Title: PSI 2016/2017 Standard Model (Review) - Lecture 12

Date: Jan 18, 2017 09:00 AM

URL: http://pirsa.org/17010016

Abstract:

...tations

...ental" of SU(3). "3"

...damental". "$\bar{3}$"

$q_i \to U_{ij} q_j$

$\bar{q}_i \to U^*_{ij} \bar{q}_j$

Young tableaux & Irreducible representations

Single quark $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ is a "fundamental" of SU(3

antiquark $\bar{q} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$ is an "antifundamental". "$\bar{3}$"

Diquark (distinguishable): $q_i^{(1)} q_j^{(2)}$

Young tableaux & Irreducible representations

Single quark $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ is a "fundamental" of $SU(3)$.

antiquark $\bar{q} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$ is an "antifundamental". "$\bar{3}$"

Diquark (distinguishable): $q_i^{(1)} q_j^{(2)} \rightarrow U_{ik} U_{jl}\, q_k^{(1)} q_l^{(2)}$

Product rep of two fundamentals: $3 \times 3 = 9$ dim. rep $\rightarrow$

Symmetric rep: $q^{(1)}_i q^{(2)}_j + q^{(1)}_j q^{(2)}_i$

Antisym. rep: $q^{(1)}_i q^{(2)}_j - q^{(1)}_j q^{(2)}_i$

wo fundamentals: $3 \times 3 = 9$ dim. rep $\longrightarrow$ reducible.

$$q_i^{(1)} q_j^{(2)} + q_j^{(1)} q_i^{(2)} \longrightarrow U_{ik} U_{jl} \left(q_k^{(1)} q_l^{(2)} + q_l^{(1)} q_k^{(2)} \right)$$

$$q_i^{(1)} q_j^{(2)} - q_j^{(1)} q_i^{(2)} \longrightarrow U_{ik} U_{jl} \left(q_k^{(1)} q_l^{(2)} - q_l^{(1)} q_k^{(2)} \right)$$

...ucible.

$$+ q^{(1)}_{l} q^{(2)}_{k})$$

6-dimensional rep (6)

$$q^{(1)}_{l} q^{(2)}_{k})$$

3-dimensional rep.

Compact notation for antisym. rep:

$$\varepsilon_{ijk}\, q_j^{(1)} q_k^{(2)} \longrightarrow \varepsilon_{ijk}\, U_{jm} U_{kn}\, q_m^{(1)} q_n^{(2)} =$$

Use: $\varepsilon_{ijk}\, U_{il} U_{jm} U_{kn} = \varepsilon_{lmn}$

$$\Rightarrow \quad \varepsilon_{ijk}\, U_{jm} U_{kn} = U_{il}^{*}\, \varepsilon_{lmn}$$

rep:

$$\varepsilon_{ijk}\, U_{jm} U_{kn}\, q^{(1)}_m q^{(2)}_n = U^*_{il}\, \varepsilon_{lmn}\, q^{(1)}_m q^{(2)}_n$$

$$U_{kn} = \varepsilon_{lmn}$$

Antisym. rep transforms as $\bar{3}$

$$U_{kn} = U^*_{il}\, \varepsilon_{lmn}$$

Compact notation for antisym. rep:

$$\varepsilon_{ijk}\, q^{(1)}_j q^{(2)}_k \longrightarrow \varepsilon_{ijk}\, U_{jm} U_{kn}\, q^{(1)}_m q^{(2)}_n = U^*_{il}$$

Use: $\varepsilon_{ijk}\, U_{il} U_{jm} U_{kn} = \varepsilon_{lmn}$

Antisym

$$\Rightarrow \quad \varepsilon_{ijk}\, U_{jm} U_{kn} = U^*_{il}\, \varepsilon_{lmn}$$

Diquark: $3 \times 3 = 6 + \bar{3}$

Young Tableaux

$\boxed{i}$ = fundamental rep with index i

$\boxed{j}$ = —" ——— " ——— j

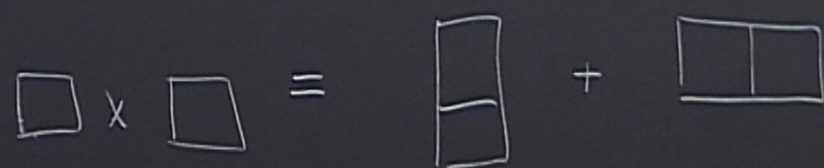
$\begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array}$ = i, j antisymmetrized

$\begin{array}{|c|c|}\hline i & j \\ \hline\end{array}$ = i, j symmetrized

Put boxes together following rules:

1. Length of rows of boxes does not increase from top to bottom
2. length of columns does not increase from left to right.
3. No more than 3 boxes in a column for $SU(3)$ (N boxes

Diquark:

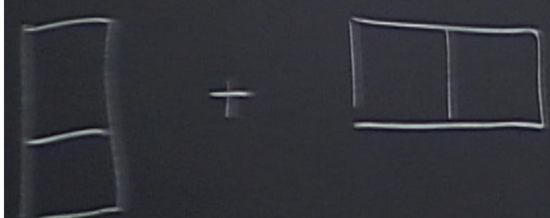


rules:

boxes does not increase from top to bottom.

does not increase from left to right.

3 boxes in a column for SU(3) (N boxes for SU(N))



Count dimension of rep. Number of ways to put indices $i, j, \cdots = 1, 2$

1. indices in a row cannot decrease

2. indices in a column must increase.

$\square = \boxed{1}, \boxed{2}, \boxed{3}$ 3 dim.

$\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} =$

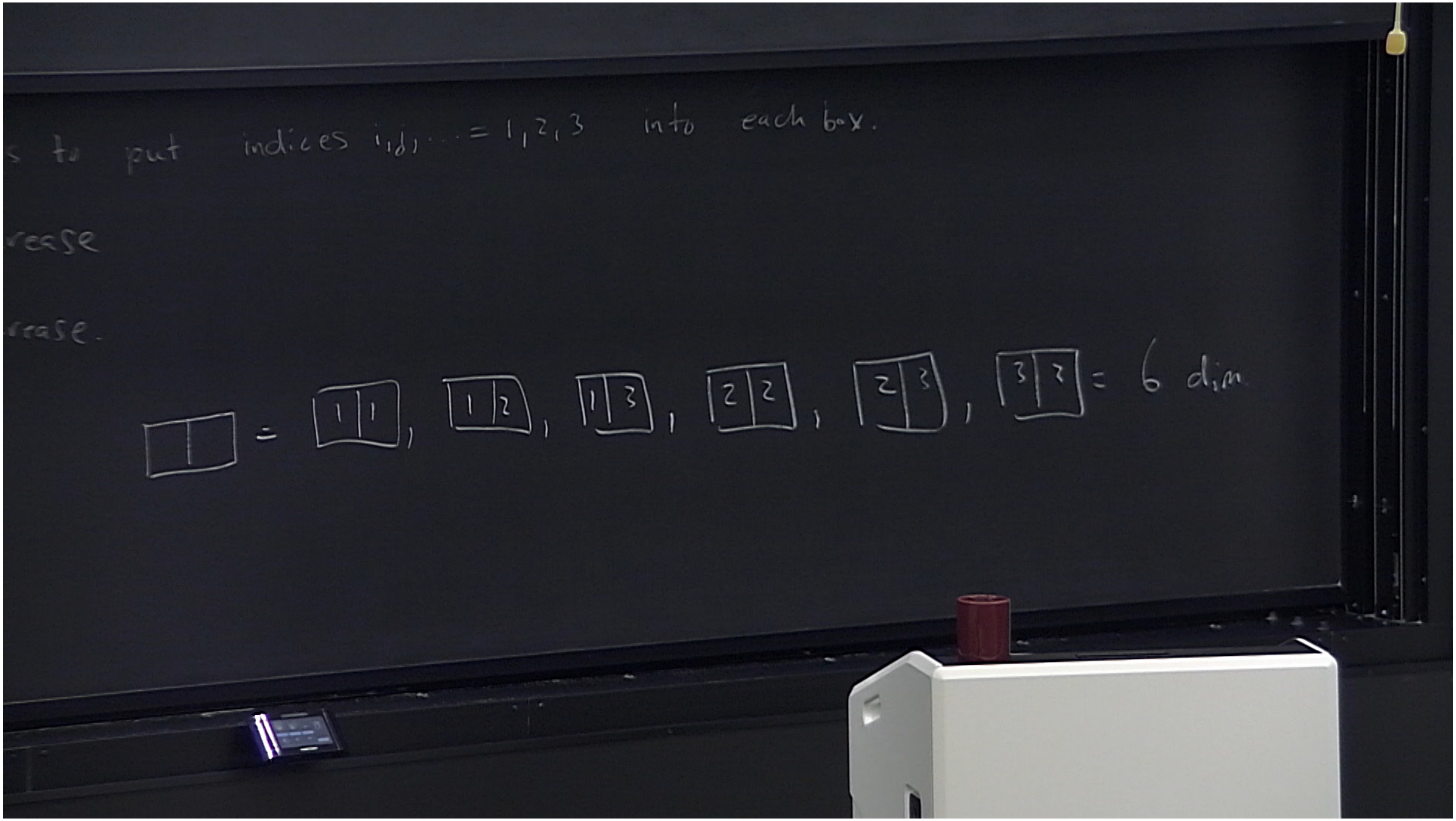
Count dimension of rep. Number of ways to put indices $i, j, \cdots = 1,$

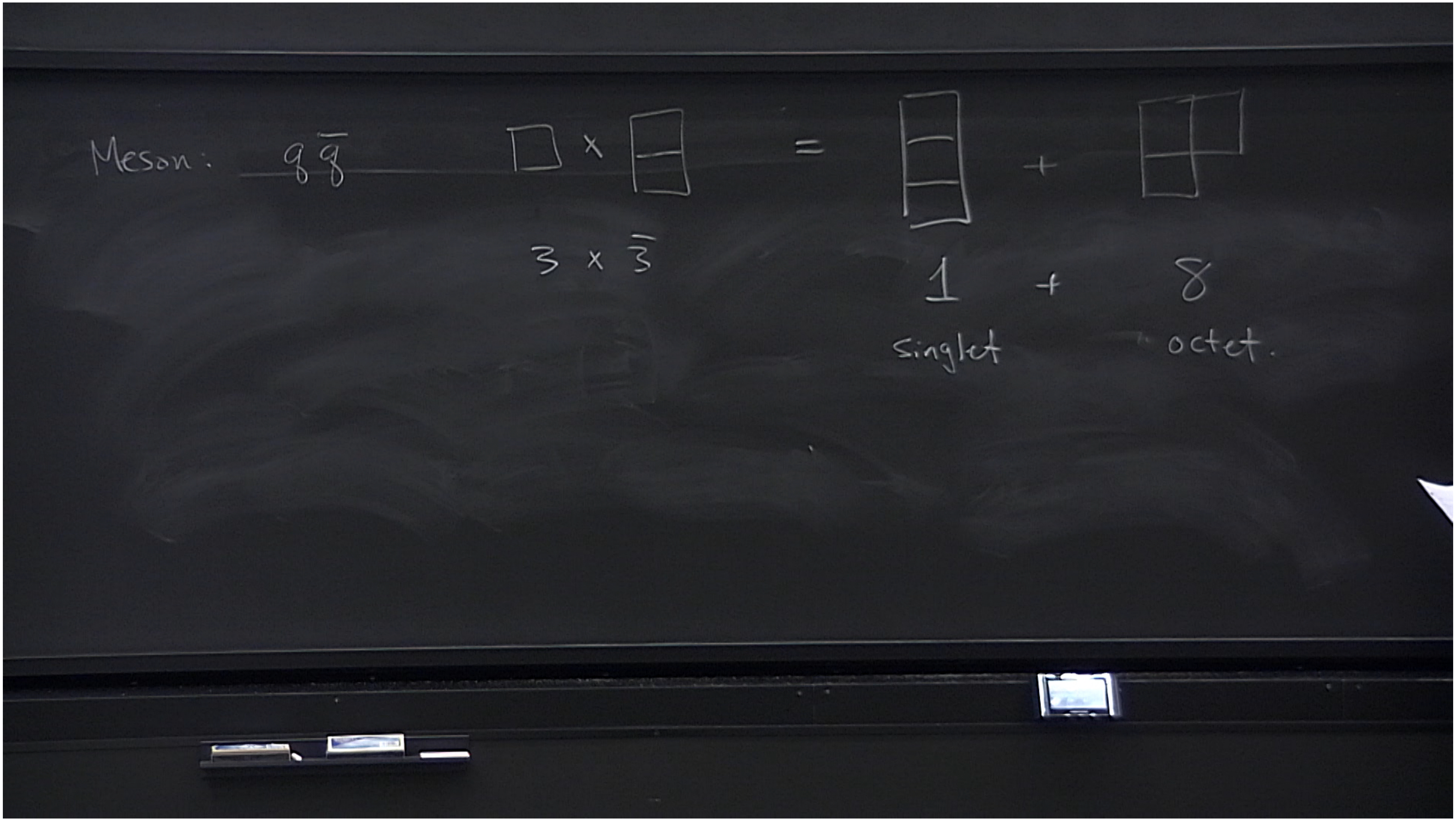
1. indices in a row cannot decrease

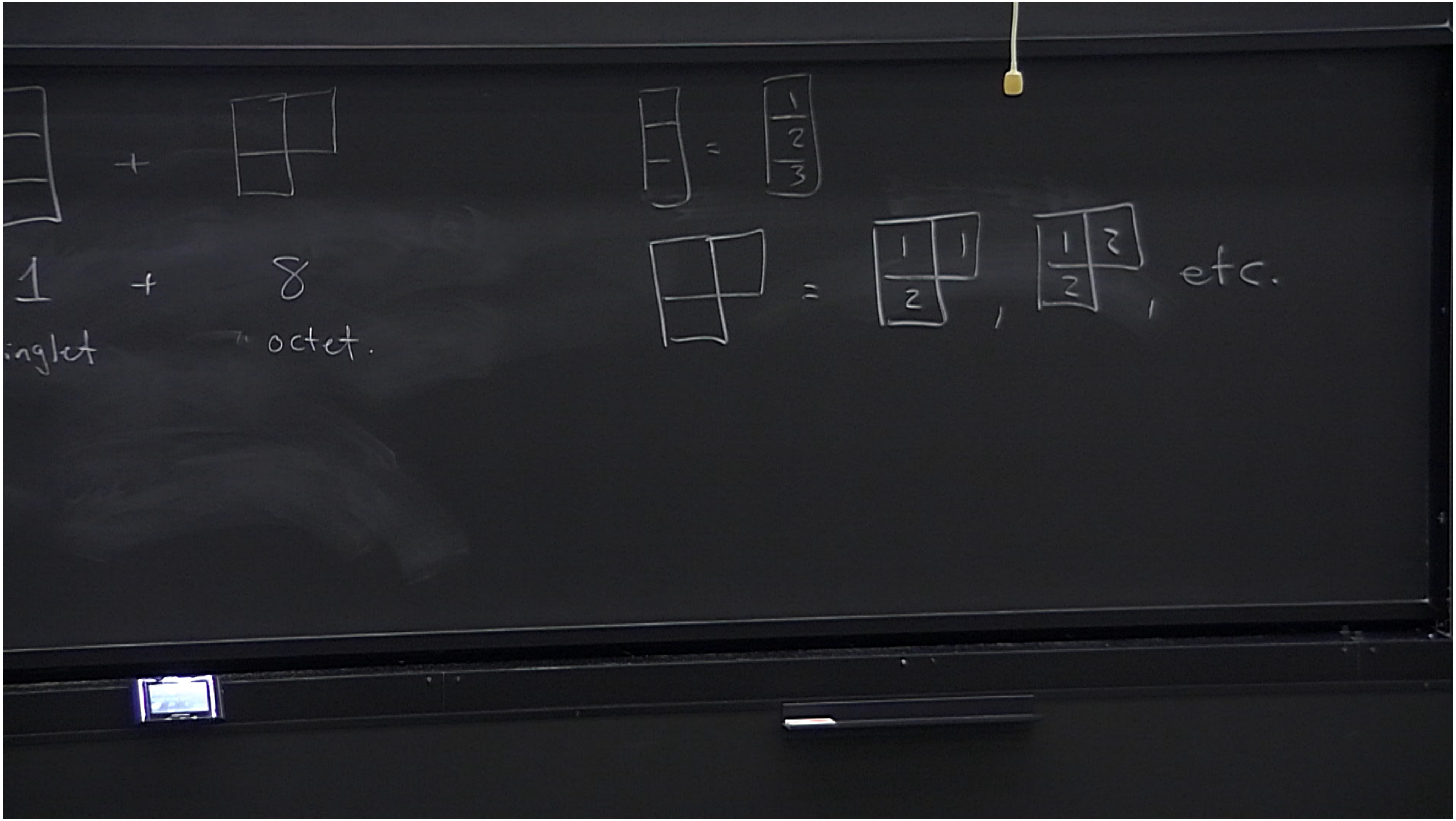
2. indices in a column must increase.

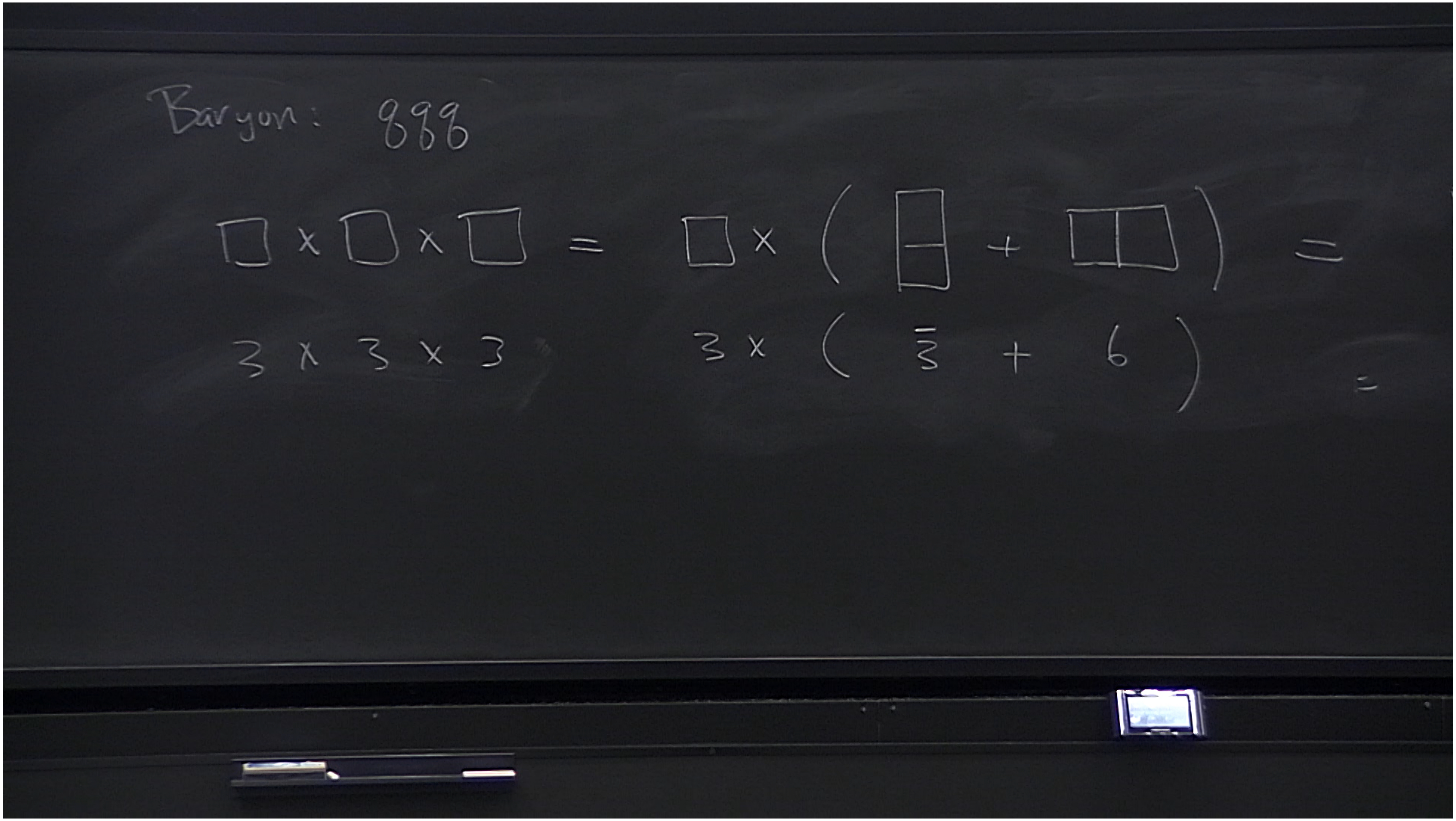
$\square = \boxed{1}, \boxed{2}, \boxed{3}$ 3 dim.

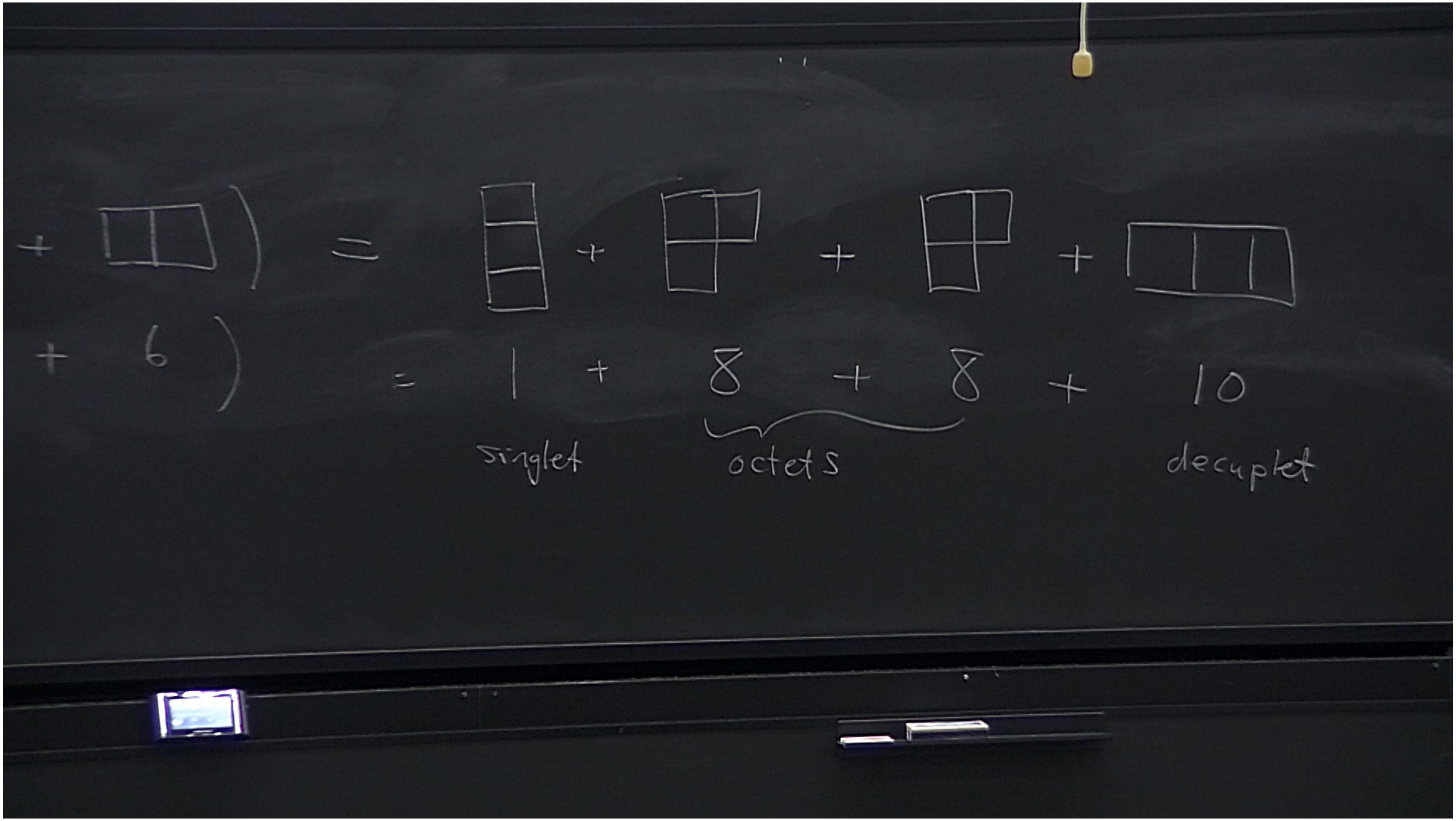
$\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} = \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}, \begin{array}{|c|}\hline 1 \\ \hline 3 \\ \hline \end{array}, \begin{array}{|c|}\hline 2 \\ \hline 3 \\ \hline \end{array}$


s to put indices i, j, ... = 1, 2, 3 into each box.
rease
rease.
= 6 dim.


Meson: $q\bar{q}$
=
$3 \times \bar{3}$
$1 + 8$
singlet
octet.


1 + 8
inglet
octet.
etc.


Baryon: qqq
3 x 3 x 3
3 x ($\bar{3}$ + 6) =


+ 6)
= 1 + 8 + 8 + 10
singlet
octets
decuplet

Mesons: $q, \bar{q}$ are distinguishable

Non-relativistic quark model: naive approx. that $q\bar{q}$ is hyd

Lowest lying states are $L=0$ spatial wavefunction. Spin

Apply Young tableaux to $SU(2)$: $\square \times \square = \begin{array}{c}\square\\[-4pt]\square\end{array} + \square\square$

$$2 \times 2 = 1 + 3$$

$$J=0 \qquad J=1$$

rox. that $q\bar{q}$ is hydrogen-like bound state.

atial wave function. Spin of meson due to spin of quarks.

$$\square \quad = \quad \begin{array}{c}\square\\[-4pt]\square\end{array} \;+\; \square\!\square$$

$$2 \qquad\qquad 1 \;+\; 3$$

$$J=0 \qquad J=1$$

$\ldots) \rightarrow$

always singlet under SU(3)color.

$\ldots (-1).$

$$P = (-1)^{L+1} = -1 \quad \text{for } L = 0.$$

Color wavefunction (SU(3) color) $\longrightarrow$ always singlet

$q\bar{q}$ has intrinsic parity (-1). $P = (-1)^{L+1} = -$

Four possibilities:

pseudoscalar $(J^P = 0^-)$ octet or singlet

vector $(J^P = 1^-)$ octet or singlet

$) \rightarrow$ always singlet under SU(3)color.

(-1). $\quad P = (-1)^{L+1} = -1$ for $L = 0$.

$)$ octet or singlet (π, K, η, η')

$1^-)$ octet or singlet $(\rho, \omega, \phi, K^*)$

Vector $(J = 1)$ octet or singlet

Octet rep. (pseudoscalar) $\qquad M_{ij} \longrightarrow U_{ik}\, U^*_{jl}\, M_{kl}$ $\qquad M$ —

Write $\quad M = M^A T^A \qquad T^A = \frac{\lambda^A}{2}$ = generators of SU(3) flavor.

$$= \frac{1}{2}\begin{pmatrix} M^3 + \frac{M^8}{\sqrt{3}} & M^1 - iM^2 & M^4 - iM^5 \\ M^1 + iM^2 & -M^3 + \frac{M^8}{\sqrt{3}} & M^6 - iM^7 \\ M^4 + iM^5 & M^6 + iM^7 & -\frac{2}{\sqrt{3}} M^8 \end{pmatrix} =$$

$^*_{jl} M_{kl}$ $\qquad M \to U M U^\dagger$

ators of SU(3) flavor.

$$\begin{pmatrix} M^5 \\ M^7 \\ M^8 \end{pmatrix} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$+ \qquad K^+$$

$$+\frac{\eta}{\sqrt{6}} \qquad K^0$$

$$\bar{K}^0 \qquad -\frac{2}{\sqrt{6}}\eta \Bigg) ,$$

$$\pi^{\pm} = \frac{M^1 \mp i M^2}{2}$$

etc.

Recall: $SU(2)$ has one diagonal generator T^3. Label components

For $SU(3)$, two diagonal generators T^3, T^8. Label compon

$$K^0: \quad \left[T^3, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K^0 \\ 0 & 0 & 0 \end{pmatrix}\right] = -\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K^0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

...abel components of rep. by eigenvalues under that generator.

Label components by eigenvalues.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K^0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \left[T^8, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K^0 \\ 0 & 0 & 0 \end{pmatrix}\right] = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K^0 \\ 0 & 0 & 0 \end{pmatrix}$$

Where do isospin & strangeness fit in?

Isospin is $SU(2)$ subgroup of $SU(3)_{flavor}$

$$U = \left(\begin{array}{c|c} V & 0 \\ \hline 0 & 1 \end{array}\right) \qquad V = SU(2) \text{ isospin rotation.}$$

3×3 $SU(3)_{flavor}$

Baryon: qqq

$$MU^{+} = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} M \begin{pmatrix} V^{+} & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \left(\begin{array}{c|c} V \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} V^{+} + \frac{\eta}{\sqrt{6}} & V \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \\ \hline (K^-, \bar{K}^0)\, V^{+} & -\frac{2}{\sqrt{6}}\eta \end{array}\right)$$

M:

$$M \to U M U^\dagger = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} M \begin{pmatrix} V^\dagger & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \left(\begin{array}{c} V \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} V^\dagger + \frac{\eta}{\sqrt{6}} \\ \hdashline (K^-, \bar{K}^0) V^\dagger \end{array} \right.$$

$$\begin{pmatrix} \cdots & \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \\ \cdots & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$\pi^\pm, \pi^0 =$ isospin triplet (adjoint rep of $SU(2)$)

$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix} =$ isospin doublet

$\eta =$ isospin singlet.

$I_3 = T^3$ isospin 3-component.

Baryon number is additional conserved quantum number.

In $SU(3)_{flavor}$, $B = \frac{1}{3}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$

Strangeness: $S = \frac{1}{\sqrt{3}} T^8 - B$

Baryon number is additional conserved quantum number.

In $SU(3)_{flavor}$,

$$B = \frac{1}{3}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Strangeness: $S = \frac{1}{\sqrt{3}} T^8 - B$ ($B = 0$ for mesons)

mesons)

states	I_3	I	T^8	S
π^+	$+1$	1	0	0
π^0	0	1	0	0
π^-	-1	1	0	0
η, η'	0	0	0	0
K^+	$1/2$	$1/2$	$\sqrt{3}$	1
K^0	$-1/2$	$1/2$	$\sqrt{3}$	1
$\bar{K}^0$	$1/2$	$1/2$	$-\sqrt{3}$	-1
K^-	$-1/2$	$1/2$	$-\sqrt{3}$	-1

Lagrangian for pseudoscalars:

$$\mathcal{L} = \frac{1}{2} \mathrm{Tr}\left[\partial_\mu M \partial^\mu M\right] - \frac{1}{2}\mu_8^2 \,\mathrm{Tr}\left[M^2\right] + \frac{1}{2}\partial_\mu \eta'$$

Kinetic terms are in canonical form:

$$\boxed{m_\pi^2 = m_K^2 = m_\eta^2} = \mu_8^2$$

$$m_{\eta'}^2 = \mu_1^2$$

lars:

$$M\Big] - \frac{1}{2}\mu_8^2 \,\mathrm{Tr}\left[M^2\right] + \frac{1}{2}\partial_\mu \eta' \partial^\mu \eta' - \frac{1}{2}\mu_1^2 \eta'^2 + \mathcal{L}_{int.}$$

l form:

$^2_8$