Title: PSI 2016/2017 Standard Model (Review) - Lecture 3

Date: Jan 05, 2017 09:00 AM

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Abstract:

Abelian Higgs model: Fermion masses

Add fermion Ψ to model

Assume chiral couplings $g_L \neq g_R$ to A_μ

Abelian Higgs model: Fermion masses

Add fermion Ψ to model

Assume chiral couplings $g_L \neq g_R$ to A_μ

$\bar{\Psi}\Psi$ forbidden by gauge symmetry.

Generate it spontaneously

Notation: g = gauge coupling

Q = charge of a field in units of g.

Quantum numbers:

ϕ : $Q_\phi = +1$

Ψ_L : $Q_L \to$ $g_L = Q_L g$

Ψ_R : $Q_R \to$ $g_R = Q_R g$

$\Big\}$ $Q_L \neq Q_R$

Ga

Gauge transformations:

$$\phi \to e^{-igQ_\phi \alpha}\,\phi$$

$$\Psi_{L,R} \to e^{-igQ_{L,R}\alpha}\,\Psi_{L,R}$$

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$$\psi_{L,R} \to e^{-igQ_{L,R}\alpha} \psi_{L,R}$$

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Gauge transformations:

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$$\psi_{L,R} \rightarrow e^{-ig Q_{L,R} \alpha} \psi_{L,R}$$

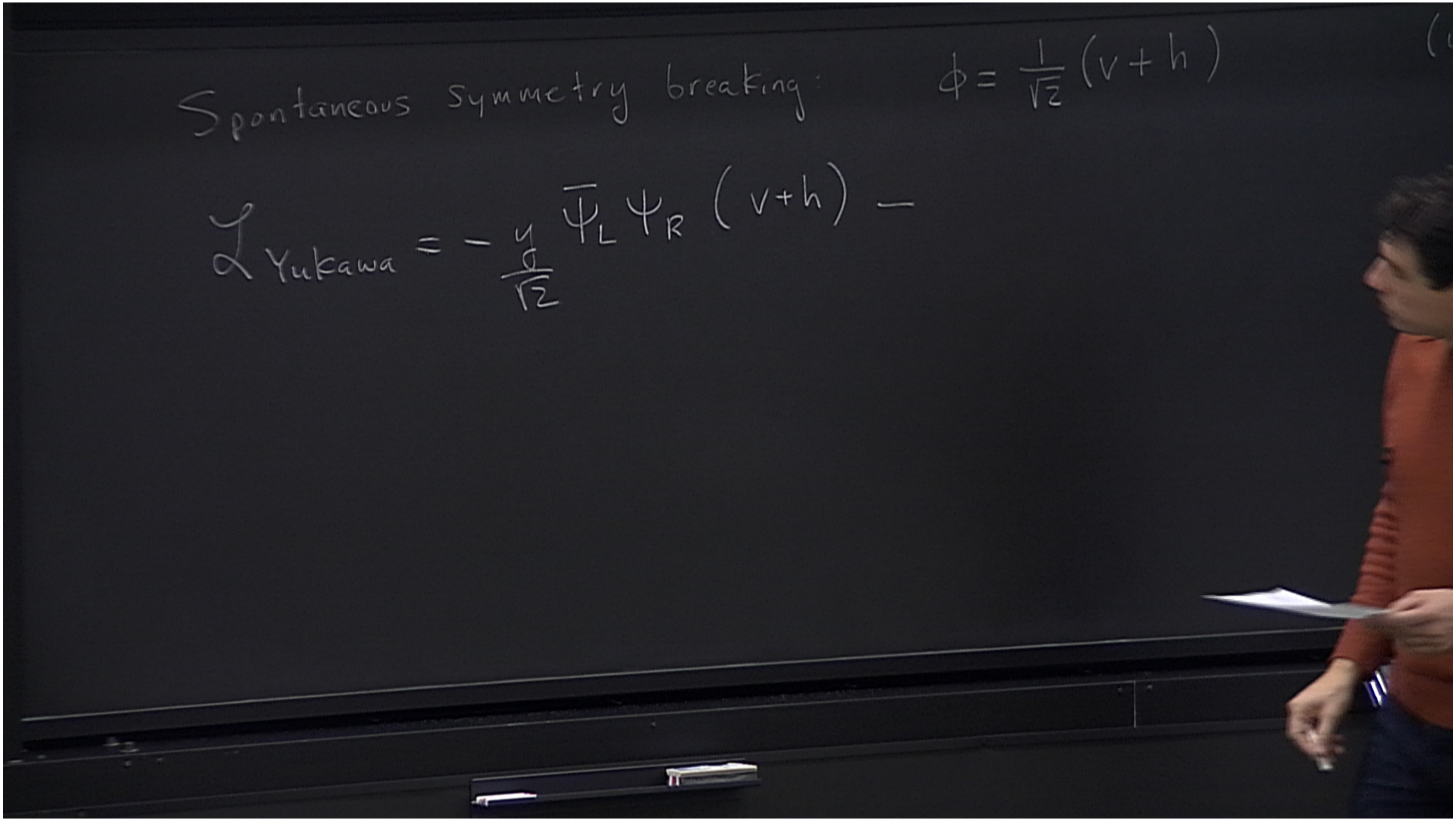
If we assume: $Q_L = Q_R + Q_\phi$, then we can have

$$\mathcal{L}_{Yukawa} = - y \bar{\psi}_L \psi_R \phi + h.c.$$

e can have a Yukawa interaction that is gauge invariant.

$+ h.c.$

$y =$ Yukawa coupling.


Spontaneous symmetry breaking:
$\phi = \frac{1}{\sqrt{2}}(v+h)$
$\mathcal{L}_{Yukawa} = -\frac{y}{\sqrt{2}}\bar{\Psi}_L \Psi_R (v+h) -$

(unitary gauge)

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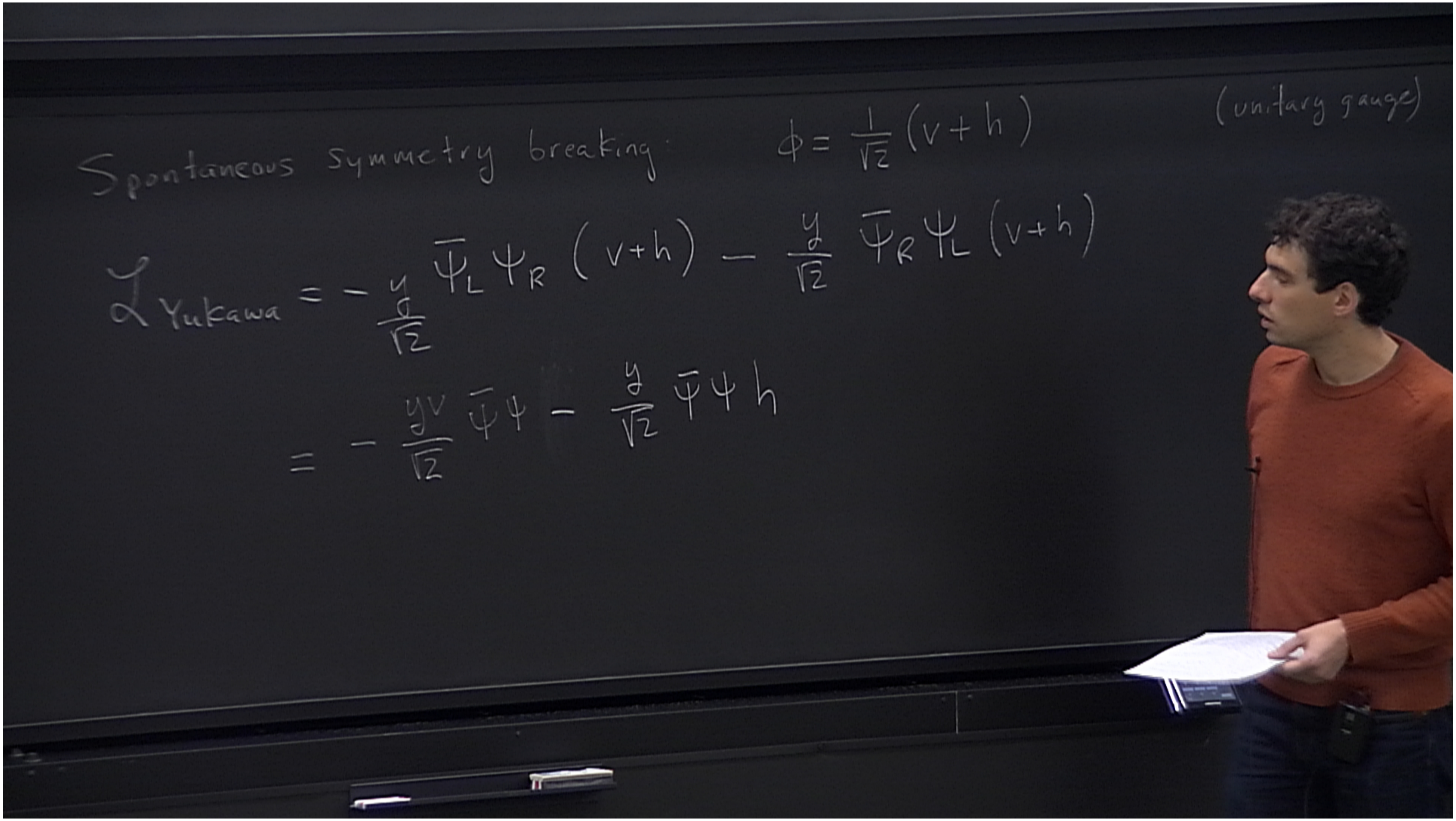
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$= -\frac{yv}{\sqrt{2}} \bar{\Psi}\Psi - \frac{y}{\sqrt{2}} \bar{\Psi}\Psi h$

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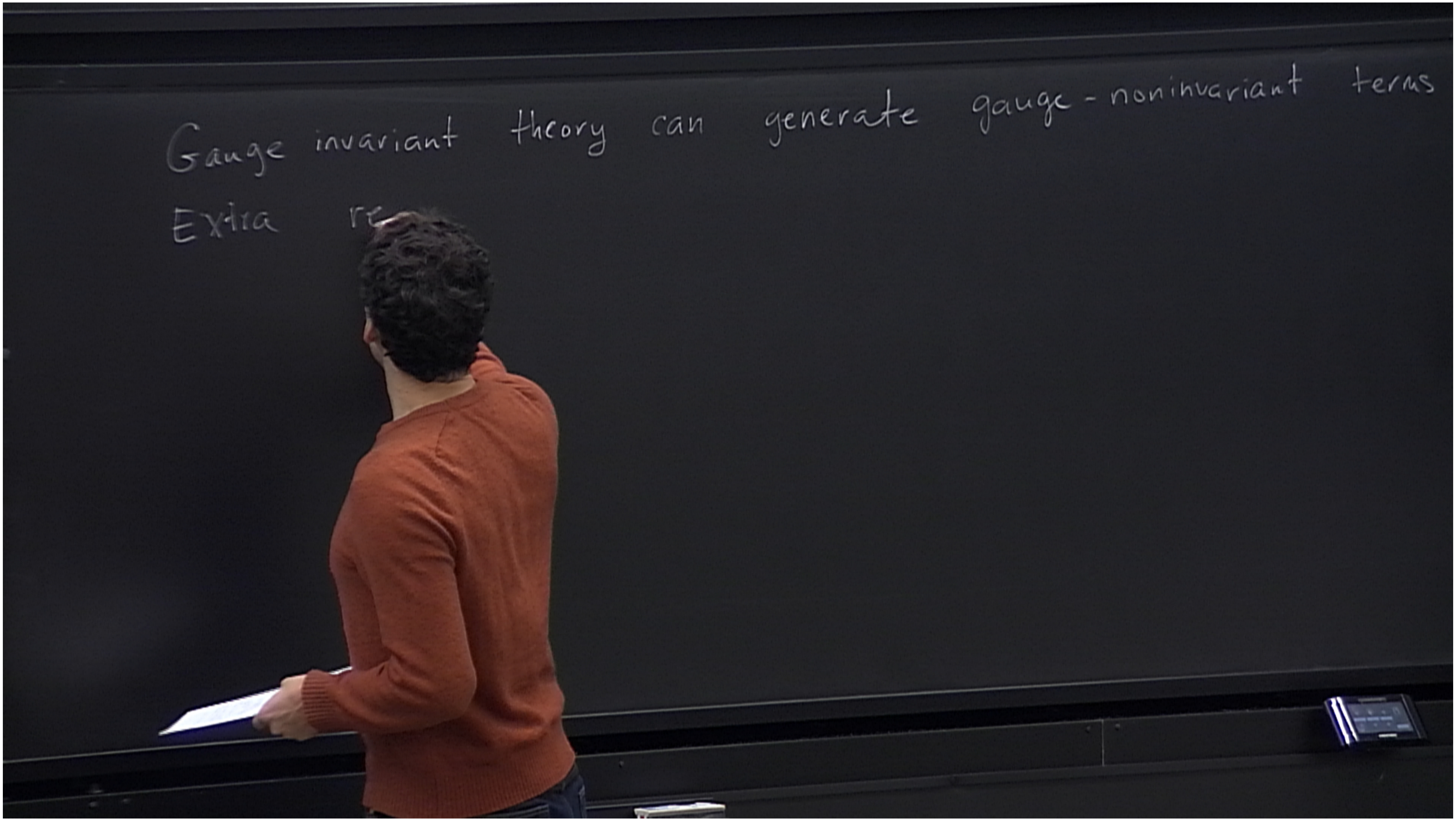
$$= -\frac{yv}{\sqrt{2}}\bar{\psi}\psi - \frac{y}{\sqrt{2}}\bar{\psi}\psi h \quad \rightarrow \quad m_\psi = \frac{yv}{\sqrt{2}}$$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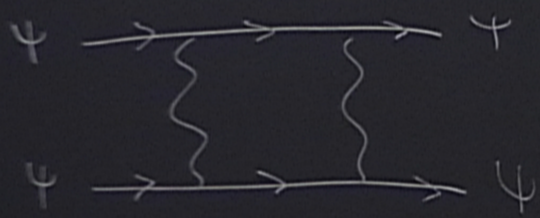
Extra residual degree of freedom h (Higgs boson) cure nonrenormalizability.



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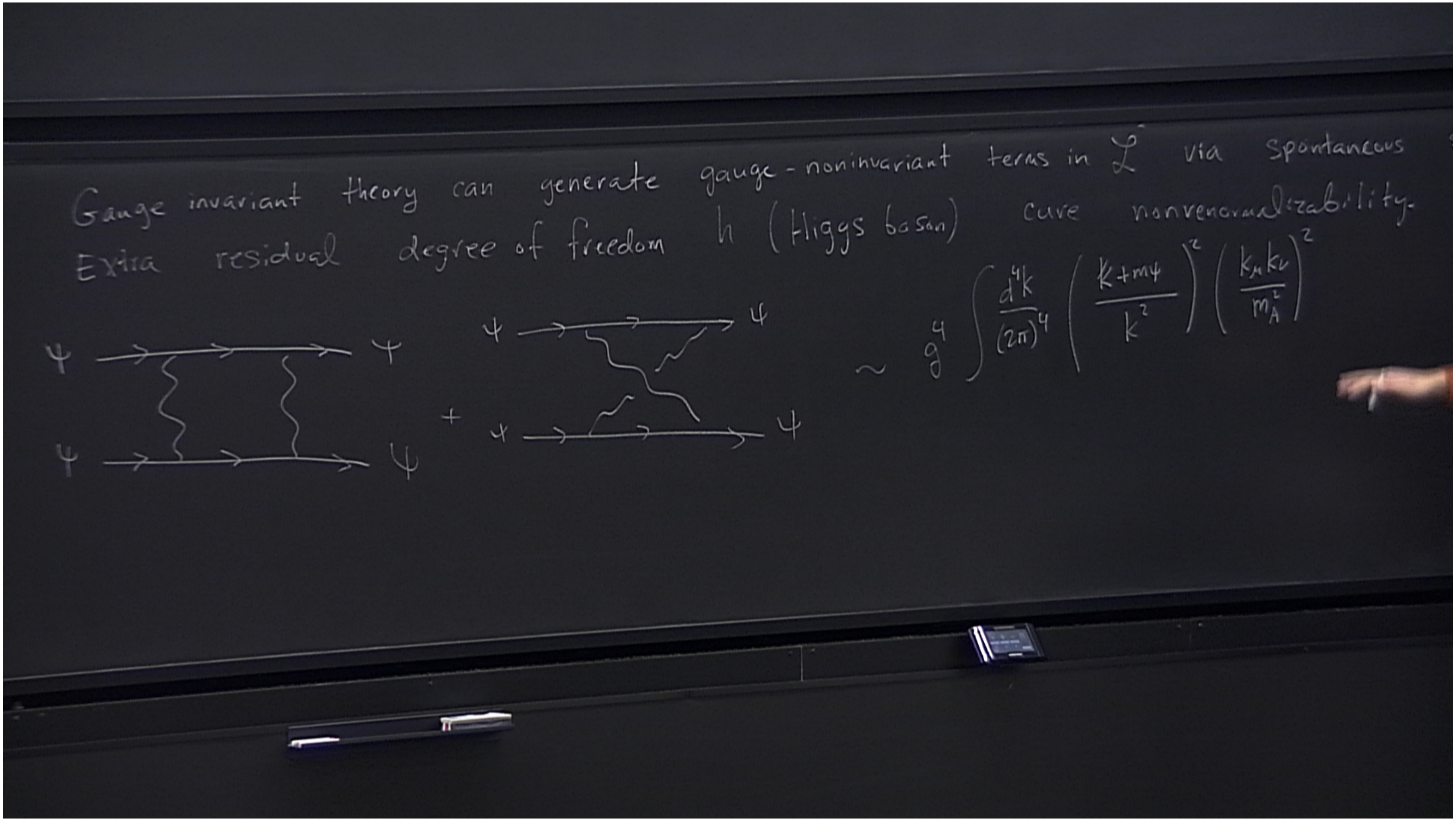
Gauge invariant theory can generate gauge-noninvariant terms in $\mathcal{L}$ via spontaneous
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Extra residual degree of freedom h (Higgs boson)
$\sim g^4 \int \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k}+m_\psi}{k^2} \right)^2 \left(\frac{k_\mu k_\nu}{m_A^2} \right)^2$
ψ ψ ψ ψ + ψ ψ ψ ψ

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$\to \psi$

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$$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{\cancelto{0}{\not{k}} + m_\psi}{k^2}\right)^2 \left(\frac{k_\mu k_\nu}{m_A^2}\right)^2 \sim g^4 \int \frac{d^4k}{(2\pi)^4} \frac{m_\psi^2}{k^4} \left(\frac{k^\mu k^\nu}{m_A^2}\right)^2$$

$\to \psi$

Gauge invariant theory can generate gauge-noninvariant terms cu

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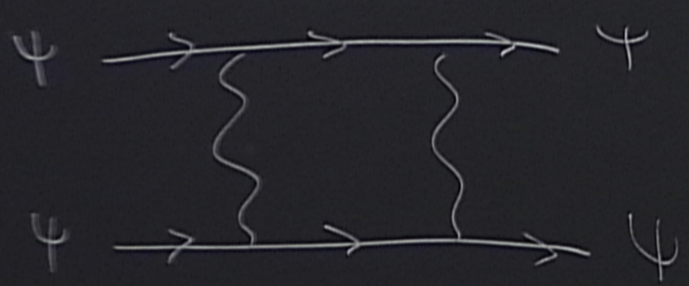

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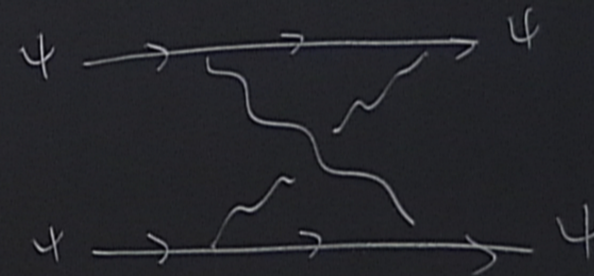
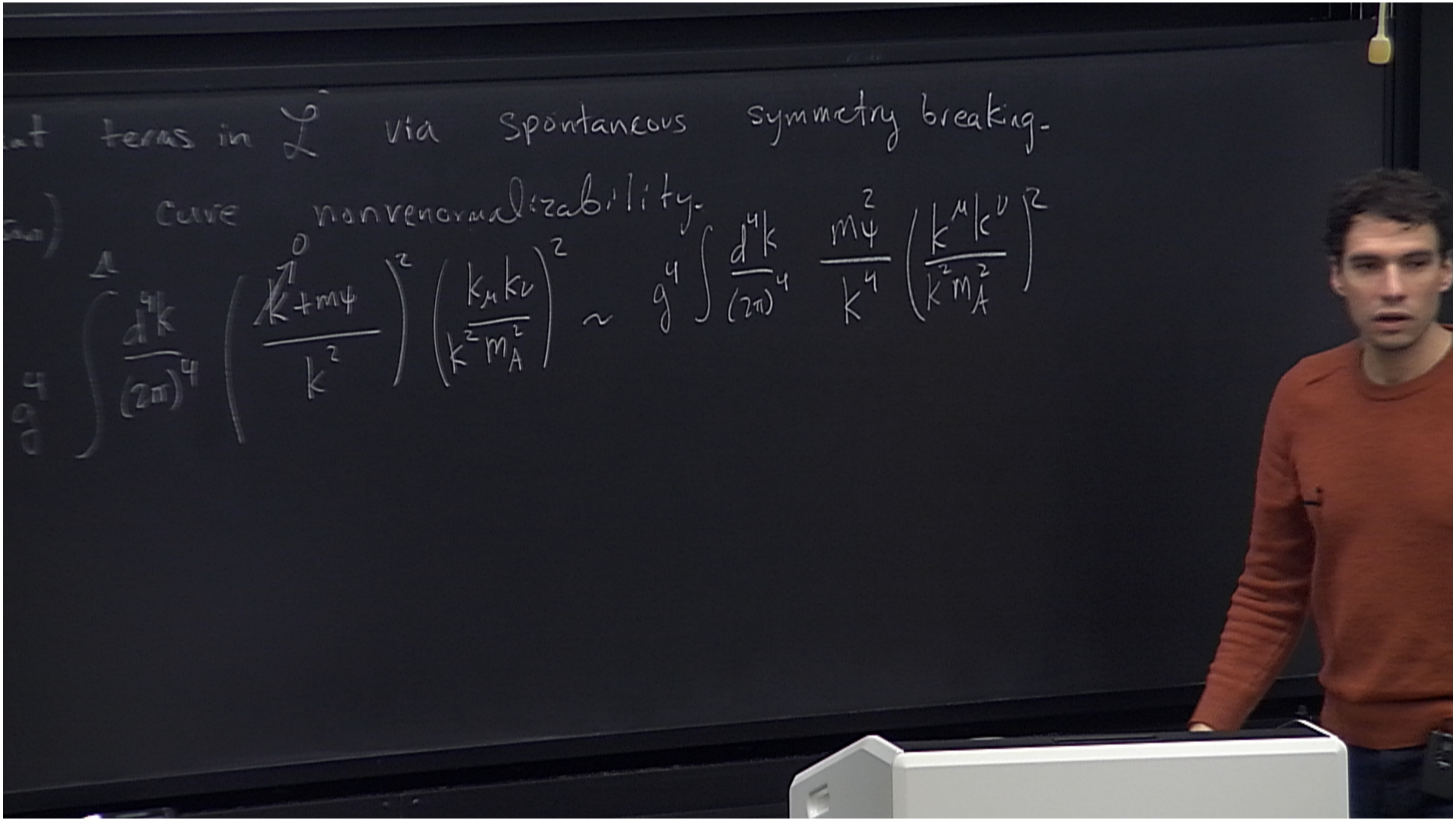


$$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4}$$

$$\frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$$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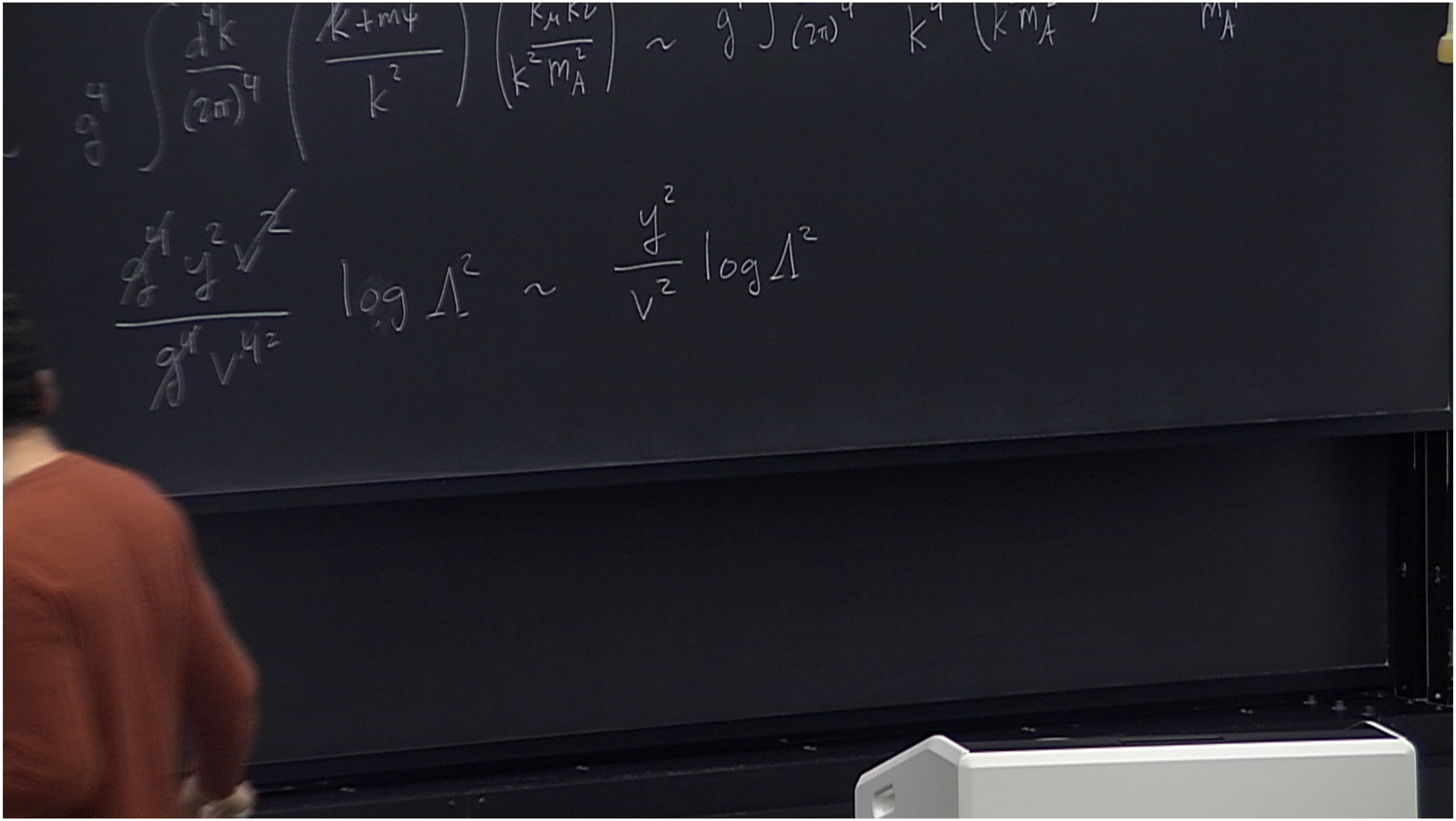
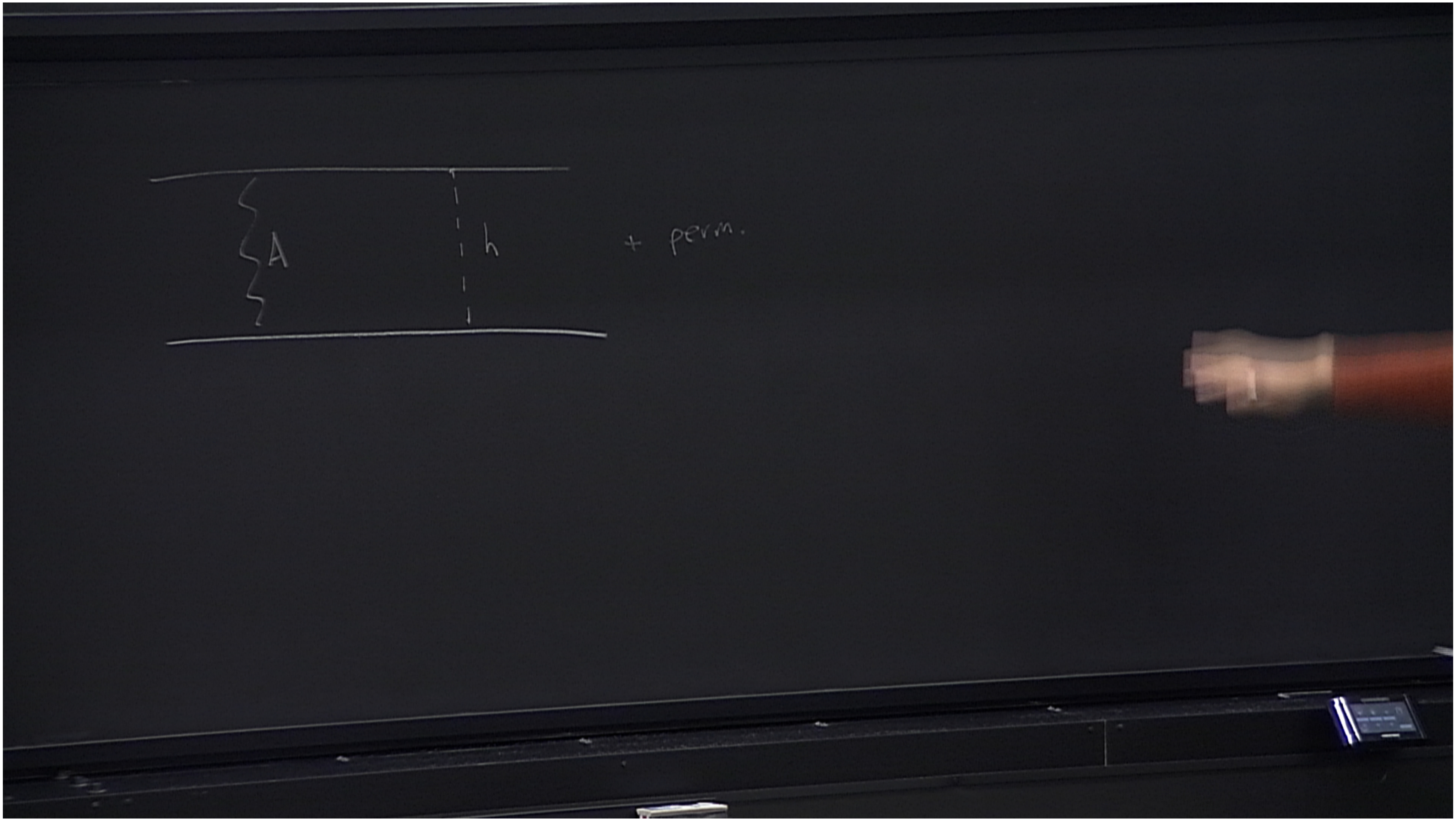
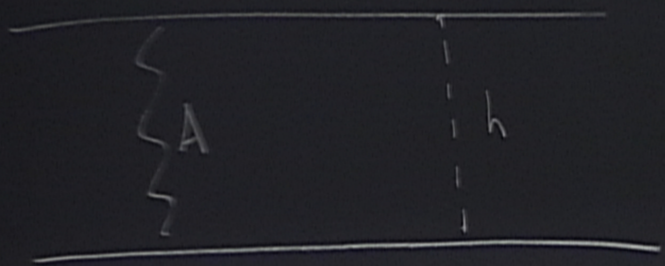
$$g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{\cancel{k}^{\,0} + m_\psi}{k^2} \right)^2 \left(\frac{k_\mu k_\nu}{k^2 m_A^2} \right)^2 \sim g^4 \int \frac{d^4k}{(2\pi)^4} \frac{m_\psi^2}{k^4} \left(\frac{k^\mu k^\nu}{k^2 m_A^2} \right)^2 \sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

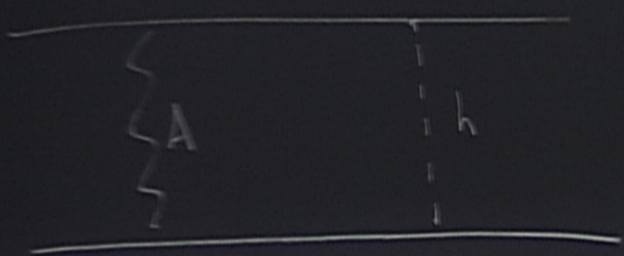

$g^4 \int \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k} + m_\psi}{k^2} \right) \left(\frac{k_\mu k_\nu}{k^2 m_A^2} \right) \sim g^4 \int \frac{}{(2\pi)^4}$
$\frac{\cancel{g^4}\, y^2 \cancel{v^2}}{\cancel{g^4}\, v^{\cancel{4}\,2}} \log \Lambda^2 \sim \frac{y^2}{v^2} \log \Lambda^2$

A
h


A
h
+ perm.



$$+ \text{ perm.} \sim g^2 y^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{k}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right) \frac{1}{k^2}$$



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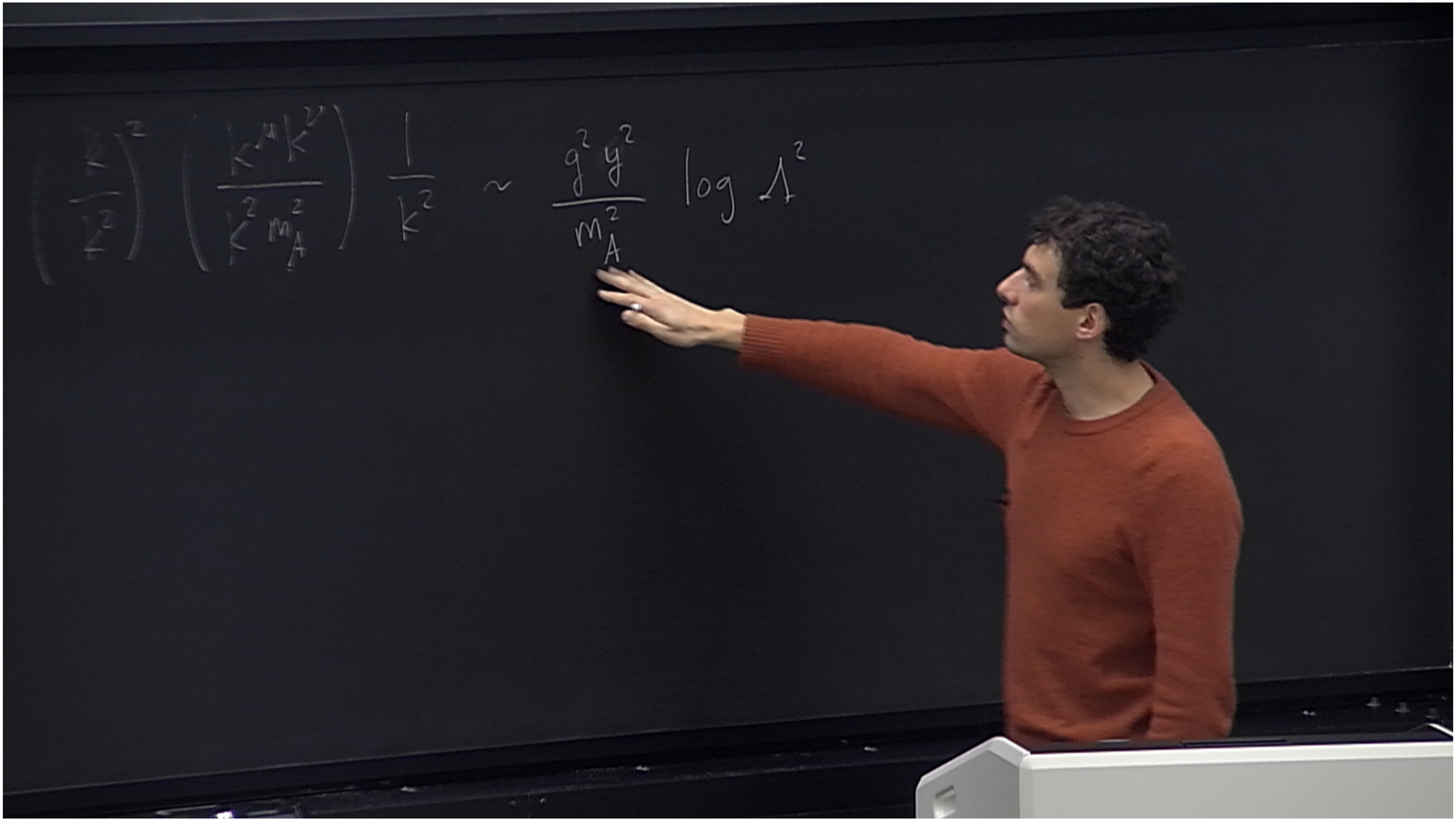
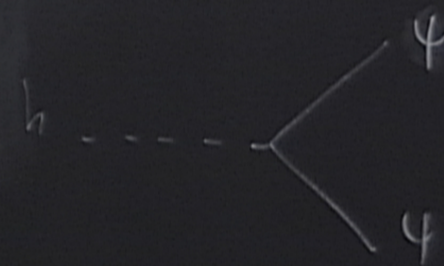

$\left(\frac{k}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right) \frac{1}{k^2} \sim \frac{g^2 y^2}{m_A^2} \log \Lambda^2$

$$\left(\frac{k}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right) \frac{1}{k^2} \sim \frac{g^2 y^2}{m_A^2} \log \Lambda^2 \sim \frac{y^2}{v^2} \log \Lambda^2$$

Prediction from spontaneously broken gauge theory:

Extra scalar: couplings are related to masses.

$$= \frac{2 i m_A^2}{v} \eta_{\mu\nu}$$



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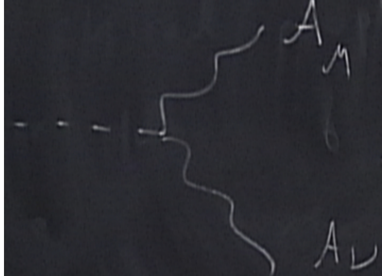

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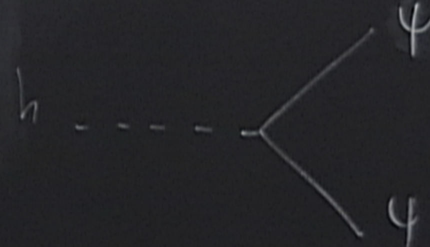


$$= \frac{i m_\psi}{v}$$

SM Lagrangian

Gauge group: $\underbrace{SU(3)_C}_{\substack{\text{QCD} \\ \text{(color)}}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak}}$

L = left

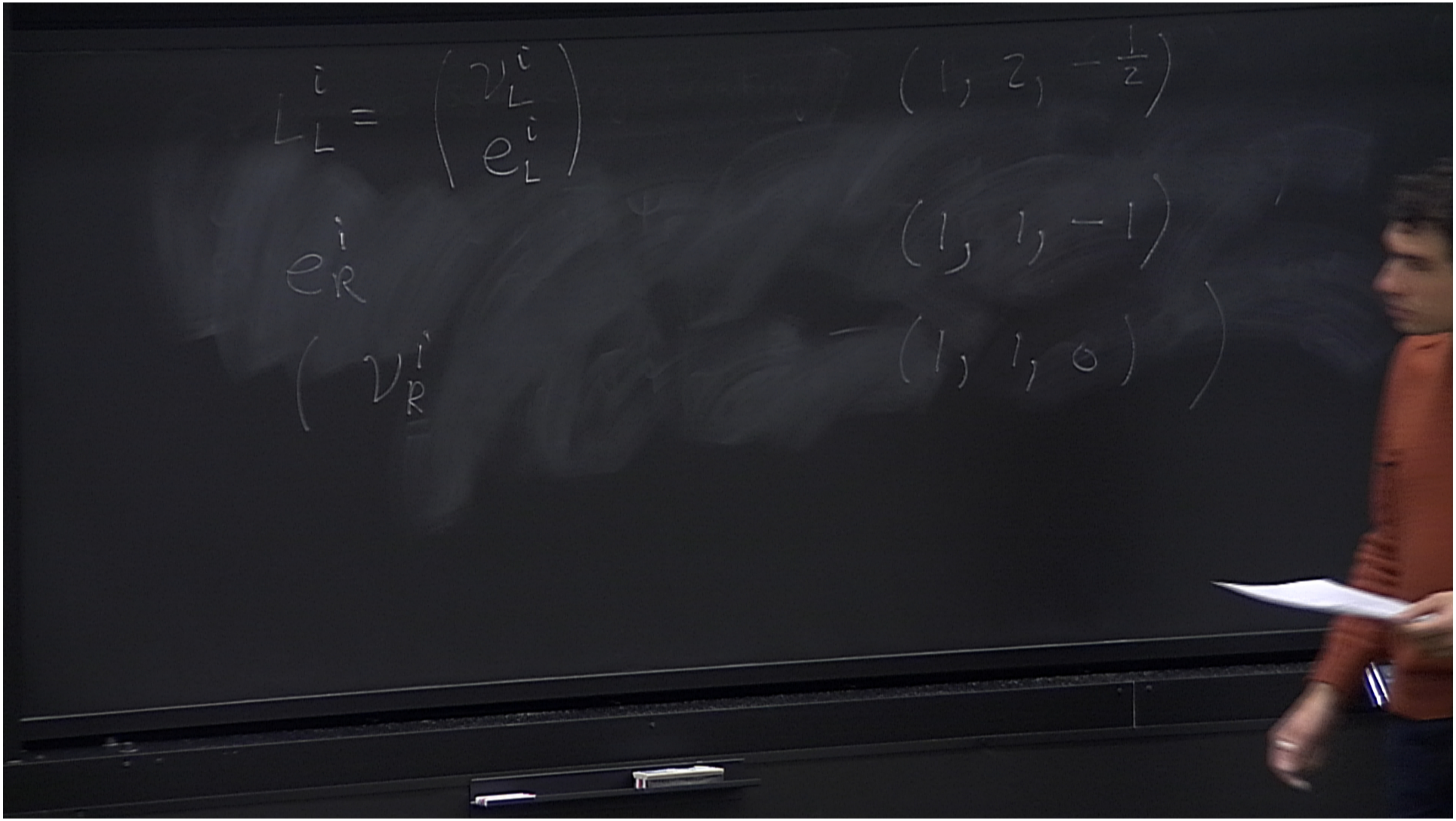
Y = hypercharge.

Field	Quantum numbers $(SU(3)_c,\ SU(2)_L,\ U(1)_Y)$
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	$\left(3,\ 2,\ \frac{1}{6}\right)$
u_R^i	$\left(3,\ 1,\ \frac{2}{3}\right)$
d_R^i	$\left(3,\ 1,\ -\frac{1}{3}\right)$

$$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \qquad (1, 2, -\tfrac{1}{2})$$

$$e_R^i \qquad (1, 1, -1)$$

$$(\nu_R^i \qquad (1, 1, 0)$$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e_R^i
$(1, 1, -1)$
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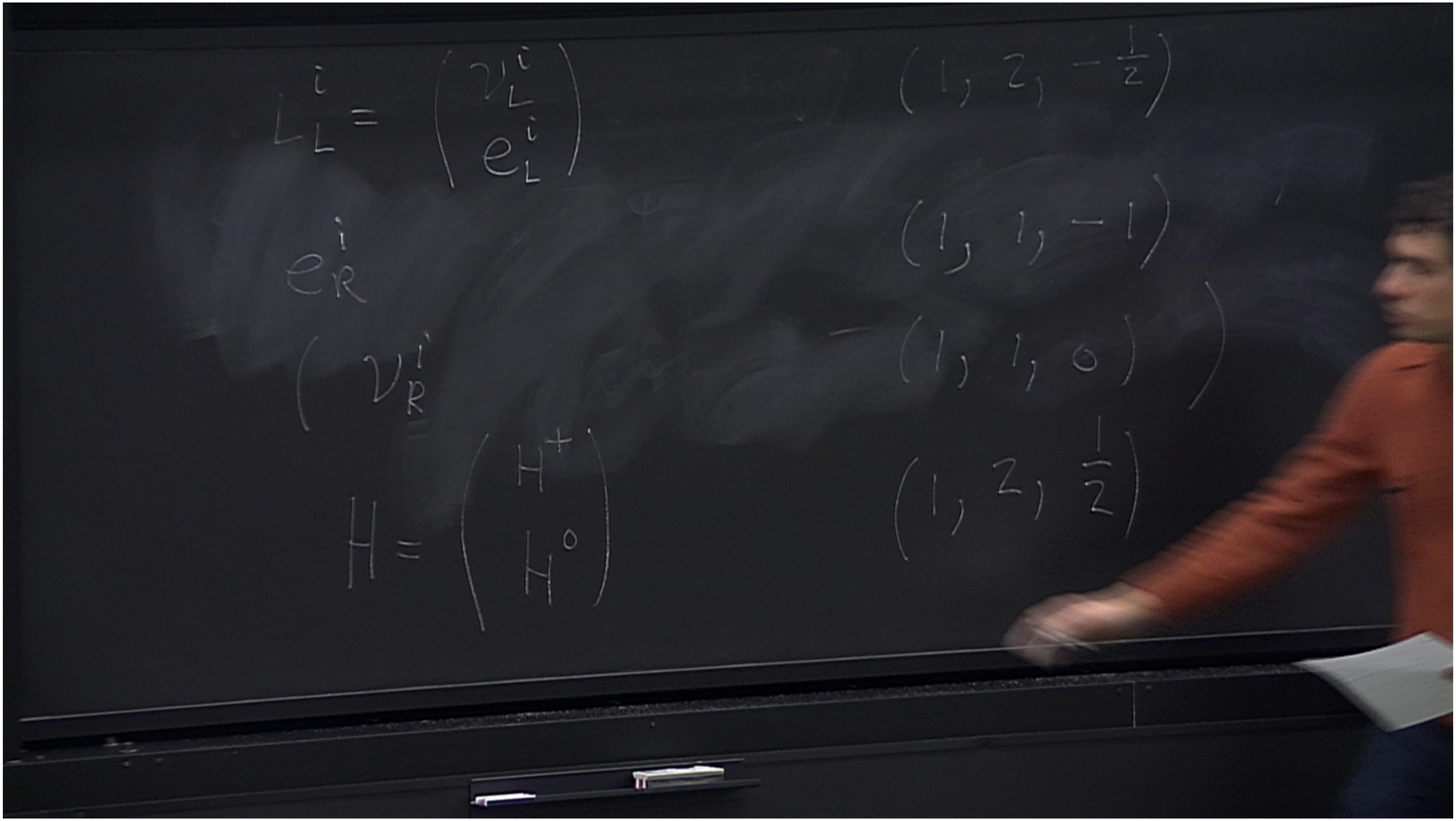
$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$
$(1, 2, -\frac{1}{2})$
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$(\nu_R^i$
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$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$
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$$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \qquad \left(1, 2, -\tfrac{1}{2}\right)$$

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$$\left(\nu_R^i \qquad (1, 1, 0) \right)$$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad \left(1, 2, \tfrac{1}{2}\right)$$

$i=1,2,3$ labels generation of each fermion.

Gauge bosons:

$SU(3)_C$: gluon field g_μ^A $A = 1, \dots, 8$, gauge coupling g_s

$SU(2)_L$: W_μ^a $a = 1, 2, 3$, g

$U(1)_Y$: B_μ , g'

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{scalar} + \mathcal{L}_{Yukawa}.$$

$$\mathcal{L}_{gauge} = -\frac{1}{2}\,\mathrm{Tr}\left[g_{\mu\nu}\, g^{\mu\nu}\right] - \frac{1}{2}\,\mathrm{Tr}\left[W_{\mu\nu} W^{\mu\nu}\right] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{\text{fermions}} = \sum_{\text{fermions } \psi} \bar{\psi}\, i \not{D}\, \psi$$

$$\mathcal{L}_{\text{scalar}} = (D_\mu H)^\dagger (D^\mu H) - V(H) \quad , \quad V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Scalar} + \mathcal{L}_{Yukawa}.$$

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$$g_{\mu\nu} = \partial_\mu g_\nu - \partial_\nu g_\mu$$

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$$g_{\mu\nu} = \partial_\mu g_\nu - \partial_\nu g_\mu + \ldots$$

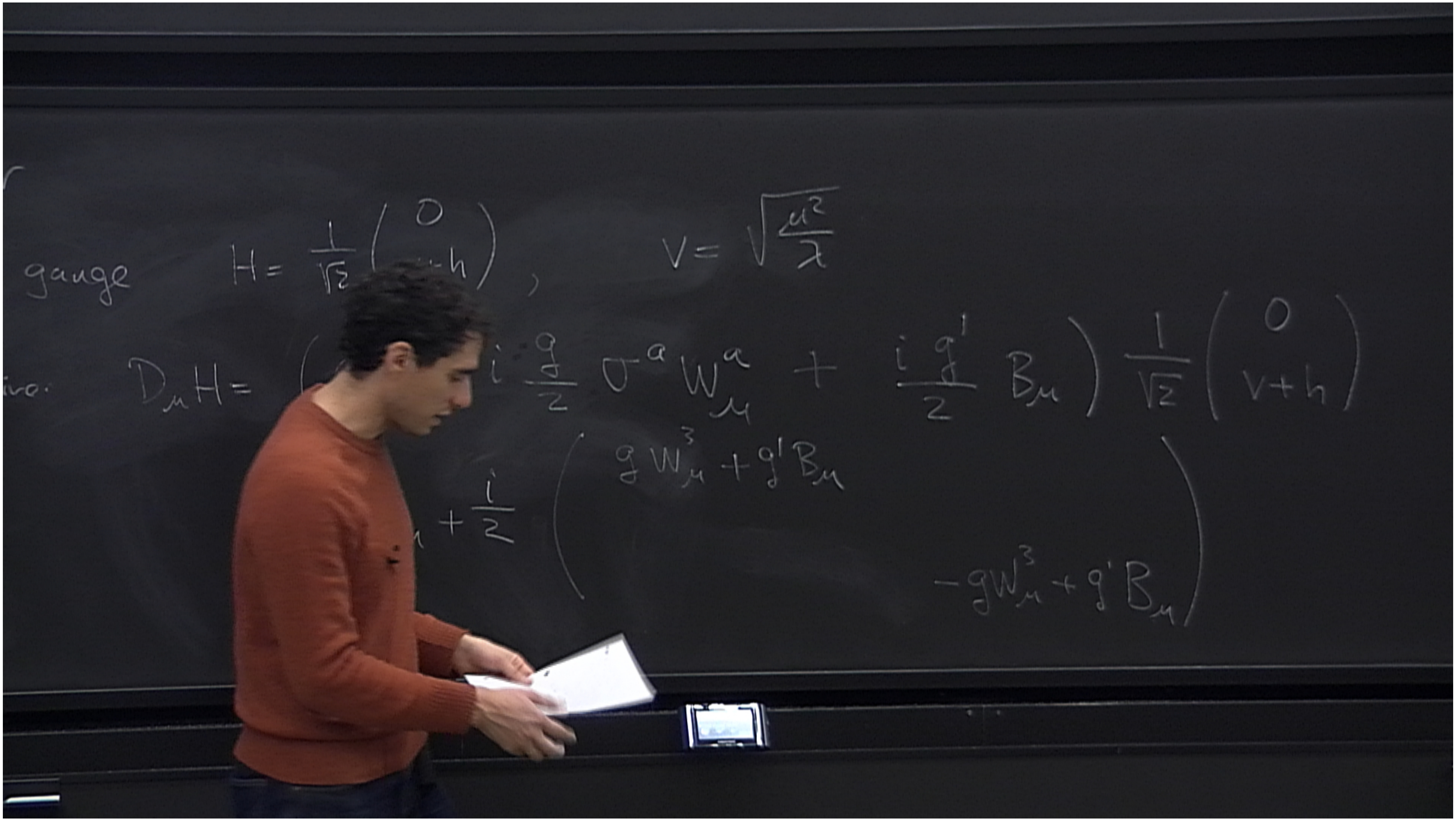
Consider $\mathcal{L}_{scalar}$

Work in unitary gauge $H = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}$, $v = \sqrt{\frac{\mu^2}{\lambda}}$

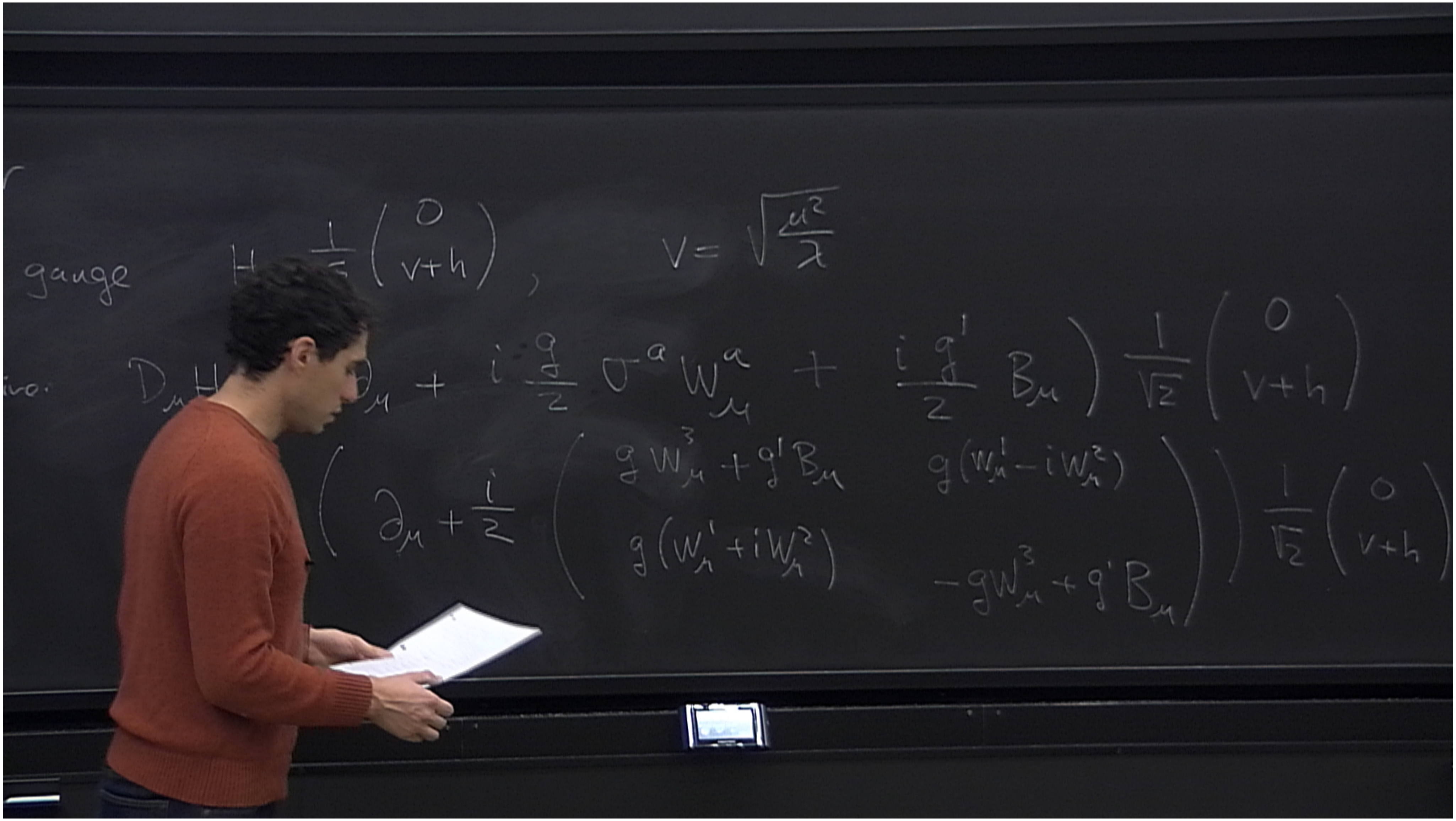
Covariant derivative. $D_\mu H = \left(\partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + \frac{i g'}{2} B_\mu \right)$

$$v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$W_\mu^a + \frac{i g'}{2} B_\mu \Big) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$



gauge
$H = \frac{1}{\sqrt{2}}$
$v = \sqrt{\frac{\mu^2}{\lambda}}$
$D_\mu H =$
$i\frac{g}{2}\sigma^a W^a_\mu + \frac{i g'}{2} B_\mu$
$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$
$+\frac{i}{2}$
$g W^3_\mu + g' B_\mu$
$-g W^3_\mu + g' B_\mu$


gauge
v+h
v = √(μ²/λ)
i g/2 σ^a W^a_μ + i g'/2 B_μ) 1/√2 (0, v+h)
(∂_μ + i/2 (g W³_μ + g' B_μ   g(W¹_μ − i W²_μ)
g(W¹_μ + i W²_μ)   −g W³_μ + g' B_μ)) 1/√2 (0, v+h)

gauge $\quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}, \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$

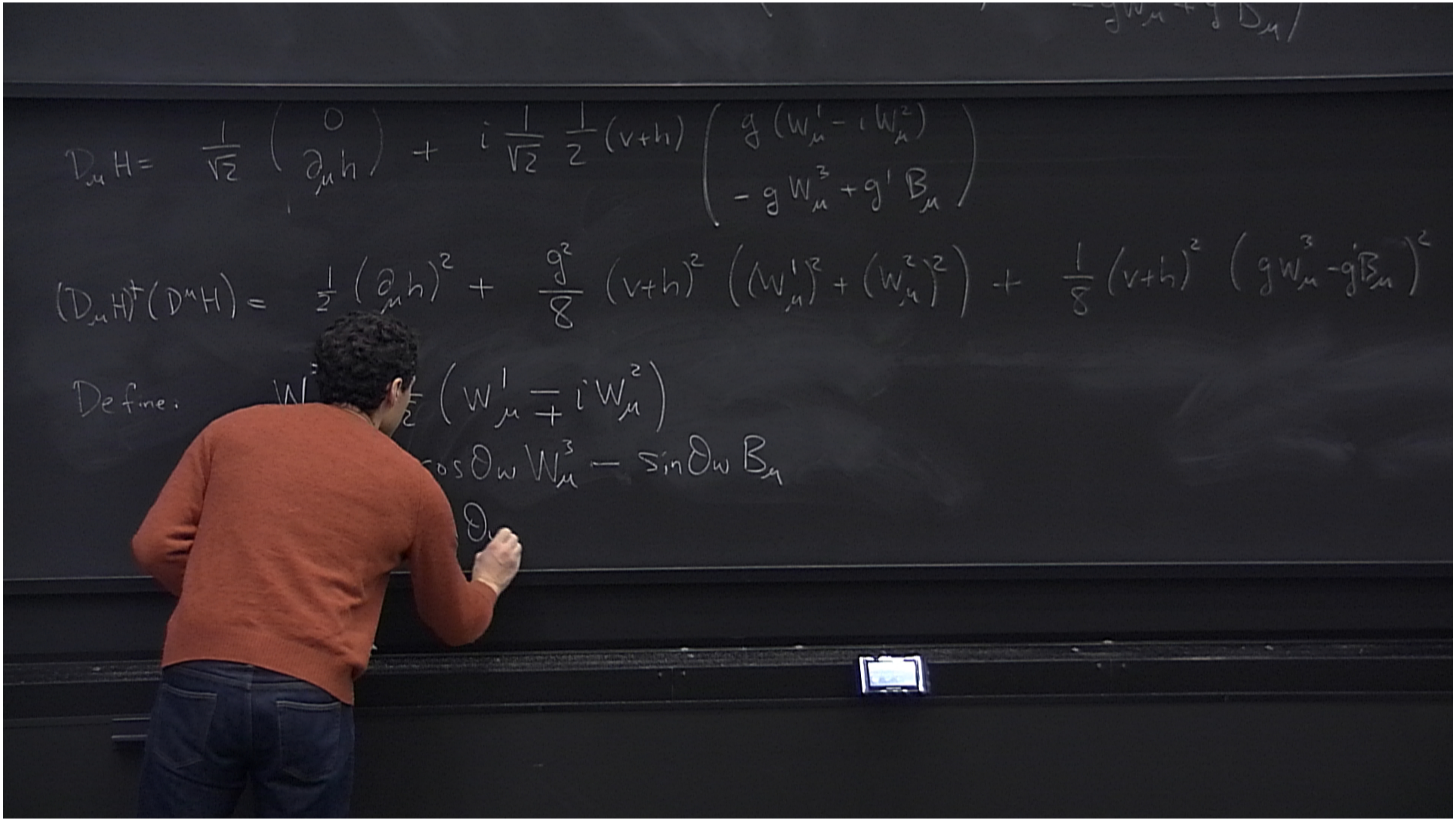
ivo. $\quad D_\mu H = \left(\partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + i\frac{g'}{2} B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$= \left(\partial_\mu + \frac{i}{2} \begin{pmatrix} gW^3_\mu + g'B_\mu & g(W^1_\mu - iW^2_\mu) \\ g(W^1_\mu + iW^2_\mu) & -gW^3_\mu + g'B_\mu \end{pmatrix} \right) \frac{1}{\sqrt{2}} \Big(v$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + i \frac{1}{\sqrt{2}} \frac{1}{2} (v+h) \begin{pmatrix} g\,(W_\mu^1 - i W_\mu^2) \\ -g W_\mu^3 + g' B_\mu \end{pmatrix}$$

$$D_\mu H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + i\frac{1}{\sqrt{2}}\frac{1}{2}(v+h)\begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ -gW_\mu^3 + g'B_\mu \end{pmatrix}$$

$$(D_\mu H)^\dagger (D^\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2\left((W_\mu^1)^2 + (W_\mu^2)^2\right) + \frac{1}{8}(v+h)^2\left(gW_\mu^3 - g'B_\mu\right)^2$$



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$(D_\mu H)^\dagger (D^\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2\left((W^1_\mu)^2 + (W^2_\mu)^2\right) + \frac{1}{8}(v+h)^2\left(gW^3_\mu - g' B_\mu\right)^2$
Define:
$(W^1_\mu \mp i W^2_\mu)$
$\cos\theta_W W^3_\mu - \sin\theta_W B_\mu$

$$D_\mu H = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \partial_\mu h\end{pmatrix} + i\frac{1}{\sqrt{2}}\frac{1}{2}(v+h)\begin{pmatrix} g(W^1_\mu - iW^2_\mu) \\ -gW^3_\mu + g' B_\mu\end{pmatrix}$$

$$(D_\mu H)^\dagger (D^\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2\left((W^1_\mu)^2+(W^2_\mu)^2\right) + \frac{1}{8}(v+h)^2\left(gW^3_\mu - g'B_\mu\right)^2$$

Define:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right)$$

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$

$$- i W_\mu^2)$$

$$+ g' B_\mu)$$

$$+ (W_\mu^2)^2) + \frac{1}{8} (v+h)^2 \left(g W_\mu^3 - g' B_\mu \right)^2$$

$$\tan\theta_W = g'/g$$

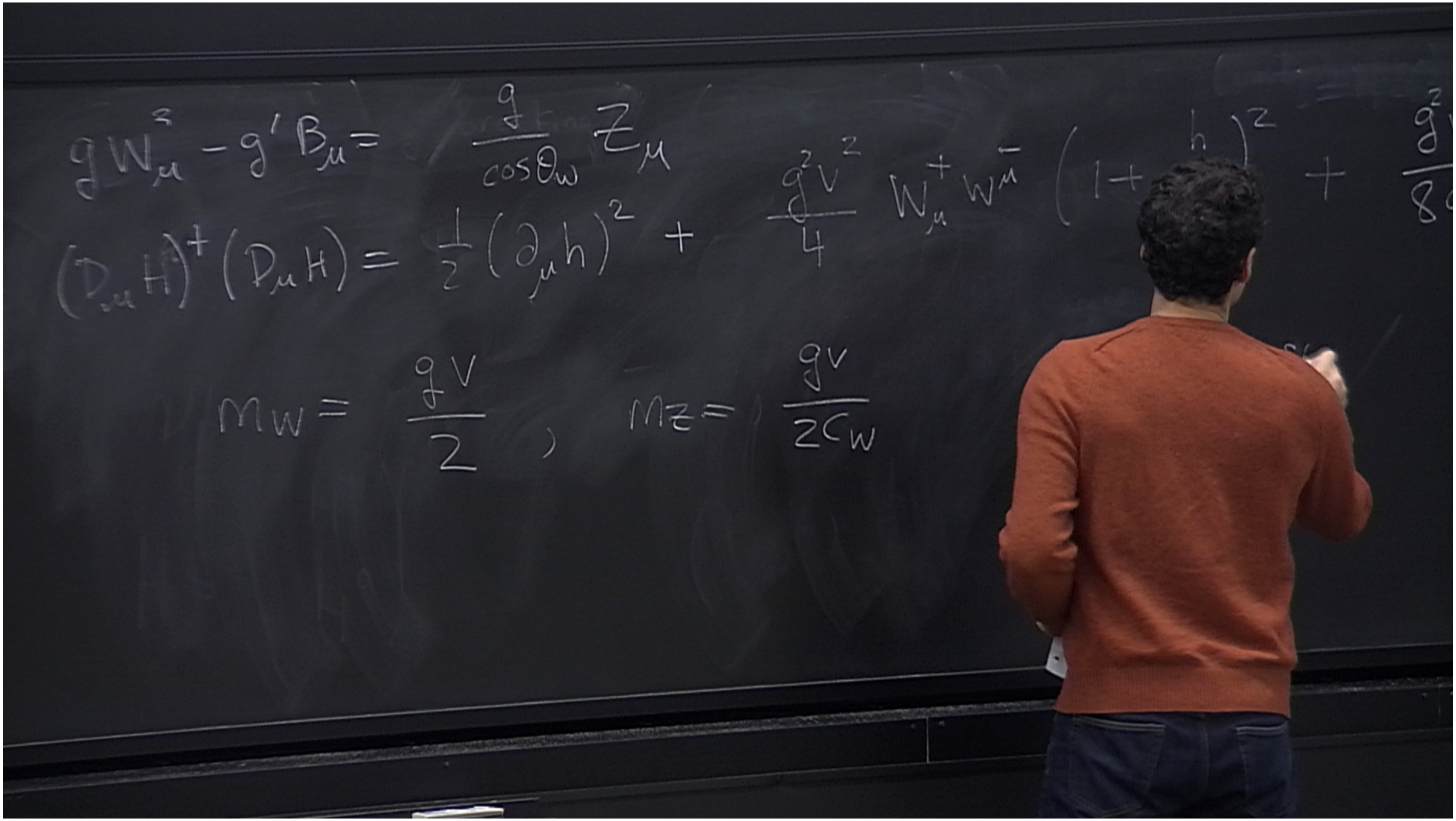
$$g' = g \tan\theta_W = g \frac{\sin\theta_W}{\cos\theta_W}$$

$$gW^3_\mu - g' B_\mu = \frac{g}{\cos\theta_W} Z_\mu$$

$$g W^3_\mu - g' B_\mu = \frac{g}{\cos\theta_W} Z_\mu$$

$$(D_\mu H)^\dagger (D_\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} \left(1 + \frac{h}{v}\right)^2 + \frac{g^2}{8}$$

$$1+\frac{h}{v}\Big)^2 + \frac{g^2 v^2}{8 c_W^2} Z_\mu Z^\mu \left(1+\frac{h}{v}\right)^2$$



$g W^3_\mu - g' B_\mu = \frac{g}{\cos\theta_W} Z_\mu$
$(D_\mu H)^\dagger (D_\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} \left(1 + \quad h\right)^2 +$
$m_W = \frac{gv}{2}$, $m_Z = \frac{gv}{2c_W}$

$$g W^3_\mu - g' B_\mu = \frac{g}{\cos\theta_w} Z_\mu$$

$$(D_\mu H)^\dagger (D_\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} \left(1 + \frac{h}{v}\right)^2 + \frac{g^2}{8} \ldots$$

$$m_W = \frac{g v}{2} \, , \qquad m_Z = \frac{g v}{2 c_W}$$

$$c_W = \cos\theta_w$$
$$s_W = \sin\theta_w$$

$$m_W = \frac{gv}{2}, \quad m_Z = \frac{gv}{2c_W}$$

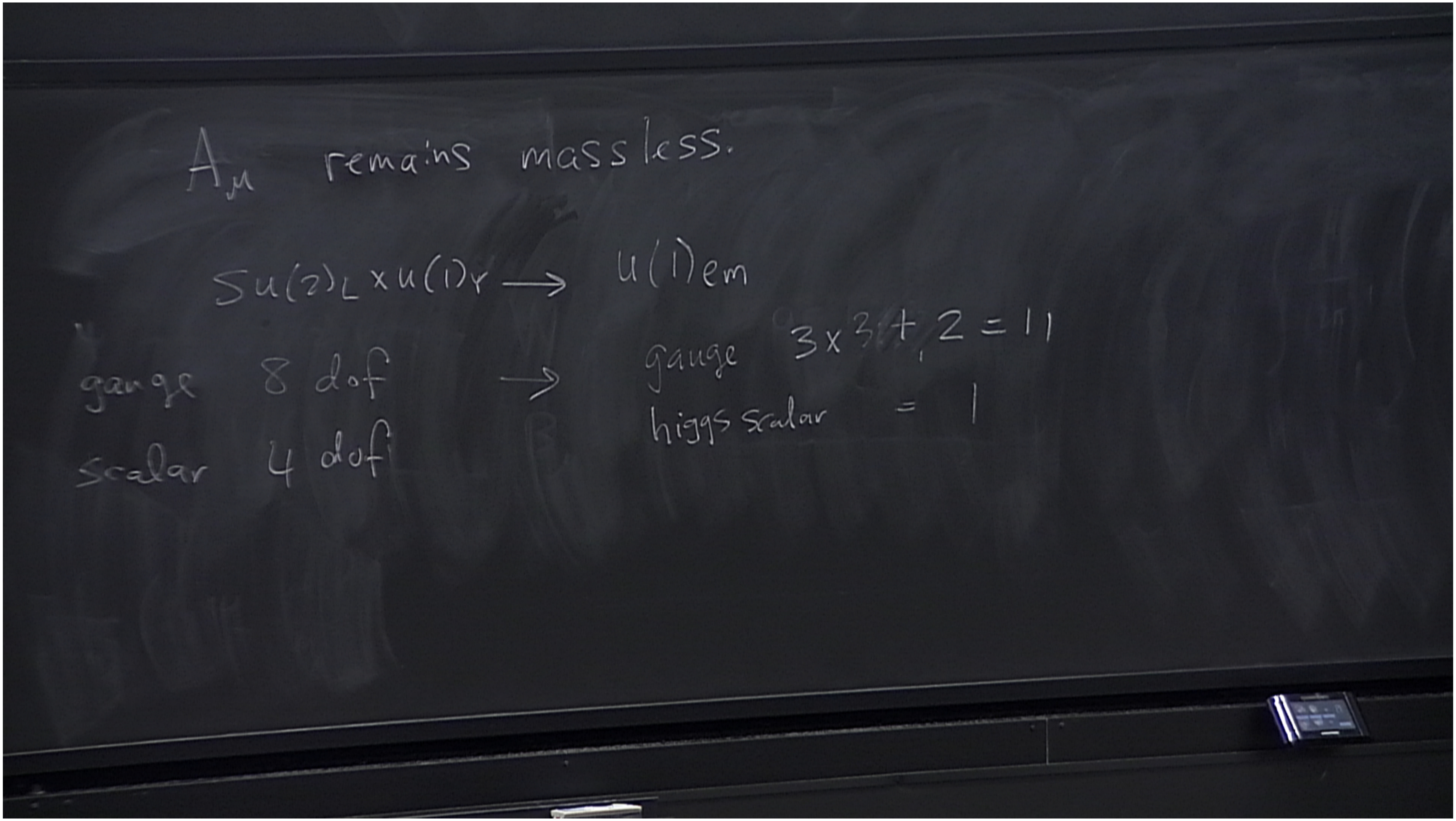
$$c_W = \cos\theta_W$$
$$s_W = \sin\theta_W$$

Define $\rho = \frac{m_W^2}{m_Z^2 c_W^2} = 1$ (at tree-level)

$$m_W = \frac{gV}{2}, \qquad m_Z = \frac{gv}{2c_W}$$

$$c_W = \cos\theta_W$$
$$s_W = \sin\theta_W$$

Define $\rho = \dfrac{m_W^2}{m_Z^2 c_W^2} = 1$ (at tree-level)


A_μ remains massless.
$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$
gauge 8 dof $\longrightarrow$ gauge $3 \times 3 + 2 = 11$
scalar 4 dof higgs scalar $= 1$