Title: Something New Under (and in) the Sun? Effective field theories for point sources and the Hydrogen atom

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Abstract: <p>This talk applies effective field theory to the back-reaction of sources with finite size but infinite mass. The main tool for calculating back-reaction is a general relation between a source's effective action and the boundary conditions of `bulkâ€™ fields in the near-source limit. As applied to the Maxwell (or Einstein) fields for point sources this boundary condition reproduces standard Gaussâ€™ Law expressions, but the same arguments imply source-dependent boundary conditions for the Schrodinger (or Dirac) field of an orbiting particle. As applied to the quantum mechanics of a particle interacting with a source through an inverse-square potential EFTs remove the guess-work from the (well-known) ambiguities in the determination of boundary conditions at the origin, and provides a simple interpretation of the classical renormalization effects that are known to arise in this case. EFT arguments show why the RG evolution associated with this classical renormalization is likely universal for a great many types of point sources. The EFT boundary conditions also modify how finite-size effects alter bound-state orbits in the Coulomb problem, and in particular give them a non-standard dependence on the mass of the orbiting particle. It is argued that this might provide a solution to the `proton-radius puzzleâ€™, in which experiments seem to indicate a different proton radius depending on whether or not it is measured using electrons or muons.</p>

Outline

(0) Intro

(1) PPEFTs

- definition
- boundary cond^{n}/s

(2) QM of the $1/r^2$ potential

(3) H atom*

- Schrödinger
- Relativistic

... potential

(3) H atom* - Schrödinger
- Relativistic

$\longleftarrow a \longrightarrow$

$M \gg \frac{1}{\ell} \gg \frac{1}{a}$.

$\frac{1}{\ell} > m$ $\frac{Z\alpha}{\ell} > m$

$\ell < (Z\alpha)^2 a_B$

Because $M \to \infty$

source is static: "bulk fields" interacting with the source's CM coordinate.

"bulk fields": A_μ, ψ

$$S_{eff} = S_B + S_p.$$

$$-i\psi^* \partial_t \psi - \frac{1}{2m}(\nabla\psi)^*(\nabla\psi) - V(x)\psi^*\psi$$

$$S_B = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - (\partial_\mu \phi^*)(\partial^\mu \phi) - m^2 \phi^*\phi \right.$$

$$S_p = S_p\left(A_\mu, \psi, y^\mu(\tau)\right) \qquad y^\mu = \begin{pmatrix} \tau \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \dot{y}^\mu = \delta^\mu_0$$

$$= \int d\tau \int d^4x\, \delta^4(x - y(\tau))\, \mathcal{L}_p(A, \psi, y)$$

$$\mathcal{L}_p = -\sqrt{-\gamma}\, M + q A_\mu \dot{y}^\mu$$

$$S_p = S_p(A_\mu, \psi, y^\mu(\tau)) \qquad y^\mu = \begin{pmatrix} \tau \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \dot y^\mu = \delta^\mu_0$$

$$= \int d\tau \int d^4x \, \delta^4(x - y(\tau)) \, \mathcal{L}_p(A, \psi, y)$$

$$\mathcal{L}_p = -\sqrt{-\gamma}\, \overset{\to\infty}{M} + q A_\mu \dot y^\mu + \ldots \quad (\text{point ptcl.})$$

$$(\gamma = g_{\mu\nu} \dot y^\mu \dot y^\nu)$$

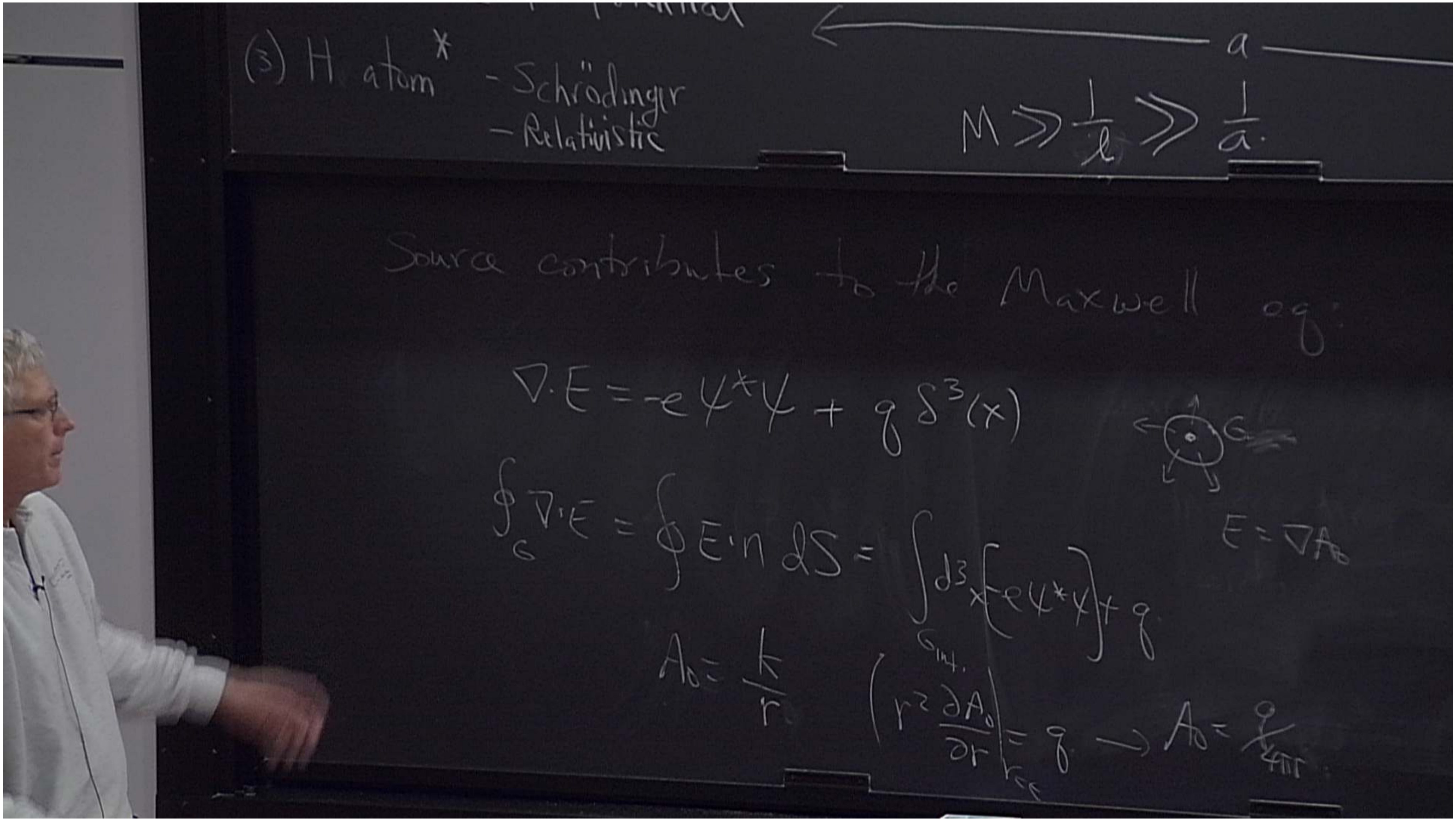
$$V_{eff} = V + h\, \delta^3(x)$$

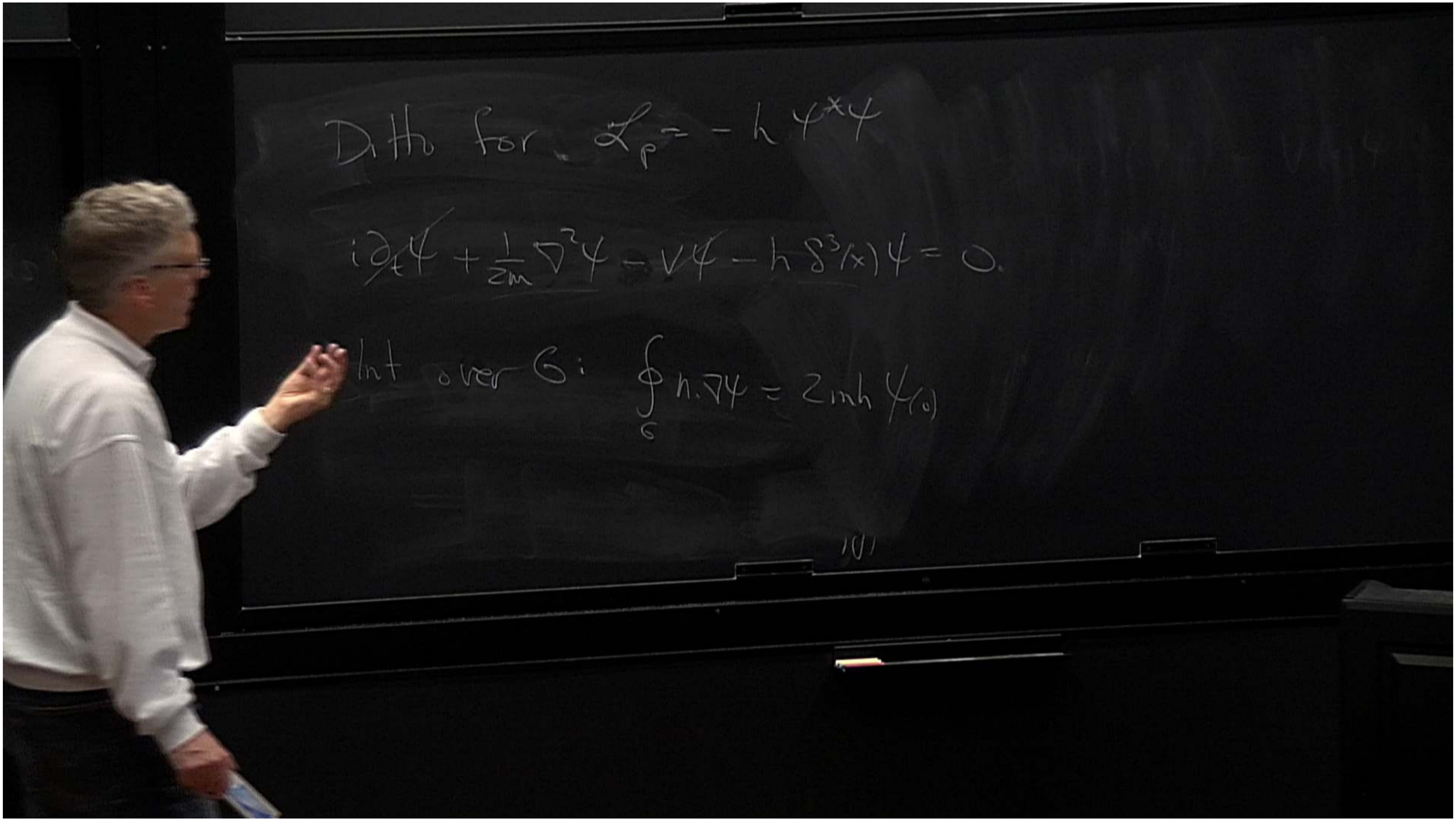
$$(\text{Schrödinger})$$

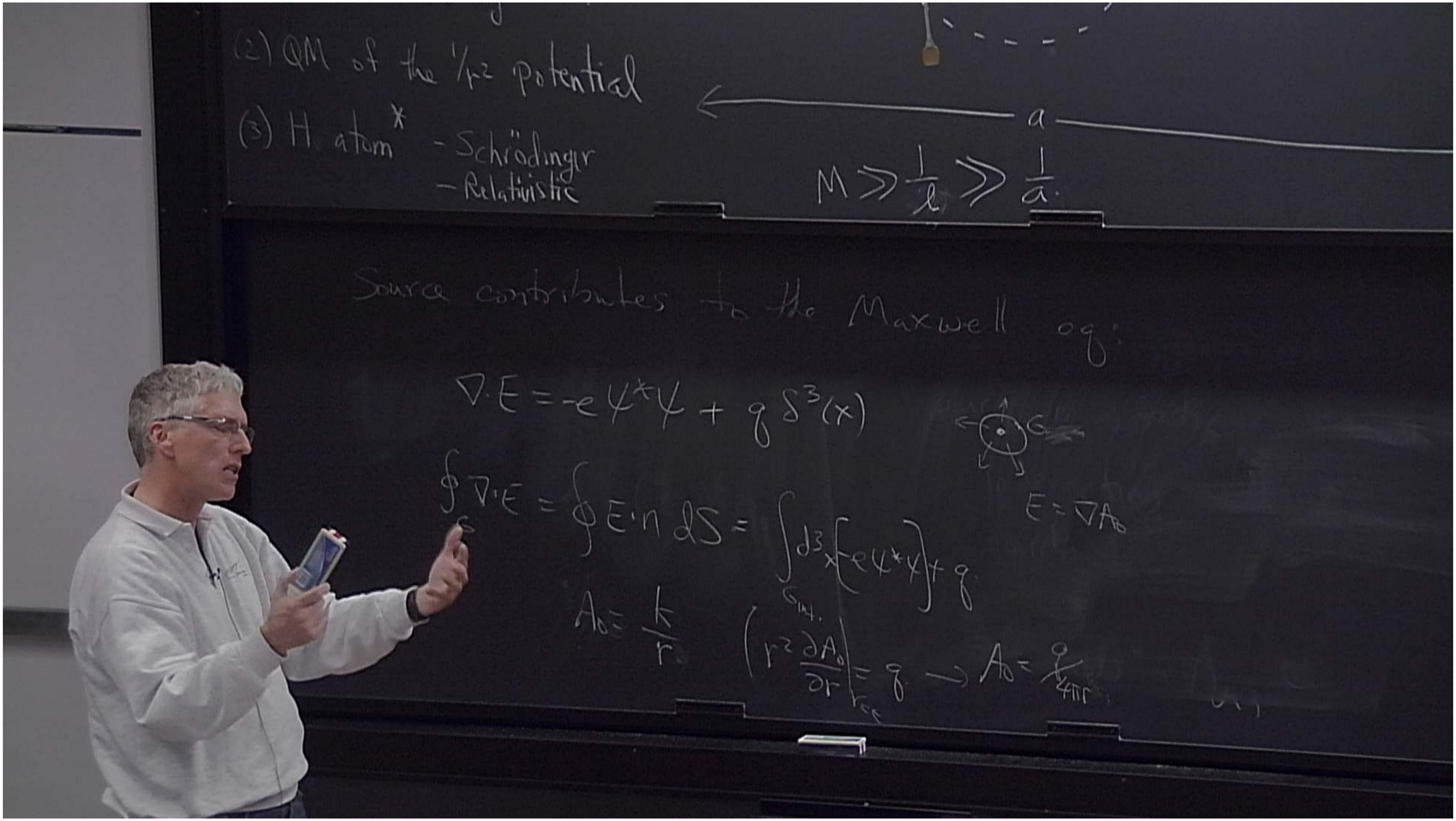
$$S_p = \int d^4x \left\{ q\left[1 + \tfrac{1}{6} r_p^2 \nabla^2\right] A_0 + \cdots - h\, \psi^* \psi + \cdots \right\} \delta^3(x)$$

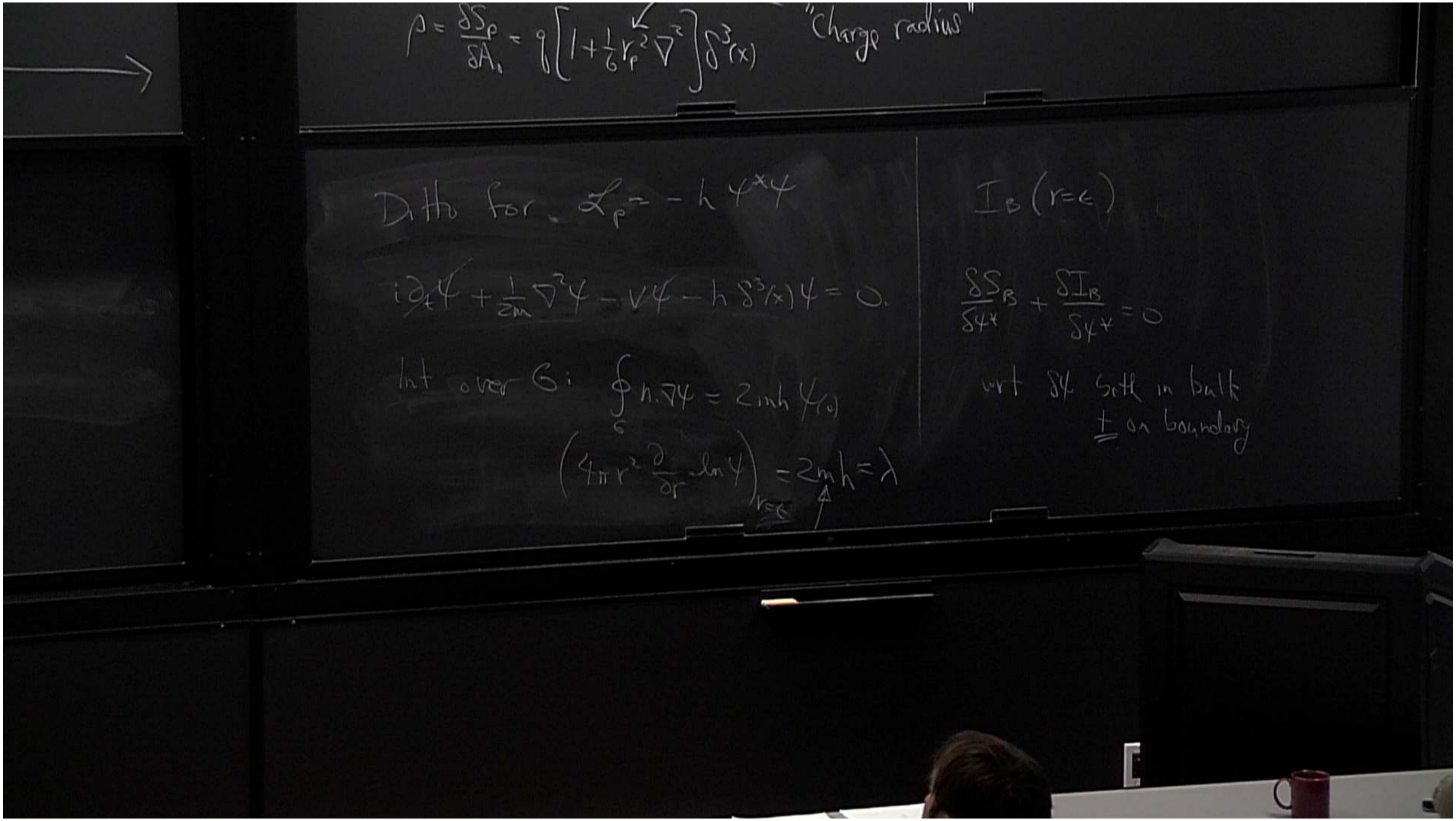
$$\rho = \frac{\delta S_p}{\delta A_0} = q\left[1 + \tfrac{1}{6} r_p^2 \nabla^2\right] \delta^3(x)$$

"Charge radius"


(3) H atom*
- Schrödinger
- Relativistic
a
$M \gg \frac{1}{\ell} \gg \frac{1}{a}$
Source contributes to the Maxwell eq:
$\nabla \cdot E = -e\psi^*\psi + q\,\delta^3(x)$
$\oint_G \nabla \cdot E = \oint E \cdot n\, dS = \int_{G_{int}} d^3x\, [-e\psi^*\psi] + q$
$E = \nabla A_0$
$A_0 = \frac{k}{r}$
$\left(r^2 \frac{\partial A_0}{\partial r}\right)\Big|_{r_G} = q \longrightarrow A_0 = \frac{q}{4\pi r}$


Ditto for $\mathcal{L}_p = -h\psi^*\psi$
$i\partial_t\psi + \frac{1}{2m}\nabla^2\psi - V\psi - h\delta^3(x)\psi = 0.$
Int over S: $\oint_S n\cdot\nabla\psi = 2mh\psi(0)$


(2) QM of the 1/r² potential
(3) H atom* - Schrödinger
- Relativistic
a
M ≫ 1/ℓ ≫ 1/a
Source contributes to the Maxwell eq:
∇·E = -e ψ*ψ + q δ³(x)
∮ ∇·E = ∮ E·n dS = ∫d³x [-e ψ*ψ] + q
E = ∇A₀
A₀ = k/r
(r² ∂A₀/∂r) = q → A₀ = q/4πr


ρ = δS_p/δA_0 = q[1 + 1/6 r_p^2 ∇^2] δ^3(x)
"Charge radius"
Ditto for 𝓛_p = − h ψ*ψ
i∂_t ψ + 1/2m ∇^2 ψ − Vψ − h δ^3(x) ψ = 0.
Int over σ: ∮_σ n·∇ψ = 2mh ψ(0)
(4πr^2 ∂/∂r ln ψ)_{r=ε} = 2mh = λ
I_B (r=ε)
δS_B/δψ* + δI_B/δψ* = 0
wrt δψ both in bulk
+ on boundary

$$S_p = \int d^4x \left\{ q\left[1 + \frac{1}{6} r_p^2 \nabla^2\right] A_0 + \cdots - h\,\psi^*\psi + \cdots \right\} \delta^3(x)$$ (Schrödinger)

$$\rho = \frac{\delta S_p}{\delta A_0} = q\left[1 + \frac{1}{6} r_p^2 \nabla^2\right] \delta^3(x)$$ "Charge radius"

Ditto for $\mathcal{L}_p = -h\,\psi^*\psi$

$$i\partial_t\psi + \frac{1}{2m}\nabla^2\psi - V\psi - h\,\delta^3(x)\,\psi = 0.$$

Int over σ: $$\oint_\sigma n\cdot\nabla\psi = 2mh\,\psi(0)$$

$$\left(4\pi r^2 \frac{\partial}{\partial r}\ln\psi\right)_{r=\epsilon} = 2mh = \lambda$$

$I_B(r=\epsilon)$

$$\frac{\delta S_B}{\delta\psi^*} + \frac{\delta I_B}{\delta\psi^*} = 0$$

wrt $\delta\psi$ Soln in bulk

$\pm$ on boundary

(2) QM of the $1/r^2$ potential

(3) H. atom* – Schrödinger
– Relativistic

$\longleftarrow a \longrightarrow$

$$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$$

NR
QM of the $\frac{1}{r^2}$ potential

$$\nu(\nu+1) = \ell(\ell+1) - 2mg$$

$$-\frac{\nabla^2}{2m}\psi - \frac{g}{r^2}\psi + \boxed{h\,\delta^3(x)\psi} = E\psi$$

for small nonzero g $\nu \gtrsim 0$ but small

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) \qquad \frac{\ell(\ell+1)}{2mr^2}$$

if $g=0$ then $R_+(r) \simeq r^{\nu}$ $\quad R = C_+R_+ + C_-R_-$

$R_-(r) \simeq r^{-\nu-1}$ $\quad$ Both $R_+, R_- \to \infty$ $r \to 0$

Self-adjoint extension

QM of the $1/r^2$ potential

(3) H atom* - Schrödinger
- Relativistic

$\longleftarrow a \longrightarrow$

$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}.$

QM of the $\frac{1}{r^2}$ potential.

$\nu(\nu+1) = \ell(\ell+1) - 2mg$

$r=\epsilon$

$$-\frac{\nabla^2}{2m}\psi - \frac{g}{r^2}\psi + \boxed{h\,\delta^3(x)\psi} = E\psi.$$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)$ $\quad \frac{\ell(\ell+1)}{2mr^2}$

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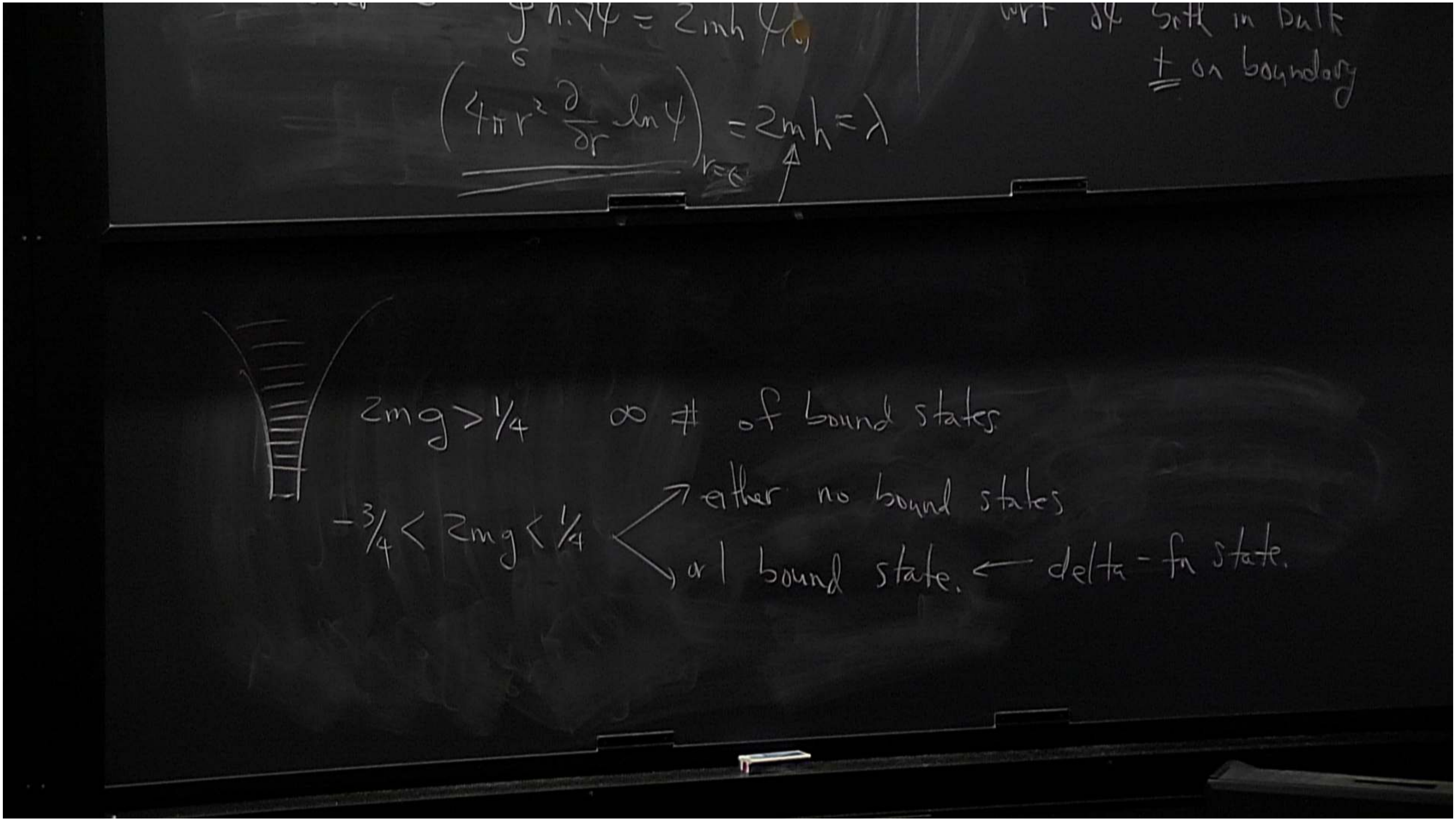
Self-adjoint extension


$\oint_\epsilon n\cdot\nabla\psi = 2mh\,\psi(\epsilon)$
$\left(4\pi r^2 \frac{\partial}{\partial r} \ln\psi\right)_{r=\epsilon} = 2mh = \lambda$
wrt δψ soln in bulk
± on boundary
2mg > 1/4 ∞ # of bound states
-3/4 < 2mg < 1/4
either no bound states
or 1 bound state. ← delta-fn state.

$$\oint_\sigma n\cdot\nabla\psi = 2mh\,\psi(0)$$

$$\left(4\pi r^2 \frac{\partial}{\partial r}\ln\psi\right)_{r=\epsilon} = 2mh = \lambda$$

wrt $\delta\psi$ Soln in bulk

$\perp$ on boundary

E_B depends on ϵ.

Bound. cond $4\pi r^2 \frac{\partial}{\partial r}\ln\psi = 2mh = \lambda$.

$R = C_+R_+ + C_-R_-$ has $C_-\neq 0$ if $\lambda\neq 0$.

Int over σ: $\oint_\sigma n\cdot\nabla\psi = 2mh\,\psi(0)$

$$\left(4\pi r^2 \frac{\partial}{\partial r}\ln\psi\right)_{r=\epsilon} = 2mh = \lambda$$

wrt $\delta\psi$: soln in bulk
$\pm$ on boundary

E_σ depends on ϵ.

Bound. cond $\boxed{4\pi r^2 \frac{\partial}{\partial r}\ln\psi} = 2mh = \lambda.$

$R = C_+R_+ + C_-R_-$ has $C_- \neq 0$ if $\lambda \neq 0$.

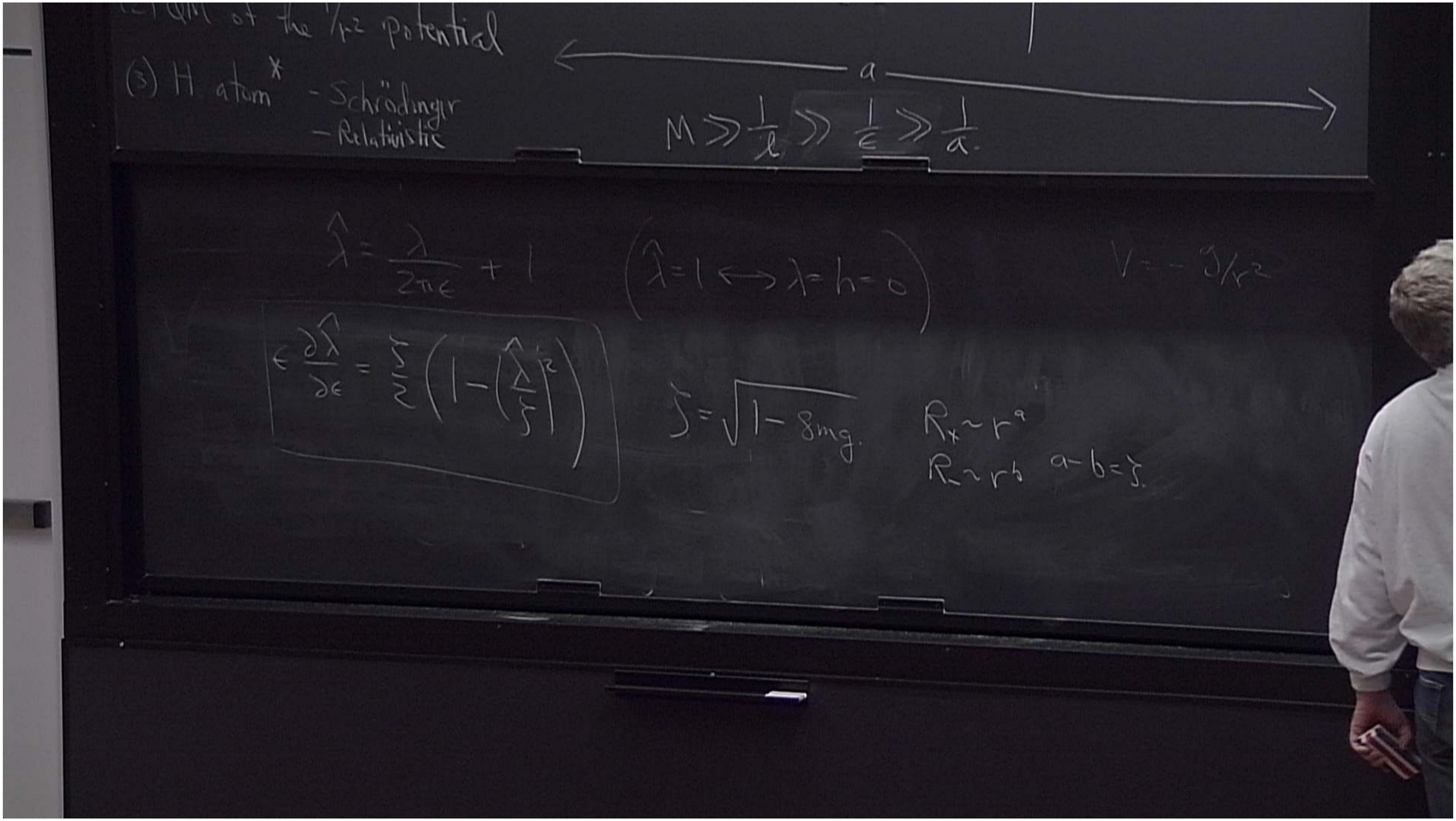
NB $\Rightarrow$ Series in $\frac{mh}{\epsilon}$ need not be small
and $2mg$

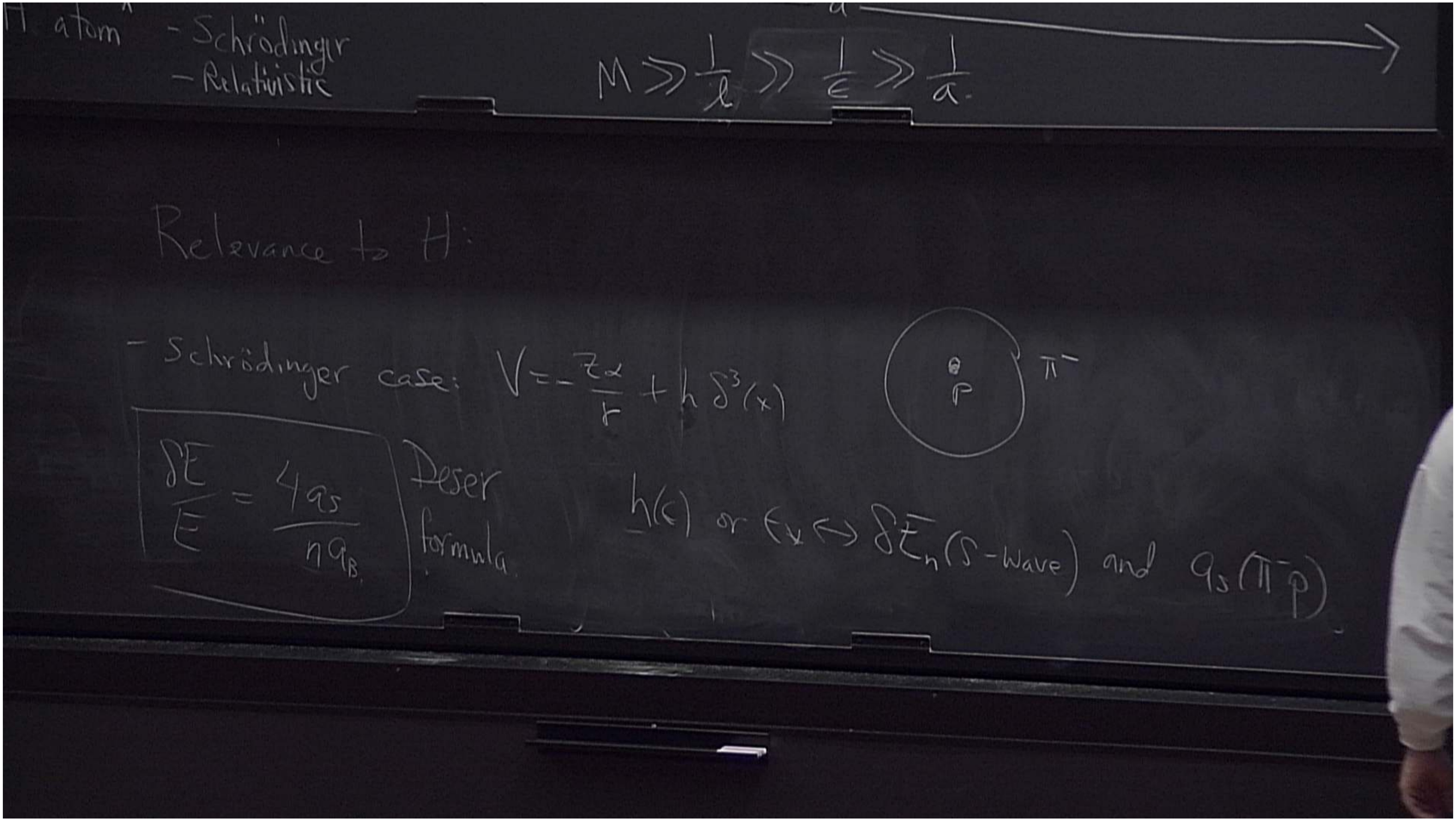
Bound-state energies:

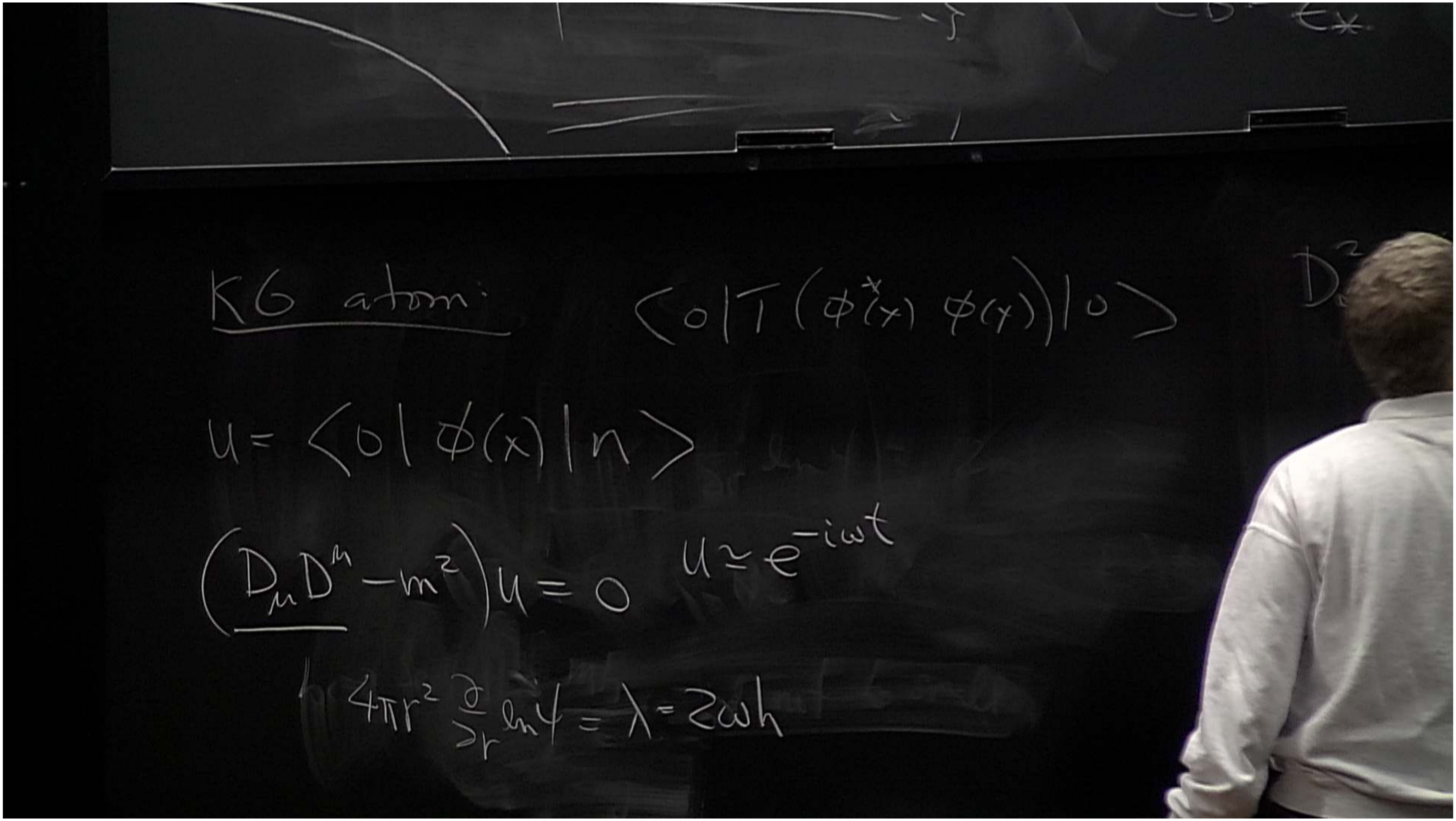
$\xrightarrow{r\to\infty} \psi \to$ normalizable @ ∞

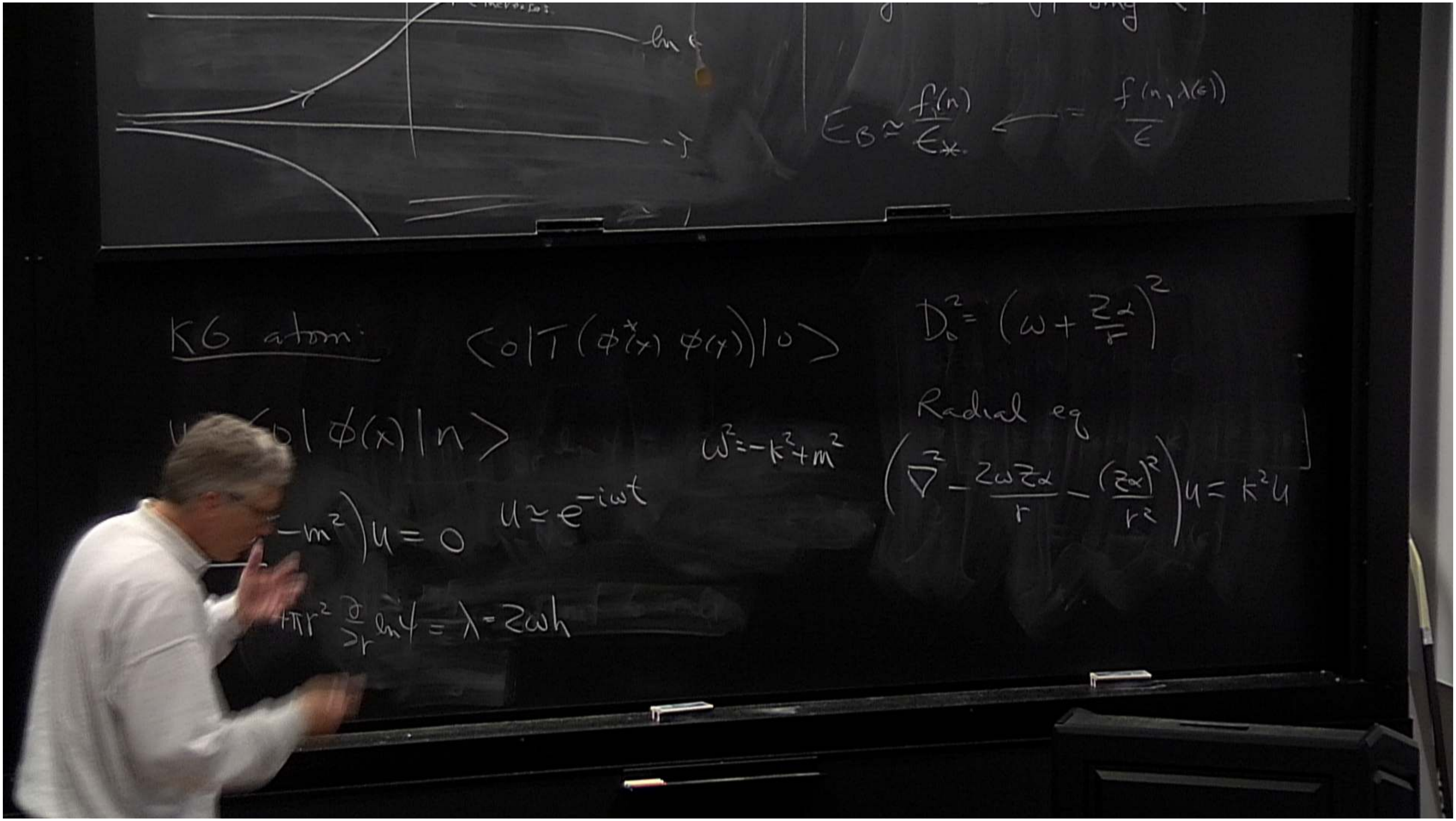
$\xrightarrow[r=\epsilon]{} \frac{C_-}{C_+} = f(\text{params}, \kappa) = \propto \lambda, \epsilon$.

$\frac{\partial}{\partial\epsilon}\Big[\quad\Big]_{\kappa,\dots} = 0 \to \lambda(\epsilon)$.

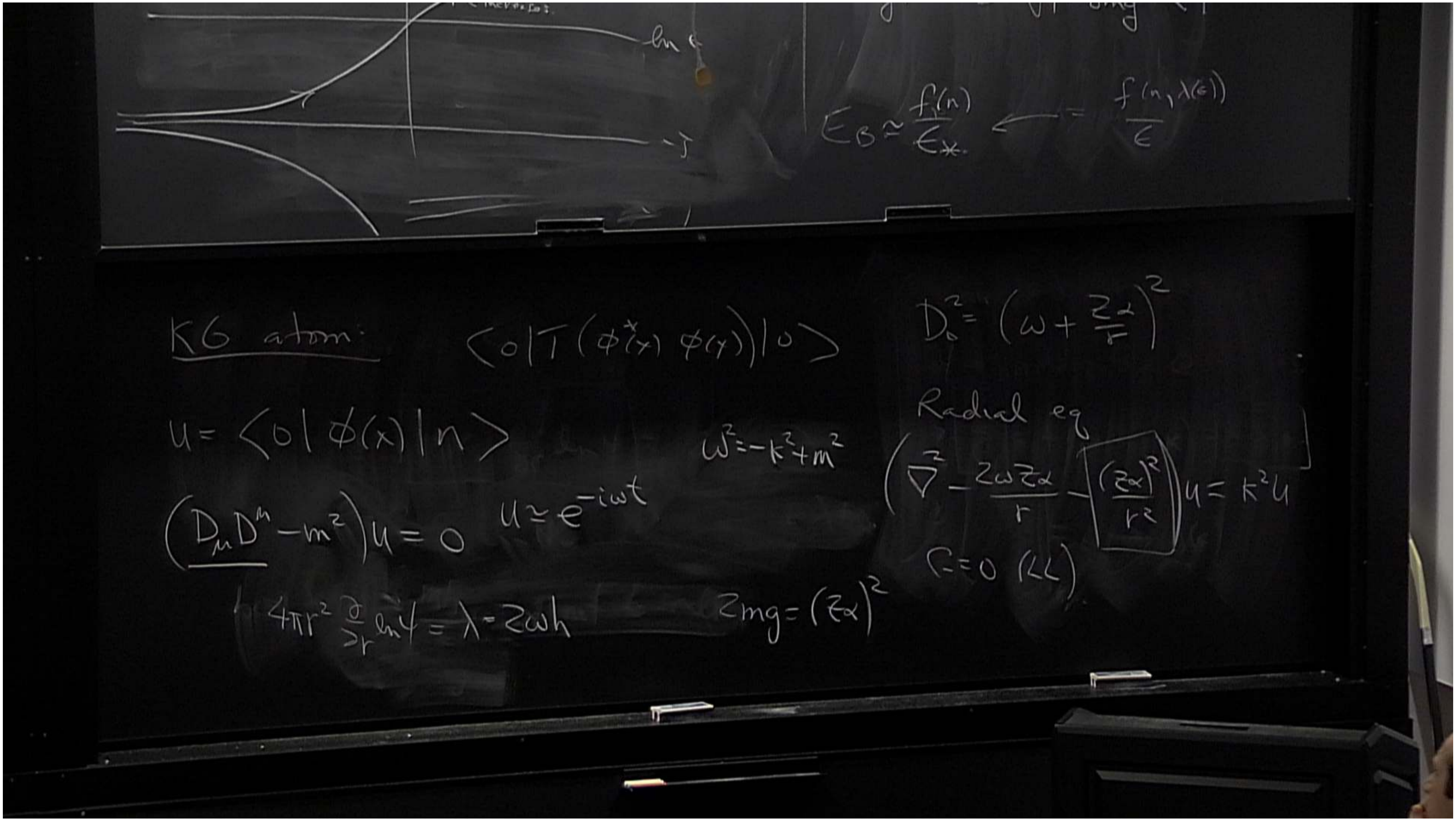

QM of the $1/r^2$ potential
(3) H atom* - Schrödinger
- Relativistic
a
$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$
$\hat{\lambda} = \frac{\lambda}{2\pi\epsilon} + 1$
$(\hat{\lambda} = 1 \longleftrightarrow \lambda = h = 0)$
$V = -g/r^2$
$\epsilon \frac{\partial \hat{\lambda}}{\partial \epsilon} = \frac{\zeta}{2}\left(1 - \left(\frac{\hat{\lambda}}{\zeta}\right)^2\right)$
$\zeta = \sqrt{1 - 8mg}$
$R_+ \sim r^a$
$R_- \sim r^b \quad a - b = \zeta$


H atom
- Schrödinger
- Relativistic
$M \gg \frac{1}{\ell} \gg \frac{1}{\epsilon} \gg \frac{1}{a}$
Relevance to H:
- Schrödinger case: $V = -\frac{Z\alpha}{r} + h\,\delta^3(x)$
p
π^-
$\frac{\delta E}{E} = \frac{4 a_s}{n a_B}$
Deser formula
$h(\epsilon)$ or $\epsilon_x \Leftrightarrow \delta E_n$ (S-wave) and $a_s(\pi^- p)$

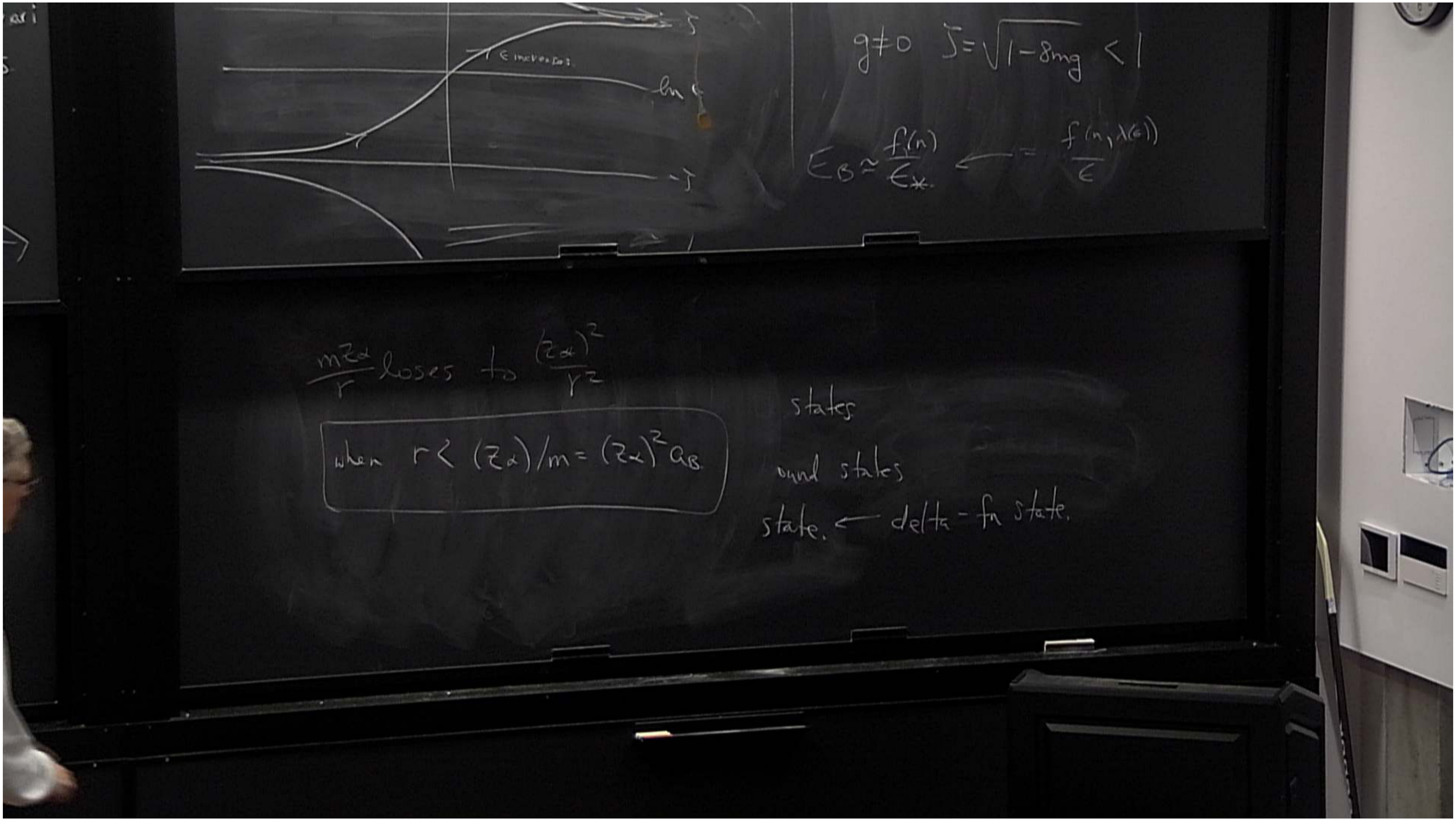

KG atom:
$\langle 0 | T(\phi^*(x)\,\phi(x)) | 0 \rangle$
$u = \langle 0 | \phi(x) | n \rangle$
$(D_\mu D^\mu - m^2)\,u = 0$
$u \simeq e^{-i\omega t}$
$4\pi r^2 \frac{\partial}{\partial r} \ln \psi = \lambda = 2\omega h$

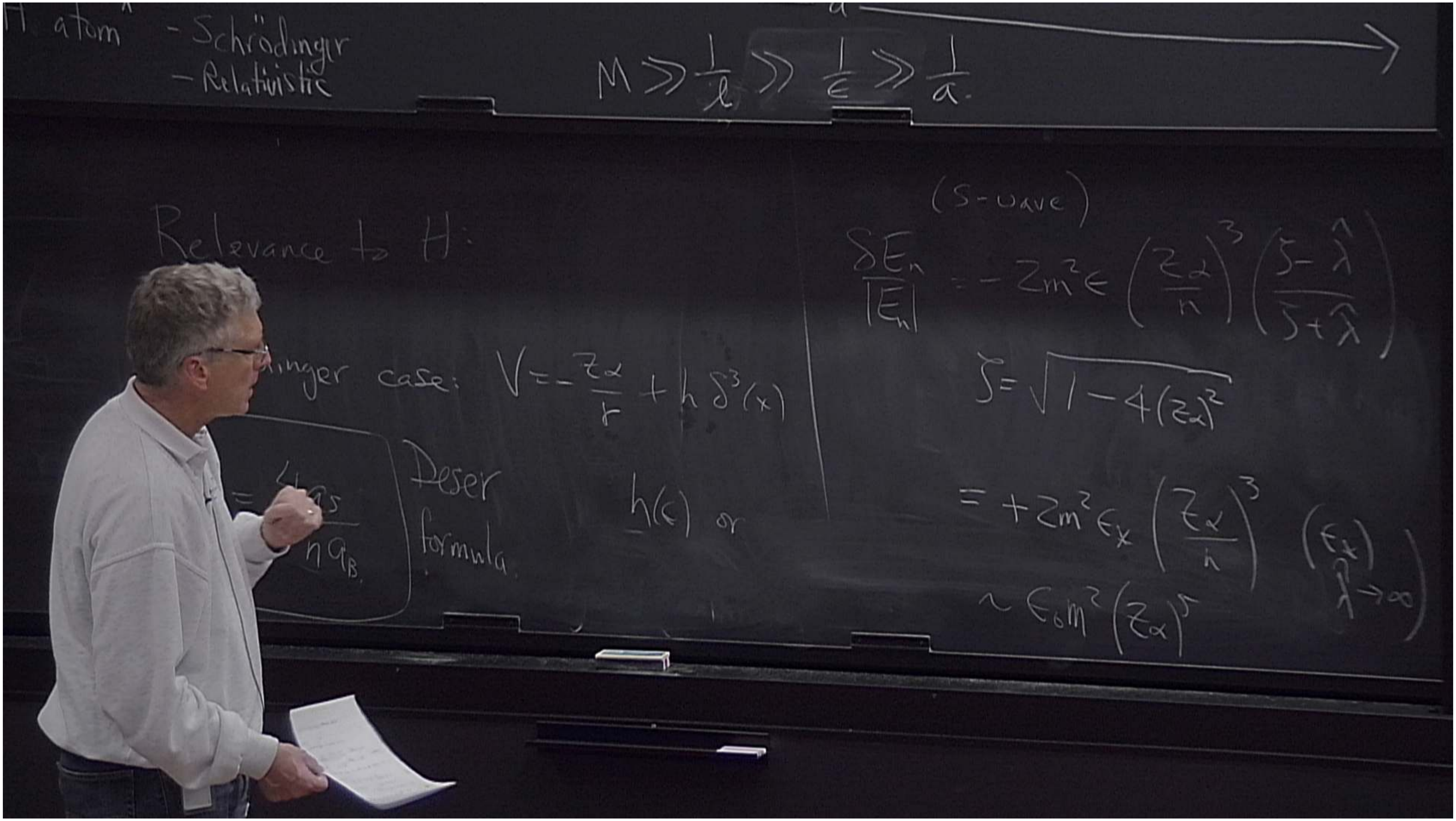


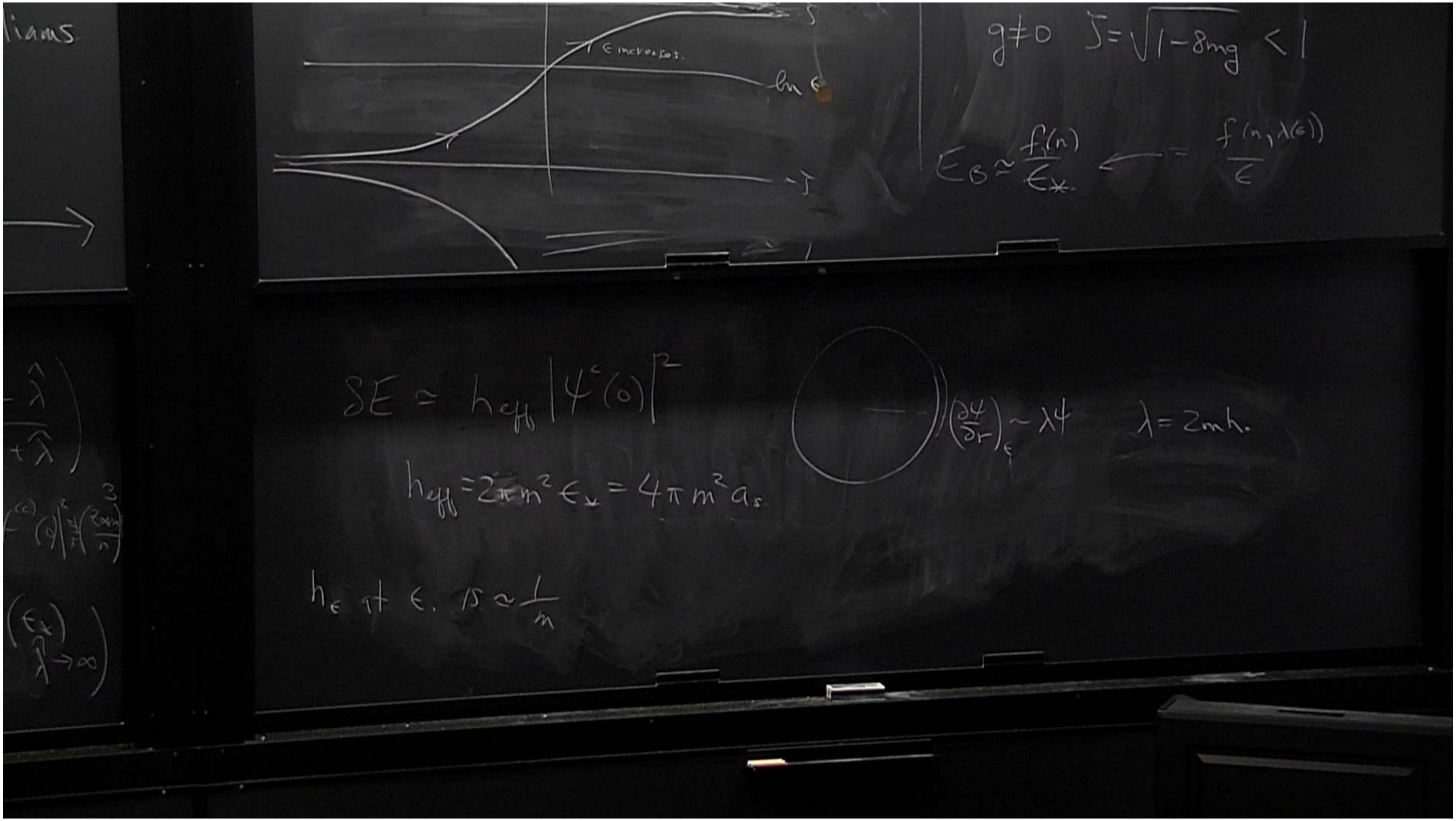
KG atom:
⟨0|T(φ*(x) φ(y))|0⟩
D₀² = (ω + Zα/r)²
Radial eq
ω² = −k² + m²
u ≃ e^{−iωt}



ln ε
E_B ≃ f_1(n)/ε_* ← f(n, λ(ε))/ε
KG atom:
⟨0|T(φ*(x) φ(y))|0⟩
u = ⟨0|φ(x)|n⟩
ω² = −k² + m²
(D_μ D^μ − m²) u = 0
u ≃ e^{−iωt}
4πr² ∂/∂r ln ψ = λ − 2ωh
2mg = (Zα)²
D_0² = (ω + Zα/r)²
Radial eq
(∇² − 2ωZα/r − (Zα)²/r²) u = k² u
r = 0 (LL)

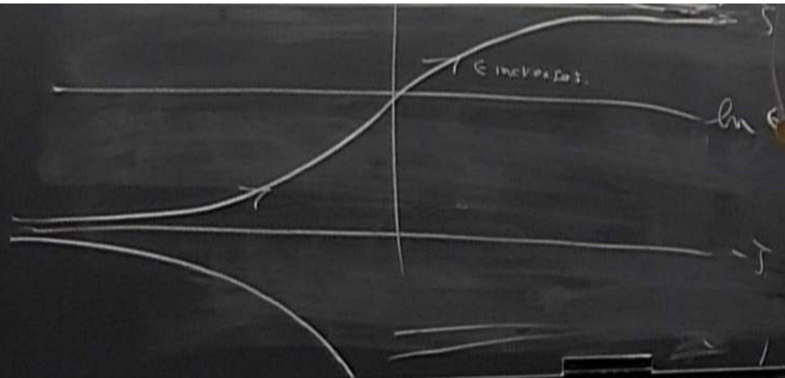

g≠0 J = √(1−8mg) < 1
ε increases
ln ε
E_B ≃ f(n)/ε_*
f(n, λ(ε))/ε
m Zα/r loses to (Zα)²/r²
when r < (Zα)/m = (Zα)² a_B
states
ound states
state. ← delta-fn state.


H atom
- Schrödinger
- Relativistic
M ≫ 1/ℓ ≫ 1/ε ≫ 1/a
Relevance to H:
(s-wave)
Deser formula
h(ε) or


ε increases
ln ε
g ≠ 0 ζ = √(1 − 8mg) < 1
ε_B ≃ f_1(n)/ε_*
f(n, λ(ε))/ε
δE ≃ h_eff |ψ^c(0)|^2
(∂ψ/∂r)_ε ~ λψ
λ = 2mh.
h_eff = 2π m^2 ε_* = 4π m^2 a_s

$g \neq 0 \quad \zeta = \sqrt{1 - 8mg} < 1$

$\epsilon_B \simeq \frac{f(n)}{\epsilon_*} \longleftarrow \frac{f(n, \lambda(\epsilon))}{\epsilon}$



$\delta E \simeq h_{eff} |\psi^c(0)|^2$

$h_{eff} = 2\pi m^2 \epsilon_* = 4\pi m^2 a_s$

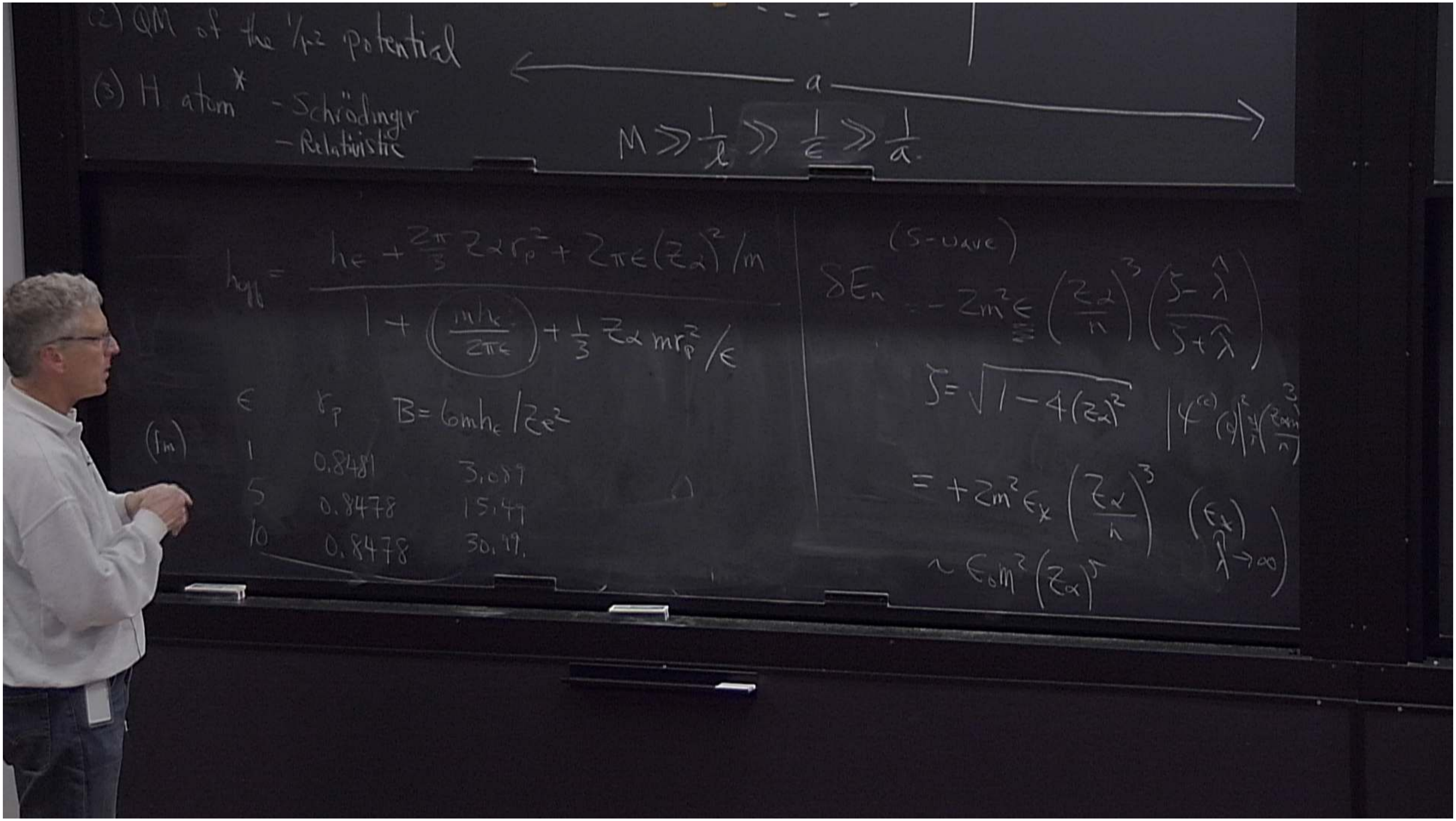
$\left(\frac{\partial \psi}{\partial r}\right)_\epsilon \sim \lambda \psi \qquad \lambda = 2mh.$

$\nabla \cdot E = -e\psi^*\psi + Ze\delta^3(x)$

h_ϵ at ϵ. $r_s \simeq \frac{1}{m}$

$h_{tot} = h_\epsilon + \frac{1}{6} Ze^2 r_p^2$

$\mathcal{L}_p = -h_\epsilon \psi^*\psi - \frac{Ze}{6} r_p^2 \nabla \cdot E$


(2) QM of the 1/r² potential
(3) H atom* - Schrödinger
- Relativistic
a
M ≫ 1/λ ≫ 1/ε ≫ 1/a
(S-wave)
ε
r_p
B = 6mh_ε / Ze²
(fm)
1
0.8481
3.089
5
0.8478
15.49
10
0.8478
30.99
= + 2m² ε_x (Zα/n)³
~ ε_0 m² (Zα)⁵