Title: PSI 2016/2017 Foundations of Quantum Mechanics (Review) - Lecture 12

Date: Jan 18, 2017 11:30 AM

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Abstract:

A Map of the Madness

	Realist		Copenhagenish
	Ontological Model	Exotic	
ψ-epistemic		Ironic many-worlds Quantum logical realism	Copenhagen QBism Rovelli's Relational QM Healy's Quantum Pragmatism Bub's "Information" Interpretation
ψ-ontic	de Broglie-Bohm Spontaneous collapse Modal interpretations	Everett/Many worlds	

I don't know where to put consistent/decoherent histories

6.1) de Broglie-Bohm Theory

- A brief history:
  - The 1st order form of dBB theory was discovered and then abandoned by de Broglie in the 1920's.
  - dBB was rediscovered, in 2nd order form, by Bohm in 1952.
  - The forgotten 1st order form was promoted by Bell in the 1970's and 80's.
  - Proponents still fight over which form is better. I will follow Bell's approach here.
- D. Dürr, S. Teufel "Bohmian Mechanics" (Springer, 2009) is a comprehensive overview of this approach.

Ontology of dBB Theory

- The goal of any interpretation is to:
  - Provide an ontology: a statement of what exists and how it behaves.
  - Save the phenomena: Explain the quantum predictions and our everyday experience in terms of the ontology.
- Bohmians typically divide the ontology into two pieces:
  - Primitive ontology: The things that determine what we experience. Usually assumed to be localized in spacetime – local beables. In dBB this is particle trajectories.
  - The rest: Needed to determine how the primitive ontology behaves. In dBB this is the quantum state.

Equations of motion

- Notation: $\vec{q}$ denotes a vector in $\mathbb{R}^3$. $\boldsymbol{q}$ denotes a vector in $\mathbb{R}^{3N}$.
- For spinless particles, we can write a quantum state as a wavefunction on $\mathbb{R}^{3N}$:

$$\psi(\boldsymbol{q},t) = \psi(\vec{q}_1, \vec{q}_2, \dots, \vec{q}_N, t) = \langle \boldsymbol{q} | \psi(t) \rangle = \langle \vec{q}_1, \vec{q}_2, \dots, \vec{q}_N | \psi(t) \rangle$$

- The wavefunction obeys the Schrödinger equation: $i\hbar \frac{\partial |\psi\rangle}{\partial t} = H|\psi\rangle$
- dBB also has an actual point in configuration space:

$$\boldsymbol{Q} = (\vec{Q}_1, \vec{Q}_2, \dots, \vec{Q}_N)$$

- This obeys the guidance equation:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}\left(\psi^* \vec{\nabla}_k \psi\right)}{\psi^* \psi}(\boldsymbol{Q})$$

Equilibrium Hypothesis and Equivariance

- One more postulate is required to obtain the same predictions as standard quantum theory - Quantum Equilibrium Hypothesis:
  - At time $t = t_0$, the probability density of the system occupying configuration point $\boldsymbol{Q}$ is:

$$\rho(\boldsymbol{Q}) = |\psi(\boldsymbol{Q})|^2$$

- Under the dBB evolution we will show that if this holds at $t = t_0$ then it holds at all times. This is known as equivariance.
- There is controversy about what $\rho(\boldsymbol{Q})$ means as dBB is applied to the *entire universe*, which only has a single configuration space point.
  - Roughly speaking, if we prepare many systems in the state $|\psi\rangle \otimes |\psi\rangle \otimes \cdots \otimes |\psi\rangle$, the probability density of configurations is $\rho(\boldsymbol{Q})$.
- Note that the quantum state is playing two *independent* roles:
  - It governs dynamics via the guidance equation.
  - It is used to set the probability density.

Bell's derivation of the guidance equation and equivariance

- Solutions of the Schrödinger equation satisfy the continuity equation:

$$\frac{\partial|\psi(\boldsymbol{q},t)|^2}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{q},t) = 0$$

where $\boldsymbol{J}(\boldsymbol{q},t)$ is the probability current:

$$\boldsymbol{J} = (\vec{J}_1, \vec{J}_2, \dots, \vec{J}_N) \qquad \vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\mathrm{Im}(\psi^*\vec{\nabla}_k\psi)(\boldsymbol{q})$$

- If we consider a preparation of $|\psi\rangle \otimes |\psi\rangle \otimes \cdots$ we want to consider $\boldsymbol{J}$ as a flow of particle density rather than probability.

- If we assume this is generated by a velocity field $\boldsymbol{v}(\boldsymbol{q})$, e.g. as in hydrodynamics, then $\boldsymbol{J} = \rho\boldsymbol{v}$, so the equation for the velocity field should be:

$$\boldsymbol{v}(\boldsymbol{q}) = \frac{\boldsymbol{J}(\boldsymbol{q})}{\rho(\boldsymbol{q})} \qquad \vec{v}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\frac{\mathrm{Im}(\psi^*\vec{\nabla}_k\psi)}{\rho}(\boldsymbol{q})$$

which gives the dBB velocities if we set $\rho(\boldsymbol{Q}) = |\psi(\boldsymbol{Q})|^2$.

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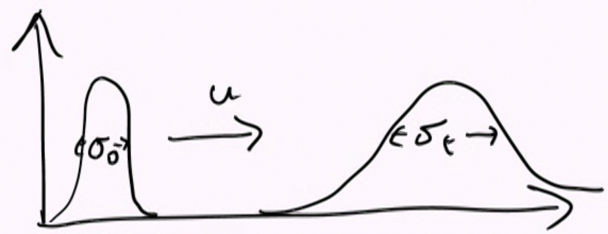
Trajectories for a 1D Gaussian Wavepacket

- Consider an initial Gaussian wavepacket moving towards the right

$$\psi(x,0) = \frac{1}{(2\pi\sigma_0^2)^{1/4}} \exp\left[-\frac{x^2}{4\sigma_0^2} + ikx\right]$$

- Under free-particle evolution this moves with group velocity $u = \frac{\hbar k}{m}$

  and spreads $\sigma_t = \sigma_0 \sqrt{1 + \frac{\hbar^2 t^2}{4m^2\sigma_0^4}}$



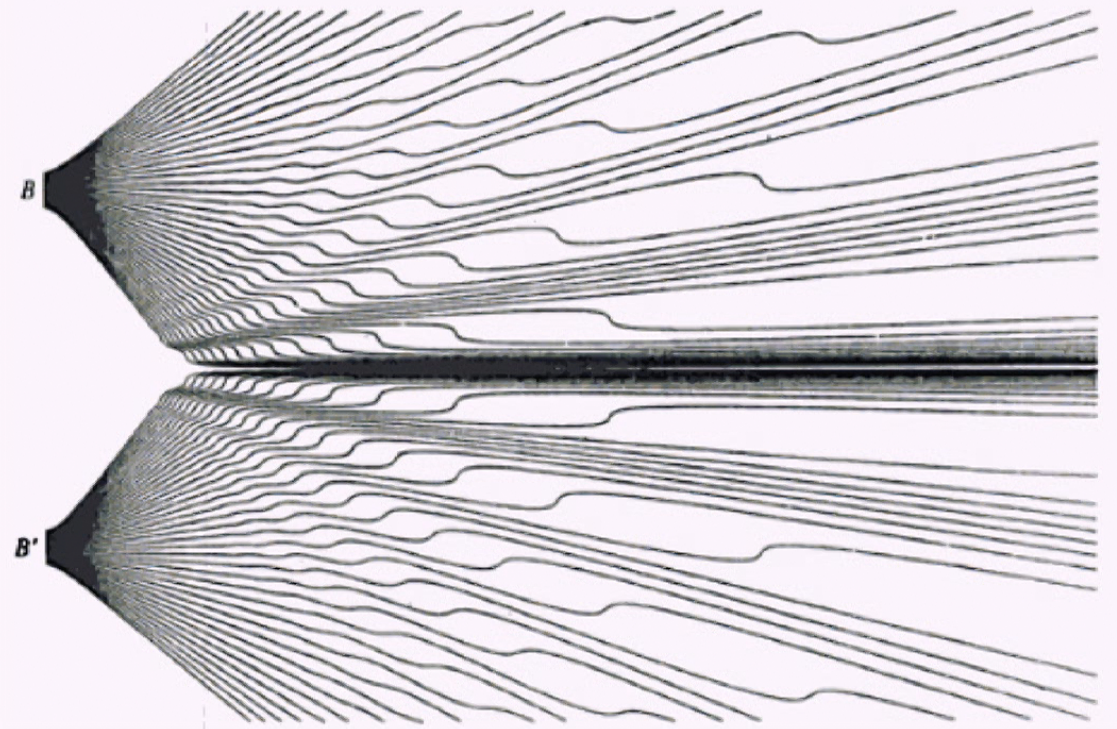
- If we consider a timescale s.t. spreading is negligible $t^2 << \frac{2m\sigma_0^2}{\hbar}$

  then the dBB velocity $\frac{dX}{dt} \simeq u$

Particle is dragged along with wavepacket at group velocity

See e.g. A. Pan, Pramana J. Phys. 74:867 (2010)

Double-Slit Trajectories



C.Philippidis et. al. Il Nuovo Cimento, vol.52B, No.1 (1979)

Model with Gaussian slits.

$$\psi(\vec{q},t) = \psi_B(\vec{q},t) + \psi_{B'}(\vec{q},t)$$

- When ψ_B, ψ_B have approximately no overlap (close to slits)

$$\vec{J} \simeq \vec{J}_B + \vec{J}_{B'}$$

$$\vec{J}_B = \frac{\hbar}{m} \operatorname{Im} \psi_B^* \vec{\nabla} \psi_B$$

$$\vec{J}_{B'} = \frac{\hbar}{m} \operatorname{Im} \psi_{B'}^* \vec{\nabla} \psi_B$$

The trajectories are as in geometric optics, i.e. perpendicular to wavefronts

- When they overlap there are cross-terms (interference) in the current, causing deflections which give the charateristic double-slit pattern.

Measurements in de Broglie-Bohm Theory

- Dividing particles into two subsets $(\boldsymbol{q}_S, \boldsymbol{q}_E)$ allows us to define a wavefunction for a subsystem called the conditional wavefunction.

$$\psi_{\boldsymbol{Q}_E}(\boldsymbol{q}_S) = \psi(\boldsymbol{q}_S, \boldsymbol{Q}_E)$$

- Generally, these do not evolve according to the Schrödinger equation, but they do if there is decoherence into localized environment states.
- Model the measurement device as a large number of particles, with outcomes represented by macroscopically distinct states with very small overlap:

with $\Phi_0(q_E)\Phi_1(q_E) \simeq 0$

- In a measurement interaction:

$$[\alpha\psi_0(\boldsymbol{q}_S) + \beta\psi_1(\boldsymbol{q}_S)]\Phi_R(\boldsymbol{q}_E) \quad \rightarrow \quad \alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

Measurements in de Broglie-Bohm Theory

$$\alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

- If the lack of position overlap between $\Phi_0(\boldsymbol{q}_E)$ and $\Phi_1(\boldsymbol{q}_E)$ persists in time then:
  - The actual configuration of the environment $\boldsymbol{Q}_E$ is either in the support of $\Phi_0(\boldsymbol{q}_E)$ or the support of $\Phi_1(\boldsymbol{q}_E)$.
  - By equivariance, it will be in the support of $\Phi_0(\boldsymbol{q}_E)$ with probability $|\alpha|^2$ and in the support of $\Phi_1(\boldsymbol{q}_E)$ with probability $|\beta|^2$.
  - The conditional state of the system will either be $\propto \psi_0(\boldsymbol{q}_S)$ or $\propto \psi_1(\boldsymbol{q}_S)$.
  - $\psi_0(\boldsymbol{q}_S)$ and $\psi_1(\boldsymbol{q}_S)$ each evolve according to the Schrödinger equation.
  - The current breaks into two terms $\boldsymbol{J} = \boldsymbol{J}_0 + \boldsymbol{J}_1$, with $\boldsymbol{J}_0 = 0$ in the support of $\Phi_1(\boldsymbol{q}_E)$ and vice versa, i.e. no cross terms in the guidance equation.
- We get an effective collapse into either $\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E)$ or $\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$ and we can use the corresponding current $\boldsymbol{J}_0$ or $\boldsymbol{J}_1$ in the guidance equation to compute subsequent evolution.

Measurements in de Broglie-Bohm Theory

- If the measurement is an (approximate) position measurement then also $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \approx 0$.
- The initial configuration $\boldsymbol{Q}_S$ of the system is either in the support of $\psi_0(\boldsymbol{q}_S)$ with probability $|\alpha|^2$ or in the support of $\psi_1(\boldsymbol{q}_S)$ with probability $|\beta|^2$.
- The measurement outcome is a deterministic function of $\boldsymbol{Q}_S$: position measurements simply reveal the pre-existing position.
- However, for other observables, e.g. momentum, $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \neq 0$, i.e. the initial configuration does not necessarily "belong" to one of the two eigenstates.
- Which measurement outcome occurs is a function of *both* $\boldsymbol{Q}_S$ and $\boldsymbol{Q}_E$.
- Momentum measurement does not measure the dBB momentum $m_k \frac{d\vec{Q}_k}{dt}$.
- The theory is deterministic: outcome uniquely determined by ontic states of system and measuring device.
- But not outcome deterministic: outcome uniquely determined by ontic state of system on its own.

Treatment of Spin

- In the minimalist Bell approach to dBB, no observables apart from position are part of the primitive ontology.
- Spin only appears in the wavefunction.
- We can write a wavefunction including spin as a spinor, e.g. for a single particle:

$$\psi_0(\vec{q}) \otimes |\uparrow\rangle + \psi_1(\vec{q}) \otimes |\downarrow\rangle \quad \rightarrow \quad \bar{\psi}(\vec{q}) = \begin{pmatrix} \psi_0(\vec{q}) \\ \psi_1(\vec{q}) \end{pmatrix}$$

- For N spin-$1/2$ particles, we would have a 2^N dimensional spinor vector.
- The guidance equation is now:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}(\bar{\psi}^* \cdot \vec{\nabla}_k \bar{\psi})}{\bar{\psi}^* \cdot \bar{\psi}} (\boldsymbol{Q}),$$

where $\cdot$ is spinor inner product.

- It is possible instead to have primitive ontic states for any complete orthonormal basis, but discrete bases require a stochastic guidance equation.

Counterintuitive Features of dBB Trajectories

- dBB trajectories display several features that violate classical intuitions about particle trajectories.
- It is important to note that, if decoherence occurs in an environmental basis that is localized in position, dBB trajectories of the system will approximately follow classical trajectories, for the reasons discussed in last two lectures.
- dBB doesn't *owe* us anything more than that. So long as:
  - It reproduces the predictions of quantum theory in measurements.
  - Macroscopic systems typically have approximately classical trajectories.

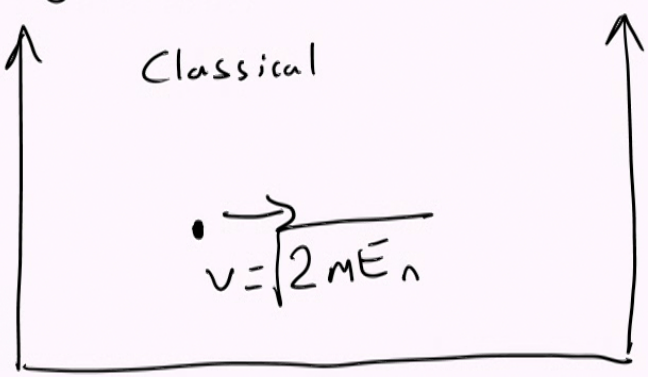
  then the theory saves the phenomena.
- Since quantum and classical predictions are different, dBB trajectories *must* differ from classical ones in some situations.
- The question is only if they are weirder than absolutely necessary to reproduce quantum theory, and whether that is a bad thing.

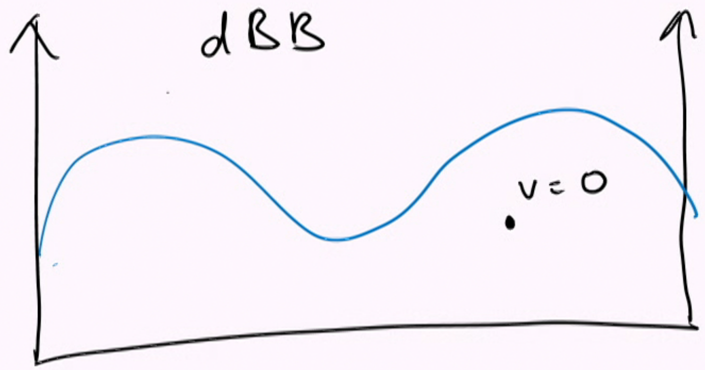
Real Stationary States

- Consider a stationary state: $\psi(\boldsymbol{q},t) = \psi_n(\boldsymbol{q})e^{-iE_n t/\hbar}$
- The current is: $\vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\mathrm{Im}(\psi_n^*\vec{\nabla}_k\psi_n)(\boldsymbol{q})$, i.e. is independent of t.
- However, if $\psi_n(\boldsymbol{q})$ is also a real valued function then:

$$\vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{2im_k}(\psi_n^*\vec{\nabla}_k\psi_n - \psi_n\vec{\nabla}_k\psi_n^*)(\boldsymbol{q}) = 0$$

- The particles are also stationary, e.g. particle in an infinite well, hydrogen atom eigenstates.





The No-Crossing Rule

- In classical mechanics, phase space trajectories do not cross (except at singularities) because equations are 2nd order and so $(\boldsymbol{q}, \boldsymbol{p})$ contains enough data to specify a unique trajectory.
- In dBB the guidance equations is 1st order and there is no back action on the quantum state from the configuration space point:
- $[\psi(\boldsymbol{q}, t_0), \boldsymbol{Q}(t_0)]$ and $[\psi(\boldsymbol{q}, t_0), \boldsymbol{Q}'(t_0)]$ specify unique trajectories.
- Trajectories associated with the same wavefunction evolution cannot cross in *configuration space*.
- This is responsible for almost all the weird features of dBB trajectories.
- Note: with decoherence into localized environment states:

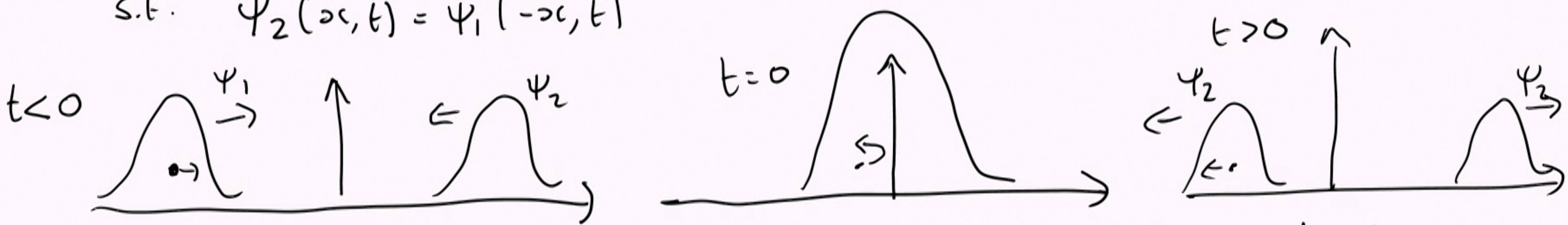
$$\alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

trajectories can cross in the *system* configuration space because $\boldsymbol{Q}_E$ is necessarily different in the two branches. This is needed to recover classical trajectories.

Empty Waves Steal the Particle

- Consider a superposition of 2 wavepackets $\psi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1(x,t) + \psi_2(x,t)\right)$

  s.t. $\psi_2(x,t) = \psi_1(-x,t)$

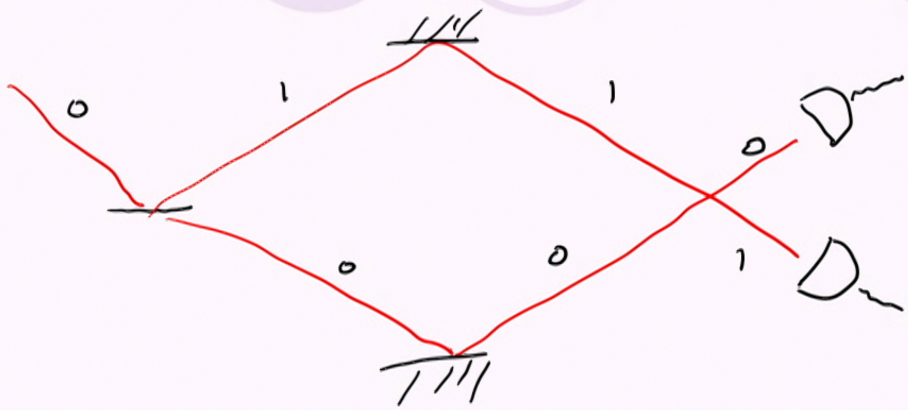


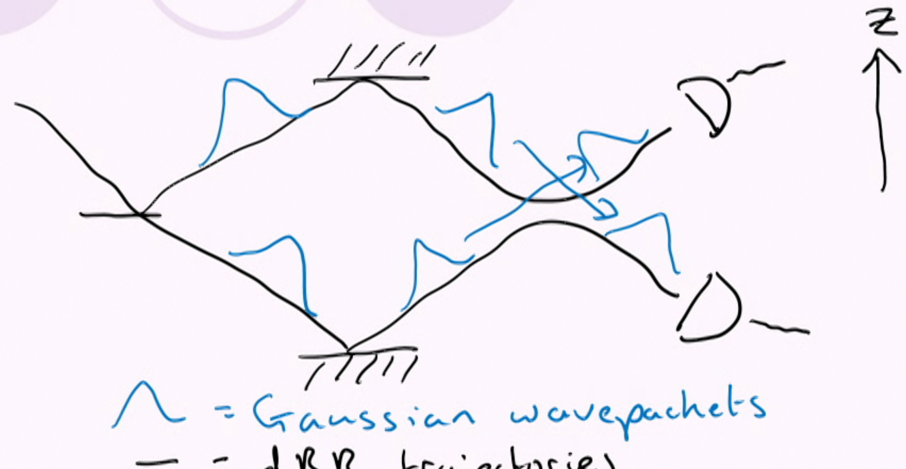
- The dBB particle will switch wavepackets during the interference due to the no-crossing rule: empty wave steals the particle.
- Here we can explicitly see that $J(0,t) = 0$ for all times b.c. $J(x,t)$ is an odd function of x

$$J(x,t) = \left[J_{11}(x,t) + J_{22}(x,t) + J_{12}(x,t) + J_{21}(x,t) \right] = \left[J_{11}(x,t) - J_{11}(-x,t) + J_{12}(x,t) - J_{12}(-x,t) \right]$$

where $J_{km}(x,t) = \frac{\hbar}{m} \operatorname{Im}\left(\psi_k^* \frac{\partial \psi_m}{\partial x} \right)$

Consequences for Mach-Zehnder





- If we remove the final beamsplitter from a Mach-Zehnder, many physicists would be inclined to say that detector 0 firing is evidence that the particle took path 0.
- The opposite happens in dBB. No crossing ⇒ the empty wave steals the particle

Detector 0 clicks ⇒ The particle travelled along path 1.

Surreal Trajectories

- To make things more dramatic, we can place a localized spin-$\frac{1}{2}$ system in path 0 initially placed in $|\uparrow\rangle$ and have the interaction

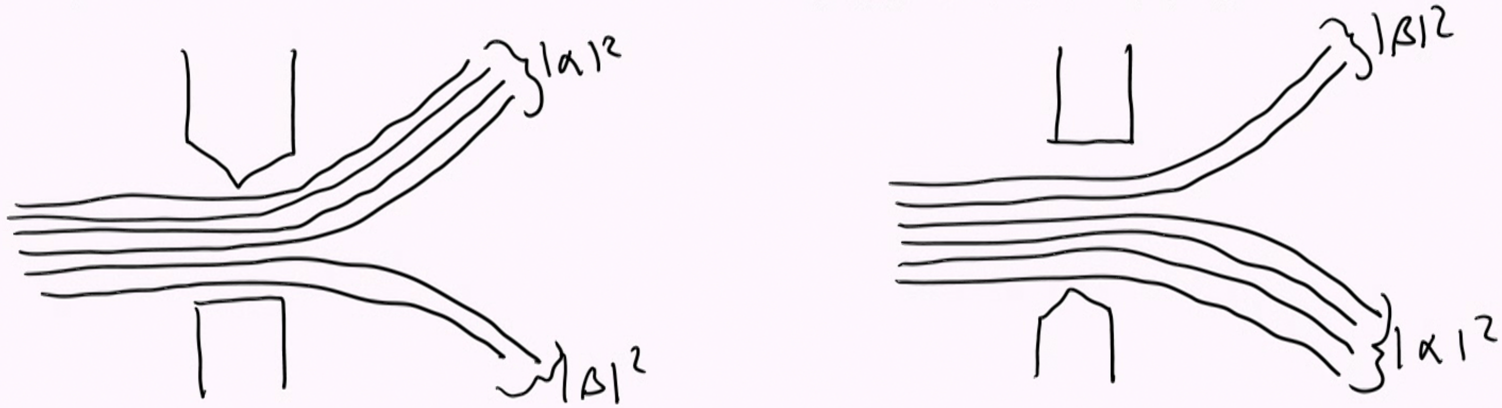
$$\psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \rightarrow \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\downarrow\rangle$$

$$\psi_1(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \rightarrow \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle$$

- Because $\vec{Q}_P$ is unaffected by this interaction the current will still be zero in the interference region.
- If we detect the particle at detector 0 and subsequently measure the spin, we will find it spin down.
- You might want to take this as evidence that the particle travelled along path 0, but the dBB trajectory is path 1.
- This can happen because the spin flip does not lead to decoherence that is localized in position.

KS Contextuality in de Broglie-Bohm

- KS Contextuality occurs in dBB because the outcome of an experiment depends on $\boldsymbol{Q}_S, \psi(\boldsymbol{q}_S), \boldsymbol{Q}_E, \Phi_R(\boldsymbol{q}_E)$, and the interaction Hamiltonian, and not on $\boldsymbol{Q}_S, \psi(\boldsymbol{q}_S)$ alone.
- Example: Stern-Gerlach measurement of $\psi(\boldsymbol{q}_S) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$



- No-crossing rule ⇒ some $\boldsymbol{q}_S$ switch between giving spin up and spin down outcomes when we rotate the magnets by 180°.
- This is *more contextual* than implied by KS, which can only be proved in $d \geq 3$.

Underdetermination

- The only property of the guidance equation needed to reproduce the quantum predictions is equivariance: $\rho(\boldsymbol{q}, t_0) = |\psi(\boldsymbol{q}, t_0)|^2 \rightarrow \rho(\boldsymbol{q}, t) = |\psi(\boldsymbol{q}, t)|^2$ for all other t.
- Any other equivariant dynamics would do just as well, e.g. (E. Deotto, G. Ghiradri, Found.Phys. 28:1-30 (1998))

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}(\psi^* \vec{\nabla}_k \psi)}{\psi^* \psi}(\boldsymbol{Q}) + \frac{\vec{J}_0(\vec{Q}_k)}{\psi^* \psi(\boldsymbol{Q})} \quad \text{with} \quad \vec{\nabla} \cdot \vec{J}_0 = 0$$

- Further:
  - We could add more primitive variables, e.g. spin with stochastic dynamics.
  - We could use a different basis, e.g. momentum.
  - We could even use a POVM, e.g. coherent states.

The Equilibrium Hypothesis

- The quantum state plays two roles in dBB:
  - Dynamical: it appears in the guidance equation.
  - Probabilistic: We set $\rho(\boldsymbol{q}, t_0) = |\psi(\boldsymbol{q}, t_0)|^2$ as a postulate – quantum equilibrium hypothesis.
- These two roles are independent, we could set the probability density to anything else.
- There is evidence (analytic and numerical) that, under suitable coarse-graining, other densities relax to $|\psi(\boldsymbol{q}, t_0)|^2$ over time, akin equilibriation in statistical mechanics.
- Valentini posits that nonequilibrium states may have occurred in the early universe.
  - This would resolve some of the underdetermination, but leads to the bold hypothesis that superluminal signaling occurs in our universe.

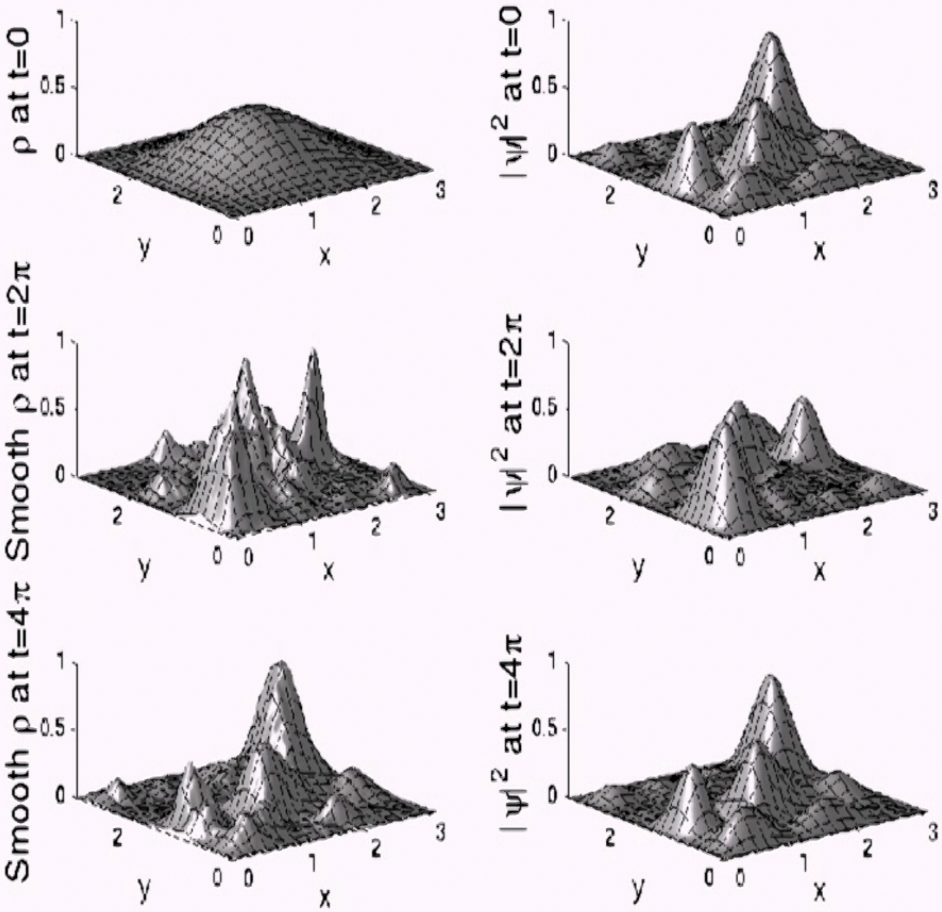


Figure 7: Smoothed $\tilde{\rho}$, compared with $|\psi|^2$, at times $t = 0$, 2π and 4π. While $|\psi|^2$ recurs to its initial value, the smoothed $\tilde{\rho}$ shows a remarkable evolution towards equilibrium.

A. Valentini, H. Westman, Proc. Roy. Soc. Lond. A 461:253-272 (2005)

Relativistic Generalizations of de Broglie-Bohm

- Generalizations of dBB to relativistic QFT have been developed. There are various versions:
  - Particle ontology vs. field ontology.
  - An ontology with particle occupation numbers requires stochastic dynamics.
  - A mixture of the two, e.g. particles for fermions and fields for bosons, only fermions and treat bosons like spin or vice versa.

- These theories cannot be *fundamentally* Lorentz invariant:
  - Under the equilibrium hypothesis, the operational predictions are Lorentz invariant.
  - But the theories violate parameter independence – there is superluminal signaling at the ontic level.
  - These effects would become observable in nonequilibrium states.

Summary

- dBB provides a coherent ontology with straightforward equations of motion, and saves the phenomena.
- Trajectories do not obey common intuitions, but arguably this must be so if they are to reproduce quantum phenomena.
- dBB arguably *more weird* than an interpretation has to be, i.e.
  - Contextual in ways that QM does not require.
  - Nonlocal in experiments that have local explanations.
  - ψ-ontic even for experiments that have good ψ-epistemic explanations.
- Taking the equilibrium hypothesis as a postulate is a fine tuning and leads to underdetermination of the theory.
- Viewing it as emergent removes the underdetermination, but leads to the bold hypothesis that we should expect to see explicit Lorentz violation, i.e. signaling, somewhere in nature.
- dBB is a good counterexample to many exaggerated claims about QM.