Title: Entanglement negativity in topologically ordered phases

Date: Dec 13, 2016 03:30 PM

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Abstract: <p>Unlike entanglement entropy and mutual information which may mix both classical and quantum correlations, entanglement negativity received extensive interest recently, for its merit of measuring the pure quantum entanglement in the system. In this talk, I will introduce the entanglement negativity in 2+1 dimensional topologically ordered phases. For a bipartitioned or tripartitioned spatial manifold, we show how the universal part of entanglement negativity depends on the presence of quasiparticles and the choice of ground states. Besides interpreting recent results in exactly solvable lattice models, we give new results on non-Abelian topologically ordered phases.</p>

Entanglement negativity in 2+1 dimensional topologically ordered phases

Xueda Wen

UIUC

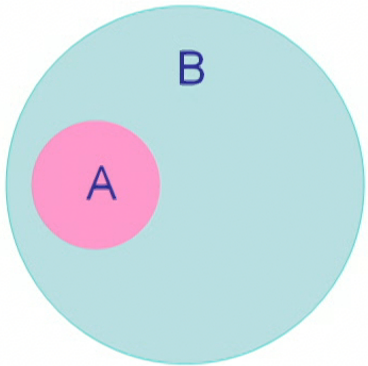
Dec. 13, 2016

Entanglement negativity in 2+1 dimensional topologically ordered phases

Q1: What is negativity?

Q2: Why are we interested in negativity?

Before we introduce the negativity



von Neumann entropy:

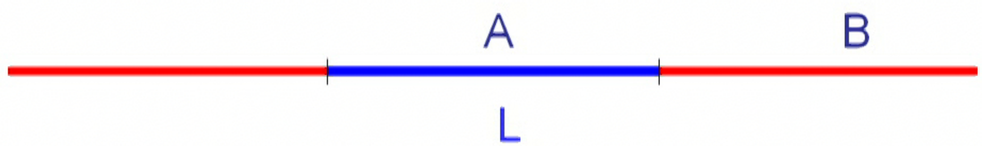
$$S_A = -\mathrm{Tr}\,\rho_A \ln \rho_A$$

Remarks:

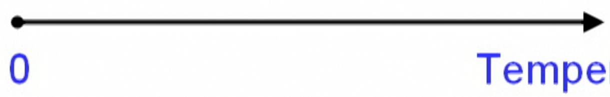
1. Good for a bipartite system in a pure state.
2. For mixed states, it may mix classical and quantum correlations.

Before we introduce the negativity

An example: 1+1 dimensional CFT

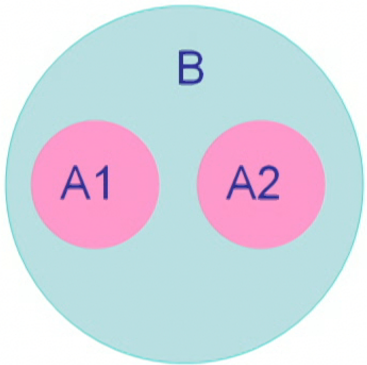


$$S_A = \frac{c}{3}\log L \qquad\qquad \frac{c}{3}\cdot\frac{\pi L}{\beta}$$



Before we introduce the negativity

Tripartite system at zero temperature

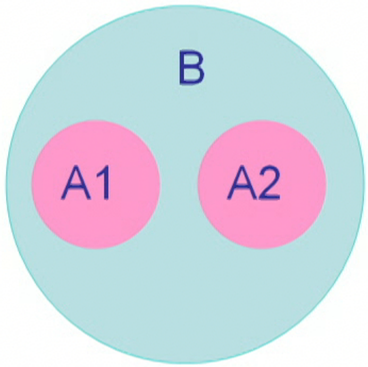


$\rho_{A_1 \cup A_2}$ is a mixed state

Entropy-based measures, such as mutual information, are no longer reliable.

$$I_{A_1 A_2} = S_{A_1} + S_{A_2} - S_{A_1 \cup A_2}$$

Why we need negativity

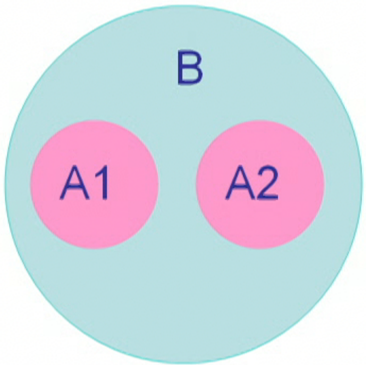


To characterize the quantum entanglement between A1 and A2, a computable quantity is: entanglement negativity.

Peres (96), Horodecki-Horodecki-Horodecki (96), Vidal-Werner, (2002); Plenio, (2005)

Definition of negativity

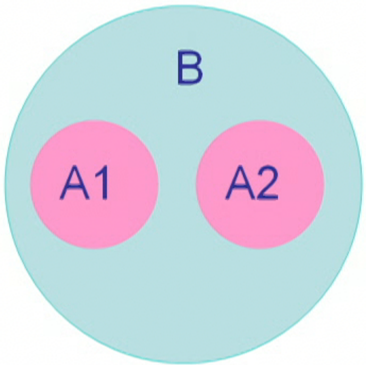
Partial transposition



$$\langle e_i^{(1)} \underline{e_j^{(2)}} | \rho_{A_1 \cup A_2}^{T_2} | e_k^{(1)} \underline{e_l^{(2)}} \rangle = \langle e_i^{(1)} \underline{e_l^{(2)}} | \rho_{A_1 \cup A_2} | e_k^{(1)} \underline{e_j^{(2)}} \rangle$$

$|e_i^{(k)}\rangle$: base of $\mathcal{H}_{A_k}$

Definition of negativity



$$||\rho^{T_2}_{A_1\cup A_2}|| = \mathrm{Tr}|\rho^{T_2}_{A_1\cup A_2}| = \sum_i |\lambda_i| = 1 - 2\sum_{\lambda_i<0} \lambda_i$$

λ_i : eigenvalue of $\rho^{T_2}_{A_1\cup A_2}$

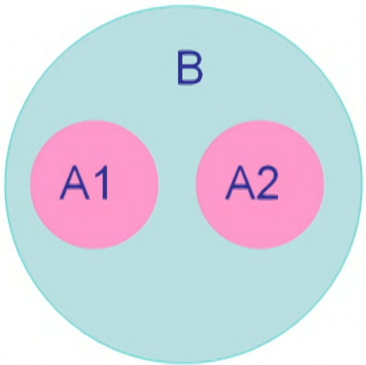
(logarithmic) Negativity

$$\mathcal{E}_{A_1A_2} = \ln ||\rho^{T_2}_{A_1\cup A_2}||$$

It measures "how much" the eigenvalues are negative, and extracts only the quantum correlations.

Peres (96), Horodecki-Horodecki-Horodecki (96),
Vidal-Werner, (2002); Plenio, (2005)

Definition of negativity



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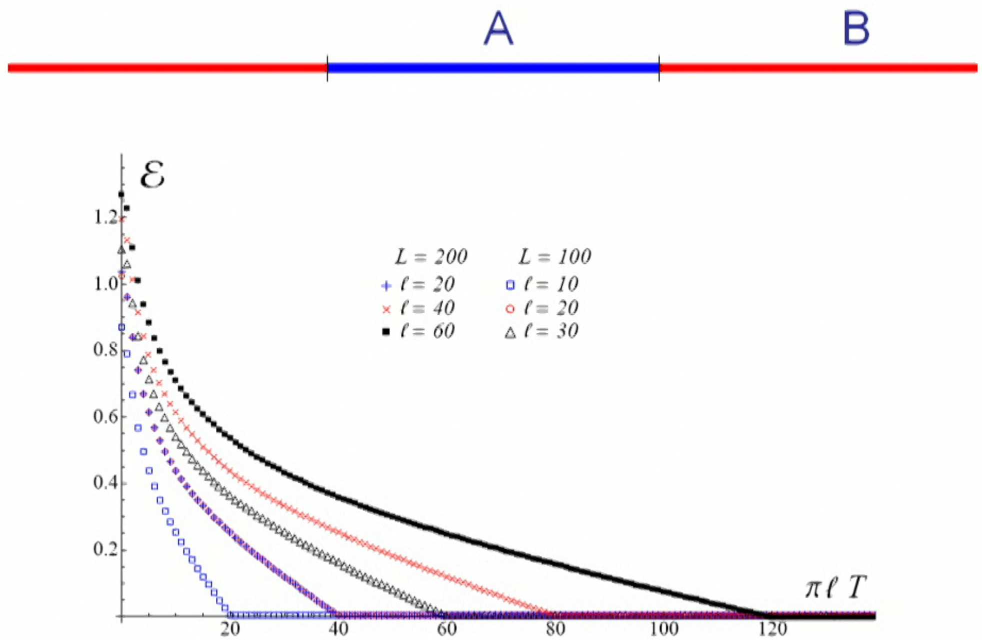
(logarithmic) Negativity

$$\mathcal{E}_{A_1A_2} = \ln ||\rho^{T_2}_{A_1\cup A_2}||$$

Believe it or not?

1+1 dimensional CFT at finite temperature



Negativity decreases as we increase temperature.

Calabrese-Cardy-Tonni, (2014)

1+1 dimensional CFT at finite temperature

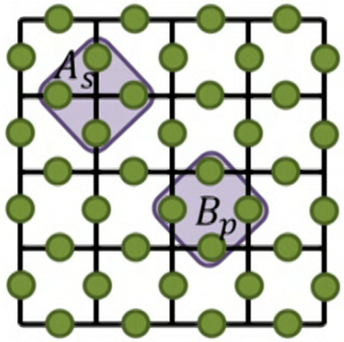
$$\mathcal{E} = \frac{c}{2} \ln \left[\frac{\beta}{\pi a} \sinh \left(\frac{\pi \ell}{\beta} \right) \right] - \frac{\pi c \ell}{2\beta}$$

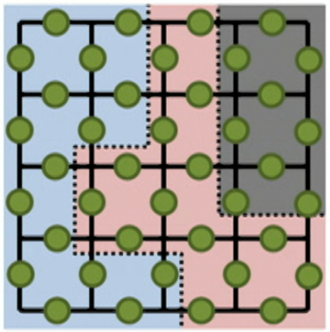
$$= \frac{3}{2} \left[S_A - S_A^{thermal} \right]$$

Calabrese-Cardy-Tonni, (2014)

Entanglement negativity and topological order

Toric Code





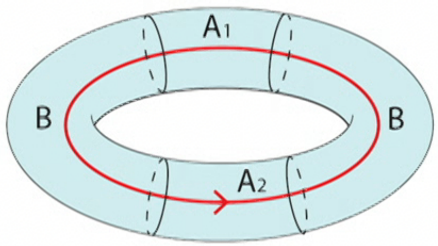
$$H = -U\sum_s A_s - J\sum_p B_p$$

The result depends on how we divide the system.

Lee-Vidal, (2013)
Castelnovo (2013)

Toric code on a torus

Case 1



$$|\Psi\rangle = \sum_a \psi_a |a\rangle$$

$$I_{A_1A_2} = -\sum_a |\psi_a|^2 \ln |\psi_a|^2$$

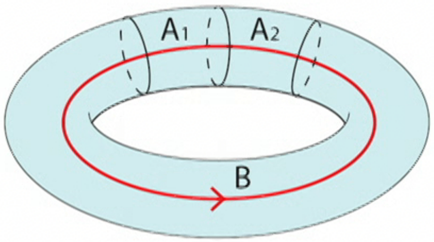
$$\mathcal{E}_{A_1A_2} = 0.$$

Q: No quantum entanglement between A1 and A2?

Lee, Vidal, (2013)
Castelnovo (2013)

Toric code on a torus

Case 2



$$|\Psi\rangle = \sum_a \psi_a |a\rangle$$

$$\mathcal{E}_{A_1 A_2} = \ln 2 \times L - \gamma$$

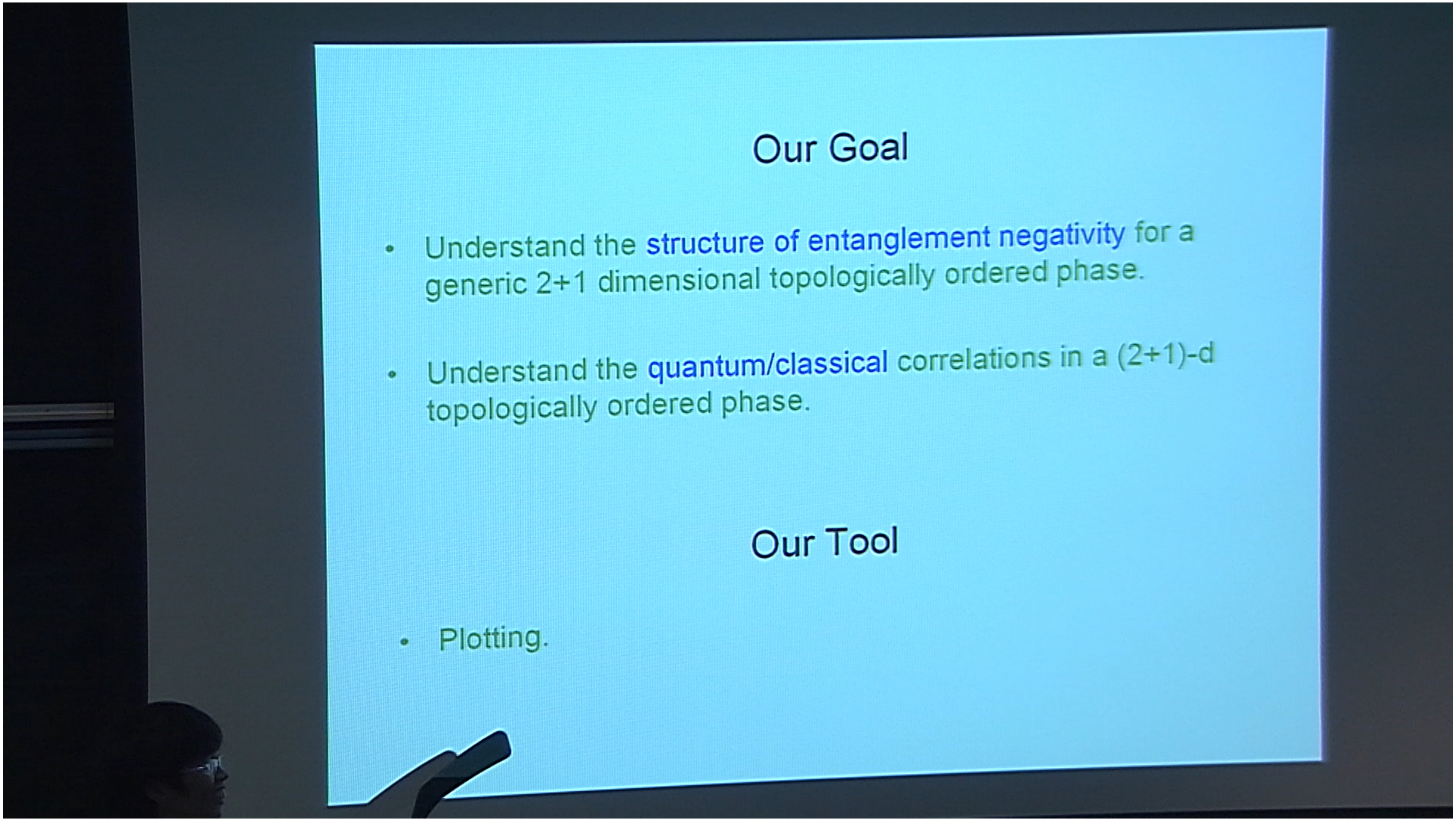
$$\gamma = \ln 2$$

Q: Why is it independent of the choice of ground state?

Lee, Vidal, (2013)
Castelnovo (2013)

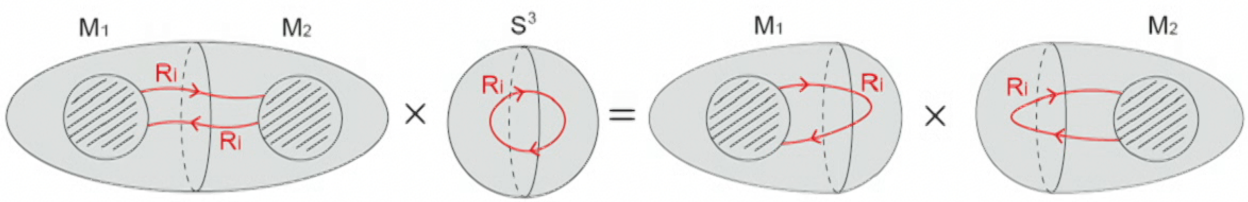
Our Goal

- Understand the structure of entanglement negativity for a generic 2+1 dimensional topologically ordered phase.
- Understand the quantum/classical correlations in a (2+1)-d topologically ordered phase.


Our Goal
Understand the structure of entanglement negativity for a generic 2+1 dimensional topologically ordered phase.
Understand the quantum/classical correlations in a (2+1)-d topologically ordered phase.
Our Tool
Plotting.

Chern-Simons theory

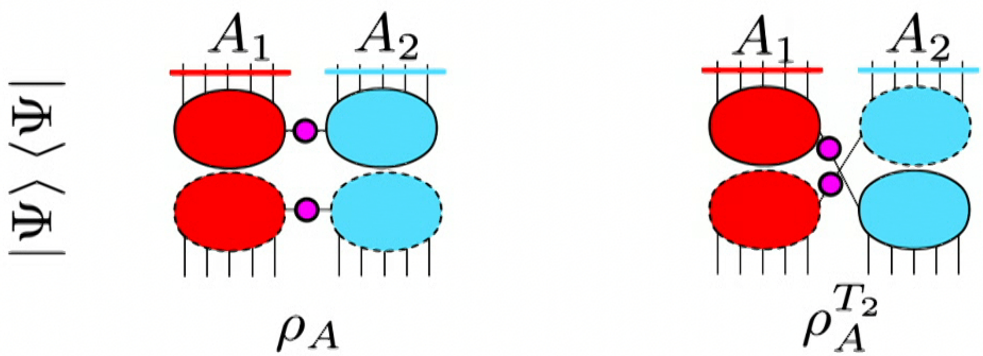
$$S_{\rm CS} = \frac{k}{4\pi}\int_M \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$



Witten, (1989)

How to do partial transposition?

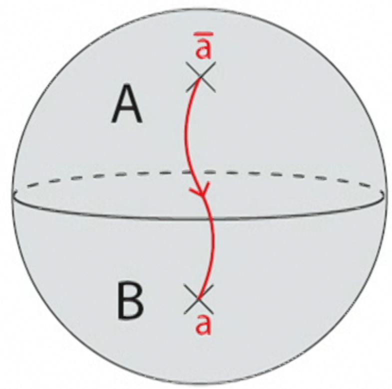
Tensor network



Calabrese-Tagliacozzo-Tonni (2013)

Chern-Simons theory

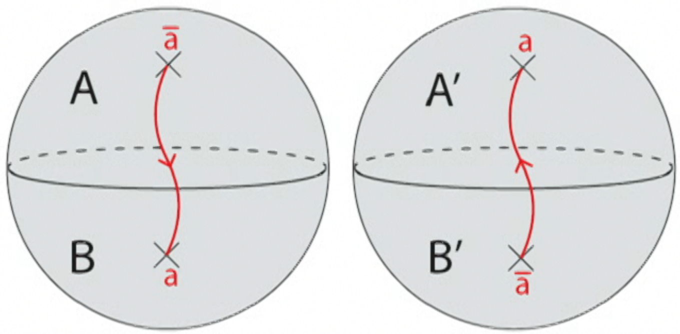
Warm up: Entanglement negativity on a bipartite sphere



$$|\Psi\rangle$$

Chern-Simons theory

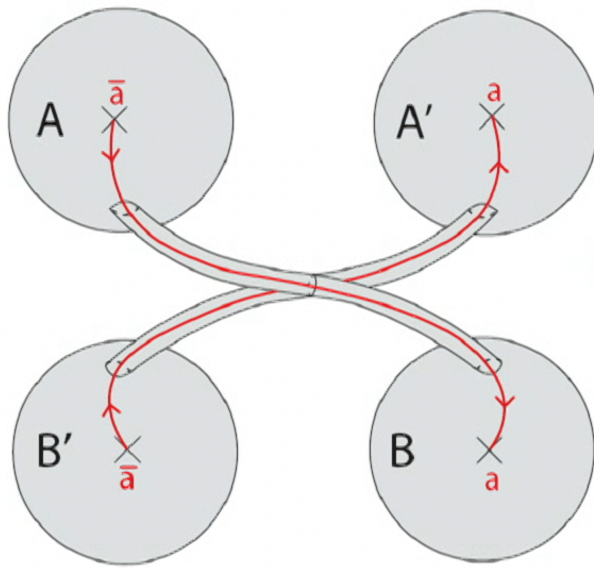
Warm up: Entanglement negativity on a bipartite sphere



$$\rho_{A\cup B} = |\Psi\rangle\langle\Psi|$$

Chern-Simons theory

Warm up: Entanglement negativity on a bipartite sphere



Q: How to evaluate

$||\rho_{A\cup B}^{T_B}||$?

$$\rho_{A\cup B}^{T_B}$$

Replica approach

$$||\rho_{A\cup B}^{T_B}|| = \lim_{n_e\to 1} \mathrm{Tr}\left(\rho_{A\cup B}^{T_B}\right)^{n_e}$$

Parity effect:

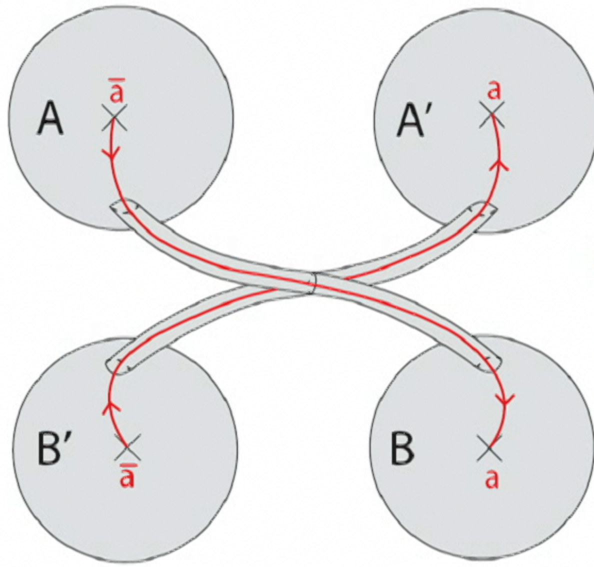
$$\mathrm{Tr}(\rho^{T_2})^{n_e} = \sum_i \lambda_i^{n_e} = \sum_{\lambda_i>0} |\lambda_i|^{n_e} + \sum_{\lambda_i<0} |\lambda_i|^{n_e},$$

$$\mathrm{Tr}(\rho^{T_2})^{n_o} = \sum_i \lambda_i^{n_o} = \sum_{\lambda_i>0} |\lambda_i|^{n_o} - \sum_{\lambda_i<0} |\lambda_i|^{n_o}.$$

$$||\rho_{A_1\cup A_2}^{T_2}|| = \sum_i |\lambda_i| = \lim_{n_e\to 1} \mathrm{Tr}\left(\rho^{T_2}\right)^{n_e}$$

Chern-Simons theory

Warm up: Entanglement negativity on a bipartite sphere



$\rho_{A\cup B}^{T_B}$

Q: How to evaluate

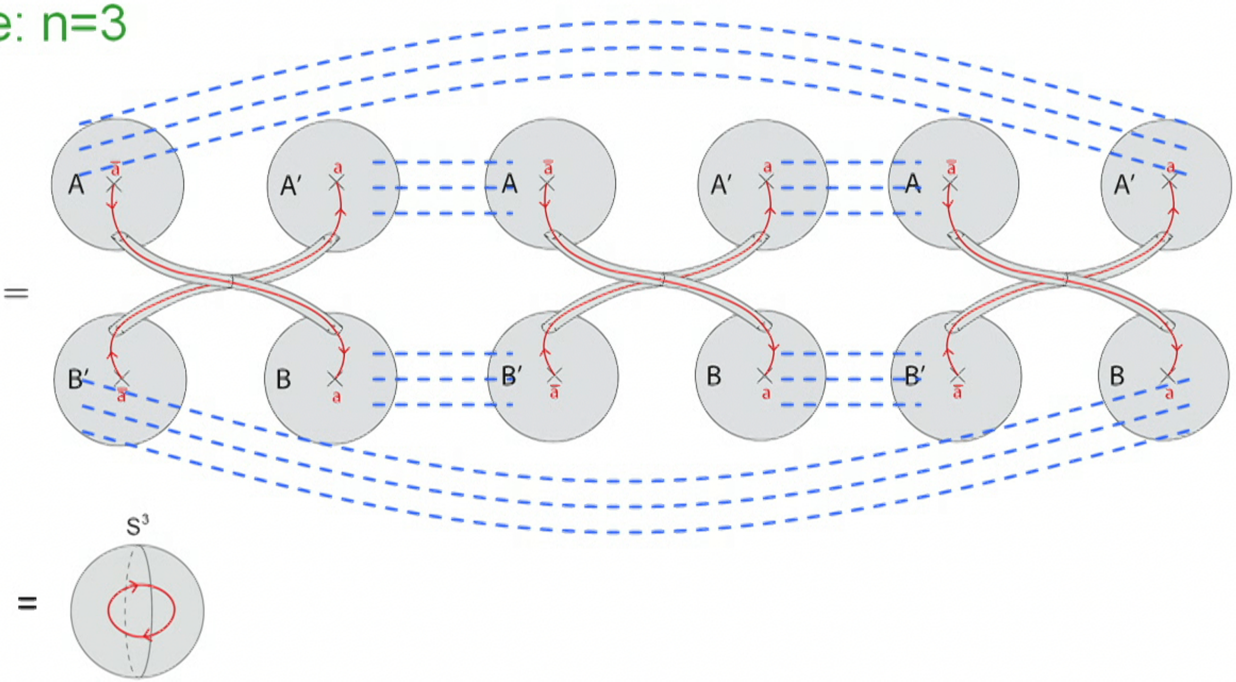
$||\rho_{A\cup B}^{T_B}||$?

Replica approach

$$||\rho_{A\cup B}^{T_B}|| = \lim_{n_e \to 1} \mathrm{Tr}\left(\rho_{A\cup B}^{T_B}\right)^{n_e}$$

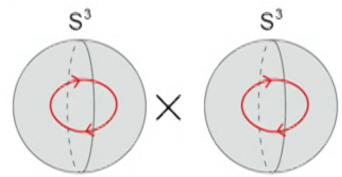
Example: n=3

$$\mathrm{Tr}\left(\rho_{A\cup B}^{T_B}\right)^2 =$$

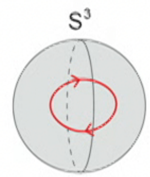


Replica approach: Parity effect

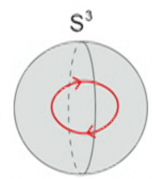
$$\mathrm{Tr}\left(\rho_{A\cup B}^{T_B}\right)^{n_e} =$$



$$\mathrm{Tr}\left(\rho_{A\cup B}^{T_B}\right)^{n_o} =$$



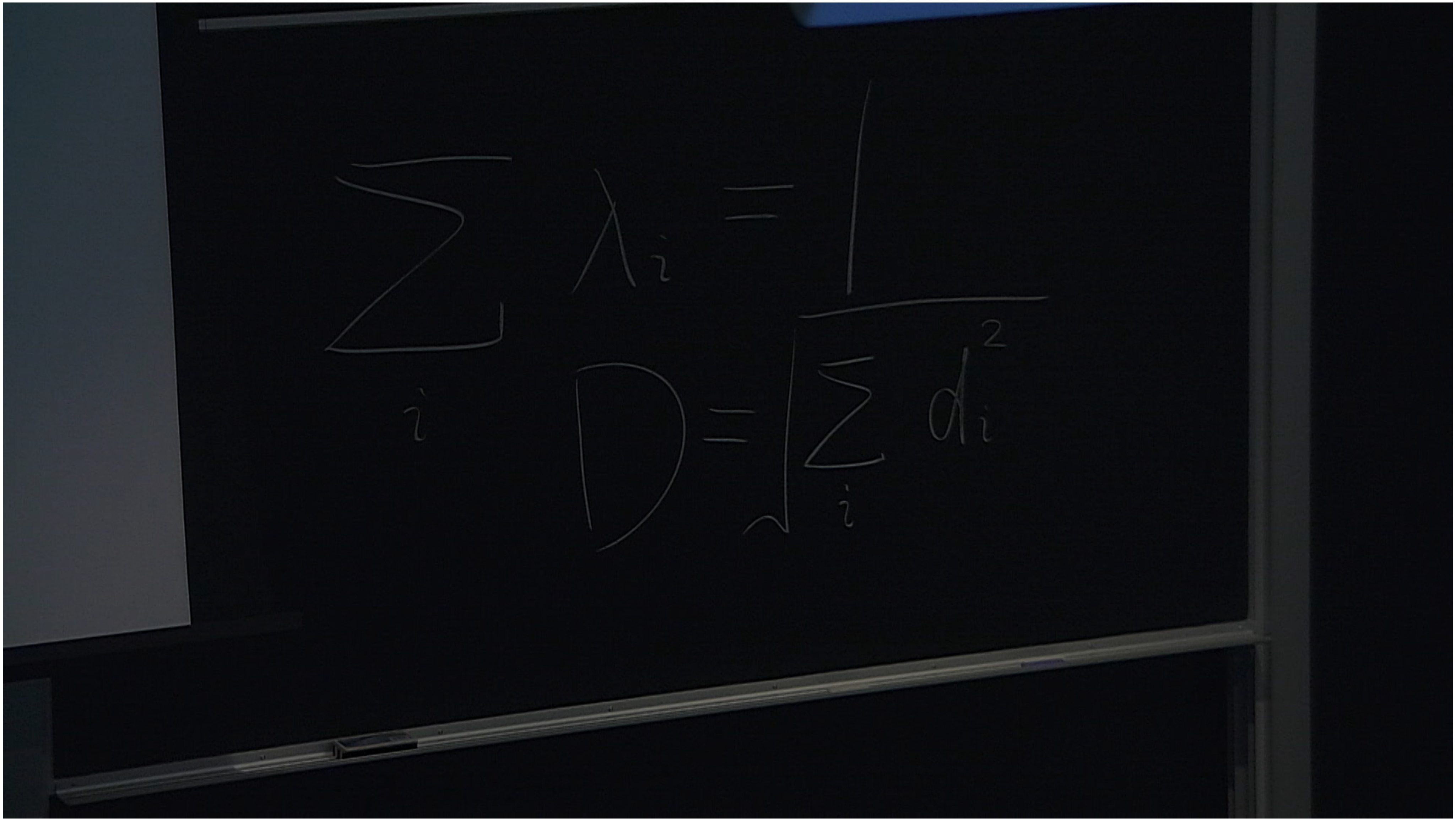
$$\mathcal{E}_{AB} = \lim_{n_e \to 1} \ln \mathrm{Tr}\left(\frac{\rho_{A\cup B}^{T_B}}{\mathrm{Tr}\rho_{A\cup B}^{T_B}}\right)^{n_e} = \ln$$



$$= \ln Z(S^3, R_a) = \ln d_a - \ln \mathcal{D}$$

The same as TEE.

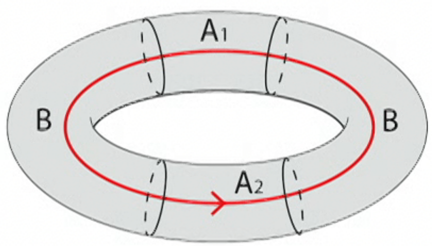
Kitaev-Preskill (2005); Levin-Wen (2005)


$\sum_i \lambda_i = \frac{1}{D}$
$D = \sqrt{\sum_i d_i^2}$

After the warm-up, let us check the more interesting cases:

Entanglement negativity between A1 and A2

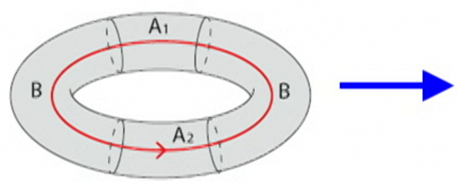
Quantum or classical correlations?



$$|\Psi\rangle = \sum_a \psi_a |a\rangle$$

Entanglement negativity between A1 and A2

Quantum or classical correlations?



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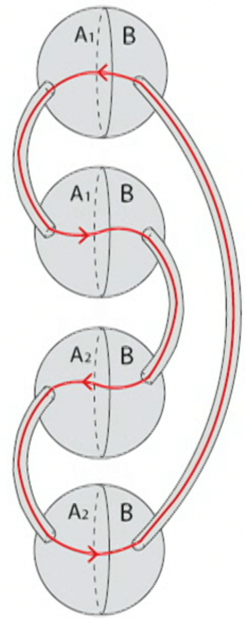
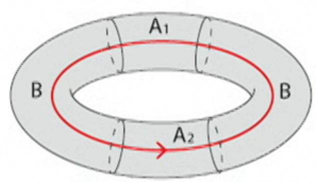


$$|\Psi\rangle$$

Entanglement negativity between A1 and A2

Quantum or classical correlations?



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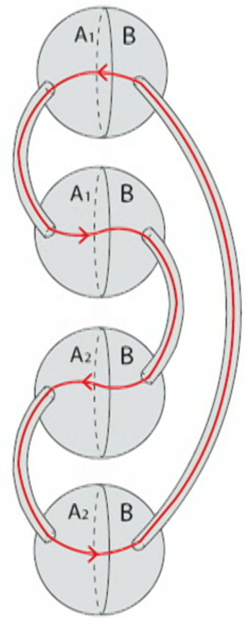
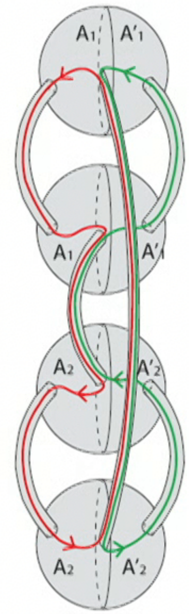


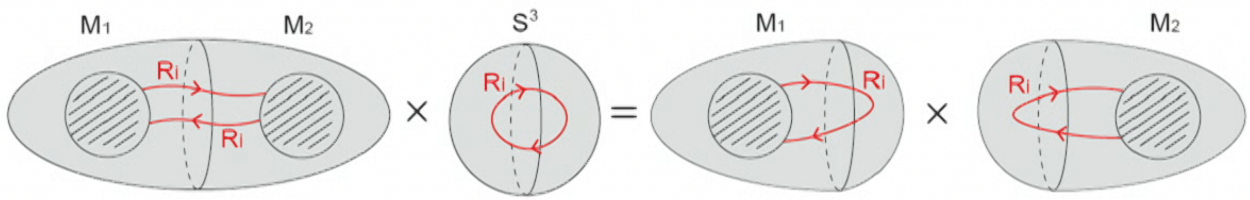
$$|\Psi\rangle$$



$$\rho_{A_1 \cup A_2} = ?$$

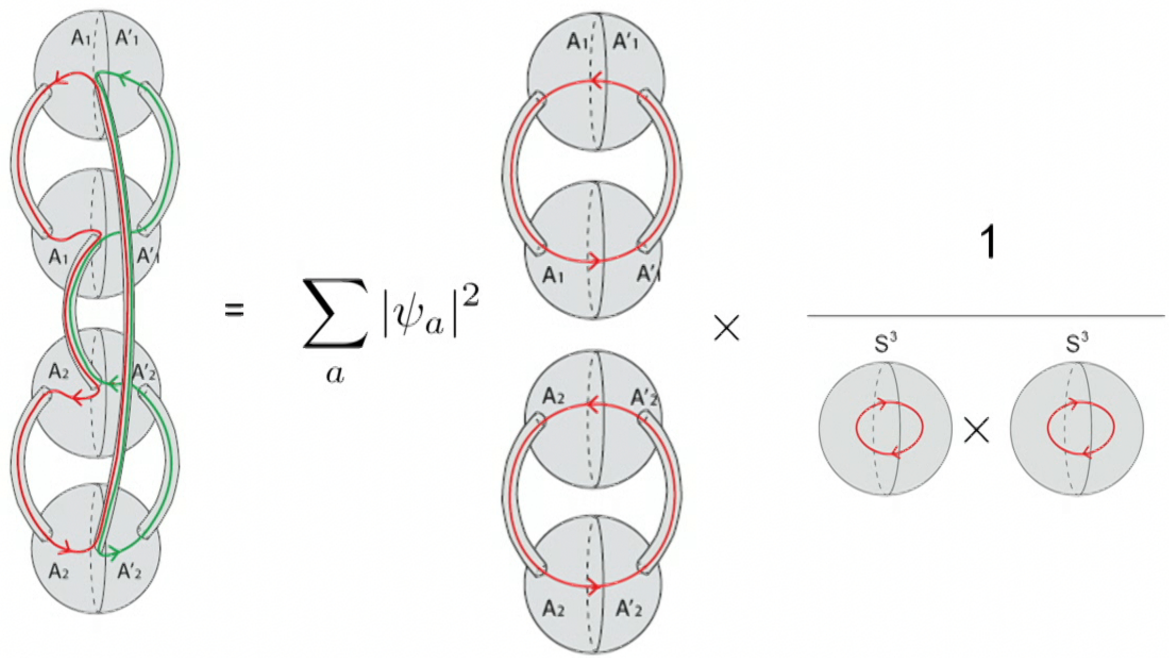
Entanglement negativity between A1 and A2

Quantum or classical correlations?



Entanglement negativity between A1 and A2

Quantum or classical correlations?



$\rho_{A_1 \cup A_2}$

Entanglement negativity between A1 and A2

Quantum or classical correlations?

$$I_{A_1A_2} = -\sum_a |\psi_a|^2 \ln |\psi_a|^2 \quad \leftrightarrow \quad \mathcal{E}_{A_1A_2} = 0.$$

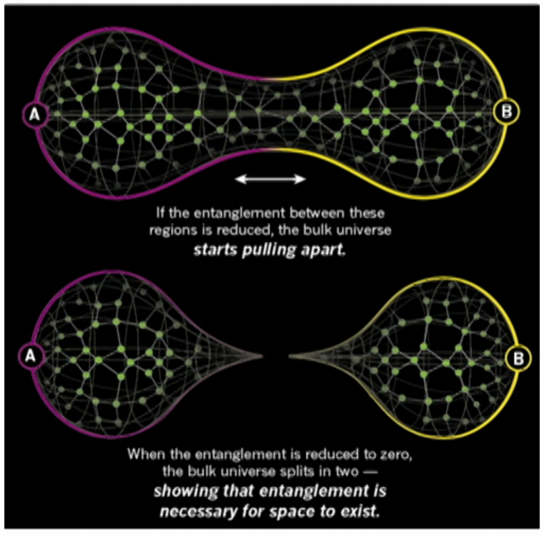
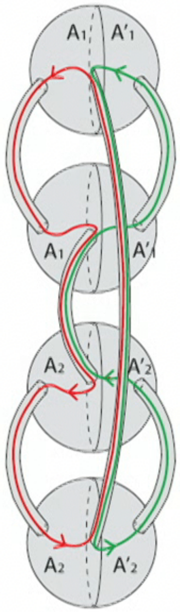


Figure from Nature 527, 290; (2015)

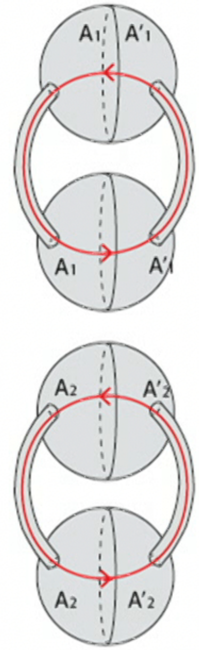
Raamsdonk, (2010)

Entanglement negativity between A1 and A2

Quantum or classical correlations?



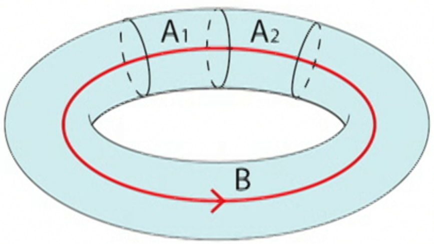
$$= \sum_a |\psi_a|^2 \frac{1}{Z(S^3, R^a)^2}$$



$\rho_{A_1 \cup A_2}$ $\rho^a_{A_1} \otimes \rho^a_{A_2}$

Classical correlations ←→ Decoupled spacetime

Another interesting case:
Toric code on a torus



$$|\Psi\rangle = \sum_a \psi_a |a\rangle$$

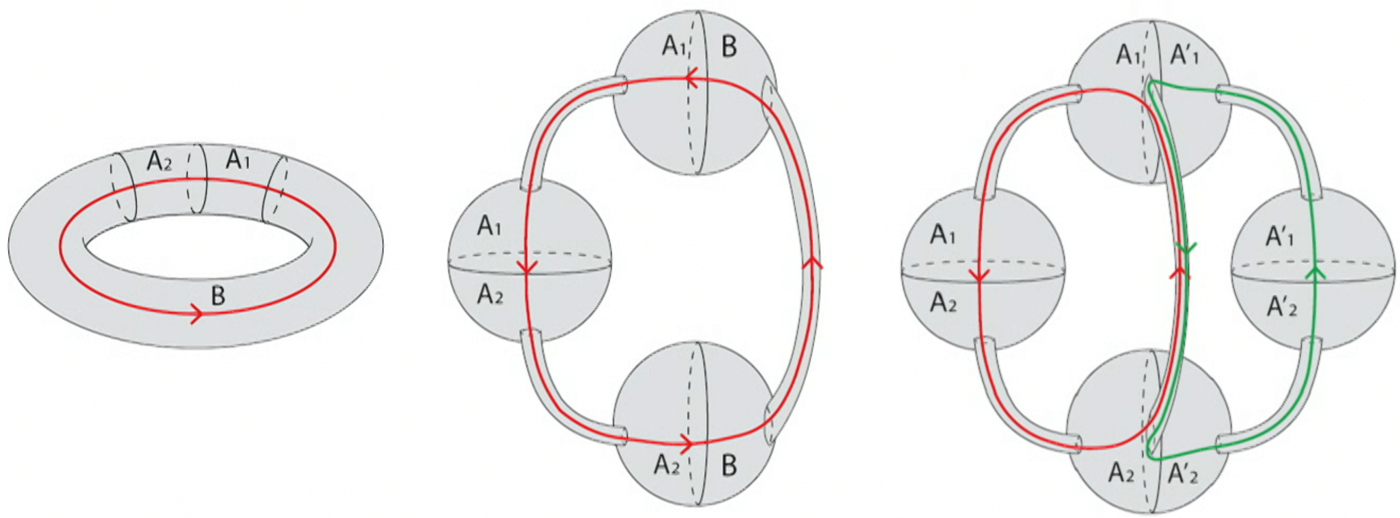
$$\mathcal{E}_{A_1 A_2} = \ln 2 \times L - \gamma$$

$$\gamma = \ln 2$$

Q: Why is it independent of the choice of ground state?

Lee-Vidal, (2013)
Castelnovo (2013)

Entanglement negativity on a torus

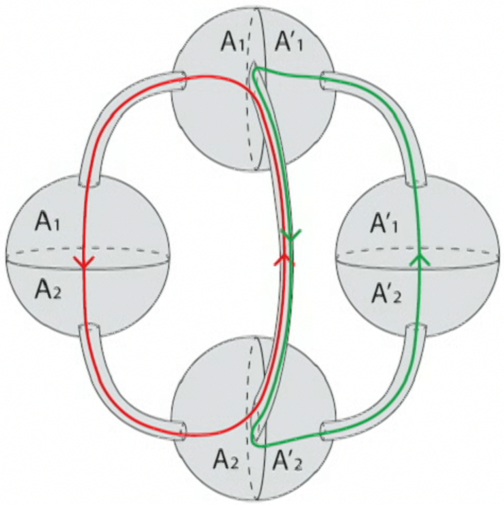


$|\Psi\rangle$ $|\Psi\rangle$ $\rho_{A_1 \cup A_2}$

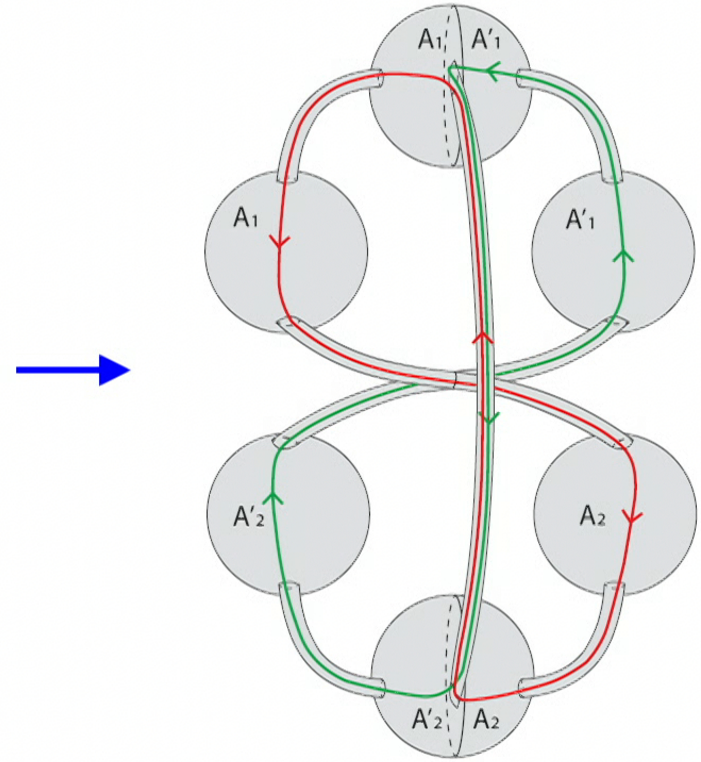
Cannot be cut into two "universes" any more.

→ There must be quantum entanglement.

Entanglement negativity on a torus



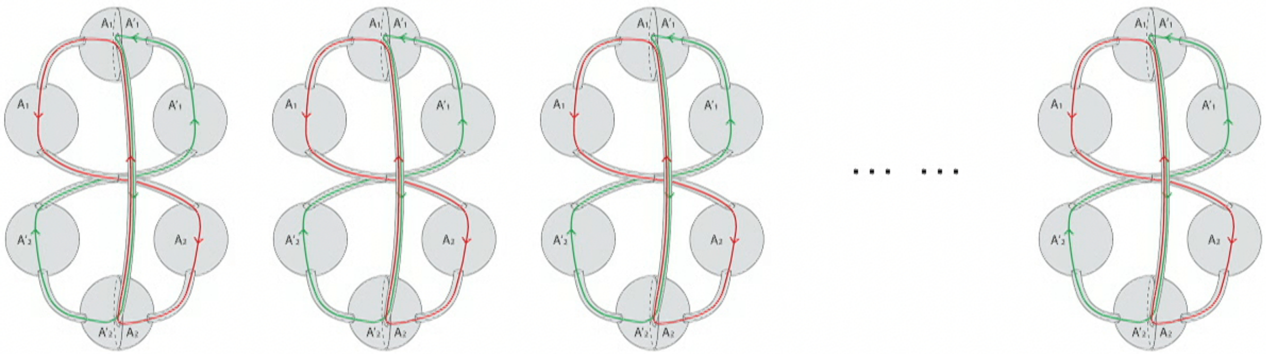
$\rho_{A_1 \cup A_2}$



$\rho^{T_{A_2}}_{A_1 \cup A_2}$

Entanglement negativity on a torus

A little bit imagination, with gluing and cut



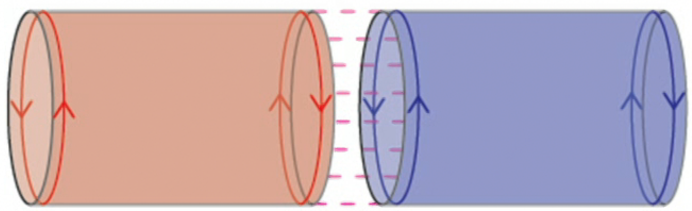
$$\mathcal{E}_{A_1 A_2} = \ln\left(\sum_a |\psi_a|^2 d_a\right) - \ln \mathcal{D}$$

Can distinguish Abelian and non-Abelian phases easily.

Wen-Chang-Ryu; arXiv:1606.04118.

(Boring) non-geometric approach: Edge theory

Homogeneous and inhomogeneous cases

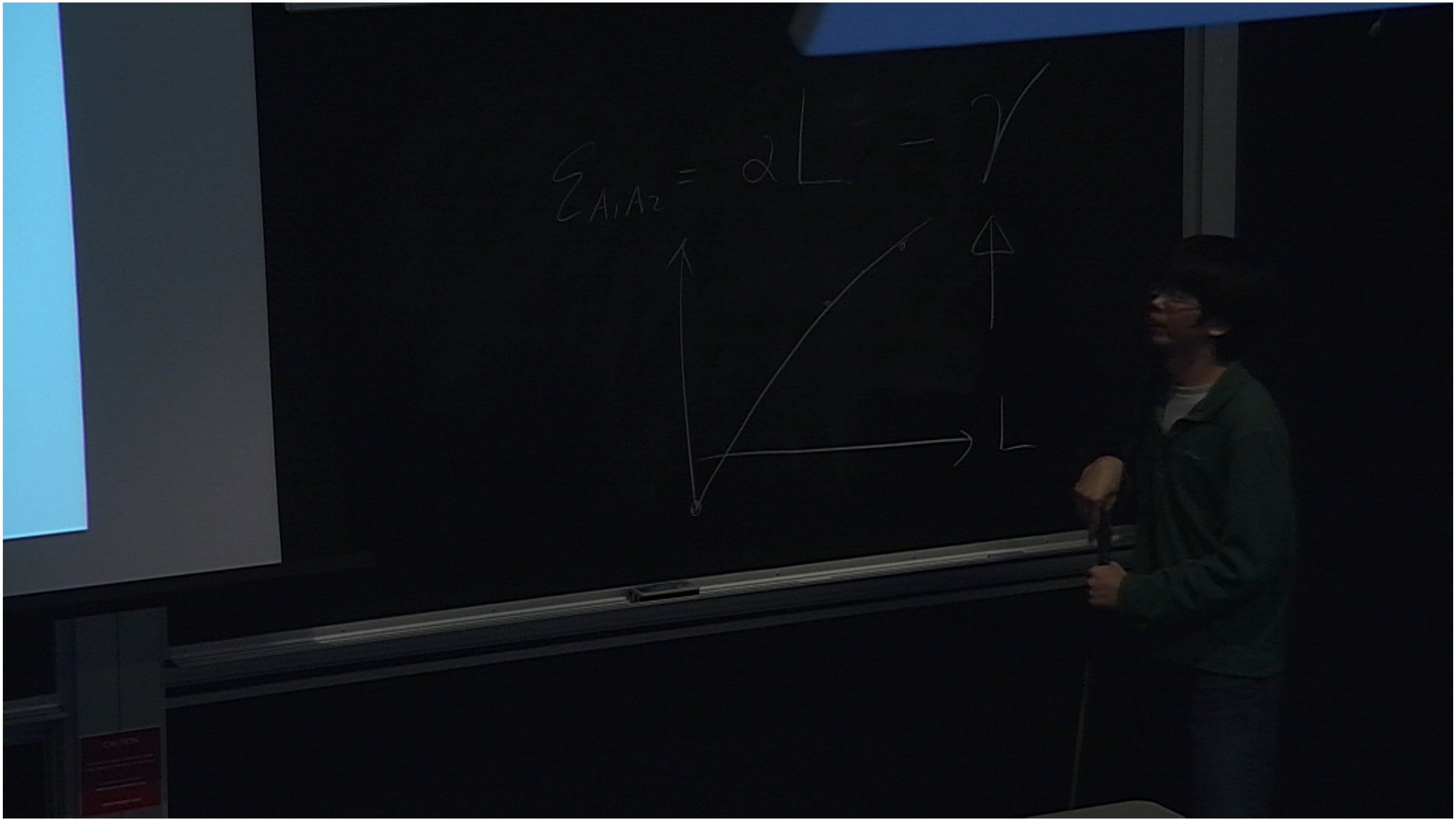


$$|h_a\rangle\rangle \equiv \sum_{N=0}^{\infty} \sum_{j=1}^{d_{h_a}(N)} |h_a, N; j\rangle \otimes \overline{|h_a, N; j\rangle}$$

Wen-Matsuura-Ryu, arXiv:1603.08534
Fliss-Wen-Parrikar-Hsieh-Han-Leigh-Hughes, to appear

Summary

- Plot negativity for (2+1)-d TO.
- Its application.
- Many future problems. (higer-d, holographic EN, etc)


$\mathcal{E}_{A_1A_2} = \alpha L - \gamma$
L