Title: Emergent Friedmann dynamics with a quantum bounce from quantum gravity condensates

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Abstract: <p>I will explain how cosmological dynamics emerge from the hydrodynamics of isotropic group field theory condensate states in the Gross-Pitaevskii approximation. The correct Friedmann equations are recovered in the classical limit for some choices of the parameters in the action for the group field theory, and quantum gravity corrections arise in the high-curvature regime causing a bounce which generically resolves the big-bang and big-crunch singularities.</p>

# Bouncing cosmologies from condensates of quantum geometry

Edward Wilson-Ewing

Albert Einstein Institute
Max Planck Institute for Gravitational Physics

Work with Daniele Oriti and Lorenzo Sindoni

CQG **33** (2016) 224001, arXiv:1602.05881 [gr-qc]
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Perimeter Institute Seminar

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# Motivation

Goal: **Extract cosmology from (loop) quantum gravity.**

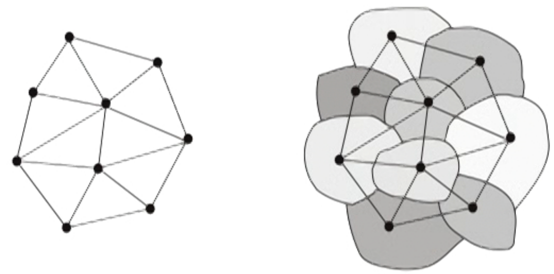
Loop quantum cosmology (LQC) —where the quantization techniques of loop quantum gravity (LQG) are applied in the symmetry-reduced minisuperspaces corresponding to homogeneous space-times— has given some potentially important insights in this direction.

However, despite its successes, the exact relation between loop quantum cosmology and full loop quantum gravity remains unclear. It is important to go beyond LQC, using any hints LQC may offer.

# Loop Quantum Gravity: Basics

Loop quantum gravity is a background independent approach to quantum gravity based on connection and triad variables.

A convenient basis for states in the canonical framework are spin networks: graphs coloured by spins on the edges and intertwiners on the nodes.



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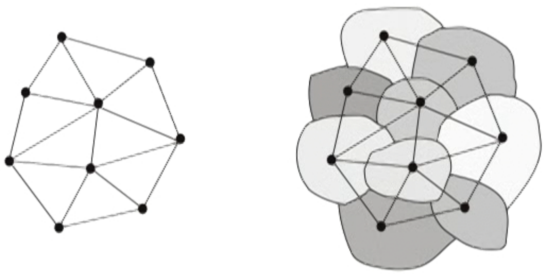

[Rovelli]

An important result is that geometrical observables like the volume and the area have a discrete spectrum. Furthermore, each node can be thought of as a polyhedron with some volume and surface areas transverse to the links: for example, a four-valent node gives a tetrahedron. In this sense, the spin network is composed of quanta of geometry. [Rovelli, Smolin; Ashtekar, Lewandowski; Freidel, Speziale; Bianchi, Dona, Speziale; Haggard; ...]

# Cosmology as a Condensate of Geometry

In any theory such as LQG which predicts that space-time is constituted of Planck-scale quanta of geometry, it is reasonable to assume that in cosmological space-times:

- there are many quanta of geometry,
- one quanta contributes a small fraction of the spatial volume,
- cosmological expansion is primarily due to quanta being added.

Furthermore, the improved dynamics of loop quantum cosmology suggest that the $N$ quanta are in fact in the same state and that each contributes the same minimal $V_{min}$ to the total volume,

$$V_{tot} = N V_{min}.$$

# Aside I: Insight from Loop Quantum Cosmology

One of the key steps in loop quantum cosmology is the definition of the field strength operator.

Since the fundamental operators of the theory are holonomies of the Ashtekar-Barbero connection $A_a^i$, it is natural to express the field strength as the holonomy of $A_a^i$ around a small loop $\Box$ of area $Ar_\Box$,

$$F \sim \frac{h_\Box(A_a^i) - \mathbb{I}}{Ar_\Box}.$$

In lattice gauge theories, one would then take the limit $Ar_\Box \to 0$, but this limit is not natural in LQC, since the spectrum of the area operator in LQG is discrete.

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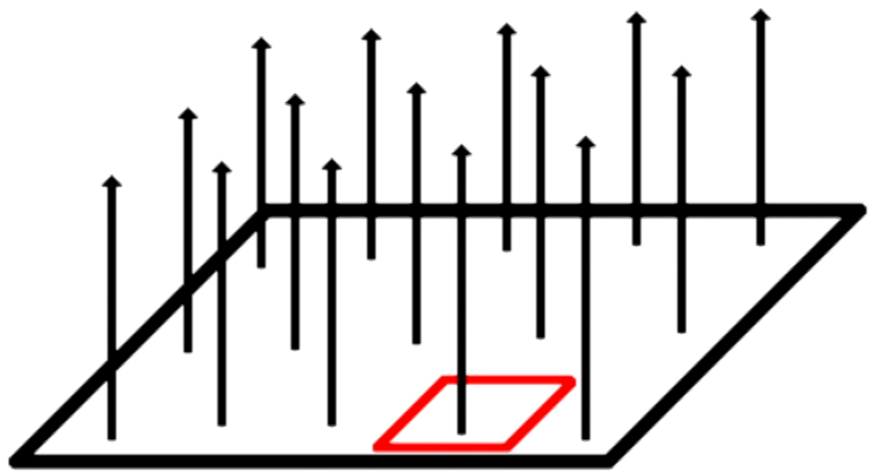
# Aside II: Choice of the Area



Instead, $Ar_\Box$ is taken to be the minimal area in LQG. Heuristically, this corresponds to assuming that the dominant contribution to the area of a surface in the cosmological space-time comes from these minimal area excitations.

Since these minimal area excitations are all identical, this suggests that the LQG states corresponding to cosmological space-times are those where the dominant contribution comes from quanta of geometry that are in the same state: **a condensate of geometry**.

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Furthermore, the improved dynamics of loop quantum cosmology suggest that the $N$ quanta are in fact in the same state and that each contributes the same minimal $V_{min}$ to the total volume,

$$V_{tot} = N V_{min}.$$

If all the quanta are indeed in the same state, this suggests using **condensate states** to extract cosmology from LQG.

This in turn directly leads to **group field theory**, a field theory for the quanta of geometry of LQG.

# Cosmology: Hydrodynamics of the Condensate

The key idea here is that the continuous cosmological space-time emerges from the coarse-graining/hydrodynamics of the group field theory (GFT) condensate state.

The microscopic dynamics of the GFT condensate state imply some effective coarse-grained Friedmann equations, which follow from the evaluation of the relevant collective cosmological observables (e.g., total spatial volume) and calculating their evolution as determined by the microscopic GFT model (with respect to some relational time).

# Outline

1. Group Field Theory with a Scalar Field
2. Condensate States
3. Effective Friedmann Equations

# Group Field Theory with a Scalar Field

Group field theory (GFT) can be seen as a second-quantized language for loop quantum gravity, where the field operators

$$\hat{\varphi}^{j_1,j_2,j_3,j_4,\iota}_{m_1,m_2,m_3,m_4}(\phi), \qquad \hat{\varphi}^{\dagger\, j_1,j_2,j_3,j_4,\iota}_{m_1,m_2,m_3,m_4}(\phi),$$

create and annihilate quanta of geometry: spin network nodes [Oriti].

The $j_i$ and $m_i$ colour the links of the (four-valent) spin network node and the intertwiner $\iota$ and the scalar field $\phi$ both live on the spin network nodes. (Connectivity is imposed via projectors on links.)

The classical GFT action $S(\varphi,\bar{\varphi})$ is typically chosen so that the perturbative expansion of the GFT partition function matches the sum over geometries of a spin foam model. In the simplest GFT actions for quantum gravity (for $V(\phi)=0$), the dominant terms are

$$S \sim \sum_{j_i,m_i,\iota_i} \int_{\phi_i} \left[\bar{\varphi}\, K_2^{(0)} \varphi + \bar{\varphi}\, K_2^{(2)} \partial_\phi^2 \varphi\right] + \sum_{j_i,m_i,\iota_i} \int_{\phi_i} \left[\bar{\varphi}^5\, \bar{\mathcal{V}}_5 + \varphi^5\, \mathcal{V}_5\right].$$

# GFT Operators

The operators in GFT have the standard second-quantized form, and in particular the number operator will be important,

$$\widehat{N} = \sum_{j_i, m_i, \iota_i} \int_\phi \hat{\varphi}^{\dagger}{}^{j_1, j_2, j_3, j_4, \iota}_{m_1, m_2, m_3, m_4}(\phi) \; \hat{\varphi}^{j_1, j_2, j_3, j_4, \iota}_{m_1, m_2, m_3, m_4}(\phi).$$

The presence of the scalar field allows for the definition of relational observables, for example the relational number operator,

$$\widehat{N}(\phi_o) = \sum_{j_i, m_i, \iota_i} \hat{\varphi}^{\dagger}{}^{j_1, j_2, j_3, j_4, \iota}_{m_1, m_2, m_3, m_4}(\phi_o) \; \hat{\varphi}^{j_1, j_2, j_3, j_4, \iota}_{m_1, m_2, m_3, m_4}(\phi_o).$$

## Condensate States

A simple family of condensate states are the Gross-Pitaevskii condensate states: coherent states of the GFT field operator which are, up to a numerical prefactor, [Gielen, Oriti, Sindoni]

$$|\sigma\rangle \sim \exp\left(\sum_{j_i,m_i,\iota} \int \mathrm{d}\phi\, \sigma^{j_i,\iota}_{m_i}(\phi)\hat{\varphi}^{\dagger\, j_i,\iota}_{\ \ m_i}(\phi)\right)|\mathbf{0}\rangle,$$

where $\sigma^{j_i,\iota}_{m_i}(\phi)$ is the condensate wave function. Note that $\sigma^{j_i,\iota}_{m_i}(\phi)$ is not normalized; rather, its norm gives the number of fundamental GFT quanta.

Importantly, the massless scalar field can be used as a relational clock: $\sigma^{j_i,\iota}_{m_i}(\phi_o)$ can be understood as the condensate wave function evaluated at the 'time' $\phi_o$.

Thus, imposing the quantum equations of motion on $|\sigma\rangle$ will give relational dynamics with respect to $\phi$.

# The Form of $\sigma^{j_i,\iota}_{m_i}(\phi)$

It is important to make choices for $\sigma^{j_i,\iota}_{m_i}(\phi)$ so that the condensate state represents a cosmological space-time. Furthermore, appropriate approximations will simplify the equations to be solved.

- We are interested in the spatially flat FLRW space-time. So **we neglect connectivity**: the main observable is the total volume where connectivity is unimportant, and the space-time is spatially flat so we do not need to worry about encoding the spatial curvature in the connectivity of the graph [Gielen, Oriti, Sindoni].
- We are only interested in isotropic observables. So **we restrict our attention to equilateral (isotropic) configurations**,

$$\sigma^{j_i,\iota}_{m_i}(\phi) \to \sigma_j(\phi),$$

  This assumption can also be motivated by the heuristic foundations of LQC.

# Relational Dynamics

The quantum equations of motion are

$$\frac{\widehat{\delta S}}{\delta \bar{\varphi}} |\sigma\rangle = 0.$$

But we expect the condensate state to only be an approximate solution to the quantum equations of motion. So, we will only impose the first Schwinger-Dyson equation [Gielen, Oriti, Sindoni],

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For the GFT action shown earlier, this gives the non-linear condensate equations of motion

$$\partial_\phi^2 \sigma_j(\phi) - m_j^2 \, \sigma_j(\phi) + w_j \, \bar{\sigma}_j(\phi)^4 = 0,$$

where the numerical values of the $m_j^2 \sim K_2^{(0)}/K_2^{(2)}$ and $w_j \sim \mathcal{V}_5/K_2^{(2)}$ depend on the parameters in the GFT action.

# The Small Interactions Regime

The Gross-Pitaevskii condensate approximation is typically most trustworthy when interactions are small. Thus, we will focus on the regime where the interaction term is negligible. To consider cases when the interaction term becomes important, it may be necessary to go beyond the Gross-Pitaevskii approximation and include interactions (i.e., connectivity information).

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As can easily be checked in the equation of motion for $\sigma_j$, the interaction term will become large when $|\sigma_j|$ becomes sufficiently large. This is the large volume limit: **interactions become important at large volumes**.

Interactions becoming important at large volumes may be related to the fact that the connectivity information has been ignored: all GFT quanta are interacting with all other quanta, not only their neighbours. Restoring connectivity information may well fix this.

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# The Mesoscopic Regime

I will now consider the mesoscopic regime where interactions are small ($|\sigma_j| \lesssim (m_j^2/w_j)^{1/3}$) and where there are enough quanta for a continuum space-time interpretation to be viable ($\sum_j |\sigma_j|^2 \gtrsim 10^3$.)

Such a regime will exist for some GFT actions (but not all), depending on the parameters in the action. For the remainder of the talk, I will take such a GFT action and only work in this mesoscopic regime.

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# Relational Dynamics

In this mesoscopic regime, rewriting $\sigma_j = \rho_j e^{i\theta_j}$, the condensate equations of motion imply that for each $j$

$$E_j = (\partial_\phi \rho_j)^2 + \rho_j^2 (\partial_\phi \theta_j)^2 - m_j^2 \rho_j^2, \qquad Q_j = \rho_j^2 \, \partial_\phi \theta_j,$$

are conserved quantities (with respect to the relational time $\phi$).

There is one remaining non-trivial equation of motion for each $j$,

$$\partial_\phi^2 \rho_j - \frac{Q_j^2}{\rho_j^3} - m_j^2 \rho_j \approx 0.$$

Importantly, $\rho_j(\phi)$ can never become zero due to the divergent repulsive potential at $\rho_j = 0$.

# Cosmological Observables

In order to extract cosmology (i.e., hydrodynamics) from the condensate state $|\sigma\rangle$, it is necessary to relate the volume $V$ and the momentum of the scalar field $\pi_\phi$ to the appropriate GFT observables.

These are

$$V(\phi) = \sum_j V_j \, \bar{\sigma}_j(\phi)\sigma_j(\phi) = \sum_j V_j \rho_j(\phi)^2,$$

$$\pi_\phi(\phi) = -\frac{i\hbar}{2} \sum_j \left[\bar{\sigma}_j(\phi)\partial_\phi \sigma_j(\phi) - \sigma_j(\phi)\partial_\phi \bar{\sigma}_j(\phi)\right] = \hbar \sum_j Q_j.$$

It immediately follows that

$$\partial_\phi \pi_\phi(\phi) = 0,$$

and so we recover the continuity equation for an FLRW space-time with a massless scalar field.

# The Condensate Friedmann Equation

The relational Friedmann equation can be derived from

$$\partial_\phi V = 2 \sum_j V_j \, \rho_j \, \partial_\phi \rho_j,$$

and the equations of motion given earlier. Using $E_j$ and $Q_j$,

$$\left(\frac{\partial_\phi V}{3V}\right)^2 = \left(\frac{2 \sum_j V_j \, \rho_j \sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2 \rho_j^2}}{3 \sum_j V_j \rho_j^2}\right)^2 .$$

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The classical Friedmann equation

$$\left(\frac{\partial_\phi V}{3V}\right)^2 = \frac{4\pi G}{3}$$

is recovered in the low curvature semi-classical limit (which here corresponds to large $\rho_j$) for $m_j^2 = 3\pi G$.

# The Singularity is Resolved

Recall that from the remaining non-trivial equation of motion,

$$\partial_\phi^2 \rho_j - \frac{Q_j^2}{\rho_j^3} - m_j^2 \rho_j \approx 0,$$

it is clear that $\rho_j$ never reaches zero due to the divergent repulsive potential $-Q_j^2/\rho_j^3$.

Since

$$V(\phi) = \sum_j V_j \rho_j^2,$$

it follows that $V(\phi)$ can never be zero.

Thus, the big-bang and big-crunch singularities are generically resolved, and, furthermore, are replaced by a bounce.

# Relation to LQC

LQC, in its construction, suggests that the appropriate condensate state is one where all the quanta are equilateral spin network nodes with $j = 1/2$. Motivated by this observation, let's consider the case where $\sigma_j(\phi)$ only has support on $j = j_o$.

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Then, using $\rho = \pi_\phi^2/2V^2$, the condensate Friedmann equation becomes

$$\left(\frac{\partial_\phi V}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{4V_{j_o}E_{j_o}}{9V},$$

with $\rho_c = 3\pi G\hbar^2/2V_{j_o}^2 \sim (6\pi/j_o^3)\rho_{\rm Pl}$.

**This is (almost) exactly the LQC effective Friedmann equation**, up to the extra term that depends on $E_{j_o}$.

# Conclusions

- Motivated by simple arguments combined with insights from LQC, we made a specific ansatz on the type of state in (the GFT reformulation of) LQG that corresponds to cosmological space-times: GFT condensate states.
- The equations of motion for the condensate states are determined by the GFT action, and their hydrodynamics give the continuity and Friedmann equations.
- The classical Friedmann equations are recovered in an appropriate semi-classical limit for some choices of parameters in the GFT action.
- The classical singularity is resolved and is generically replaced by a bounce. Also, the LQC effective Friedmann equations are (almost) recovered for a natural choice of the condensate wave function.

# Outlook

There are many open questions, including:

- Study other condensate wave functions and GFT actions [Gielen; Pithis, Sakellariadou],
- Details of the semi-classical regime at late times [Gielen],
- Calculate error in higher order Schwinger-Dyson equations,
- Allow for scalar fields with non-vanishing potentials $V(\phi)$,
- Include spatial curvature, anisotropies and perturbations,
- Study the regime of large interactions [de Cesare, Pithis, Sakellariadou, Tomov],
- Include connectivity information in the analysis,
- Understand the physical interpretation of the $E_j$ term,
- And many more. . .