Title: PSI 2016/2017 Gravitational Physics (Review) - Lecture 8

Date: Jan 12, 2017 10:15 AM

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Abstract:

LECTURE 8: Many Black Holes?

Start with an odd metric.

$$ds^2 = \frac{1}{A^2(x+y)^2}\left[F(y)\,dt^2 - \frac{dy^2}{F(y)} - \frac{dx^2}{G(x)} - G(x)\,d\phi^2\right]$$

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periodic.

$$F(y) = -1 + y^2 - 2GMAy^3$$

$$G(x) = 1 - x^2 - 2GMAx^3 \qquad (= -F(-x))$$

Solves vacuum Einstein eqns.

$x + y = 0$ singular (?)

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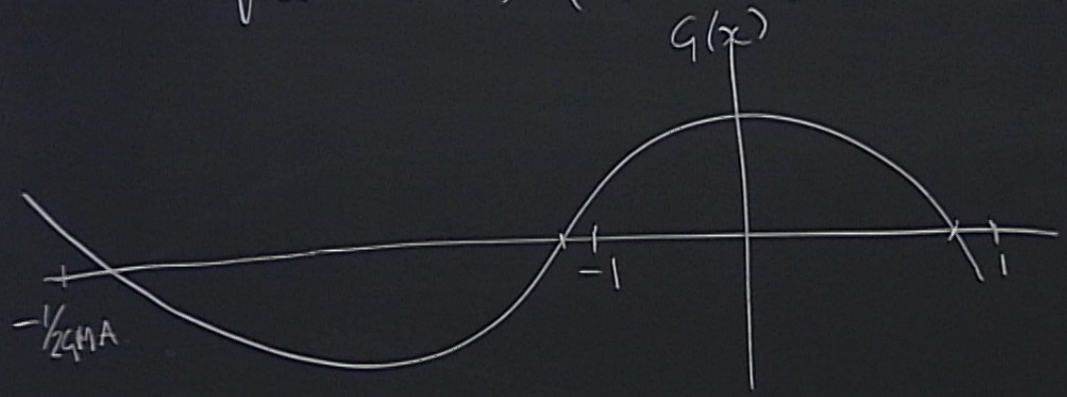
$x/y \approx \pm 1/2GMA$.

Require $F, G > 0$ for our patch of spacetime.



x looks like $\cos\theta$, & $\{x, \phi\}$ part of metric topologically S^2.

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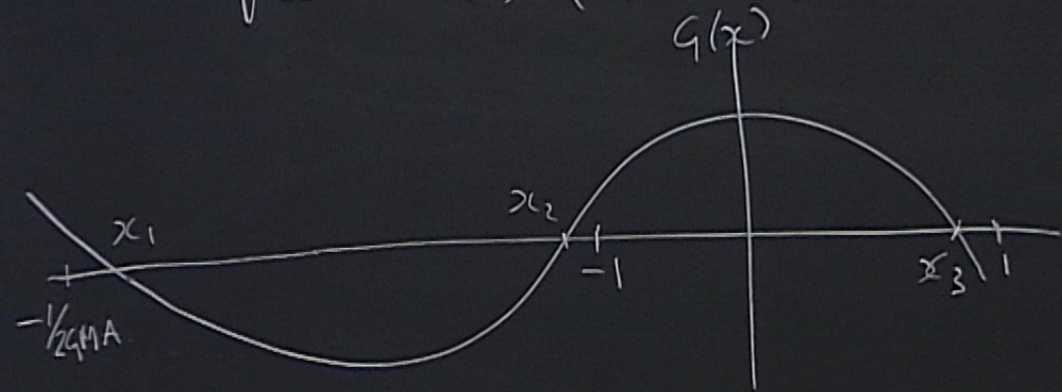

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Want 2 roots for G ie $GMA < 1/3\sqrt{3}$

& $x \in [x_2, x_3]$

- $F(y) > 0 \Rightarrow y \in [-x_2, -x_1]$ (or $< -x_3$)

$F \to 0$ suggest a horizon, $y \approx 1/2 GMA$ & ≈ 1

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Black hole? Spherical topology.

topologically S^2.

Change coords $r = 1/Ay$ $\hat{t} = t/A$.

$$F(y) = \frac{1}{A^2 r^2}\left[1 - A^2 r^2 - 2\frac{GM}{r}\right] = \frac{f_{(SDS)}(r)}{A^2 r^2}$$

$$dy^2 = \frac{dr^2}{A^2 r^4} \quad \& \quad ds^2 = \frac{1}{(1+Arx)^2}\left[f(r)d\hat{t}^2 - \frac{dr^2}{f(r)} - r^2 d\hat{\Omega}^2\right]$$

Looks like distorted SCH-DS

2 horizons - black hole & cosmological?

But we do not have a Λ, so explore for $M=0$.

$$ds^2 = \frac{1}{(1+Ar\cos\theta)^2}\left[(1-A^2r^2)dt^2 - \frac{dr^2}{1-A^2r^2} - r^2 d\Omega_{II}^2\right]$$

$(x=\cos\theta)$

Define new coords

$$\rho = \frac{r\sin\theta}{1 + Ar\cos\theta}$$

$$\lambda = \frac{\sqrt{1 - A^2 r^2}}{1 + Ar\cos\theta}.$$

Then (EX).

$$ds^2 = \lambda^2 dt^2 - \frac{d\lambda^2}{A^2} - d\rho^2 - \rho^2 d\varphi^2$$

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$$ds^2 = \lambda^2 dt^2 - \frac{d\lambda^2}{A^2} - d\rho^2 - \rho^2 d\varphi^2$$

Now define $\tau = \frac{\lambda}{A} \sinh At$

$$b = \frac{\lambda}{A} \cosh At$$

$x = \rho \cos\theta$

$y = \rho \sin\theta$

& get $ds^2 = d\tau^2 - db^2 - dx^2 - dy^2$

$$x = \rho\cos\theta$$
$$y = \rho\sin\theta$$

& get $ds^2 = d\tau^2 - d\zeta^2 - dx^2 - dy^2$

Origin of original spacetime, $r=0$, $\rho=0$ & $\lambda=1$

or $\tau = \frac{1}{A}\sinh At$, $\zeta = \frac{1}{A}\cosh At$

$$X^\mu = \frac{1}{A}(\sinh At, \cosh At, 0, 0).$$

$$\dot{X}^\mu = (\cosh At, \sinh At, 0, 0) \quad (t \text{ affine})$$

$$\ddot{X}^\mu = A^2 X^\mu \qquad |\ddot{X}^\mu|^2 = -A^2$$

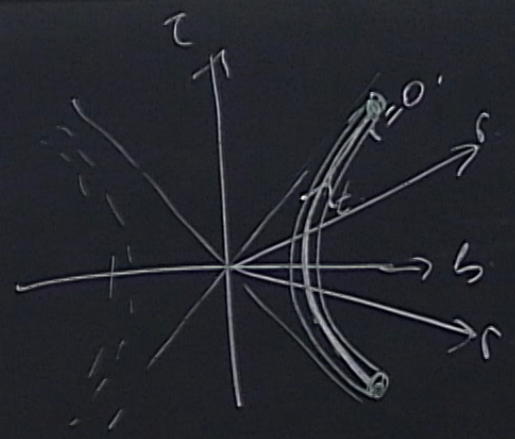
- uniformly accelerating observer

A represents acceleration.

(t affine)

ener

RINDLER



Horizon is limit of what observer can see

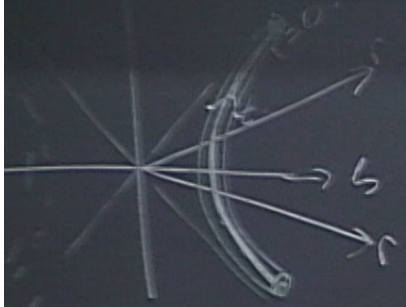
Shows how to extend horizons.

Shows how to extend across acceleration horizons. Check $\{x, \varphi\}$ sections

$$d\hat{\Omega}^2 = \frac{dx^2}{G(x)} + G(x)\, d\varphi^2$$

For spherical topology G has two zeros.

Consider $x \to x_2, x_3$



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$$d\hat{\Omega}^2 \propto \frac{dx^2}{G(x)} + G(x)\, d\varphi^2$$

For spherical topology G has two zeros.

Consider $x \to x_2, x_3$

$$G(x_i) \simeq G'(x_i)(x - x_i)$$

Define $\theta_i = 2\sqrt{\frac{(x-x_i)}{G'(x_i)}}$

$$\Rightarrow \quad d\theta_i = \frac{dx}{\sqrt{G_i'(x-x_i)}} \simeq \frac{dx}{\sqrt{G(x)}}$$

θ_i is a proper angular coord near poles.

But $d\tilde{\Omega} \propto d\theta_i^2 + G_i'(x-x_i)\,d\varphi^2$

$$= d\theta_i^2 + \theta_i^2 \frac{G_i'^2}{4} d\varphi^2$$

For nonsingularity, $\frac{|G_i'|}{2}\varphi$ has periodicity 2π.

But $d\hat{s}^2 \propto d\theta_i^2 + G_i'(x-x_i)\,d\varphi^2$

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For nonsingularity, $\frac{|G_i'|}{2}\varphi$ has periodicity 2π.

topologically S^2.

Take $GMA \ll 1$

$$G_i' = -2x_i - 6GMA\, x_i^2$$

where $1 - x_i^2 = 2GMA\, x_i^3$

$$x_i = \pm 1 - GMA$$

$$G_i' = \pm 2 - 4GMA$$

ogically S^2.

$MA \ll 1$

$-2x_i - 6GMA\, x_i^2$

$1 - x_i^2 = 2GMA\, x_i^3$

$\pm 1 - GMA$

$= \pm 2 - 4GMA$

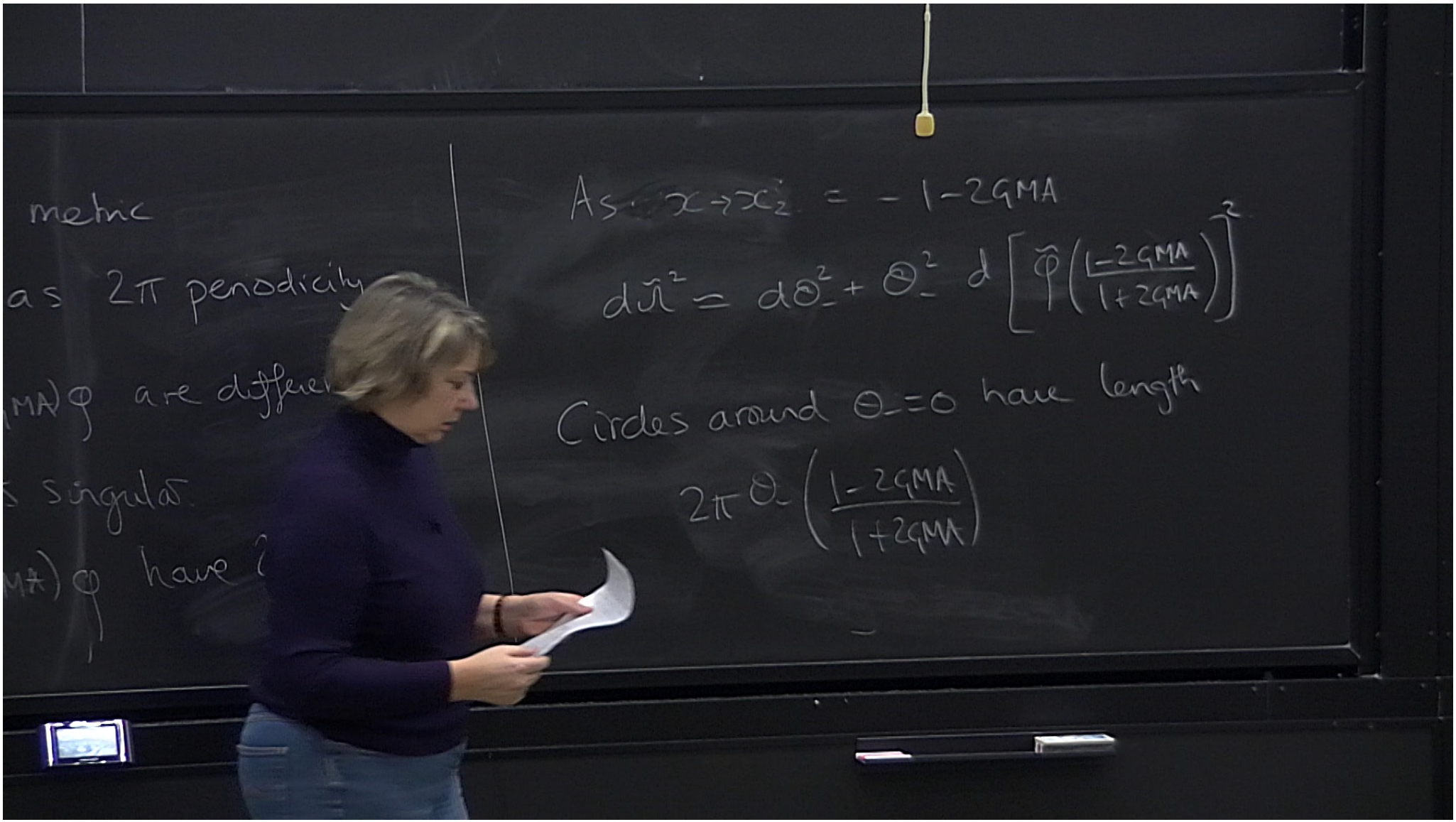
To have regular metric

$\tilde{\varphi}_i = \frac{|G_i|}{2}\varphi$ has 2π periodicity

But $\tilde{\varphi}_\pm = (1 \pm 2GMA)\varphi$ are different

i.e. At least 1 pole is singular

Let $\tilde{\varphi} = (1 + 2GMA)\varphi$ have 2π periodicity


metric
as 2π periodicity
(MA) φ are differen
s singular
(MA) φ have
As x → x_2 = -1-2GMA
dΩ̃² = dθ_-² + θ_-² d[φ̃((1-2GMA)/(1+2GMA))]²
Circles around θ_- = 0 have length
2π θ_-((1-2GMA)/(1+2GMA))

metric

as 2π periodicity

$MA)\varphi$ are different

singular

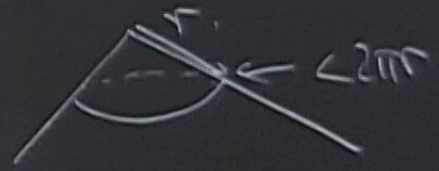
$MA)\varphi$ have 2π periodicity

As $x \to x_2$ $= -(1-2GMA$

$$d\tilde{\Omega}^2 = d\theta_-^2 + \theta_-^2\, d\left[\tilde{\varphi}\left(\frac{1-2GMA}{1+2GMA}\right)\right]^2$$

Circles around $\theta_- = 0$ have length

$$2\pi\,\theta_-\left(\frac{1-2GMA}{1+2GMA}\right)$$



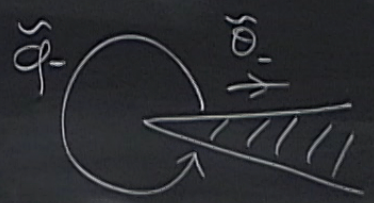
cf a cone

Black hole? Spherical topology

or for $\tilde{\varphi}_- = \dfrac{(1-2GMA)}{(1+2GMA)}\,\tilde{\varphi}_+$

has $\Delta\hat{\varphi} = 2\pi\left(1 - \dfrac{4GMA}{1+2GMA}\right)$

$\simeq 2\pi(1-4GMA)$



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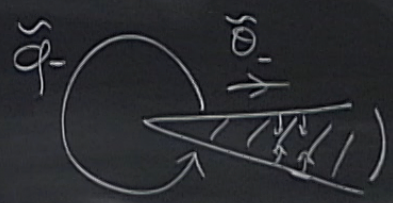

$\delta = 8\pi GMA$

DEFICIT ANGLE.

Deficit angles

Deficit angles are sourced by <u>cosmic strings</u> - linear, string sources, codimension 2. Have sharply localized energy-momentum.

$$T^{\mu}{}_{\nu} \sim \mu \, (1, 1, 0, 0) \, \delta^2(\underline{x})$$

μ: mass per unit length

$t \quad z \quad \rho \quad \varphi$

Deficit angle $\delta = 8\pi G \mu$.

For our metric, string has tension $\mu = MA$.

$\mu \sim$ Force applied $\rightarrow$ $F = MA$

$\frac{f_{(SCH)}(r)}{A^2 r^2}$

$d\hat{t}^2 - dr^2 - r^2 d\hat{\Omega}^2]$

Looks like distorted SCH-DS

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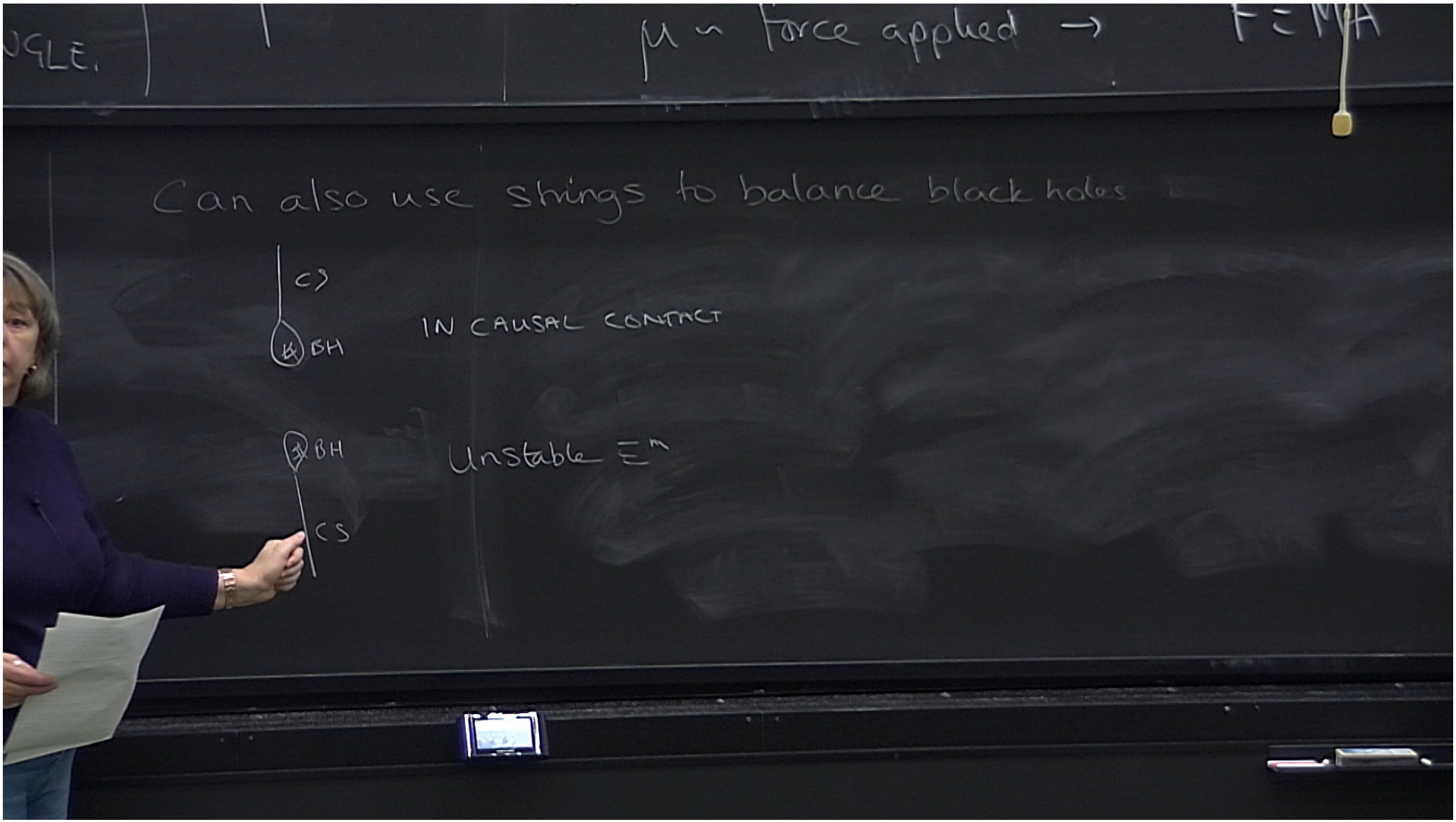
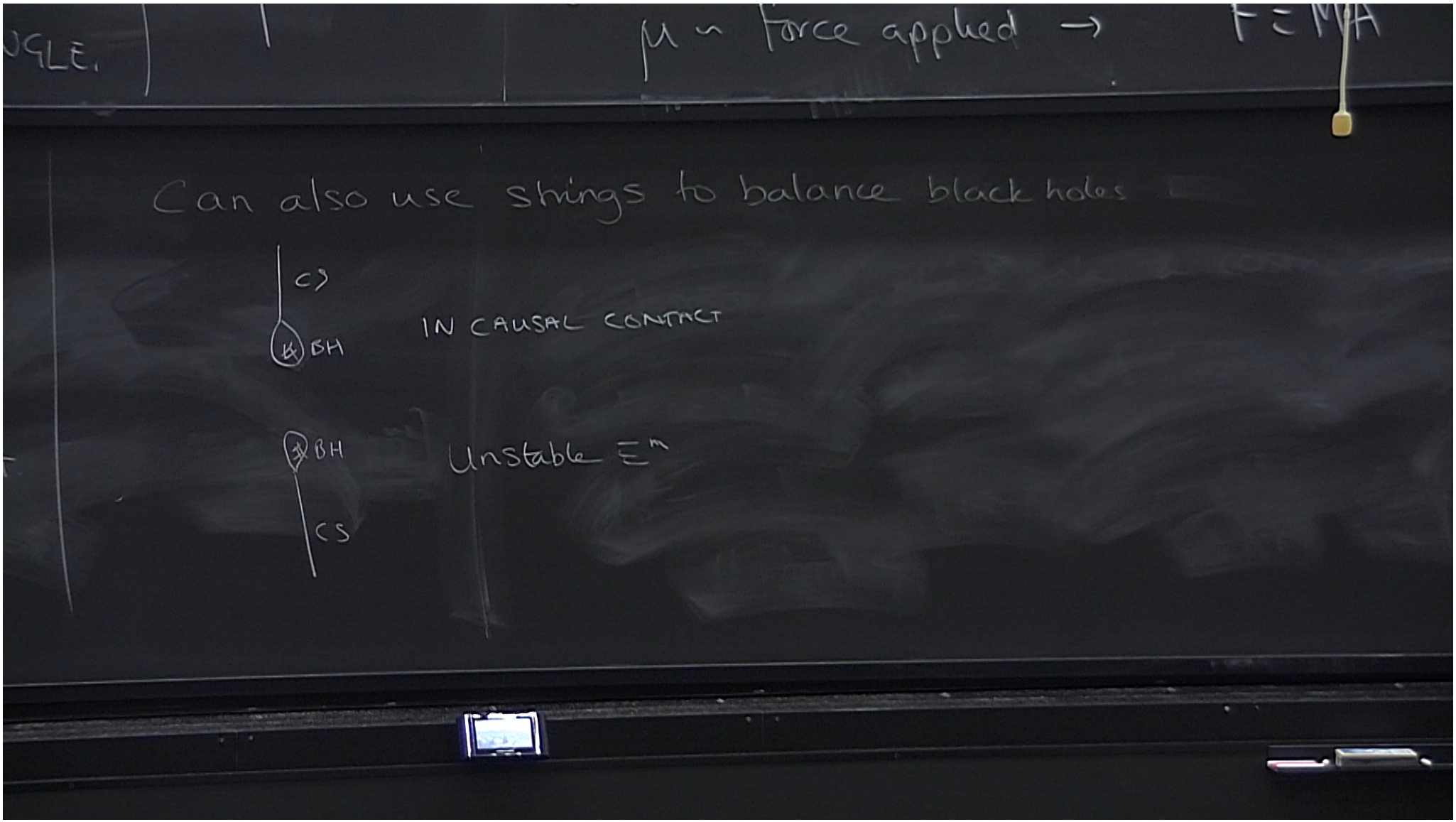

μ ~ Force applied →
F = MA
Can also use strings to balance black holes
CS
BH
IN CAUSAL CONTACT
BH
Unstable E^m
CS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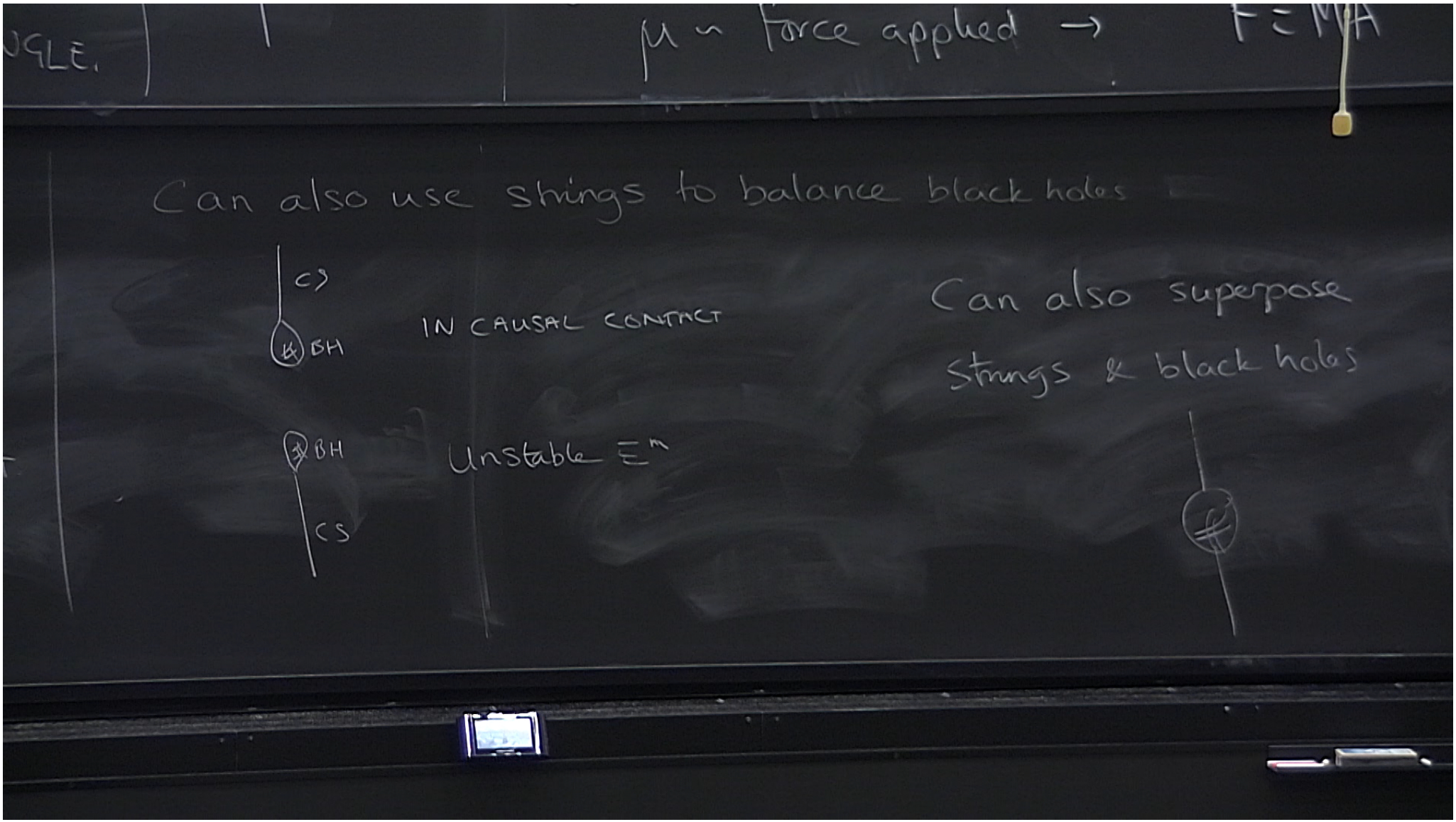
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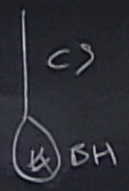

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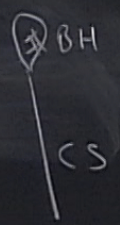

Unstable E^m

Can also superpose strings & black holes

Same δ.