Title: PSI 2016/2017 Gravitational Physics (Review) - Lecture 12

Date: Jan 18, 2017 10:15 AM

URL: http://pirsa.org/17010030

Abstract:

LECTURE 12: Warped extra dimensions

If we have >1 extra dimension, our internal manifold can have its own symmetry group. $\{k_i^m\}$

lies in internal manifold

labels k.v.

e.g. $\xi_1 = \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi$

$\xi_2 = \cos\phi\,\partial_\theta - \cot\theta\sin\phi\,\partial_\phi$

$\xi_3 = \partial_\phi$

With an internal S^2 compactification.

$$[d\psi + A_\mu dx^\mu]^2 \quad \text{in 5D} \rightarrow$$

$$[dy^m + k_i^m A^i]^2$$

coords on internal manifold, here $\{\theta, \phi\}$.

With an internal S^2 compactification.

$[d\psi + A_\mu dx^\mu]^2$ in 5D $\rightarrow$

$[dy^m + k_i^m A^i]^2$

coords on internal manifold, here $\{\theta, \phi\}$.

A^i $i = 1,2,3$

gauge fields.

$$= -\frac{1}{2}\Box h_{ab} + R_{d(a} h^{d}{}_{b)} - R_{acbd} h^{cd}$$

$$+ \nabla_{(a} \nabla^{c} \left(h_{b)c} - \frac{1}{2} h g_{b)c} \right)$$

$$= -\frac{1}{2} \Delta_L h_{ab} \quad \text{(Lichnerowicz)}$$

Black String Instability

Consider linear perturbation theory $g \to g + h$.

Recall: $\delta R_{ab} = \nabla_c \delta \Gamma^c{}_{ab} - \nabla_b \delta \Gamma^c{}_{ac} = 0$

$$= \frac{1}{2} \nabla_c \nabla_a h^c{}_b + \frac{1}{2} \nabla_c \nabla_b h^c{}_a - \frac{1}{2} \Box h_{ab} - \frac{1}{2} \nabla_a \nabla_b h$$

$$= \frac{1}{2} \nabla_a \nabla_c h^c{}_b + \frac{1}{2} \nabla_b \nabla_c h^c{}_a - \frac{1}{2} \Box h_{ab} - \frac{1}{2} \nabla_a \nabla_b h$$

$$+ \frac{1}{2} R^c{}_{dca} h^d{}_b + \frac{1}{2} R_{bdca} h^{cd} + \frac{1}{2} R^c{}_{dcb} h^d{}_a + \frac{1}{2} R_{adcb} h^{cd}.$$

$$= -\frac{1}{2}\Box h_{ab} + R_{d(a} h^{d}{}_{b)} - R_{acbd} h^{cd}$$

$$+ \nabla_{(a} \nabla^{c} \left(h_{b)c} - \frac{1}{2} h g_{b)c} \right)$$

$$= -\frac{1}{2} \Delta_L h_{ab} \qquad \text{(Lichnerowicz)}$$

Can choose gauge s.t. $\nabla^c \left(h_{cb} - \frac{1}{2} h g_{cb} \right) = 0$

De-Donder gauge

$\ldots_{cb} h^{cd}$

$$= -\frac{1}{2}\Box h_{ab} + R_{d(a} h^{d}{}_{b)} - R_{acbd} h^{cd}$$

$$+ \nabla_{(a} \nabla^{c} \left(h_{b)c} - \frac{1}{2} h g_{b)c} \right)$$

$$= -\frac{1}{2} \Delta_L h_{ab} \quad \text{(Lichnerowicz)}$$

Can choose gauge s.t. $\nabla^c \left(h_{cb} - \frac{1}{2} h g_{cb} \right) = 0$

De-Donder gauge. IF $T_{\mu\nu}, \delta T_{\mu\nu} = 0$,

can also take $h^{\lambda}{}_{\lambda} = 0$

$cb\, h^{cd}$.

Still have gauge freedom

$$X^a \to X^a + \chi^a$$

s.t. $\Box \chi^a + R^a{}_b \chi^b = 0.$

$\ldots + \frac{1}{2} R_{bdca} h^{cd} + \frac{1}{2} R^c{}_{dcb} h^d{}_a + \frac{1}{2} R_{adcb} h^{cd}$.

can also take

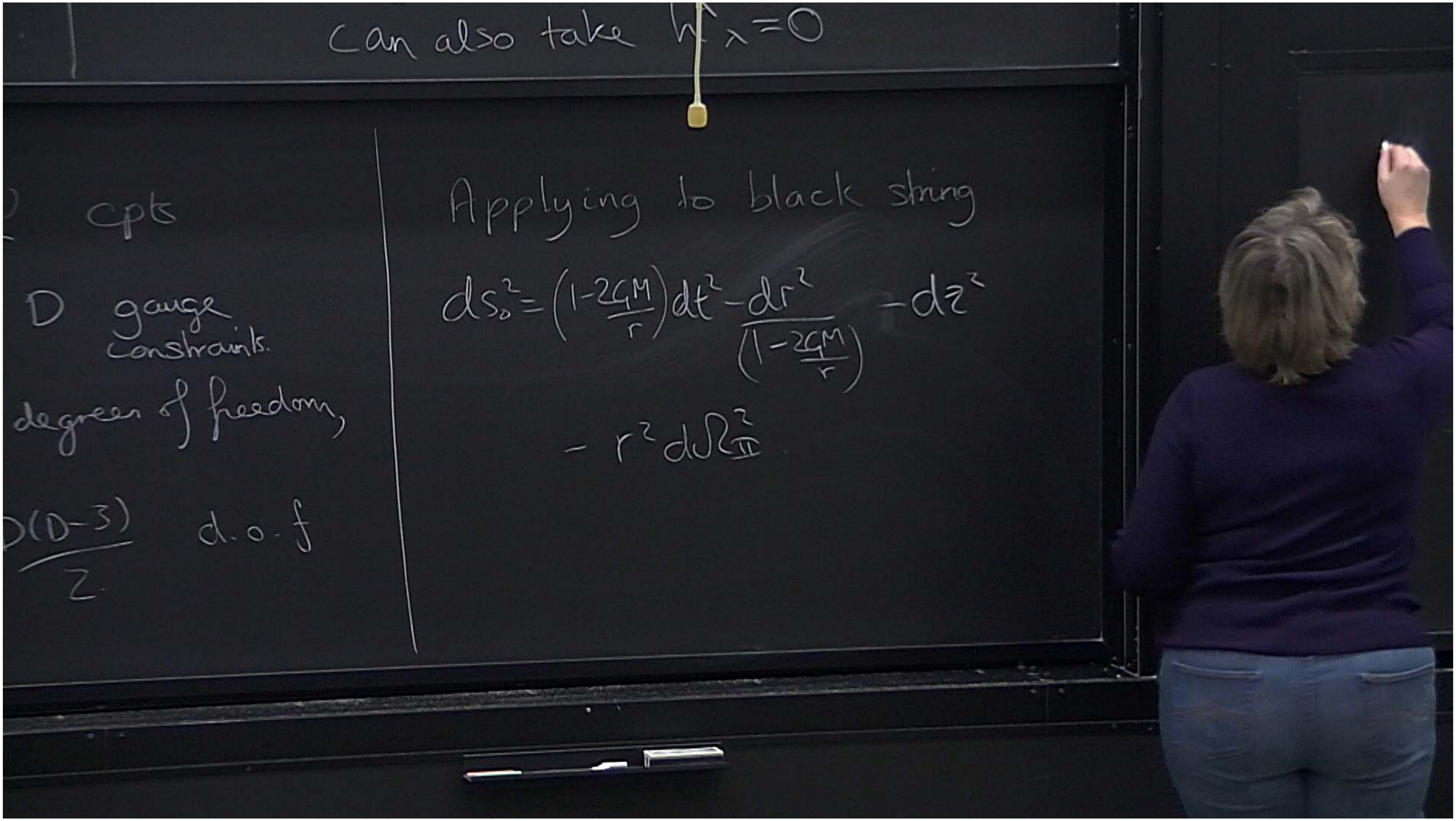
...dom

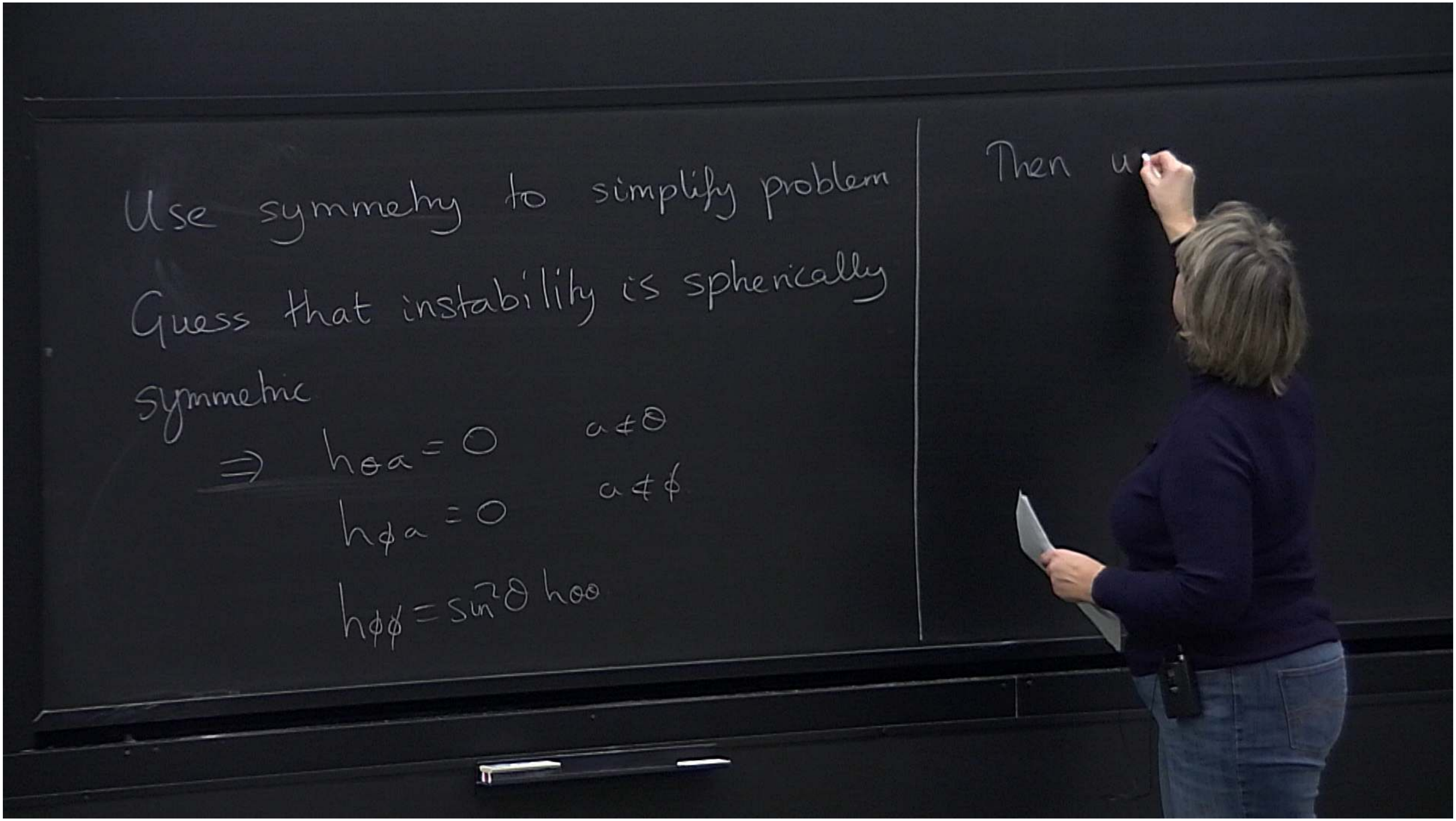
i.e. h_{ab} has $\frac{D(D+1)}{2}$ cpts

subtract $D_a(h^{ab} - \frac{1}{2} h g^{ab}) = 0$ = D gauge constraints

have X^a = D gauge degrees of freedom,

Physical graviton has $\frac{D(D-3)}{2}$ d.o.f


can also take $h^{\lambda}{}_{\lambda}=0$
cpts
D gauge constraints.
degrees of freedom,
$\frac{D(D-3)}{2}$ d.o.f
Applying to black string
$ds_0^2=\left(1-\frac{2GM}{r}\right)dt^2-\frac{dr^2}{\left(1-\frac{2GM}{r}\right)}-dz^2$
$-r^2d\Omega_{II}^2$


Use symmetry to simplify problem
Guess that instability is spherically symmetric
$\Rightarrow h_{\theta a} = 0 \quad a \neq \theta$
$h_{\phi a} = 0 \quad a \neq \phi$
$h_{\phi\phi} = \sin^2\theta \, h_{\theta\theta}$
Then u

em

lly

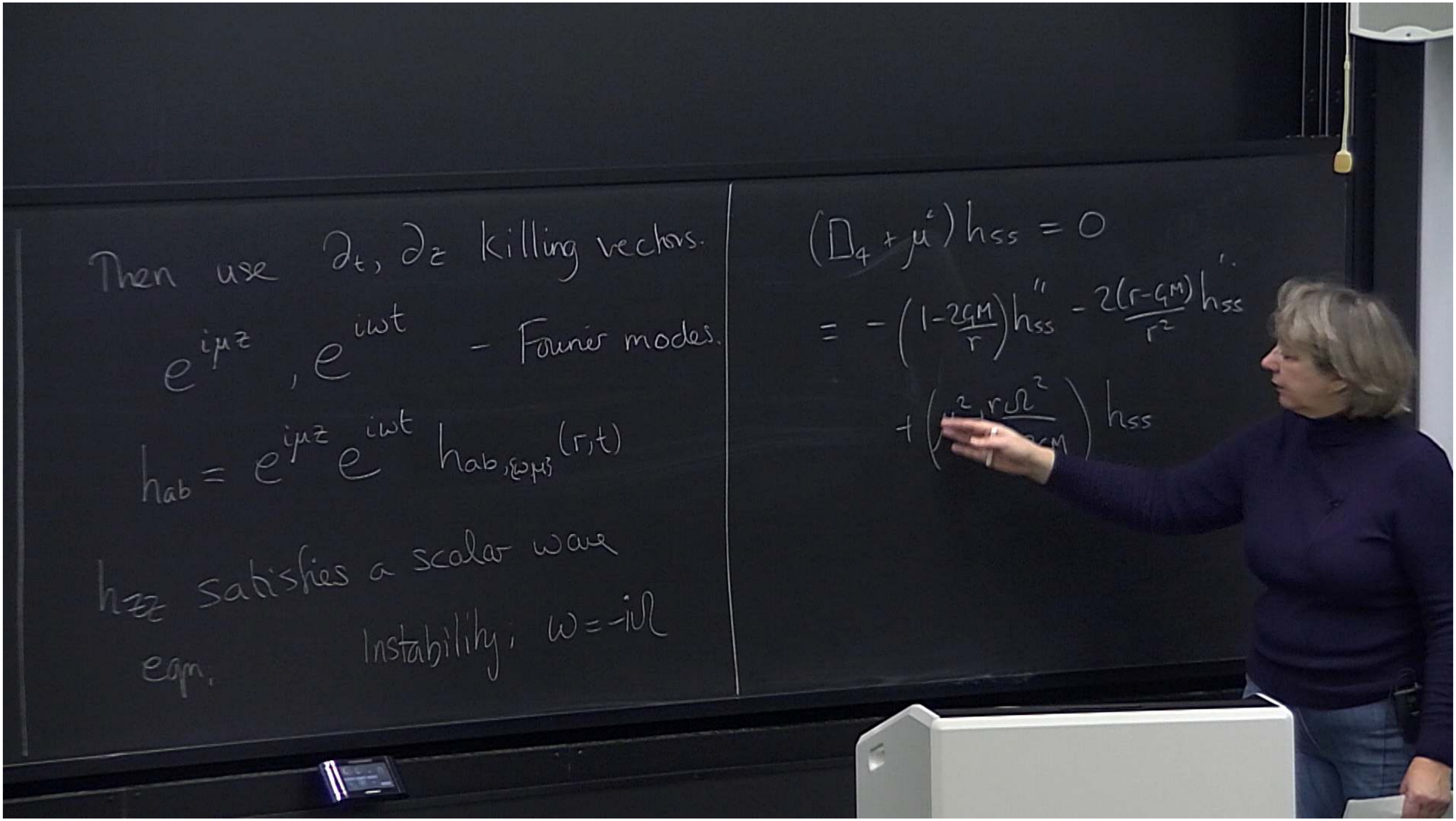
Then use ∂_t, ∂_z killing vectors.

$e^{i\mu z}, e^{i\omega t}$ – Fourier modes.

$$h_{ab} = e^{i\mu z} e^{i\omega t} h_{ab,\{\omega\mu\}}(r,t)$$

h_{zz} satisfies a scalar wave eqn.

$(\Box_4 + \mu^2) h_{S}$


Then use ∂_t, ∂_z killing vectors.
$e^{i\mu z}$, $e^{i\omega t}$ - Fourier modes.
$h_{ab} = e^{i\mu z} e^{i\omega t} h_{ab,\{\omega\mu\}}(r,t)$
h_{zz} satisfies a scalar wave eqn.
Instability: $\omega = -i\Omega$
$(\Box_4 + \mu^2) h_{ss} = 0$
$= -\left(1-\frac{2GM}{r}\right) h_{ss}'' - \frac{2(r-GM)}{r^2} h_{ss}'$
h_{ss}

ectors.

modes.

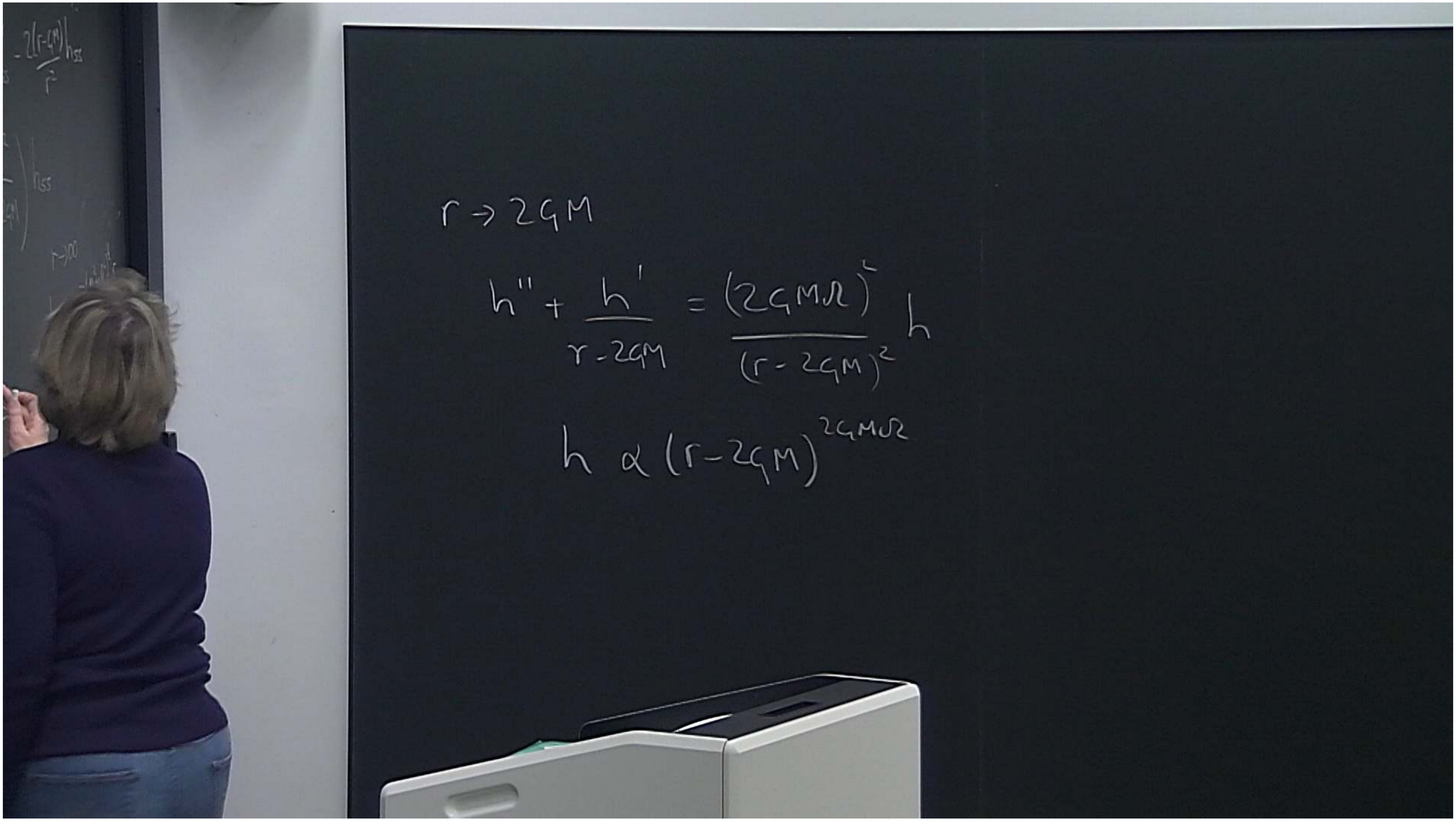
$$(\Box_4 + \mu^2) h_{ss} = 0$$

$$= -\left(1-\frac{2GM}{r}\right) h_{ss}'' - \frac{2(r-GM)}{r^2} h_{ss}'$$

$$+ \left(\mu^2 + \frac{r\Omega^2}{r-2GM}\right) h_{ss}$$

$r \to \infty$

$$h \propto e^{-(\mu^2+\Omega^2)^{1/2} r}$$


$r \to 2GM$
$h'' + \frac{h'}{r-2GM} = \frac{(2GM\Omega)^2}{(r-2GM)^2} h$
$h \propto (r-2GM)^{2GM\Omega}$

ng vectors.

uier modes.

(r,t)

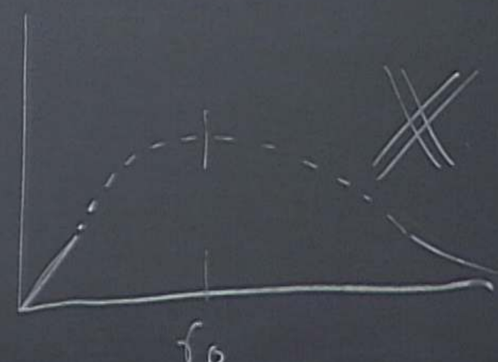
oau

$\omega = -i\Omega$

$$(\Box_4 + \mu^2)\, h_{ss} = 0$$

$$= -\left(1-\frac{2GM}{r}\right) h''_{ss} - \frac{2(r-GM)}{r^2}\, h'_{ss}$$

$$+\left(\mu^2 + \frac{r\,\Omega^2}{r-2GM}\right) h_{ss}$$



$r \to \infty$

$$h \propto e^{-(\mu^2+\Omega^2)^{1/2} r}$$

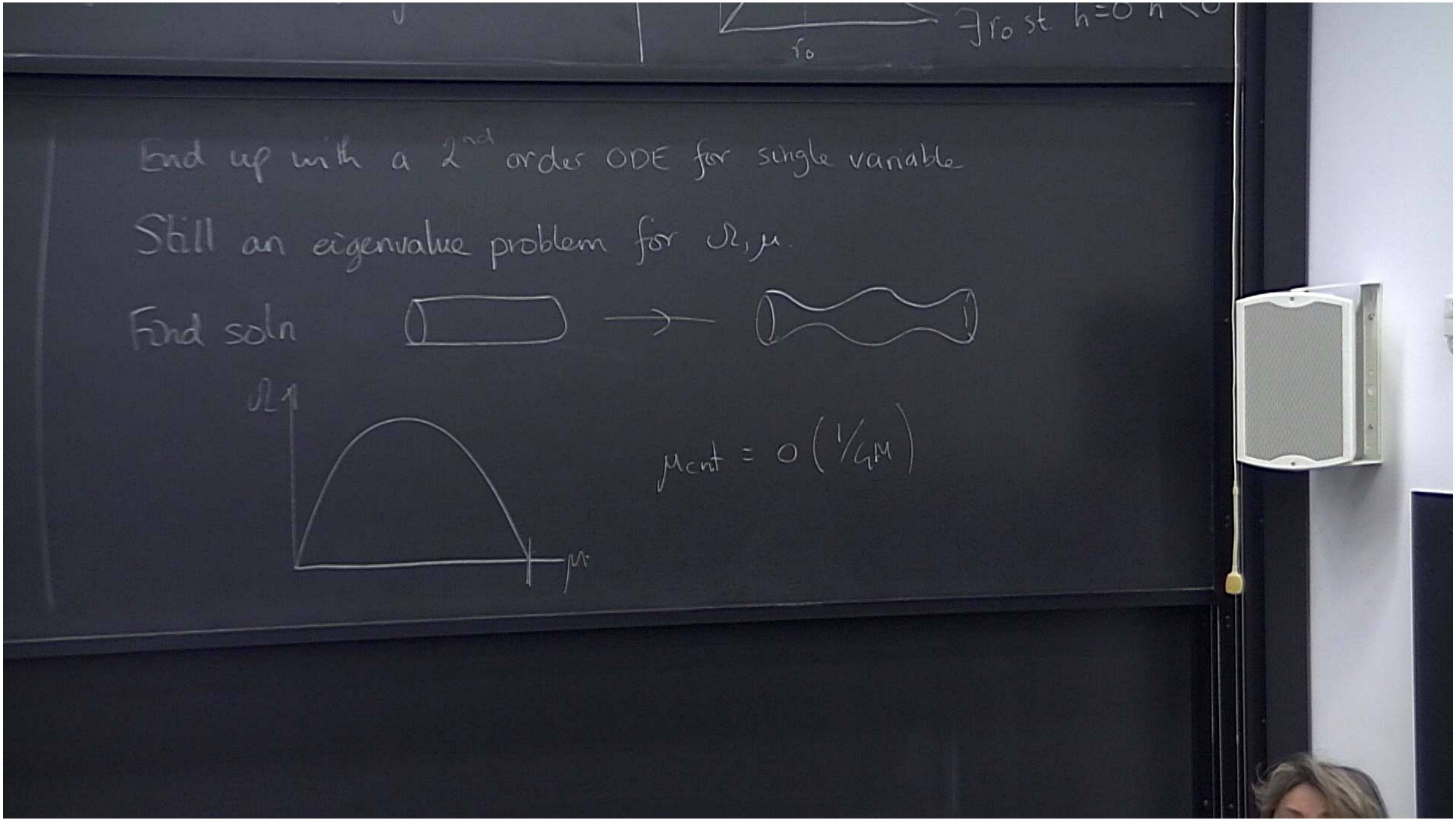
$\exists\, r_0$ st. $h' = 0$ $h'' < 0$

Similar argument for $h_{5\mu}$

Remaining

$$h_{\mu\nu} = e^{\Omega t} e^{i\mu z} \begin{pmatrix} h_{tt} & h_{tr} & \multicolumn{2}{c}{0} \\ h_{tr} & h_{rr} & & \\ & & K & 0 \\ \multicolumn{2}{c}{0} & 0 & K\sin^2\theta \end{pmatrix}$$

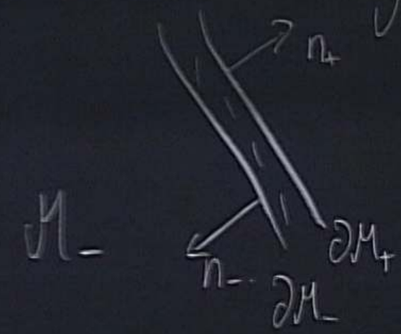
End up with


End up with a 2nd order ODE for single variable
Still an eigenvalue problem for Ω, μ.
Find soln
Ω
μ
μcrit = O(1/GM)
r0

Now consider different type of extra dimension.

Consider a domain wall in AdS

$$S = -\frac{1}{16\pi G}\int_{M_+ \cup M_-}(R+2\Lambda)\sqrt{g}\,d^5x - \frac{1}{8\pi G}\int_{\partial M_+ \cup \partial M_-}\sqrt{h}\,K + \int_{\Sigma}\sigma\sqrt{h}$$



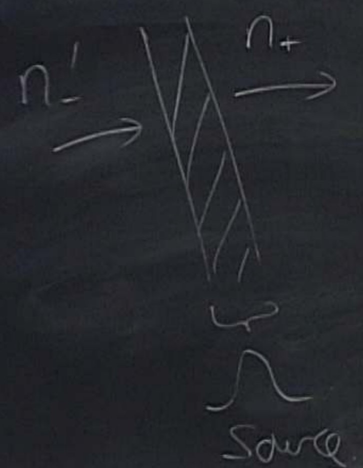
$$\Sigma = \partial\mathcal{M}_+ = \partial\mathcal{M}_-$$

$\dfrac{\delta S}{\delta g^{ab}}$ gives bulk Einstein eqns.

plus boundary term.

$$\left.\frac{K_{ab} - K h_{ab}}{-16\pi G}\right|_{\mathcal{M}_+ \& \mathcal{M}_-} = \frac{1}{2}\sigma h_{ab}$$

Views Σ as boundary But can also view
Σ as limit of a localised source



$$T_{\mu\nu} \simeq S_{\mu\nu}\, \delta(X^\mu - X^\mu(\sigma))$$

$$\Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G S_{ab} \quad \text{— } \underline{\text{ISRAEL EQNS}}$$

$$= 8\pi G \sigma h_{ab}$$

ADS $\quad ds^2 = e^{-2kz}\eta_{\mu\nu}dx^\mu dx^\nu - dz^2$

$z>0$

$n_+ = \frac{\partial}{\partial z}$ & take Σ at $z=0$.

$K_{+\mu\nu} = X^a_{,\mu} X^b_{,\nu} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = -k\eta_{\mu\nu}$

For $z<0$ take $ds^2 = e^{2kz}\eta_{\mu\nu}dx^\mu dx^\nu - dz^2$ $\quad(\mathbb{Z}_2\text{ symm})$

$K_{-\mu\nu} = -K_{+\mu\nu}$

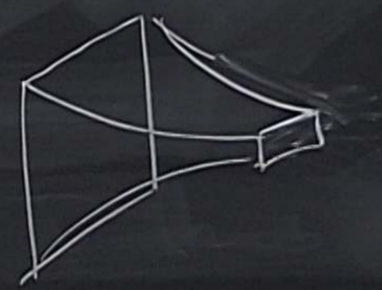
$$\Delta K_{\mu\nu} - \Delta K \eta_{\mu\nu} = -2k\eta_{\mu\nu} + 8k\eta_{\mu\nu} = 6k\eta_{\mu\nu} = 8\pi G \sigma \eta_{\mu\nu}$$

Domain wall has tension $\sigma = 6k/8\pi G$

Take $\eta_{\mu\nu} \to g_{\mu\nu}$, Einstein also solves 5D E-eqns

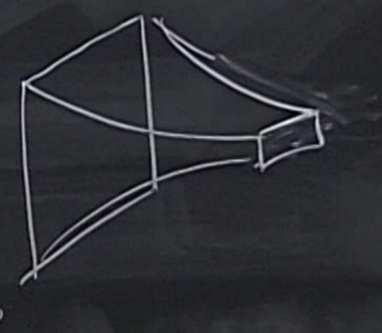
4D gravity sits inside 5D theory

Perturbation theory, including $\delta T_{\mu\nu}$ on brane, confirms gravity is localised at low energies.



$z=0$

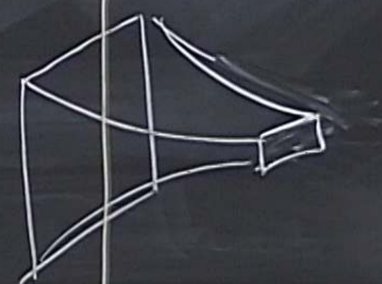
Perturbation theory, including $\delta T_{\mu\nu}$ on brane, confirms gravity is localised at low energies.



Opposite of KK theory.

$K_{-\mu\nu} = -K_{+\mu\nu}$

Perturbation theory, including $\delta T_{\mu\nu}$ on brane, confirms gravity is localised at low energies.



$z=0$

Opposite of KK theory.

RANDALL-SUNDRUM.

$$-\frac{1}{16\pi G_D}\int R_D\sqrt{g_D}\,d^Dx = -\frac{1}{16\pi G_D}\int (R_4 + \cdots)\sqrt{g_4}\sqrt{g_{D-4}}\,d^4x\,d^{D-4}y$$

$$B^2(y)\,g_{\mu\nu}dx^\mu dx^\nu - A^2(y)\,dy^2_{D-4}$$

$$-\frac{V_{D-4}}{16\pi G_D}\int R_4\sqrt{g_4}\,d^4x$$

RANDALL - SUNDRUM.

i.e. $G_N = G_D / V_{D-4}$ - renormalises Newton const.

- Alternate way to give hierarchy between gravity & gauge

To motivate localization on brane.

Domain wall $\phi \sim \eta \tanh(\sqrt{\lambda}\eta z)$

Couple fermion:

$$\mathcal{L}_\Psi = i\bar{\Psi}\Gamma^a D_a \Psi - g\phi\bar{\Psi}\Psi$$

$$i\gamma^5 \underline{\Psi}' = g\eta \tanh(\sqrt{\lambda}\eta z)\,\underline{\Psi}$$

$$\underline{\Psi} \propto \psi_0 \left[\operatorname{sech} \sqrt{\lambda}\eta z\right]^{g/\sqrt{\lambda}} \quad \text{if } i\gamma^5\psi_0 = -\psi_0 .$$

is lowest energy soln. Then $\psi_0(x)$ if $\partial\!\!\!/\,\psi_0 = 0$.

Illustrates warped extra dims

- hierarchy
- localization