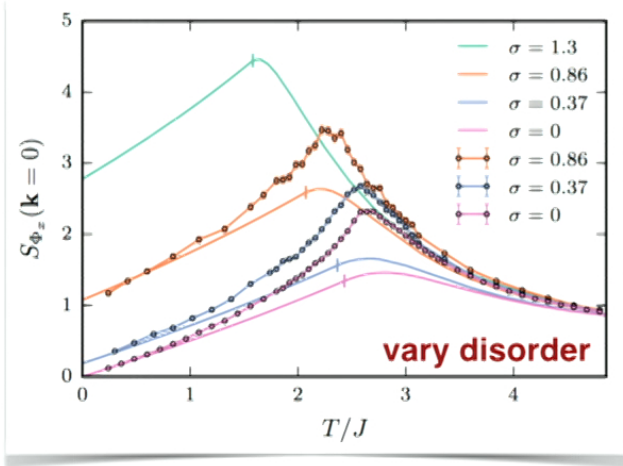
CDW structure factor



increase $\sigma \longrightarrow S_{\Phi_x}(T=0) \neq 0$
 increase $J_z \longrightarrow T_{max}$ and T_c closer

Agreement between large N and Monte Carlo

CDW structure factor



increase $\sigma \longrightarrow S_{\Phi_x}(T=0) \neq 0$
 increase $J_z \longrightarrow T_{max}$ and T_c closer

Agreement between large N and Monte Carlo

2. CDW + SC + disorder

- Thermal evolution of CDW structure factor:
disorder and interlayer coupling are important

CDW + SDW + disorder: The model

Nie, Maharaj, Fradkin, Kivelson
arXiv: 1701.02751

$$\text{CDW} \quad \rho(\mathbf{r}) = \bar{\rho} + \left[\Phi_x(\mathbf{r})e^{i\mathbf{Q}_x \cdot \mathbf{r}} + \Phi_y(\mathbf{r})e^{i\mathbf{Q}_y \cdot \mathbf{r}} \right] + c.c. + \dots$$

$$\text{SDW} \quad \mathbf{S}(\mathbf{r}) = \mathbf{S}_x(\mathbf{r})e^{i\mathbf{K}_x \cdot \mathbf{r}} + \mathbf{S}_y(\mathbf{r})e^{i\mathbf{K}_y \cdot \mathbf{r}} + c.c. + \dots$$

$$H = H_{\mathbf{S}}[\mathbf{S}_x, \mathbf{S}_y] + H_{\Phi}[\Phi_x, \Phi_y] + H_{\text{int}}[\mathbf{S}_x, \mathbf{S}_y, \Phi_x, \Phi_y]$$

$$H_{\Phi} = \alpha_{\Phi} \left[|\Phi_x|^2 + |\Phi_y|^2 \right] + J_{\Phi} \left[|\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 \right] + U_{\Phi} \left[|\Phi_x|^2 + |\Phi_y|^2 \right]^2 + \gamma_{\Phi} |\Phi_x|^2 |\Phi_y|^2$$

$$+ \left[h^* (\Phi_x + \Phi_y) + c.c. \right] \quad h |\mathbf{S}|^2$$

$$H_{\mathbf{S}} = \alpha_s \left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2 \right] + J_s \left[|\nabla \mathbf{S}_x|^2 + |\nabla \mathbf{S}_y|^2 \right] + U_s \left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2 \right]^2 + \gamma_s |\mathbf{S}_x|^2 |\mathbf{S}_y|^2$$

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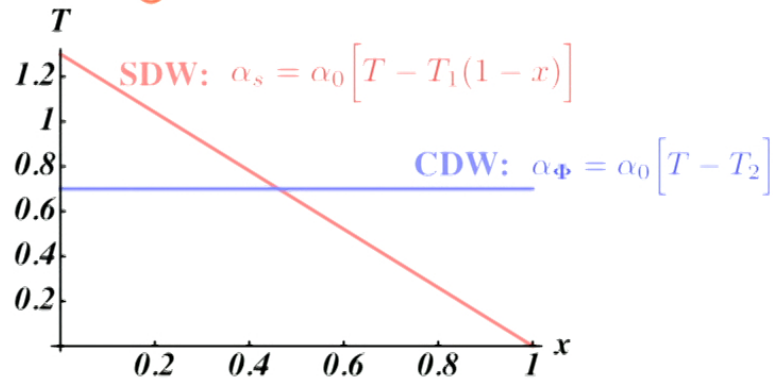
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CDW + SDW + disorder: The model

Nie, Maharaj, Fradkin, Kivelson
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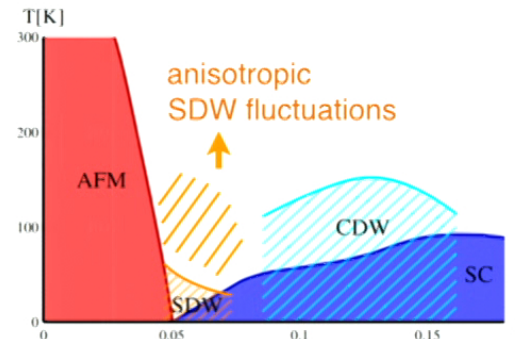
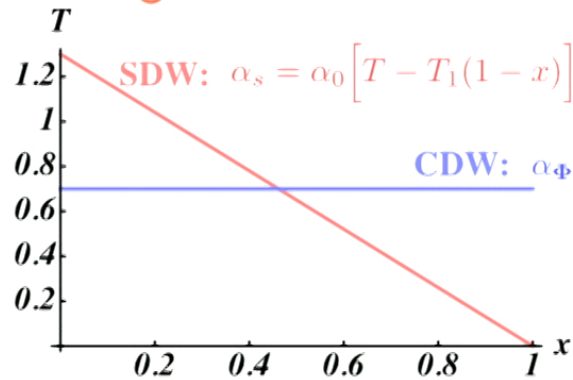
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CDW + SDW + disorder: The model

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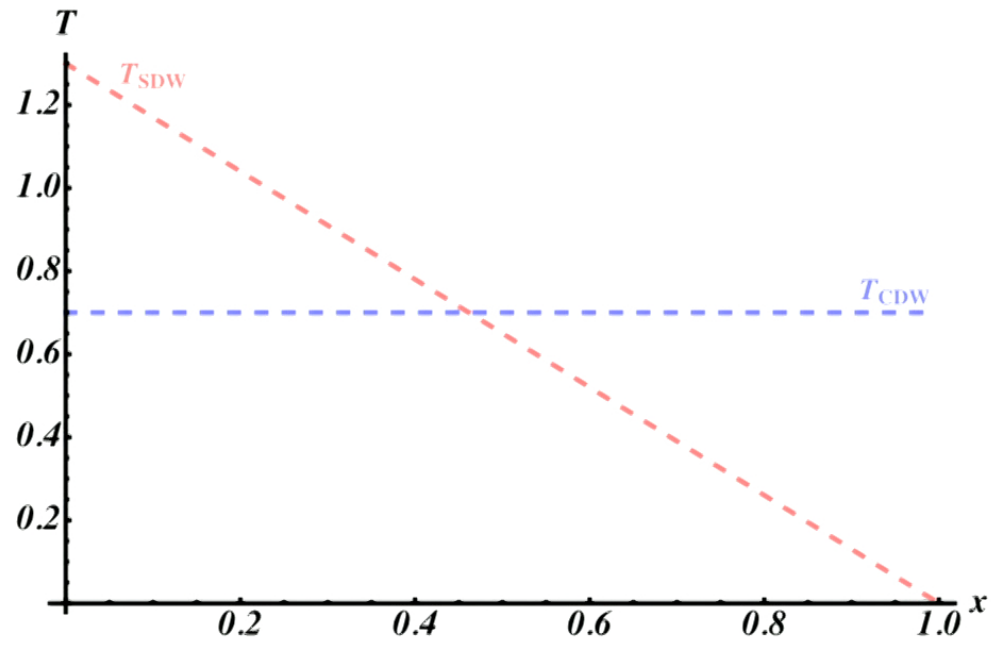
$$H_\Phi = \alpha_\Phi \left[|\Phi_x|^2 + |\Phi_y|^2 \right] + J_\Phi \left[|\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 \right] + U_\Phi \left[|\Phi_x|^2 + |\Phi_y|^2 \right]^2 + \gamma_\Phi |\Phi_x|^2 |\Phi_y|^2 \\ + \left[h^*(\Phi_x + \Phi_y) + c.c. \right]$$

$$H_S = \alpha_s \left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2 \right] + J_s \left[|\nabla \mathbf{S}_x|^2 + |\nabla \mathbf{S}_y|^2 \right] + U_s \left[|\mathbf{S}_x|^2 + |\mathbf{S}_y|^2 \right]^2 + \gamma_s |\mathbf{S}_x|^2 |\mathbf{S}_y|^2$$

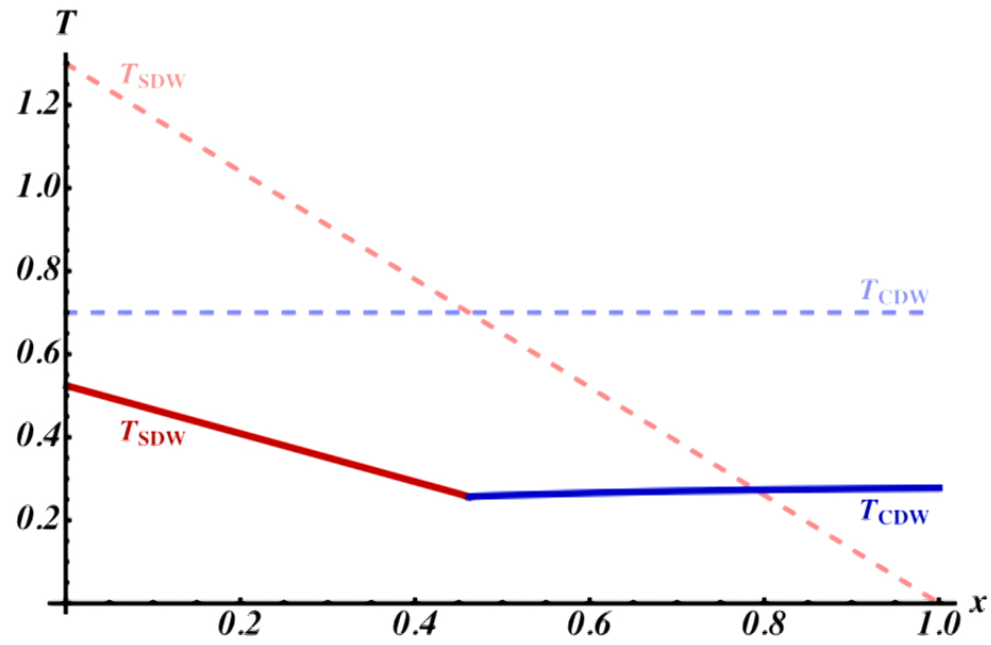
$$H_{\text{int}} = v \left[|\Phi_x|^2 |\mathbf{S}_x|^2 + |\Phi_y|^2 |\mathbf{S}_y|^2 \right] + w \left[|\Phi_x|^2 |\mathbf{S}_y|^2 + |\Phi_y|^2 |\mathbf{S}_x|^2 \right]$$

$v > w$: SDW \perp CDW

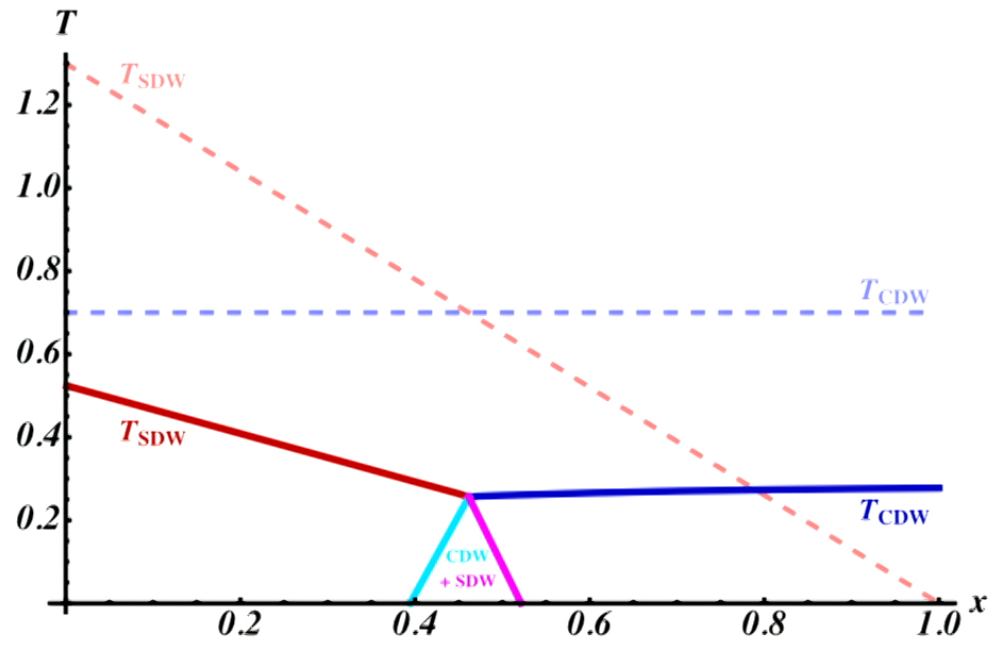
Result: zero disorder ($\sigma = 0$)



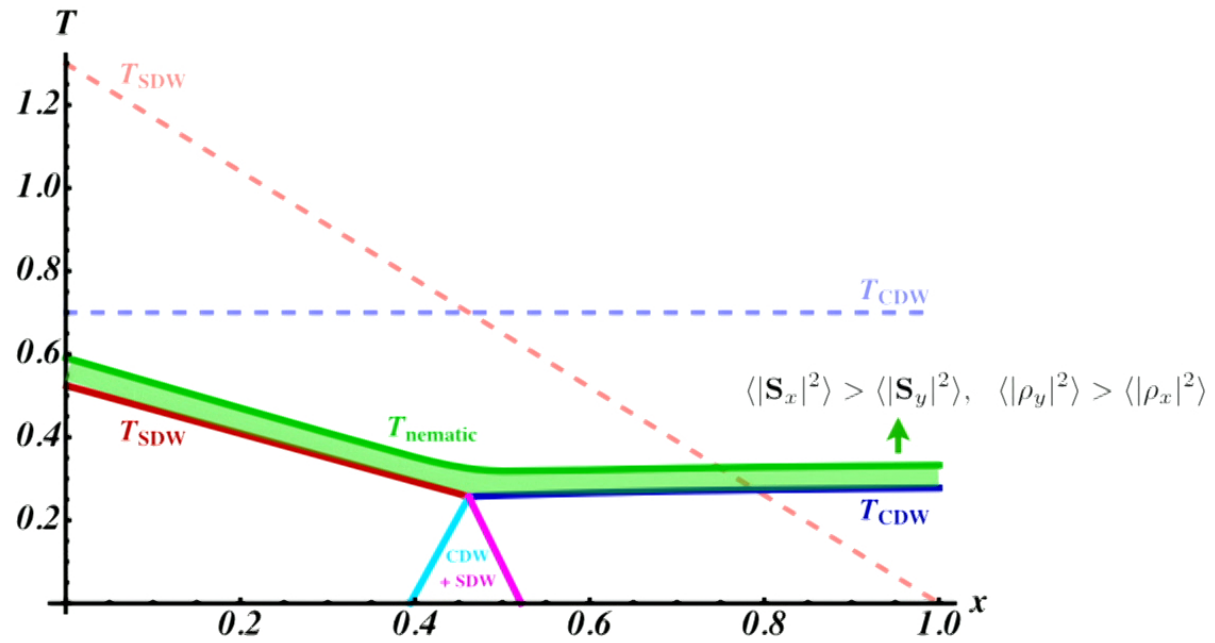
Result: zero disorder ($\sigma = 0$)



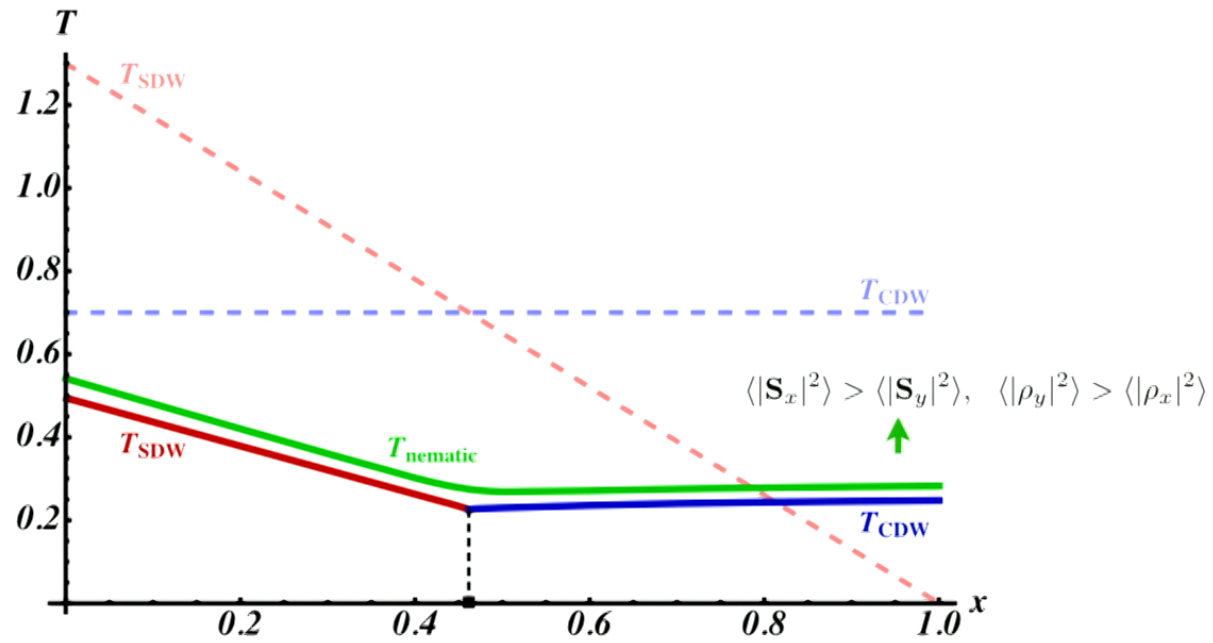
Result: zero disorder ($\sigma = 0$)



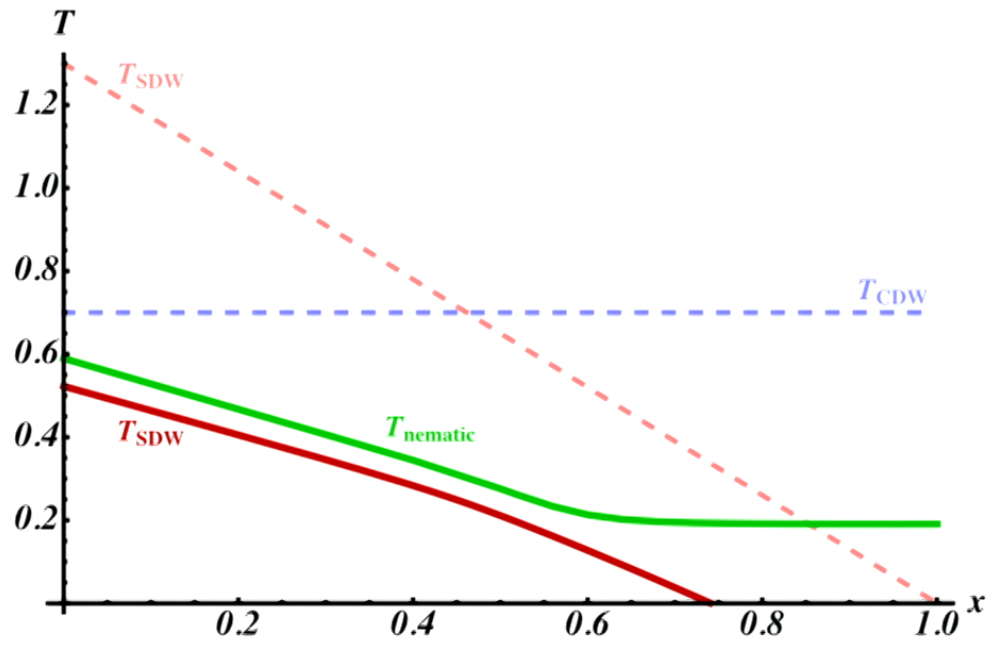
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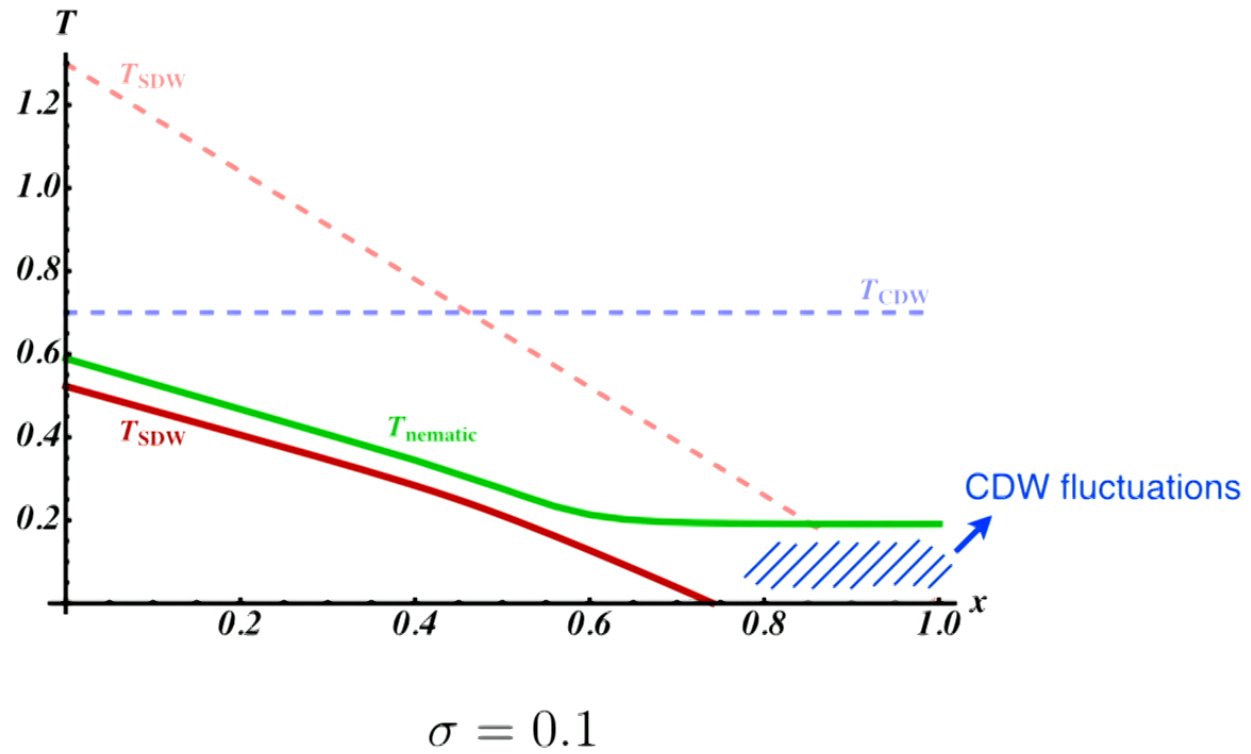


Result: finite disorder ($\sigma \neq 0$)

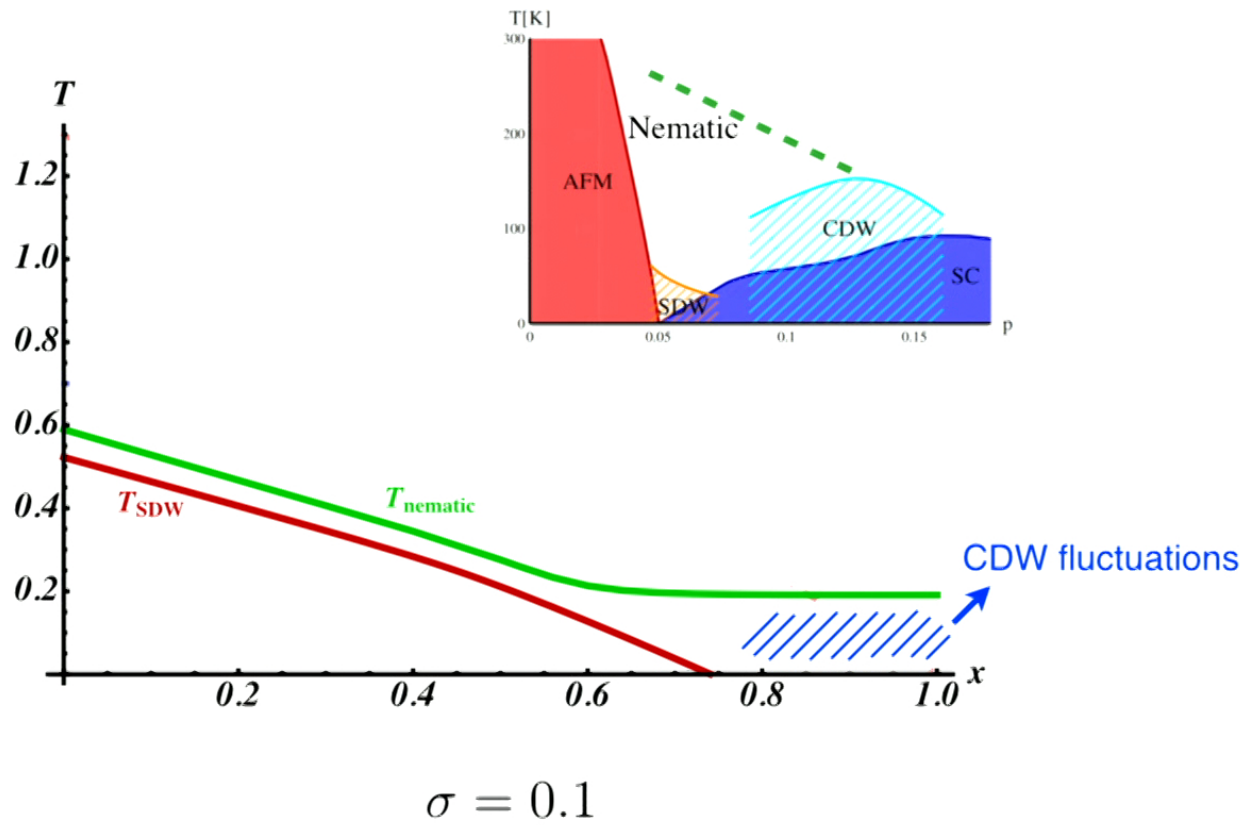


$\sigma = 0.1$

Result: finite disorder ($\sigma \neq 0$)



Result: finite disorder ($\sigma \neq 0$)



3. CDW + SDW + disorder

- A universal nematic transition across the entire doping range, from partial melting of density waves
(thermal & random-field disorder for CDW)

Summary

1. CDW + disorder

Short-ranged CDW and long-ranged nematicity

2. CDW + SC + disorder

Disorder and interlayer coupling (dimensionality)

3. CDW + SDW + disorder

One nematic phase, driven by CDW and SDW