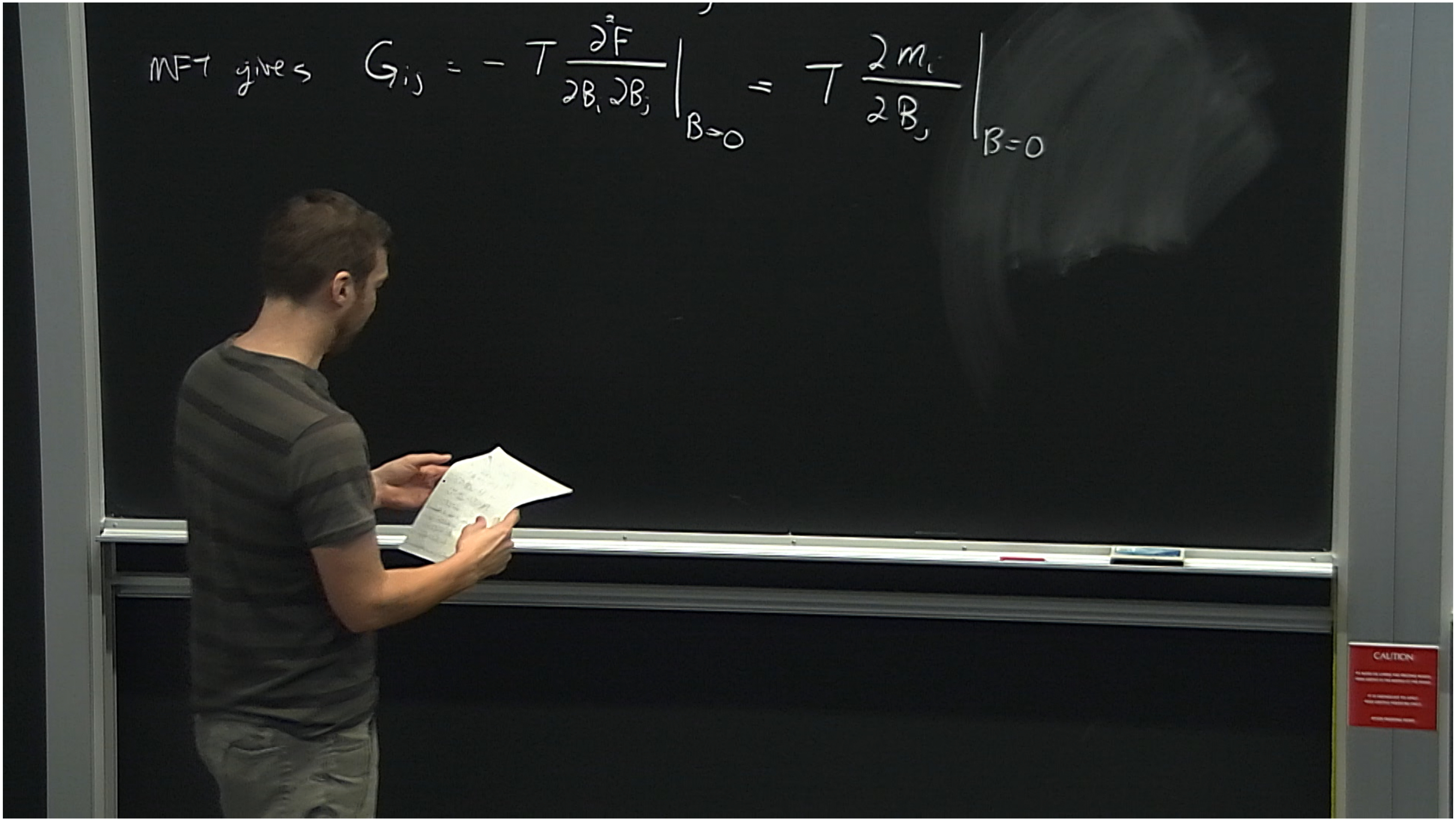
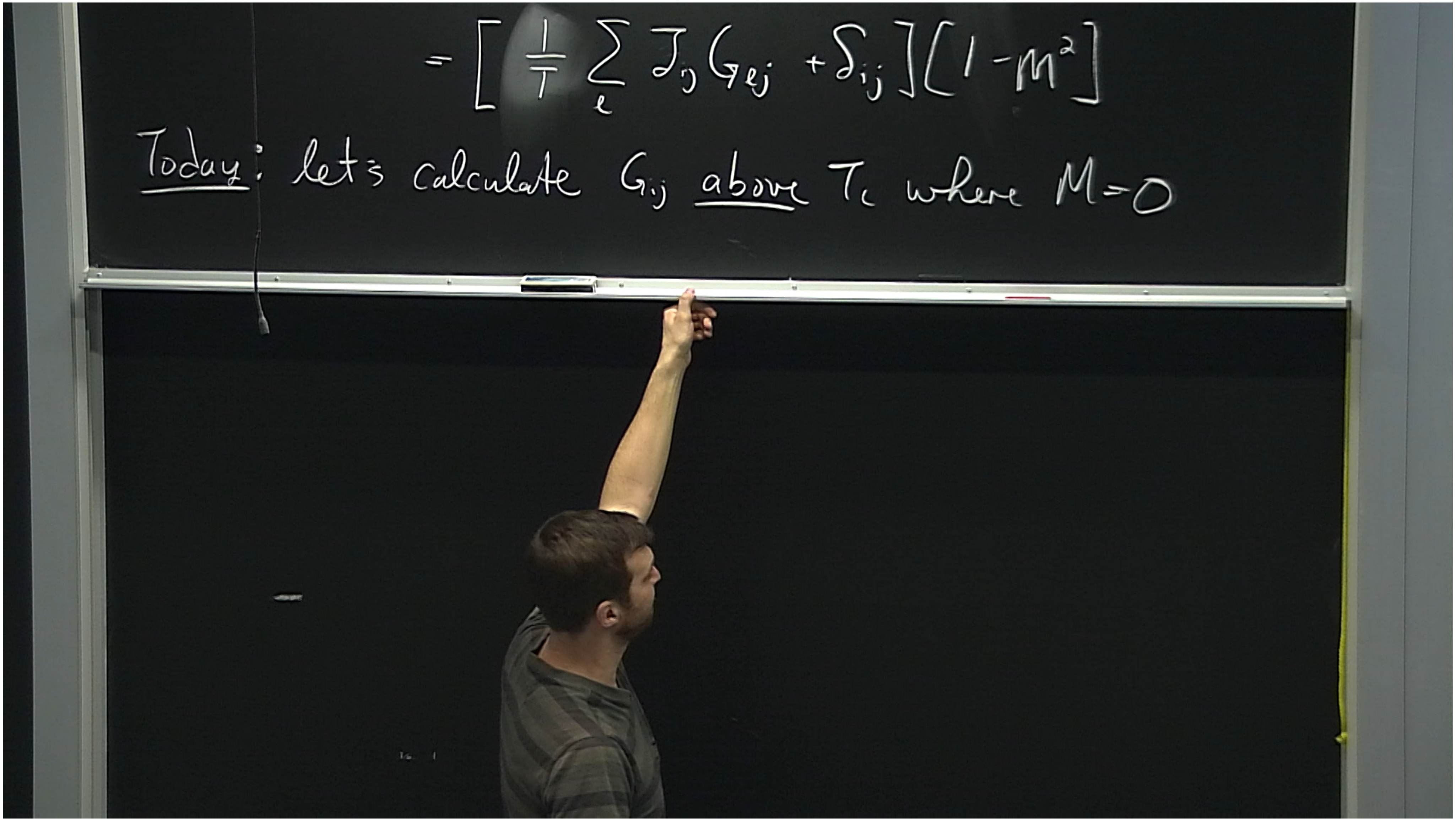
Title: 2016/2017 Statistical Mechanics 2 - Roger Melko - Lecture 5

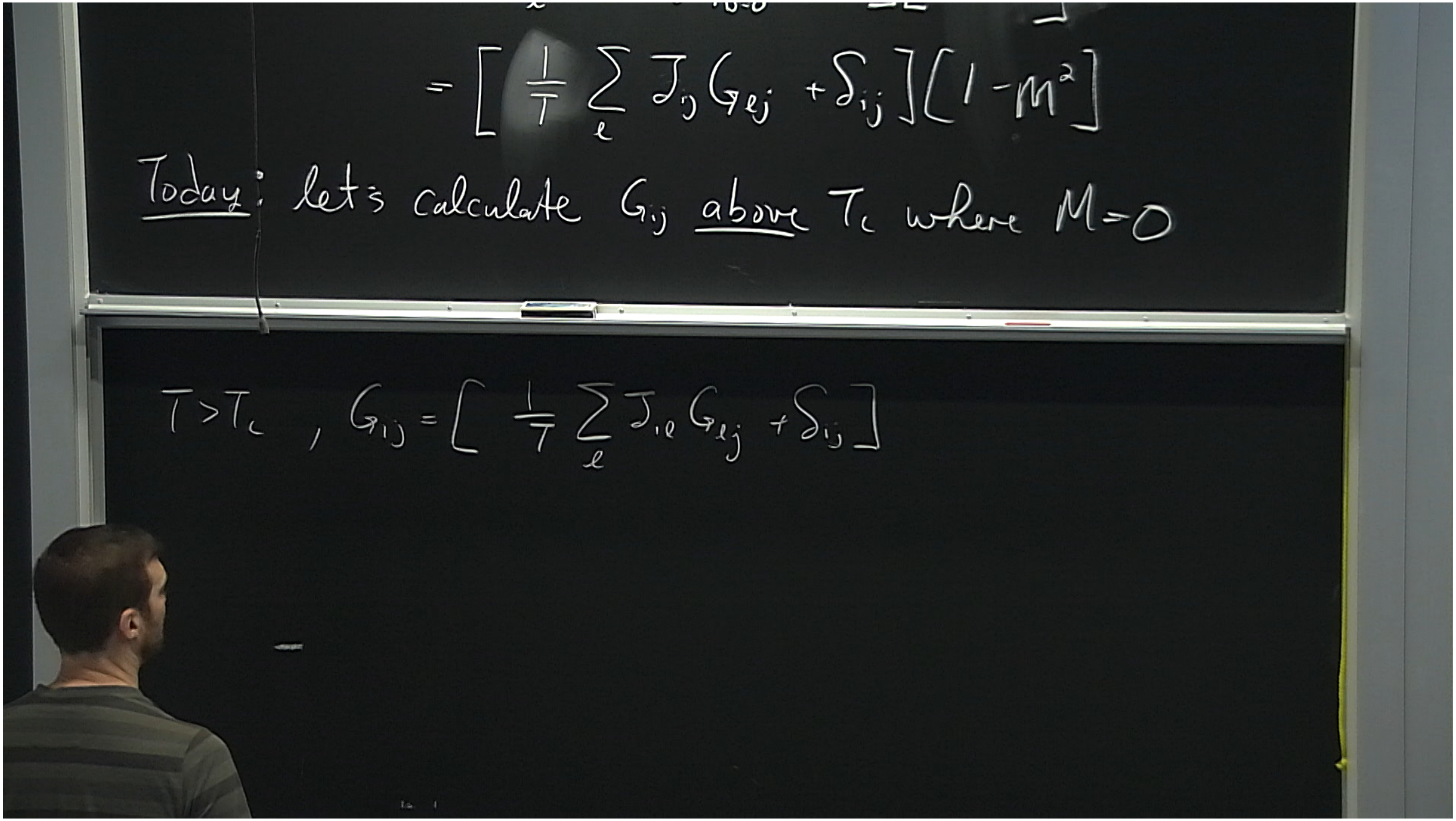
Date: Jan 18, 2017 10:30 AM

URL: http://pirsa.org/17010051

Abstract:


MFT gives $G_{ij} = -T \frac{\partial^2 F}{\partial B_i \partial B_j}\Big|_{B=0} = T \frac{\partial m_i}{\partial B_j}\Big|_{B=0}$
CAUTION


$= \left[\frac{1}{T} \sum_{l} J_{il} G_{lj} + \delta_{ij} \right] \left[1 - m^2 \right]$
Today: let's calculate G_{ij} above T_c where $M = 0$



$= \left[\frac{1}{T} \sum_{l} J_{ij} G_{lj} + \delta_{ij} \right] \left[1 - m^2 \right]$
Today: let's calculate G_{ij} above T_c where $M=0$
$T > T_c$, $G_{ij} = \left[\frac{1}{T} \sum_{l} J_{il} G_{lj} + \delta_{ij} \right]$

$T > T_c$, $G_{ij} = \left[\frac{1}{T} \sum_{\ell} J_{i\ell} G_{\ell j} + \delta_{ij} \right]$

$$G_{ij} - \frac{1}{T} \sum_{\ell} J_{i\ell} G_{\ell j} = \delta_{ij}$$

$$\sum_{\ell} \left[\delta_{i\ell} - \frac{1}{T} J_{i\ell} \right] G_{\ell j} = \delta_{ij}$$

↙ Solve this equation using Fourier transform.

Introduce the lattice F.T.

$T > T_c$, $G_{ij} = \left[\frac{1}{T} \sum_{l} J_{il} G_{lj} + \delta_{ij} \right]$

$$G_{ij} - \frac{1}{T} \sum_{l} J_{il} G_{lj} = \delta_{ij}$$

$$\sum_{l} \left[\delta_{il} - \frac{1}{T} J_{il} \right] G_{lj} = \delta_{ij}$$

↙ Solve this equation using Fourier transform.

Introduce the lattice F.T.

$$\varphi_i = \frac{1}{N} \sum_{k} \varphi_k \, e^{i \vec{k} \cdot \vec{r}_i}$$

$T > T_c$, $G_{ij} = \left[\frac{1}{T} \sum_{l} J_{il} G_{lj} + \delta_{ij} \right]$

$$G_{ij} - \frac{1}{T} \sum_{l} J_{il} G_{lj} = \delta_{ij}$$

$$\sum_{l} \left[\delta_{il} - \frac{1}{T} J_{il} \right] G_{lj} = \delta_{ij}$$

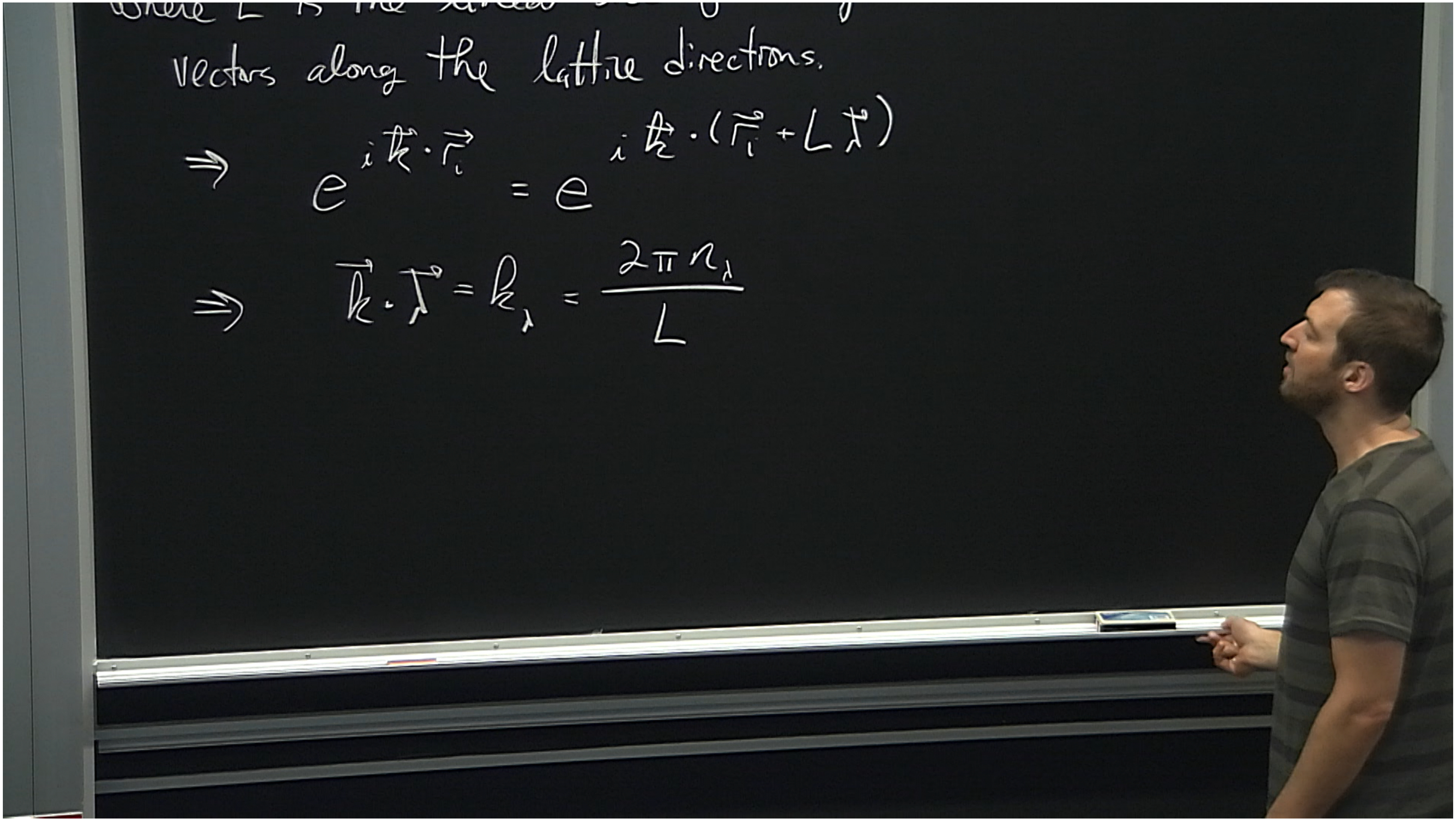
Solve this equation using Fourier transform.

<u>Introduce the lattice F.T.</u>

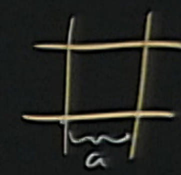
$$\varphi_i = \frac{1}{N} \sum_{k} \varphi_k e^{i \vec{k} \cdot \vec{r}_i}$$

assume Periodic Boundary conditions

$$\varphi(\vec{r}_i) \equiv \varphi_i = \varphi(\vec{r}_i + L\vec{\lambda})$$


vectors along the lattice directions.
$\Rightarrow \quad e^{i\vec{k}\cdot\vec{r}_i} = e^{i\vec{k}\cdot(\vec{r}_i + L\hat{\lambda})}$
$\Rightarrow \quad \vec{k}\cdot\hat{\lambda} = k_\lambda = \frac{2\pi n_\lambda}{L}$

$$\Rightarrow \quad e^{i\vec{k}\cdot\vec{r}_i} = e^{i\vec{k}\cdot(\vec{r}_i + L\vec{\lambda})}$$

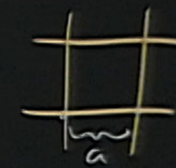


$$\Rightarrow \quad \vec{k}\cdot\vec{\lambda} = k_\lambda = \frac{2\pi n_\lambda}{L}$$ and n_λ is an integer

<u>where</u> $\vec{r}_i = \sum_{\vec{\lambda}} l_i \, a \, \vec{\lambda}$ where $\left\{ \begin{array}{l} a \text{ is the lattice "constant"} \\ \text{and } l_i \text{ is an integer.} \end{array} \right.$

vectors along the lattice directions.

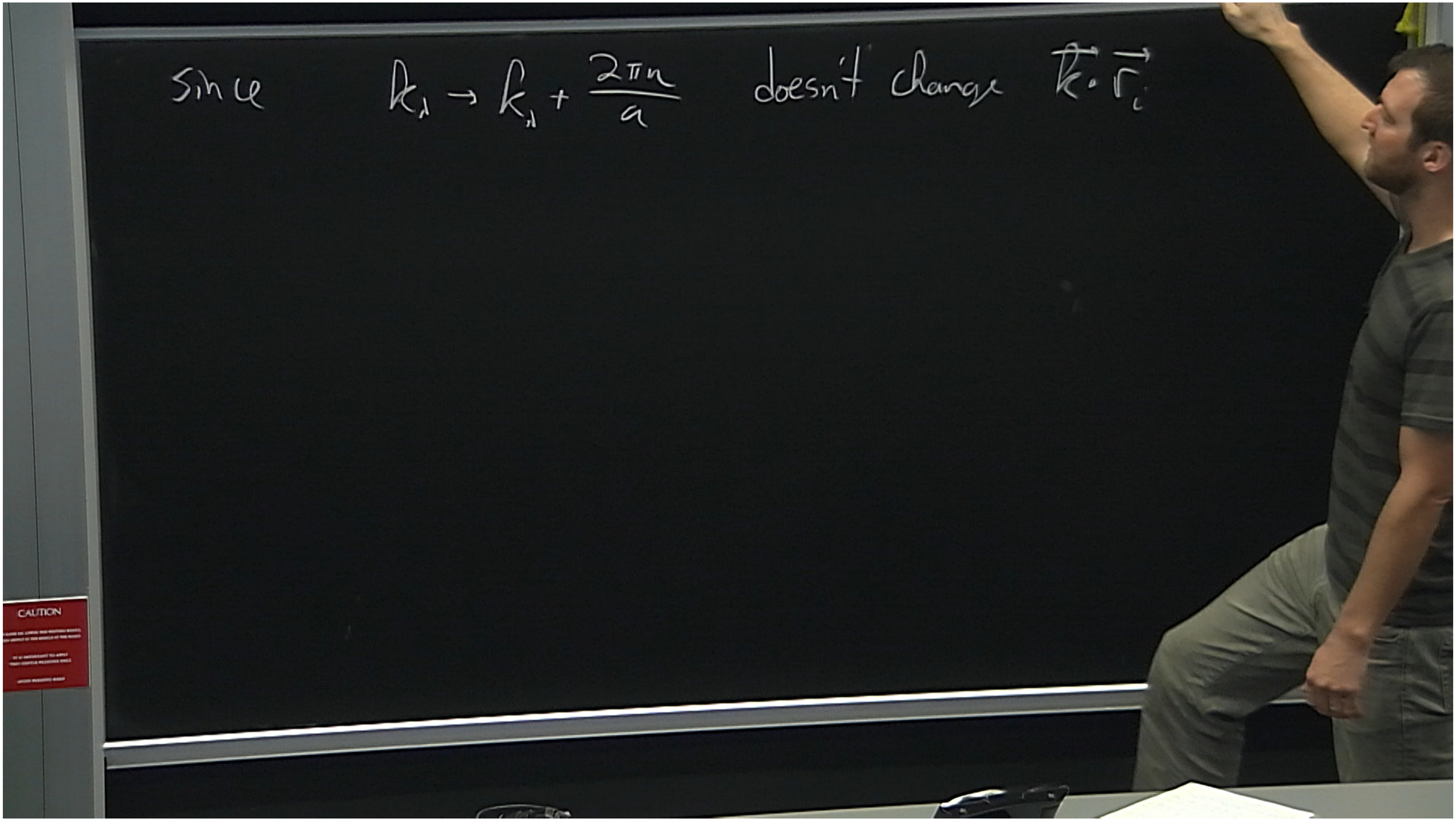
$$\Rightarrow \quad e^{i\vec{k}\cdot\vec{r}_i} = e^{i\vec{k}\cdot(\vec{r}_i + L\hat{\lambda})}$$

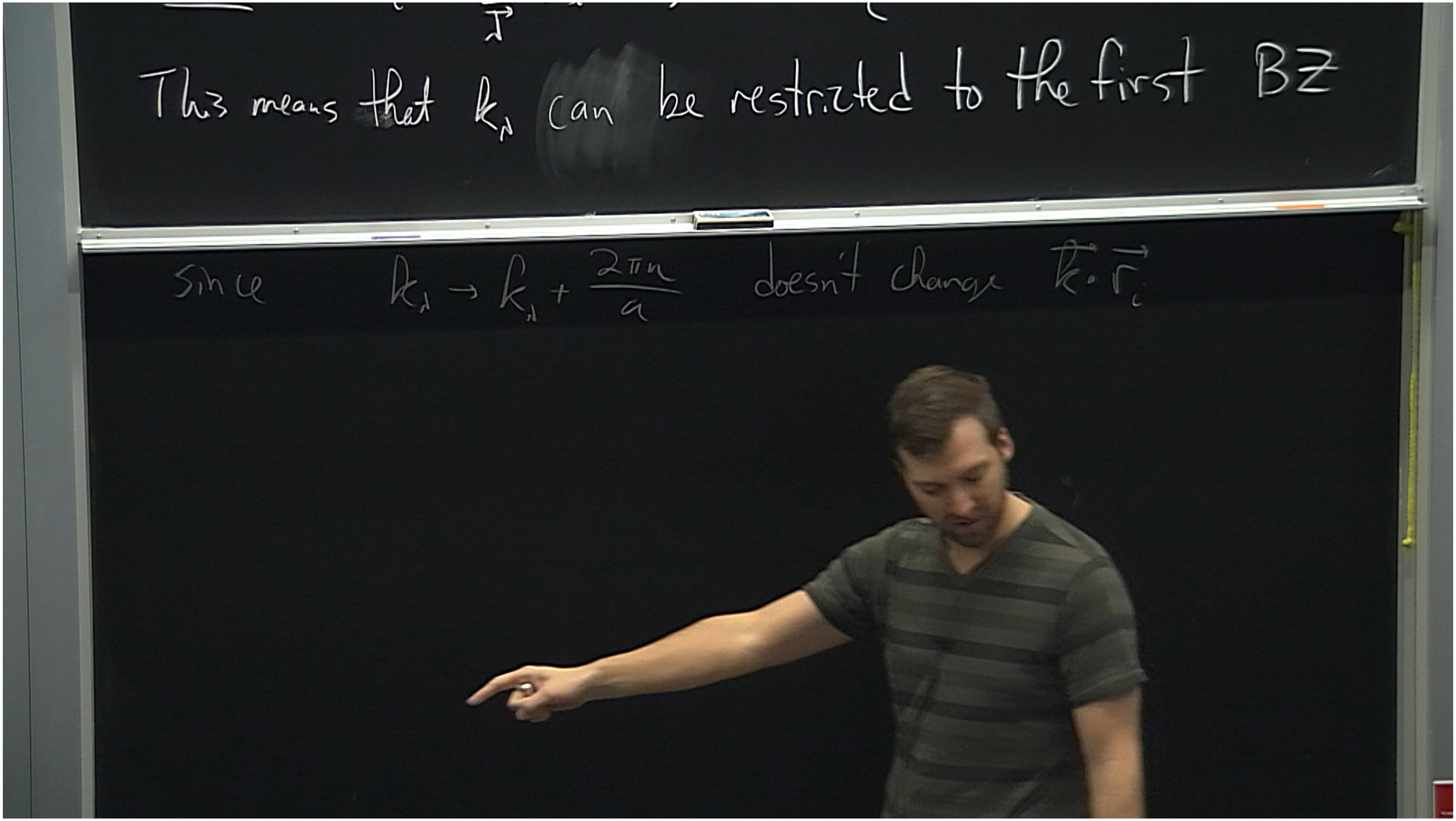


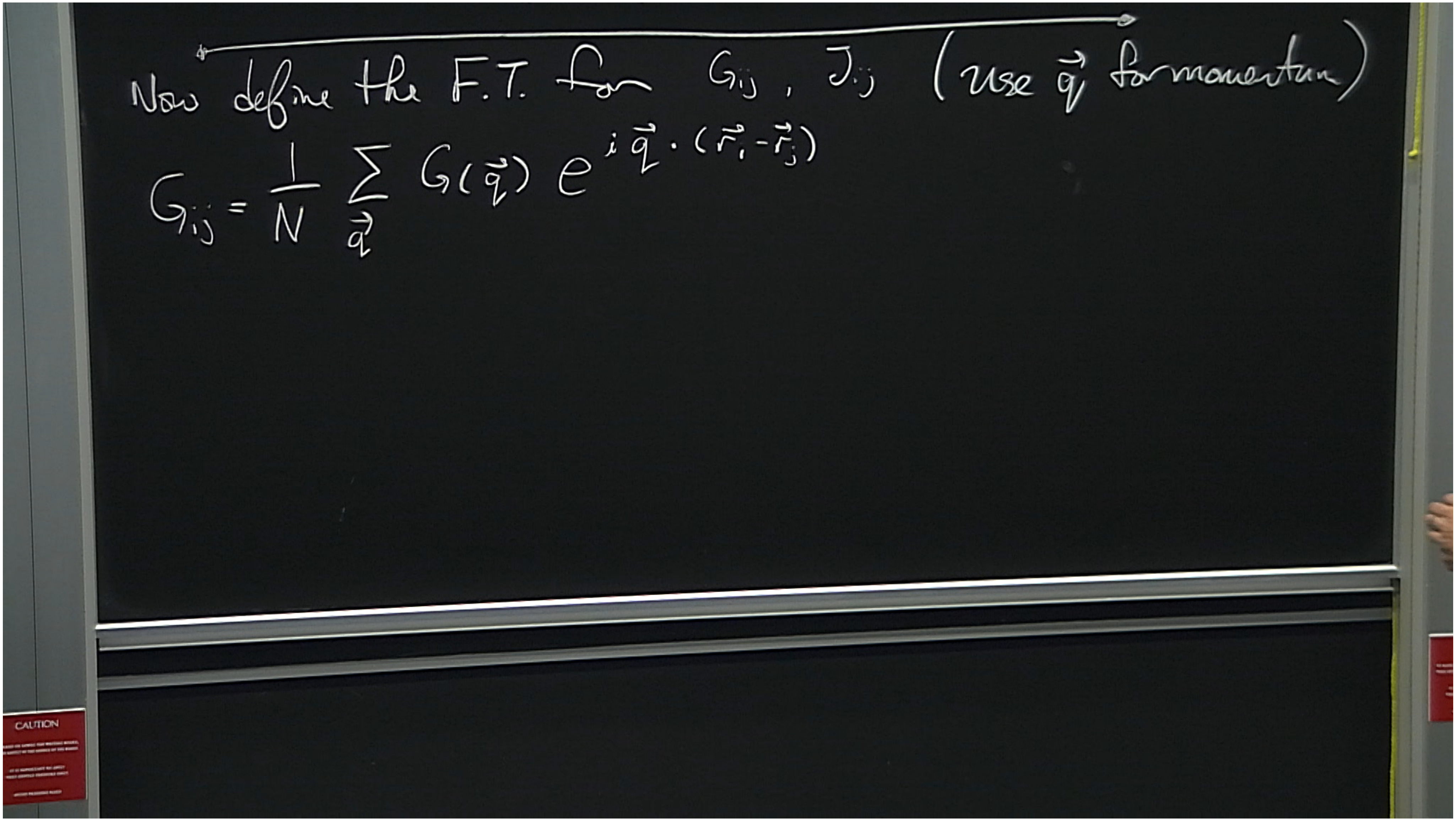
$$\Rightarrow \quad \vec{k}\cdot\hat{\lambda} = k_\lambda = \frac{2\pi n_\lambda}{L}$$ and n_λ is an integer

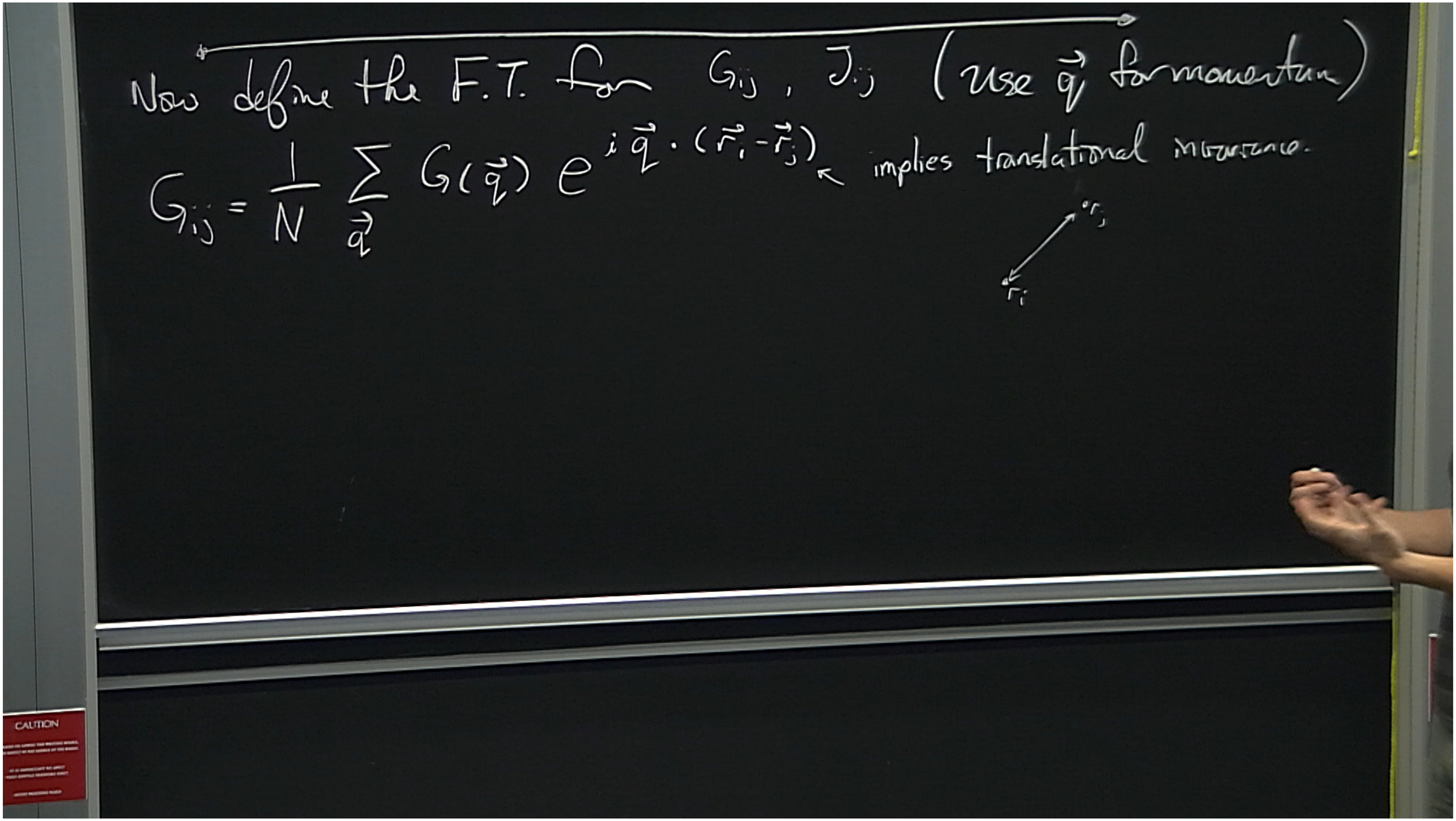
where $\vec{r}_i = \sum_{\hat{\lambda}} l_i \, a \, \hat{\lambda}$ where { a is the lattice "constant" and l_i is an integer.

This means that k_λ can be restricted to the first BZ


since $k_x \to k_x + \frac{2\pi n}{a}$ doesn't change $\vec{k}\cdot\vec{r}_i$
CAUTION


This means that k_λ can be restricted to the first BZ
Since $k_\lambda \to k_\lambda + \frac{2\pi n}{a}$ doesn't change $\vec{k}\cdot\vec{r}_i$

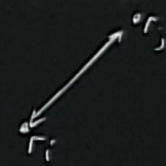

Now define the F.T. for G_{ij}, J_{ij} (use $\vec{q}$ for momentum)
$G_{ij} = \frac{1}{N} \sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$
CAUTION


Now define the F.T. for G_{ij} , J_{ij} (use $\vec{q}$ for momentum)
$G_{ij} = \frac{1}{N} \sum_{\vec{q}} G(\vec{q})\, e^{i \vec{q} \cdot (\vec{r}_i - \vec{r}_j)}$ ← implies translational invariance.
r_j
r_i

Now define the F.T. for G_{ij} , J_{ij} (use $\vec{q}$ for momentum)

$$G_{ij} = \frac{1}{N} \sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

← implies translational invariance.



$$J_{ij} = \frac{1}{N} \sum_{\vec{q}} J(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

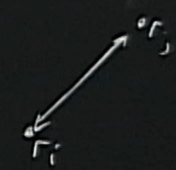
$$G_{ij} - \frac{1}{T} \sum_{l} J_{il} G_{lj} = \delta_{ij}$$

sub in the above expressions.

Now define the F.T. for G_{ij}, J_{ij} (use $\vec{q}$ for momentum)

$$G_{ij} = \frac{1}{N} \sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

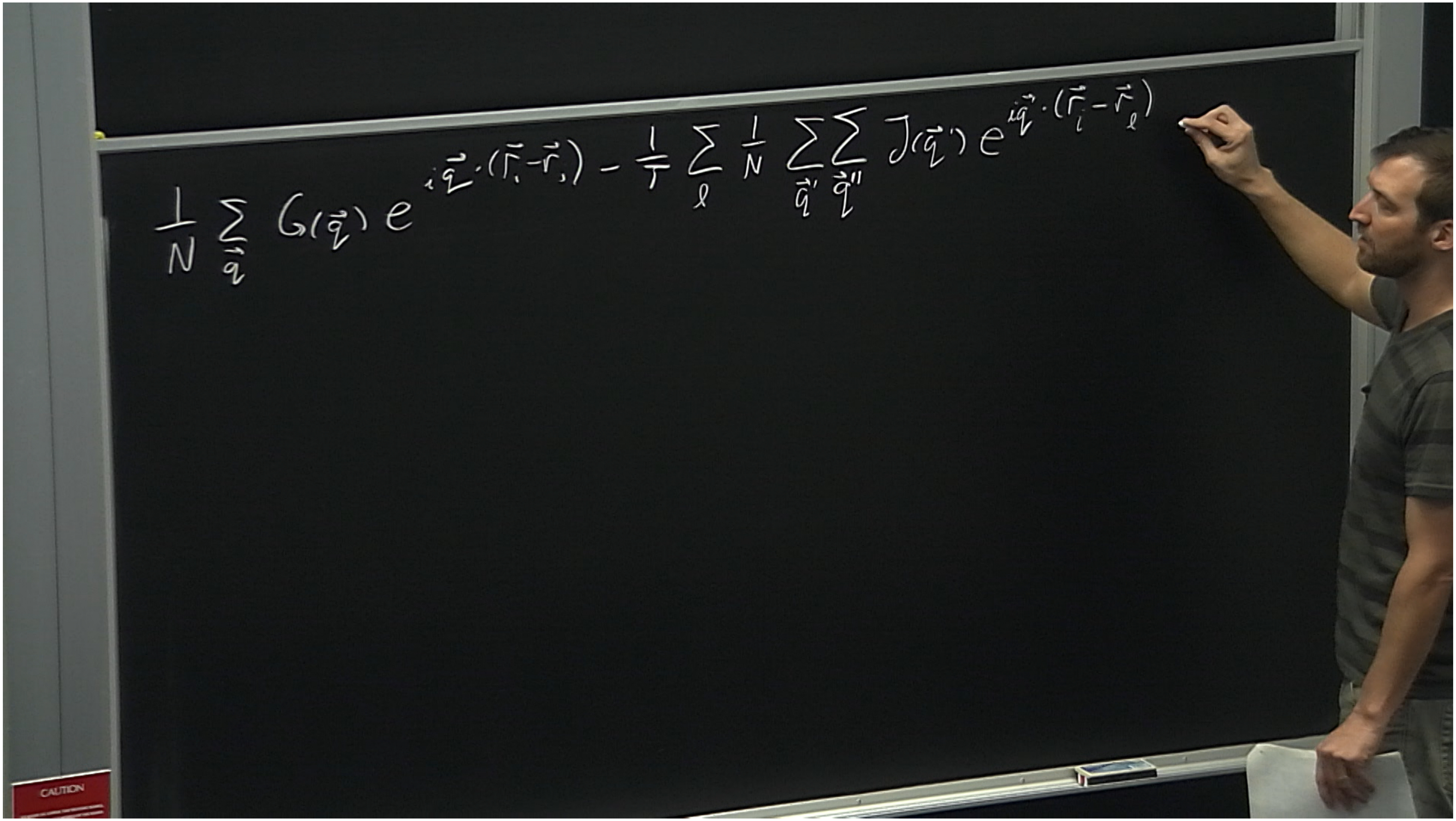
← implies translational invariance.



$$J_{ij} = \frac{1}{N} \sum_{\vec{q}} J(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

$$G_{ij} - \frac{1}{T} \sum_{l} J_{il} G_{lj} = \delta_{ij}$$

sub in the above expressions.


$\frac{1}{N}\sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)} - \frac{1}{T}\sum_{l}\frac{1}{N}\sum_{\vec{q}'}\sum_{\vec{q}''} J(\vec{q}')\, e^{i\vec{q}'\cdot(\vec{r}_i-\vec{r}_l)}$
CAUTION

$$\frac{1}{N}\sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)} - \frac{1}{T}\sum_{\ell}\frac{1}{N}\sum_{\vec{q}'}\sum_{\vec{q}''} J(\vec{q}')\, e^{i\vec{q}'\cdot(\vec{r}_i-\vec{r}_\ell)}\, G(\vec{q}'')$$

$$e^{i\vec{q}''\cdot(\vec{r}_j-\vec{r}_\ell)}$$

$$\frac{1}{N}\sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)} - \frac{1}{T}\sum_{l}\frac{1}{N^2}\sum_{\vec{q}'}\sum_{\vec{q}''} J(\vec{q}')\, e^{i\vec{q}'\cdot(\vec{r}_i-\vec{r}_l)}\, G(\vec{q}'')$$

$$e^{i\vec{q}''\cdot(\vec{r}_j-\vec{r}_l)}$$

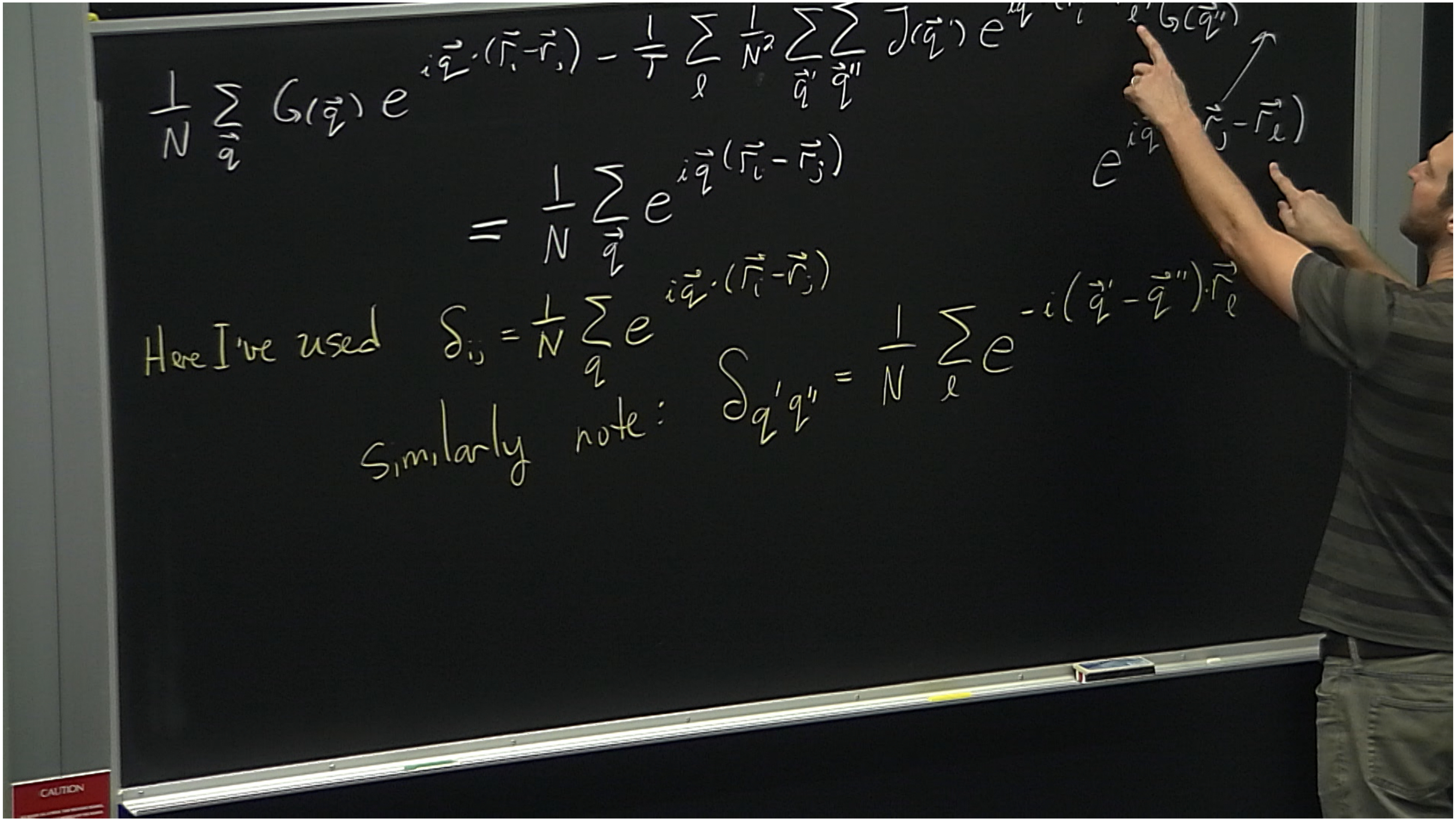
$$= \frac{1}{N}\sum_{\vec{q}} e^{i\vec{q}(\vec{r}_i-\vec{r}_j)}$$

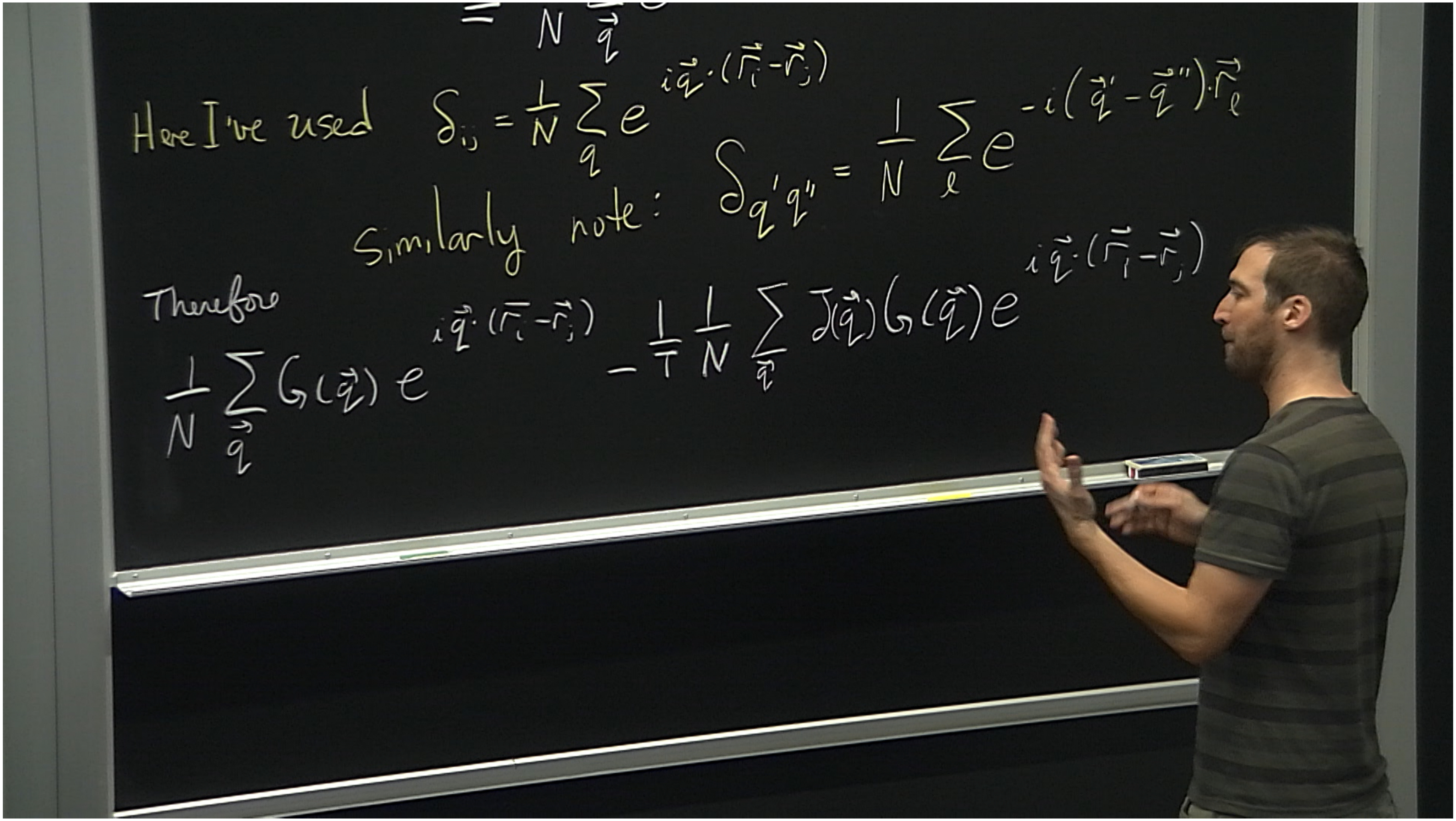
$N \sum_{\vec{q}}$ ... $e^{i\vec{q}''(\vec{r}_j - \vec{r}_\ell)}$

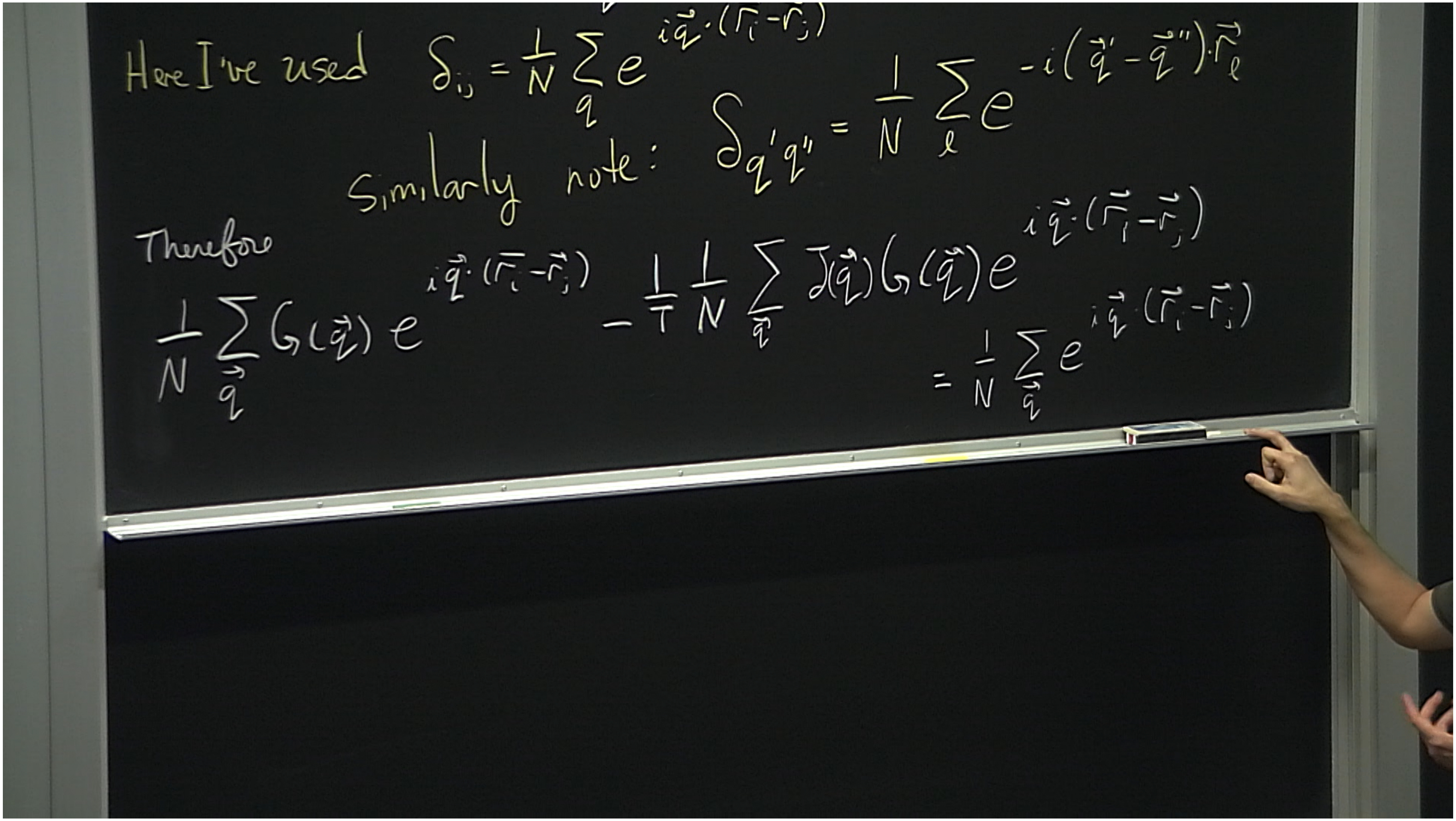
$$= \frac{1}{N}\sum_{\vec{q}} e^{i\vec{q}(\vec{r}_i - \vec{r}_j)}$$

Here I've used $\delta_{ij} = \frac{1}{N}\sum_{q} e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$

Similarly note: $\delta_{q'q''} = \frac{1}{N}\sum_{\ell} e^{-i(\vec{q}' - \vec{q}'')\cdot\vec{r}_\ell}$


Here I've used
Similarly note:
$\frac{1}{N}\sum_{\vec q} G(\vec q)\, e^{i\vec q\cdot(\vec r_i-\vec r_j)} - \frac{1}{T}\sum_{\ell}\frac{1}{N^2}\sum_{\vec q'}\sum_{\vec q''} J(\vec q')\, e$
$= \frac{1}{N}\sum_{\vec q} e^{i\vec q(\vec r_i-\vec r_j)}$
$\delta_{ij} = \frac{1}{N}\sum_{q} e^{i\vec q\cdot(\vec r_i-\vec r_j)}$
$\delta_{q'q''} = \frac{1}{N}\sum_{\ell} e^{-i(\vec q'-\vec q'')\cdot\vec r_\ell}$


Here I've used $\delta_{ij} = \frac{1}{N}\sum_{q} e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$
Similarly note: $\delta_{q'q''} = \frac{1}{N}\sum_{\ell} e^{-i(\vec{q}' - \vec{q}'')\cdot\vec{r}_\ell}$
Therefore
$\frac{1}{N}\sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} - \frac{1}{T}\frac{1}{N}\sum_{\vec{q}} J(\vec{q})\, G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$

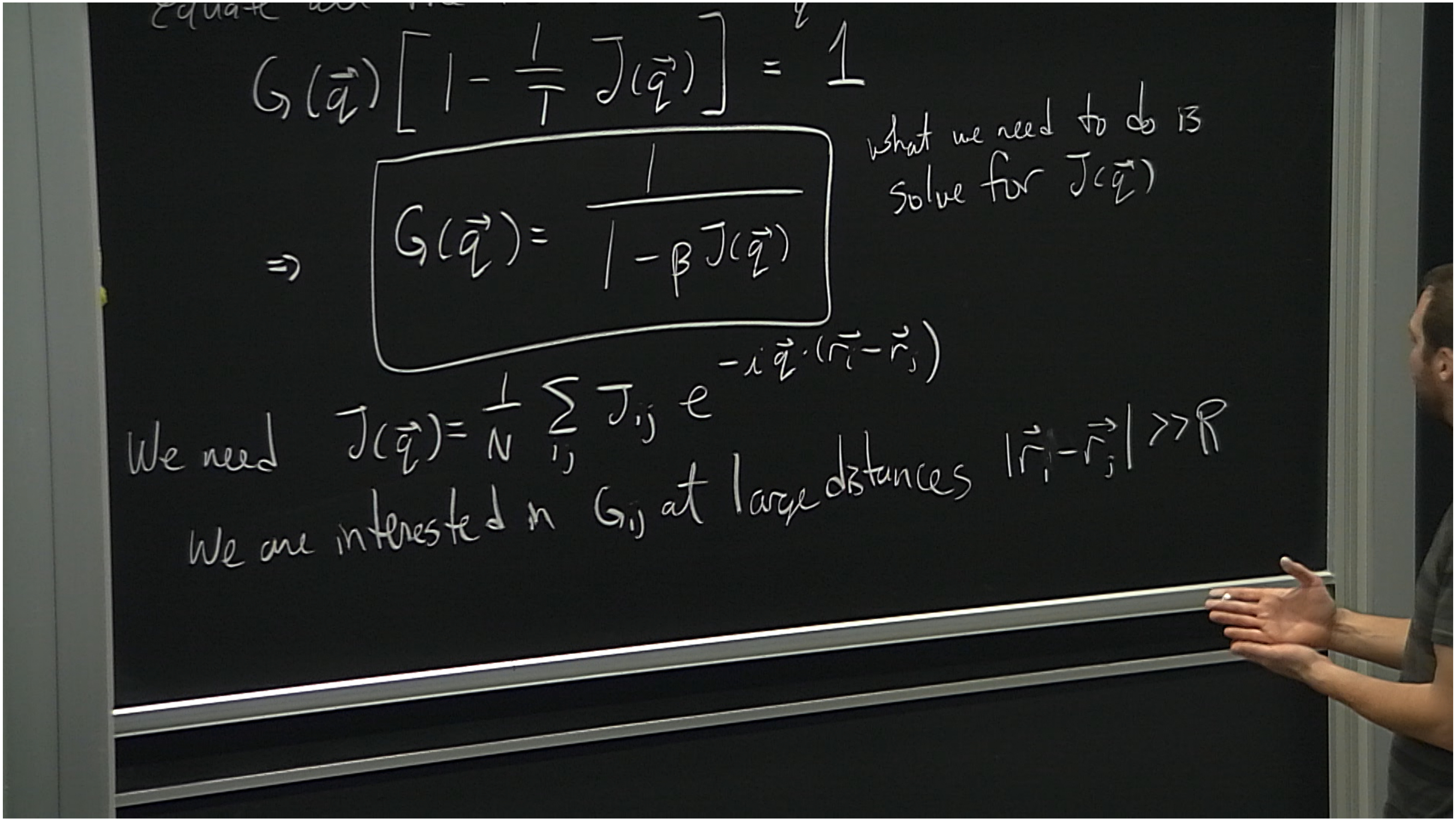


Here I've used $\delta_{ij} = \frac{1}{N}\sum_q e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$
Similarly note: $\delta_{q'q''} = \frac{1}{N}\sum_\ell e^{-i(\vec{q}' - \vec{q}'')\cdot\vec{r}_\ell}$
Therefore
$\frac{1}{N}\sum_{\vec{q}} G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} - \frac{1}{T}\frac{1}{N}\sum_{\vec{q}} J(\vec{q})\, G(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$
$= \frac{1}{N}\sum_{\vec{q}} e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$

Equate all the terms inside $\sum_{\vec{q}}$ to get

$$G(\vec{q})\left[1-\frac{1}{T}J(\vec{q})\right]=1$$

$$\Rightarrow \quad \boxed{G(\vec{q})=\frac{1}{1-\beta J(\vec{q})}}$$


$G(\vec{q})\left[1 - \frac{1}{T} J(\vec{q})\right] = 1$
what we need to do is solve for $J(\vec{q})$
$\Rightarrow \quad G(\vec{q}) = \frac{1}{1 - \beta J(\vec{q})}$
We need $J(\vec{q}) = \frac{1}{N} \sum_{i,j} J_{ij}\, e^{-i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$
We are interested in G_{ij} at large distances $|\vec{r}_i - \vec{r}_j| \gg R$

$$G(\vec{q})\left[1-\frac{1}{T}J(\vec{q})\right]=1$$

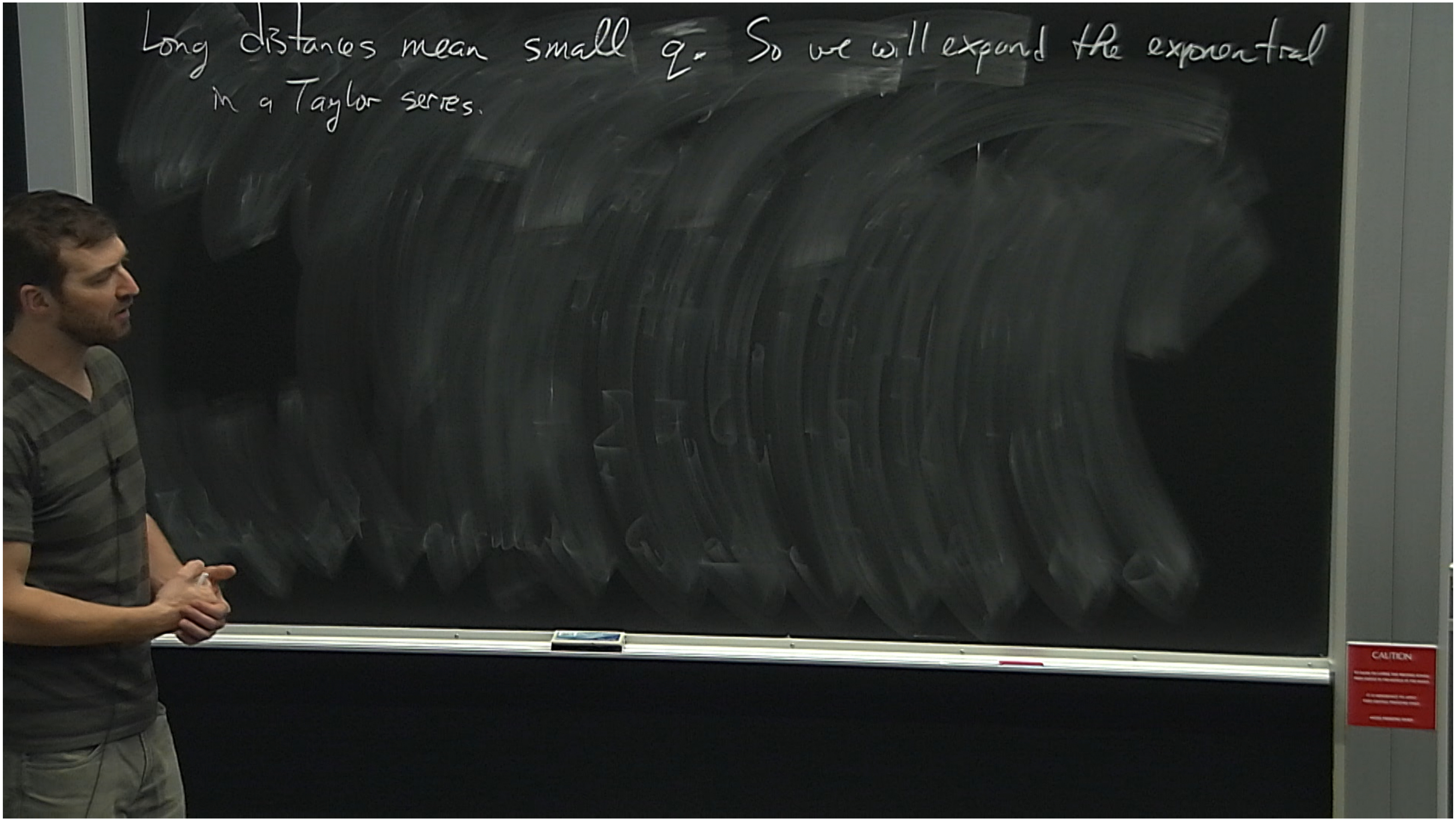
$$\Rightarrow \quad \boxed{G(\vec{q})=\frac{1}{1-\beta J(\vec{q})}}$$

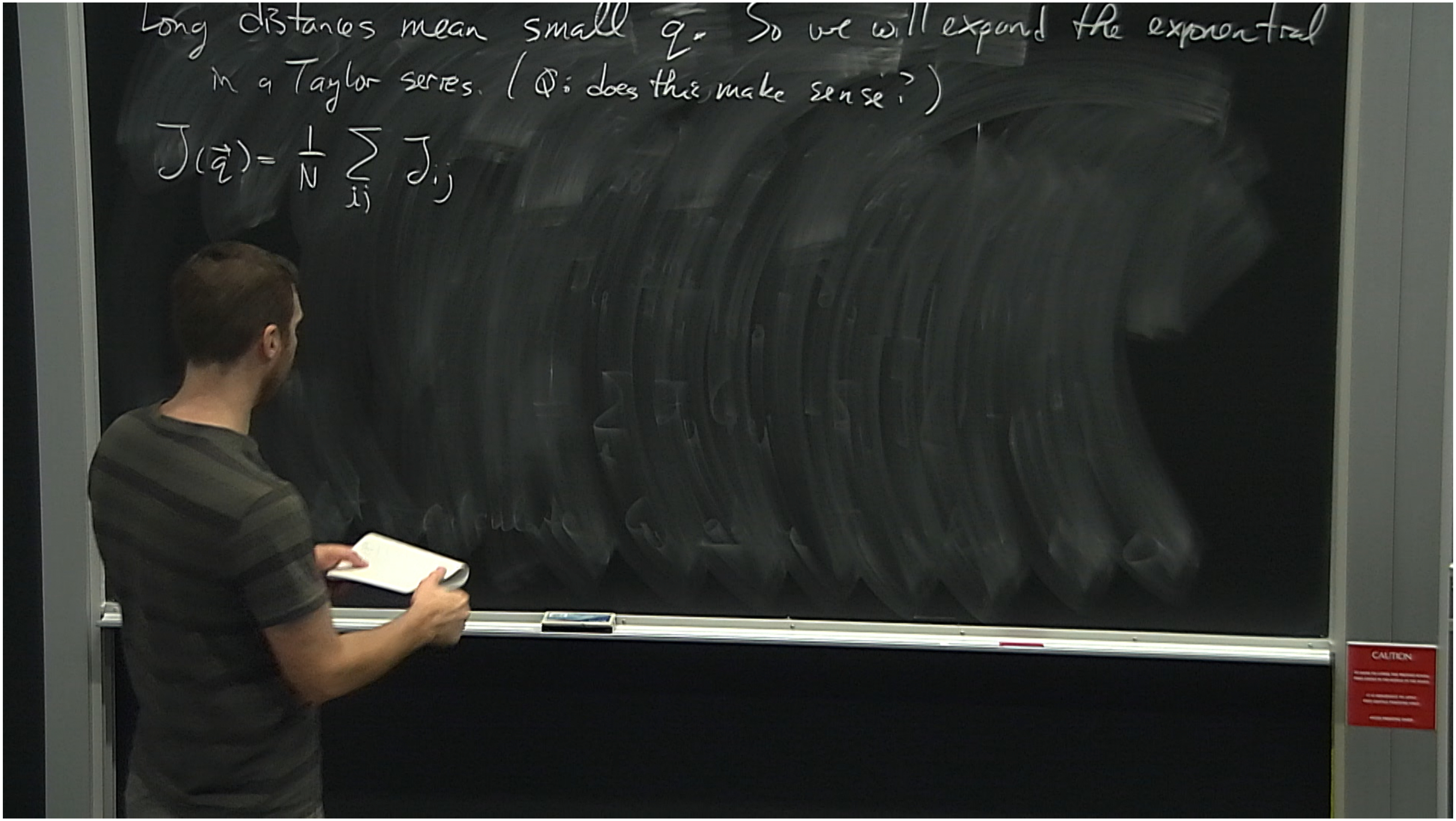
what we need to do is solve for $J(\vec{q})$

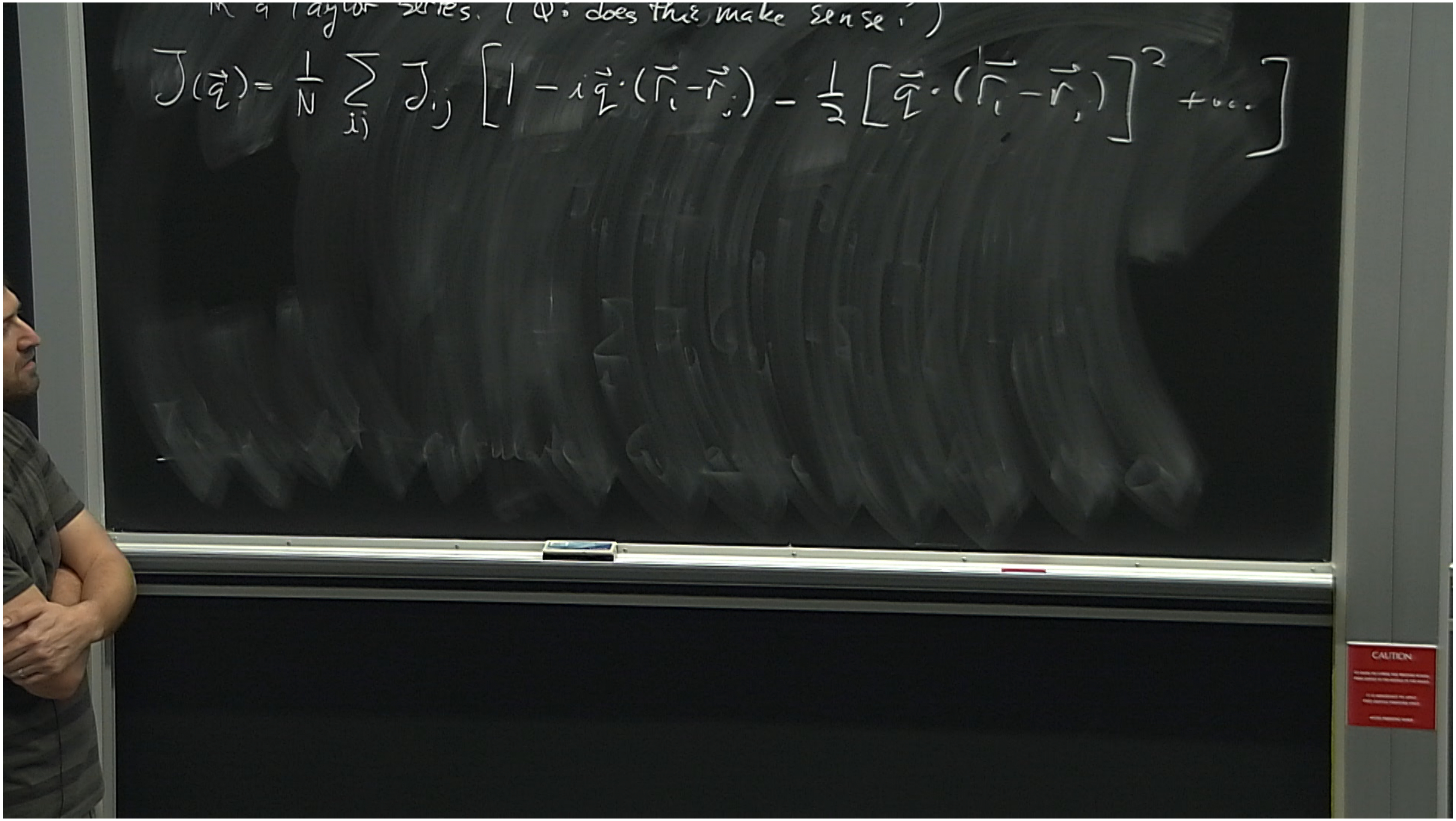
We need $J(\vec{q})=\frac{1}{N}\sum_{i,j} J_{ij}\, e^{-i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)}$

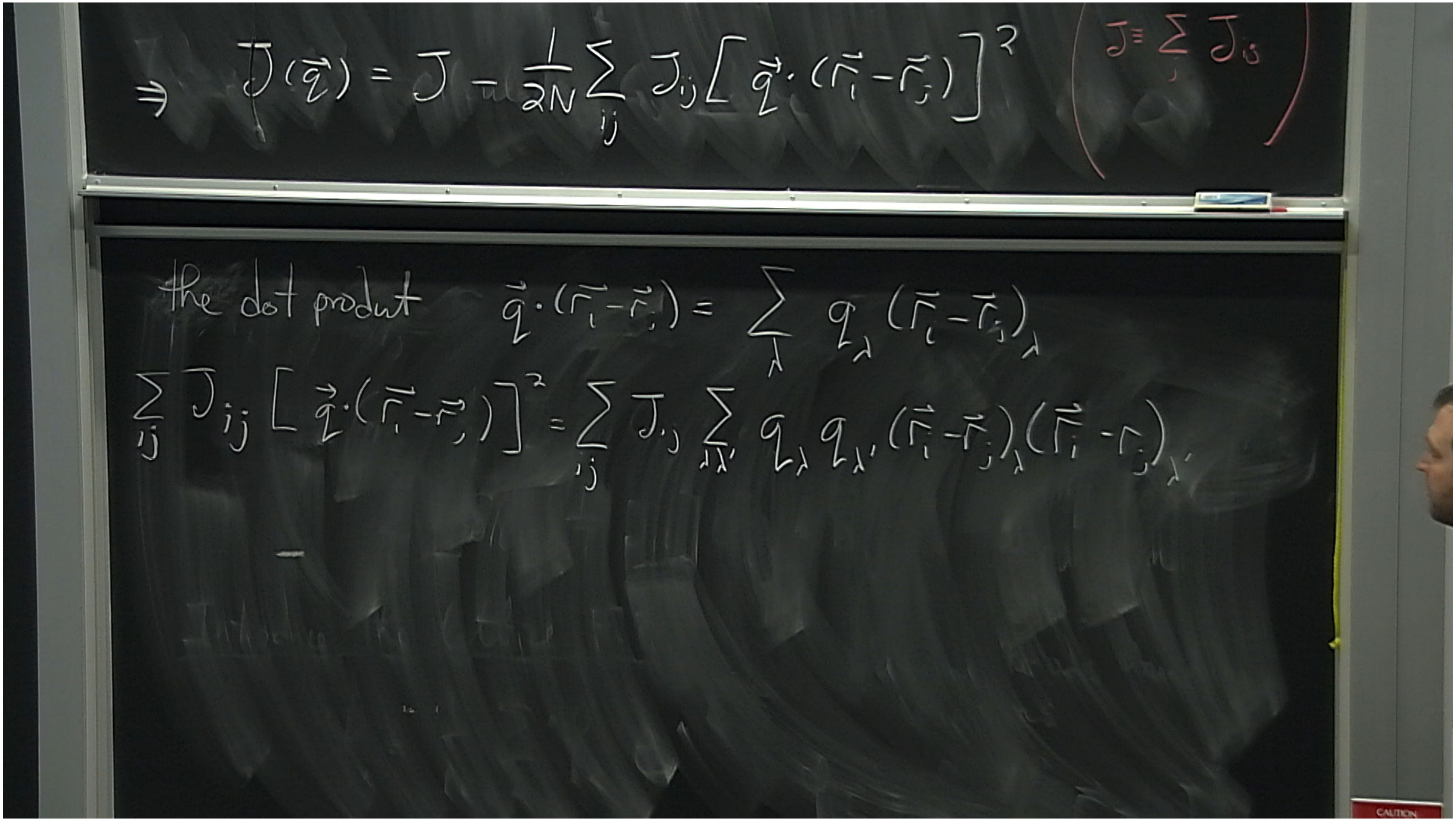
We are interested in G_{ij} at large distances $|\vec{r}_i-\vec{r}_j| \gg R$

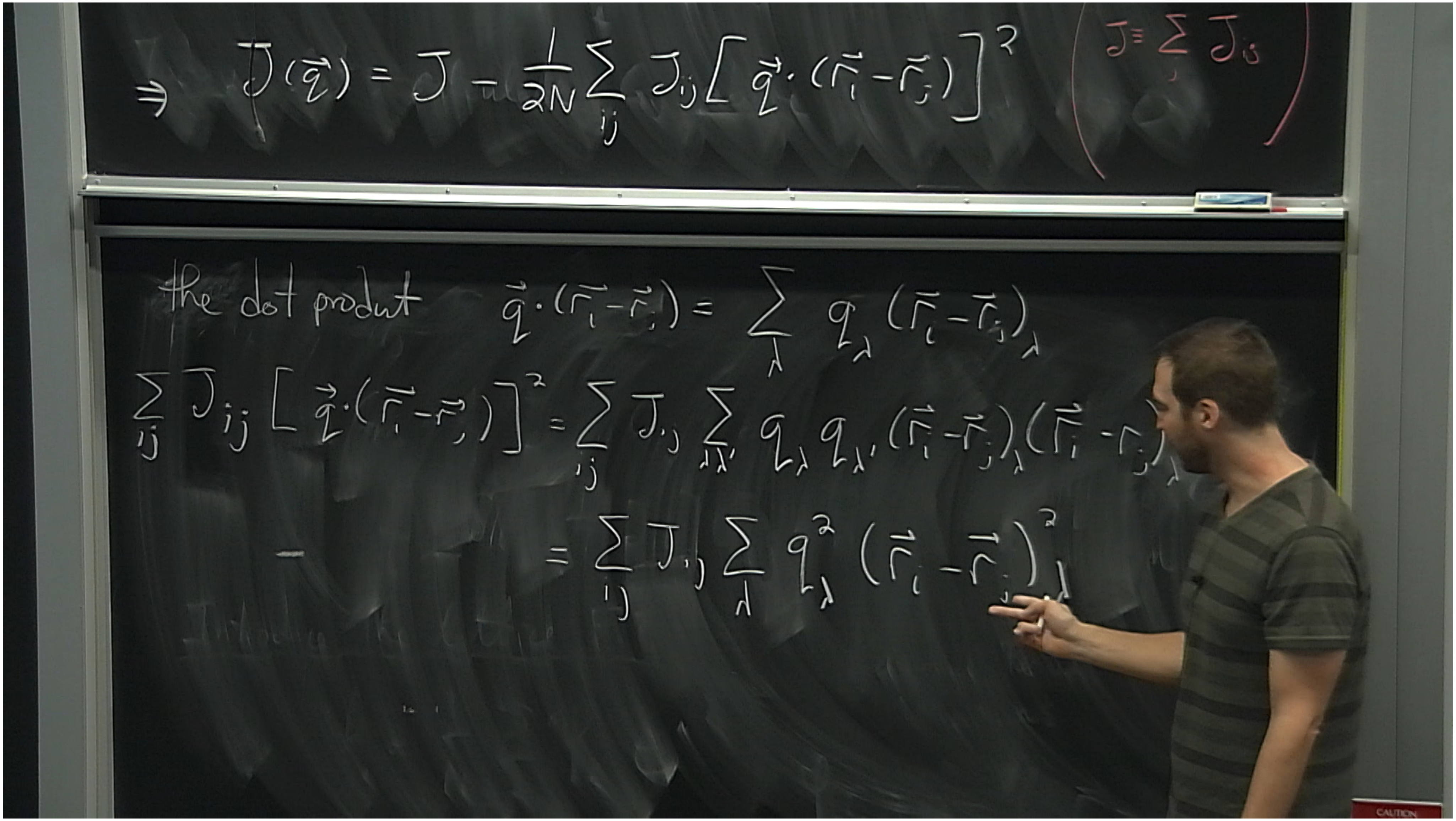
where R is the "effective range" of interactions J_{ij} (e.g. $R\sim a$)


Long distances mean small q. So we will expand the exponential
in a Taylor series.


Long distances mean small q. So we will expand the exponential
in a Taylor series. (Q: does this make sense?)
J(q⃗) = 1/N Σ_ij J_ij


in a Taylor series. (Q: does this make sense?)
$J(\vec{q}) = \frac{1}{N} \sum_{ij} J_{ij} \left[1 - i\vec{q}\cdot(\vec{r}_i - \vec{r}_j) - \frac{1}{2}\left[\vec{q}\cdot(\vec{r}_i - \vec{r}_j)\right]^2 + \dots \right]$
CAUTION


$\Rightarrow \ J(\vec{q}) = J - \frac{1}{2N}\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2$
$\left(J \equiv \sum_j J_{ij} \right)$
the dot product $\vec{q}\cdot(\vec{r}_i-\vec{r}_j) = \sum_\lambda q_\lambda (\vec{r}_i-\vec{r}_j)_\lambda$
$\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2 = \sum_{ij} J_{ij} \sum_{\lambda\lambda'} q_\lambda q_{\lambda'} (\vec{r}_i-\vec{r}_j)_\lambda (\vec{r}_i-\vec{r}_j)_{\lambda'}$


$\Rightarrow \quad J(\vec{q}) = J - \frac{1}{2N}\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2$
$\left(J \equiv \sum_j J_{ij} \right)$
the dot product $\vec{q}\cdot(\vec{r}_i-\vec{r}_j) = \sum_\lambda q_\lambda (\vec{r}_i-\vec{r}_j)_\lambda$
$\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2 = \sum_{ij} J_{ij} \sum_{\lambda\lambda'} q_\lambda q_{\lambda'} (\vec{r}_i-\vec{r}_j)_\lambda (\vec{r}_i-\vec{r}_j)_{\lambda'}$
$= \sum_{ij} J_{ij} \sum_\lambda q_\lambda^2 (\vec{r}_i-\vec{r}_j)_\lambda^2$

the dot product $\vec{q}\cdot(\vec{r}_i-\vec{r}_j) = \sum_{\lambda} q_{\lambda}(\vec{r}_i-\vec{r}_j)_{\lambda}$

$$\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2 = \sum_{ij} J_{ij}\sum_{\lambda\lambda'} q_{\lambda} q_{\lambda'}(\vec{r}_i-\vec{r}_j)_{\lambda}(\vec{r}_i-\vec{r}_j)_{\lambda'}$$

$$= \sum_{ij} J_{ij}\sum_{\lambda} q_{\lambda}^2(\vec{r}_i-\vec{r}_j)_{\lambda}^2 \quad \circledast$$

$$= \frac{1}{d}\sum_{ij} J_{ij}|\vec{r}_i-\vec{r}_j|^2 q^2 \quad (d \text{ dimensions})$$

call $K = \frac{1}{Nd}\sum_{ij} J_{ij}|\vec{r}_i-\vec{r}_j|^2 \Rightarrow \boxed{J(\vec{q}) \simeq J - \frac{1}{2}Kq^2}$

the dot product $\vec{q}\cdot(\vec{r}_i-\vec{r}_j) = \sum_\lambda q_\lambda (\vec{r}_i-\vec{r}_j)_\lambda$

$$\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2 = \sum_{ij} J_{ij} \sum_{\lambda\lambda'} q_\lambda q_{\lambda'} (\vec{r}_i-\vec{r}_j)_\lambda (\vec{r}_i-\vec{r}_j)_{\lambda'}$$

$$= \sum_{ij} J_{ij} \sum_\lambda q_\lambda^2 (\vec{r}_i-\vec{r}_j)_\lambda^2 \quad (*)$$

$$= \frac{1}{d}\sum_{ij} J_{ij} |\vec{r}_i-\vec{r}_j|^2 q^2 \quad (d \text{ dimensions})$$

call $K = \frac{1}{Nd}\sum_{ij} J_{ij}|\vec{r}_i-\vec{r}_j|^2 \Rightarrow \boxed{J(\vec{q}) \simeq J - \frac{1}{2}Kq^2}$

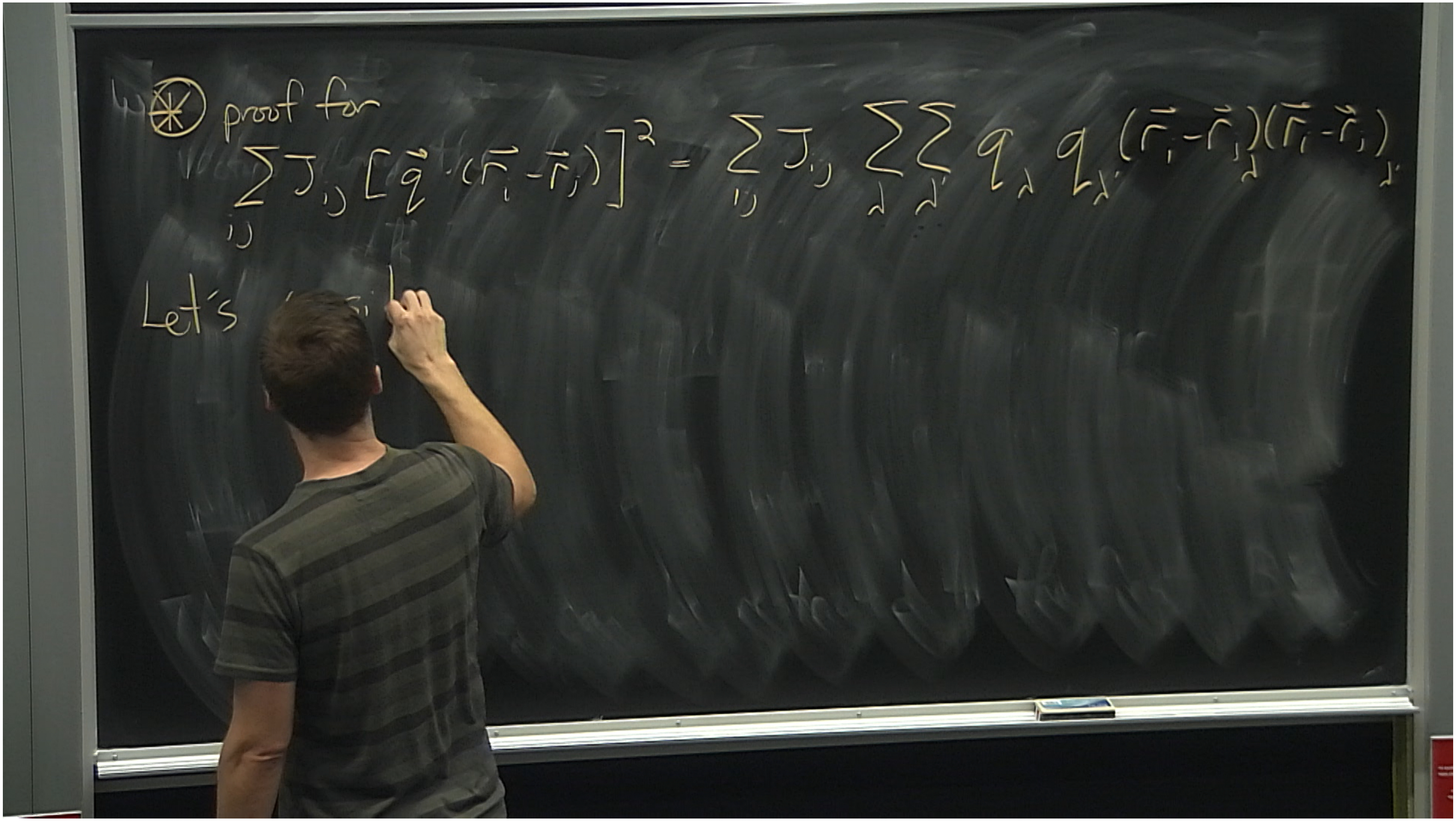
the dot product $\vec{q}\cdot(\vec{r}_i-\vec{r}_j) = \sum_{\lambda} q_{\lambda}(\vec{r}_i-\vec{r}_j)_{\lambda}$

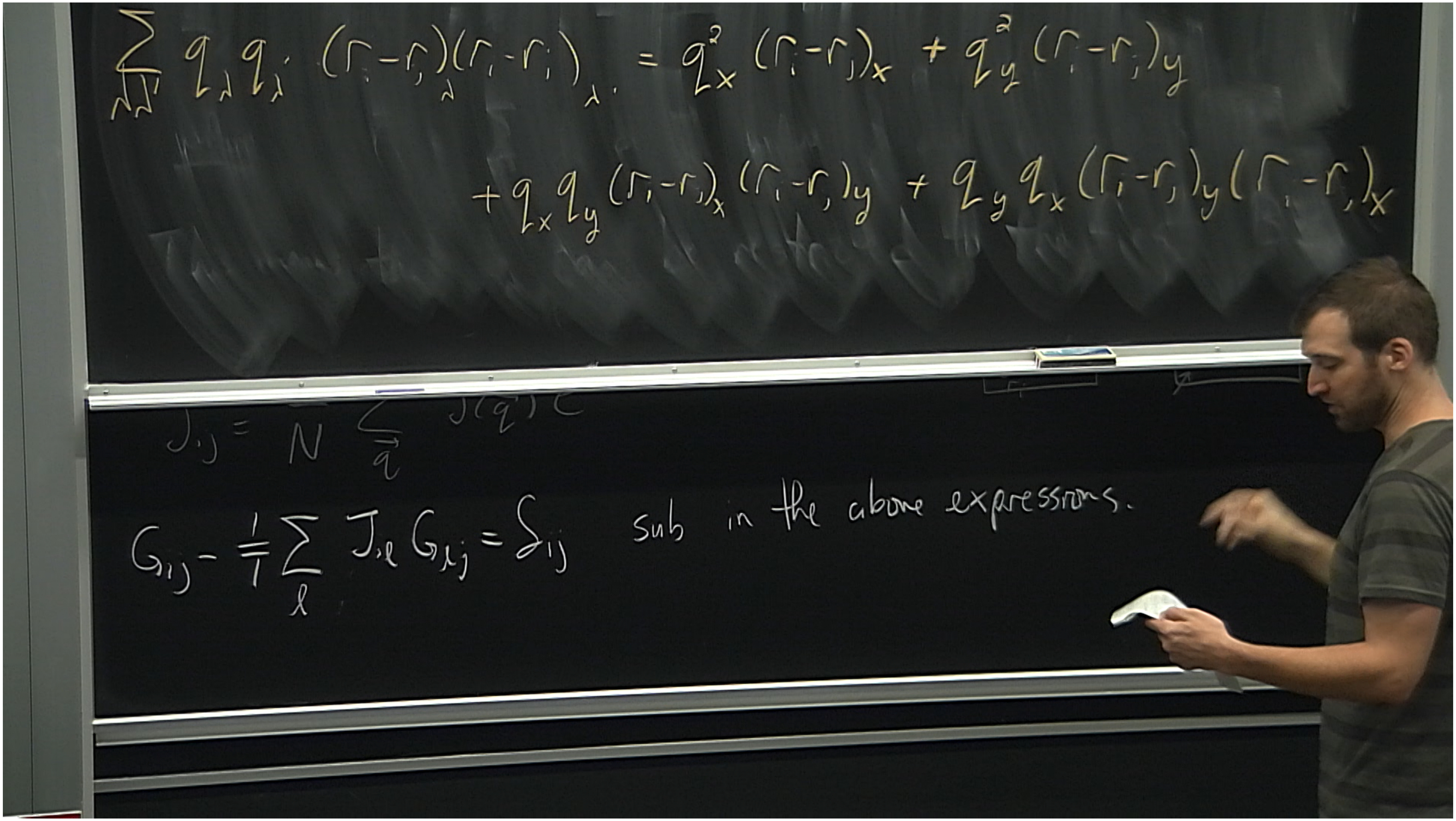
$$\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2 = \sum_{ij} J_{ij} \sum_{\lambda\lambda'} q_{\lambda} q_{\lambda'} (\vec{r}_i-\vec{r}_j)_{\lambda}(\vec{r}_i-\vec{r}_j)_{\lambda'}$$

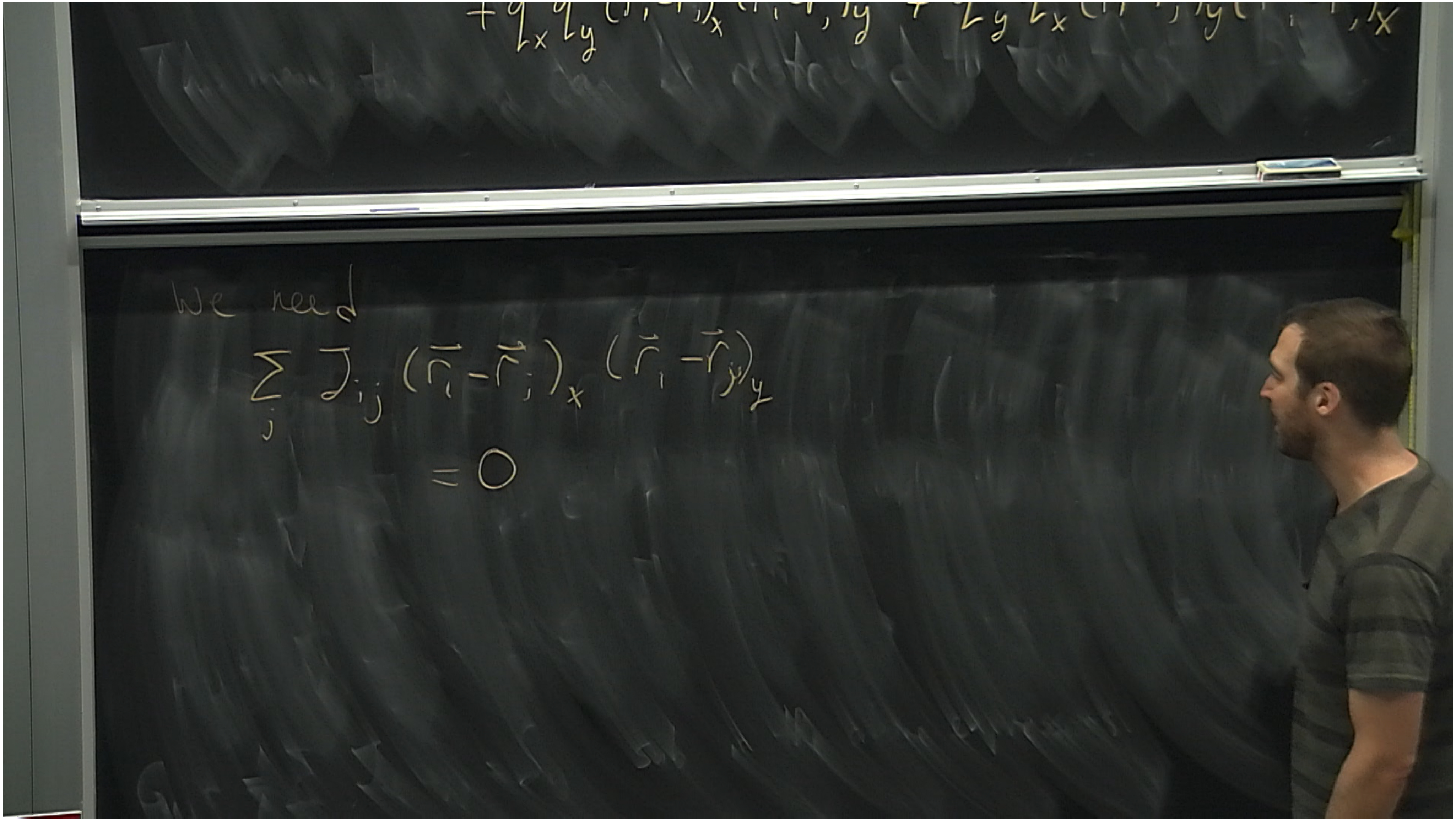
$$= \sum_{ij} J_{ij} \sum_{\lambda} q_{\lambda}^2 (\vec{r}_i-\vec{r}_j)_{\lambda}^2 \quad \circledast$$

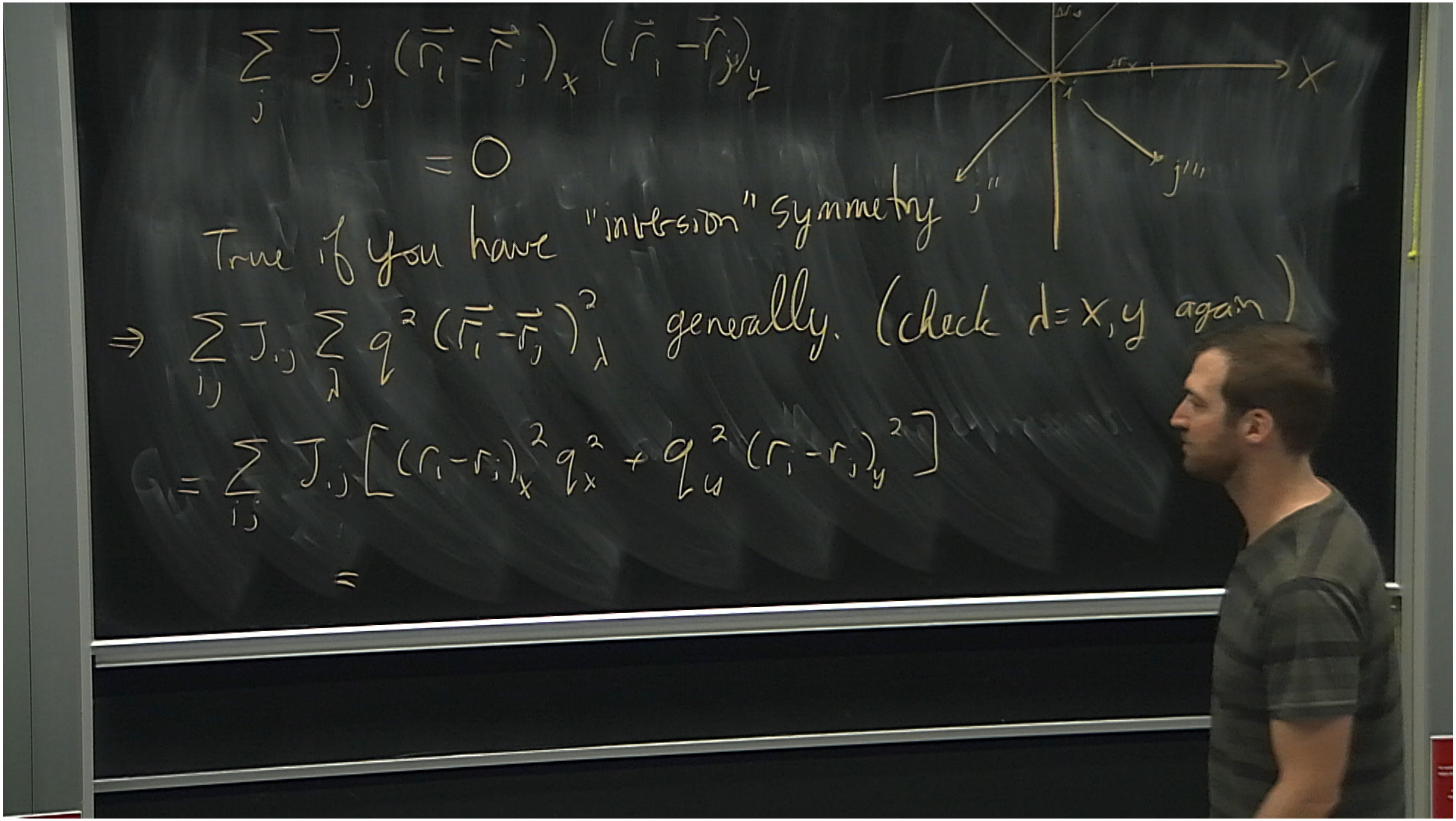
$$= \frac{1}{d}\sum_{ij} J_{ij} |\vec{r}_i-\vec{r}_j|^2 q^2 \quad \text{(d dimensions)}$$

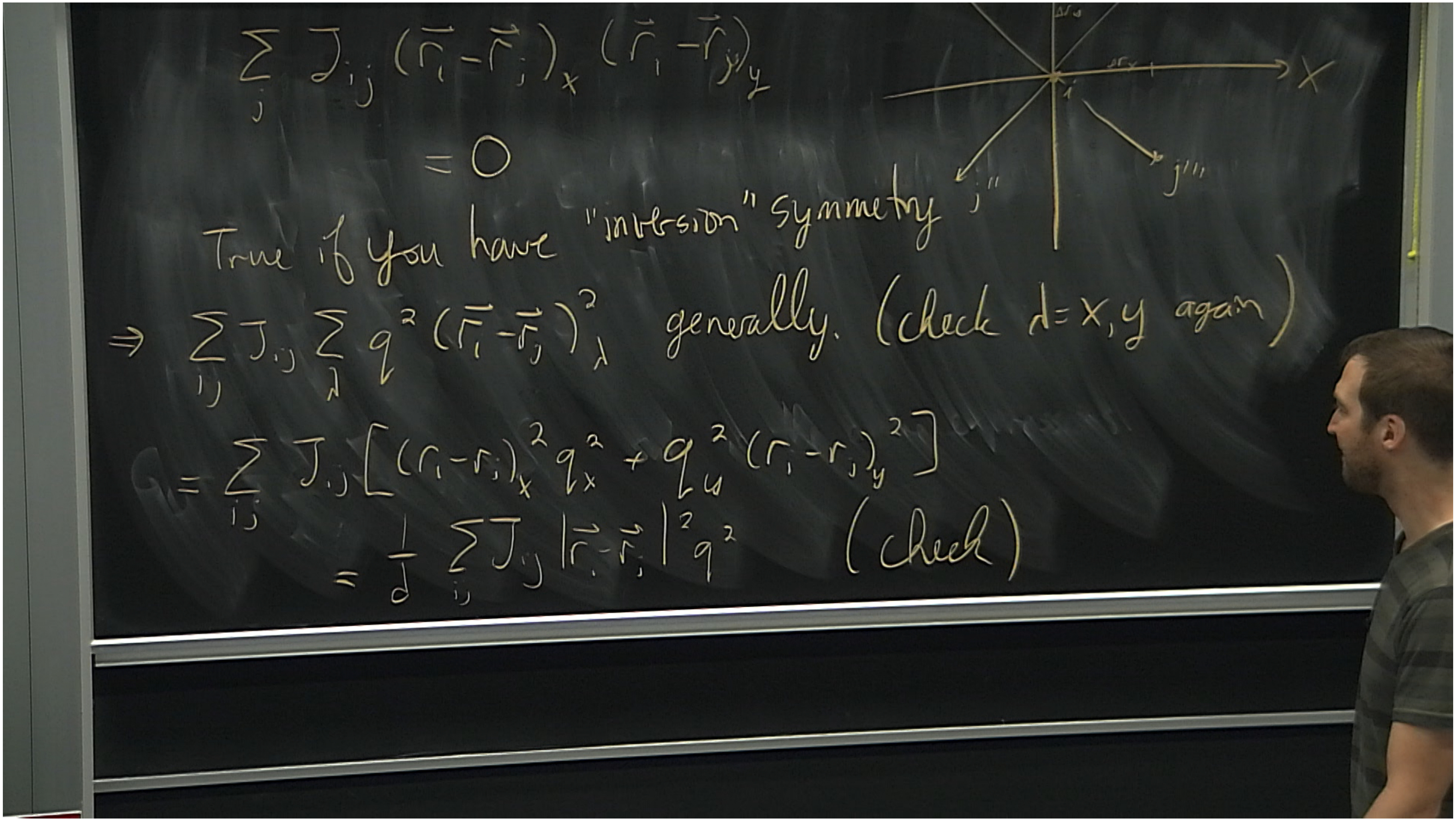
call $K = \frac{1}{Nd}\sum_{ij} J_{ij}|\vec{r}_i-\vec{r}_j|^2 \Rightarrow \boxed{J(\vec{q}) \simeq J - \frac{1}{2} K q^2}$


proof for
$\sum_{ij} J_{ij} [\vec{q} \cdot (\vec{r}_i - \vec{r}_j)]^2 = \sum_{ij} J_{ij} \sum_{\lambda} \sum_{\lambda'} q_\lambda q_{\lambda'} (\vec{r}_i - \vec{r}_j)_\lambda (\vec{r}_i - \vec{r}_j)_{\lambda'}$
Let's


$\sum_{\lambda\lambda'} q_\lambda q_{\lambda'} (r_i - r_j)_\lambda (r_i - r_j)_{\lambda'} = q_x^2 (r_i - r_j)_x + q_y^2 (r_i - r_j)_y$
$+ q_x q_y (r_i - r_j)_x (r_i - r_j)_y + q_y q_x (r_i - r_j)_y (r_i - r_j)_x$
$G_{ij} - \frac{1}{T} \sum_l J_{il} G_{lj} = \delta_{ij}$
sub in the above expressions.


We need
$\sum_j J_{ij} (\vec{r}_i - \vec{r}_j)_x (\vec{r}_i - \vec{r}_j)_y$
$= 0$

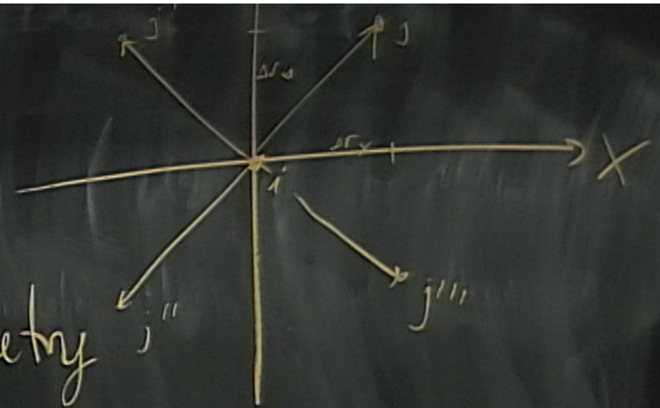

$\sum_j J_{ij} (\vec{r}_i - \vec{r}_j)_x (\vec{r}_i - \vec{r}_j)_y$
$= 0$
True if you have "inversion" symmetry
$\Rightarrow \sum_{ij} J_{ij} \sum_\lambda q^2 (\vec{r}_i - \vec{r}_j)^2_\lambda$ generally. (check $\lambda = x, y$ again)
$= \sum_{ij} J_{ij} \left[(r_i - r_j)_x^2 q_x^2 + q_y^2 (r_i - r_j)_y^2 \right]$
$=$
X
i
j''
j'''


$\sum_j J_{ij} (\vec r_i - \vec r_j)_x (\vec r_i - \vec r_j)_y$
$= 0$
True if you have "inversion" symmetry
$\Rightarrow \sum_{ij} J_{ij} \sum_\lambda q^2 (\vec r_i - \vec r_j)^2_\lambda$ generally. (check $\lambda = x, y$ again)
$= \sum_{ij} J_{ij} \left[(r_i - r_j)_x^2 q_x^2 + q_y^2 (r_i - r_j)_y^2 \right]$
$= \frac{1}{d} \sum_{ij} J_{ij} |\vec r_i - \vec r_j|^2 q^2$ (check)
X
i
j''
j'''

We need Δr_x Δr_y

$$\sum_j J_{ij} (\vec{r}_i - \vec{r}_j)_x (\vec{r}_i - \vec{r}_j)_y$$

$$= 0$$

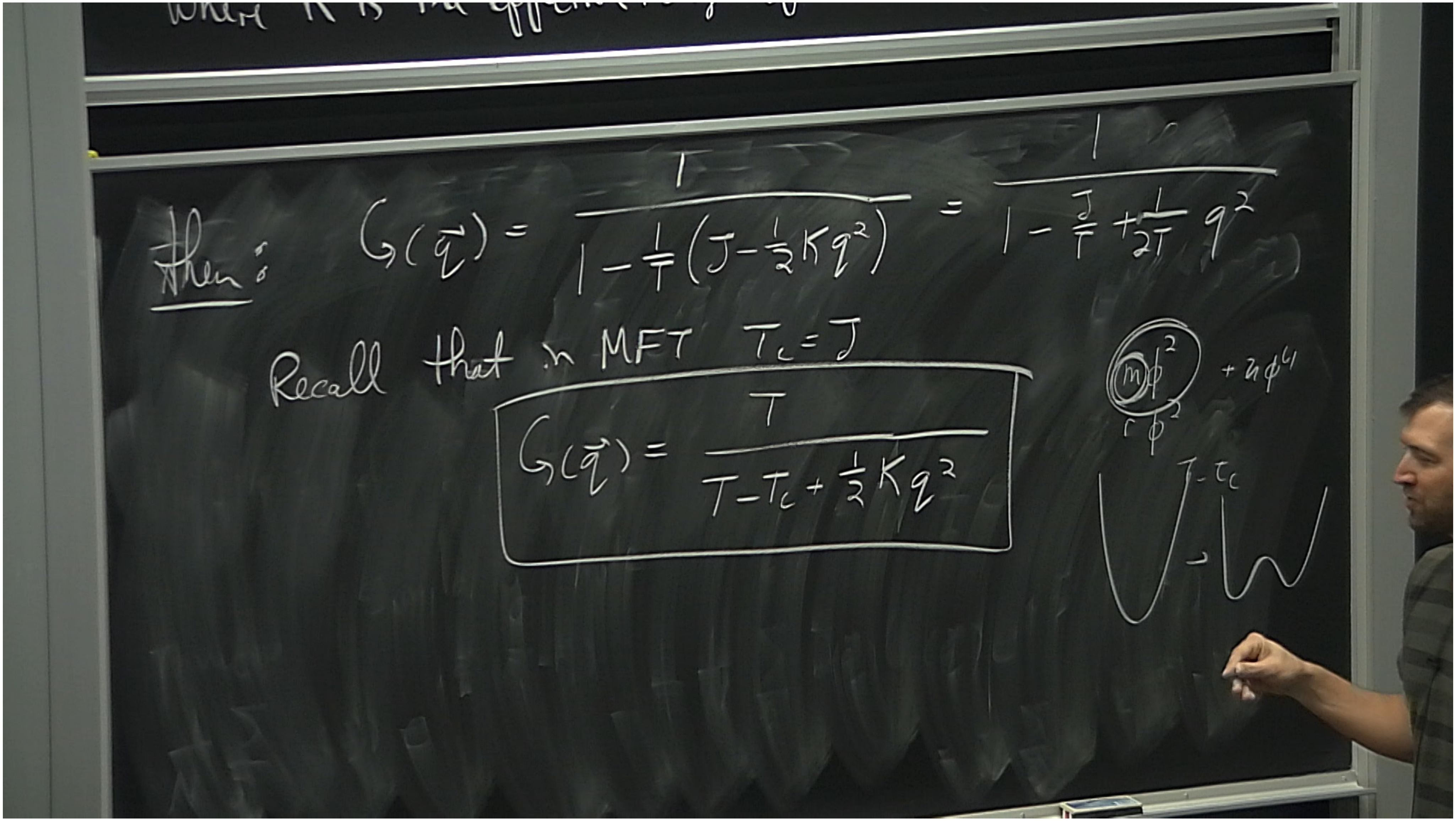


True if you have "inversion" symmetry

$$\Rightarrow \sum_{ij} J_{ij} \sum_{\lambda} q^2 (\vec{r}_i - \vec{r}_j)^2_{\lambda}$$ generally. (check $\lambda = x, y$ again)

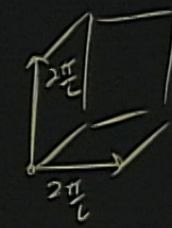
$$= \sum_{ij} J_{ij} \left[(r_i - r_j)_x^2 q_x^2 + q_y^2 (r_i - r_j)_y^2 \right]$$

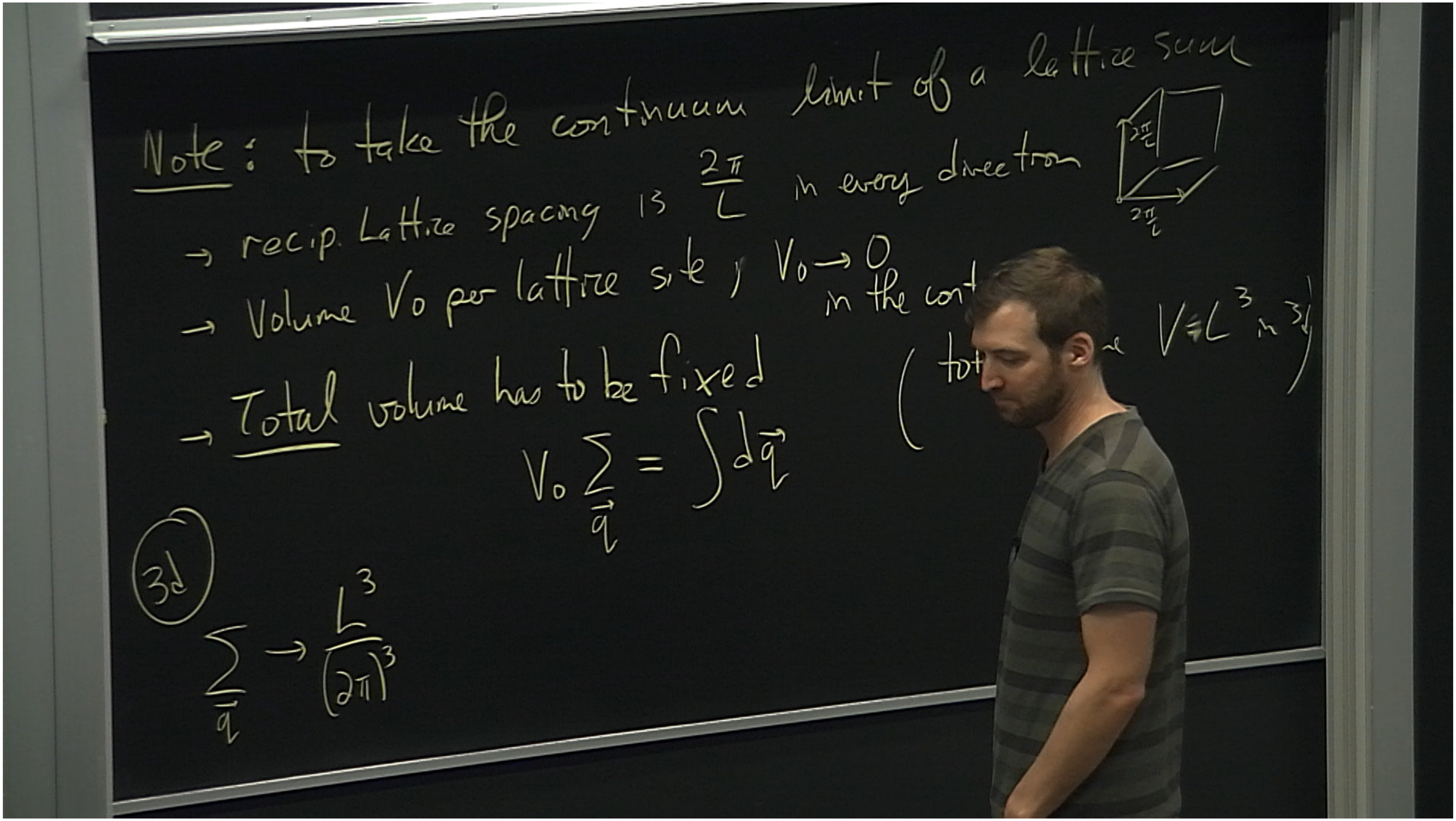
$$= \frac{1}{d} \sum_{ij} J_{ij} |\vec{r}_i - \vec{r}_j|^2 q^2$$ (check) d-dim hypercube.


then:
$G(\vec{q}) = \frac{1}{1 - \frac{1}{T}(J - \frac{1}{2}Kq^2)} = \frac{1}{1 - \frac{J}{T} + \frac{1}{2T}q^2}$
Recall that in MFT $T_c = J$
$G(\vec{q}) = \frac{T}{T - T_c + \frac{1}{2}Kq^2}$
$m\phi^2 + u\phi^4$
$r\phi^2$
$T - T_c$

Note: to take the continuum limit of a lattice sum

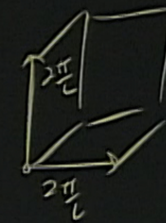
→ recip. Lattice spacing is $\frac{2\pi}{L}$ in every direction





Note: to take the continuum limit of a lattice sum
→ recip. Lattice spacing is 2π/L in every direction
2π/L
2π/L
→ Volume V0 per lattice site, V0 → 0 in the cont
(tot e V = L^3 in 3d)
→ Total volume has to be fixed
V0 Σ_q = ∫ dq
3d
Σ_q → L^3/(2π)^3

Note: to take the continuum limit of a lattice sum



→ recip. Lattice spacing is $\frac{2\pi}{L}$ in every direction

→ Volume V_0 per lattice site ; $V_0 \to 0$ in the continuum

→ Total volume has to be fixed (total volume $V = L^3$ in 3d)

$$V_0 \sum_{\vec{q}} = \int d\vec{q}$$

(3d)

$$\sum_{\vec{q}} \to \frac{L^3}{(2\pi)^3} \int d\vec{q} \qquad \text{and} \qquad \frac{L^3}{N} = a^3$$

$$G(\vec{r}) = a^3 \int \frac{d\vec{q}}{(2\pi)^3}\, G(\vec{q})\, e^{i\vec{q}\cdot\vec{r}}$$

$$= \frac{a^3}{(8\pi^3)} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, \sin\theta \int_0^{\infty} dq\, q^2\, G(\vec{q})\, e^{iqr\cos\theta}$$

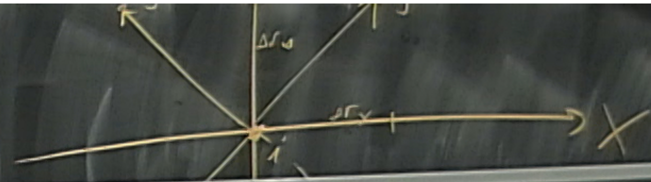
$$G(\vec{r}) = \frac{a^3 T}{\pi^2 r K} \int_0^\infty \frac{dq \cdot q \cdot \sin(qr)}{\left(q - i\sqrt{2\frac{T-T_c}{K}}\right)\left(q + i\sqrt{2\frac{T-T_c}{K}}\right)}$$

$$= \frac{a^3 T}{\pi^2 r K} \frac{1}{2} \int_{-\infty}^{\infty} dq \, q \, \frac{\frac{1}{2i}\left(e^{iqr} - e^{-iqr}\right)}{\left(q - i\sqrt{2\frac{T-T_c}{K}}\right)\left(q + i\sqrt{2\frac{T-T_c}{K}}\right)}$$

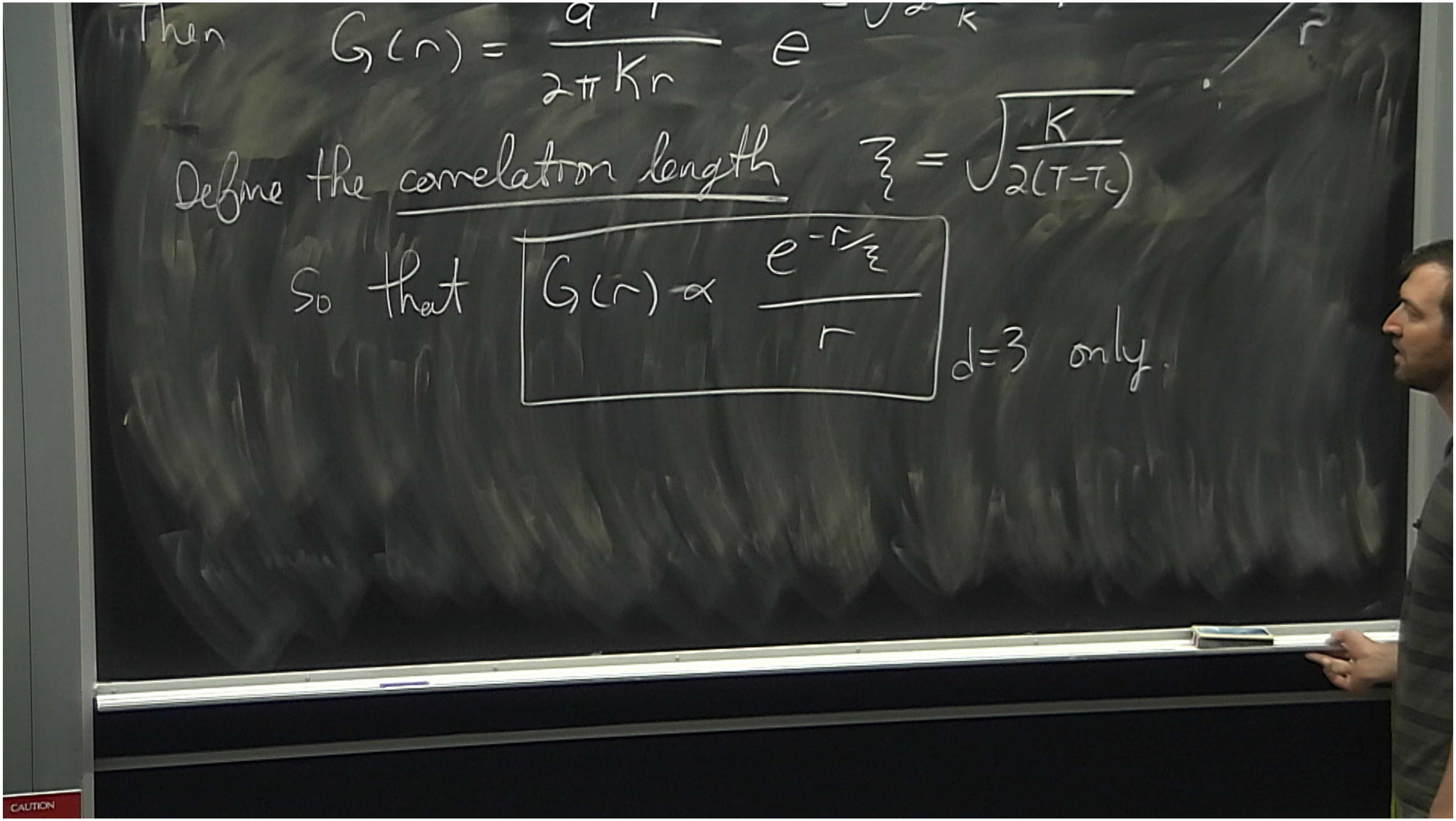
use contour integration

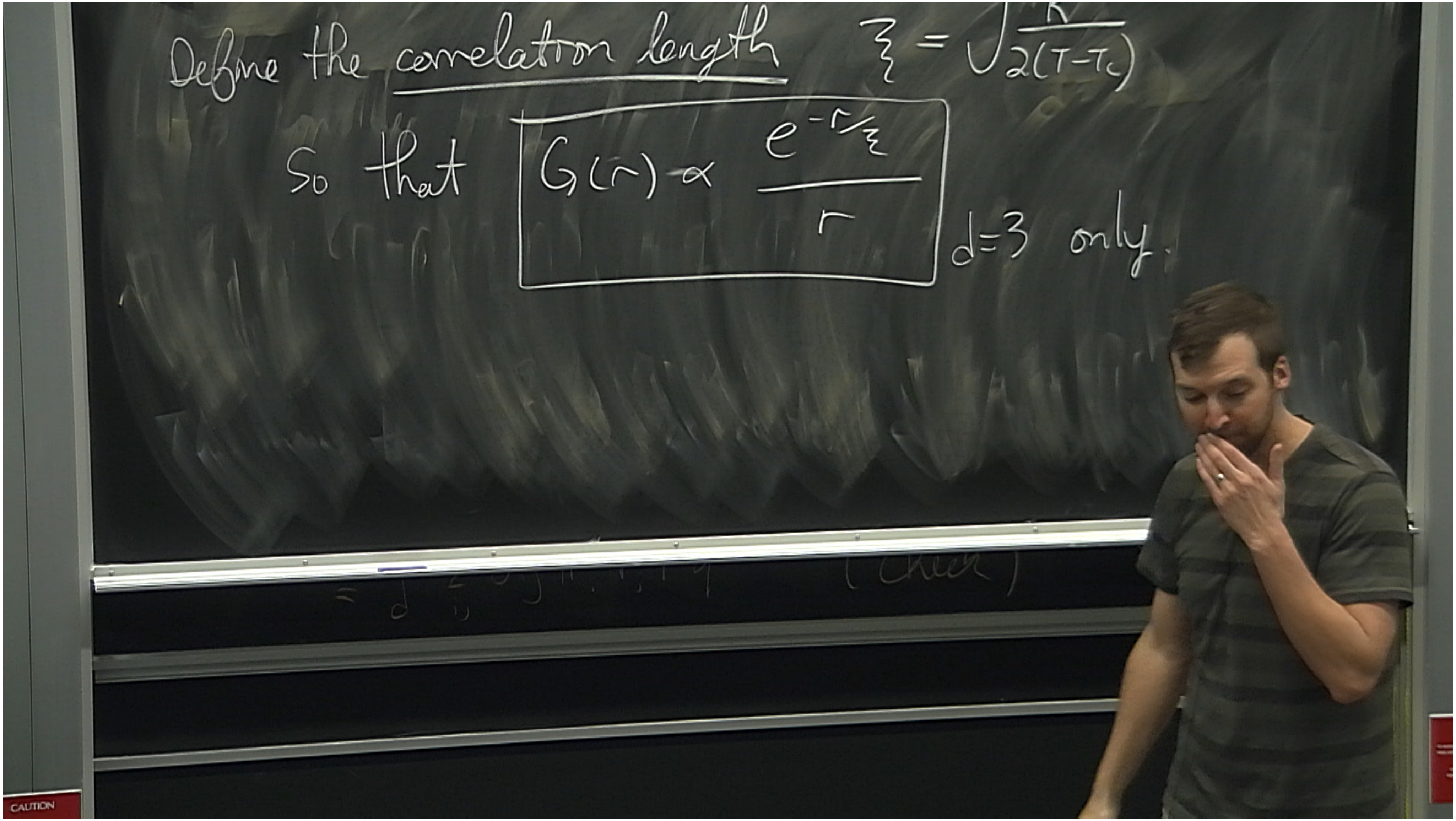
$$\int_{-\infty}^{\infty} \frac{z e^{\pm iz}}{(z^2 + a)} dz = \pm i\pi e^{-\sqrt{a}}$$

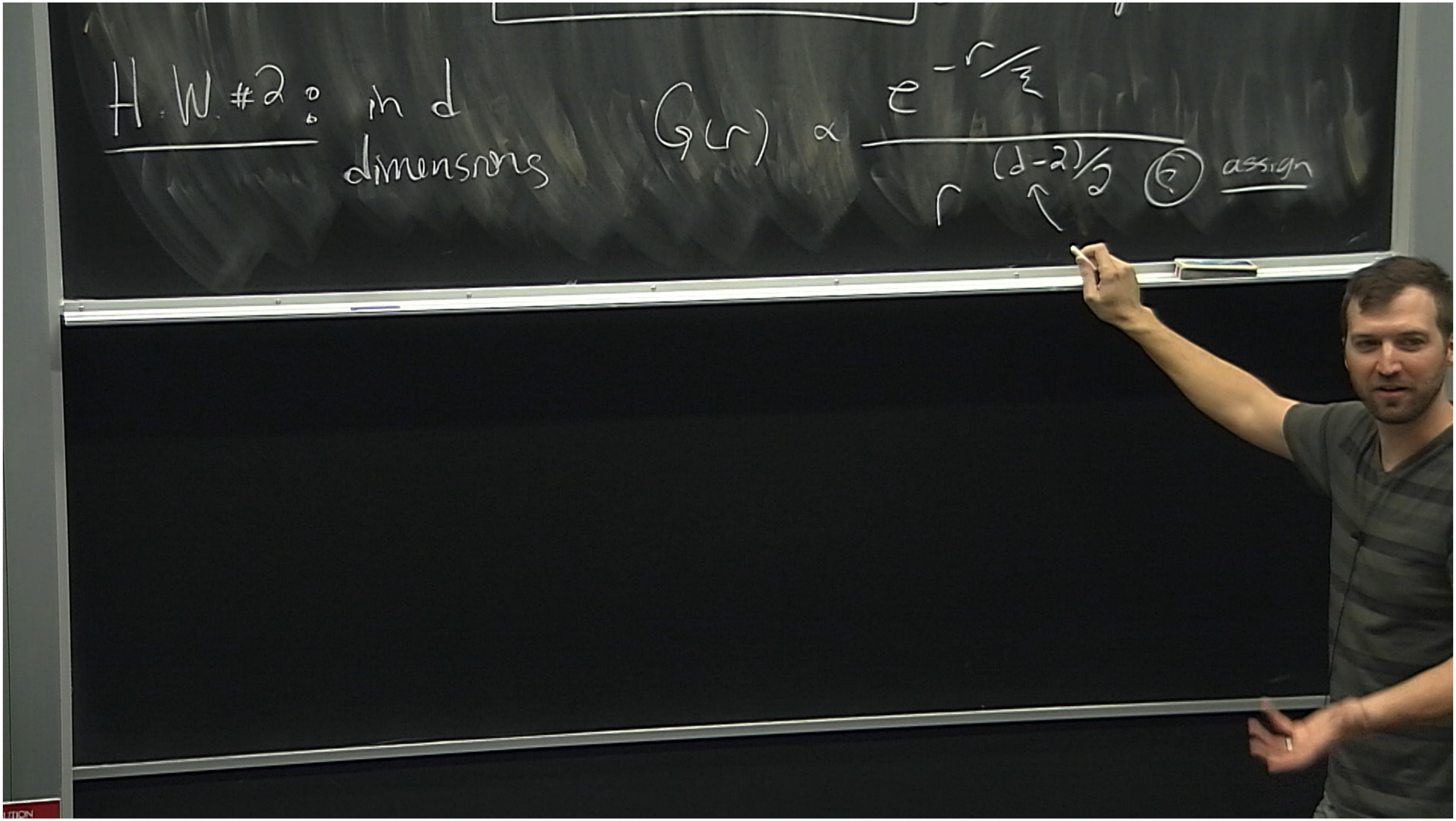
$$\sum_j J_{ij} (\vec{r}_i - \vec{r}_j)_x (\vec{r}_i - \vec{r}_j)_y$$



Then $$G(r) = \frac{a^3 T}{2\pi K r} e^{-\sqrt{2\frac{T-T_c}{K}}\, r}$$


Then $G(r) = \frac{a}{2\pi K r} e$
Define the correlation length $\xi = \sqrt{\frac{K}{2(T-T_c)}}$
So that $G(r) \propto \frac{e^{-r/\xi}}{r}$
d=3 only.
CAUTION

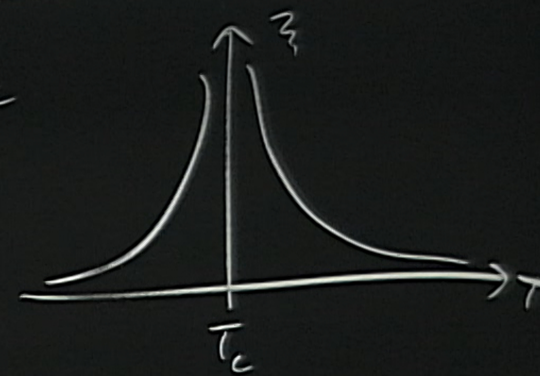

Define the correlation length $\xi = \sqrt{\frac{\kappa}{2(T-T_c)}}$
So that $G(r) \propto \frac{e^{-r/\xi}}{r}$
d=3 only.
CAUTION


H.W. #2: in d dimensions
G(r) ∝ e^{-r/ξ} / r^{(d-2)/2}
assign

So the correlation length diverges at T_c

as $\xi \sim (T - T_c)^{-\nu}$

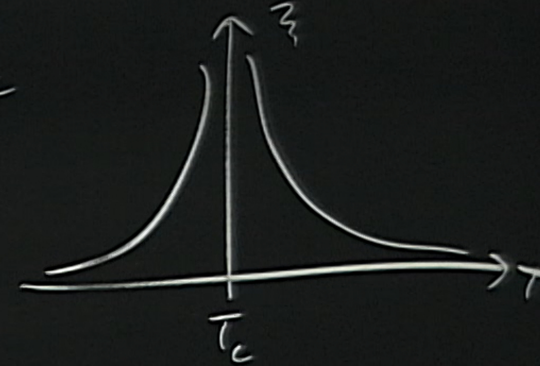
and $\nu = \frac{1}{2}$ in <u>MFT</u>



So the correlation length diverges at T_c

as $\xi \sim (T-T_c)^{-\nu}$

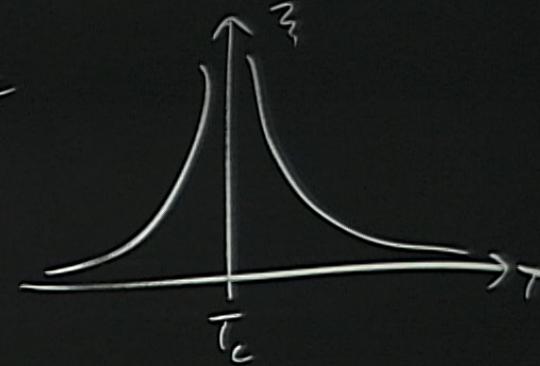
and $\nu = \frac{1}{2}$ in <u>MFT</u>



(note $\xi \to \infty$ @ T_c gives us "criticality" at a continuous transition. For First-order this is not true)

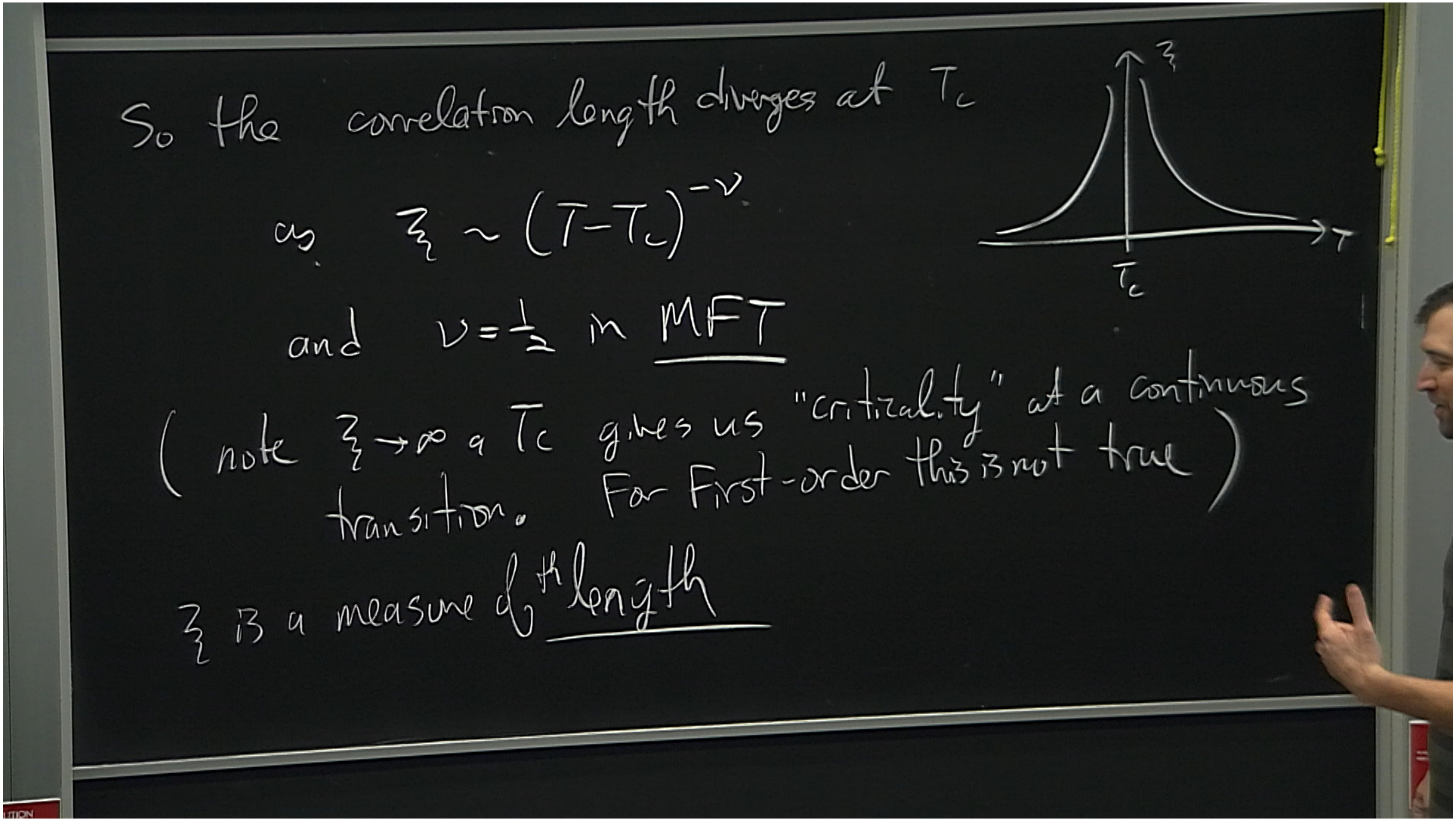
So the correlation length diverges at T_c

as $\xi \sim (T-T_c)^{-\nu}$



and $\nu = \frac{1}{2}$ in <u>MFT</u>

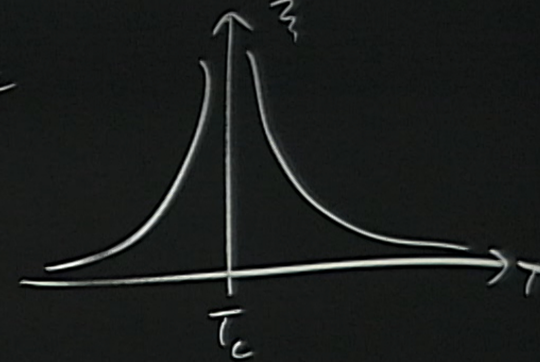
(note $\xi \to \infty$ @ T_c gives us "criticality" at a continuous transition. For First-order this is not true)


So the correlation length diverges at T_c
as $\xi \sim (T-T_c)^{-\nu}$
and $\nu = \frac{1}{2}$ in MFT
(note $\xi \to \infty$ @ T_c gives us "criticality" at a continuous transition. For First-order this is not true)
ξ is a measure of the length
ξ
T
T_c

So the correlation length diverges at T_c

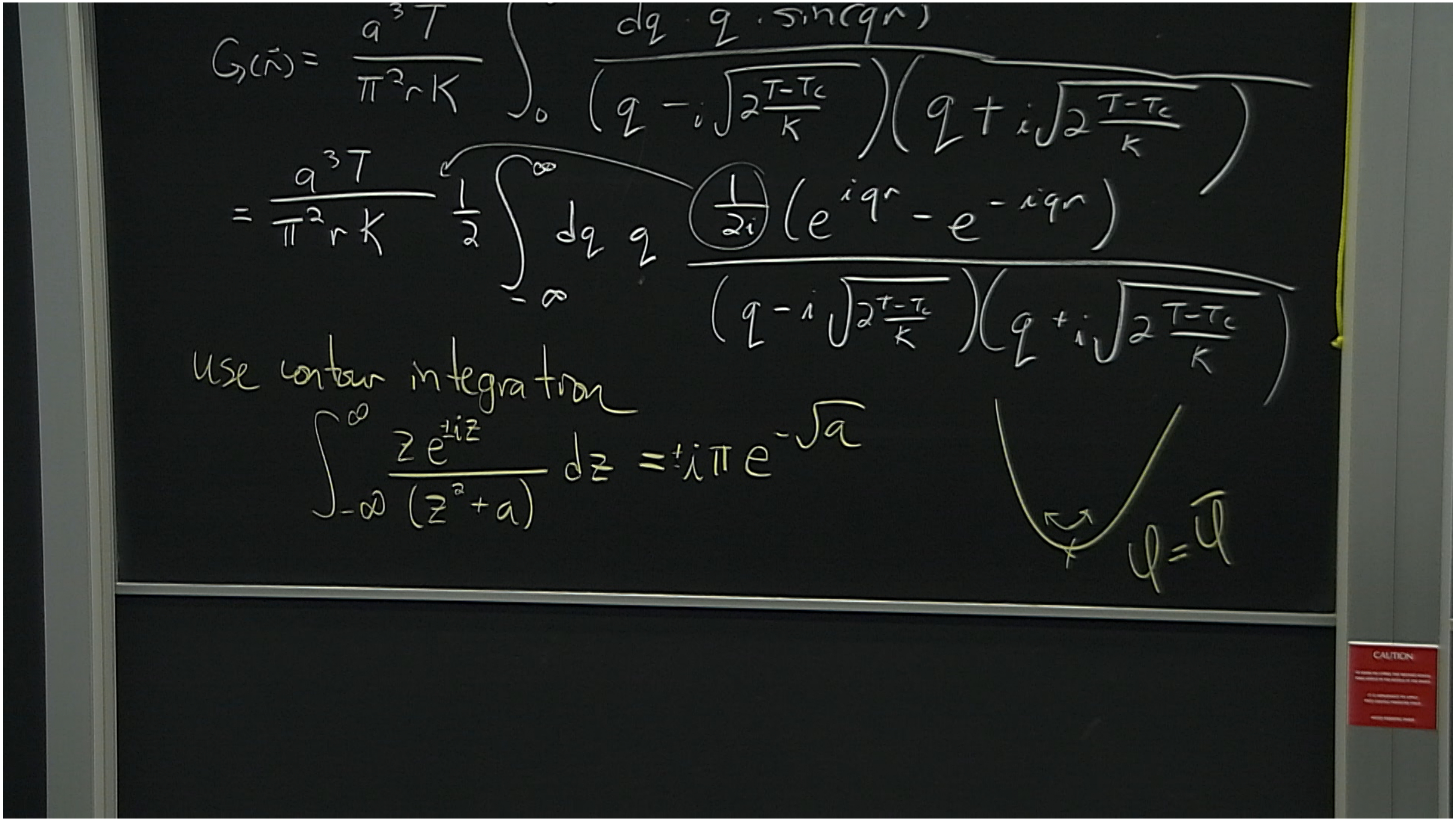
as $\xi \sim (T-T_c)^{-\nu}$

and $\nu = \frac{1}{2}$ in MFT



(note $\xi \to \infty$ @ T_c gives us "criticality" at a continuous transition. For First-order this is not true)

ξ is a measure of the length over which two particles are correlated with probability 1.


$G(\vec{r}) = \frac{a^3 T}{\pi^2 r K} \int_0^\infty \frac{dq \cdot q \cdot \sin(qr)}{\left(q - i\sqrt{2\frac{T-T_c}{K}}\right)\left(q + i\sqrt{2\frac{T-T_c}{K}}\right)}$
$= \frac{a^3 T}{\pi^2 r K} \frac{1}{2} \int_{-\infty}^{\infty} dq\, q \frac{\frac{1}{2i}\left(e^{iqr} - e^{-iqr}\right)}{\left(q - i\sqrt{2\frac{T-T_c}{K}}\right)\left(q + i\sqrt{2\frac{T-T_c}{K}}\right)}$
Use contour integration
$\int_{-\infty}^{\infty} \frac{z e^{\pm iz}}{(z^2+a)} dz = \pm i\pi e^{-\sqrt{a}}$
$\varphi = \bar{\varphi}$