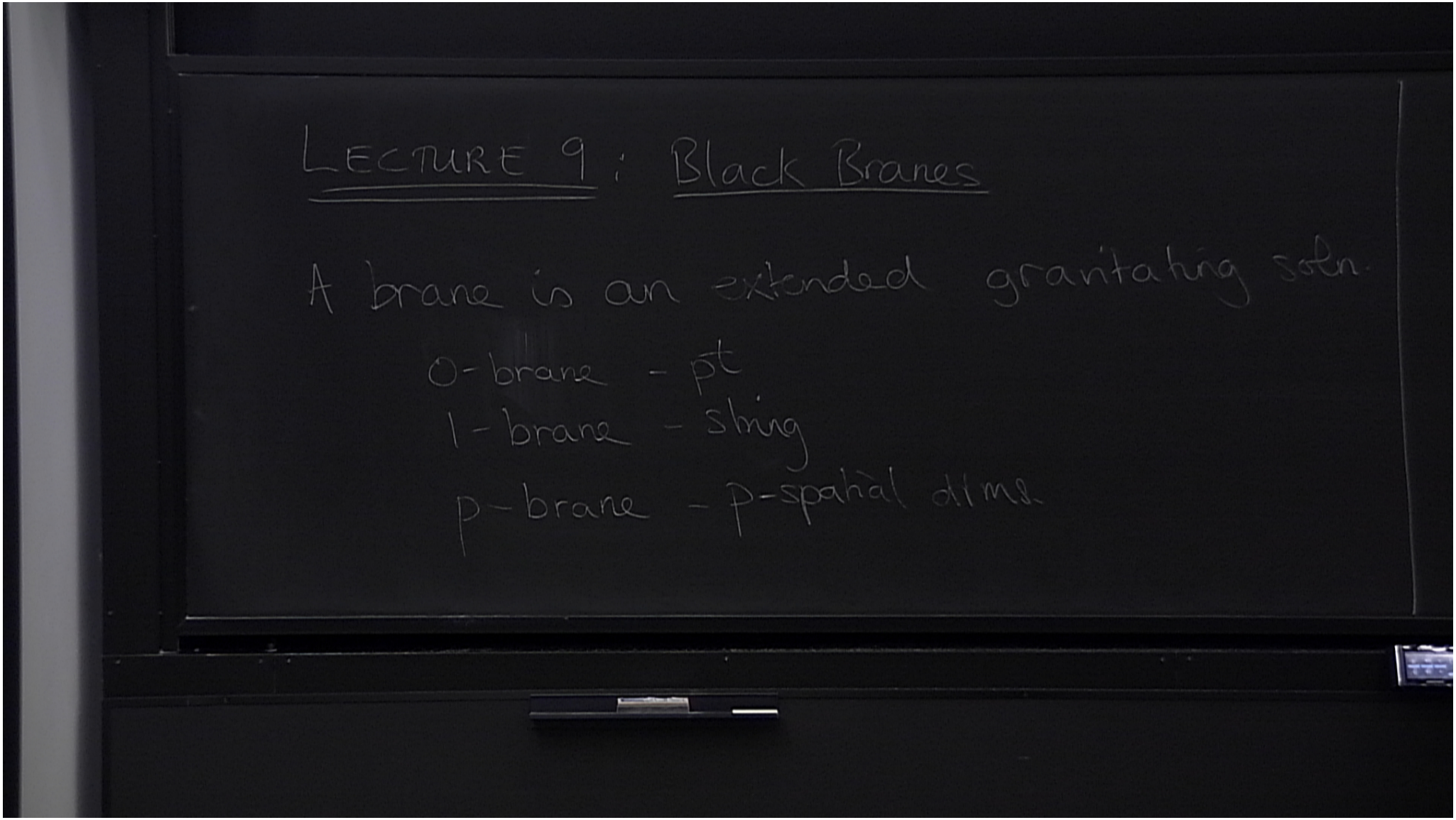
Title: PSI 2016/2017 Gravitational Physics (Review) - Lecture 9

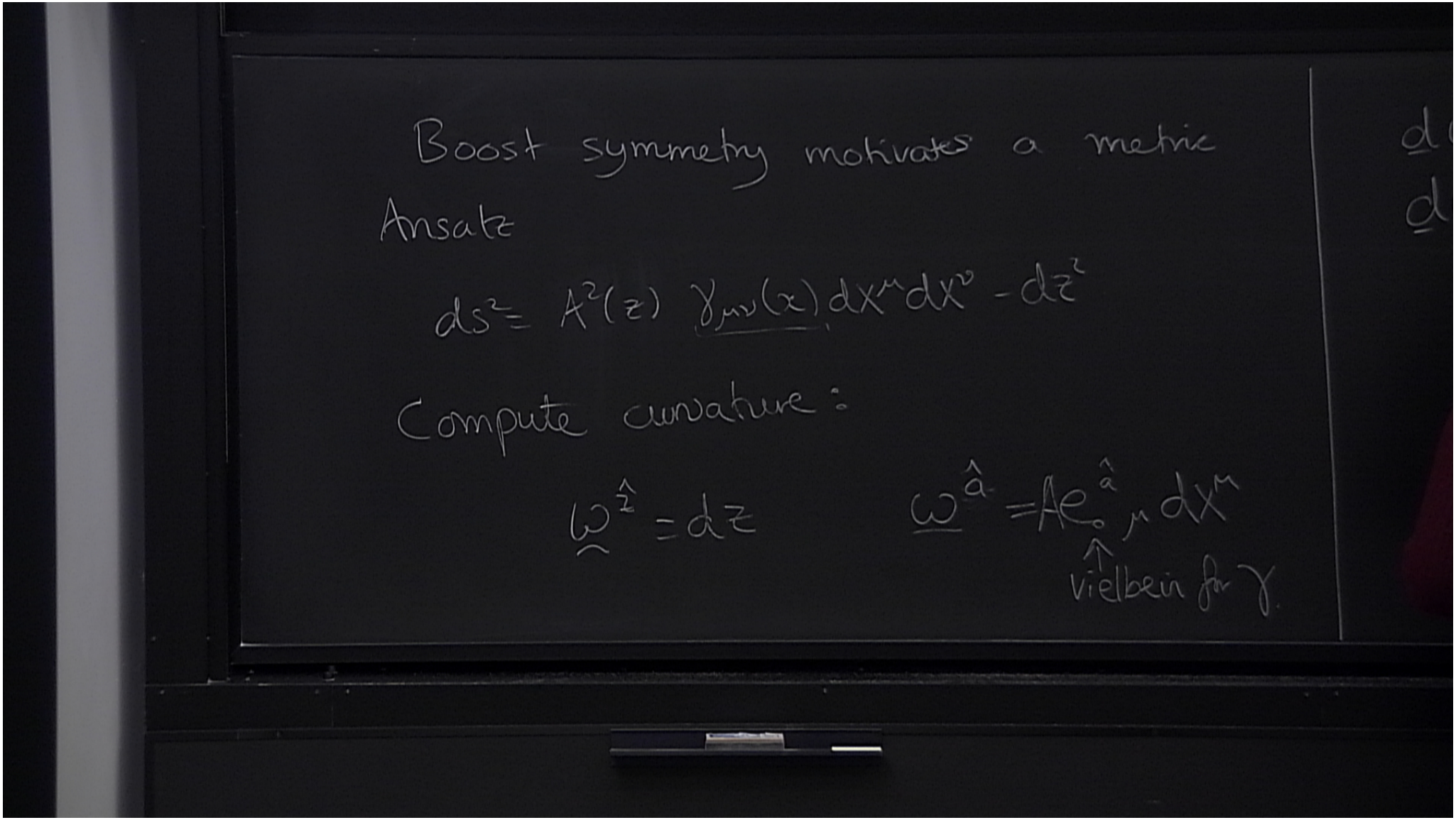
Date: Jan 13, 2017 10:15 AM

URL: http://pirsa.org/17010027

Abstract:




LECTURE 9: Black Branes
A brane is an extended gravitating soln.
0-brane - pt
1-brane - string
p-brane - p-spatial dims.


Boost symmetry motivates a metric
Ansatz
$ds^2 = A^2(z)\,\gamma_{\mu\nu}(x)\,dx^\mu dx^\nu - dz^2$
Compute curvature:
$\omega^{\hat{z}} = dz$
$\omega^{\hat{a}} = A e^{\hat{a}}{}_\mu\, dx^\mu$
Vielbein for $\gamma$.

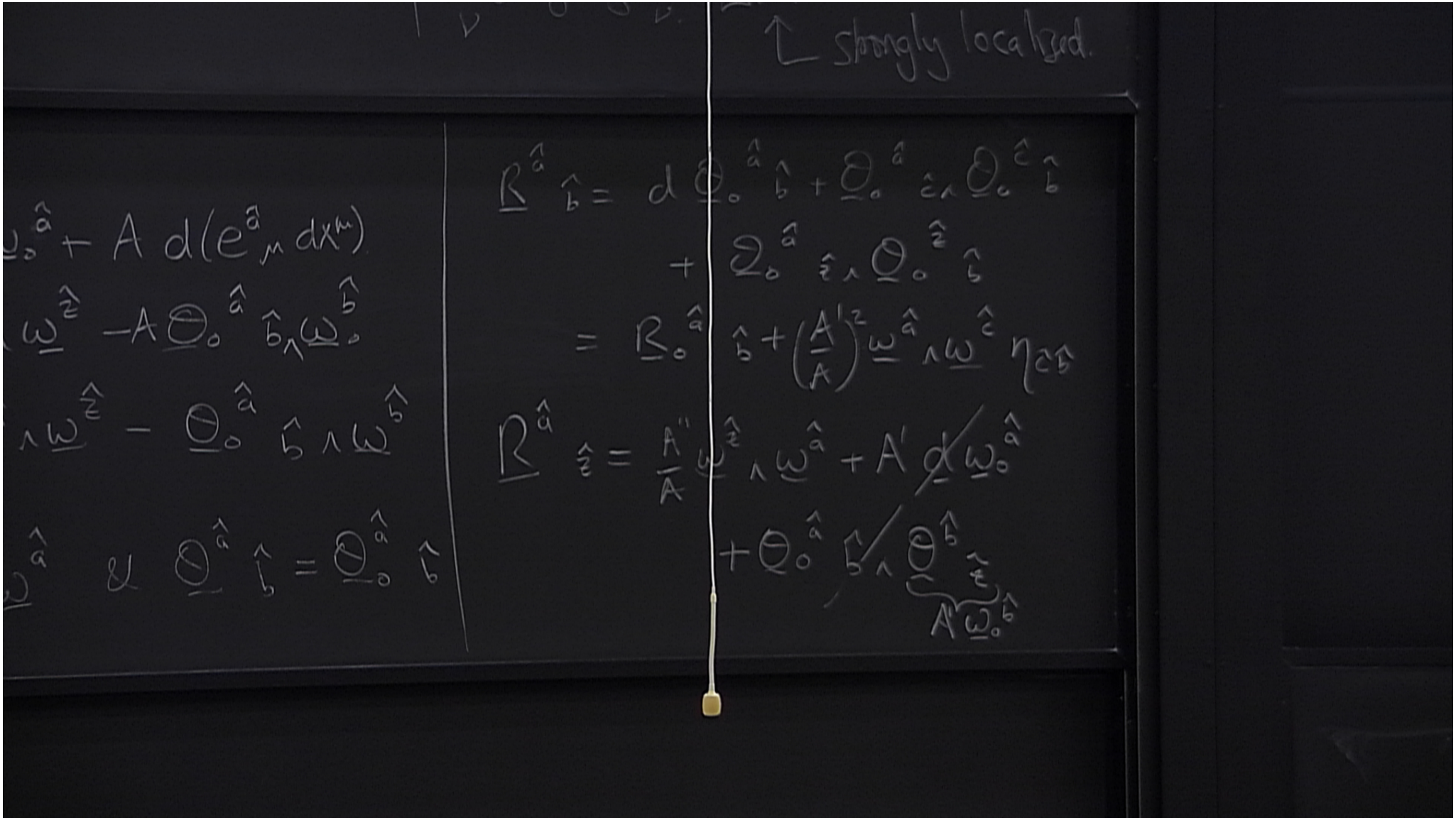
$$\underline{d\omega}^{\hat{z}} = 0$$

$$\underline{d\omega}^{\hat{a}} = A' dz \wedge \underline{\omega}_0{}^{\hat{a}} + A\, d(e^{\hat{a}}{}_\mu dx^\mu)$$

$$= -\frac{A'}{A}\underline{\omega}^{\hat{a}} \wedge \underline{\omega}^{\hat{z}} - A\underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}} \wedge \underline{\omega}_0{}^{\hat{b}}$$

$$= -\frac{A'}{A}\underline{\omega}^{\hat{a}} \wedge \underline{\omega}^{\hat{z}} - \underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}} \wedge \underline{\omega}^{\hat{b}}$$

i.e. $\underline{\Theta}^{\hat{a}}{}_{\hat{z}} = \frac{A'}{A}\underline{\omega}^{\hat{a}}$ & $\underline{\Theta}^{\hat{a}}{}_{\hat{b}} = \underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}}$


↑ strongly localized.
$\omega_0^{\hat{a}} + A\, d(e^{\hat{a}}_{\mu} dx^{\mu})$
$\omega^{\hat{a}}$ & $\Theta^{\hat{a}}{}_{\hat{b}} = \Theta_0^{\hat{a}}{}_{\hat{b}}$
$R^{\hat{a}}{}_{\hat{b}} = d\Theta_0^{\hat{a}}{}_{\hat{b}} + \Theta_0^{\hat{a}}{}_{\hat{c}} \wedge \Theta_0^{\hat{c}}{}_{\hat{b}}$
$R^{\hat{a}}{}_{\hat{z}} = \frac{A''}{A} \omega^{\hat{z}} \wedge \omega^{\hat{a}} + A' d\omega_0^{\hat{a}}$
$A' \omega_0^{\hat{b}}$

$$\Rightarrow R^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}} = \frac{1}{A^2} R_0{}^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}} + \left(\frac{A'}{A}\right)^2 \left( \delta^{\hat{a}}{}_{\hat{c}} \eta_{\hat{b}\hat{d}} - \delta^{\hat{a}}{}_{\hat{d}} \eta_{\hat{b}\hat{c}} \right)$$

$$R^{\hat{a}}{}_{\hat{z}\hat{b}\hat{z}} = -\frac{A''}{A} \delta^{\hat{a}}{}_{\hat{b}}$$

In co-ord basis

$$R^{\mu\nu}{}_{\lambda\tau} = \frac{1}{A^2} R^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 \left( \delta^{\mu}_{\lambda} \delta^{\nu}_{\tau} - \delta^{\mu}_{\tau} \delta^{\nu}_{\lambda} \right)$$

$$R^{\mu z}{}_{\nu z} = \frac{A''}{A} \delta^{\mu}_{\nu}$$

$\omega^{\hat{a}} = A e^{\hat{a}}_{\ \mu}\, dx^{\mu}$

↑ Vielbein for $\gamma$

i.e. $\Theta^{\hat{a}}_{\ \hat{z}} = \frac{A'}{A}\,\omega^{\hat{a}}$ & $\Theta^{\hat{a}}_{\ \hat{i}} = \Theta^{\hat{a}}_{\ \hat{b}}$

$+\Theta_0{}^{\hat{a}}\ \Gamma_{\lambda}\ \Theta^{\hat{\lambda}}_{\ \hat{z}}$

$A\,\omega^{\hat{a}}_{\ 0}$

$\hat{a} + \left(\frac{A'}{A}\right)^2 \left(\delta^{\hat{a}}_{\ \hat{z}}\,\eta_{\hat{c}\hat{a}} - \delta^{\hat{a}}_{\ \hat{a}}\,\eta_{\hat{c}\hat{z}}\right)$

$\lambda\tau + \left(\frac{A'}{A}\right)^2 \left(\delta^{\mu}_{\lambda}\delta^{\nu}_{z} - \delta^{\mu}_{z}\delta^{\nu}_{\lambda}\right)$

$\frac{A''}{A}\,\delta^{\mu}_{\ \nu}$

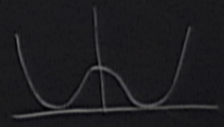
$R^{z}_{\ z} = (D-1)\,A''/A$

$R^{\mu}_{\ \nu} = A^{-2} R^{\mu}_{0\ \nu} + \left[(D-2)\left(\frac{A'}{A}\right)^2 + \frac{A''}{A}\right]\delta^{\mu}_{\nu}$

Look at $D=4$

DOMAIN WALL

Consider $\mathcal{L}_{\phi} = \frac{1}{2}(\partial\phi)^2 - \underbrace{\frac{\lambda}{2}(\phi^2-\eta^2)}_{V(\phi)}$



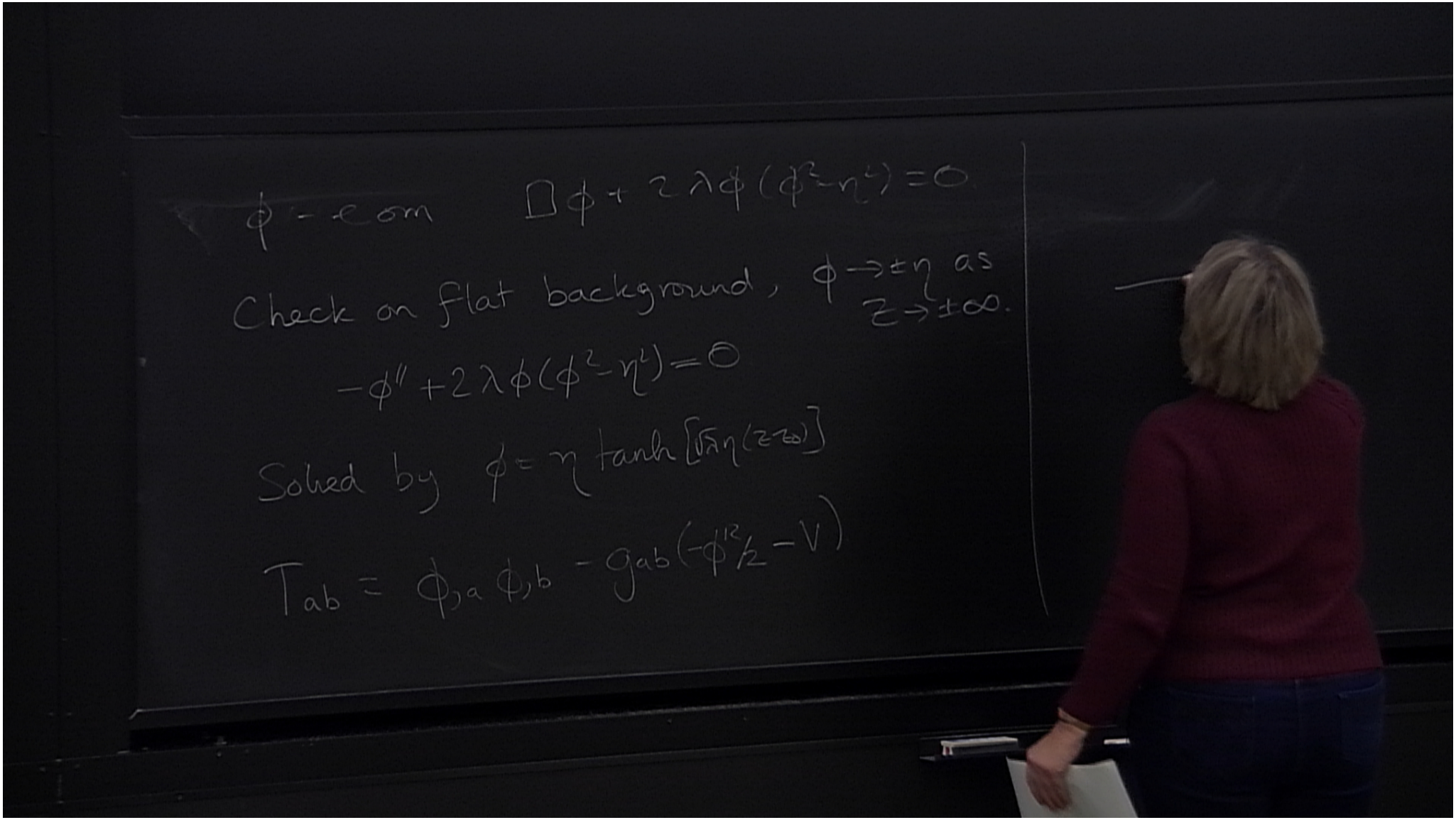
$V$ has 2 vacua, $\phi = \pm\eta$ Vacuum nontrivial

$\phi$ - eom $\quad \Box\phi + 2\lambda\phi(\phi^2 - \eta^2) = 0$

Check on flat background, $\phi \to \pm\eta$ as $z \to \pm\infty$.

$$-\phi'' + 2\lambda\phi(\phi^2 - \eta^2) = 0$$

Solved by $\phi = \eta \tanh[\sqrt{\lambda}\eta(z - z_0)]$


$\phi$ - eom $\Box\phi + 2\lambda\phi(\phi^2-\eta^2)=0$
Check on flat background, $\phi \to \pm\eta$ as $z\to\pm\infty$.
$-\phi'' + 2\lambda\phi(\phi^2-\eta^2)=0$
Solved by $\phi = \eta \tanh[\sqrt{\lambda}\eta(z-z_0)]$
$T_{ab} = \phi_{,a}\phi_{,b} - g_{ab}(-\phi'^2/2 - V)$

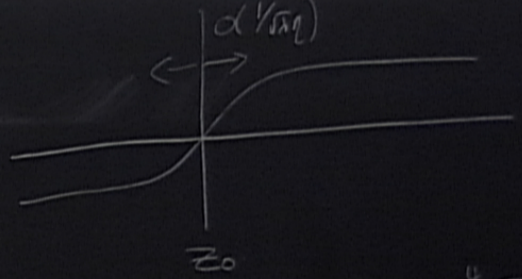
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$$-\phi'' + 2\lambda\phi(\phi^2-\eta^2)=0$$

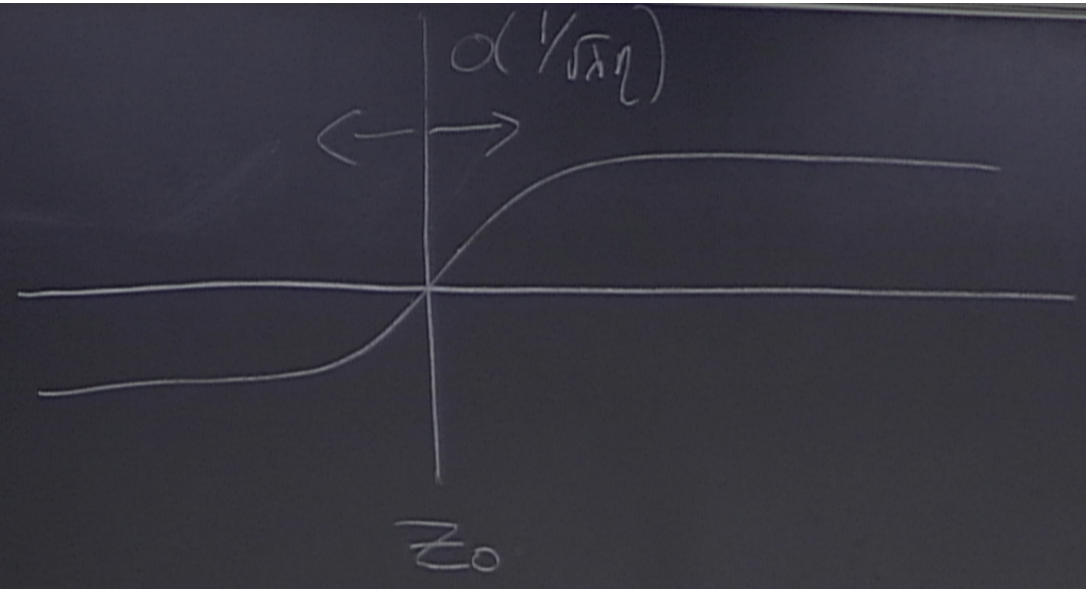
Solved by $\phi = \eta \tanh[\sqrt{\lambda}\eta(z-z_0)]$

$$T_{ab} = \phi_{,a}\phi_{,b} - g_{ab}\left(-\phi'^2/2 - V\right)$$
$$= \phi'^2 \delta^z_a \delta^z_b + \phi'^2/2\, g_{ab} + V g_{ab}$$



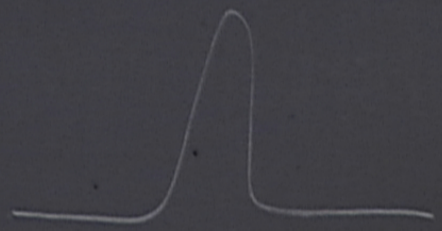

$$T^0_0 = T^x_x = T^y_y = \lambda\eta^4 \operatorname{sech}^4\sqrt{\lambda}\eta(z-z_0) = 2V.$$

$$T^z_z = 0$$




$$T^0_0 = T^x_x = T^y_y = \lambda\eta^4 \operatorname{sech}^4 \sqrt{\lambda}\eta(z - z_0) = 2V.$$

$$T^z_z = 0$$

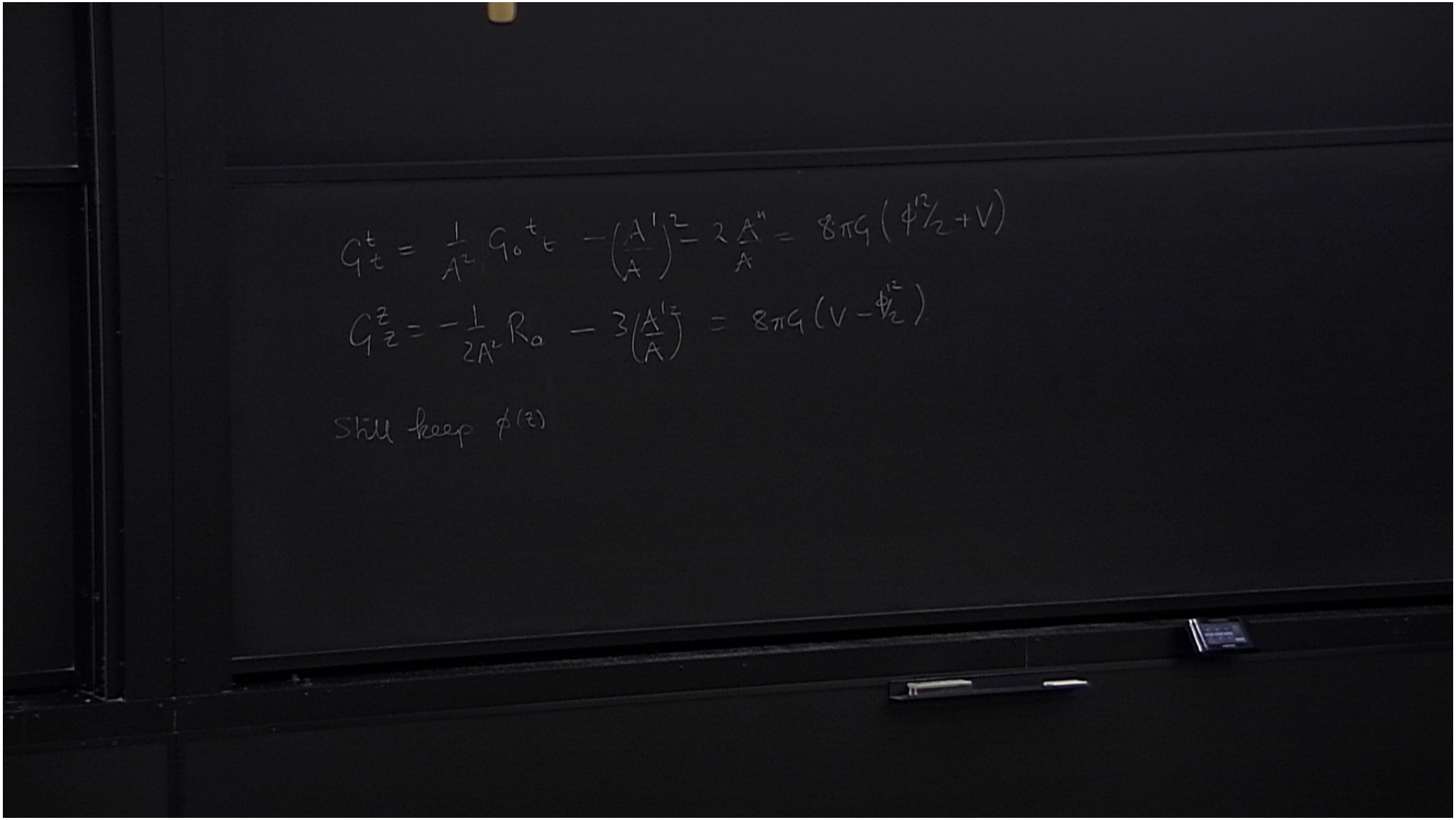


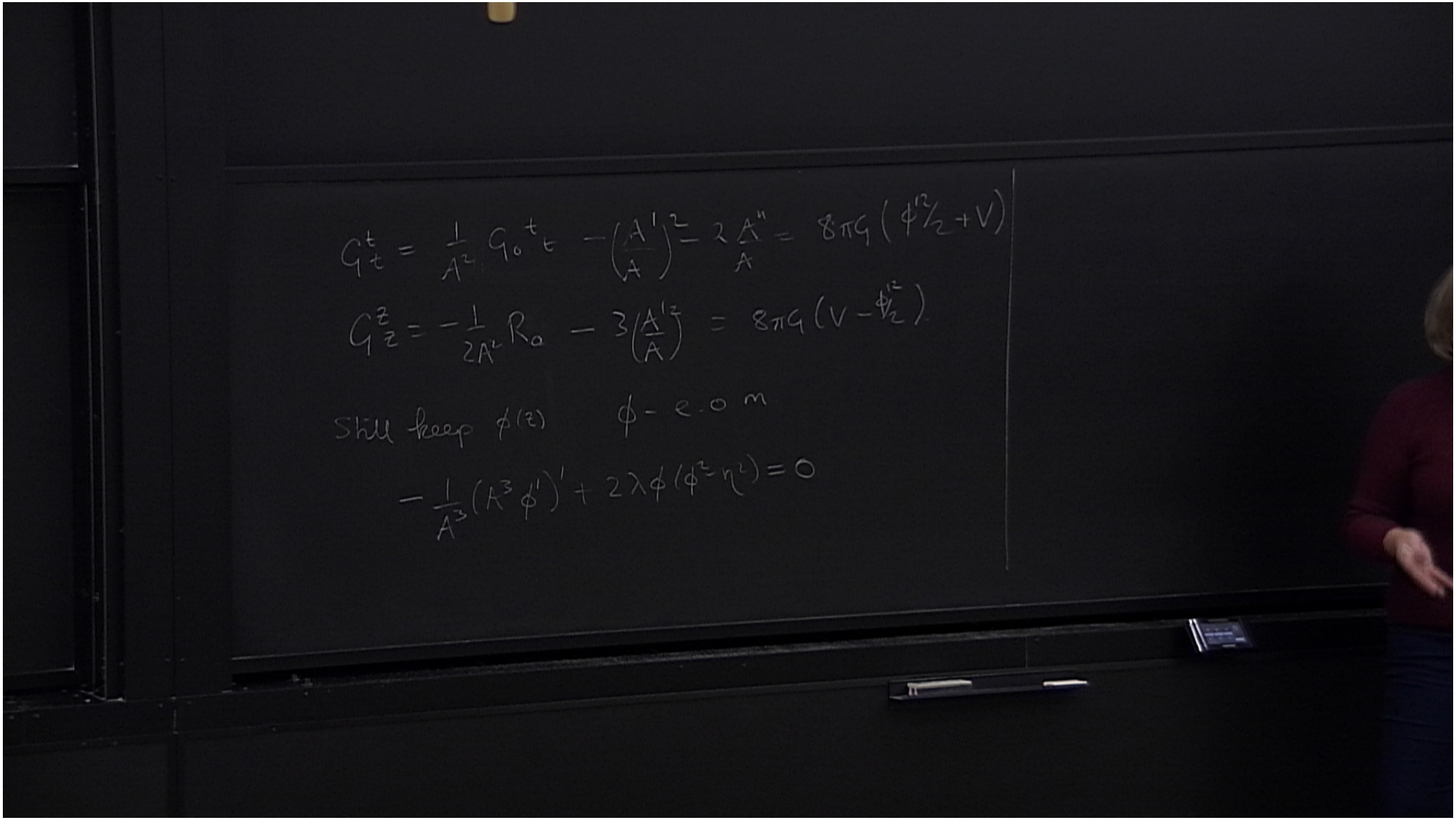
$$\int T^0_0 \, dz = \frac{4}{3} \sqrt{\Lambda} \, \eta^3 \sim \sigma$$

Energy per unit area, finite.

But, area is $\infty$!

$= 2V$.


$G^t_t = \frac{1}{A^2} G_0{}^t{}_t - \left(\frac{A'}{A}\right)^2 - 2\frac{A''}{A} = 8\pi G\left(\phi'^2/2 + V\right)$
$G^z_z = -\frac{1}{2A^2} R_0 - 3\left(\frac{A'}{A}\right)^2 = 8\pi G\left(V - \phi'^2/2\right)$
Still keep $\phi(z)$


$G^t_t = \frac{1}{A^2} G_0{}^t{}_t - \left(\frac{A'}{A}\right)^2 - 2\frac{A''}{A} = 8\pi G\left(\phi'^2/2 + V\right)$
$G^z_z = -\frac{1}{2A^2} R_0 - 3\left(\frac{A'}{A}\right)^2 = 8\pi G\left(V - \phi'^2/2\right)$
Still keep $\phi(z)$ $\phi$-e.o.m
$-\frac{1}{A^3}(A^3\phi')' + 2\lambda\phi(\phi^2 - v^2) = 0$

$$G^t_t = \frac{1}{A^2} G_0{}^t{}_t - \left(\frac{A'}{A}\right)^2 - 2\frac{A''}{A} = 8\pi G\left(\phi'^2/2 + V\right)$$

$$G^z_z = -\frac{1}{2A^2} R_0 - 3\left(\frac{A'}{A}\right)^2 = 8\pi G\left(V - \phi'^2/2\right)$$

Still keep $\phi(z)$ $\phi$-e.o.m

$$-\frac{1}{A^3}\left(A^3\phi'\right)' + 2\lambda\phi\left(\phi^2 - v^2\right) = 0$$

Weak fiel

Characteris

$8\pi G V = \frac{1}{2} \cdot 8$

Weak field limit.

Characteristic scales:

$$8\pi G V = \frac{1}{2} \cdot \underbrace{8\pi G \eta^2}_{\epsilon \ll 1} \cdot \underbrace{\lambda \eta^2}_{1/w^2} \cdot \operatorname{sech}^4(\sqrt{\lambda}\eta z)$$

$\epsilon \ll 1$ grav effect of wall

$1/w^2$, wall width

Choose $\sqrt{\lambda}\eta = 1$ (choice of units)

$\partial_z^2 h_2 + V)$

)

Weak field limit.

Characteristic scales:

$$8\pi G V = \frac{1}{2} \cdot \underbrace{8\pi G \eta^2}_{\epsilon \ll 1} \cdot \underbrace{\lambda \eta^2}_{1/w^2,\ \text{wall width}} \cdot \operatorname{sech}^4(\sqrt{\lambda}\eta z)$$

grav effect of wall

Choose $\sqrt{\lambda}\eta = 1$ (choice of units)

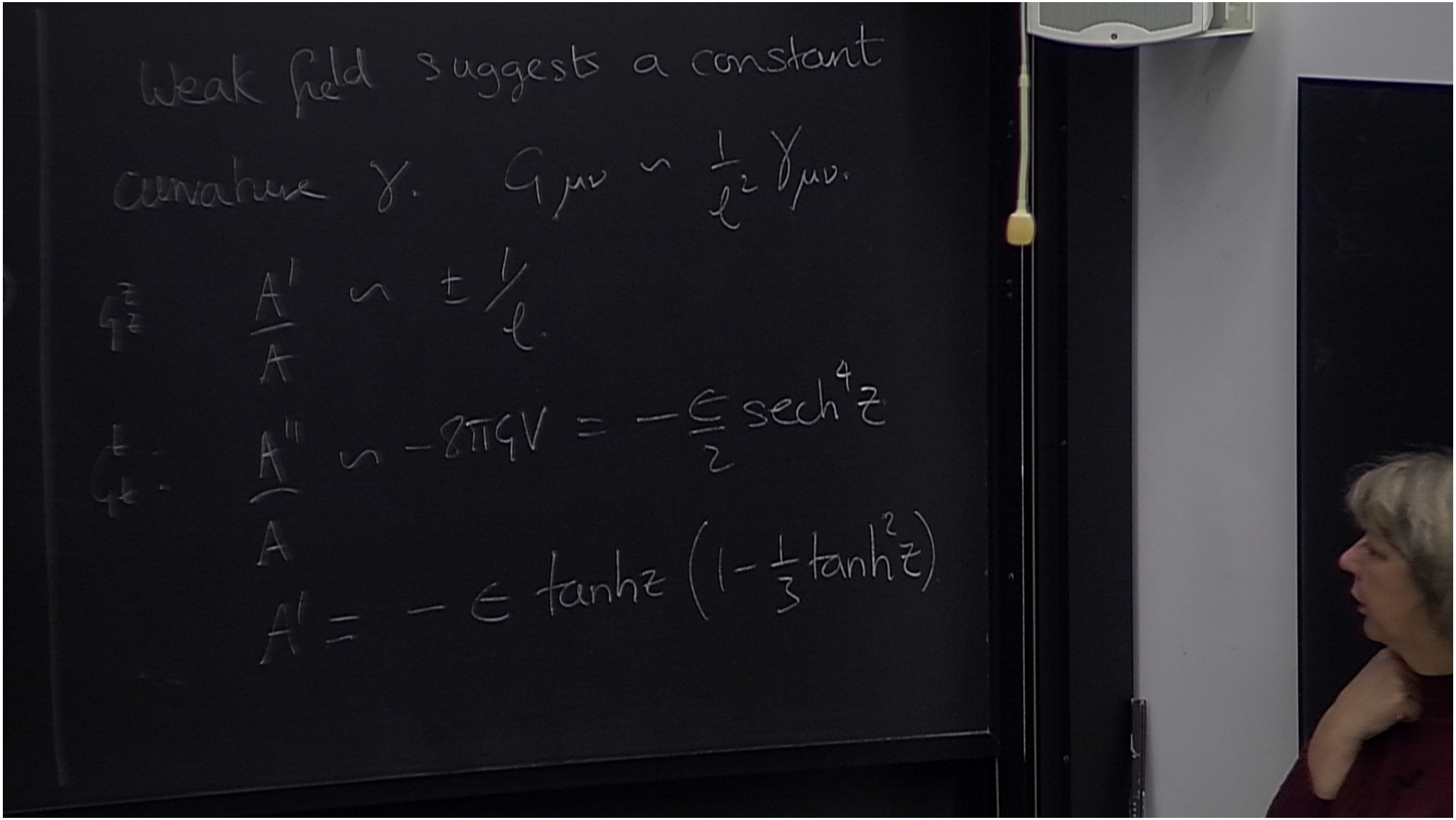
Weak fie

curvature

$G^z_z$ : $\frac{A'}{A}$

$G^t_t$ : $\frac{A''}{A}$

$A' =$


Weak field suggests a constant
curvature $\gamma$. $g_{\mu\nu} \sim \frac{1}{\ell^2}\gamma_{\mu\nu}$.
$\frac{A'}{A} \sim \pm\frac{1}{\ell}$
$\frac{A''}{A} \sim -8\pi G V = -\frac{\epsilon}{2}\operatorname{sech}^4 z$
$A' = -\epsilon \tanh z \left(1 - \frac{1}{3}\tanh^2 z\right)$

$$-\frac{1}{A^3}(A^3\phi')' + 2\lambda\phi(\phi^2-\eta^2) = 0$$

grav effect of wall

wall width

Choose $\sqrt{\lambda}\eta = 1$ (choice of units)

$$ds^2 \simeq \left(1-|z|/\ell\right)^2\left[dt^2 - \ell^2\cosh^2 t/\ell\, d\Omega_{\mathbb{H}}^2\right] - dz^2$$

$A(z)$ — peak of width $\sim O(1/\sqrt{\lambda}\eta)$

Write $\rho = \ell\left(1-\frac{z}{\ell}\right)\cosh t/\ell$

$(z>0)$

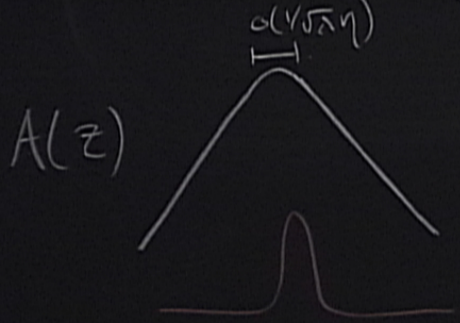
$\tau = \ell\left(1-\frac{z}{\ell}\right)\sinh t/\ell$

$$\rightarrow d\tau^2 - d\rho^2 - \rho^2 d\Omega_{\mathbb{H}}^2$$

$\frac{1}{A^3}(R\phi)$

Choose

$$ds^2 \simeq \left(1 - |z|/\ell\right)^2 \left[dt^2 - \ell^2 \cosh^2 t/\ell \, d\Omega_{\mathbb{H}}^2\right] - dz^2$$

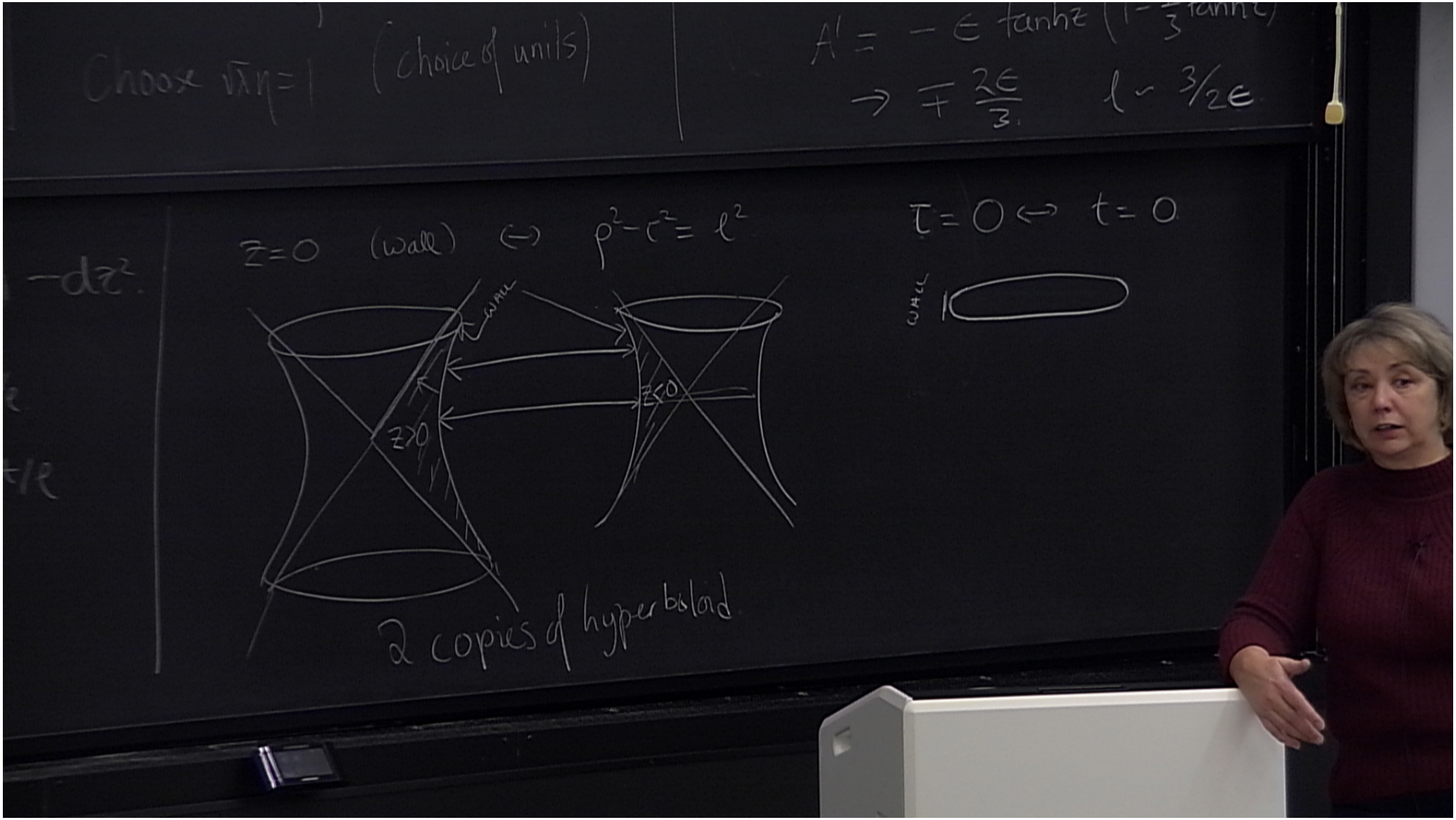


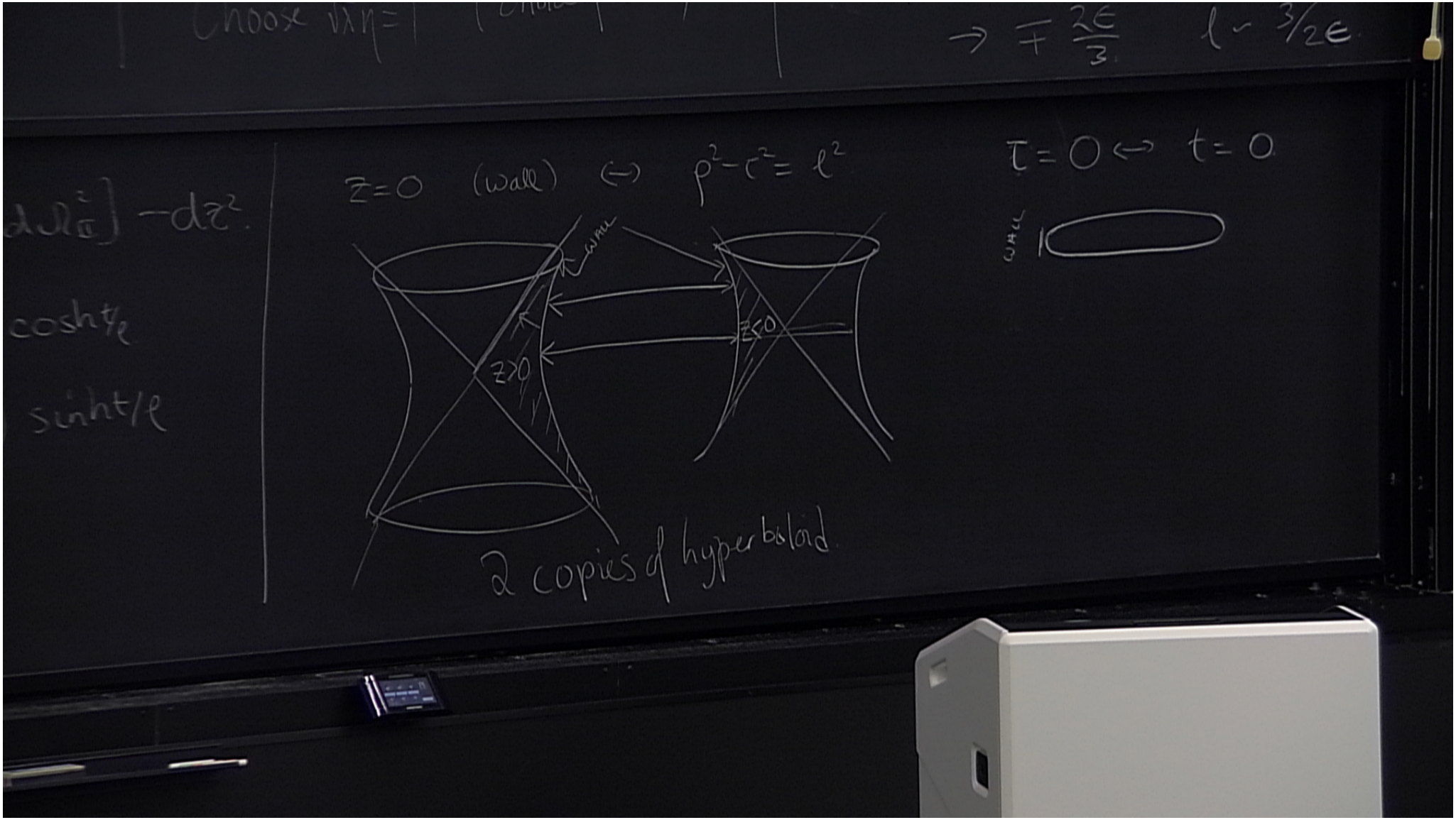

$(z>0)$

Write $\rho = \ell\left(1 - \frac{z}{\ell}\right)\cosh t/\ell$

$\tau = \ell\left(1 - \frac{z}{\ell}\right)\sinh t/\ell$

$$\rightarrow d\tau^2 - d\rho^2 - \rho^2 d\Omega_{\mathbb{H}}^2$$


Choose $\sqrt{\lambda}\eta = 1$ (choice of units)
$A' = -\epsilon \tanh z$
$\rightarrow \mp \frac{2\epsilon}{3}$
$\ell \sim 3/2\epsilon$
$-dz^2$
$z=0$ (wall) $\leftrightarrow$ $\rho^2 - t^2 = \ell^2$
$\tau = 0 \leftrightarrow t = 0$
WALL
WALL
$z>0$
$z<0$
2 copies of hyperboloid


Choose
$\rightarrow \mp \frac{2\epsilon}{3}$
$l \sim 3/2\epsilon$
$d\Omega_{\Pi}^2] - dz^2$
$\cosh t/\ell$
$\sinh t/\ell$
$z=0$ (wall) $\leftrightarrow$ $\rho^2 - r^2 = \ell^2$
wall
$z<0$
$z>0$
2 copies of hyperboloid
$\tau = 0 \leftrightarrow t = 0$
wall

More generally, if $\gamma_{\mu\nu}$ is an Einstein manifold,

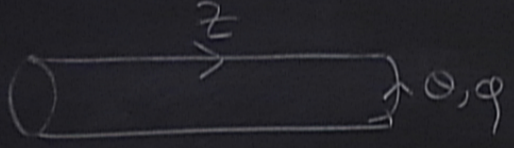
& $A=1$

$$ds^2 = \gamma_{\mu\nu} dx^\mu dx^\nu - dz^2$$

Solves Einstein eqns.

e.g black string:

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2 - dz^2$$

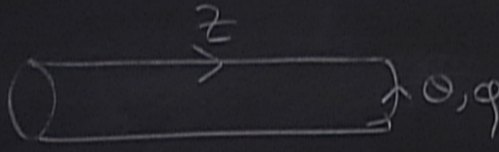



& can add more flat directions to get a general black brane.

More generally

$$ds^2 = A^2(r)\underbrace{g_{\mu\nu}dx^\mu dx^\nu}_{(1+p)\text{-dim}^l.\ g(x)} - dr^2 - C^2(r)\underbrace{\gamma_{\alpha\beta}dy^\alpha dy^\beta}_{n\text{-dim}^l\ \gamma(y)}$$

$$\underline{R}^{\hat{a}}{}_{\hat{b}} = d\underline{\theta}_o{}^{\hat{a}}{}_{\hat{b}} + \underline{\theta}_o{}^{\hat{a}}{}_{\hat{c}} \wedge \underline{\theta}_o{}^{\hat{c}}{}_{\hat{b}}$$




& can add more flat directions to get a general black brane.

More generally

$$ds^2 = A^2(r)\underbrace{g_{\mu\nu}dx^\mu dx^\nu}_{(1+p)\text{-dim}^l.\ g(x)} - dr^2 - C^2(r)\underbrace{\gamma_{\alpha\beta}dy^\alpha dy^\beta}_{n\text{-dim}^l\ \gamma(y)}$$

$$\underline{\omega}^{\hat{\mu}} = A\,\underline{\omega}_0{}^{\hat{\mu}} \qquad \underline{\omega}^{\hat{a}} = C\,\underline{\omega}_0{}^{\hat{a}}$$

Clear that g & γ do not mix at connection level.

$$\underset{\sim}{\Theta}^{\hat{\mu}}{}_{\hat{\nu}} = \underset{\sim}{\Theta}_0{}^{\hat{\mu}}{}_{\hat{\nu}} \qquad \underset{\sim}{\Theta}^{\hat{\alpha}}{}_{\hat{\beta}} = \underset{\sim}{\Theta}_0{}^{\hat{\alpha}}{}_{\hat{\beta}}$$

$$\underset{\sim}{\Theta}^{\hat{\mu}}{}_{\hat{r}} = \frac{A'}{A}\,\underset{\sim}{\omega}^{\hat{\mu}} \qquad \underset{\sim}{\Theta}^{\hat{\alpha}}{}_{\hat{r}} = \frac{C'}{C}\,\underset{\sim}{\omega}^{\hat{\alpha}}$$

$\underline{\omega}^{\hat{A}} = A\,\underline{\omega}_0^{\hat{A}}$ $\qquad$ $\underline{\omega}^{\hat{a}} = C\,\underline{\omega}_0^{\hat{a}}$

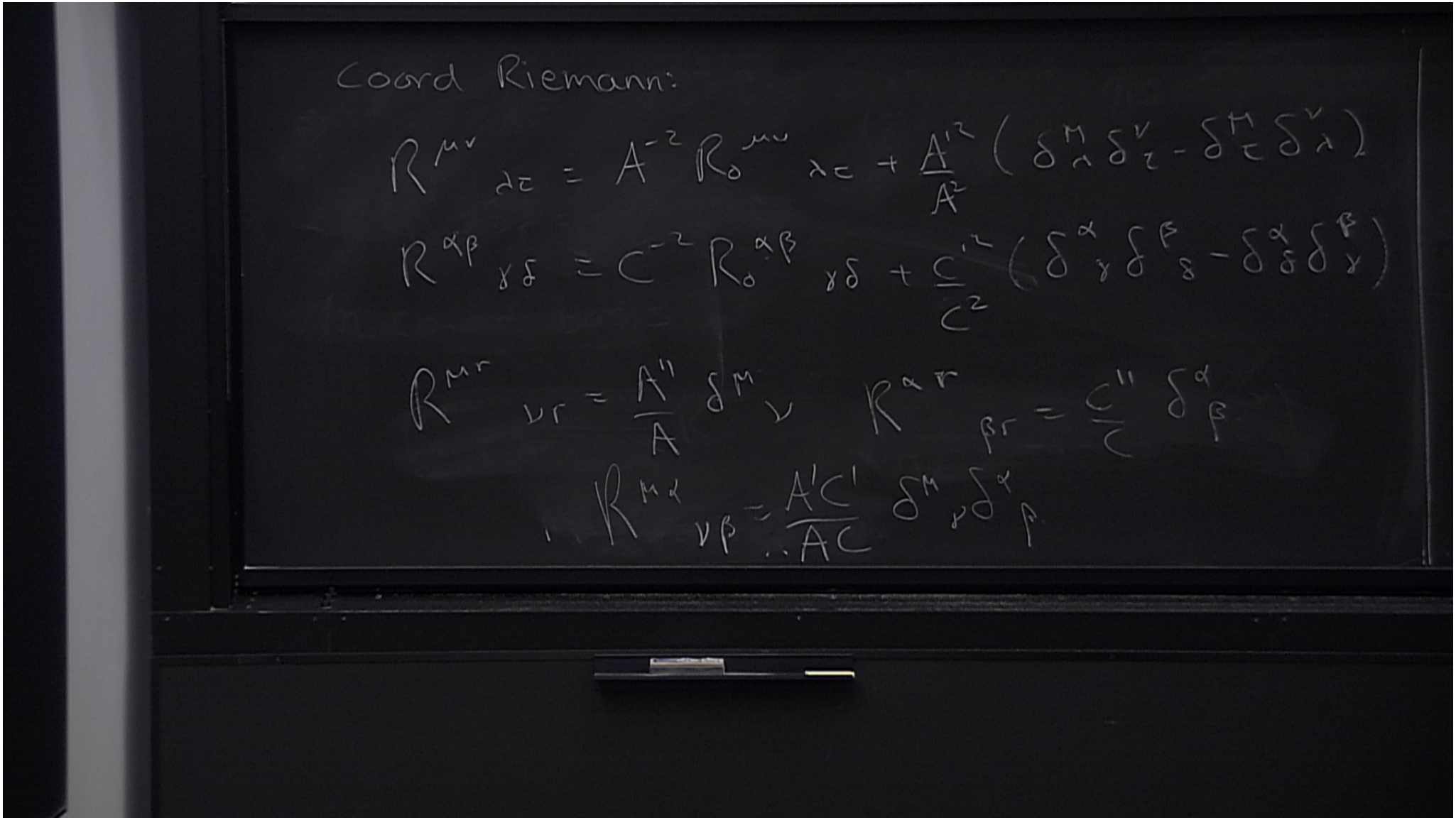
most curvature 2-forms same or similar

$$\underline{R}^{\hat{a}}{}_{\hat{\beta}} = \underline{R}_0{}^{\hat{a}}{}_{\hat{\beta}} + \left(\frac{C'}{C}\right)^2 \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}_{\hat{\beta}}$$

$$\underline{R}^{\hat{a}}{}_{\hat{r}} = \frac{C''}{C}\,\underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{a}}$$

PLUS $\underline{R}^{\hat{M}}{}_{\hat{a}} = \underline{\Theta}^{\hat{M}}{}_{\hat{r}} \wedge \underline{\Theta}^{\hat{r}}{}_{\hat{a}} = \dfrac{A'C'}{AC}\,\underline{\omega}^{\hat{M}} \wedge \underline{\omega}_{\hat{a}}$

$R^2 = (D-1)\,A''/A$ } Look at $D=4$.


Coord Riemann:
$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_0{}^{\mu\nu}{}_{\lambda\tau} + \frac{A'^2}{A^2}\left(\delta^\mu_\lambda \delta^\nu_\tau - \delta^\mu_\tau \delta^\nu_\lambda\right)$
$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_0{}^{\alpha\beta}{}_{\gamma\delta} + \frac{C'^2}{C^2}\left(\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right)$
$R^{\mu r}{}_{\nu r} = \frac{A''}{A}\delta^\mu{}_\nu \qquad R^{\alpha r}{}_{\beta r} = \frac{C''}{C}\delta^\alpha_\beta$
$\therefore R^{\mu\alpha}{}_{\nu\beta} = \frac{A'C'}{AC}\delta^\mu{}_\nu \delta^\alpha{}_\beta$

$$R^{\mu}{}_{\nu} = A^{-2} R_{(3)}{}^{\mu}{}_{\nu} + \left(\frac{A''}{A} + p\frac{A'^2}{A^2}\right)\delta^{\mu}_{\nu} + n\frac{A'C'}{AC}\delta^{\mu}_{\nu}$$

$$R^{\alpha}{}_{\beta} = C^{-2} R_{(\gamma)}{}^{\alpha}{}_{\beta} + \left(\frac{C''}{C} + (n-1)\frac{C'^2}{C^2}\right)\delta^{\alpha}_{\beta} + (p+1)\frac{A'C'}{AC}\delta^{\alpha}_{\beta}$$

$$R^{r}{}_{r} = (p+1)\frac{A''}{A} + n\frac{C''}{C}$$

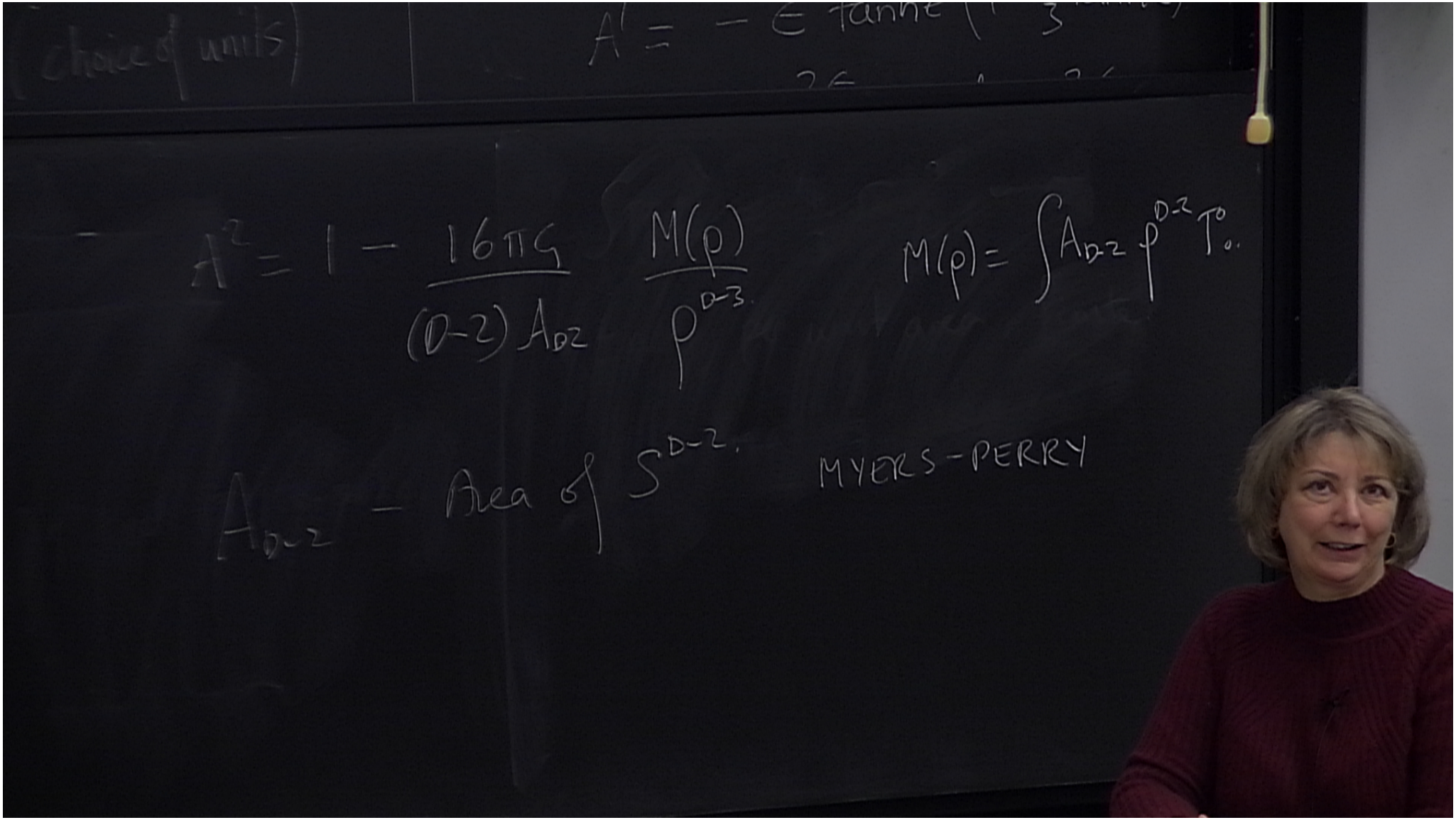
Building block for higher dim solns.

e.g. "SCH" $ds^2 = A^2 dt^2 - dr^2 - C^2 d\Omega_{D-2}^2$

Let $C = \rho$ & $dr = d\rho / A$

$$\frac{dC}{dr} = A \quad , \quad \frac{d^2C}{dr^2} = A A'(\rho)$$

$$G_0^0 = -(D-2)\left[\frac{2A'A}{\rho} + (D-3)\left(\frac{A^2-1}{\rho^2}\right)\right] = \frac{\left[(A^2-1)\rho^{D-3}\right]'}{\rho^{D-2}}(D-2) = 8\pi G P_0$$


choice of units
$A^2 = 1 - \frac{16\pi G}{(D-2)A_{D-2}} \frac{M(\rho)}{\rho^{D-3}}$
$M(\rho) = \int A_{D-2}\, \rho^{D-2}\, T^0{}_0$
$A_{D-2}$ – Area of $S^{D-2}$.
MYERS-PERRY