Title: PSI 2016/2017 Gravitational Physics (Review) - Lecture 13

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URL: http://pirsa.org/17010031

Abstract:

LECTURE 13: Gravitational Instantons

Usually in QFT are dealing with perturbative processes: 'real' interacting particles. There is another type of important quantum process that is non-perturbative, tunneling, that we also need to describe

Perturbative $\sim O(\hbar)$

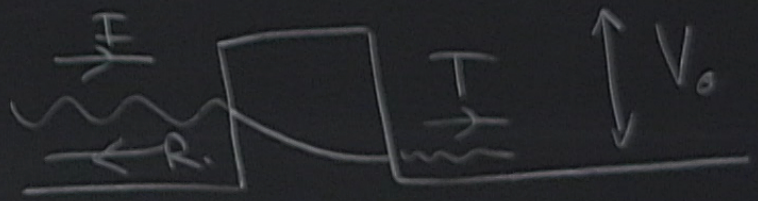
Non-" $\sim e^{-a(1/\hbar)}$

Tunneling usually described as a process in Euclidean time: the Instanton

Example: False-Vacuum Decay

Coleman PRD 15 2929 (77)
+ de Luccia PRD 21 3305 (80)

Recall QM tunneling



Fin

Finite transmission probability

$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \propto e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar} \int_0^d \sqrt{2m(V_0 - E)}\, dx$$

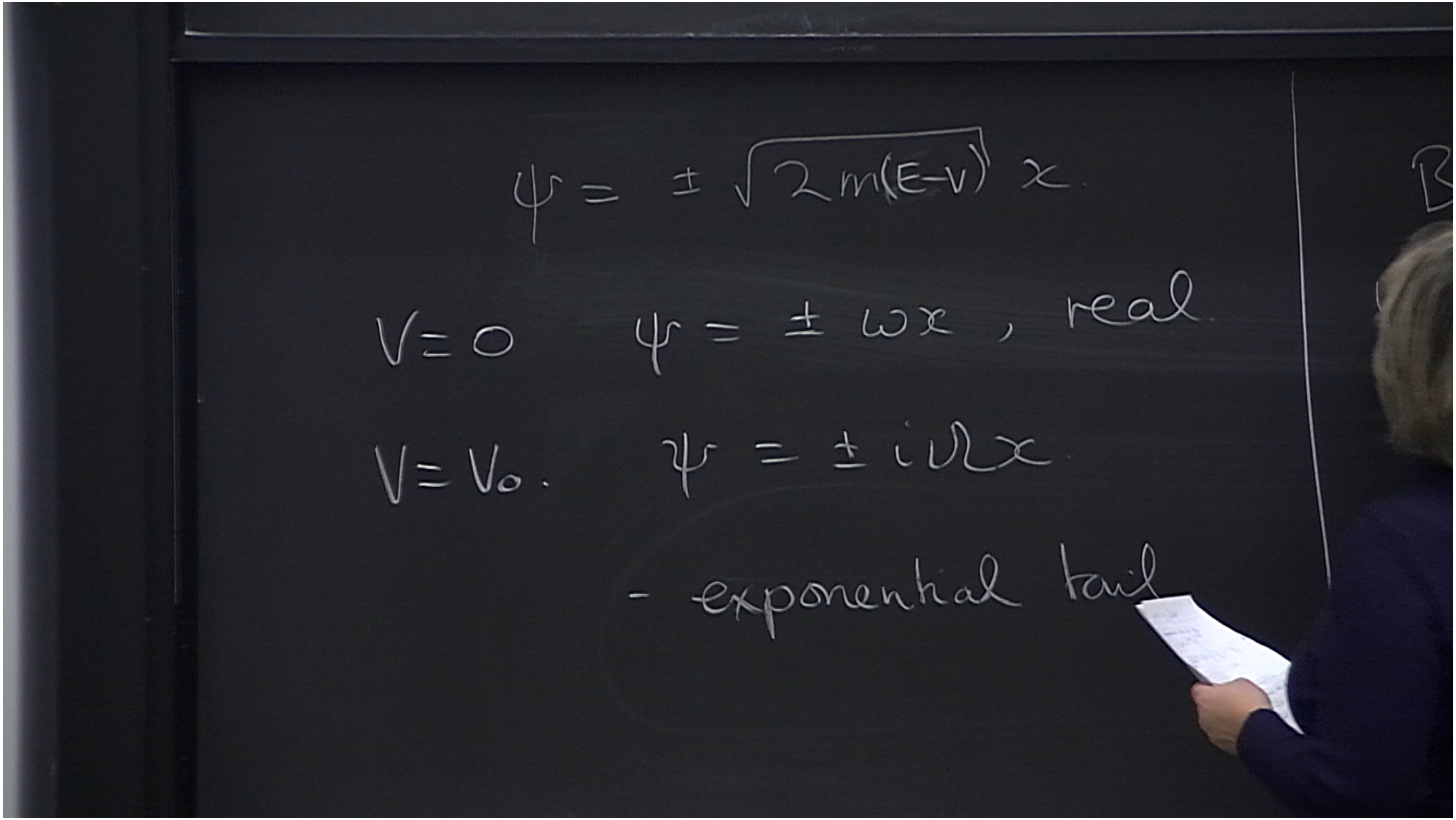
Approximate wave fn:

$$\Psi = e^{i\psi(x)/\hbar} e^{-iEt/\hbar}.$$

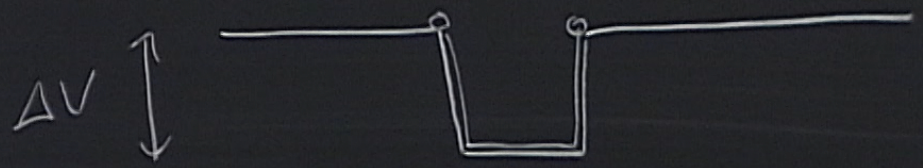
$$-\frac{\hbar^2}{2m}\Psi'' + V\Psi = \left[-\frac{i\hbar}{2m}\psi'' + \frac{\psi'^2}{2m} + V\right]\Psi$$

$$= E\Psi$$

$$\Rightarrow \psi'^2 = 2m(E-V)$$


$\psi = \pm \sqrt{2m(E-V)}\, x$
$V=0 \quad \psi = \pm \omega x$, real
$V=V_0 \quad \psi = \pm i\nu x$
- exponential tail

But consider a particle moving in a potential well $-\Delta V$ $(V-E)$



$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 = \Delta V$$

$\Rightarrow \quad \psi'^2 = 2m(E-V)$

particle moving

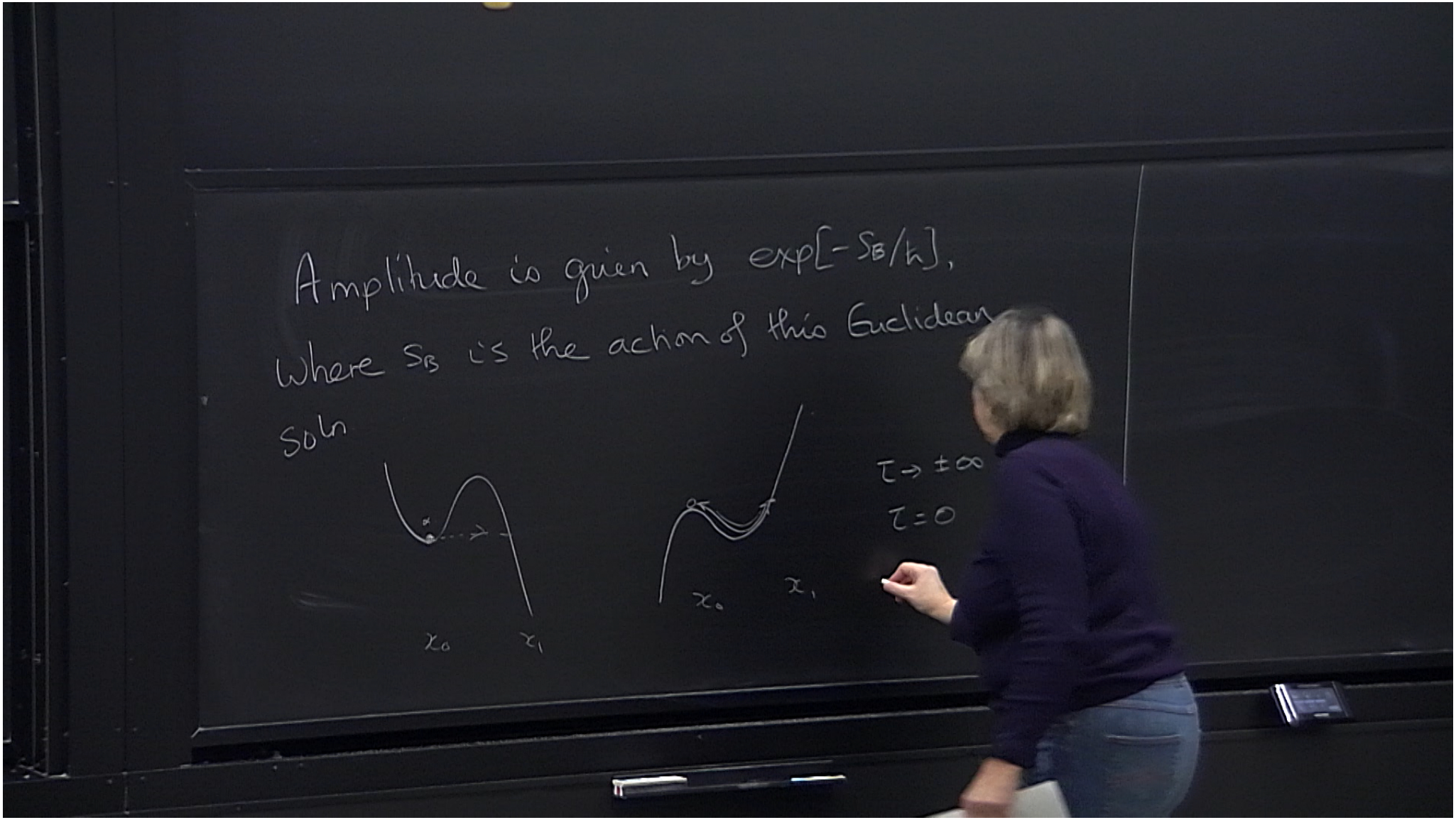
$-\Delta V \quad (V-E)$

$$\int \sqrt{2\Delta V}\, dx = \int \sqrt{2\Delta V}\cdot\sqrt{2\Delta V}\, dx$$

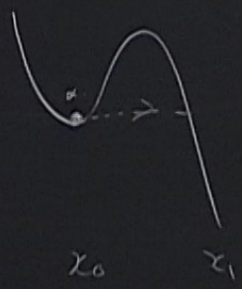
$$= \int \left[\Delta V + \frac{1}{2}\left(\frac{dx}{dx}\right)^2\right] dx$$

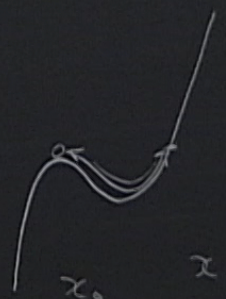
The exponent in tunneling is action of a ptcle moving in inverted potential in Euclidean time.

$$L = \frac{1}{2}\dot{x}^2 - V \xrightarrow{t\to ix} -\frac{1}{2}\left(\frac{dx}{dx}\right)^2 - V = -\left[\frac{1}{2}\left(\frac{dx}{dx}\right)^2 - (-V)\right]$$


Amplitude is given by exp[-S_E/ħ],
where S_E is the action of this Euclidean
soln
x_0
x_1
τ → ±∞
τ = 0
x_0
x_1

Amplitude is given by $\exp[-S_B/\hbar]$, where S_B is the action of this Euclidean soln





$\tau \to \pm\infty \quad x = x_0$

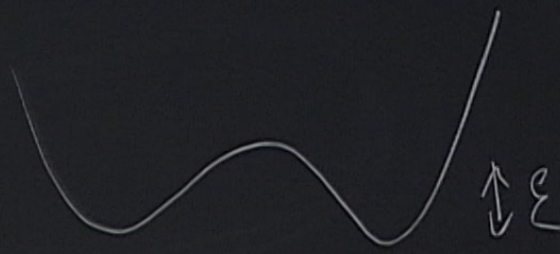
$\tau = 0 \quad x = x_1$

"BOUNCE"

False vacuum decay

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \frac{\lambda}{2}(\phi^2 - \eta^2)^2 - \frac{\epsilon(\phi-\eta)}{2\eta}$$



$\phi = \eta$ $V = 0$ – VACUUM.

$\phi = -\eta$ $V = \varepsilon$, but $V' = 0$, $V'' > 0$.

"FALSE VACUUM"

False vacuum is <u>metastable</u>,

eventually ϕ will tunnel to true vac

Look for Euclidean soln

$$\frac{\partial^2\phi}{\partial\tau^2} + \nabla^2\phi = \frac{\partial V}{\partial\phi} = 2\lambda\phi(\phi^2 - \eta^2) - \epsilon/\eta$$

Look for Euclidean soln

$$\frac{\partial^2\phi}{\partial\tau^2} + \nabla^2\phi = \frac{\partial V}{\partial\phi} = 2\lambda\phi(\phi^2 - \eta^2) - \underbrace{\epsilon/2\eta}_{\text{SMALL}}$$

$\tau, \underline{x}$

1st order phase transition,

look for spherically symmetric soln,

$\phi = \phi(\rho)$ $\rho^2 = \tau^2 + \underline{x}^2$.

Look for Euclidean soln

$$\frac{\partial^2\phi}{\partial\tau^2} + \nabla^2\phi = \frac{\partial V}{\partial\phi} = 2\lambda\phi(\phi^2 - \eta^2) - \underbrace{\epsilon/2\eta}_{\text{SMALL}}$$

$\tau, \underline{x}$

1st order phase transition,

look for spherically symmetric soln.

$\phi = \phi(\rho)$ $\rho^2 = \tau^2 + \underline{x}^2$.

$\phi'' + \frac{3\phi'}{\rho} = 2\lambda\phi(\phi^2 -$

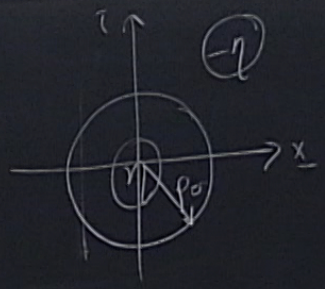
$\phi'' + \frac{3}{\rho}\phi' \simeq 2\lambda\phi(\phi^2 - \eta^2)$

$\phi = -\eta \tanh[\sqrt{\lambda}\eta(\rho - \rho_0)]$

good approx if $\rho_0 \gg 1/\sqrt{\lambda}\eta$

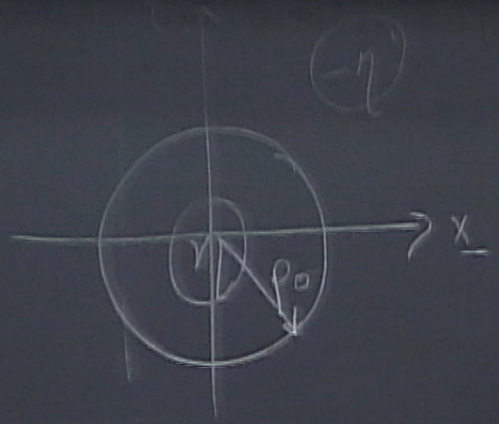
$\tau \to \pm\infty \quad \phi \to -\eta$

$\tau, \underline{x} \to 0. \quad \phi \simeq +\eta$



At ρ_0 have wall

$$I_E = \int \frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V.$$



At ρ_0 have wall.

$$I_E = \int \frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V.$$

$$\simeq \int \frac{1}{2}\phi'^2 + V$$

$$= \underbrace{\sigma}_{\text{tension}} \times \underbrace{2\pi^2\rho_0^3}_{\text{area}}.$$

Action for false vacuum

$$\epsilon \int_0^{\infty} 2\pi^2\rho^3 d\rho.$$

$$+\epsilon \int_{\rho_0}^{\infty} 2\pi^2\rho^3 d\rho$$

$\phi = \phi(\rho)$ $\rho^2 = \tau^2 + \underline{x}^2$.

Action for $S_B = I_E - I_{FV}$.

$$= 2\pi^2 \rho_0^3 \sigma - \varepsilon \frac{\pi^2}{2} \rho_0^4$$

To find preferred radius - take $\frac{\partial S_B}{\partial \rho_0} = 0$

$$6\pi^2 \rho_0^2 \sigma = 2\varepsilon \pi^2 \rho_0^3$$

$$\Rightarrow \quad \rho_0 = 3\sigma/\varepsilon$$

Action for $S_B = I_E - I_{F.V.}$

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To find preferred radius – take $\frac{\partial S_B}{\partial \rho_0} = 0$.

$$6\pi^2 \rho_0^2 \sigma = 2\varepsilon \pi^2 \rho_0^3$$

$$\Rightarrow \quad \rho_0 = 3\sigma/\varepsilon \gg 1$$

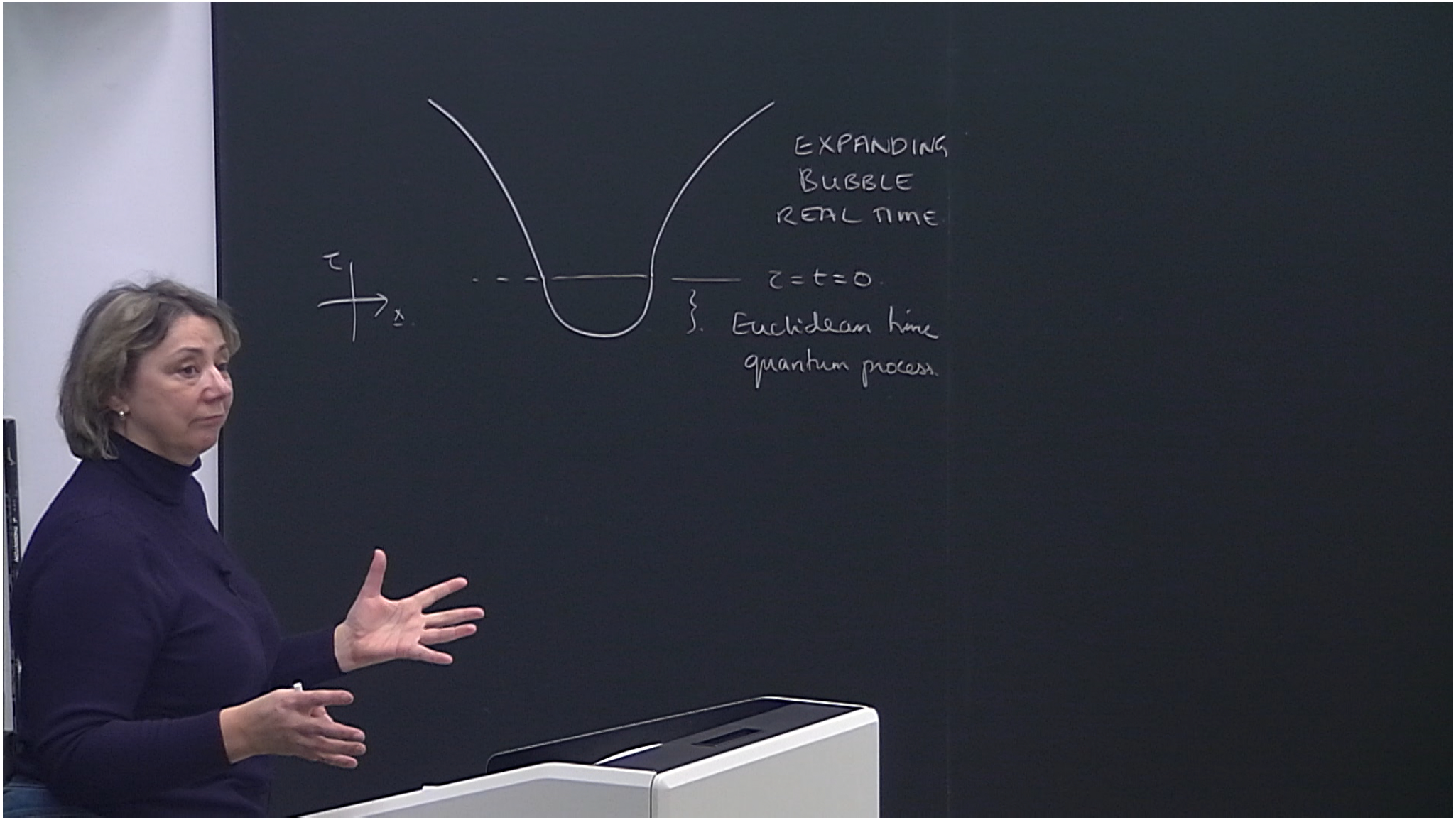
consistent.

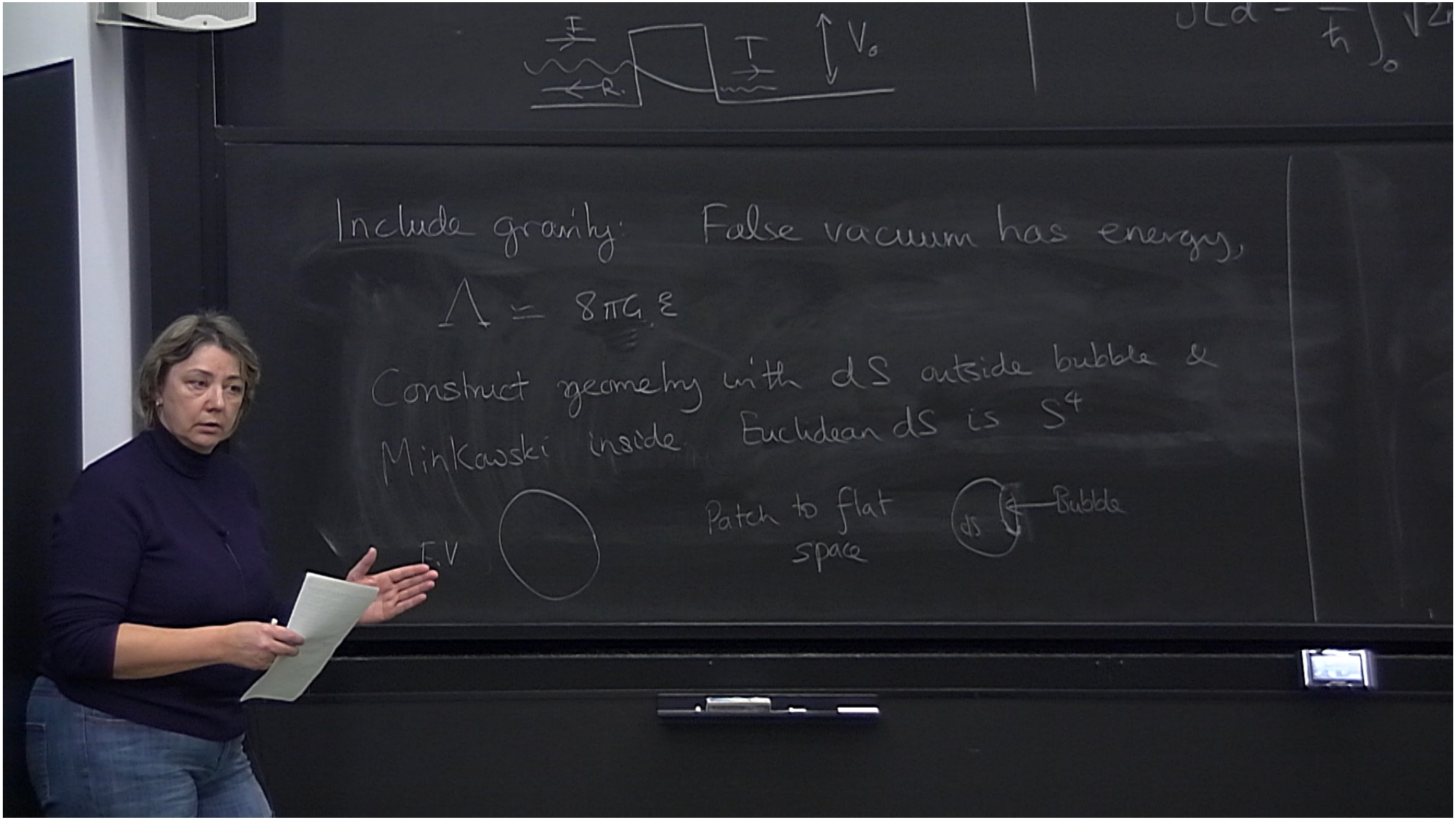
$$\Rightarrow S_B = \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$$

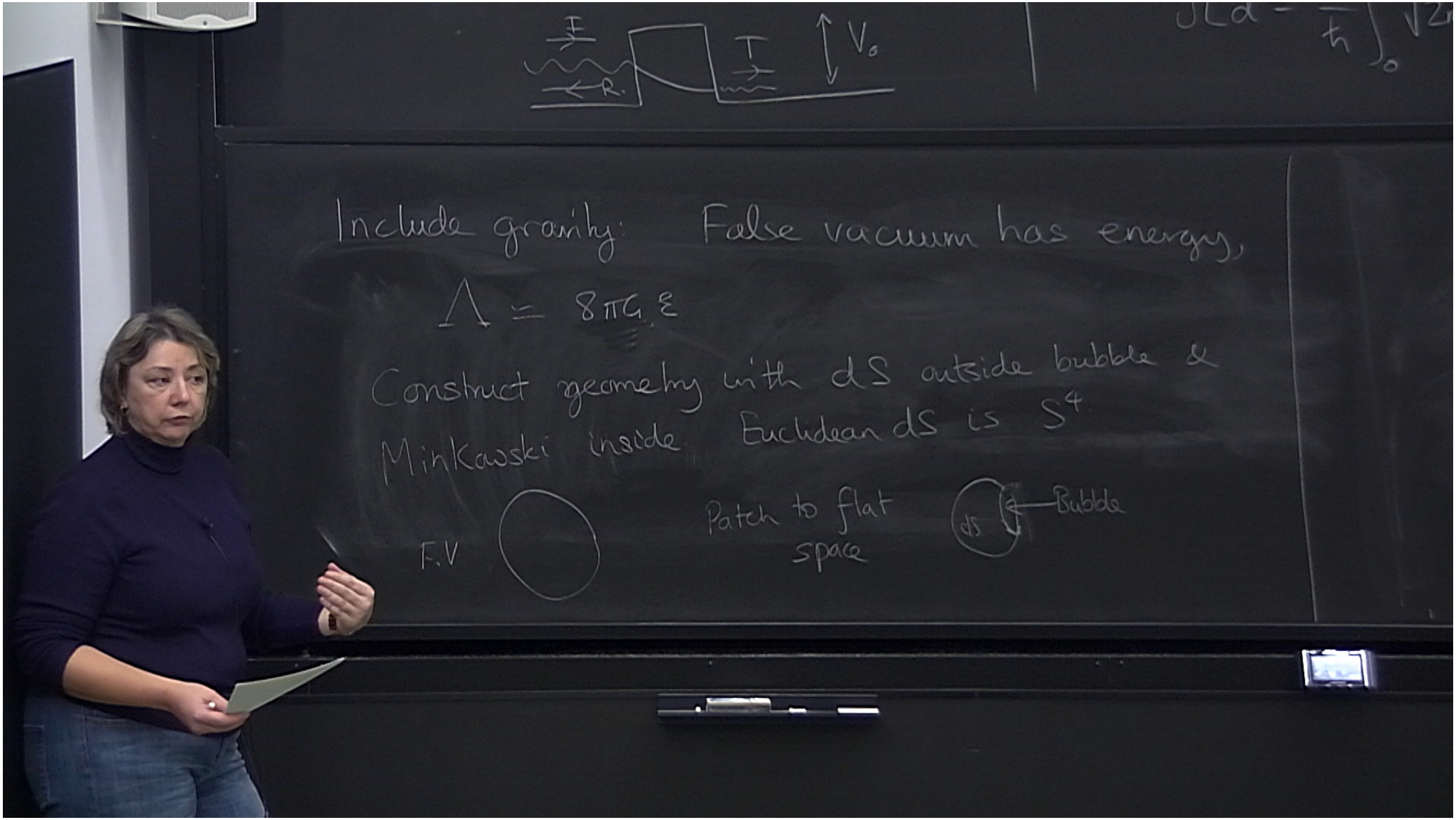
Once bubble has formed, it expands, find Lorentzian solution by analytic continuation

Euclide

τ^2


EXPANDING
BUBBLE
REAL TIME
τ
x
τ = t = 0
Euclidean time
quantum process


I
R
I
V_0
Include gravity: False vacuum has energy,
$\Lambda = 8\pi G_N \mathcal{E}$
Construct geometry with dS outside bubble &
Minkowski inside. Euclidean dS is S^4
F.V
Patch to flat space
dS
Bubble


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Construct geometry with dS outside bubble &
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$\Rightarrow \dot{\rho}^2 = 2m(E-V)$

energy,

bble &

de

Application of Israel eqns

$\mathcal{M}_+ = dS$, boundary - bubble wall

$\mathcal{M}_- = M.S.T.$ " " "

Calculat extrinsic curvature.

$\mathcal{M}_-$: $ds^2 = d\rho^2 + \rho^2 d\Omega^2_{III}$ Wall at ρ_0, normal $\frac{\partial}{\partial \rho}$.

Application of Israel eqns

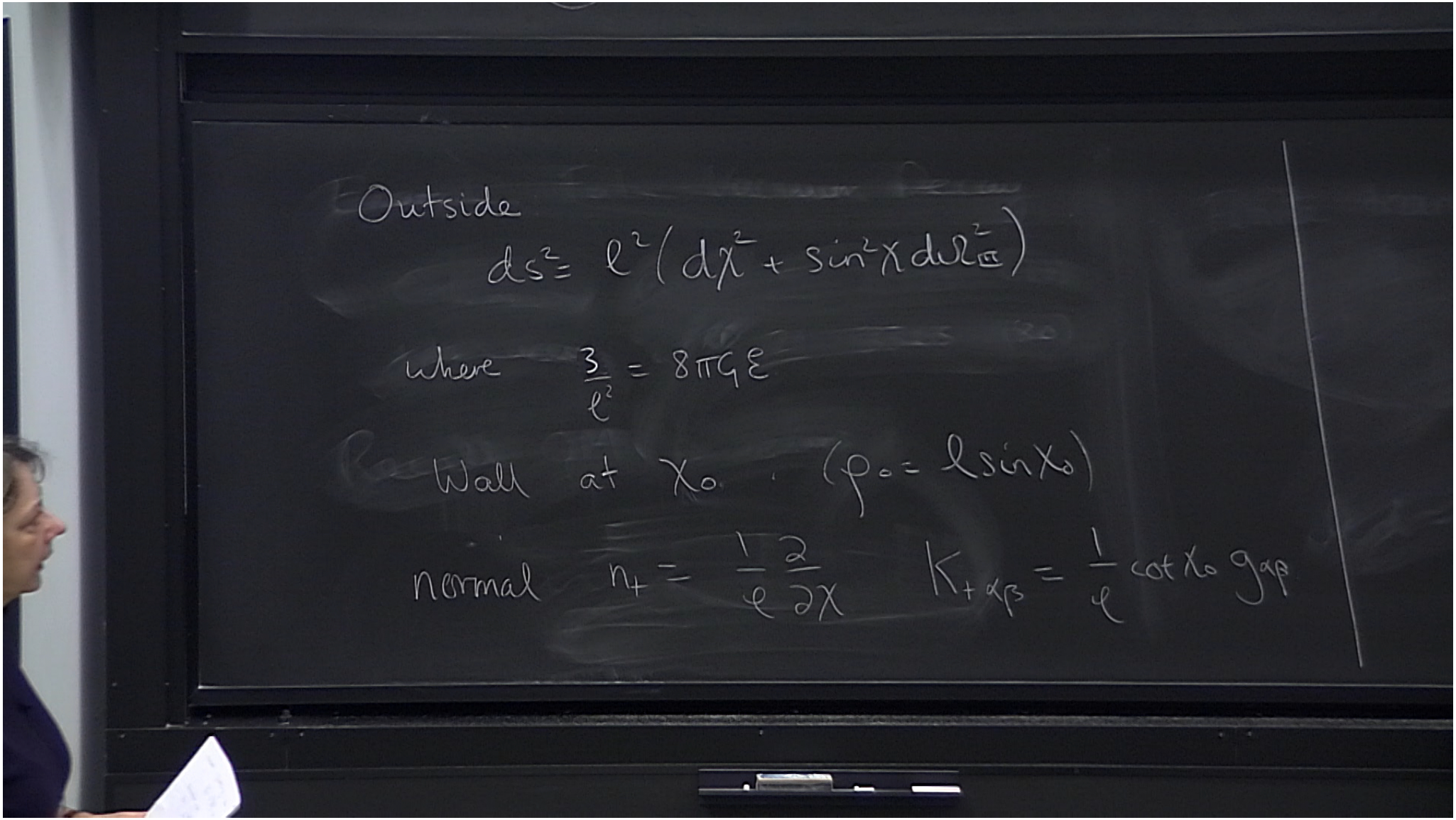
$M_+ = dS$, boundary - bubble wall

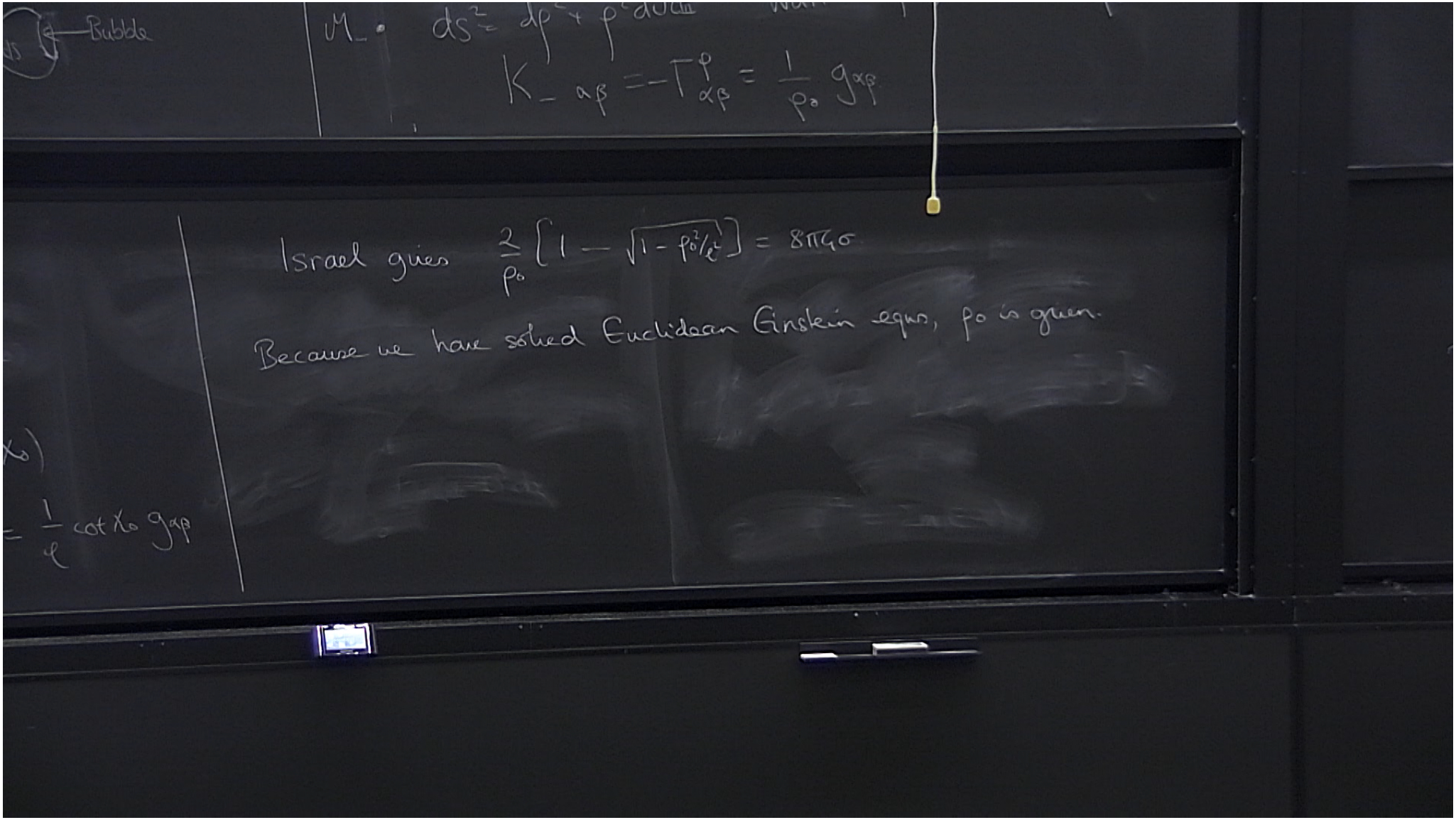
$M_- = M.S.T.$ " " "

Calculat extrinsic curvature.

M_- $ds^2 = d\rho^2 + \rho^2 d\Omega^2_{III}$ Wall at ρ_0, normal $\frac{\partial}{\partial \rho}$.

$$K_{-\,\alpha\beta} = -\Gamma^{\rho}_{\alpha\beta} = \frac{1}{\rho_0} g_{\alpha\beta}$$


Outside
$ds^2 = \ell^2 (d\chi^2 + \sin^2\chi\, d\Omega_{(2)}^2)$
where $\frac{3}{\ell^2} = 8\pi G \varepsilon$
Wall at χ_0 : $(\rho_0 = \ell \sin\chi_0)$
normal $n_+ = \frac{1}{\ell}\frac{\partial}{\partial\chi}$ $K_{+\alpha\beta} = \frac{1}{\ell}\cot\chi_0\, g_{\alpha\beta}$


Bubble
$K_{-\alpha\beta} = -\Gamma^{\rho}_{\alpha\beta} = \frac{1}{\rho_0} g_{\alpha\beta}$
Israel gives $\frac{2}{\rho_0}\left[1 - \sqrt{1 - \rho_0^2/\ell^2}\right] = 8\pi G\sigma$
Because we have solved Euclidean Einstein eqns, ρ_0 is given.
$\frac{1}{\ell}\cot\chi_0\, g_{\alpha\beta}$

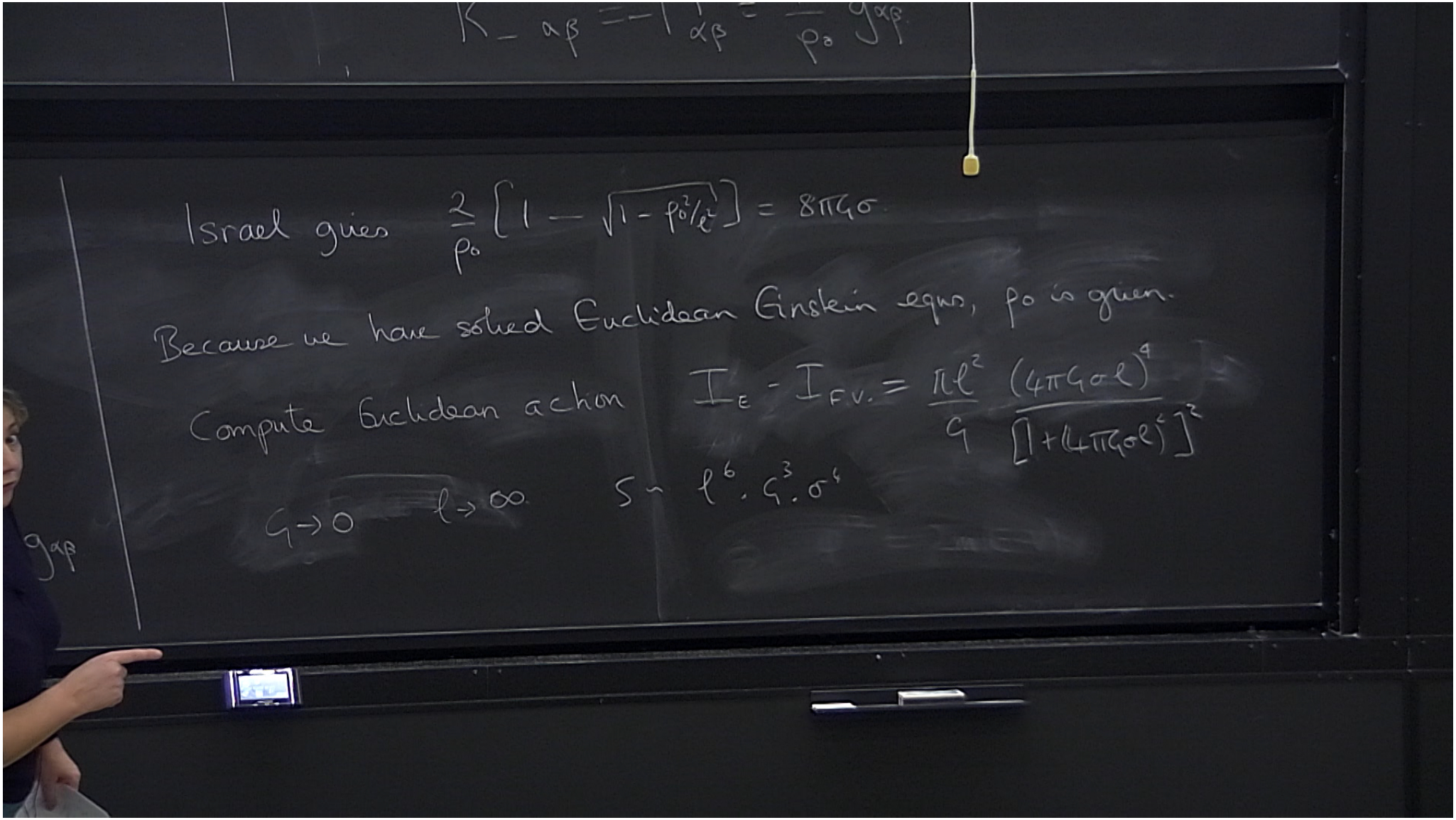
$K_{-\alpha\beta} = -\frac{1}{\alpha\beta} = \frac{1}{\rho_0} g_{\alpha\beta}$

Israel gives $\frac{2}{\rho_0}\left[1 - \sqrt{1 - \rho_0^2/\ell^2}\right] = 8\pi G\sigma$

Because we have solved Euclidean Einstein eqns, ρ_0 is given.

Compute Euclidean action $I_E - I_{F.V.} = \frac{\pi\ell^2}{9} \frac{(4\pi G\sigma\ell)^4}{\left[1 + (4\pi G\sigma\ell)^2\right]^2}$

$g_{\alpha\beta}$


$K_{-\alpha\beta} = -\frac{1}{\rho_0} g_{\alpha\beta}$
Israel gives $\frac{2}{\rho_0}\left[1-\sqrt{1-\rho_0^2/\ell^2}\right] = 8\pi G\sigma$
Because we have solved Euclidean Einstein eqns, ρ_0 is given.
Compute Euclidean action $I_E - I_{F.V.} = \frac{\pi\ell^2}{q}\frac{(4\pi G\sigma\ell)^4}{\left[1+(4\pi G\sigma\ell)^2\right]^2}$
$G \to 0 \quad \ell \to \infty \quad S \sim \ell^6 \cdot G^3 \cdot \sigma^4$
$g_{\alpha\beta}$

$K_{-\alpha\beta} = -\Gamma_{\alpha\beta} = \frac{}{\rho_0} g_{\alpha\beta}$

Israel gives $\frac{2}{\rho_0}\left[1 - \sqrt{1 - \rho_0^2/\ell^2}\right] = 8\pi G\sigma$

Because we have solved Euclidean Einstein eqns, ρ_0 is given.

Compute Euclidean action $I_E - I_{F.V.} = \frac{\pi\ell^2}{4}\frac{(4\pi G\sigma\ell)^4}{\left[1+(4\pi G\sigma\ell)^2\right]^2}$

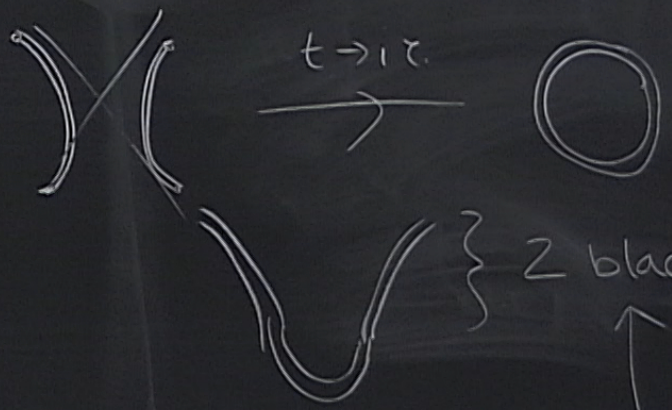
$G \to 0 \qquad \ell \to \infty \qquad S \sim \ell^6 \cdot G^3 \cdot \sigma^4$

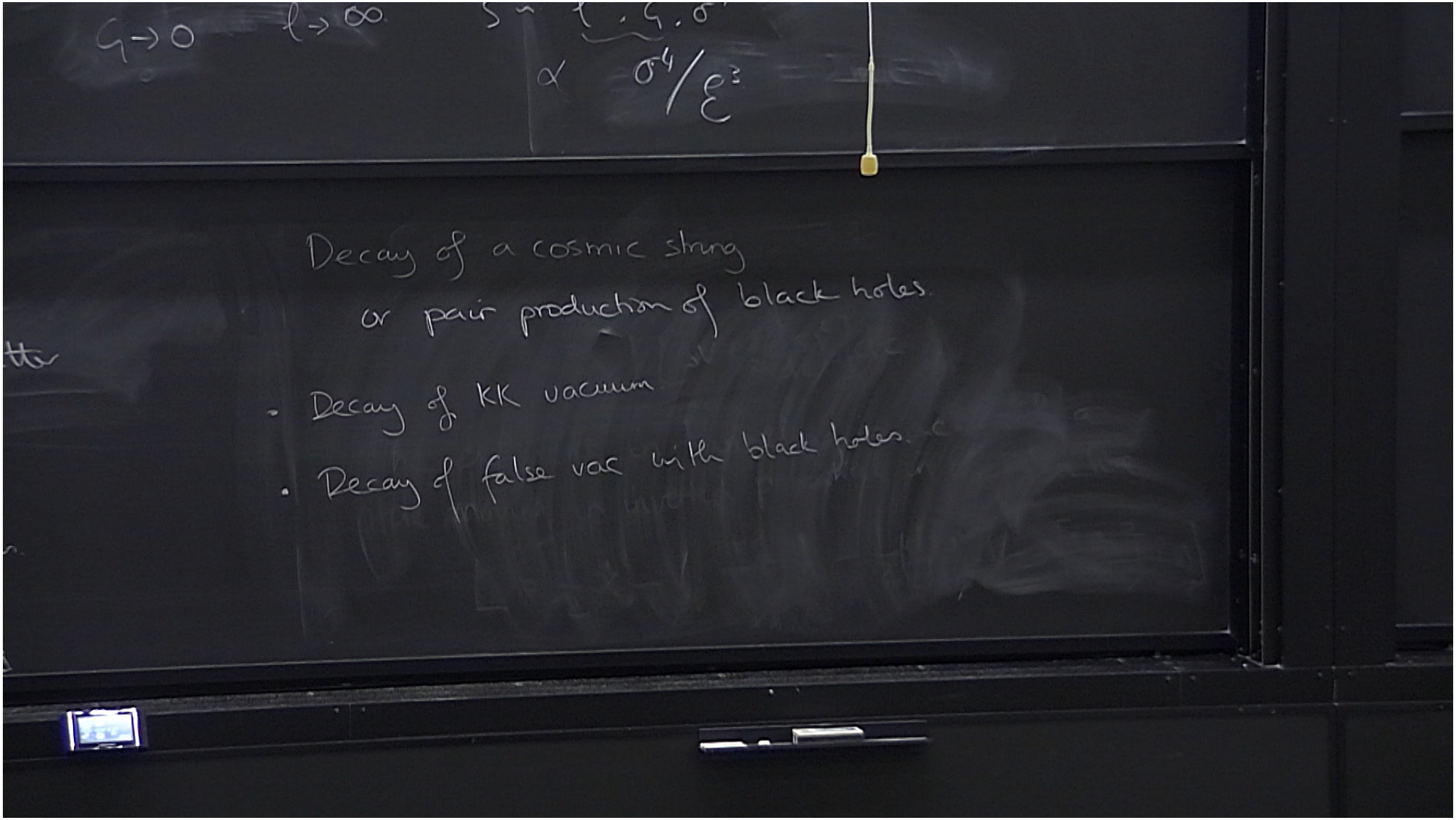
$\alpha \quad \sigma^4/\varepsilon^3$

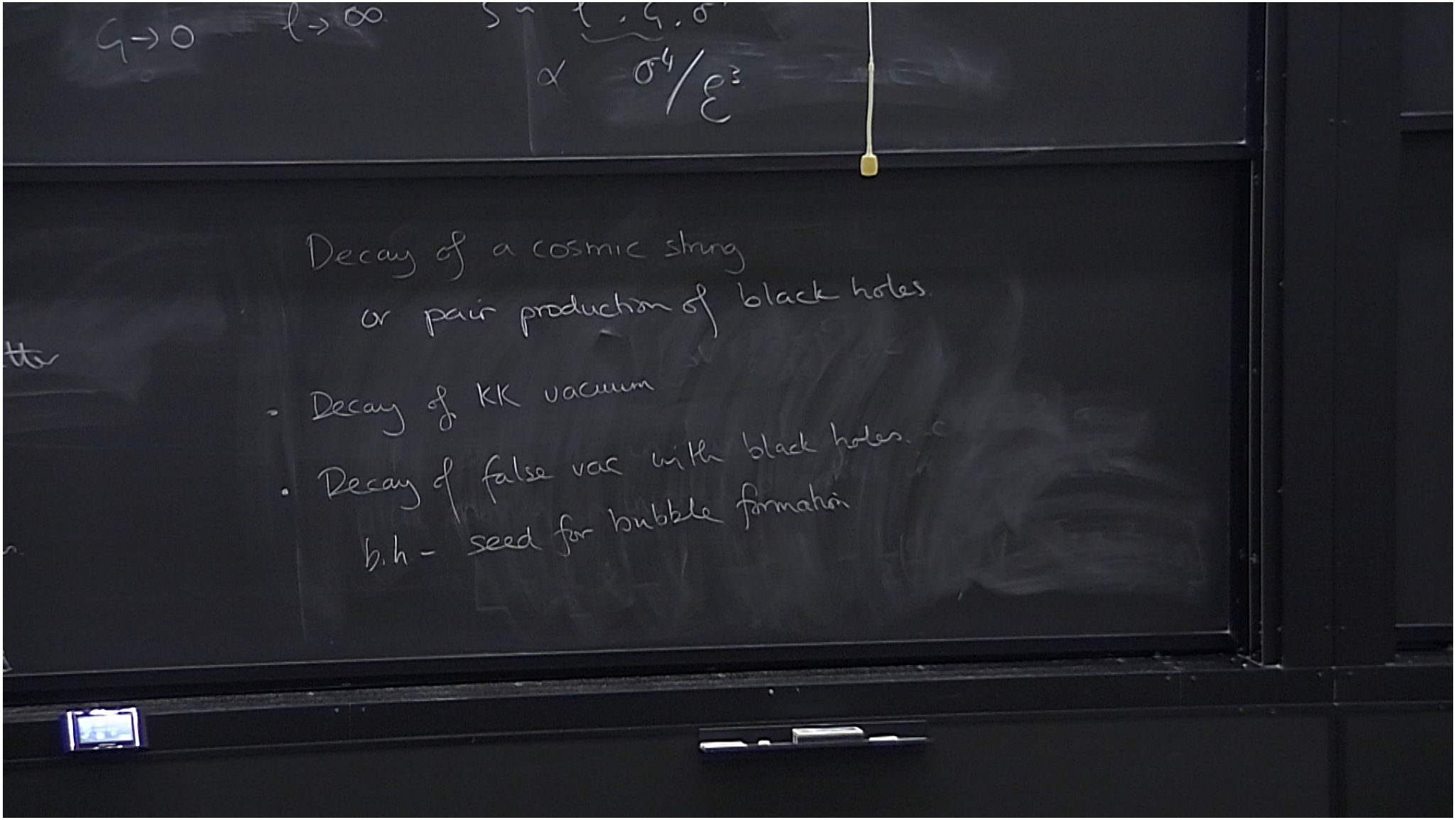
$g_{\alpha\beta}$

More generally, a gravitational instanton is a soln to Euclidean Einstein eqns with or without matter

- C-metric




$q \to 0$
$l \to \infty$
$\alpha \quad \sigma^4/\epsilon^3$
Decay of a cosmic string
or pair production of black holes.
• Decay of KK vacuum
• Decay of false vac with black holes.


$q \to 0$ $l \to \infty$
$\propto \sigma^4/\epsilon^3$
Decay of a cosmic string
or pair production of black holes.
- Decay of KK vacuum
- Decay of false vac with black holes.
b.h - seed for bubble formation