Title: PSI 2016/2017 Standard Model (Review) - Lecture 13

Date: Jan 19, 2017 09:00 AM

URL: http://pirsa.org/17010017

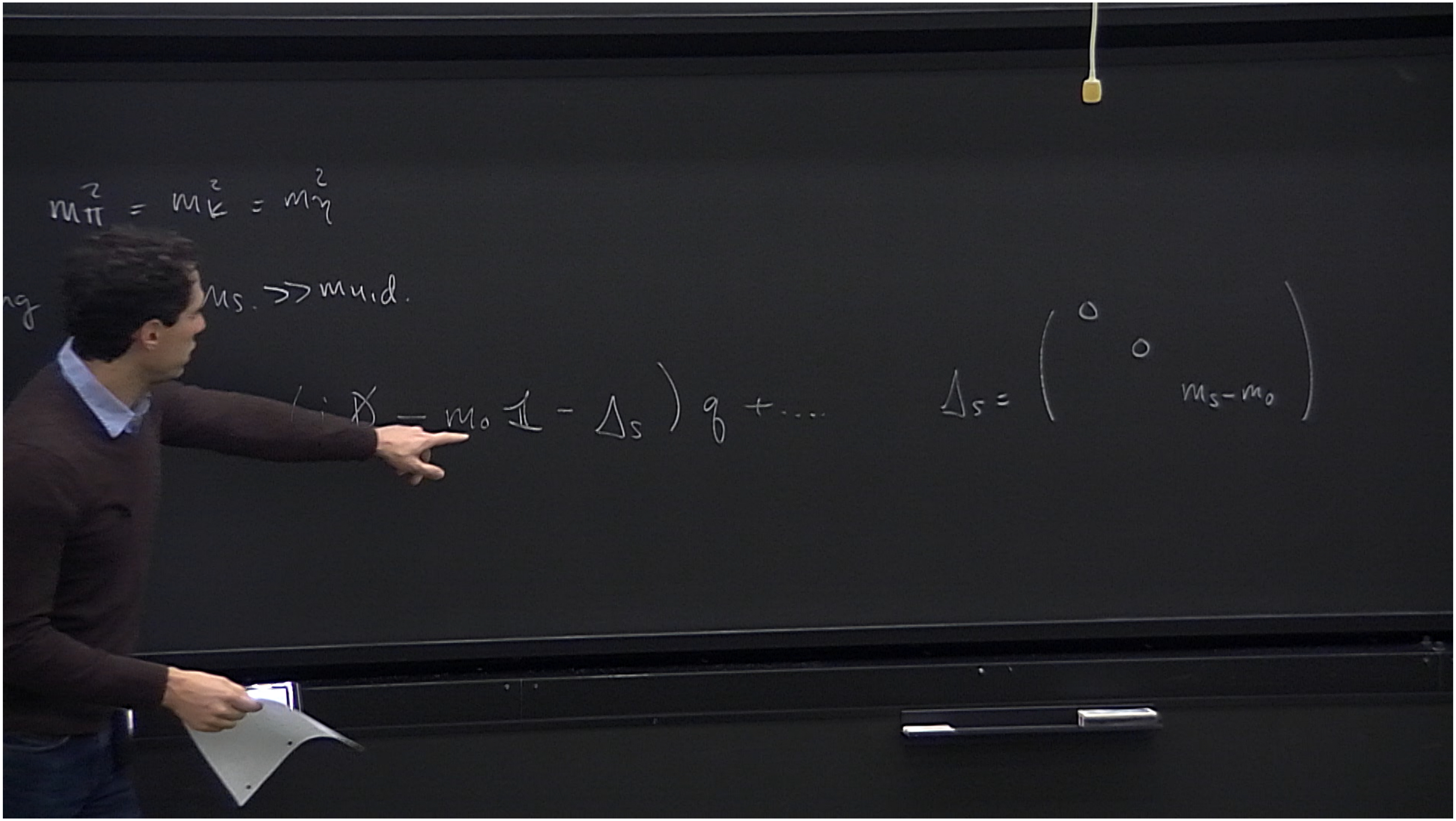
Abstract:

SU(3) flavor - breaking

Mass relation for $J^P = 0^-$ octet: $m_\pi^2 = m_K^2 = m_\eta^2$

Largest source of SU(3) flavor - breaking is $m_s \gg m_{u,d}$.

$$\mathcal{L} = \bar{q}\left(i\not{D} - \begin{pmatrix} m_0 & & \\ & m_0 & \\ & & m_s \end{pmatrix}\right) q + \dots \quad = \quad \bar{q}\left(i\not{D} - m_0 \mathbb{1}\right.$$


$m_\pi^2 = m_K^2 = m_\eta^2$
ng
$m_s \gg m_{u,d}$.
$(i\not{D} - m_0 \mathbb{1} - \Delta_s)\, q + \dots$
$\Delta_s = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_s - m_0 \end{pmatrix}$

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$$= \bar{q}\left(i\not{D} - m_0 \mathbb{1} - \Delta_s\right) q + \dots \qquad \Delta_s = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_s - m_0 \end{pmatrix}$$

$SU(3)_{flavor}$ - breaking

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$m_0 = m_{u,d}$ universal mass for u,d

...n.d.

$$\not{\partial} - m_0 \mathbb{1} - \Delta_s) q + \dots$$

$$\Delta_s = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_s - m_0 \end{pmatrix} \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_s \end{pmatrix}$$

Trick: treat Δ_s as spurion.

If we assume $\Delta_s \longrightarrow U \Delta_s U^\dagger$, then $\mathcal{L}$ is again

$$\bar{q} \Delta_s q \rightarrow \bar{q} U^\dagger U \Delta_s U^\dagger U q = \bar{q} \Delta_s q$$

s spurion

$\longrightarrow U \Delta_s U^\dagger$, then $\mathcal{L}$ is again $SU(3)$ flavor-preserving.

$\Delta_s q \rightarrow \bar{q} U^\dagger U \Delta_s U^\dagger U q = \bar{q} \Delta_s q$

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$$\bar{q}\Delta_s q \rightarrow \bar{q} U^\dagger U \Delta_s U^\dagger U q = \bar{q}\Delta_s q$$

Include Δ_s in effective Lagrangian for meson fields:

$$\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\left[\partial_\mu M\,\partial^\mu M\right] - \frac{1}{2}\mu_8^2\,\mathrm{Tr}\left[M^2\right] - \frac{1}{2}\alpha\,\mathrm{Tr}\left[\Delta_s M^2\right]$$

$$+\frac{1}{2}\partial_\mu\eta'\partial^\mu\eta' - \frac{1}{2}\mu_1^2\eta'^2 - \frac{1}{2}\beta\,\eta'\,\mathrm{Tr}\left[\Delta_s M^2\right] - \frac{1}{2}\gamma$$

$$\mathcal{L} = \text{... kinetic terms ...} - \frac{1}{2}\mu_8^2\left(\pi^{0\,2} + 2\pi^+\pi^-\right) - \left(\mu_8^2 \right.$$

$$-\frac{1}{2}\left(\eta, \eta'\right)\begin{pmatrix} \mu_8^2 + \frac{2}{3}\alpha m_s & -\frac{\beta m_s}{\sqrt{6}} \\ -\frac{\beta m_s}{\sqrt{6}} & \mu_1^2 + \gamma m_s \end{pmatrix}\Bigg($$

$$M^2] - \frac{1}{2}\alpha\, \mathrm{Tr}[\Delta_s M^2]$$

$$\frac{1}{2}\beta\, \eta'\, \mathrm{Tr}[\Delta_s M^2] - \frac{1}{2}\gamma\, \mathrm{Tr}[\Delta_s]\, \eta'^2 + \mathcal{L}_{int}$$

$$(\pi^{0\,2} + 2\pi^+\pi^-) - \left(\mu_8^2 + \frac{\alpha m_s}{2}\right)\left(K^0\bar{K}^0 + K^+K^-\right)$$

$$) \begin{pmatrix} \mu_8^2 + \frac{2}{3}\alpha m_s & -\frac{\beta m_s}{\sqrt{6}} \\ -\frac{\beta m_s}{\sqrt{6}} & \mu_1^2 + \gamma m_s \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

Strange mass has two effects:

(1) Splits degeneracy between octet states

(2) Mixing between "octet η" & "singlet η'".

Mixing angle is small ($\sim 20°$)

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$$\boxed{4 m_K^2 = m_\pi^2 + 3 m_\eta^2}$$

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satisfied to $\sim 5\%$

Repeat w/ Baryon octet

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$

Flavor physics

Hadrons can also include c, b quarks.

Lightest pseudoscalar ($J^P = 0^-$) states:

Charmed meson:

$D^0, \bar{D}^0$ — $c\bar{u}, \bar{c}u$

D^+, D^- — $c\bar{d}, \bar{c}d$

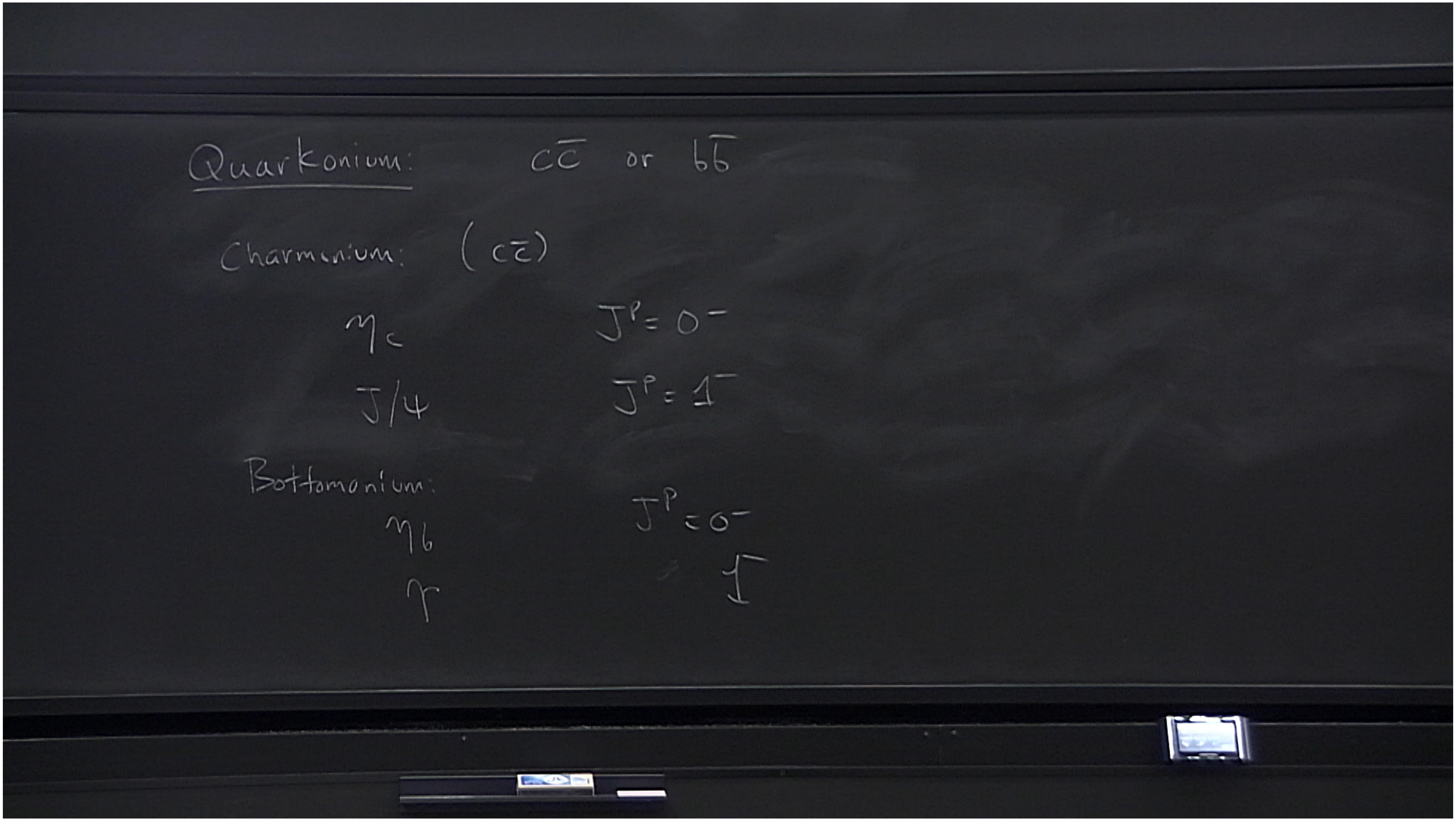
D_s^+, D_s^- — $c\bar{s}, s\bar{c}$

} 1800 – 2000 MeV

Bottom (beauty) mesons:

B^+, B^-	$u\bar{b}, \bar{u}b$	5-6 GeV.
$B^0, \bar{B}^0$	$d\bar{b}, \bar{d}b$	
$B_s^0, \bar{B}_s^0$	$s\bar{b}, \bar{s}b$	
B_c^+, B_c^-	$c\bar{b}, \bar{c}b$	

Also vector ($J^P = 1^-$) states: just add "*"


Quarkonium: $c\bar{c}$ or $b\bar{b}$
Charmonium: $(c\bar{c})$
η_c $J^P = 0^-$
J/ψ $J^P = 1^-$
Bottomonium:
η_b $J^P = 0^-$
Υ 1^-

Other approx. conserved quantum numbers:

charm C: c has $C=+1$, $\bar{c}$ has $C=-1$

bottomness B: b has $B=-1$, $\bar{b}$ has $B=+1$

Processes that violate S, C, B must occur via weak interaction.

Test consistency of CKM matrix as sole source of flavor violation.

Important class of observables: Flavor-changing neutral currents (FCNCs)

$s \leftrightarrow d, \quad b \leftrightarrow d, \quad b \leftrightarrow s, \quad c \leftrightarrow u$

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Important class of observables: Flavor-changing neutral currents (FCNCs)

$s \leftrightarrow d$, $b \leftrightarrow d$, $b \leftrightarrow s$, $c \leftrightarrow u$

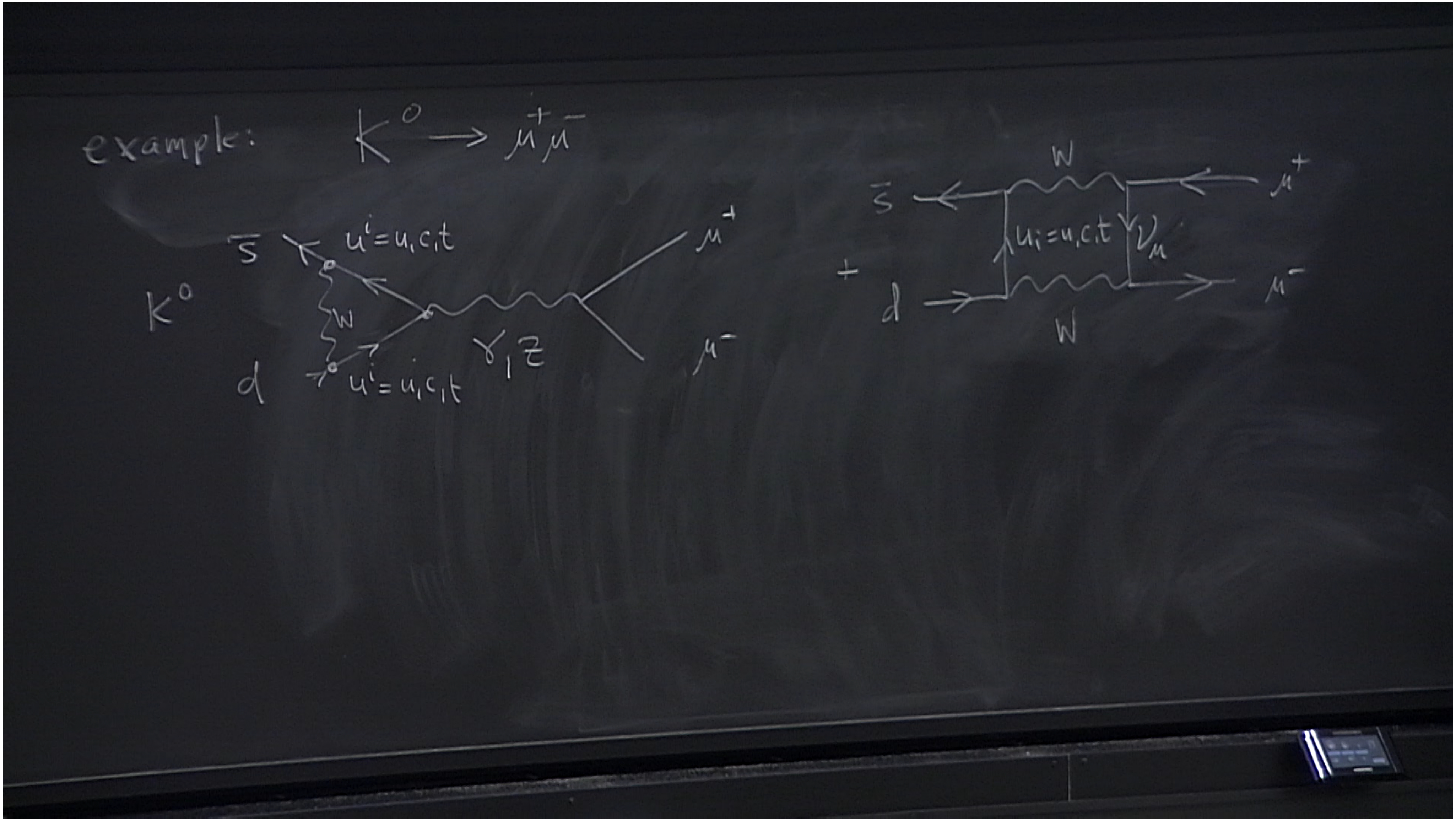
Forbidden at tree-level since NC is flavor-conserving. Must occur at one-lo ust occur via weak interaction.

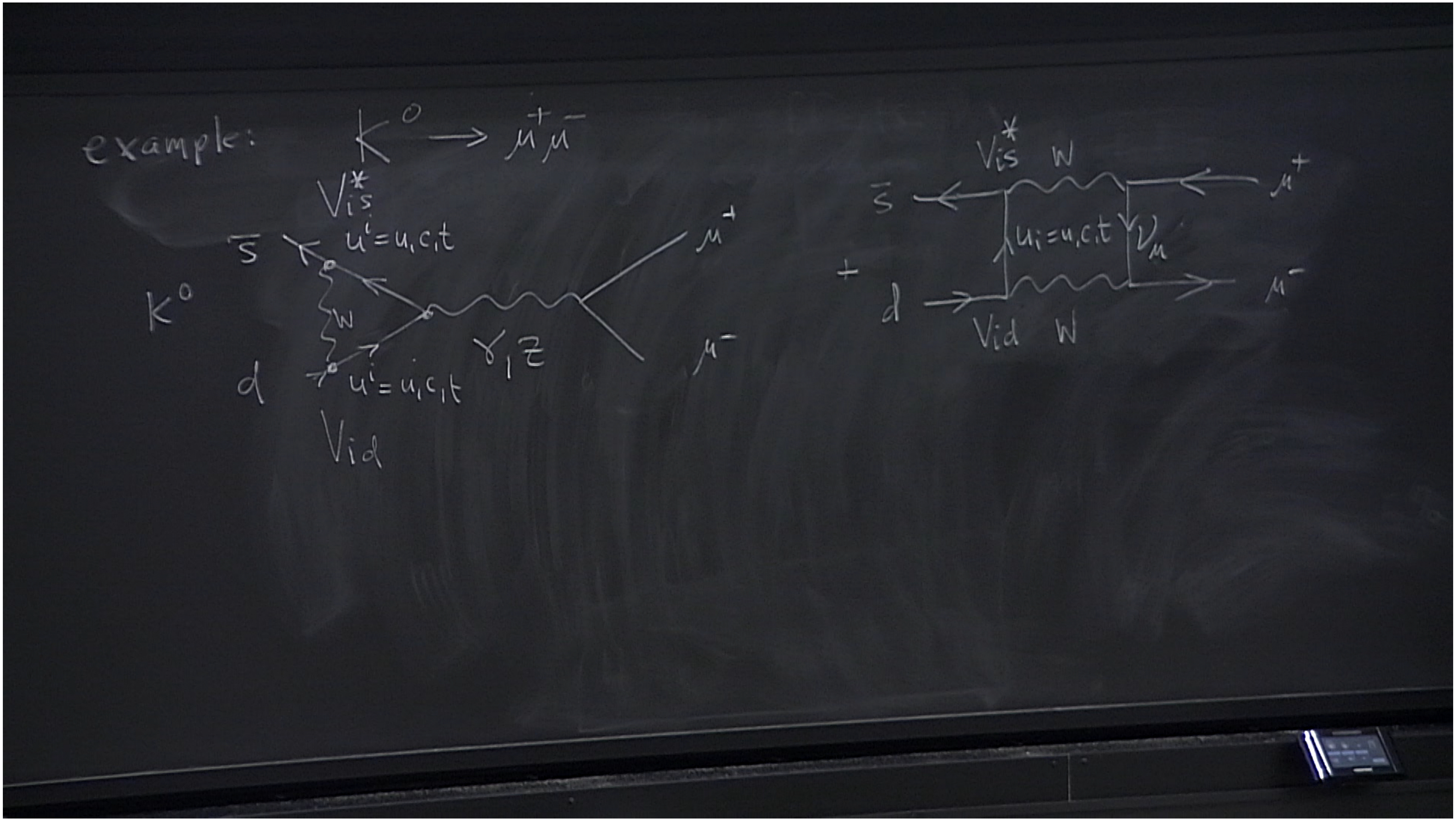
sole source of flavor violation.

or - changing neutral currents (FCNCs)

$c \longleftrightarrow u$

is flavor-conserving Must occur at one-loop order in SM.


example: $K^0 \longrightarrow \mu^+\mu^-$
K^0
$\bar{s}$
d
$u^i = u, c, t$
W
γ, Z
$u^i = u, c, t$
μ^+
μ^-
+
$\bar{s}$
d
W
$u_i = u, c, t$
ν_μ
W
μ^+
μ^-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V_{id}
+
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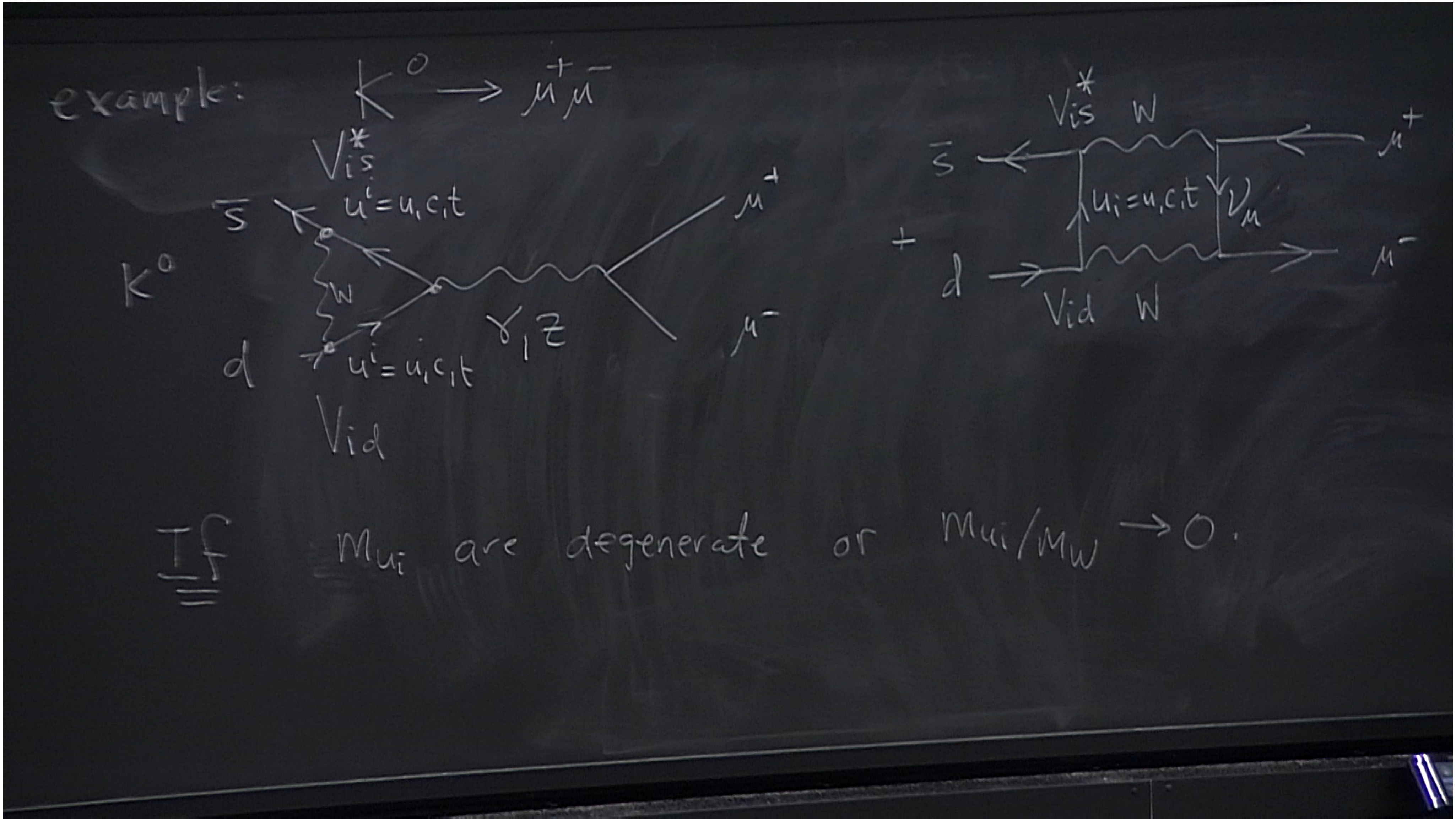
ν_μ
μ^+
μ^-
d
V_{id} W

$\leftarrow \mu^+$

$\propto \sum_{i=u,c,t} V_{id} V_{is}^* \; f\left(m_{ui}/m_W \right)$

$\rightarrow \mu^-$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ν_μ
μ^+
μ^-
d
V_{id} W
If m_{u_i} are degenerate or $m_{u_i}/m_W \to 0$.

μ^+

$$\propto \sum_{i=u,c,t} V_{id} V_{is}^* \, f\left(m_{ui}/m_W \right)$$

μ^-

Then $$i\mathcal{M} \propto \sum_i V_{id} V_{is}^* = \left(V^\dagger V\right)_{sd} = \delta_{21} = 0 .$$

This is called the GIM mechanism (Glashow - Iliopoulis - Maia[ni]

In the limit of massless quarks (compared to m_W), all FCNCs

t-quark is very massive ; but contribution is suppressed for $s \leftrightarrow d$

$$|V_{td} V_{ts}| \approx 3 \times 10^{-4}$$

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FCNCs are loop suppressed & GIM-suppressed (or CKM-suppressed)

...ed the GIM mechanism (Glashow - Iliopoulis - Maiani)

... of massless quarks (compared to m_W), all FCNCs vanish.

... very massive ; but contribution is suppressed for $s \leftrightarrow d$.

$$|V_{td} V_{ts}| \approx 3 \times 10^{-4}$$

loop suppressed & GIM-suppressed (or CKM-suppressed)

Brief history of kaons

(1) Longevity of kaons: $\Longrightarrow$ strangeness & eightfold way (1961) $\Rightarrow$

(2) Theta-tau puzzle: parity violation.

~1950, two particles discovered: θ, τ same mass.

$$\theta^+ \to \pi^+ \pi^0$$

$$\tau^+ \to \pi^+ \pi^0 \pi^0, \ \pi^+ \pi^+ \pi^-$$

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f kaons

f kaons: $\Longrightarrow$ strangeness & eightfold way (1961) $\Rightarrow$ quarks

tau puzzle: parity violation.

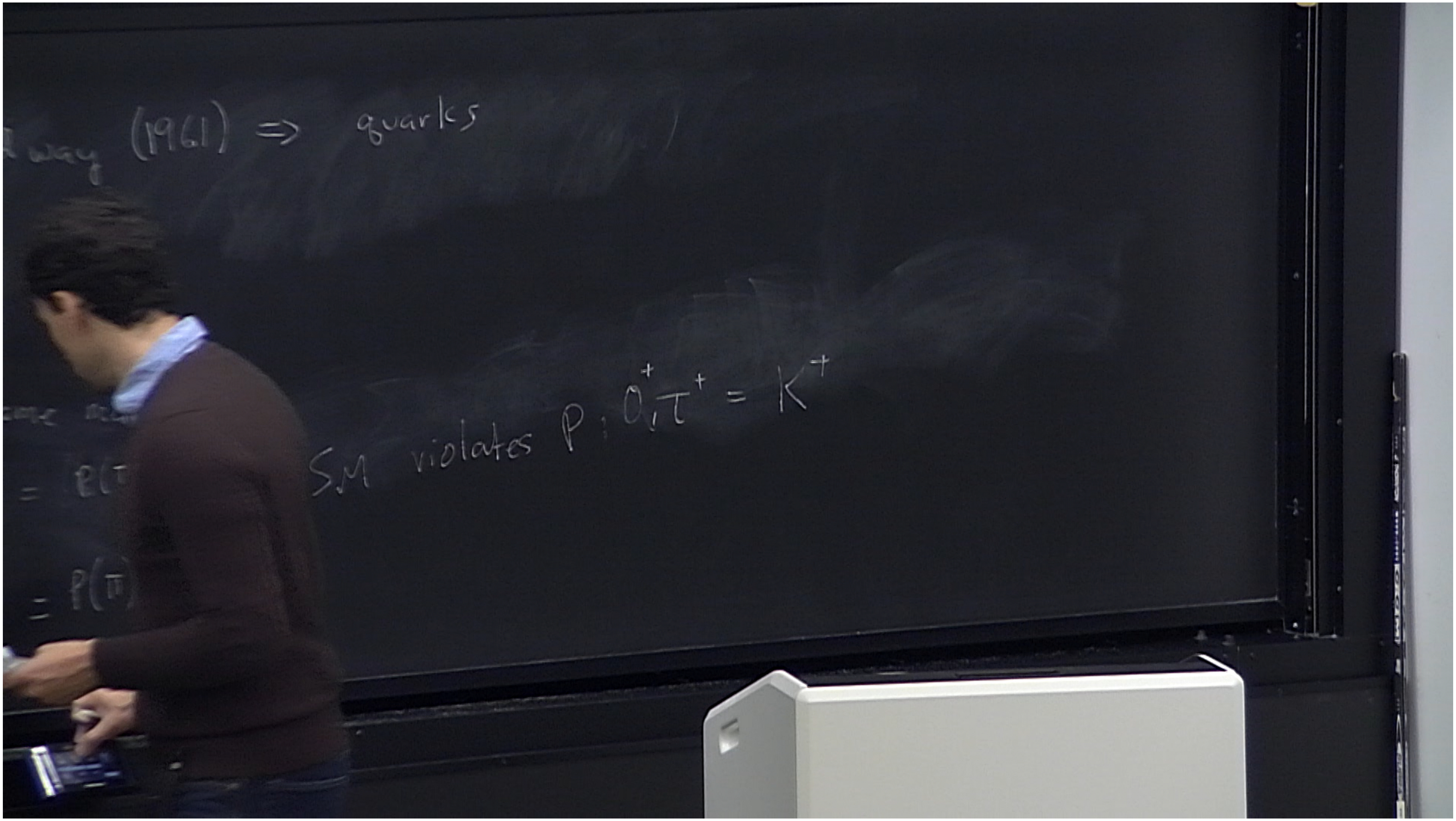
, two particles discovered: θ, τ same mass.

$P = +1 = P(\pi)^2$

$\theta^+ \to \pi^+ \pi^0$

$P = -1 = P(\pi)^3$

$\tau^+ \to \pi^+ \pi^0 \pi^0, \pi^+ \pi^+ \pi^-$


way (1961) => quarks
SM violates P: θ+, τ+ = K+

same mass.

$= P(\pi)^2$

$= P(\pi)^3$

SM violates P: $\theta^+, \tau^+ = K^+$

(3) Neutral kaon mixing, charm quark.

We have $K^0, \bar{K}^0$ which are eigenstates of strangeness $S = -1, +1$

$P|K^0\rangle = -|K^0\rangle$, $C|K^0\rangle = |\bar{K}^0\rangle$, same for $|\bar{K}^0\rangle$

$CP|K^0\rangle = -|\bar{K}^0\rangle$, $CP|\bar{K}^0\rangle = -|K^0\rangle$

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$$CP|K^0\rangle = -|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = -|K^0\rangle$$

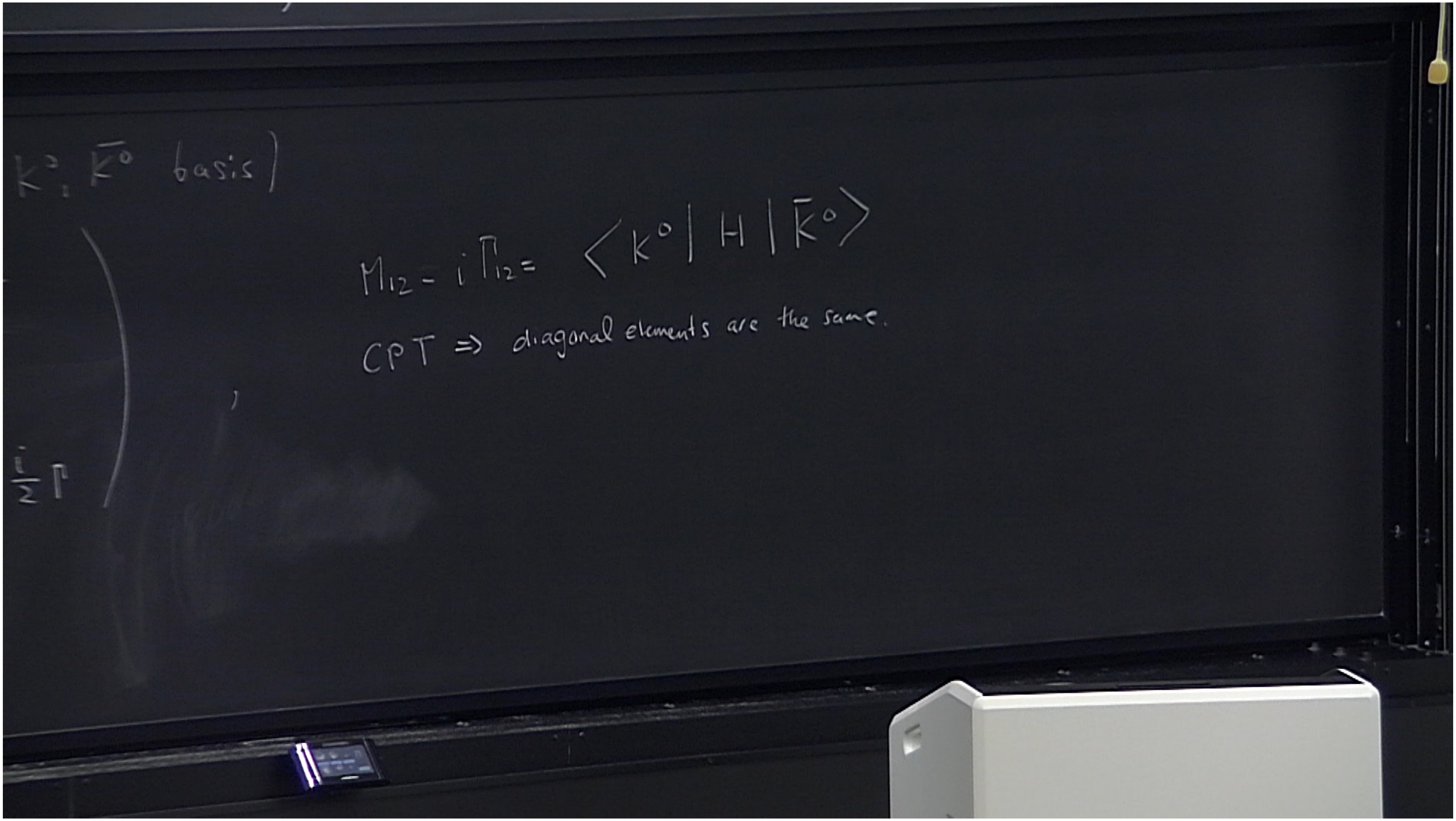
CP eigenstates:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

Neutral kaon Hamiltonian: (in $K^0, \bar{K}^0$ basis)

$$H = \begin{pmatrix} m_K - \frac{i}{2}\Gamma & m_{12} - \frac{i}{2}\Gamma_{12} \\ m_{12}^* - \frac{i}{2}\Gamma_{12}^* & m_K - \frac{i}{2}\Gamma \end{pmatrix}$$


K⁰, K̄⁰ basis)
M₁₂ − i Γ₁₂ = ⟨K⁰ | H | K̄⁰⟩
CPT ⇒ diagonal elements are the same.
i/2 Γ

If CP is conserved, $M_{12} = M_{12}^*$, $\Gamma_{12} = \Gamma_{12}^*$

$$\begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}}_{U} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

$$H \to U^\dagger H U = \begin{pmatrix} m_K + M_{12} - \frac{i}{2}(\Gamma + \Gamma_{12}) & 0 \\ 0 & m_K - M_{12} - \frac{i}{2}(\Gamma - \Gamma_{12}) \end{pmatrix}$$

Mass splitting:

$$\Delta m_K = m_1 - m_2 = 2M_{12}$$

GIM-suppressed

$$0$$

$$\ldots_K - M_{12} - \frac{i}{2}\left(\Gamma - \Gamma_{12}\right)$$

(4) CP violation.

Observationally: K_L (long) & K_S (short)

$$K_S \longrightarrow \underbrace{\pi^+\pi^-,\ \pi^0\pi^0}_{\text{CP-even}} \quad \text{with a small lifetime} \sim 10^{-10}\,s$$

$$K_L \rightarrow \underbrace{\pi\pi\pi}_{\text{CP-odd}},\ \ldots \quad \text{with long lifetime} \sim 5\times 10^{-8}$$

If CP were conserved: $K_S = K_1$

$K_L = K_2$

If CP were conserved: $K_S = K_1$

$K_L = K_2$

$m_K - 3m_\pi \sim 90$ MeV

If CP were conserved: $K_S = K_1$

$K_L = K_2$

Observed by Cronin, Fitch
(Turlay & Christenson)

$K_L \to \pi\pi$

$- 3 m_\pi \sim 90$ MeV

CP-odd

$$|K_{S,L}\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_{1,2}\rangle + \varepsilon\,|K_{2,1}\rangle\right)$$

CP-admixtu

$$\varepsilon \propto \mathrm{Im}(M_{12})$$