Title: Something New Under (and in) the Sun? Effective field theories for point sources and the Hydrogen atom

Date: Jan 10, 2017 01:00 PM

URL: http://pirsa.org/17010063

Abstract: <p>This talk applies effective field theory to the back-reaction of sources with finite size but infinite mass. The main tool for calculating back-reaction is a general relation between a source's effective action and the boundary conditions of `bulkâ€™ fields in the near-source limit. As applied to the Maxwell (or Einstein) fields for point sources this boundary condition reproduces standard Gaussâ€™ Law expressions, but the same arguments imply source-dependent boundary conditions for the Schrodinger (or Dirac) field of an orbiting particle. As applied to the quantum mechanics of a particle interacting with a source through an inverse-square potential EFTs remove the guess-work from the (well-known) ambiguities in the determination of boundary conditions at the origin, and provides a simple interpretation of the classical renormalization effects that are known to arise in this case. EFT arguments show why the RG evolution associated with this classical renormalization is likely universal for a great many types of point sources. The EFT boundary conditions also modify how finite-size effects alter bound-state orbits in the Coulomb problem, and in particular give them a non-standard dependence on the mass of the orbiting particle. It is argued that this might provide a solution to the `proton-radius puzzleâ€™, in which experiments seem to indicate a different proton radius depending on whether or not it is measured using electrons or muons.</p>

Outline

(0) Intro

(1) PPEFTs
   - definition
   - boundary cond^n/s

(2) QM of the $1/r^2$ potential

(3) H atom * - Schrödinger
            - Relativistic

the potential

$$\xleftarrow{\qquad\qquad} a \xrightarrow{\qquad\qquad\qquad}$$

③ H atom* — Schrödinger
         — Relativistic

$$M \gg \frac{1}{\ell} \gg \frac{1}{a}.$$

$$\frac{1}{\ell} > m \qquad \frac{z\alpha}{\ell} > m$$

$$\ell < (z\alpha)^2 a_B$$

Because $M \to \infty$

source is static: "bulk fields"
interacting with the source's CM
coordinate.

"bulk fields": A_μ, ψ

$$S_{eff} = S_B + S_P$$

$$-i\psi^* \partial_t \psi \; -\frac{1}{2m}(\nabla\psi)^*(\nabla\psi) - V(x)\psi^*\psi$$

$$S_B = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - (\partial_\mu\phi^*)(\partial^\mu\phi) - m^2\phi^*\phi \right.$$

$$S_P = S_P(A_\mu, \psi, y^\mu(\tau)) \qquad y^\mu = \begin{pmatrix} \tau \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \dot{y}^\mu = \delta^\mu_0$$

$$= \int d\tau \int d^4x \; \delta^4(x - y(\tau)) \, \mathcal{L}_P(A, \psi, \dot{y})$$

$$\mathcal{L}_P = -\sqrt{-\gamma} \, M + q \, A_\mu \dot{y}^\mu$$

$$S_p = S_p(A_\mu, \psi, y^\mu(\tau)) \qquad y^\mu = \begin{pmatrix} \tau \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \dot{y}^\mu = \delta_0^\mu$$

$$= \int d\tau \int d^4x \; \delta^4(x - y(\tau)) \, \mathcal{L}_p(A, \psi, g)$$

$$\mathcal{L}_p = -\sqrt{-\gamma} \; M^{\xrightarrow{\infty}} + q A_\mu \dot{y}^\mu + \dots \quad (\text{point ptcl}_o)$$

$$(\gamma = g_{\mu\nu} \dot{y}^\mu \dot{y}^\nu)$$

$$V_{eff} = V + h \, \delta^3(x)$$
$$(\text{Schrödinger})$$

$$S_p = \int d^4x \left\{ q \left[1 + \tfrac{1}{6} r_p^2 \nabla^2 \right] A_0 + \dots - h \, \psi^* \psi + \dots \right\} \delta^3(x)$$
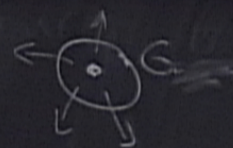
$$\rho = \frac{\delta S_p}{\delta A_0} = q \left[1 + \tfrac{1}{6} r_p^2 \nabla^2 \right] \delta^3(x) \qquad \text{"charge radius"}$$

(3) H atom* — Schrödinger
            — Relativistic

$$M \gg \frac{1}{e} \gg \frac{1}{a}$$

Source contributes to the Maxwell eq.

$$\nabla \cdot E = -e\psi^*\psi + q\,\delta^3(x)$$

$$\oint_G \nabla \cdot E = \oint E \cdot n\, dS = \int_{G \text{int.}} d^3x \left[e\psi^*\psi\right] + q$$

$$E = \nabla A_0$$

$$A_0 = \frac{k}{r} \qquad \left(r^2 \frac{\partial A_0}{\partial r}\Big|_{r \to \infty} = q \longrightarrow A_0 = \frac{q}{4\pi} \right)$$

Ditto for $\mathcal{L}_p = -h \psi^* \psi$

$$i\partial_t \psi + \frac{1}{2m} \nabla^2 \psi - V\psi - h \delta^3(x))\psi = 0.$$

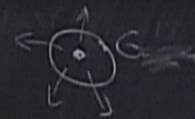Int over G: $\oint_G n \cdot \nabla \psi = 2mh \psi_{(0)}$

(1)

(2) QM of the $1/r^2$ potential

(3) H atom* — Schrödinger
— Relativistic

$$M \gg \frac{1}{e} \gg \frac{1}{a}$$

Source contributes to the Maxwell eq:

$$\nabla \cdot E = -e\psi^*\psi + q\,\delta^3(x)$$

$$\oint_S \nabla \cdot E = \oint_S E \cdot n \, dS = \int d^3x \left(e\psi^*\psi\right) + q$$

$$A_0 = \frac{k}{r} \qquad \left(r^2 \frac{\partial A_0}{\partial r}\bigg|_{r \ll} = q \longrightarrow A_0 = \frac{q}{4\pi r}\right)$$

$$E = \nabla A_0$$

$$\rho = \frac{\delta S_\rho}{\delta A_0} = q\left[1 + \frac{1}{6}r_\rho^2 \nabla^2\right]\delta^3(x) \qquad \text{"charge radius"}$$

Ditto for $\mathcal{L}_\rho = -h\psi^*\psi$

$$i\partial_t\psi + \frac{1}{2m}\nabla^2\psi - V\psi - h\,\delta^3(x)\psi = 0.$$

Int over 6: $\oint_6 \hat{n}\cdot\nabla\psi = 2mh\,\psi_{(0)}$

$$\left(4\pi r^2 \frac{\partial}{\partial r}\ln\psi\right)_{r=\epsilon} = 2mh = \lambda$$

$I_B(r=\epsilon)$

$$\frac{\delta S_B}{\delta\psi} + \frac{\delta I_B}{\delta\psi} = 0$$

wrt $\delta\psi$ set $\downarrow$ in bulk
$\underline{=}$ on boundary

$$S_p = \int d^4x \left\{ q\left[1 + \tfrac{1}{6}r_p^2 \nabla^2\right]A_0 + \cdots - h\,\psi^*\psi + \cdots \right\} \delta^3(x) \quad \text{(Schrödinger)}$$

$$\rho = \frac{\delta S_p}{\delta A_0} = q\left[1 + \tfrac{1}{6}r_p^2 \nabla^2\right]\delta^3(x) \qquad \text{"Charge radius"}$$

Diff. for $\mathcal{L}_p = -h\,\psi^*\psi$

$$i\partial_t \psi + \tfrac{1}{2m}\nabla^2\psi - V\psi - h\,\delta^3(x)\psi = 0.$$

Int over G: $\oint_G n\cdot\nabla\psi = 2mh\,\psi(0)$

$$\left(4\pi r^2 \frac{\partial}{\partial r}\ln\psi\right)_{r=\epsilon} = 2mh = \lambda$$

$I_B(r=\epsilon)$

$$\frac{\delta S_B}{\delta\psi^*} + \frac{\delta I_B}{\delta\psi^*} = 0$$

wrt $\delta\psi$ sett in bulk

$\pm$ on boundary

(2) QM of the $1/r^2$ potential

(3) H atom*   - Schrödinger

         - Relativistic

$$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

a

NR

QM of the $\frac{1}{r^2}$ potential

$$-\frac{\nabla^2}{2m}\psi - \frac{g}{r^2}\psi + h\delta^3(x)\psi = E\psi$$

$$\frac{1}{r^2}\partial_r\left(r^2 \frac{\partial\psi}{\partial r}\right) \qquad \frac{\ell(\ell+1)}{2mr^2}$$

Self-adjoint extension

$$\nu(\nu+1) = \ell(\ell+1) - 2mg$$

for small nonzero g $\nu \gtrsim 0$ but small

if $g=0$ then $R_+(r) \simeq r^\nu$ $R = C_+ R_+ + C_- R_-$

$R_-(r) \simeq r^{-\nu-1}$ Both $R_+, R_- \to \infty$ $r \to 0$

QM of the $1/r^2$ potential

(3) H atom* — Schrödinger
— Relativistic

$$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

QM of the $\frac{1}{r^2}$ potential.

$$\nu(\nu+1) = \ell(\ell+1) - 2mg$$

$$-\frac{\nabla^2}{2m}\psi - \frac{g}{r^2}\psi + h\,\delta^3(x)\psi = E\psi.$$

$r=0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right)$$

$$\frac{\ell(\ell+1)}{2mr^2}$$

for small nonzero g $\nu \approx 0$ but small.

if $g=0$ then $R_+(r) \simeq r^{\nu}$ $R = C_+ R_+ + C_- R_-$

$R_-(r) \simeq r^{-\nu-1}$ Both $R_+, R_- \to \infty$ $r \to 0$

Self-adjoint extension

$$\oint_6 \hat{n} \cdot \nabla \psi = 2mh \psi_{(6)}$$

$$\left(4\pi r^2 \frac{\partial}{\partial r} \ln \psi \right)_{r=6} = 2mh = \lambda$$

w.r.t. $\delta \psi$ Self in bulk
$\pm$ on boundary

$2mg > 1/4$ ∞ # of bound states.

$-\frac{3}{4} < 2mg < 1/4$ $\Big\langle$ either no bound states
$\alpha 1$ bound state. $\leftarrow$ delta-fn state.

$$\oint_\delta \hat{n} \cdot \nabla \psi = 2mh\, \psi_{(\omega)}$$

$$\left(4\pi r^2 \frac{\partial}{\partial r} \ln \psi \right)_{r=\epsilon} = 2mh = \lambda$$

wrt $\delta\psi$; Set ψ in bulk
$\pm$ on boundary

E_6 depends on ϵ.

Bound. cond $\quad 4\pi r^2 \frac{\partial}{\partial r} \ln \psi = 2mh = \lambda.$

$\quad R = C_+ R_+ + C_- R_- \qquad$ has $C_- \neq 0$ if $\lambda \neq 0.$

Int over G: $\oint_G n \cdot \nabla \psi = 2mh \, \psi_{(0)}$

$\left(4\pi r^2 \frac{\partial}{\partial r} \ln \psi \right)_{r=\epsilon} = 2mh = \lambda$

wrt $\delta\psi$ s.th in bulk
$\pm$ on boundary

E_G depends on ϵ.

Bound. cond $\boxed{4\pi r^2 \frac{\partial}{\partial r} \ln \psi} = 2mh = \lambda$.

$R = C_+ R_+ + C_- R_-$ has $C_- \neq 0$ if $\lambda \neq 0$.

bc $\Rightarrow$ series in $\frac{mh}{\epsilon}$ need not to small

and $2mg$

Bound-state energies:

$\xrightarrow{r\to\infty} \psi \to$ normalizable @ ∞

$\xrightarrow{r=0} \frac{C_-}{C_+} = f(params, \kappa) = \propto \lambda, \epsilon$

$\frac{\partial}{\partial \epsilon} \left[\; \right]_{k,..} = 0 \longrightarrow \lambda(\epsilon)$.

QM of the $1/r^2$ potential

③ H atom* — Schrödinger
           — Relativistic

$$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

a

$$\hat{\lambda} = \frac{\lambda}{2\pi\epsilon} + 1 \qquad \left(\hat{\lambda} = 1 \iff \lambda = h = 0\right) \qquad V = -g/r^2$$

$$\boxed{\epsilon \frac{\partial \hat{\lambda}}{\partial \epsilon} = \frac{s}{2}\left(1 - \left(\frac{\hat{\lambda}}{s}\right)^2\right)}$$

$$s = \sqrt{1 - 8mg}$$

$R_+ \sim r^a$

$R_- \sim r^b$ $a - b = s$

"H atom" — Schrödinger
— Relativistic

$$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

Relevance to H:

— Schrödinger case: $V = -\frac{Z\alpha}{r} + h\,\delta^3(x)$

$$\boxed{\frac{\delta E}{E} = 4\frac{a_s}{n\,a_B}}$$ Deser formula

$h(\epsilon)$ or $\epsilon_* \leftrightarrow \delta E_n$ (S-wave) and $a_s\,(\pi^- p)$
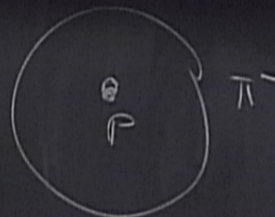
KG atom: $\langle 0|T(\phi^*(x)\,\phi(y))|0\rangle$

$u = \langle 0|\phi(x)|n\rangle$

$(D_\mu D^\mu - m^2)u = 0 \qquad u \approx e^{-i\omega t}$

$4\pi r^2 \dfrac{\partial}{\partial r}\ln\psi = \lambda = 2\omega h$

$E_B \approx \dfrac{f(n)}{\epsilon_*}$ ⟵ $= \dfrac{f(n, \lambda(\epsilon))}{\epsilon}$

KG atom: $\langle 0 | T(\phi^*(x)\, \phi(x)) | 0 \rangle$

$D_0^2 = \left(\omega + \dfrac{Z\alpha}{r}\right)^2$

$\langle 0 | \phi(x) | n \rangle$

$\omega^2 = -k^2 + m^2$

Radial eq

$\left(\nabla^2 - \dfrac{2\omega Z\alpha}{r} - \dfrac{(Z\alpha)^2}{r^2}\right) u = k^2 u$

$-m^2) u = 0 \qquad u \approx e^{-i\omega t}$

$4\pi r^2 \dfrac{\partial}{\partial r} \ln \psi = \lambda = Z\omega h$

$$E_B \approx \frac{f(n)}{\epsilon_*} \longleftarrow \cdot = \frac{f(n, \lambda(\epsilon))}{\epsilon}$$
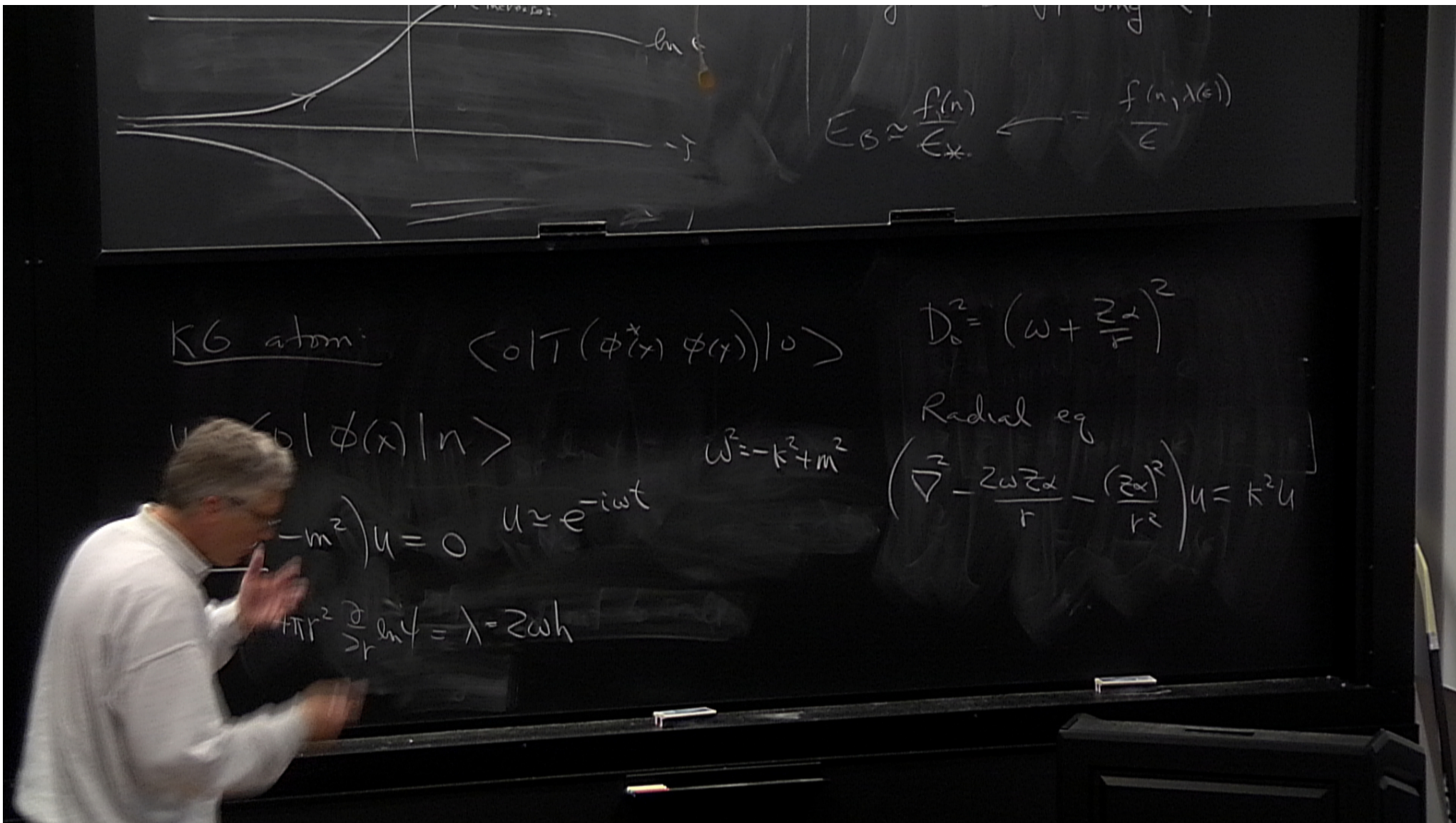
KG atom:

$$\langle 0 | T(\phi^\dagger(x) \phi(y)) | 0 \rangle$$

$$D_0^2 = \left(\omega + \frac{Z\alpha}{r}\right)^2$$

$$u = \langle 0 | \phi(x) | n \rangle$$

$$\omega^2 = -k^2 + m^2$$

Radial eq

$$\left(D_\mu D^\mu - m^2\right) u = 0 \qquad u \approx e^{-i\omega t}$$

$$\left(\nabla^2 - \frac{2\omega Z\alpha}{r} - \frac{(Z\alpha)^2}{r^2}\right) u = k^2 u$$

$$4\pi r^2 \frac{\partial}{\partial r} \ln \psi = \lambda - 2\omega h \qquad 2mg = (Z\alpha)^2$$

$$r = 0 \quad (RL)$$

$$g \neq 0 \quad \bar{s} = \sqrt{1 - 8mg} < 1$$
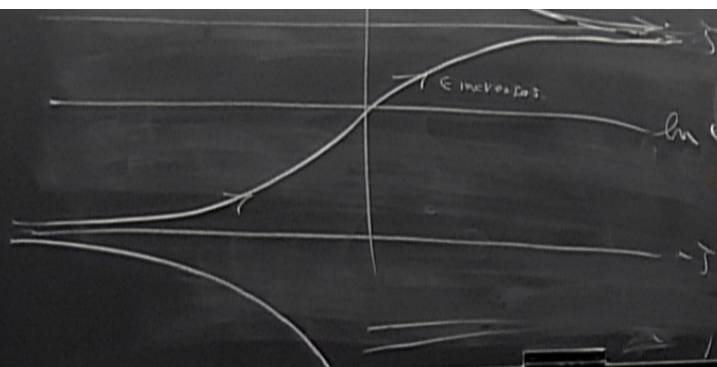
$$E_B \approx \frac{f(n)}{\epsilon_*} \longleftarrow \quad = \frac{f(n, \lambda(\epsilon))}{\epsilon}$$

$$\frac{mZ\alpha}{r} \text{ loses to } \frac{(Z\alpha)^2}{r^2}$$

when $r < (Z\alpha)/m = (Z\alpha)^2 a_B$

states

ound states

state. ← delta-fn state.

"H atom" — Schrödinger
         — Relativistic

$$M \gg \frac{1}{l} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

Relevance to H:

...inger case: $V = -\frac{Z\alpha}{r} + h \delta^3(x)$

$= \frac{4 a_s}{n \, a_B}$ Deser formula $h(\epsilon)$ or

(S-wave)

$$\frac{\delta E_n}{|E_n|} = -2m^2 \epsilon \left(\frac{Z\alpha}{n}\right)^3 \left(\frac{\varsigma - \lambda}{\varsigma + \lambda}\right)$$

$$\varsigma = \sqrt{1 - 4(Z\alpha)^2}$$

$$= +2m^2 \epsilon_\times \left(\frac{Z\alpha}{n}\right)^3 \qquad \left(\begin{array}{c} \epsilon_\times \\ \lambda \to \infty \end{array}\right)$$

$$\sim \epsilon_0 m^2 (Z\alpha)^5$$

liams.

ϵ increases.

$\ln \epsilon$

$g \neq 0 \quad j = \sqrt{1 - 8mg} < 1$

$\epsilon_B \approx \dfrac{f(n)}{\epsilon_*} \longleftarrow = \dfrac{f(n, \lambda(\epsilon))}{\epsilon}$

$\delta E \approx h_{eff} |\psi^c(0)|^2$

$h_{eff} = 2\pi m^2 \epsilon_* = 4\pi m^2 a_s$

$\left(\dfrac{\partial \psi}{\partial r}\right)_\epsilon \sim \lambda \psi \qquad \lambda = 2mh.$

h_ϵ at ϵ. $r \approx \dfrac{1}{m}$

liams.

ϵ increases.

$\ln \epsilon$

$g \neq 0 \quad \bar{J} = \sqrt{1 - 8mg} < 1$

$\epsilon_B \approx \dfrac{f(n)}{\epsilon_*} \longleftarrow = \dfrac{f(n, \lambda(\epsilon))}{\epsilon}$

$\delta E \approx h_{eff} |\psi^c(0)|^2$

$\left(\dfrac{\partial \psi}{\partial r}\right)_\epsilon \sim \lambda \psi \qquad \lambda = 2mh.$

$h_{eff} = 2\pi m^2 \epsilon_* = 4\pi m^2 a_s.$

$\nabla \cdot E = -e \psi^* \psi + Ze \delta^3(x)$

$h_\epsilon \neq \epsilon. \quad \beta \simeq \dfrac{1}{m}$

$\mathcal{L}_p = -h_\epsilon \psi^* \psi - \dfrac{Ze r_p^2}{6} \nabla \cdot E$

$h_{tot} = h_\epsilon + \dfrac{1}{6} Ze^2 r_p^2$

(2) QM of the $1/r^2$ potential

(3) H atom * — Schrödinger
            — Relativistic

$$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

$$h_{\text{eff}} = \frac{h_c + \frac{2\pi}{3} Z\alpha r_p^2 + 2\pi \epsilon (Z\alpha)^2/m}{1 + \left(\frac{m h_c}{2\pi\epsilon}\right) + \frac{1}{3} Z\alpha m r_p^2/\epsilon}$$

ϵ (fm)	r_p	$B = 6 m h_c / Z e^2$
1	0.8481	3.089
5	0.8478	15.49
10	0.8478	30.99

(S-wave)

$$\delta E_n = -2m^2 \epsilon \left(\frac{Z\alpha}{n}\right)^3 \left(\frac{s-1}{s+1}\right)$$

$$s = \sqrt{1 - 4(Z\alpha)^2} \quad \left| \psi^{(0)}(0) \right|^2 = \frac{1}{\pi}\left(\frac{Z\alpha m}{n}\right)^3$$

$$= +2m^2 \epsilon_x \left(\frac{Z\alpha}{n}\right)^3 \left(\epsilon_x\right)_{\lambda \to \infty}$$

$$\sim \epsilon_0 m^2 (Z\alpha)^5$$

$) H$ atom* $-$ Schrödinger
           $-$ Relativistic

$$a$$

$$M \gg \frac{1}{e} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

$$h_{eff} = \frac{\left(h\epsilon\right) + \frac{2\pi}{3} Z\alpha r_p^2 + \left(2\pi \epsilon \left(Z\alpha\right)^2\right)/m}{\left(\frac{m h\epsilon}{2\pi\epsilon}\right) + \frac{1}{3} Z\alpha m r_p^2/\epsilon}$$

$$\epsilon \qquad r_p$$

$$(fm) \qquad 1$$

$$\sim 0mh\epsilon/Ze^2$$

$$5$$

$$10$$

$$G(0,0,t-t')$$

$$\omega \simeq m$$

$$\boxed{\frac{1}{\epsilon} \ll m}$$