Title: Breaking inflation with inhomogeneous initial conditions

Date: Dec 13, 2016 11:00 AM

URL: http://pirsa.org/16120032

Abstract: <p>Inflation is proposed as a means of explaining why the Universe is currently so homogeneous on larger scales, solving both the horizon and flatness problems in early universe cosmology. However, if inflation itself requires homogeneous conditions to get started, then inflation is not a solution to the horizon problem. Most work up until now has focussed on a dynamical systems approach to classifying the stability of inflationary models, but recently Numerical Relativity (NR) has been used to simulate the actual evolution of the inflaton field, leading to new insights. I will describe a recent work (https://arxiv.org/abs/1608.04408) in which we used NR to consider the robustness of generic small and large field inflationary models to initial inhomogeneities in the inflaton field and the extrinsic curvature of the metric.</p>

Inflation

Why does the universe look the same in all directions?

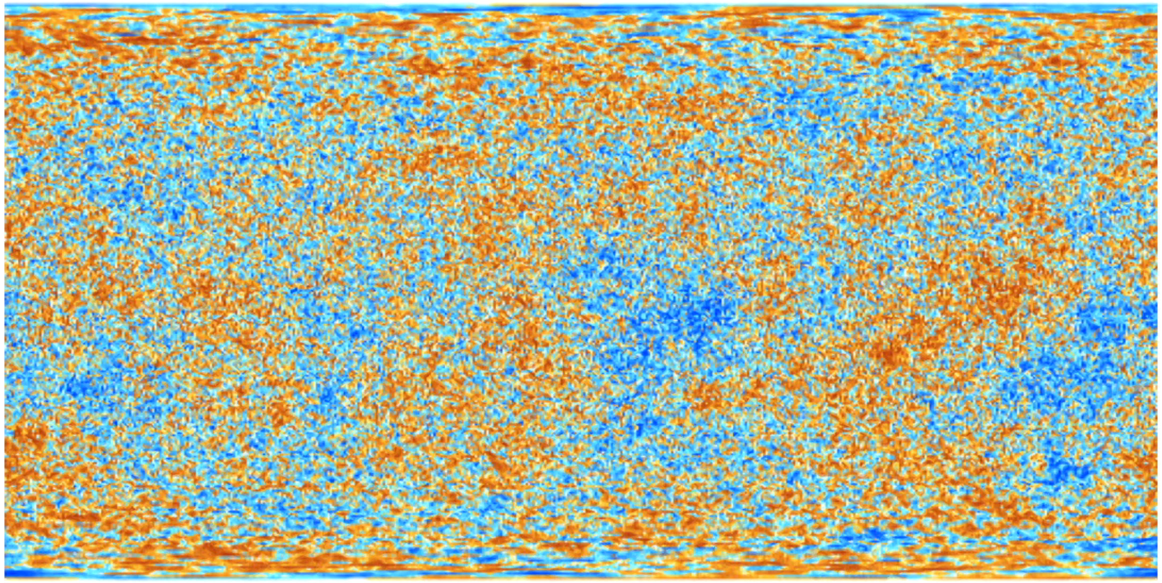


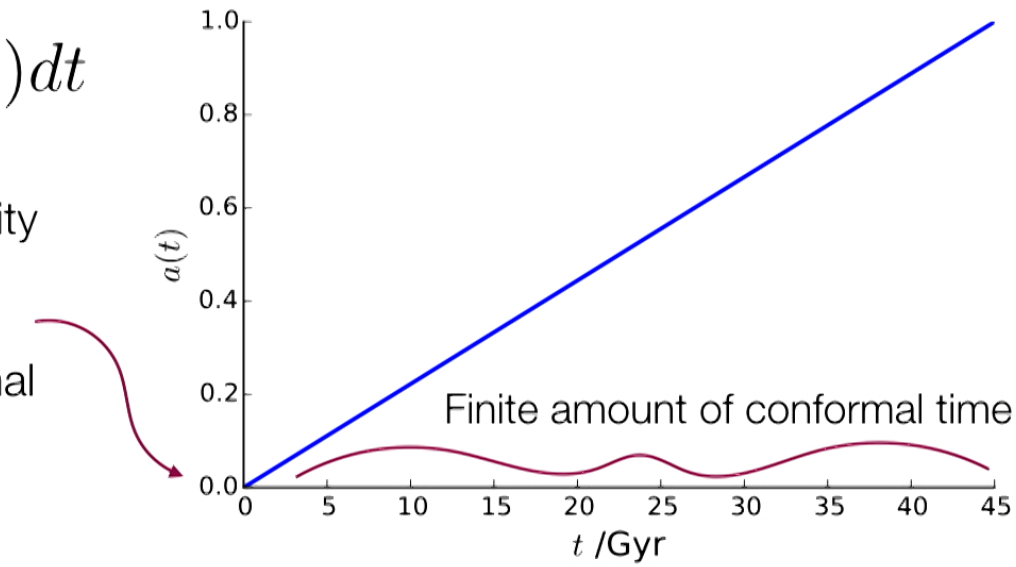
Image ESA/Planck

Horizon problem - no time for causal contact

Using Einstein equation for known energy sources - work out the historic evolution of the scale factor

$$d\tau = a(t)dt$$

Singularity at zero conformal time

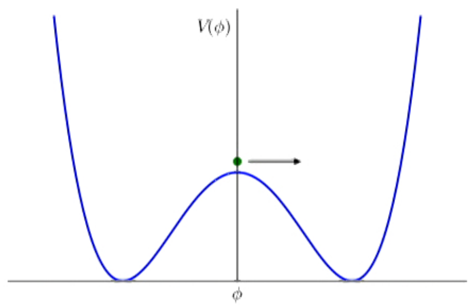


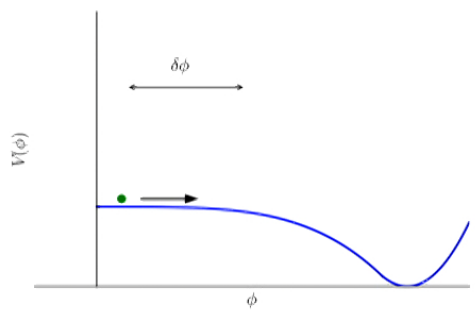
Scalar fields and slow roll

- Scalar field with canonical kinetic term, non trivial potential

$$L_\phi = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi)$$

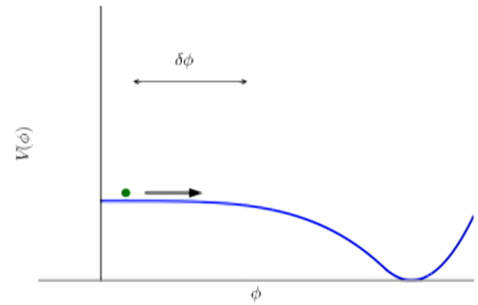
- Originally Higgs type potential was proposed, later "flattened" regions to achieve **slow roll**

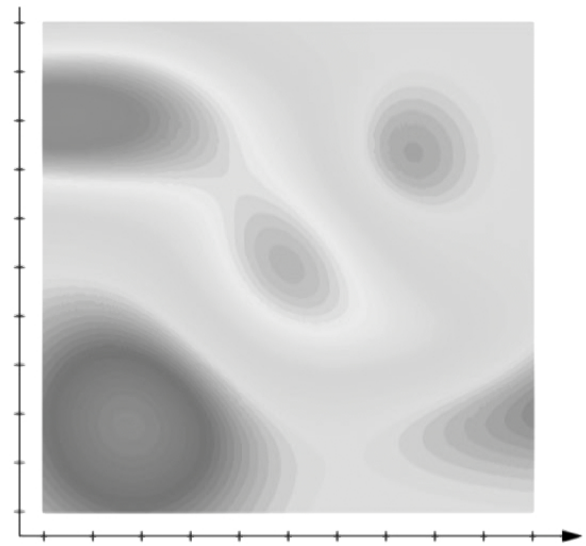




The initial condition problem for inflation

- Slow roll calculations assume (approx) homogeneity
- In scalar field
- In the metric
- Why bother with inflation if things are already homogeneous?





Large field (versus Small field) inflationary models

- Large field $\delta\phi \gg M_{pl}$
  - High energy scale inflation
  - Slow roll is (local) attractor in phase space

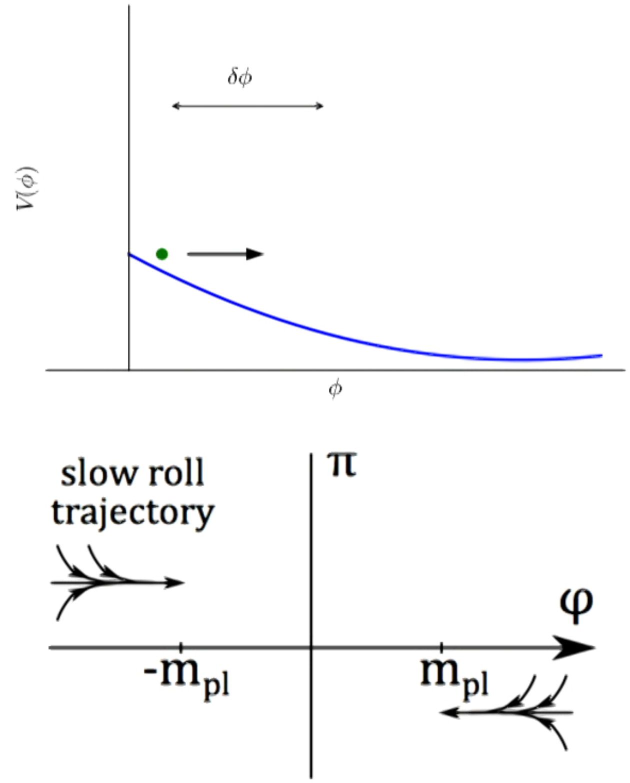
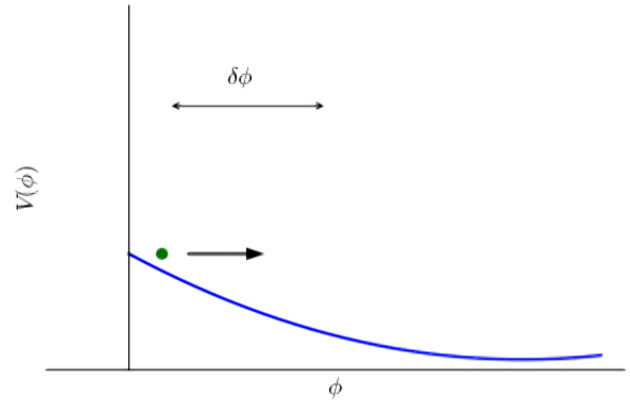
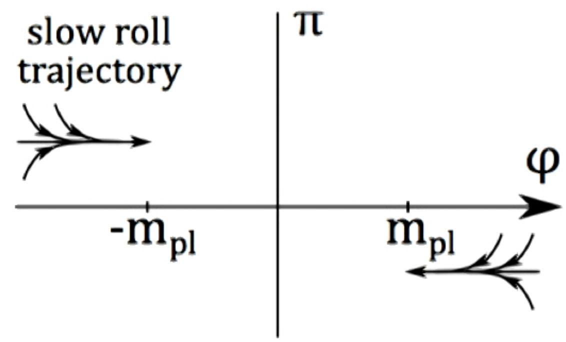


Diagram from:
Initial Conditions for Inflation: A Short Review
Robert Brandenberger, arXiv 1601.01918

Large field (versus Small field) inflationary models

- Large field $\delta\phi \gg M_{pl}$
  - High energy scale inflation
  - Slow roll is (local) attractor in phase space
  - But difficult to “fit” into an effective theory / quantum theory of gravity





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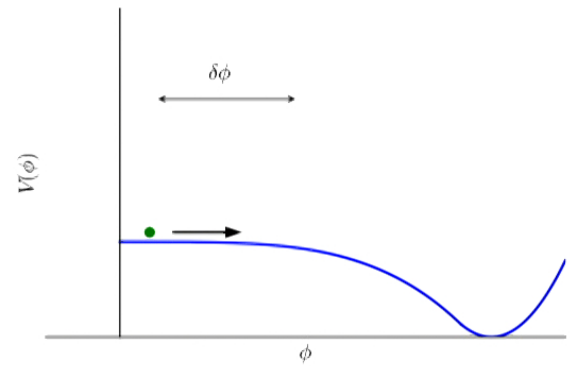
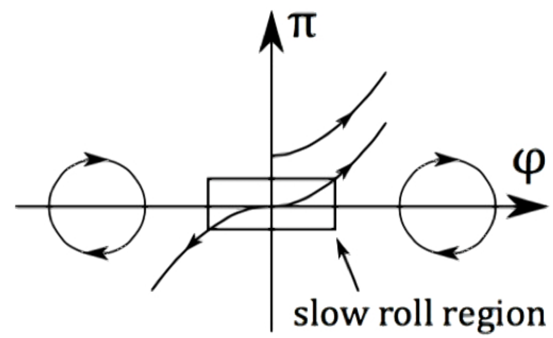
(Large field versus) Small field inflationary models

- Small field $\delta\phi \ll M_{pl}$
  - Low energy scale
  - Liked by string theorists
  - Not an attractor in field space





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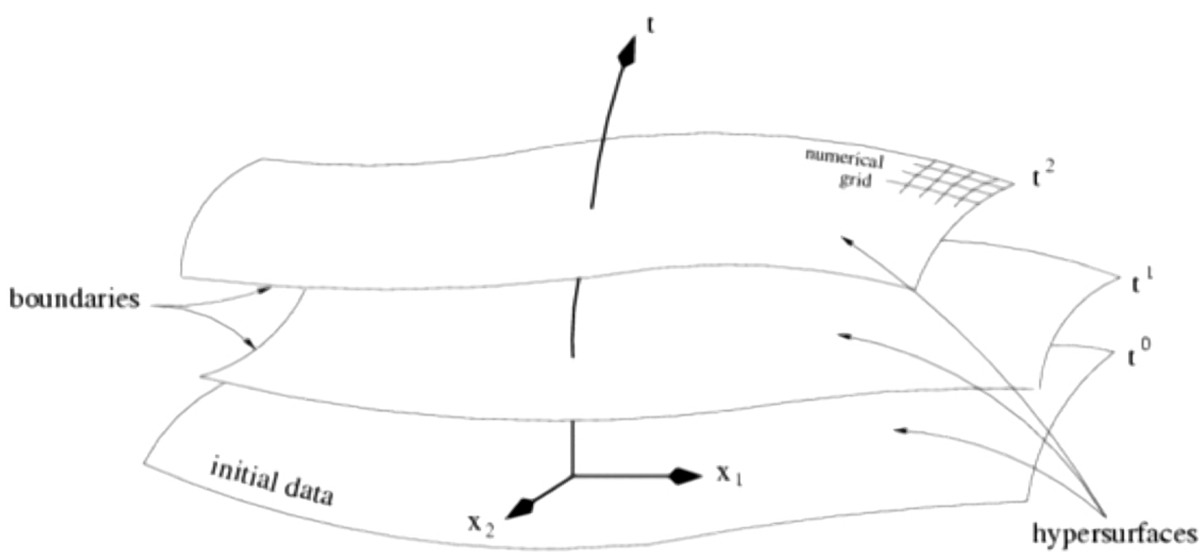
Numerical General Relativity

- Foliation of 4 dimensional spacetime into spatial hypersufaces evolved over a local time coordinate



Numerical General Relativity

- Solves the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

- Or more specifically the 3+1 ADM decomposition of the Einstein equation which evolves the quantities:
  - The spatial metric (field values)

    $$\gamma_{ij} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}$$

  - The extrinsic curvature (conjugate momenta of metric values)

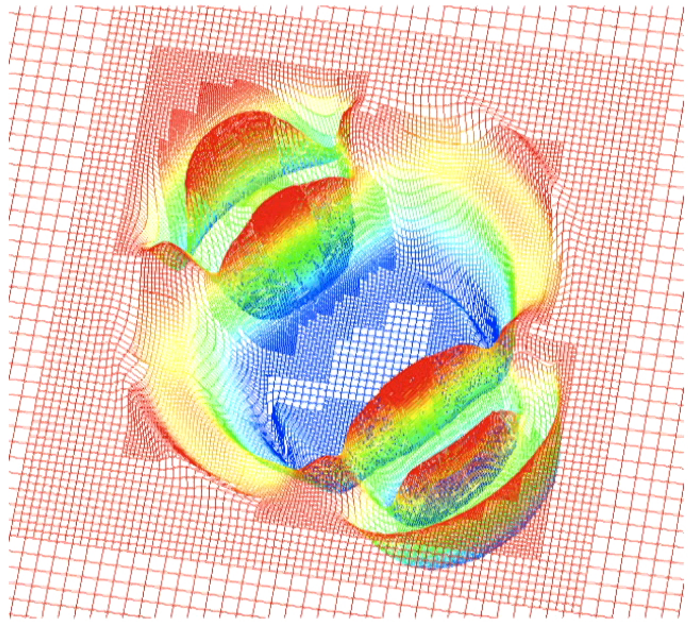
    $$K_{ij} = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix}$$

- Also evolve the energy momentum source: $\nabla_a \nabla^a \phi = \frac{dV}{d\phi}$

GRChombo - NR with adaptive mesh refinement

- Small emergent features well-resolved at all times
- More problem fits within a given memory footprint

www.grchombo.org



"GRChombo : Numerical Relativity with Adaptive Mesh Refinement"
Class.Quant.Grav. 32 (2015) 24, 245011
arXiv:1503.03436

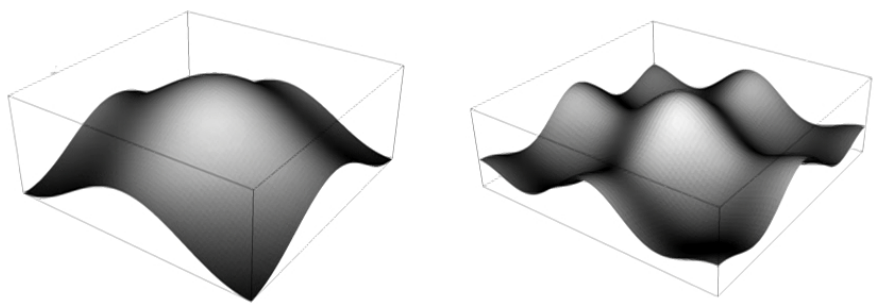
Numerical GR simulations of inflation

- Dalia S. Goldwirth and Tsvi Piran
  - Inhomogeneity and the onset of inflation
    Phys. Rev. Lett. 64, 2852
- P. Laguna, H. Kurki-Suonio, and R. A. Matzner
  - Inhomogeneous Inflation: Numerical Evolution
    arXiv:astro-ph/9306009
- William E. East, Matthew Kleban, Andrei Linde and Leonardo Senatore
  - Beginning Inflation in an inhomogeneous universe
    arXiv:1511.05143


Robustness of Inflation to Inhomogeneous Initial Conditions
Katy Clough, Eugene A. Lim, Brandon S. DiNunno, Willy Fischler, Raphael Flauger, Sonia Paban
arXiv:1608.04408

Inhomogeneous inflation

- Investigate dynamically effects of perturbations on different models
- General initial conditions may have metric as well as matter fluctuations



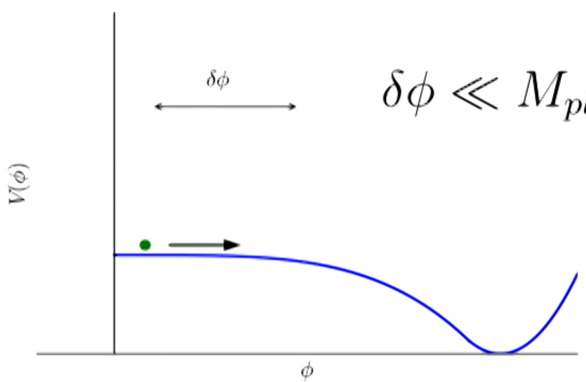
K Clough, EA Lim,
B DiNunno, W Fischler, R Flauger, and S Paban
Robustness of Inflation to Inhomogeneous Initial Conditions
arXiv 1608.04408

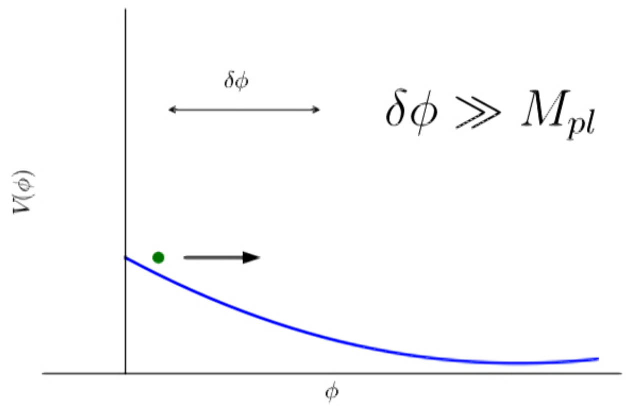
Scalar field inflaton

- Scalar field minimally coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - U(\phi) - \frac{R}{16\pi G} \right]$$

- Inflaton model -> Small and large field models





NR / Cosmology translation guide

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} dx^i dx^j \qquad ds^2 = a^2(t)(-dt^2 + dx^2 + dy^2 + dz^2)$$

$$K_{ij} = -\frac{\partial_t \gamma_{ij}}{2\alpha} \quad \tilde{\gamma}_{ij} = \chi^2 \gamma_{ij} \qquad H = \frac{\dot{a}}{a}$$

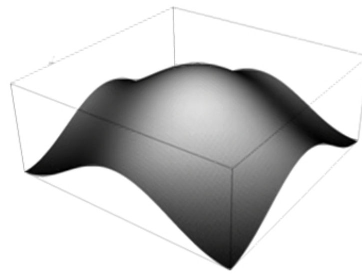
In a homogeneous universe, with conformal time t and zero shift

Conformally flat	***Isotropic***
Conformal factor χ	***Scale factor $a = 1/\chi$***
Trace of Extrinsic Curvature K	***Hubble constant $H = -K/3$***

Initial conditions - constant K

- Simple superposition of plane waves in 3 directions, periodic boundary conditions

$$\phi(t=0,\mathbf{x}) = \phi_0 + \frac{\Delta\phi}{N}\sum_{n=1}^{N}\left(\cos\frac{2\pi n x}{L} + \cos\frac{2\pi n y}{L} + \cos\frac{2\pi n z}{L}\right)$$



- BSSN formalism, dynamical moving puncture gauge
- Solve Momentum constraints and Hamiltonian constraint for initial spatial metric and extrinsic curvature

Initial conditions - constant K

- Conformally flat, transverse-traceless part of extrinsic curvature is zero, K set via average energy density

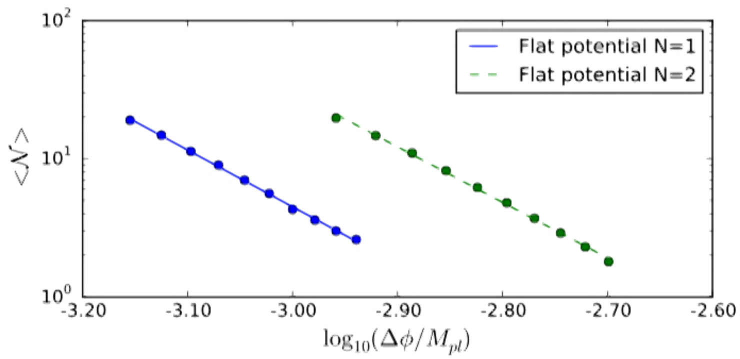
$$\tilde{\gamma}_{ij} = \delta_{ij} \qquad \tilde{A}_{ij} = 0 \qquad K = -\sqrt{24\pi G\langle\rho\rangle}$$

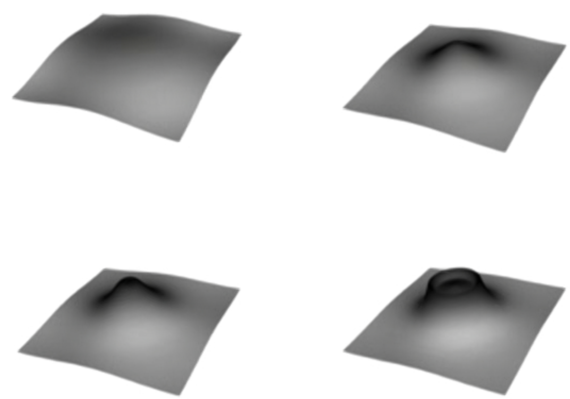
- Relax conformal factor χ to satisfy Hamiltonian constraint, periodic BCs

$$\chi \tilde{D}^2 \chi - \frac{3}{2}\tilde{\gamma}^{ij}\tilde{D}_i\chi\tilde{D}_j\chi + \frac{\chi^2\tilde{R}}{4} + \frac{K^2}{6} - \frac{1}{4\chi}\tilde{A}_{ij}\tilde{A}^{ij} = 4\pi G\rho$$

Small field inflation - not very robust

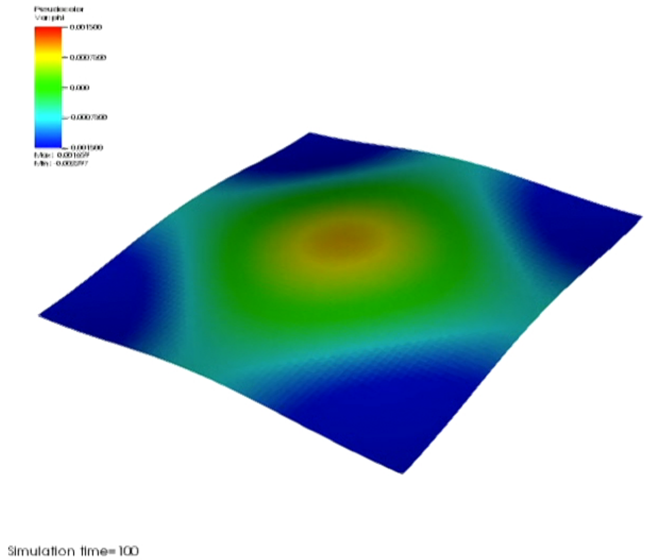
- Increase amplitude of inhomogeneities, decrease number of e-folds
- Failure mode - “falling off the hill”
- Fails even for subdominant gradient energies (but only one mode)

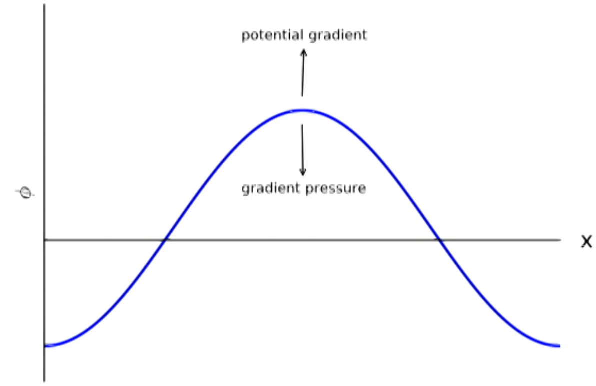


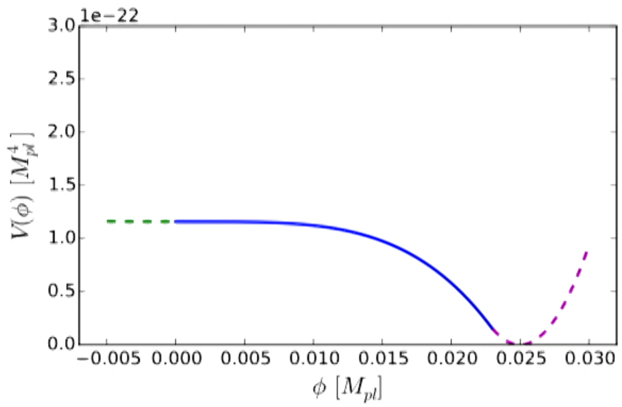


But a bit more robust than we expected

- Can be well understood by considering scalar field dynamics, without gravity







Small field - can predict “pull-back effect”

- Klein Gordon Eqn

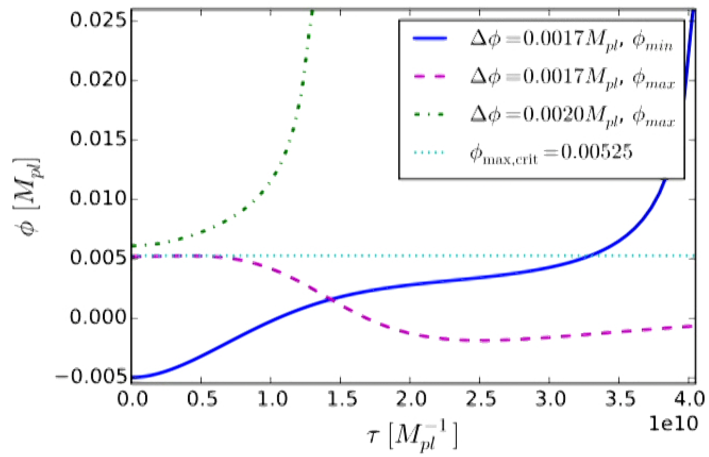
$$\partial_t^2\phi - \gamma^{ij}\partial_i\partial_j\phi + \frac{dV}{d\phi} = 0$$

- Inflation fails where

$$-\nabla^2\phi < \left|\frac{dV}{d\phi}\right|$$

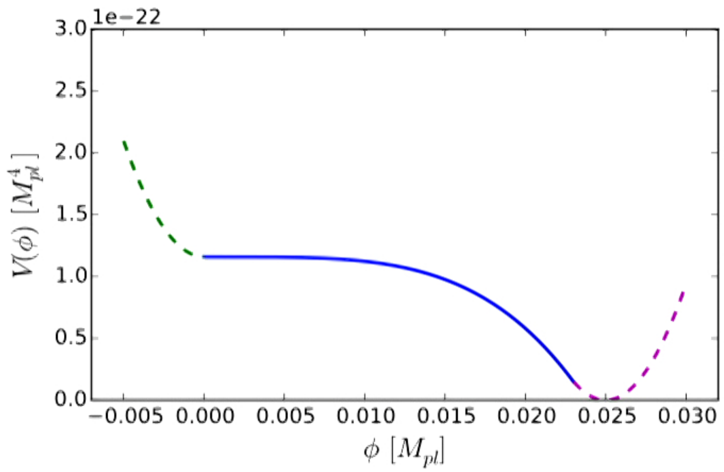
- In terms of our potential and initial conditions

$$\Delta\phi > \sqrt{\frac{8\pi^3}{27}\frac{n\mu^2}{M_{pl}}}$$



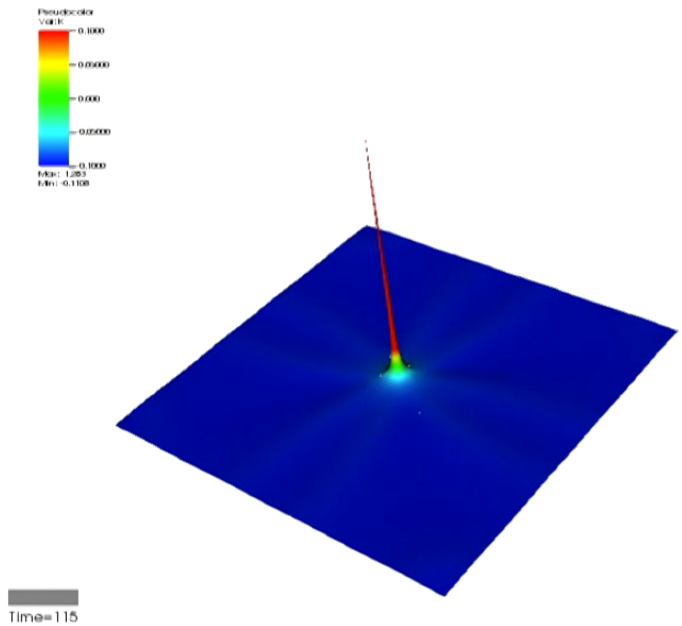
Small field - shape of potential important

- Convex vs concave
- ``Cliffs" resulted in additional KE and aided failure



Large field inflation - constant K - very robust

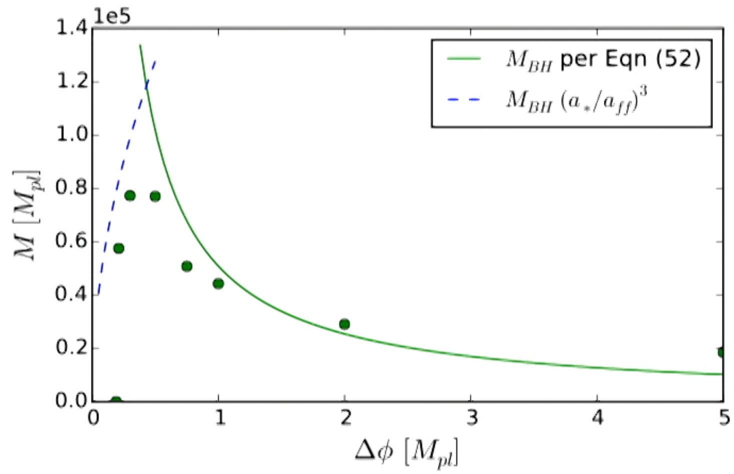
- Even very large inhomogeneities do not kill inflation
- BH forms but ultimately “inflated out”
- Confirms results of previous studies



East et. al.
Beginning inflation in an inhomogeneous universe
arXiv 1511.05143
Kurki-Sunio et. al.
Inhomogeneous inflation: Numerical evolution
astro-*ph/93.06009*
Laguna et. al.
Inhomogeneous inflation: The Initial value problem
PhysRevD.44.3077

Large field inflation - constant K - black holes

- Initial increase in BH mass with increase in amplitude of fluctuations
- Cannot make a "Giant Death Black Hole" which eats whole Hubble patch



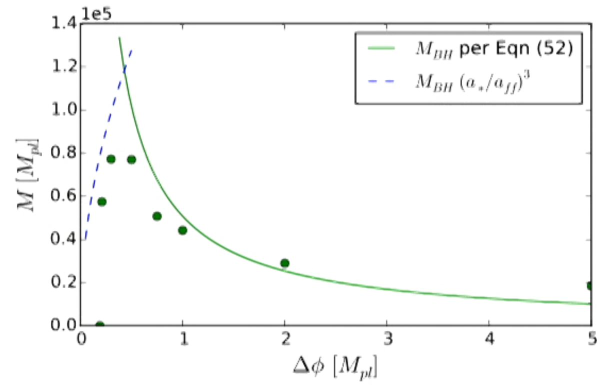
Large field inflation - constant K - black holes

- Two timescales
  - Free fall timescale a_ff

$$a_{ff} = \text{constant}$$

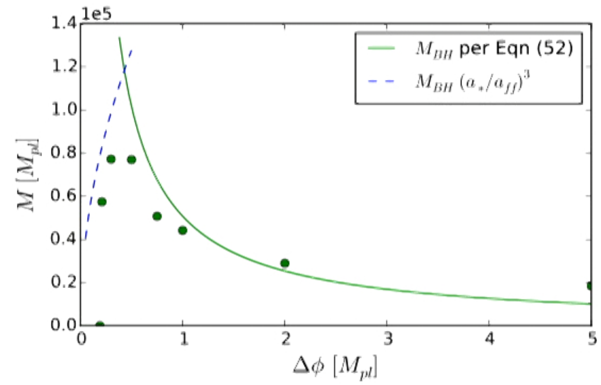
  - Transition to de Sitter

$$a^* \propto \Delta\phi$$



- a_ff > a* early transition to de Sitter results in smaller mass
- a_ff < a* accrete from within initial Hubble radius $\rho \propto \Delta\phi^2 \quad H^{-1} \propto \frac{1}{\Delta\phi}$

Large field inflation - constant K - black holes



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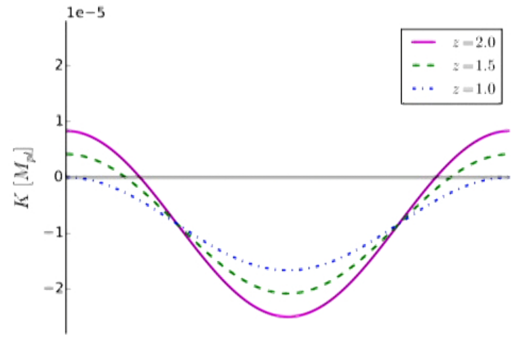
$$M_{\rm BH} \approx \frac{4\sqrt{3}}{(8\pi)^2}\left(\frac{M_{pl}}{\Delta\phi}\right)\left(\frac{M_{pl}^2}{\sqrt{V_0}}\right)M_{pl}$$

Initial conditions - Varying K

- A simple case of non constant K is achieved by setting the kinetic part of the energy density:

$$\eta = \frac{1}{\alpha}(\dot{\phi} - \beta^k \partial_k \phi)$$



to a constant: $\eta = -\frac{C}{12\pi G}$

- Then the momentum constraint:

$$\tilde{D}_j \tilde{A}^{ij} - \frac{3}{\chi}\tilde{A}^{ij}\tilde{D}_j \chi - \frac{2}{3}\tilde{\gamma}^{ij}\tilde{D}_j K = 8\pi G \eta \tilde{\gamma}^{ij}\partial_j \phi$$

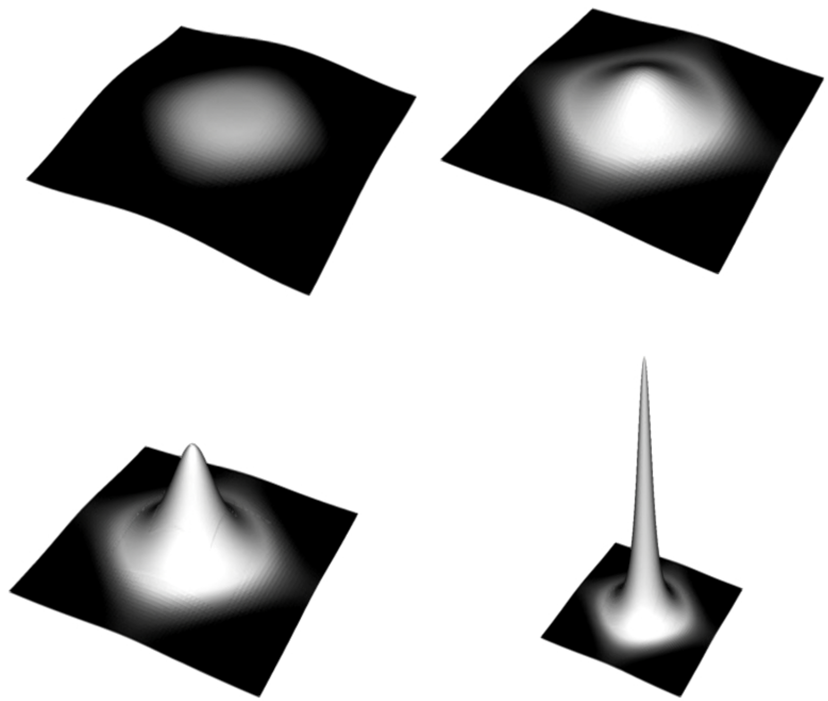
is solved by: $K = -C\phi + K_0$

Large field - varying extrinsic curvature

- BHs form more readily in initially collapsing regions

- But - some part of spacetime always remains inflationary if initially

$$\langle K \rangle < 0$$



Large field - varying extrinsic curvature

- Previous analytical studies

Barrow et. al.
Closed universe - Their future evolution and final state
mnras/216.2.395

Kleban et. al.
Inhomogeneous Anisotropic Cosmology
arXiv 1602.03520

- Toroidal topologies continue to expand if start with net overall expansion
  - Only certain topologies can have non negative ${}^{(3)}R$ (S^3 and $S^2 \times S^1$)
  - Barrow and Tippler showed that maximal hypersurfaces require positive ${}^{(3)}R$
  - For an initially expanding spacetime to recollapse, it must pass through a maximal hypersurface
  - Closed universes with toroidal topology are not in this category of topologies and therefore cannot recollapse

Key findings

- Small field not robust to subdominant gradient energies
- Large field robust, even to dominant gradient energies
- No “giant death black holes” if space approximately flat, with periodic boundary conditions
- Varying K does not kill inflation if expanding on average and space approximately flat, confirming previous analytical studies

K Clough, EA Lim,
B DiNunno, W Fischler, R Flauger, and S Paban
Robustness of Inflation to Inhomogeneous Initial Conditions
arXiv 1608.04408

Future work

- More general initial conditions - non conformally flat, traceless part of K_ij non zero
- Multiple modes
- Multi-field inflation
- Constrain initial inhomogeneities using observables on CMB