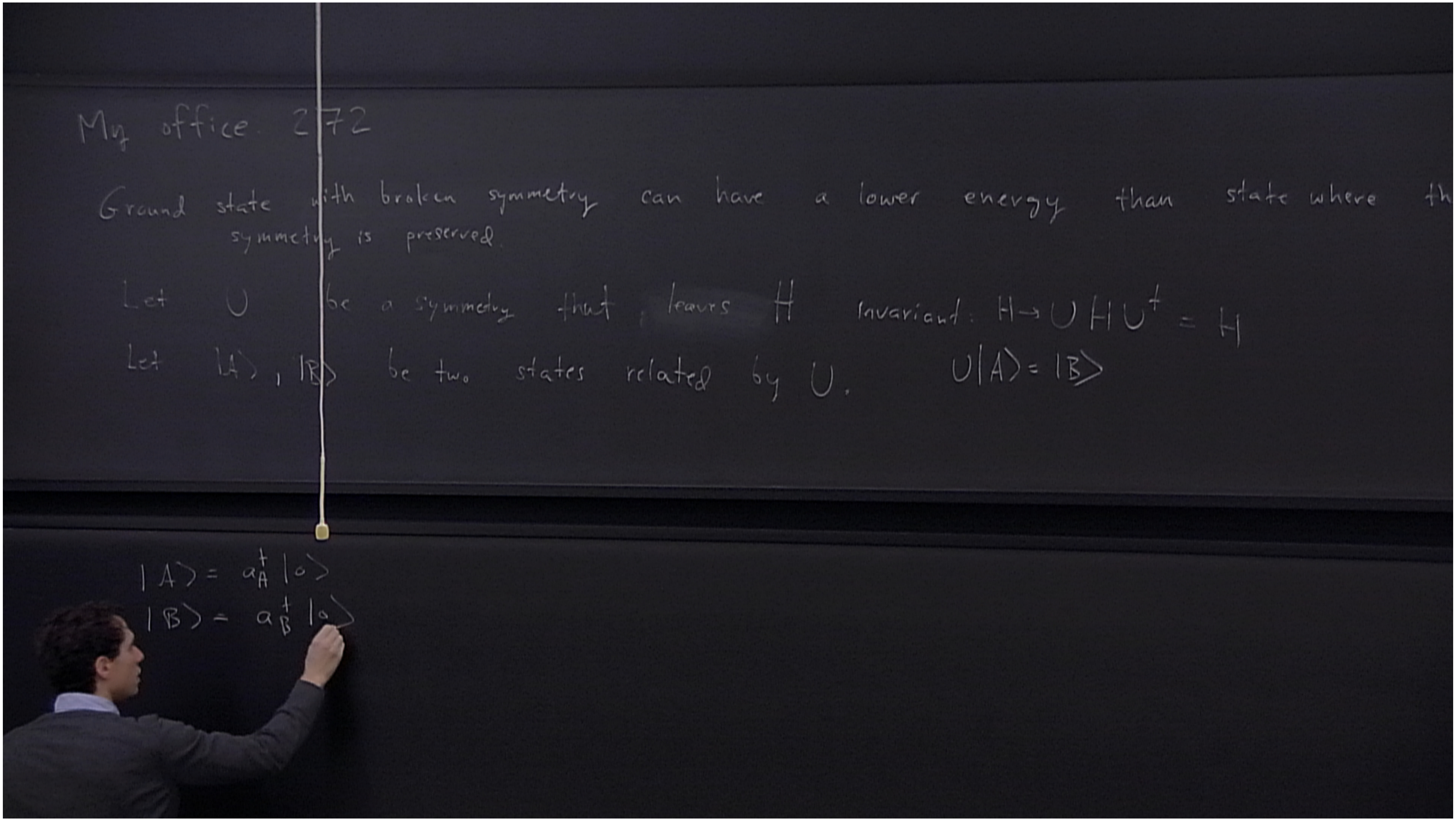
Title: PSI 2016/2017 Standard Model (Review) - Lecture 2

Date: Jan 04, 2017 09:00 AM

URL: http://pirsa.org/17010006

Abstract:

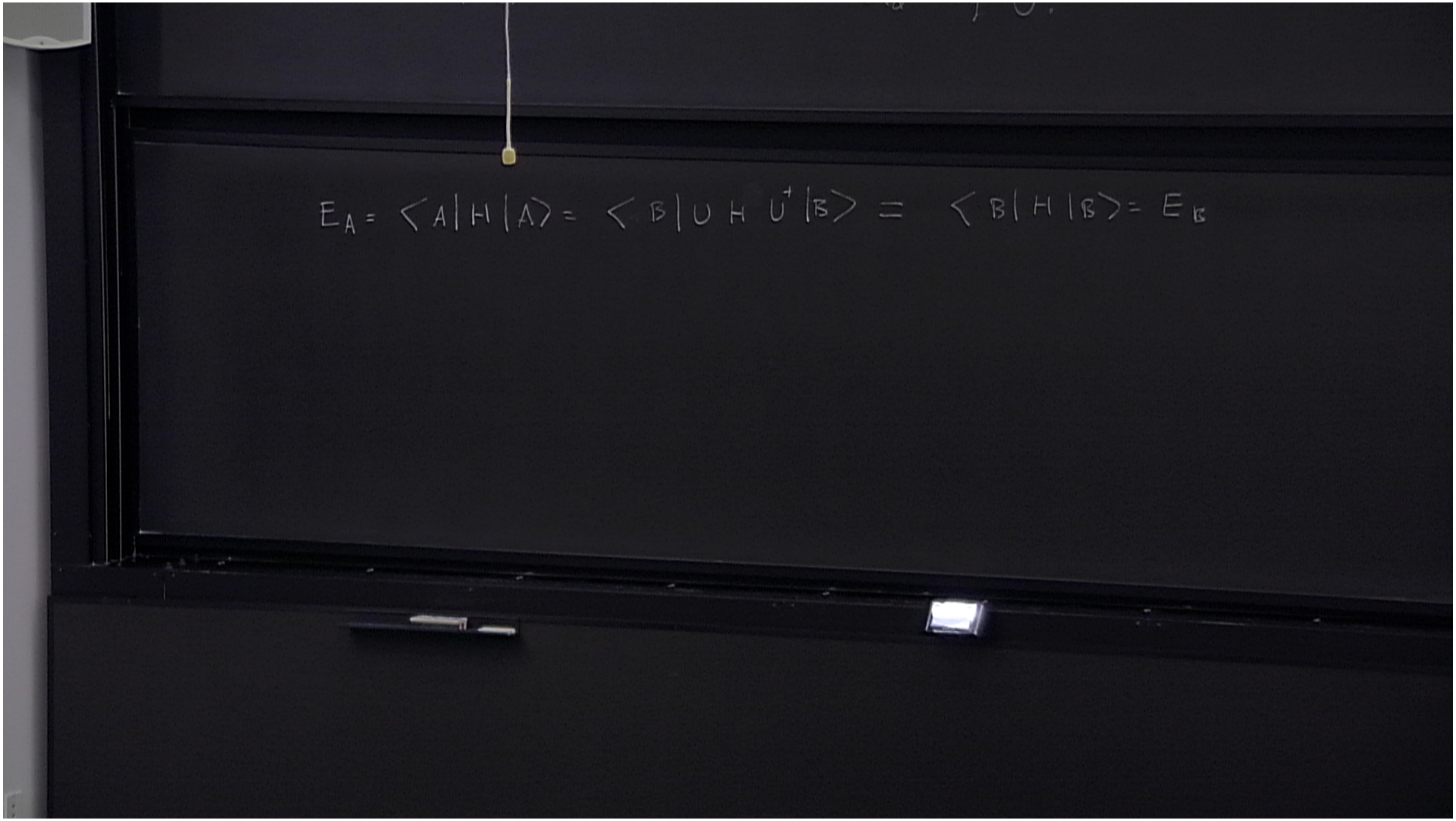

My office: 2 72
Ground state with broken symmetry can have a lower energy than state where th
symmetry is preserved.
Let U be a symmetry that leaves H invariant: H → U H U† = H
Let |A⟩, |B⟩ be two states related by U.
U|A⟩ = |B⟩
|A⟩ = a†A |0⟩
|B⟩ = a†B |0⟩

$$|A\rangle = a_A^\dagger |0\rangle$$

$$|B\rangle = a_B^\dagger |0\rangle$$

Must have: $U a_A^\dagger U^\dagger = a_B^\dagger$

$$U|A\rangle = U a_A^\dagger |0\rangle = \underbrace{U a_A^\dagger U^\dagger}_{a_B^\dagger} \underbrace{U|0\rangle}_{=|0\rangle} = |B\rangle$$


$E_A = \langle A | H | A \rangle = \langle B | U H U^{\dagger} | B \rangle = \langle B | H | B \rangle = E_B$

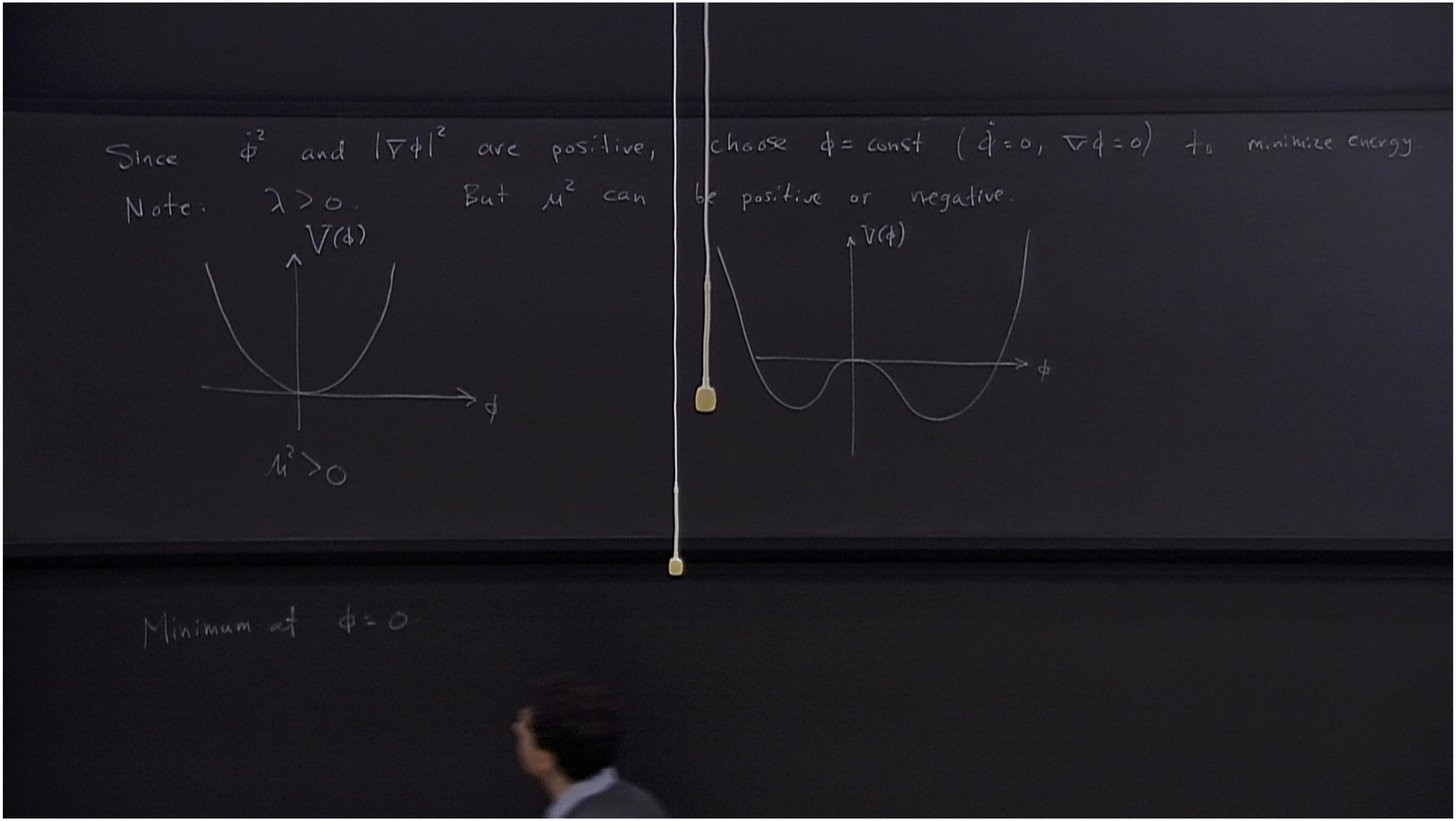
Discrete symmetry

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4$$

Assume Z_2 symmetry: $\phi \to -\phi$, $\mathcal{L}$ is invariant.

Hamiltonian: $H = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + V(\phi)$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$


Since $\dot{\phi}^2$ and $|\nabla\phi|^2$ are positive, choose $\phi = \text{const}$ ($\dot{\phi} = 0$, $\nabla\phi = 0$) to minimize energy.
Note: $\lambda > 0$. But μ^2 can be positive or negative.
$V(\phi)$
ϕ
$\mu^2 > 0$
$V(\phi)$
ϕ
Minimum at $\phi = 0$.

Minimum at $\phi = 0$.

$\langle 0 | \phi | 0 \rangle = 0$

Minimum at $\phi \neq 0$.

$\langle 0 | \phi | 0 \rangle = v$

$$0 = \frac{\partial V}{\partial \phi} = \mu^2 \phi + \lambda \phi^3 \Big|_{\phi = v} \implies v = \pm \sqrt{\frac{-\mu^2}{\lambda}}$$

v = vacuum expectation value (of ϕ)

Define a shifted field $\phi = v + \phi'$ to expand about minimum in broken ca

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi')^2 - \frac{1}{2}\mu^2 (v+\phi')^2 - \frac{1}{4}\lambda (v+\phi')^4$$

$$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\,\phi'^2 - \lambda v\,\phi'^3 - \frac{\lambda}{4}\,\phi'^4$$

No Z_2 symmetry for ϕ'. Mass of ϕ' is $m^2_{\phi'} = -2\mu^2 = 2\lambda v^2$

Define a shifted field $\phi = v + \phi'$ to expand about minimum in broken case. (take $v>0$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi')^2 - \frac{1}{2}\mu^2 (v+\phi')^2 - \frac{1}{4}\lambda (v+\phi')^4$$

$$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

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Original symmetry is not manifest, but reflected in fact that 3 constants are related in terms of two constants.

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Abelian (global) symmetry

complex scalar ϕ with $U(1)$ symmetry $\quad \phi \to e^{i\alpha}\phi$

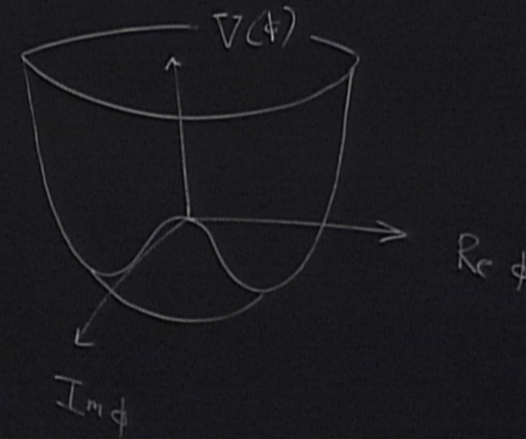
$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

Assuming $\lambda > 0$, $\mu^2 > 0 \rightarrow$ spontaneous symmetry breaking.

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$0 = \frac{\partial V}{\partial \phi} = (-\mu^2 + \lambda \phi^\dagger \phi)\, \phi^\dagger$$

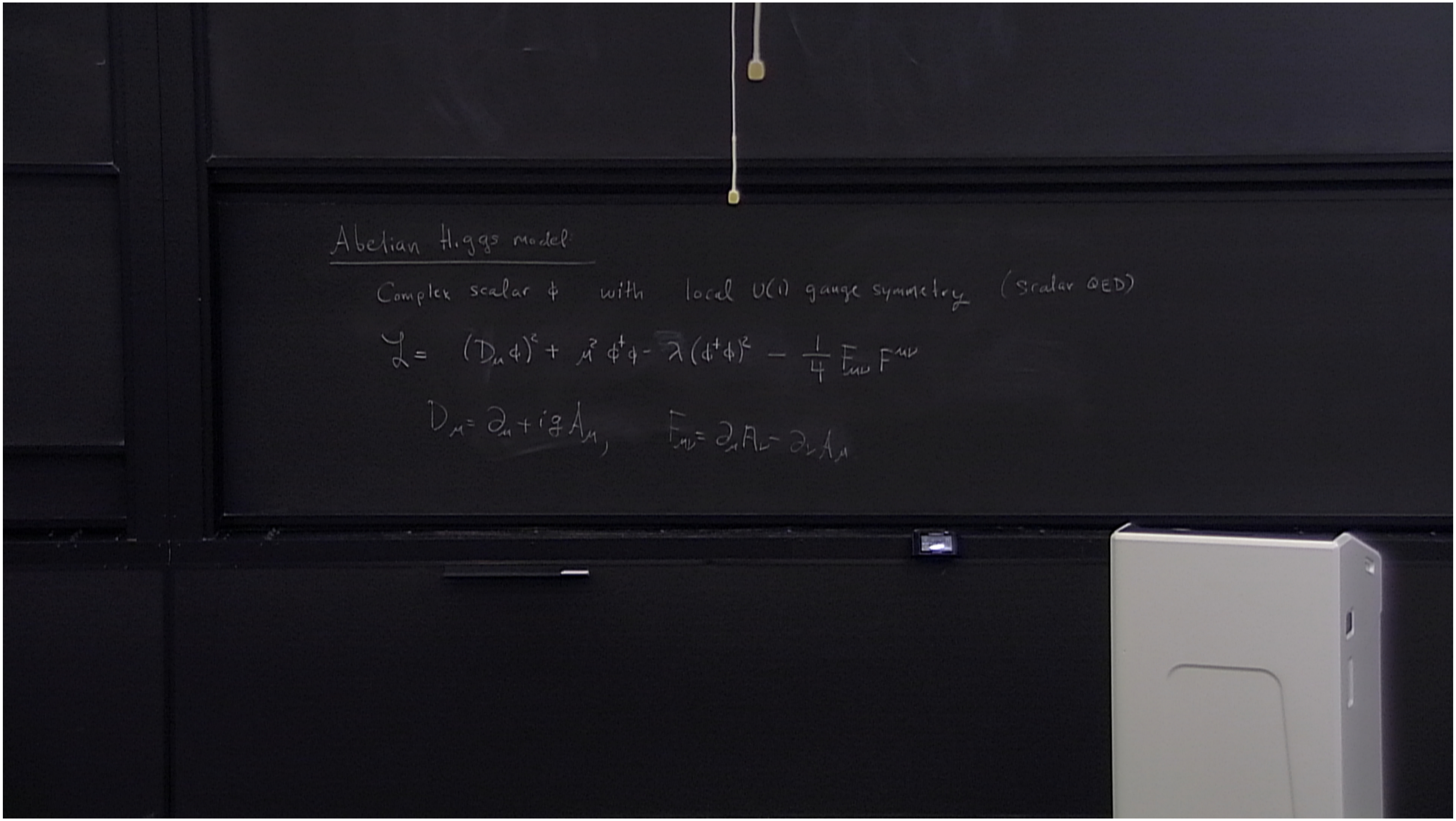
$$|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$$



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1')^2 + \frac{1}{2}(\partial_\mu \phi_2')^2 - \mu^2 \phi_1'^2 - \lambda v \phi_1' (\phi_1'^2 + \phi_2'^2) - \frac{\lambda}{4}(\phi_1'^2 + \phi_2'^2)^2$$

ϕ_1' has mass $m^2_{\phi_1'} = 2\mu^2$

ϕ_2' is massless


Abelian Higgs model
Complex scalar ϕ with local $U(1)$ gauge symmetry (scalar QED)
$\mathcal{L} = (D_\mu \phi)^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$D_\mu = \partial_\mu + i g A_\mu , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$, minimum at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$

Shifted fields $\phi_1 = \phi_1' + v$, $\phi_2 = \phi_2'$

New part is covariant derivative.

$$\mathcal{L} \supset |D_\mu \phi|^2 = \left| (\partial_\mu + i g A_\mu) \left(\frac{v + \phi_1' + i \phi_2'}{\sqrt{2}} \right) \right|^2$$

Abelian (global) symmetr

Remove mixing term: $\sim A_\mu \partial^\mu \phi_2$!

Expand ϕ in polar coordinates $\phi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right) e^{i\xi(x)/v} \simeq \frac{1}{\sqrt{2}}\left(v + h(x) + i\,\xi(x)\right) + \dots$

Remove $\xi(x)$ by gauge transformation: $\phi(x) \to e^{-i\xi/v}\,\phi = \frac{1}{\sqrt{2}}\left(v + h(x)\right)$ Unitary gauge.

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial_\mu \xi$$

$$D_\mu \phi = \left(\partial_\mu + igA_\mu\right)\phi \to \frac{1}{\sqrt{2}}\left(\partial_\mu + igA'_\mu\right)(v+h)$$

$$\mathcal{L} = \frac{1}{2}\left|\left(\partial_\mu + i g A_\mu\right)(v+h)\right|^2 + \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}g^2 v^2 A_\mu A^\mu + g^2 v A_\mu A^\mu h + \frac{1}{2} g^2 h^2 A_\mu A^\mu$$

$$\underbrace{\left(\frac{\mu^2}{2} - \frac{3}{2}\lambda v^2\right)}_{-\mu^2} h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4 - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$v^2 = \frac{\mu^2}{\lambda}$$

$$\mathcal{L}_{int} = g^2 v A_\mu A^\mu h + \frac{1}{2} g^2 h^2 A_\mu A^\mu - \lambda v h^3 - \frac{\lambda}{4} h^4 \quad \Big\} \text{ interactions fixed in terms of } m_h, m_A, \lambda$$

Would-be massless goldstone boson ξ gets "eaten" by A_μ

2 scalar d.o.f. + 2 photon d.o.f. $\longrightarrow$ 1 scalar d.o.f. + 3 d.o.f. for A_μ

ϕ_1, ϕ_2 $\qquad\qquad\qquad\qquad\qquad\qquad h$