Title: Non-holonomic tomography and detecting state-preparation and measurement correlated errors

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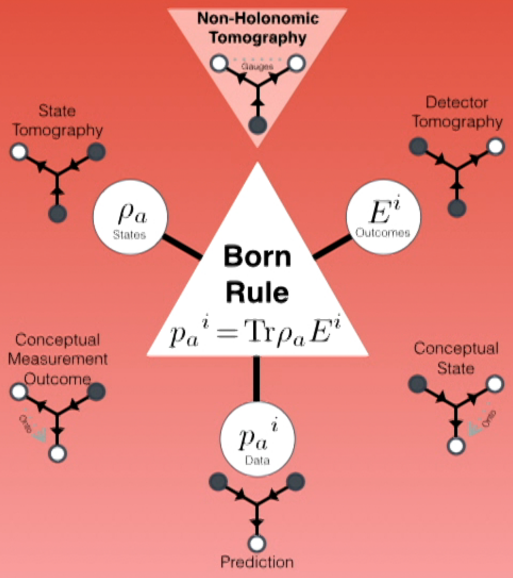
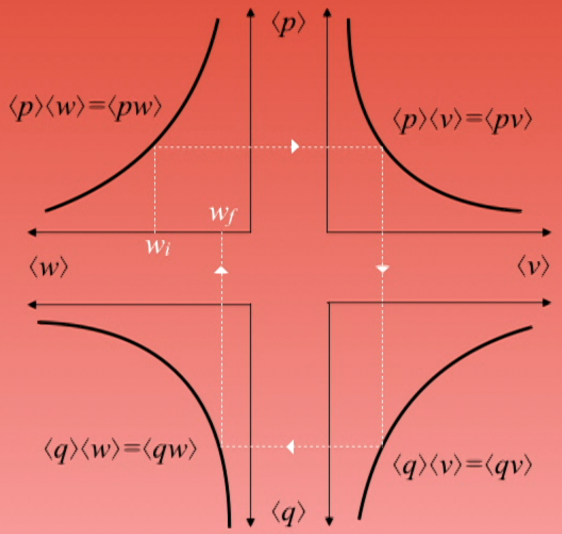
Abstract: <p>Quantum tomography is an important tool for characterizing the parameters of unknown states, measurements, and gates. Standard quantum tomography is the practice of estimating these parameters with known measurements, states, or both, respectively. In recent years, it has become important to address the issue of working with systems where the ``devices" used to prepare states and make measurements both have significant errors. Of particular concern to me is whether such state-preparation and measurement errors are correlated with each other. In this talk, I will share a solution to assessing such correlations with an object called a partial determinant. Further, I will show how this technique suggests a perspective for such correlated quantum states and observables (over the space of device settings) is analogous to the non-holonomic perspectives of thermodynamic heat and work (over the macroscopic state space.)</p>

Non-Holonomic Tomography and Correlated Errors





The Born Rule

$$p = \mathrm{Tr}\rho E$$

$$p \longleftarrow \frac{n}{N}$$

$$S = \mathrm{Tr}\rho\Sigma \qquad \Sigma = E - \bar{E}$$

Tomography

$$S_a{}^i = \mathrm{Tr}\rho_a\Sigma^i$$

$$a = 1,\ldots,N$$

$$i = 1,\ldots,M$$

$M \geq d^2 : \left\{\Sigma^i\right\}$ & $S_a{}^i \Longrightarrow \rho_a$ State Tomography

$N \geq d^2 : \left\{\rho_a\right\}$ & $S_a{}^i \Longrightarrow \Sigma^i$ Detector Tomography

SPAM Errors

What if both the state-preparations device and measurements device have errors?

① $$S_a{}^i \not\Rightarrow \rho_a \ \& \ \Sigma^i$$

② $$S_a{}^i = \langle \mathrm{Tr}\rho\Sigma\rangle_a{}^i$$

② Correlated SPAM

“the data”

$$\langle \mathrm{Tr}\rho\Sigma\rangle_a{}^i \overset{?}{\neq} \mathrm{Tr}\langle\rho\rangle_a\langle\Sigma\rangle^i$$

Not available in principle!

Effectively Uncorrelated

Definition

${S_a}^i$ is effectively uncorrelated if there exist $\{\rho_a\}$ and $\{\Sigma^i\}$ such that ${S_a}^i = \mathrm{Tr}\rho_a\Sigma^i$.

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$S_a{}^i$ is effectively (d-)uncorrelated if there exist $\{\rho_a\}$ and $\{\Sigma^i\}$ such that $S_a{}^i = \mathrm{Tr}\rho_a\Sigma^i$.

d^2-dimensional

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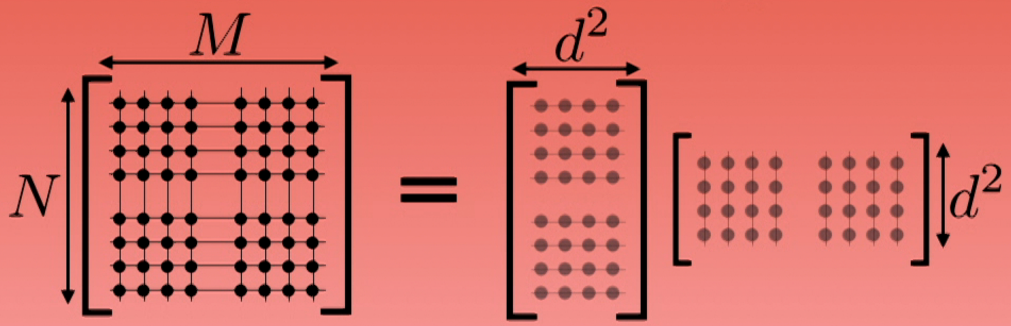
d^2-dimensional

$$S = PW$$



$$\Longleftrightarrow \quad \mathrm{rank}\, S \leq d^2$$

① SPAM Gauge

$$S = PW$$

state-preparation and measurement parameters are not uniquely determined

$$(P, W) \longrightarrow (PG, G^{-1}W)$$

$G \in \mathrm{GL}(d^2) \longleftarrow d^4 \longleftarrow$ "Blame"

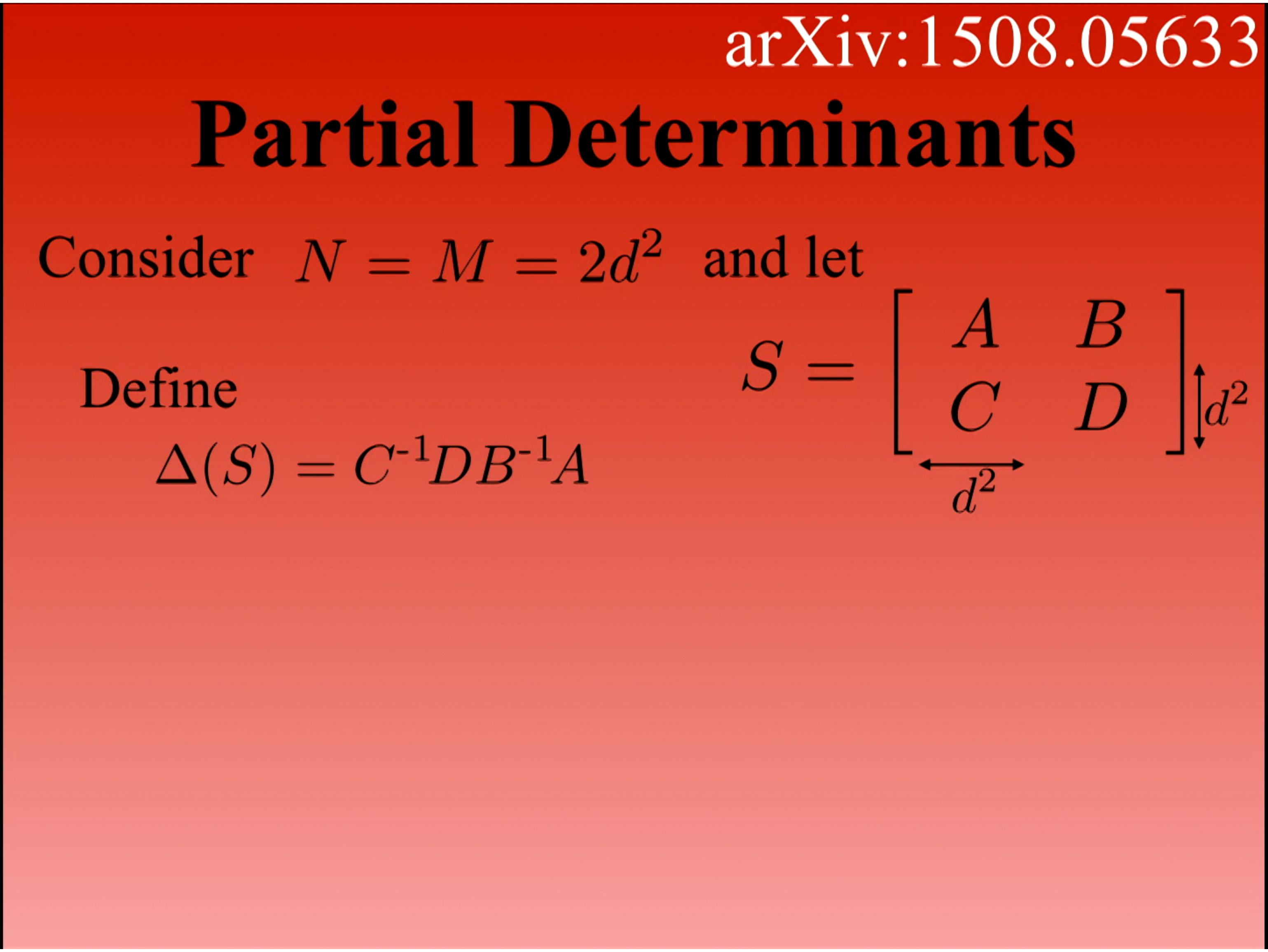
$\mathrm{U}(d) \longleftarrow d^2 \longleftarrow$ Trivial Choice of Vector Basis

arXiv:1508.05633

Partial Determinants

Consider $N = M = 2d^2$ and let

$$S = \begin{array}{cc} \left[\begin{array}{cc} A & B \\ C & D \end{array}\right] & \updownarrow d^2 \\ \underset{d^2}{\longleftrightarrow} & \end{array}$$



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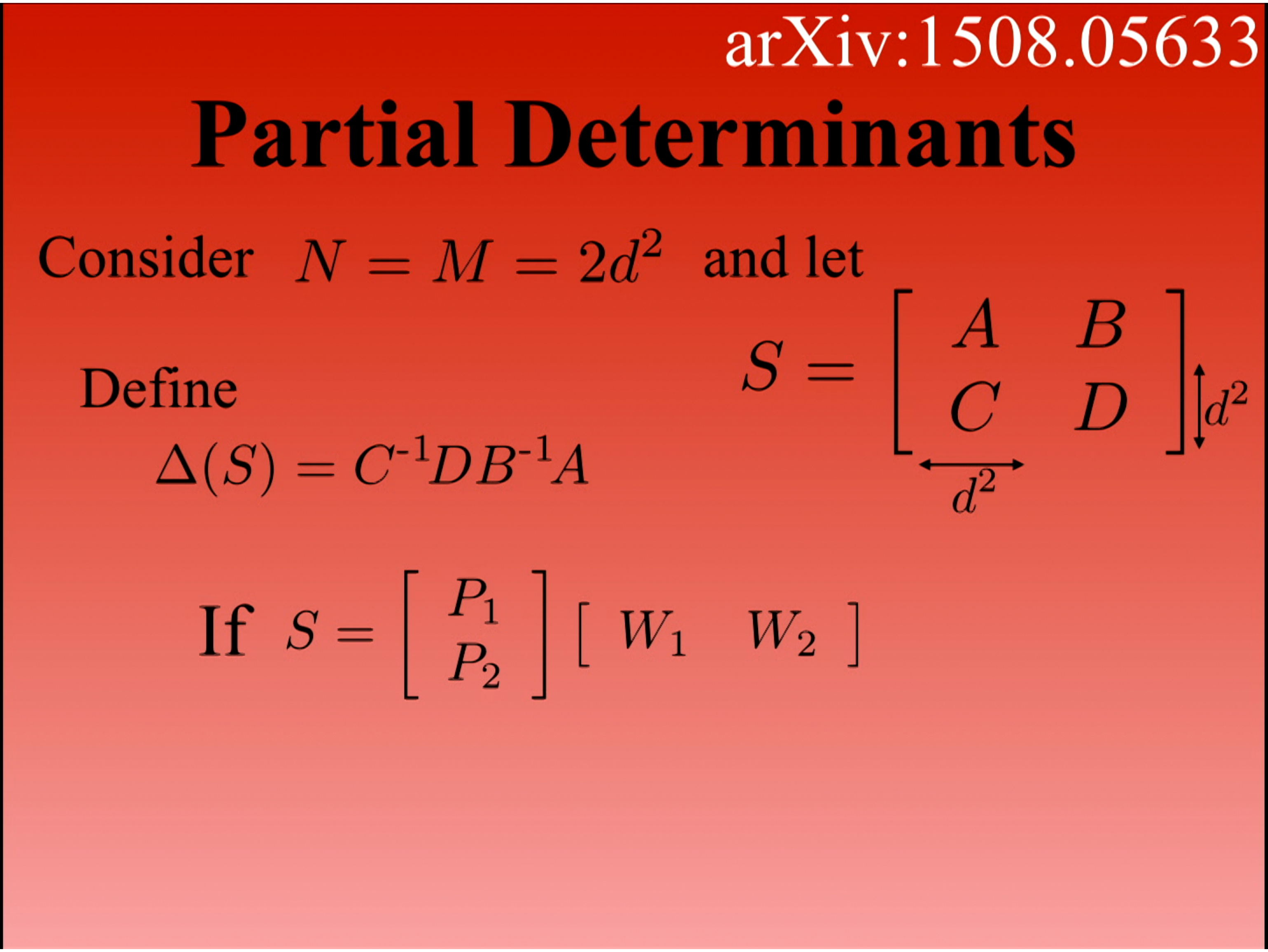
Partial Determinants

Consider $N = M = 2d^2$ and let

$$S = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \updownarrow d^2 \qquad \xleftrightarrow{\quad} \; d^2$$

Define

$$\Delta(S) = C^{-1} D B^{-1} A$$



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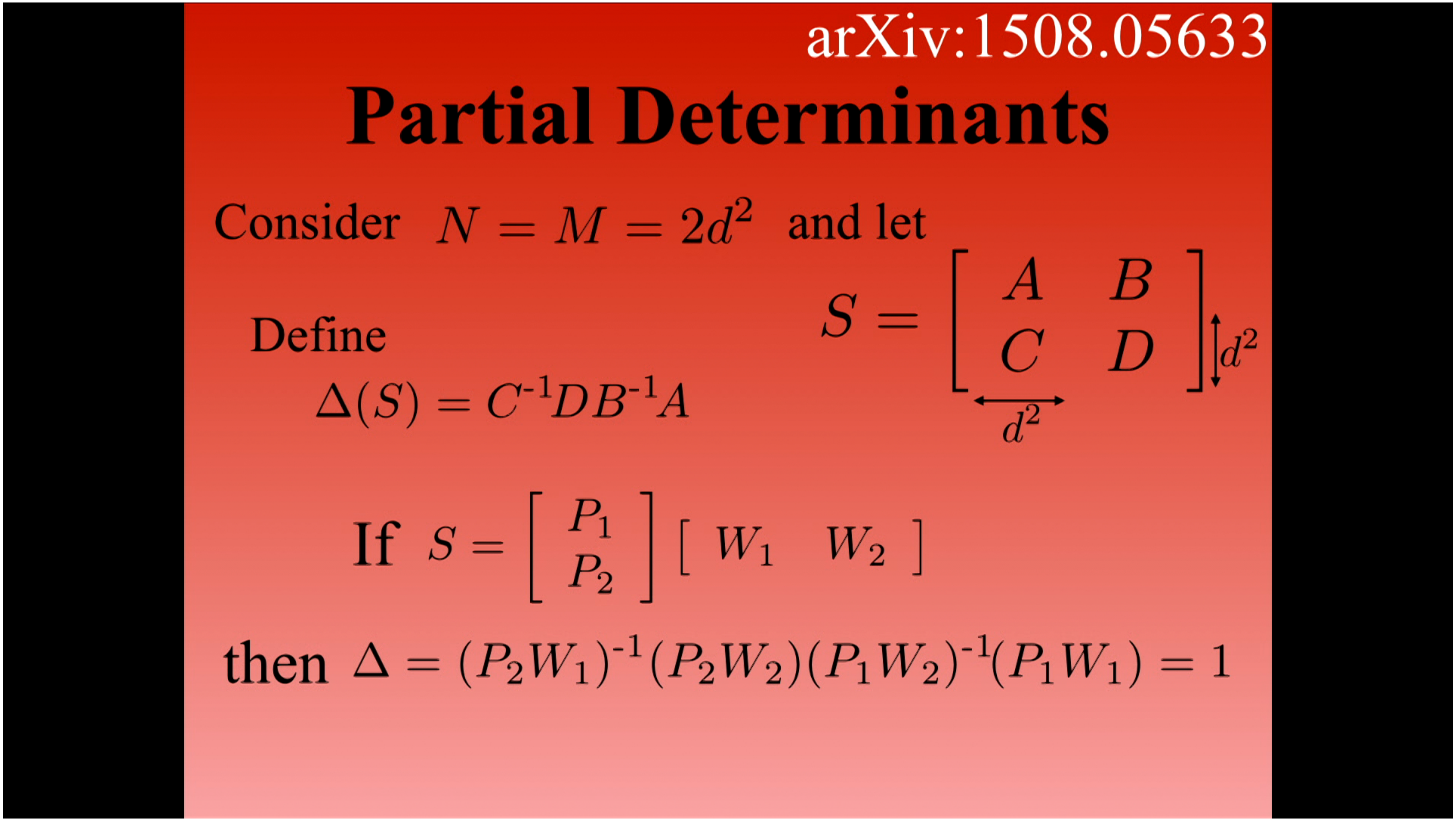
If $S = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \begin{bmatrix} W_1 & W_2 \end{bmatrix}$



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If $S = \left[\begin{array}{c} P_1 \\ P_2 \end{array}\right]\left[\begin{array}{cc} W_1 & W_2 \end{array}\right]$

then $\Delta = (P_2W_1)^{-1}(P_2W_2)(P_1W_2)^{-1}(P_1W_1) = 1$

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This is necessary *and* sufficient!

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Partial Determinants

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$$\xleftrightarrow[d^2]{}$$

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d^4 correlation parameters!

↓

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This is necessary *and* sufficient!

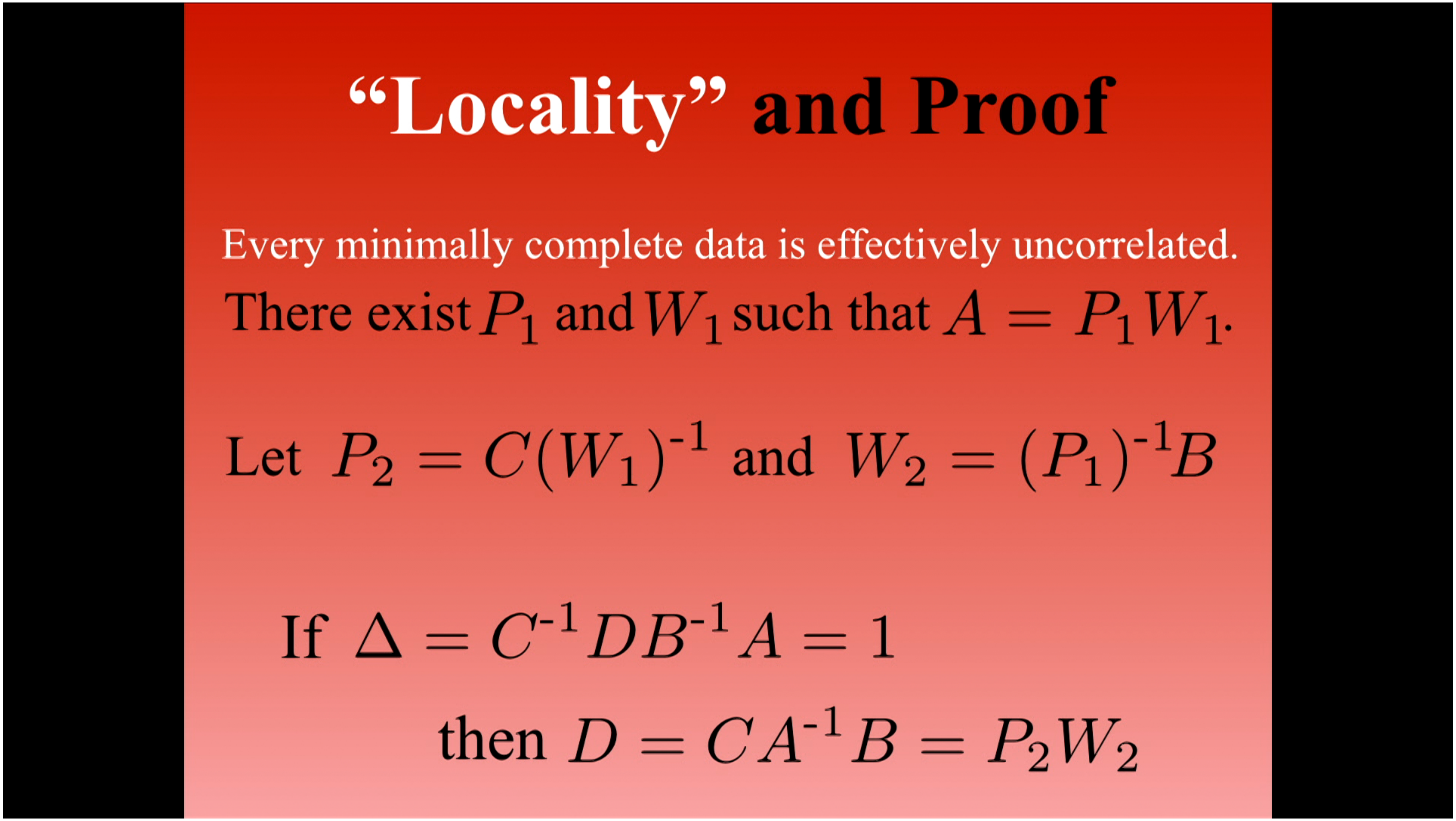
“Locality” and Proof

There exist P_1 and W_1 such that $A = P_1 W_1$.

Let $P_2 = C(W_1)^{-1}$ and $W_2 = (P_1)^{-1} B$

If $\Delta = C^{-1} D B^{-1} A = 1$

then $D = C A^{-1} B = P_2 W_2$



“Locality” and Proof

Every minimally complete data is effectively uncorrelated.

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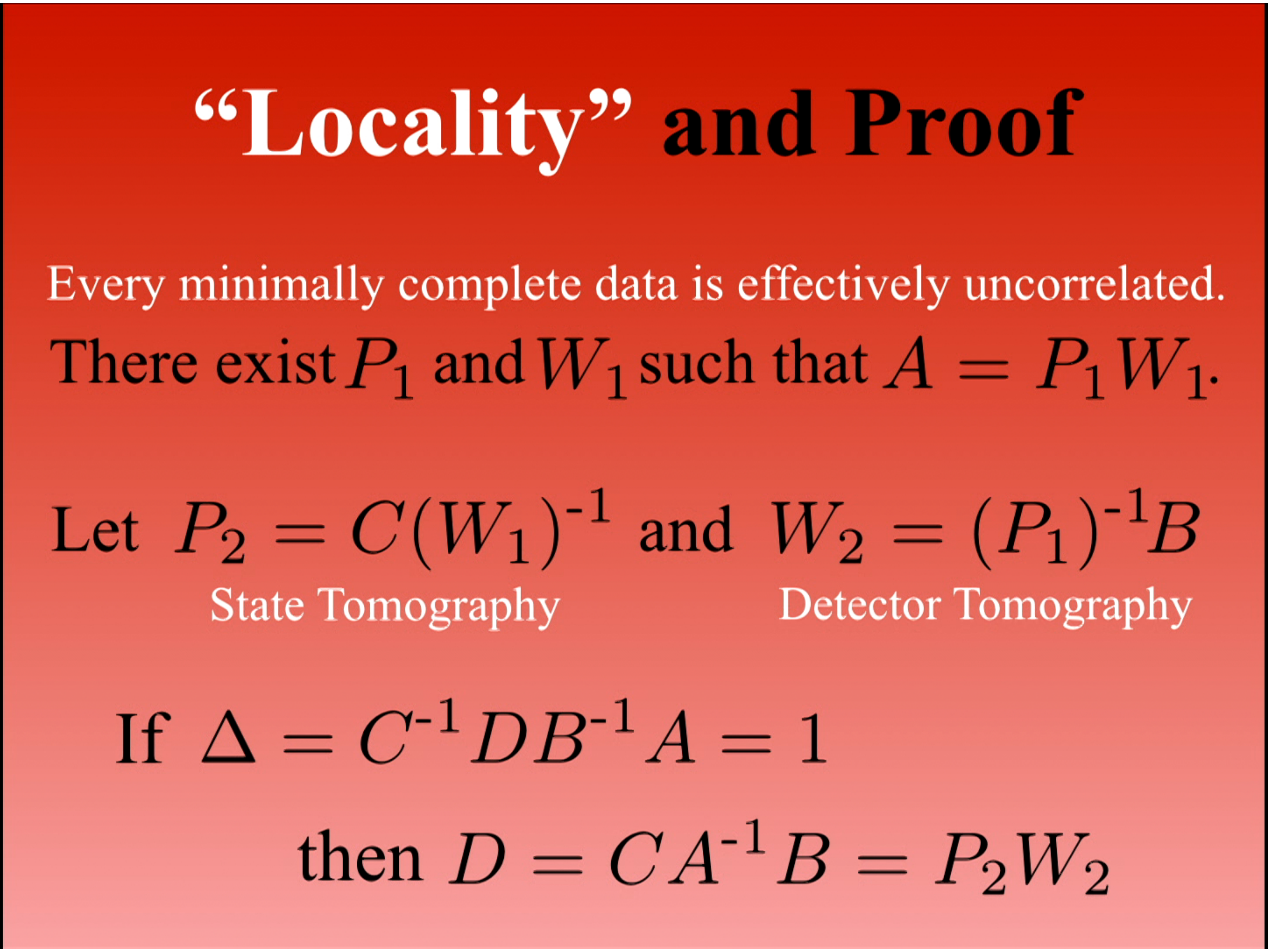

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State Tomography

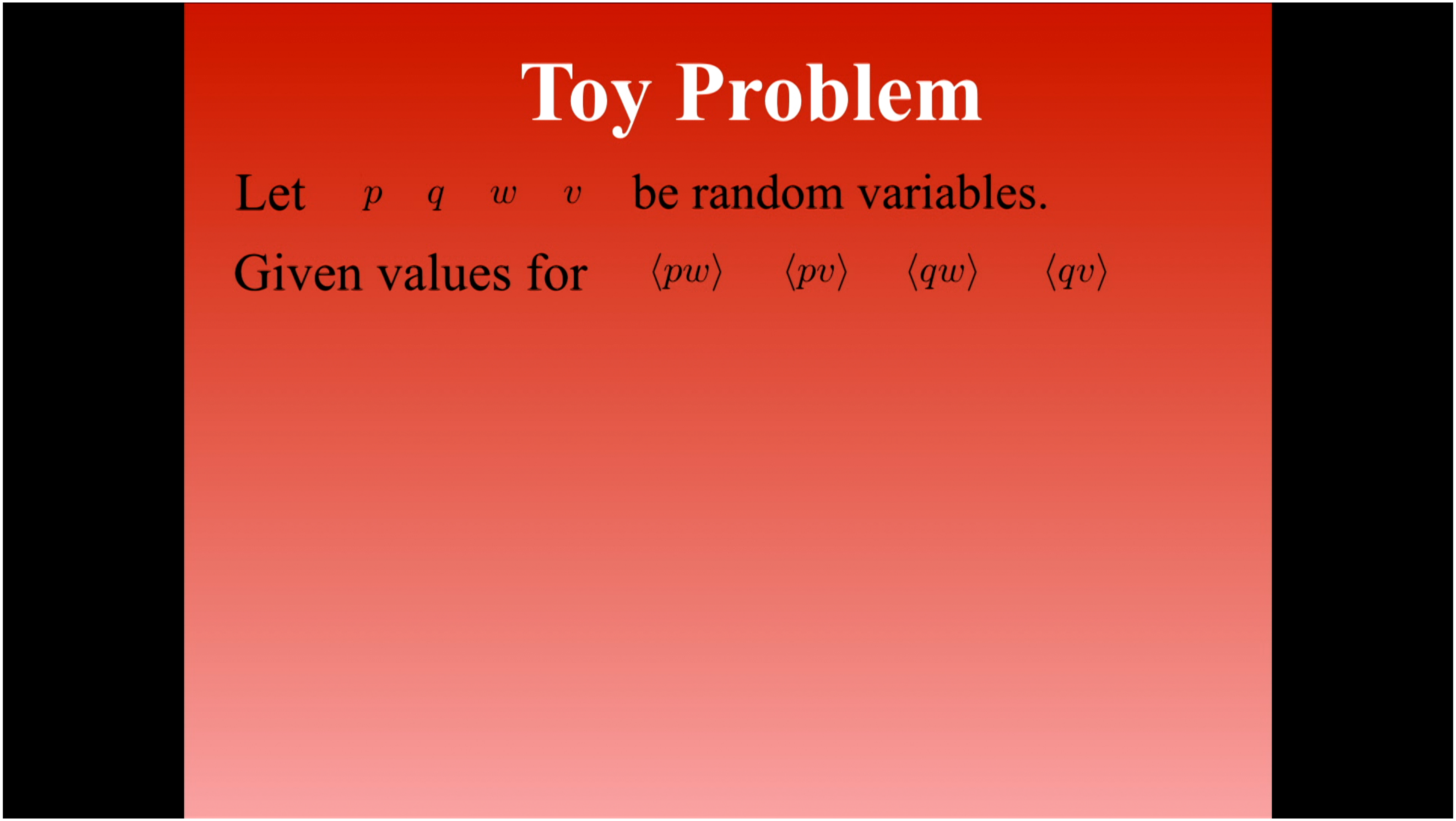
Detector Tomography

If $\Delta = C^{-1} D B^{-1} A = 1$

then $D = C A^{-1} B = P_2 W_2$

Toy Problem

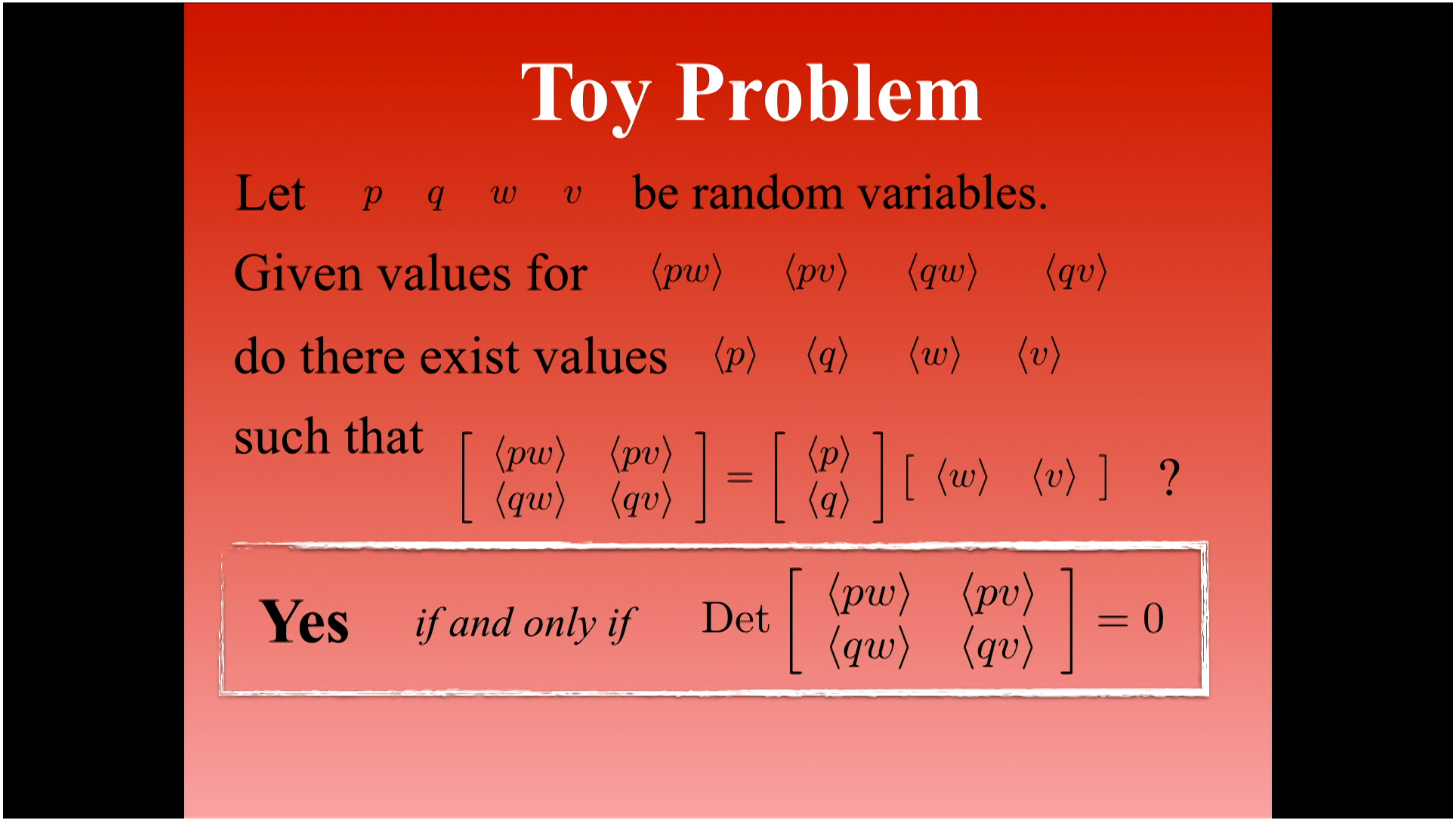
Let p q w v be random variables.



Toy Problem

Let p q w v be random variables.

Given values for $\langle pw \rangle$ $\langle pv \rangle$ $\langle qw \rangle$ $\langle qv \rangle$



Toy Problem

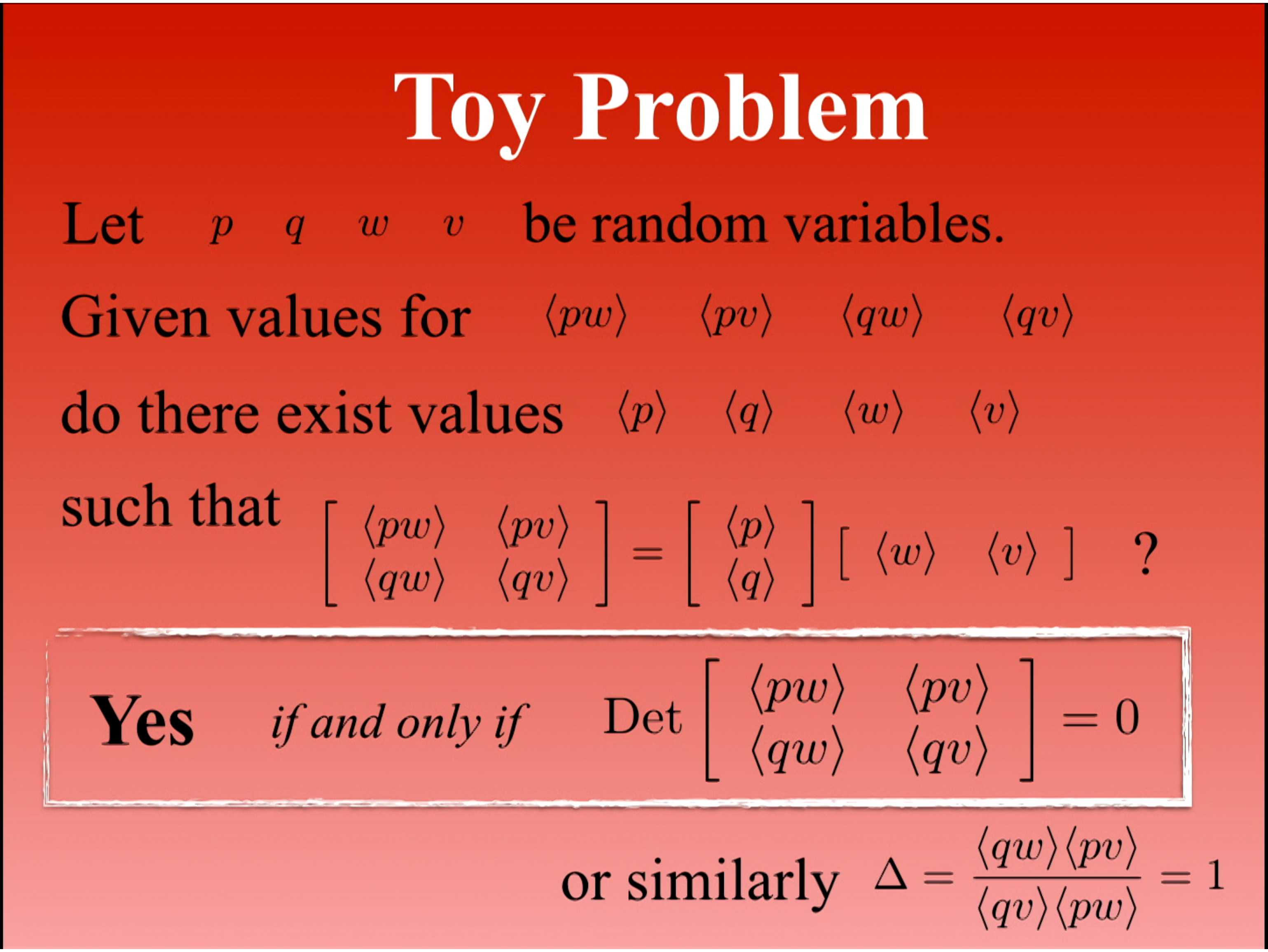
Let p q w v be random variables.

Given values for $\langle pw\rangle$ $\langle pv\rangle$ $\langle qw\rangle$ $\langle qv\rangle$

do there exist values $\langle p\rangle$ $\langle q\rangle$ $\langle w\rangle$ $\langle v\rangle$

such that $$\begin{bmatrix} \langle pw\rangle & \langle pv\rangle \\ \langle qw\rangle & \langle qv\rangle \end{bmatrix} = \begin{bmatrix} \langle p\rangle \\ \langle q\rangle \end{bmatrix} \begin{bmatrix} \langle w\rangle & \langle v\rangle \end{bmatrix} \quad ?$$

Yes *if and only if* $\mathrm{Det}\begin{bmatrix} \langle pw\rangle & \langle pv\rangle \\ \langle qw\rangle & \langle qv\rangle \end{bmatrix} = 0$



Toy Problem

Let $p \quad q \quad w \quad v$ be random variables.

Given values for $\langle pw \rangle \quad \langle pv \rangle \quad \langle qw \rangle \quad \langle qv \rangle$

do there exist values $\langle p \rangle \quad \langle q \rangle \quad \langle w \rangle \quad \langle v \rangle$

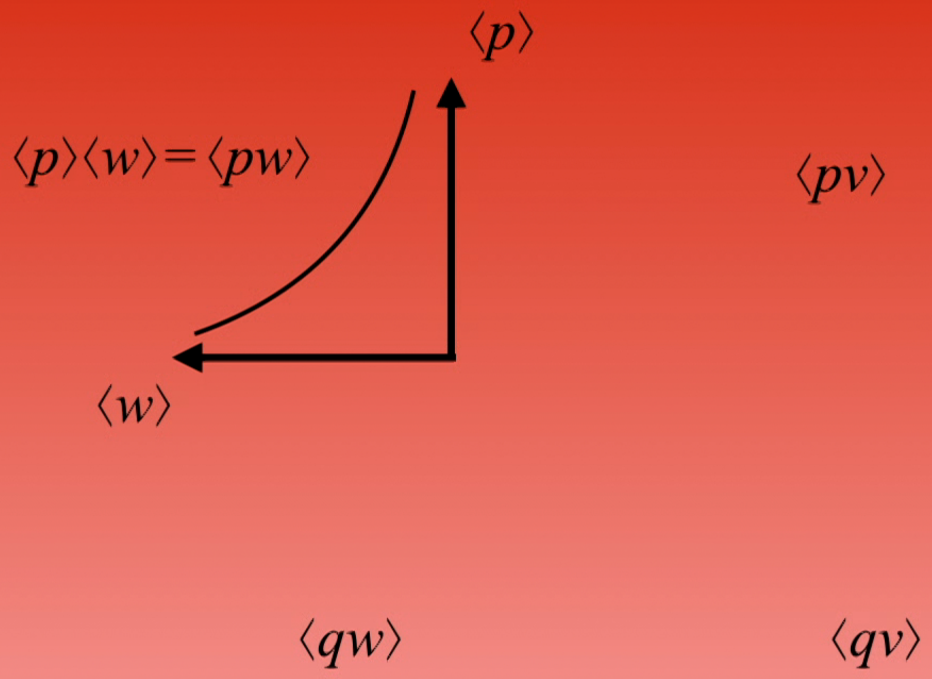
such that
$$\begin{bmatrix} \langle pw \rangle & \langle pv \rangle \\ \langle qw \rangle & \langle qv \rangle \end{bmatrix} = \begin{bmatrix} \langle p \rangle \\ \langle q \rangle \end{bmatrix} \begin{bmatrix} \langle w \rangle & \langle v \rangle \end{bmatrix} \quad ?$$

Yes *if and only if* $\mathrm{Det} \begin{bmatrix} \langle pw \rangle & \langle pv \rangle \\ \langle qw \rangle & \langle qv \rangle \end{bmatrix} = 0$

or similarly $\Delta = \dfrac{\langle qw \rangle \langle pv \rangle}{\langle qv \rangle \langle pw \rangle} = 1$

arXiv:1608.08067

Loops and Holonomy



$\langle p \rangle$

$\langle p \rangle \langle w \rangle = \langle pw \rangle$

$\langle pv \rangle$

$\langle w \rangle$

$\langle qw \rangle$

$\langle qv \rangle$

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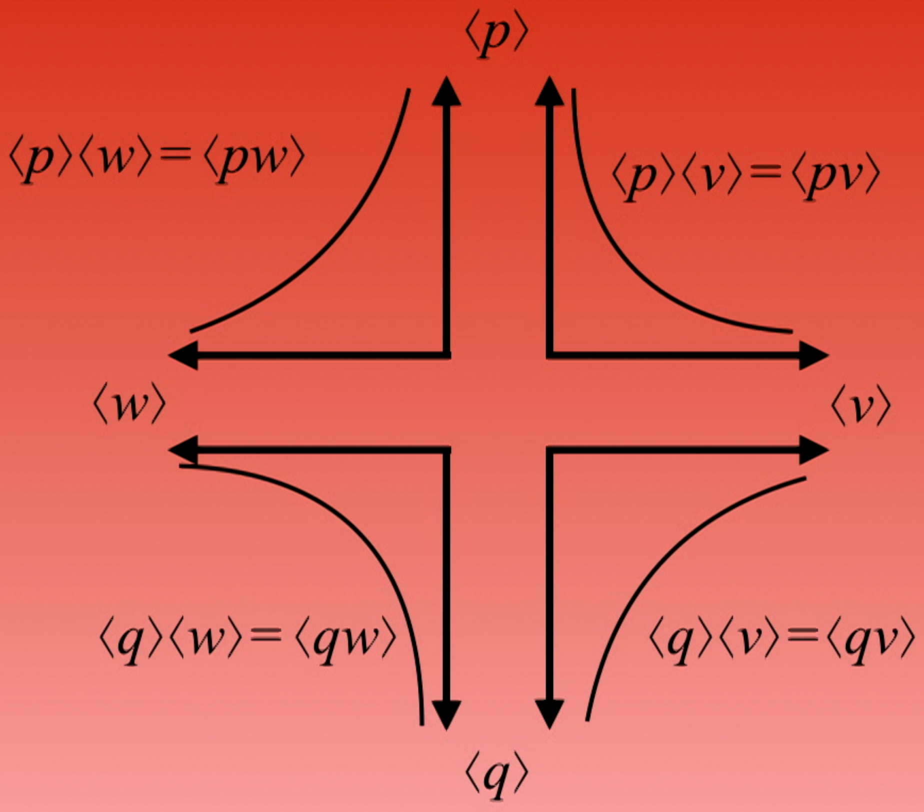
Loops and Holonomy



$\langle p \rangle$

$\langle p \rangle \langle w \rangle = \langle pw \rangle$

$\langle p \rangle \langle v \rangle = \langle pv \rangle$

$\langle w \rangle$

$\langle v \rangle$

$\langle q \rangle \langle w \rangle = \langle qw \rangle$

$\langle q \rangle \langle v \rangle = \langle qv \rangle$

$\langle q \rangle$

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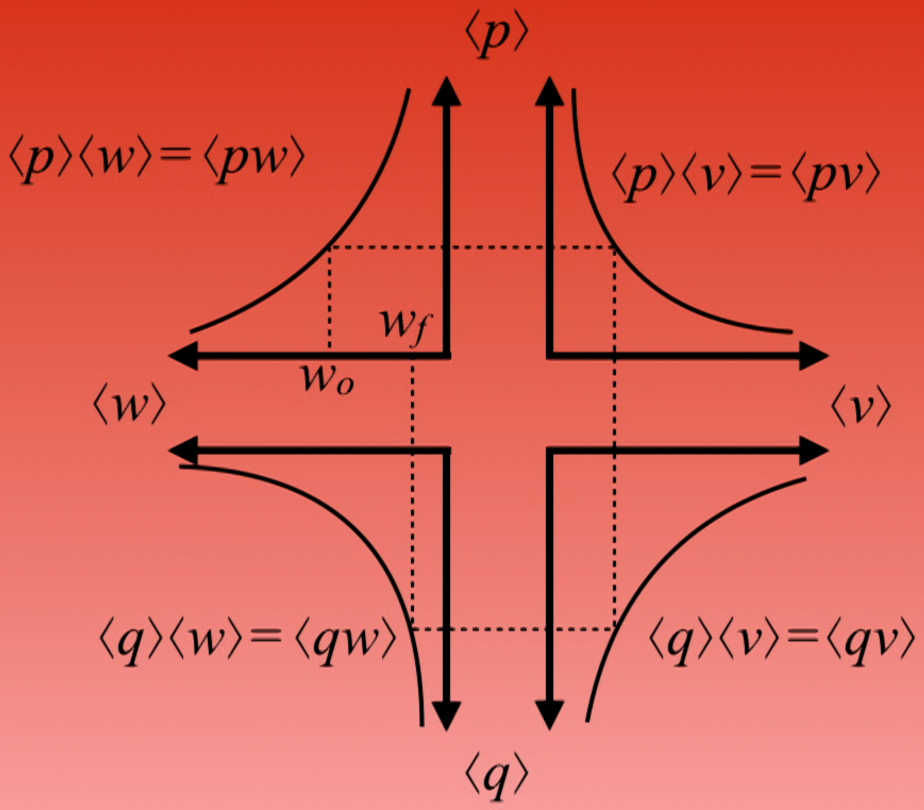
Loops and Holonomy



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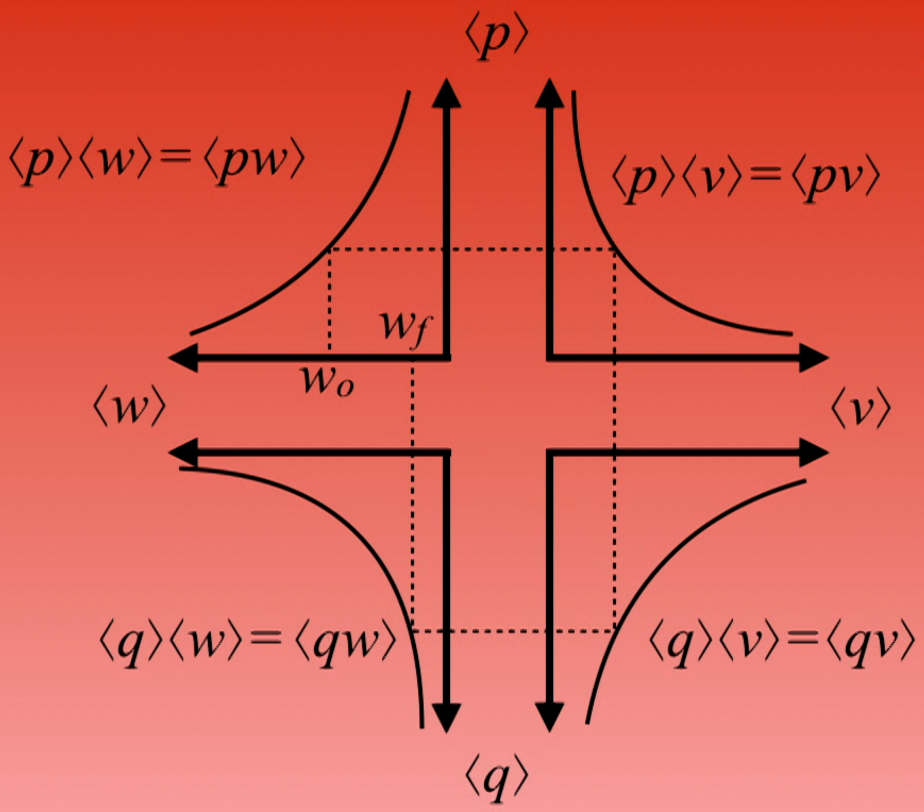
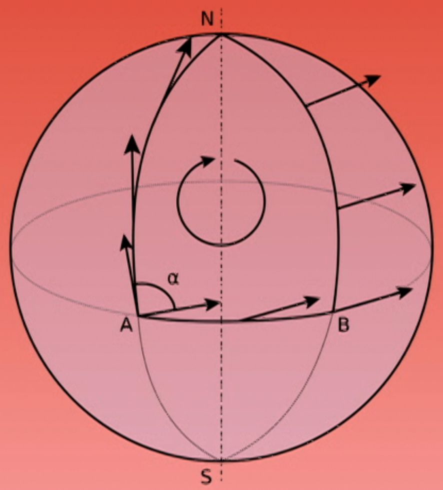
Loops and Holonomy



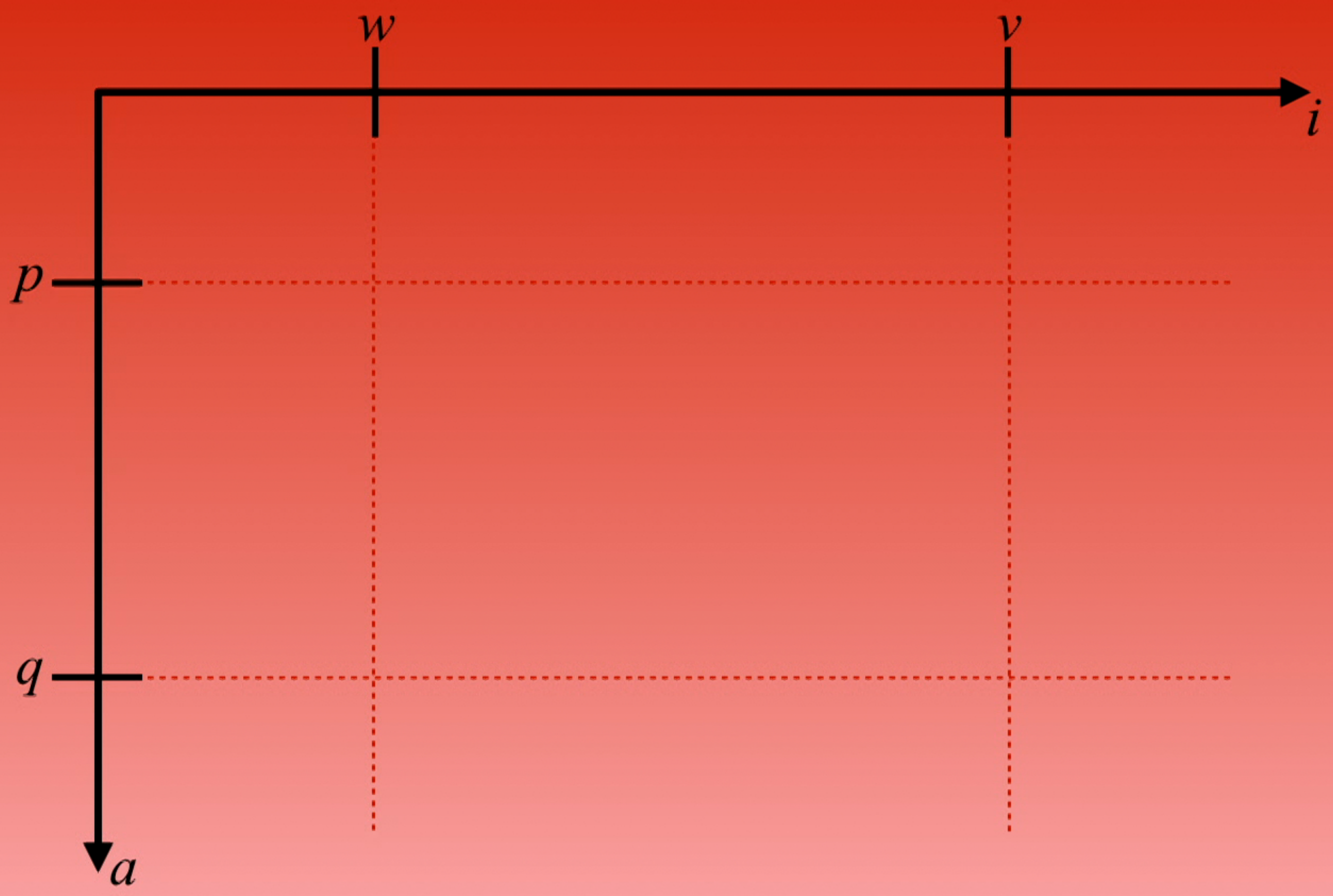
$$\Delta = \frac{w_f}{w_o} = \frac{\langle qw\rangle\langle pv\rangle}{\langle qv\rangle\langle pw\rangle}$$

Gauge Invariant

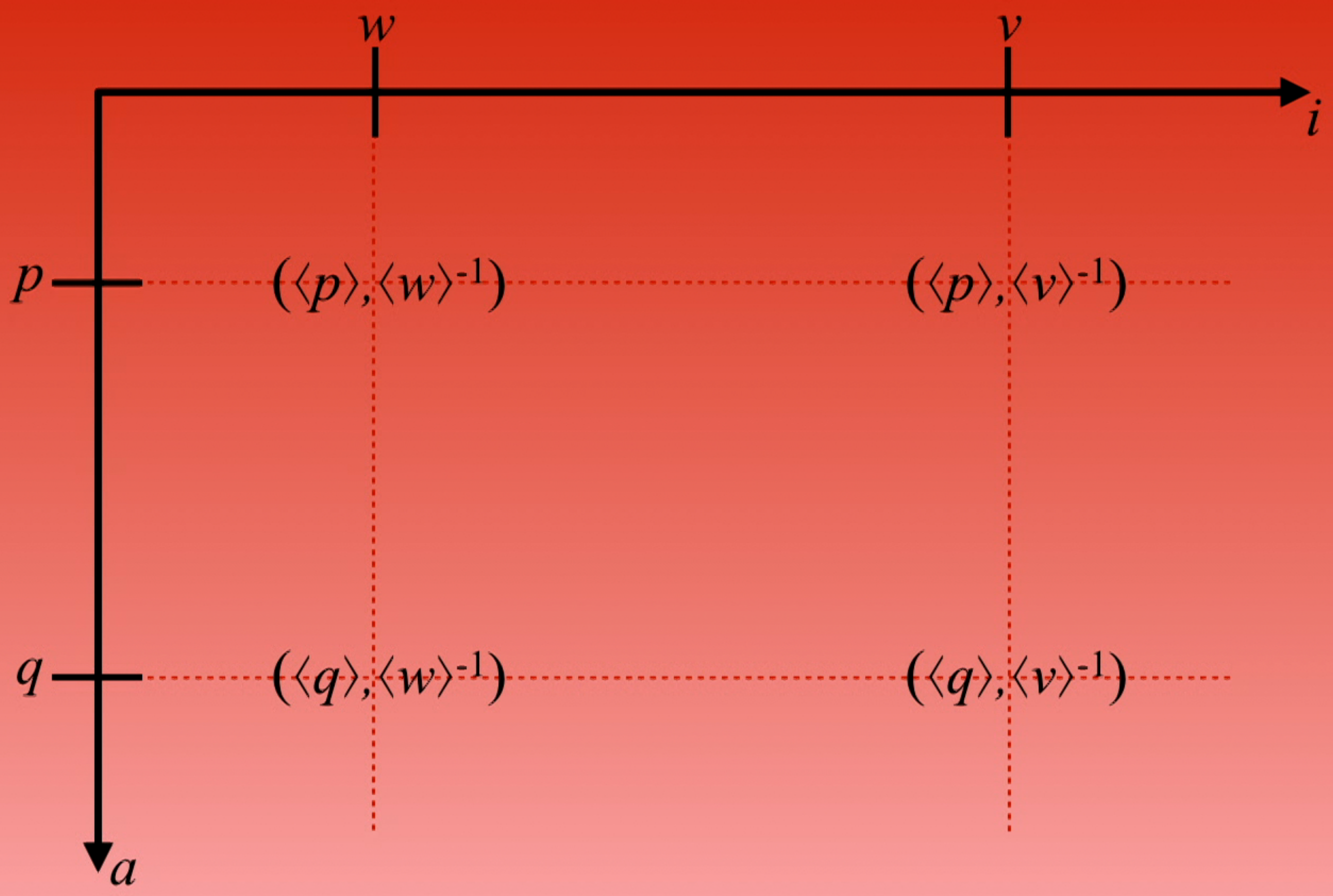


“ $\Delta = e^{\alpha}$ ”

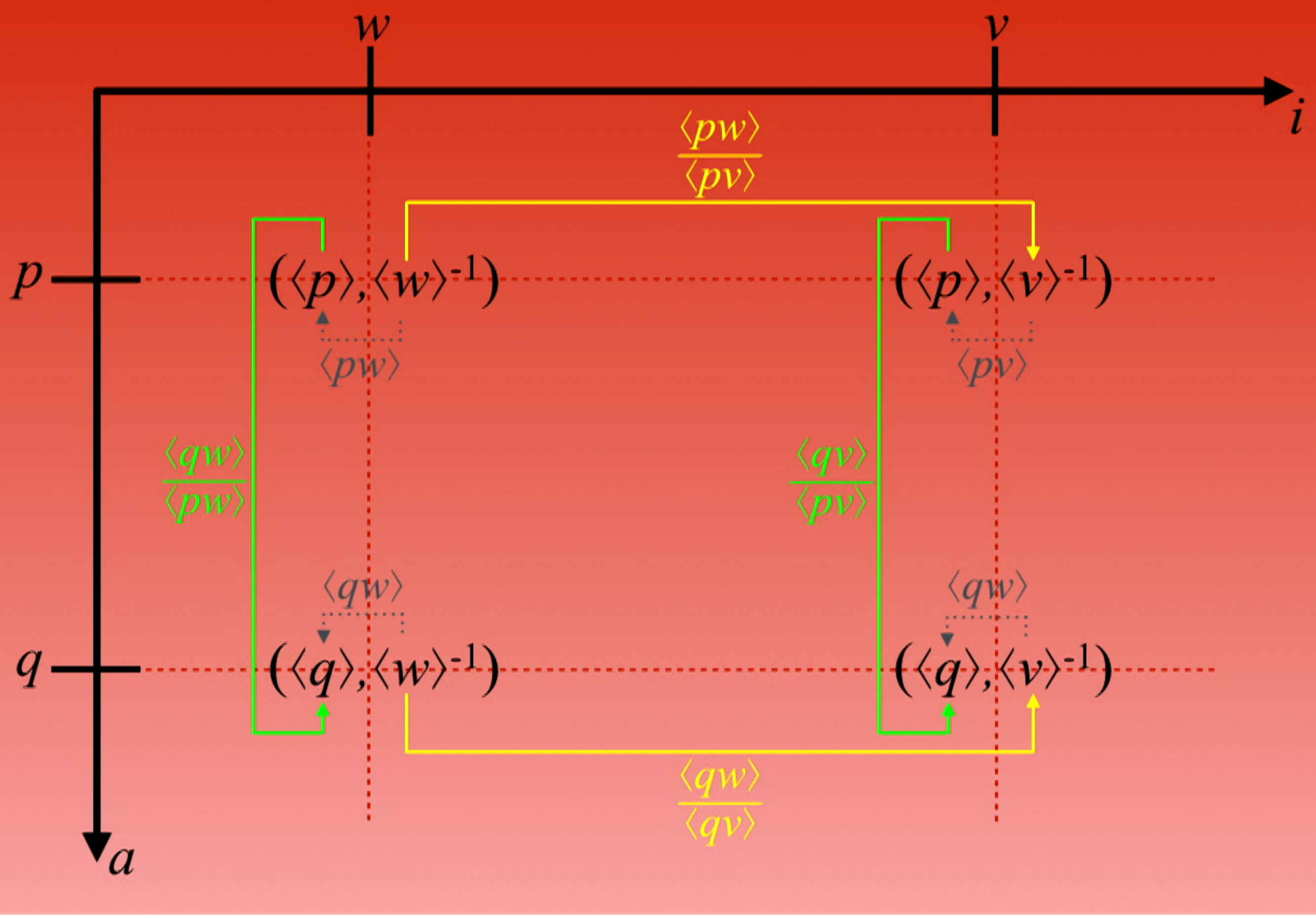
Setting Space



Setting Space



Setting Space



w v i

$\frac{\langle pw\rangle}{\langle pv\rangle}$

p $(\langle p\rangle,\langle w\rangle^{-1})$ $(\langle p\rangle,\langle v\rangle^{-1})$

$\langle pw\rangle$ $\langle pv\rangle$

$\frac{\langle qw\rangle}{\langle pw\rangle}$ $\frac{\langle qv\rangle}{\langle pv\rangle}$

$\langle qw\rangle$ $\langle qw\rangle$

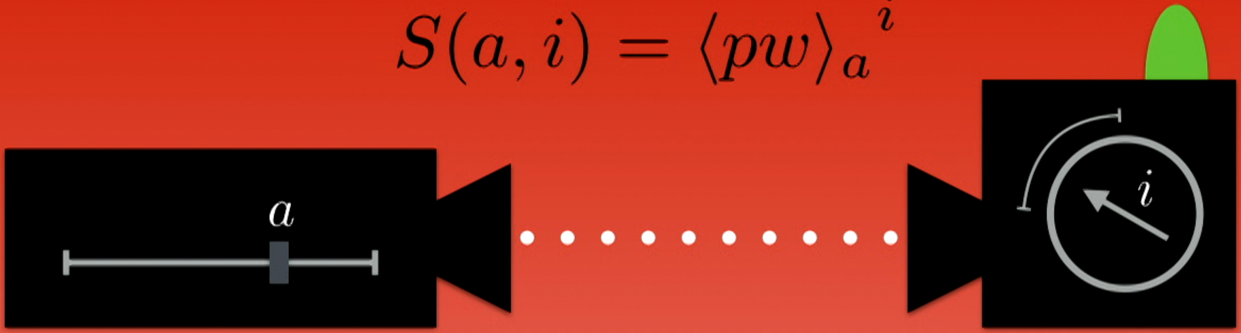
q $(\langle q\rangle,\langle w\rangle^{-1})$ $(\langle q\rangle,\langle v\rangle^{-1})$

$\frac{\langle qw\rangle}{\langle qv\rangle}$

a

Data Susceptibilities

$$S(a,i) = \langle pw \rangle_a{}^i$$

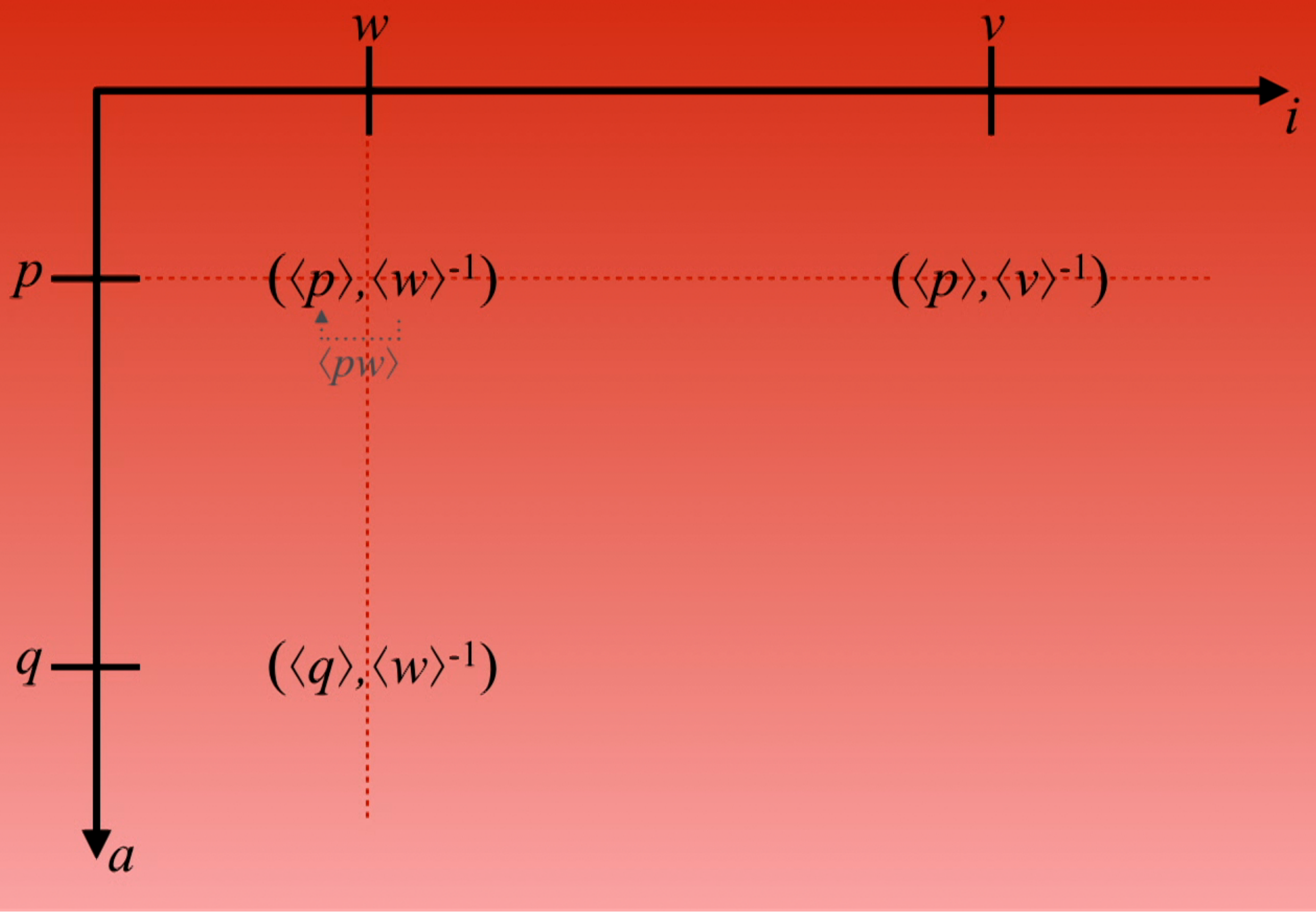


$$\chi = \frac{\partial}{\partial a}\bigg|_i \log S \qquad \xi = -\frac{\partial}{\partial i}\bigg|_a \log S$$

$$d\log S = \chi da - \xi di$$

$$d\log\langle pw\rangle = \text{đ}\log\langle p\rangle - \text{đ}\log\langle w\rangle^{-1}$$

“Tomography Lines” $\Delta = e^{\int_\gamma \tau}$



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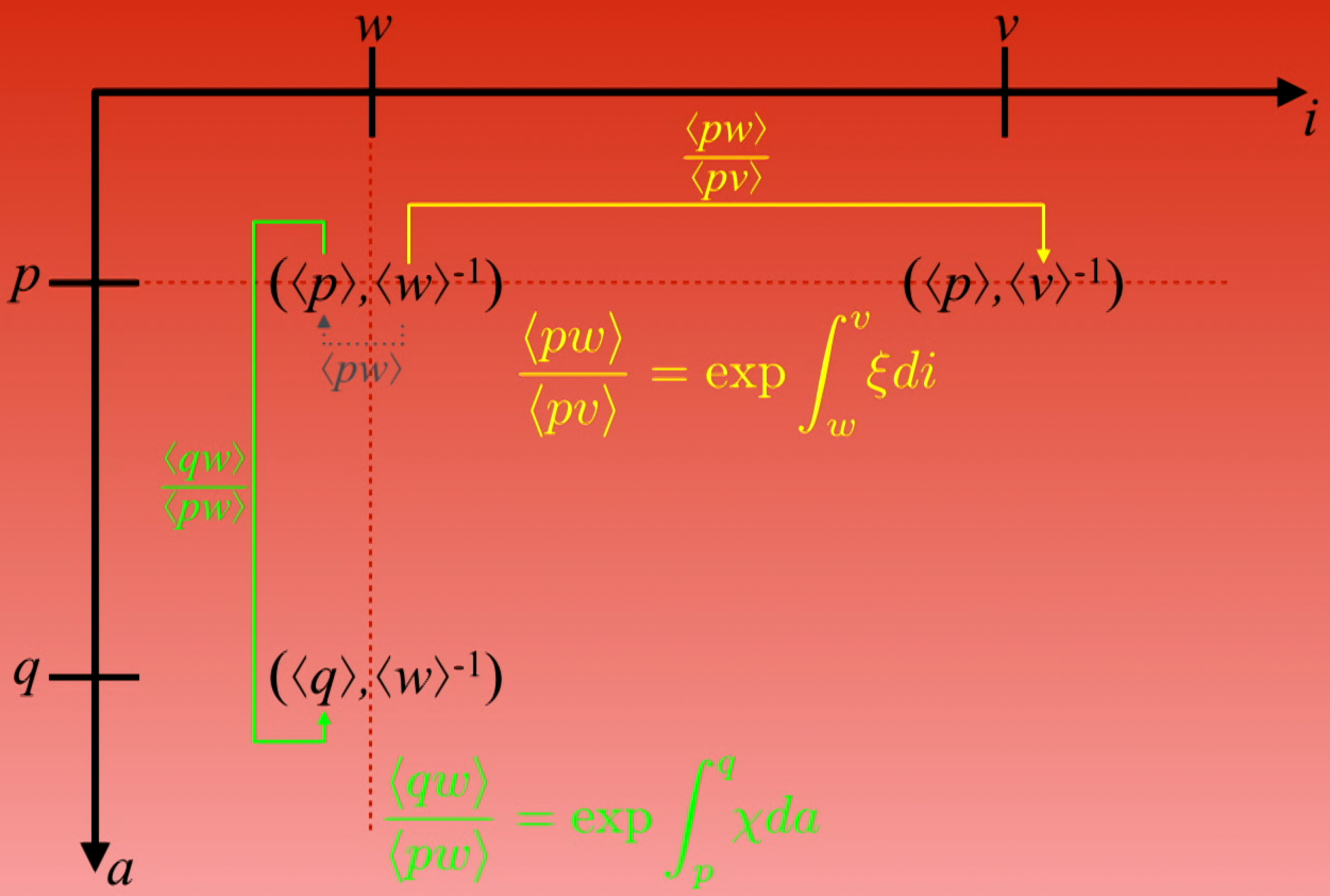
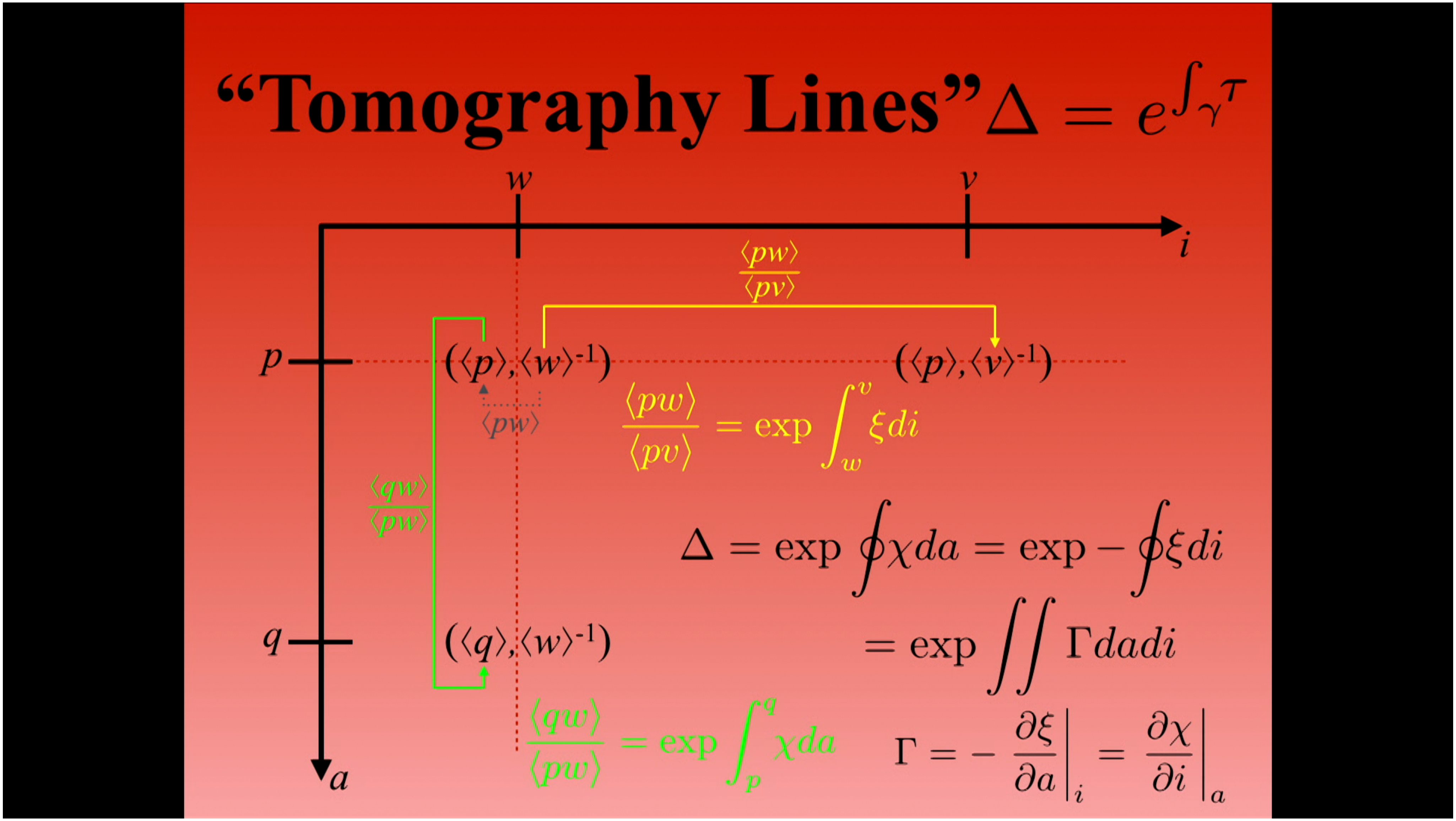


$$\frac{\langle pw\rangle}{\langle pv\rangle} = \exp \int_w^v \xi \, di$$

$$\frac{\langle qw\rangle}{\langle pw\rangle} = \exp \int_p^q \chi \, da$$

"Tomography Lines" $\Delta = e^{\int_\gamma \tau}$



$$\frac{\langle pw\rangle}{\langle pv\rangle} = \exp\int_w^v \xi di$$

$$\Delta = \exp\oint \chi da = \exp - \oint \xi di$$

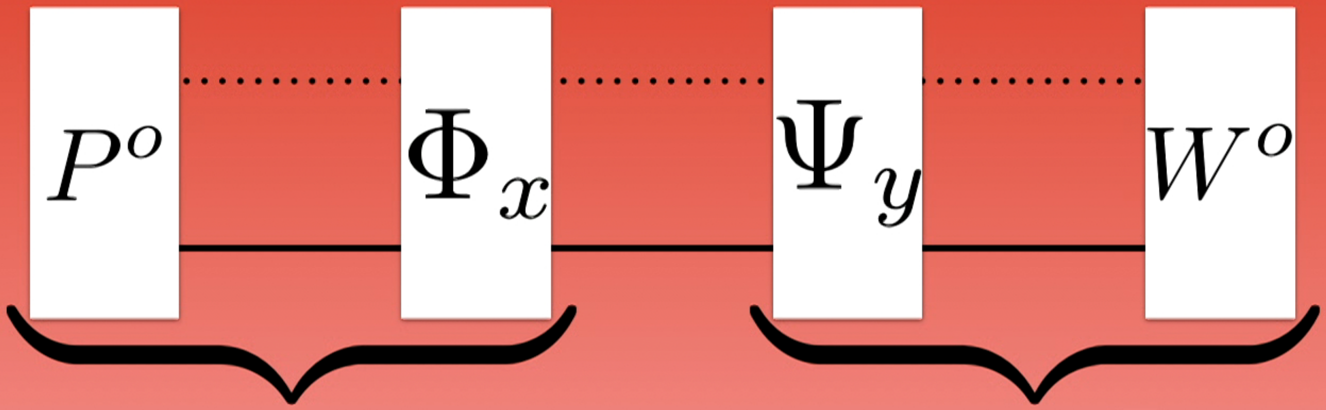
$$= \exp\iint \Gamma dadi$$

$$\frac{\langle qw\rangle}{\langle pw\rangle} = \exp\int_p^q \chi da$$

$$\Gamma = - \left.\frac{\partial\xi}{\partial a}\right|_i = \left.\frac{\partial\chi}{\partial i}\right|_a$$

Potential Application

Non-Markonianity in Quantum Information Processing



$$P_x = P^o {\Phi_x}^\top \qquad W_y = {\Psi_y}^\top W^o$$

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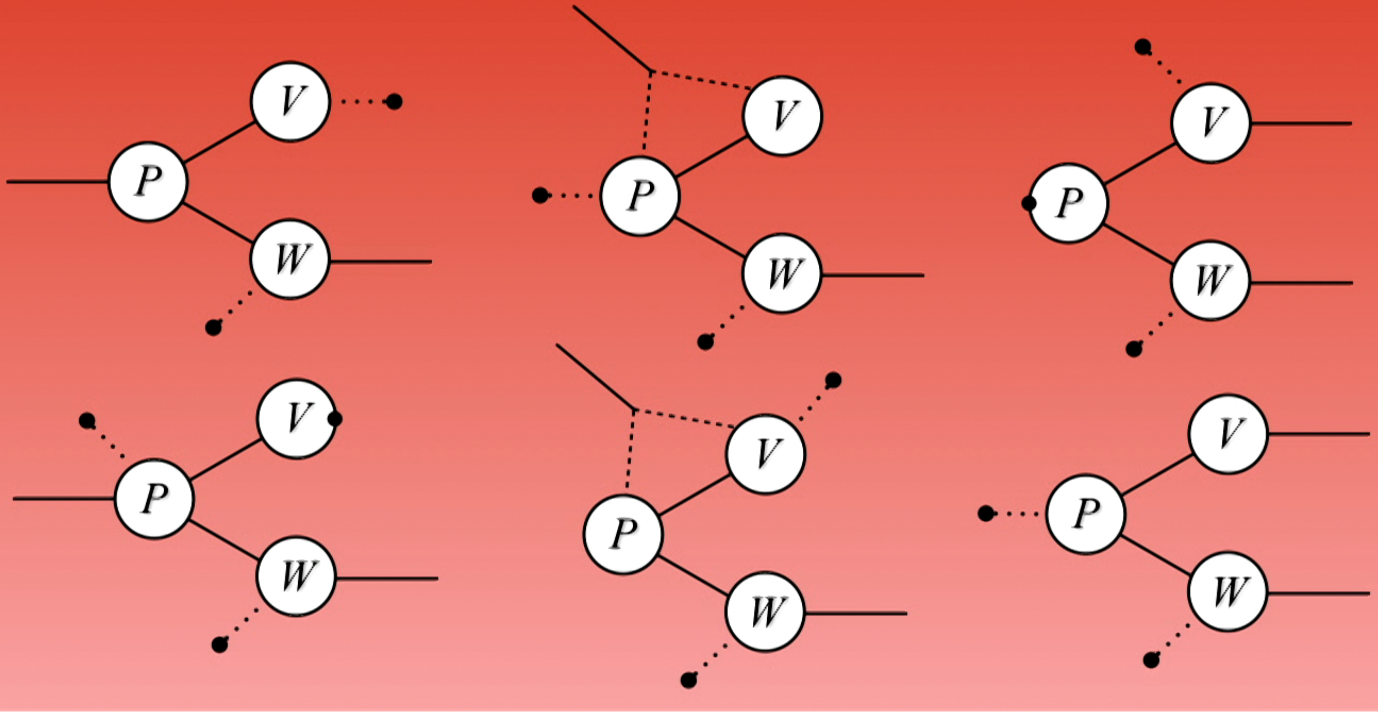
Multi-Qudit Systems

$$S_A{}^{ij} = \mathrm{Tr}\rho_A \Sigma^i \otimes \Sigma^j \qquad S_A{}^{ij} = P_A{}^{\mu\nu} V_\mu{}^i W_\nu{}^j$$

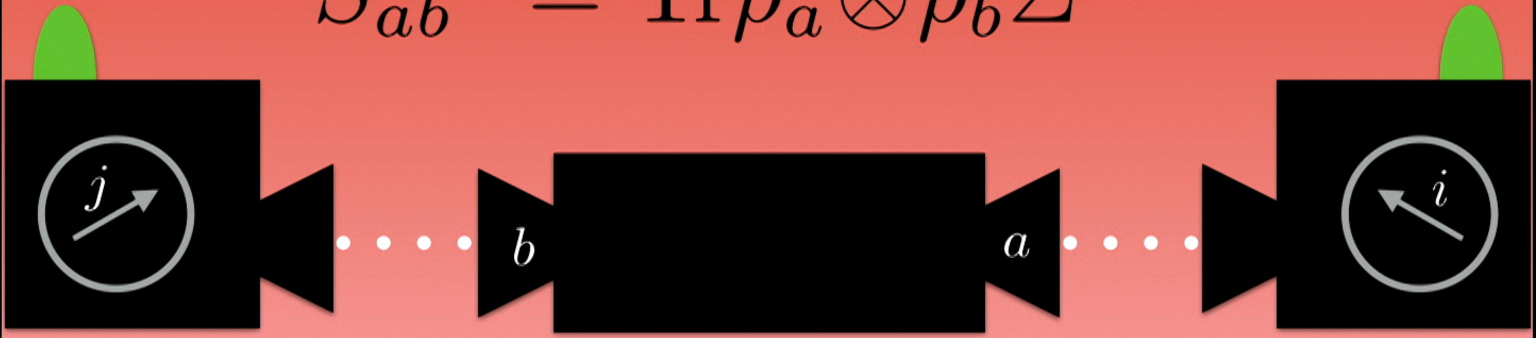


Potential Application

Faked Bell Inequalities in Quantum Key Distribution

$$S_A{}^{ij} = \mathrm{Tr}\rho_A \Sigma^i \otimes \Sigma^j$$

$$S_{ab}{}^{I} = \mathrm{Tr}\rho_a \otimes \rho_b \Sigma^{I|ab}$$



Potential Application

Compressed Sensing and Low-rank Tomography

$$\begin{bmatrix} A & B \\ \underset{\underset{r}{\longleftrightarrow}}{C} & D \end{bmatrix} \Big\updownarrow r$$

$$C^{-1}DB^{-1}A = 1$$

Potential Application

Compressed Sensing and Low-rank Tomography

$$\begin{array}{l} \left[\begin{array}{cc} A & B \\ C & D \end{array}\right] \updownarrow r \\ \quad \underset{r}{\longleftrightarrow} \end{array} \qquad\qquad C^{-1}DB^{-1}A = 1$$

$$\bar{C}D\bar{B}A - (\det B)(\det C)1 = 0$$

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Thank You!