Title: The ABCs of color codes

Date: Dec 12, 2016 04:00 PM

URL: http://pirsa.org/16120028

Abstract: <p>To build a fully functioning quantum computer, it is necessary to encode quantum information to protect it from noise. Topological codes, such as the color code, naturally protect against local errors and represent our best hope for storing quantum information. Moreover, a quantum computer must also be capable of processing this information. Since the color code has many computationally valuable transversal logical gates, it is a promising candidate for a future quantum computer architecture.</p>

<p>In the talk, I will provide an overview of the color code. First, I will establish a connection between the color code and a well-studied model - the toric code. Then, I will explain how one can implement a universal gate set with the subsystem and the stabilizer color codes in three dimensions using techniques of code switching and gauge fixing. Next, I will discuss the problem of decoding the color code. Finally, I will explain how one can find the optimal error correction threshold by analyzing phase transitions in certain statistical-mechanical models.</p>

<p>The talk is based on http://arxiv.org/abs/1410.0069, http://arxiv.org/abs/1503.02065 and recent works with M. Beverland, F. Brandao, N. Delfosse, J. Preskill and K. Svore.</p>

THE ABC OF COLOR CODES

Aleksander Kubica

INSTITUTE FOR QUANTUM INFORMATION AND MATTER

arXiv:1410.0069, 1503.02065

ongoing work w/ M. Beverland, F. Brandao, N. Delfosse, J. Preskill, K. Svore

1

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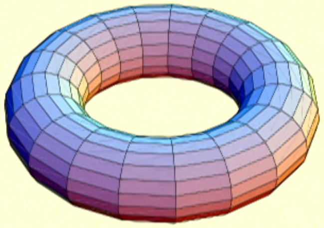
- Want to perform computation - logical operations should be fault-tolerant and easy to implement.

TOPOLOGICAL QUANTUM CODES

- Topological quantum codes - (geometrically) local generators, encode information in non-local degrees of freedom.



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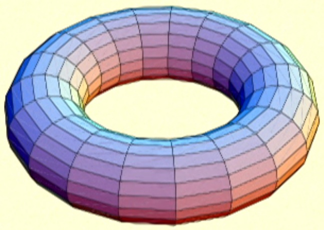

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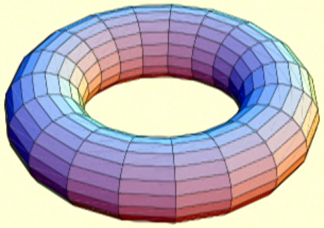

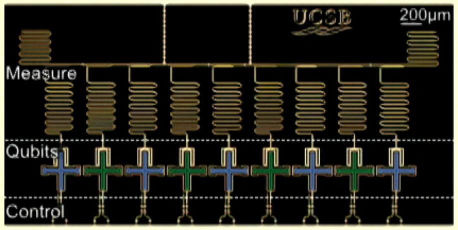

Kelly et al., Nature 519, 66–69 (2015)

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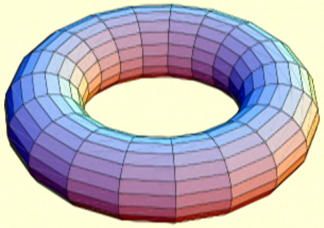

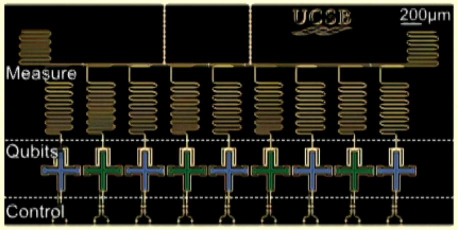

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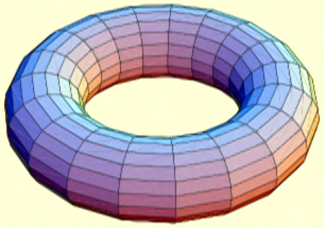

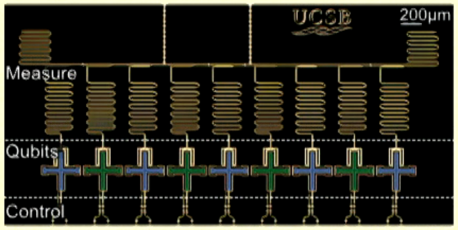

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- Side remark: topological codes as (exactly solvable) toy models for classification of quantum phases.

OUTLINE

COLOR CODE

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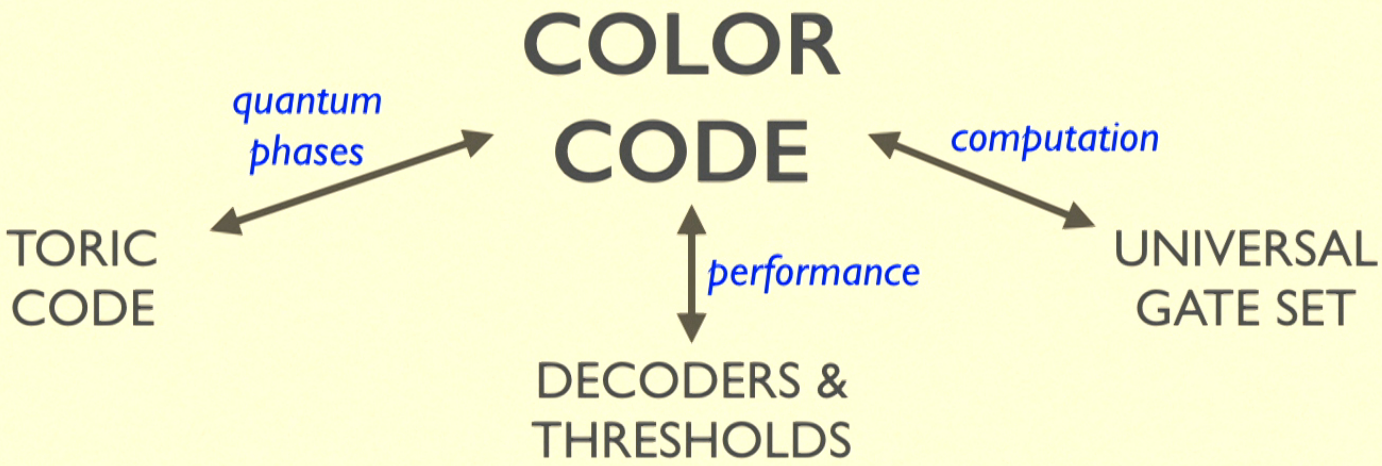
COLOR CODE

TORIC CODE ⟷ *quantum phases* ⟷ COLOR CODE

COLOR CODE ⟷ *computation* ⟷ UNIVERSAL GATE SET

OUTLINE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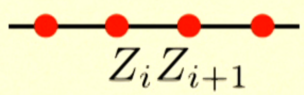

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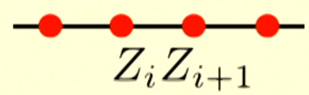

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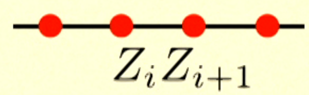

- Repeated stabilizer measurements needed to perform error correction.
- Generalization: subsystem code specified by a gauge group (might be non-Abelian).

TOPOLOGICAL QUANTUM CODE: 2D TORIC CODE

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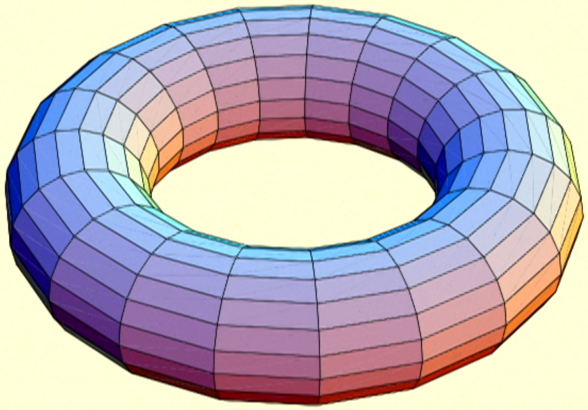

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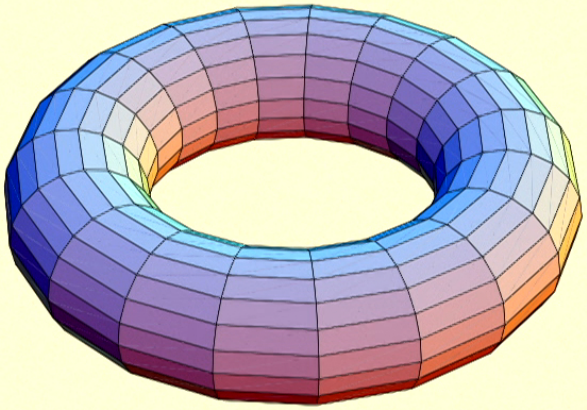
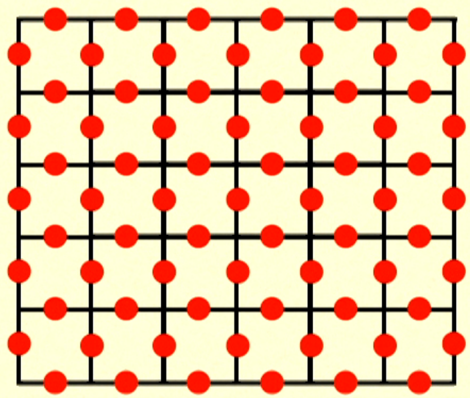


- qubits on edges
- X-vertex and Z-plaquette terms



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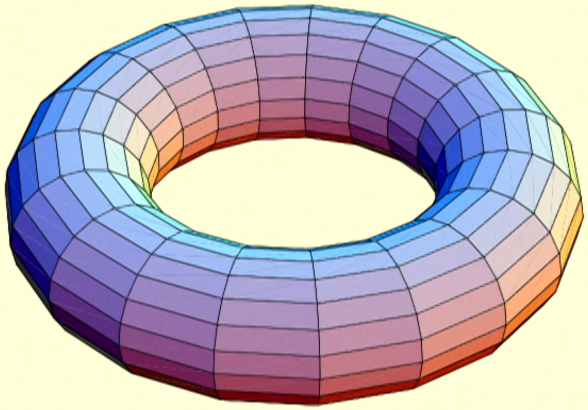

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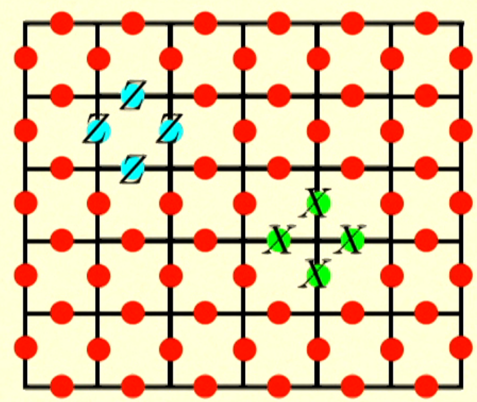

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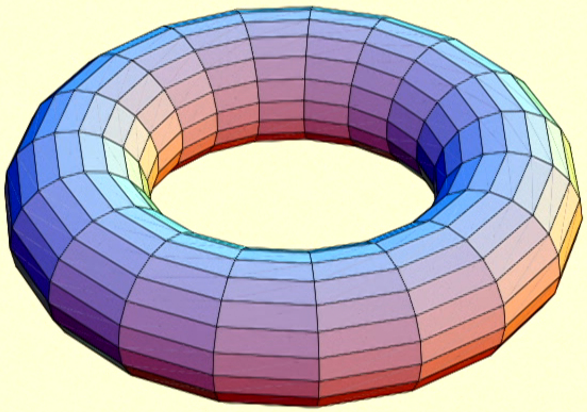

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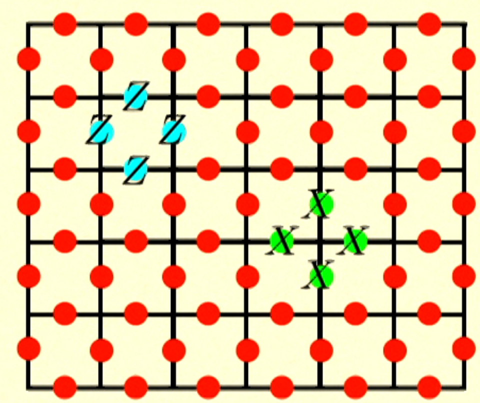

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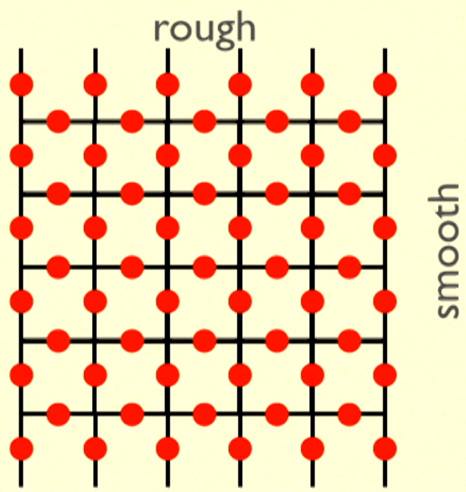
- degeneracy(C) = 2^2

2D TORIC CODE WITH BOUNDARIES

- 2 dim toric code with boundaries: rough and smooth



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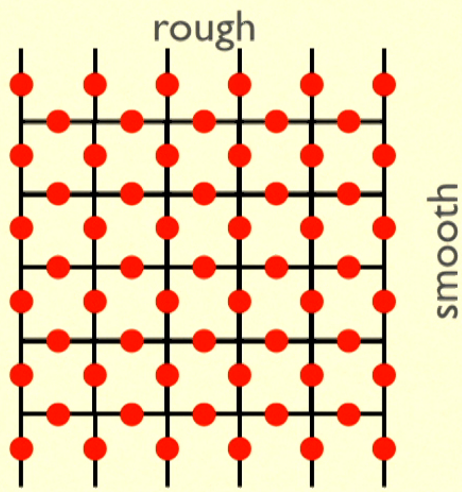

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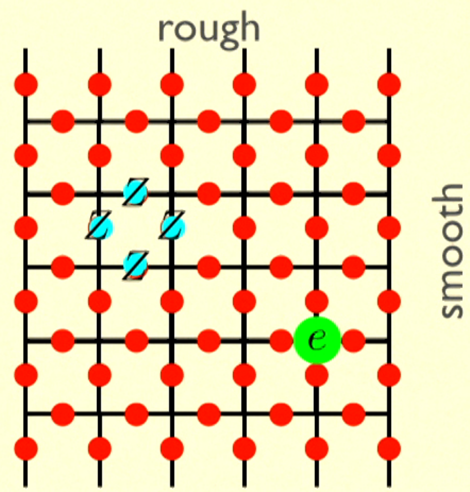

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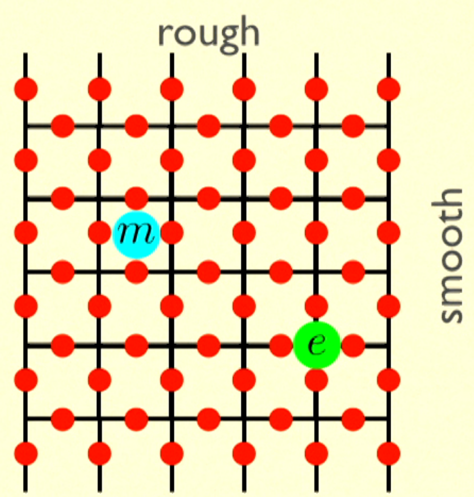

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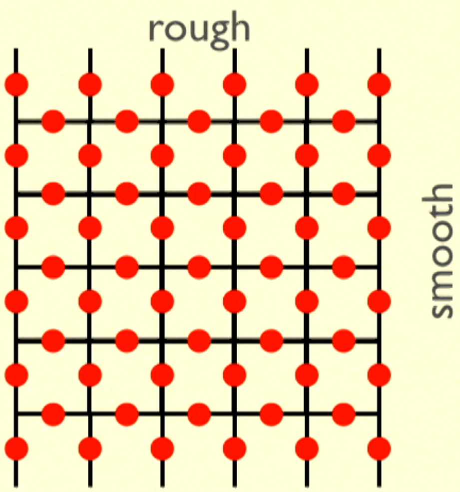
- 1 logical qubit and string-like logical operators



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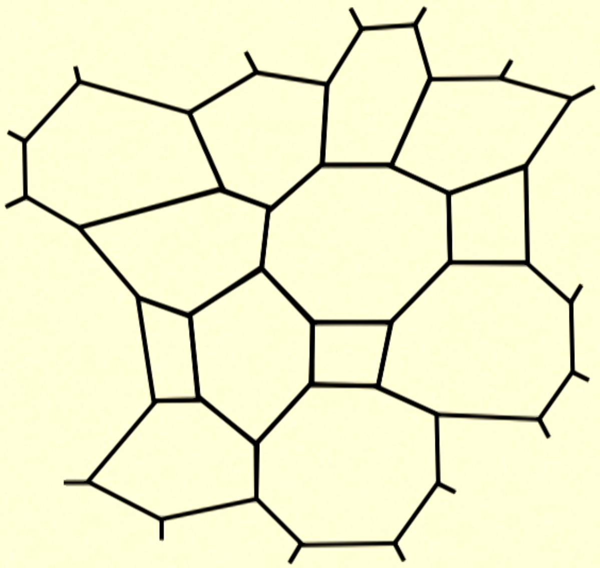

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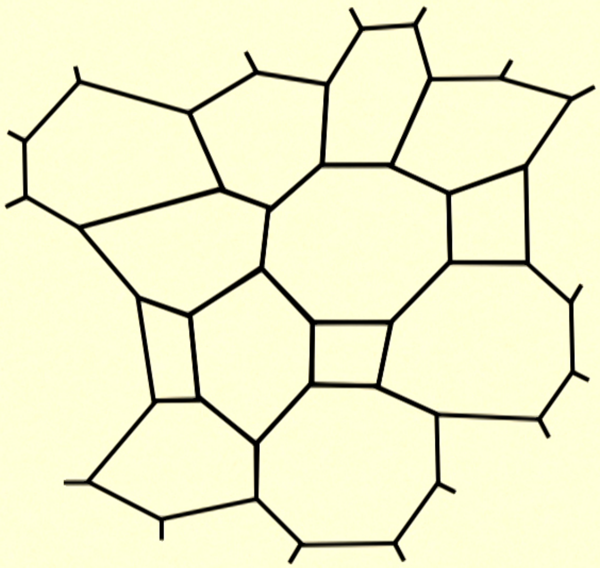

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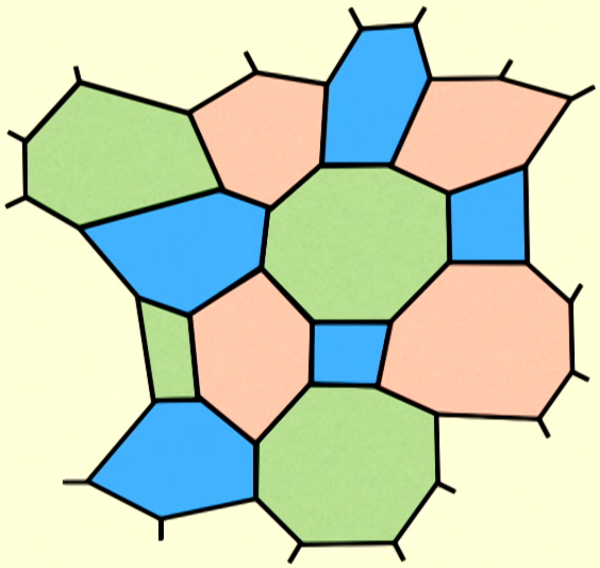

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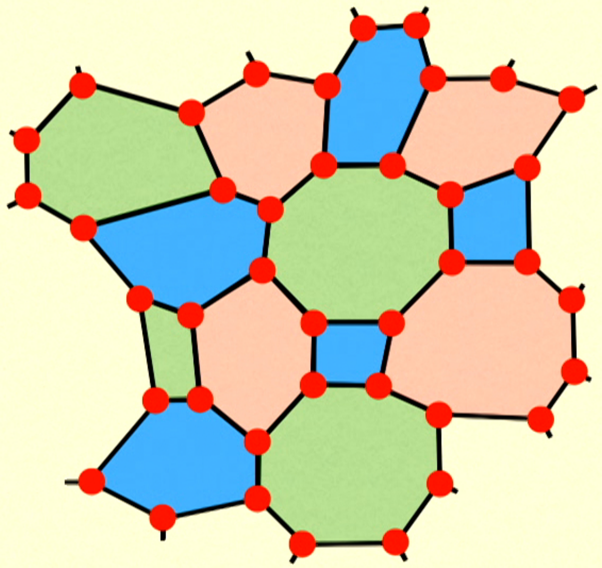

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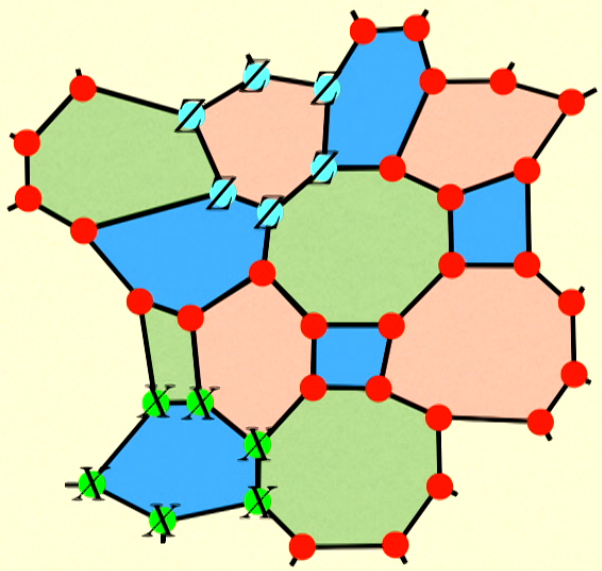

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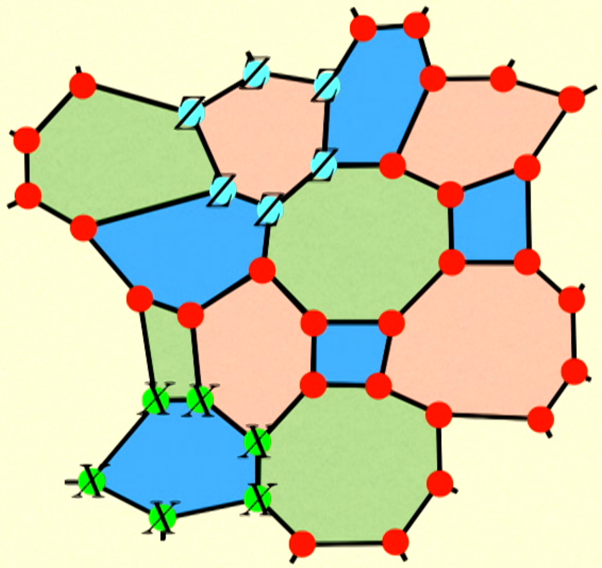

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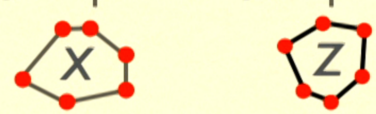

$$\forall p, p' : \; [X(p), Z(p')] = 0$$

- code space C = ground space of H
- degeneracy(C) = 2^{4g}, where g - genus

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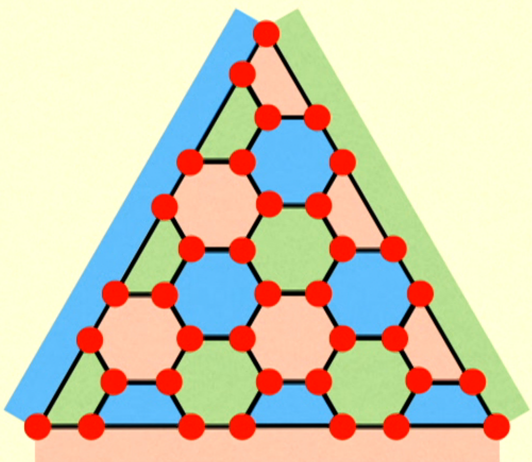

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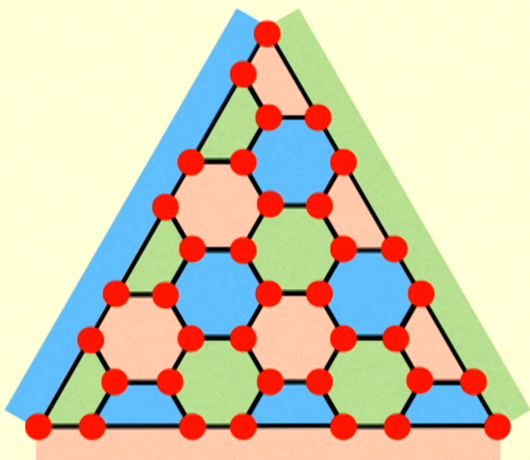

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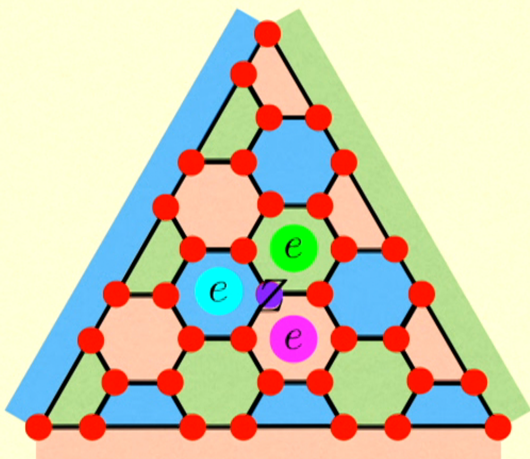

- **Main results (Kubica, Yoshida, Pastawski'15):**
  - in $d \geq 2$ dim, color code equivalent to toric code
  - explicit construction of unitary mapping

CLASSIFICATION OF QUANTUM PHASES OF MATTER

arXiv:1503.02065

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- **Yoshida'10, Bombin'11**: 2D stabilizer Hamiltonians w/ local interactions, translation and scale symmetries are equivalent to toric code.
- **Our results**: beyond 2D, any lattice, include boundaries, explicit mapping!

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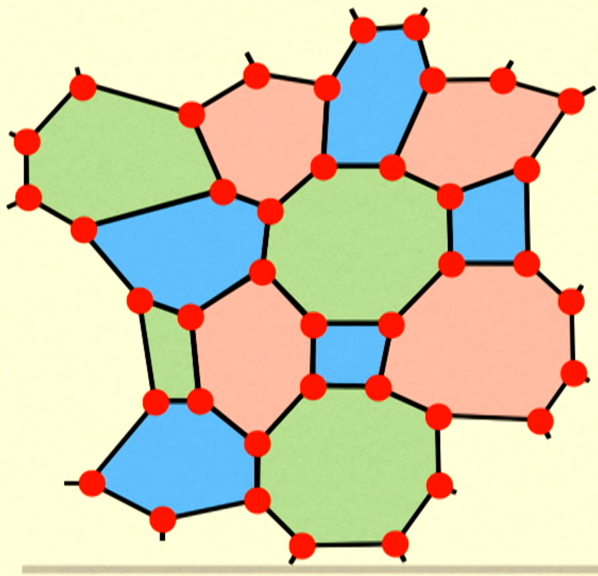
TRANSFORMATION IN 2D

- Color code encodes twice as many logical qubits as toric code. This suggests that CC = 2 x TC.



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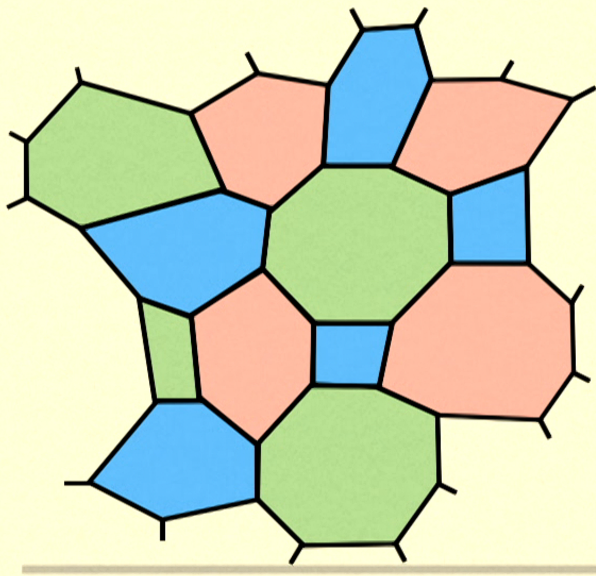

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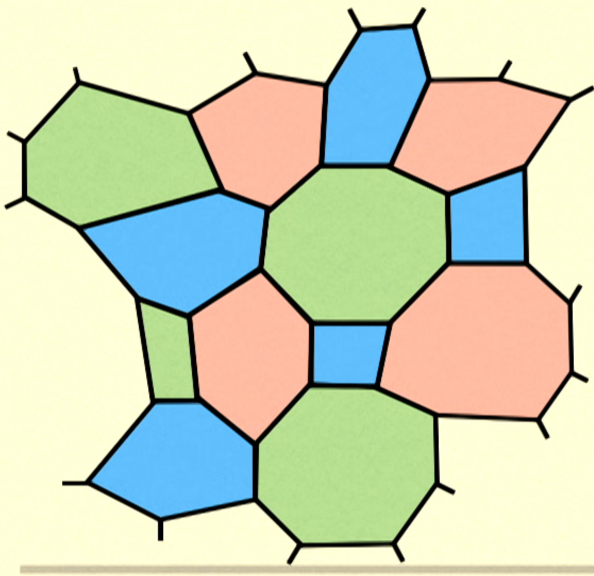

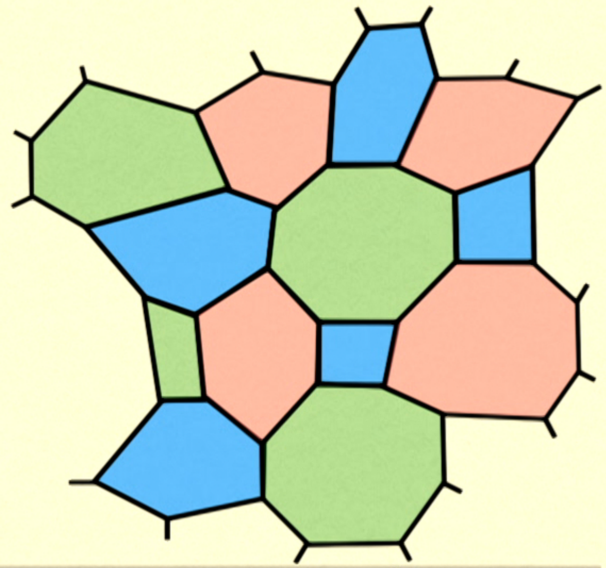

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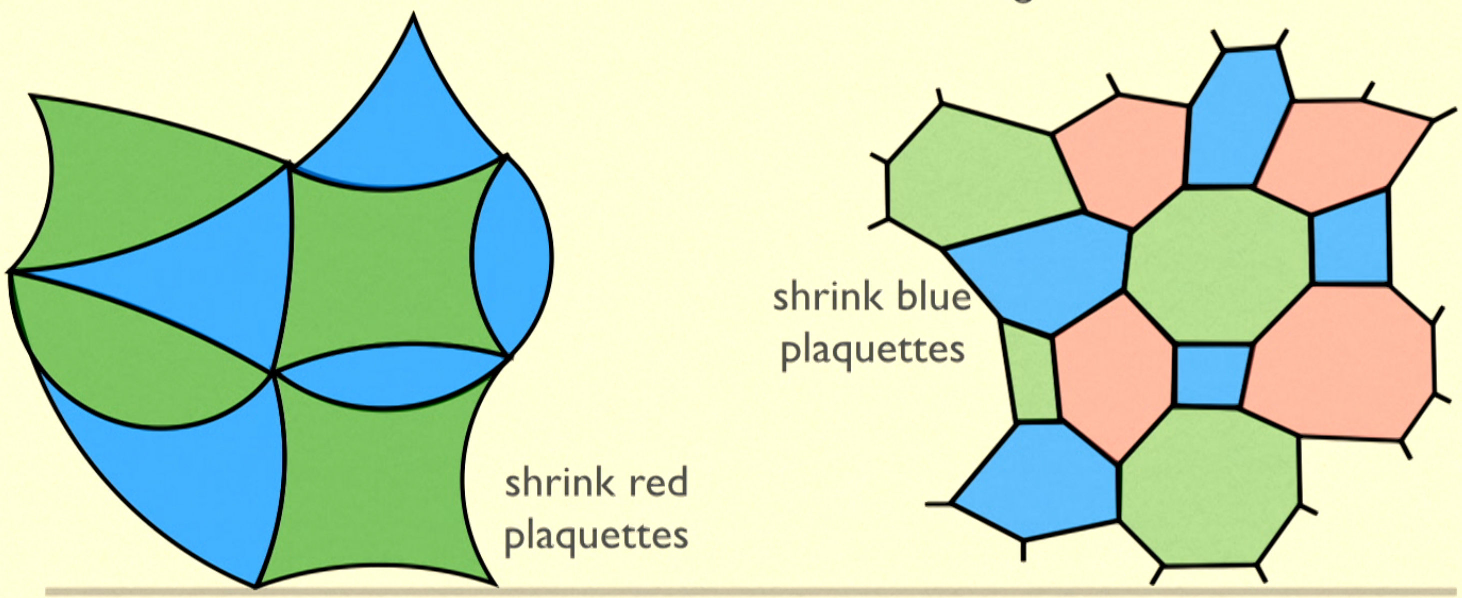

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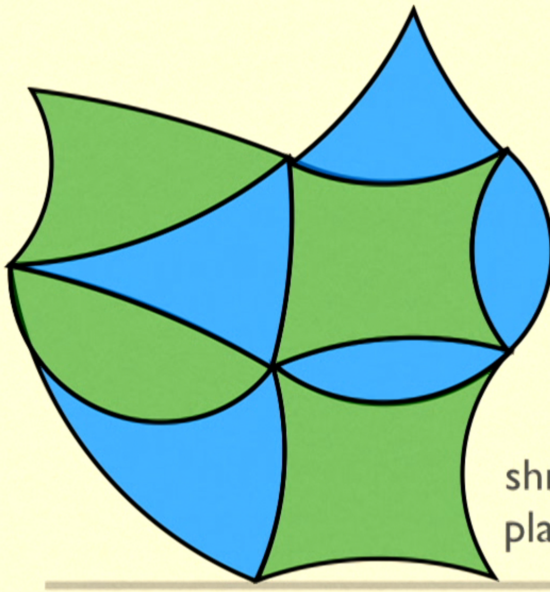
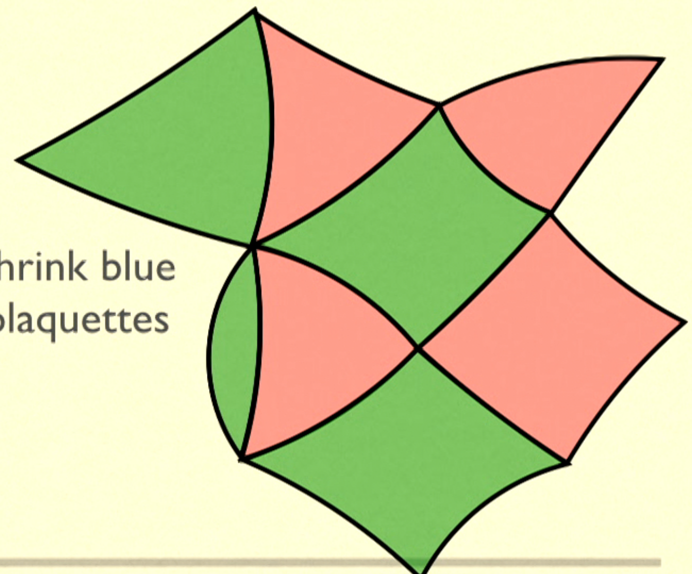


shrink red plaquettes

shrink blue plaquettes



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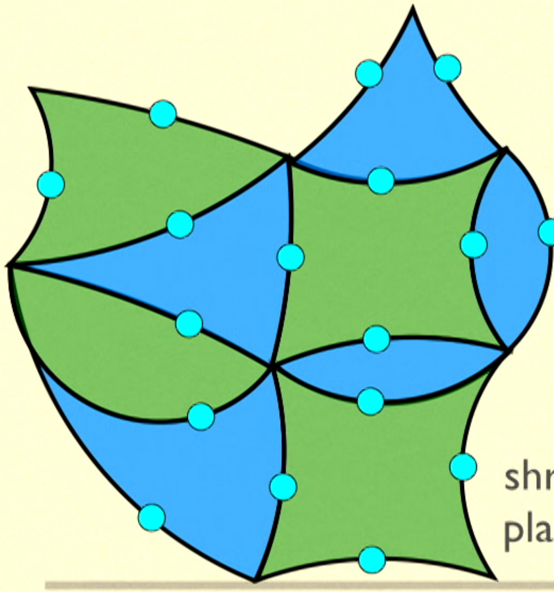

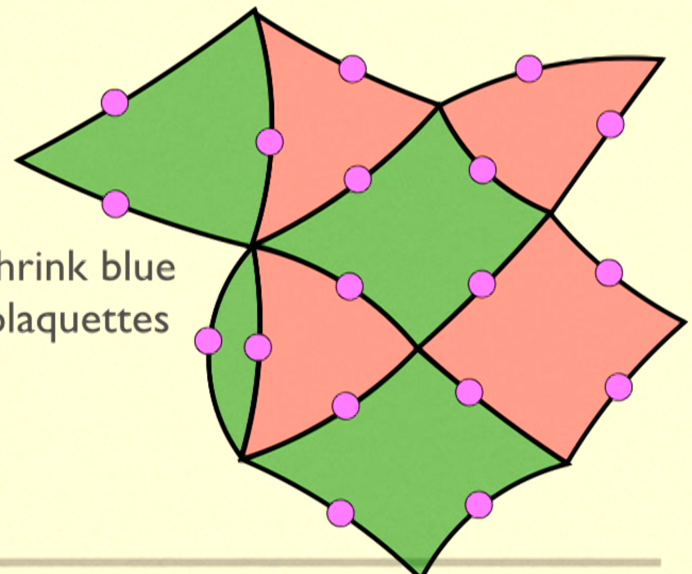

arXiv:1503.02065

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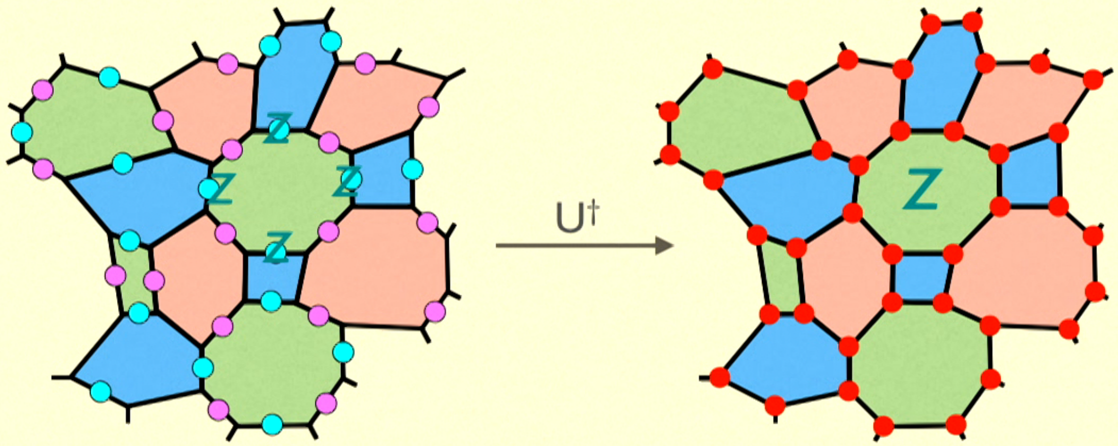

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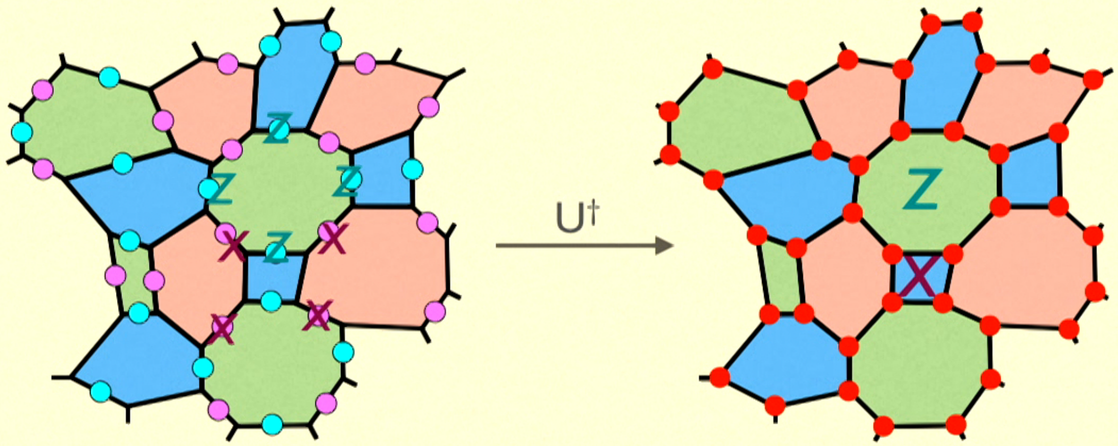

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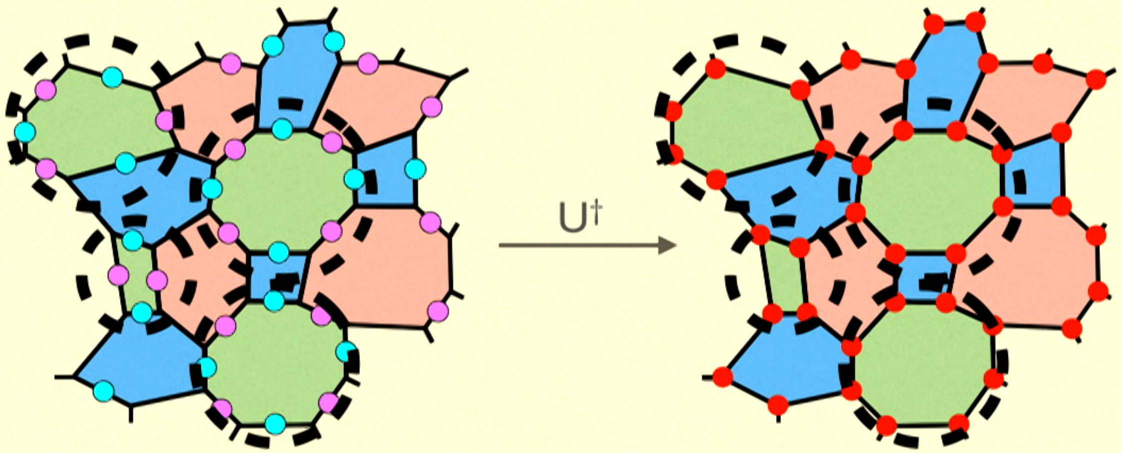

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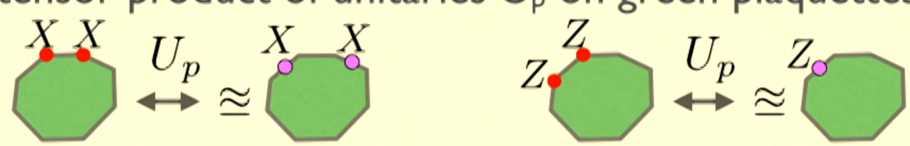

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BOUNDARIES AND CONDENSATION OF ANYONS

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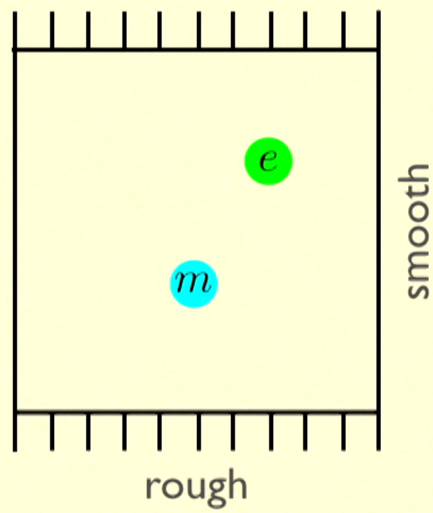

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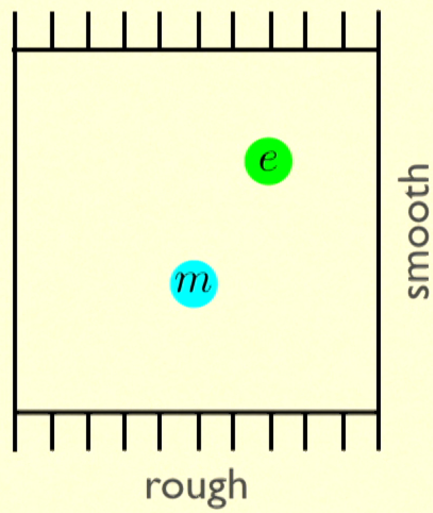

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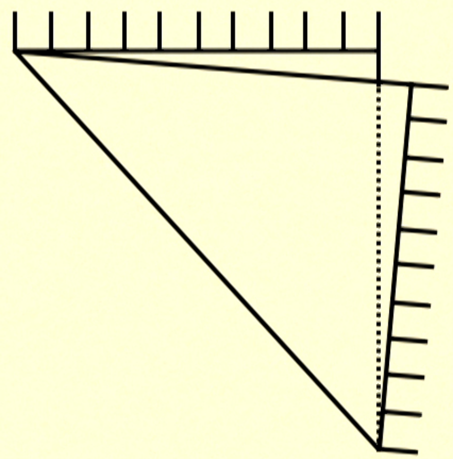

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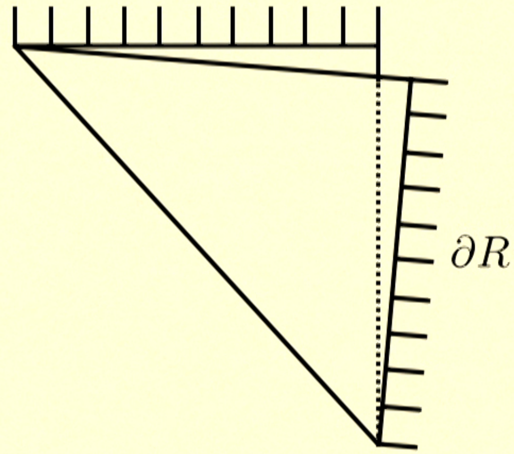

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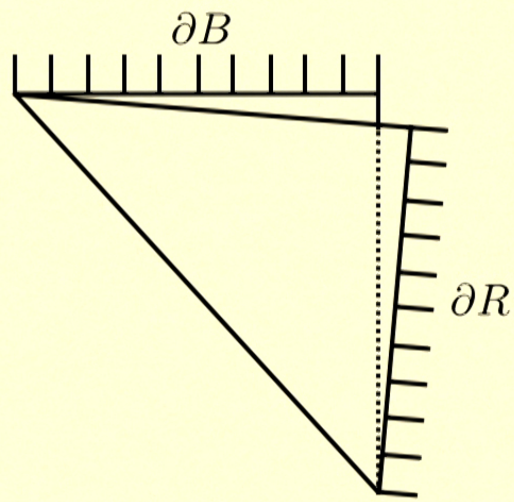

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- Correspondence between anyonic excitations in toric code and color code:

$e_1 \equiv R_X \qquad e_2 \equiv B_X$
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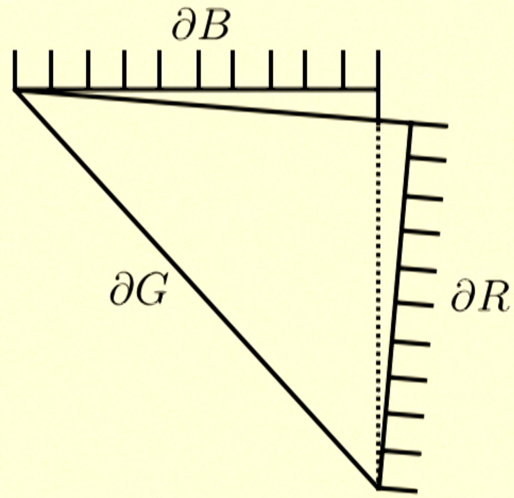

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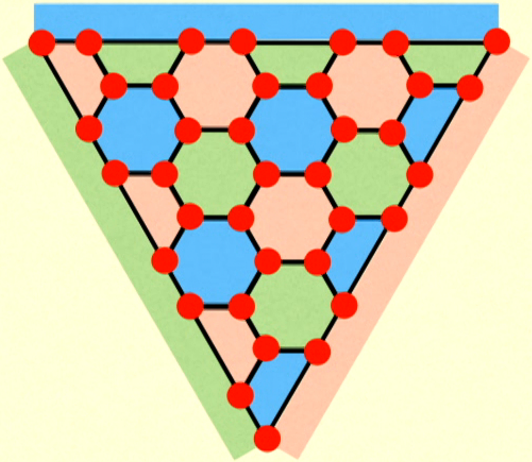

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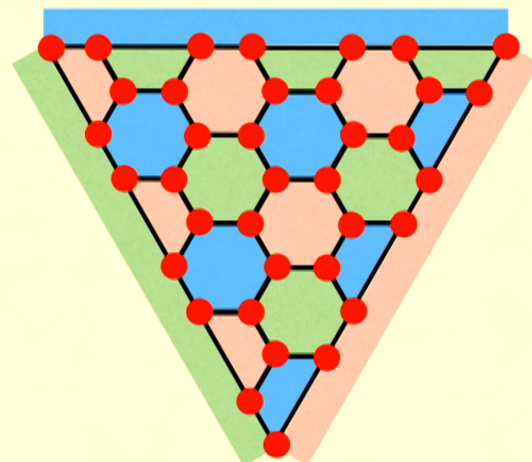

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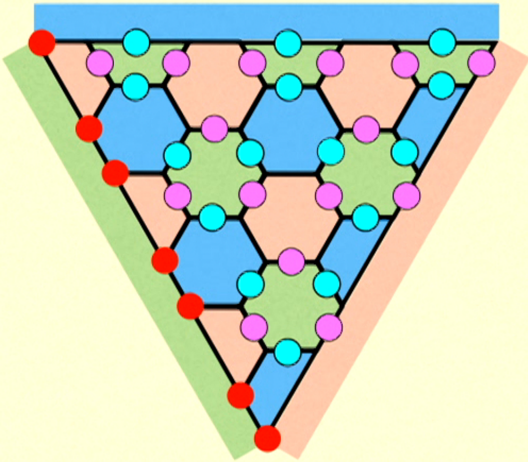

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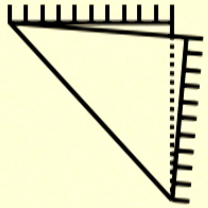

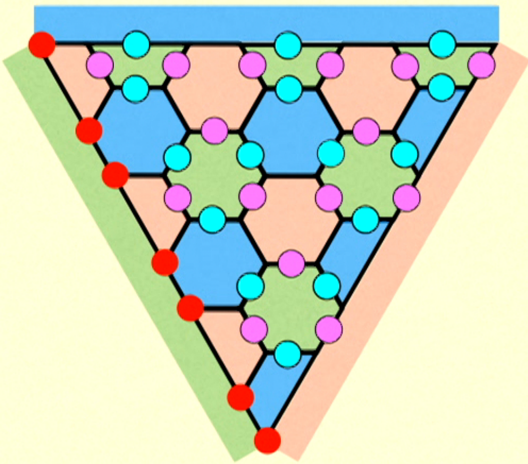

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EQUIVALENCE IN D DIMENSIONS WITH OR WITHOUT BOUNDARIES

Theorem: there exists a unitary $U = \bigotimes_\delta U_\delta$, which is a tensor product of local terms with disjoint support, such that U transforms the color code into $n = \binom{d}{k-1}$ decoupled copies of the toric code.

$$U\left[CC_k(\mathcal{L})\right]U^\dagger = \bigotimes_{i=1}^{n} TC_{k-1}(\mathcal{L}_i)$$

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Theorem: color code with point-like excitations in d dim with d+1 boundaries is equivalent to d copies of toric code attached along (d-1)-dim boundary.

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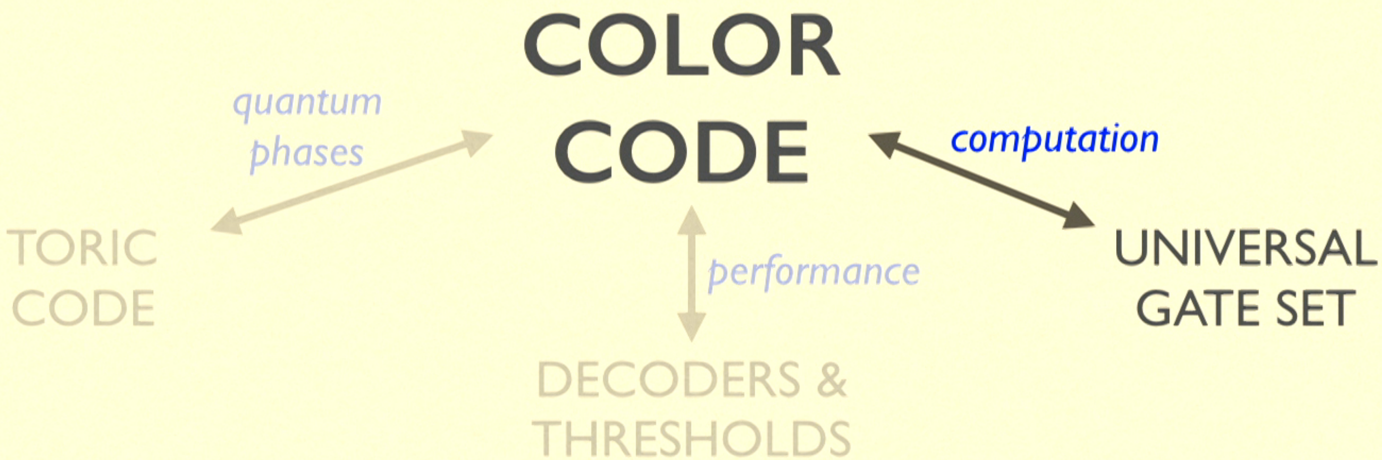

- **Main results:**
  - **Kubica, Beverland'15:** explicit construction of logical R_d gate; universality with without magic-state distillation
  - **Kubica, Yoshida, Pastawski'15:** fault-tolerant implementation of d-qubit control-Z gate in toric code
  - **Beverland, Kubica, Brandao, Svore (in prep.):** estimating the cost of universality by switching between color codes

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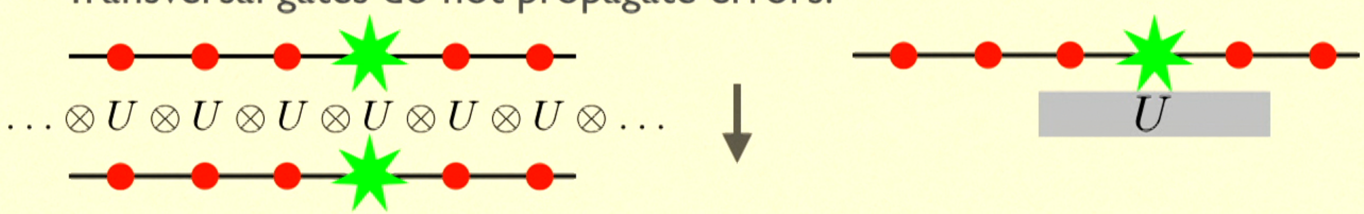

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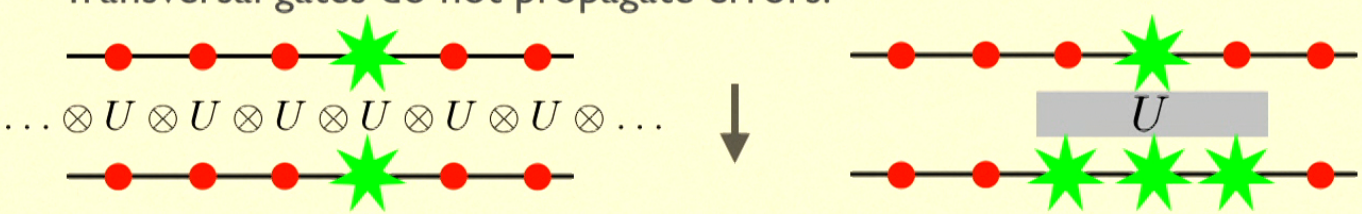

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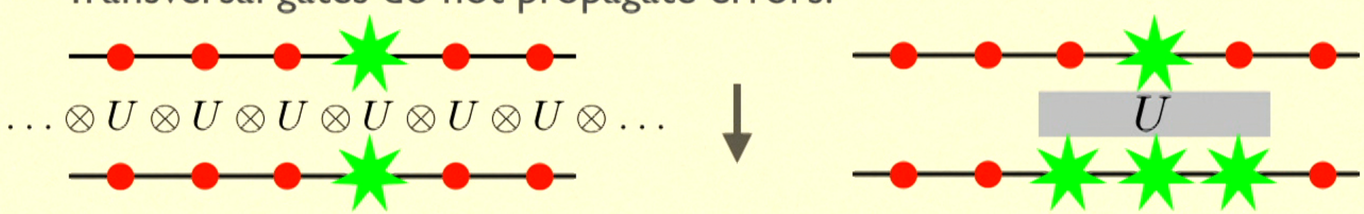

- **Zeng et al.'07, Eastin&Knill'09**: for any nontrivial local-error-detecting quantum code, the set of transversal logical unitaries is not universal.

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NO-GO RESULTS

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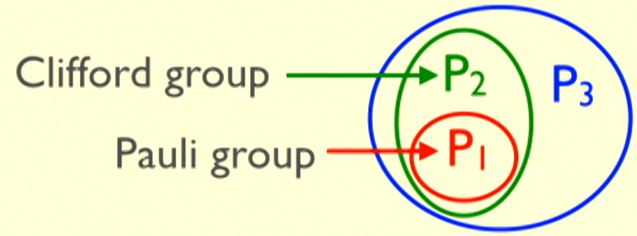

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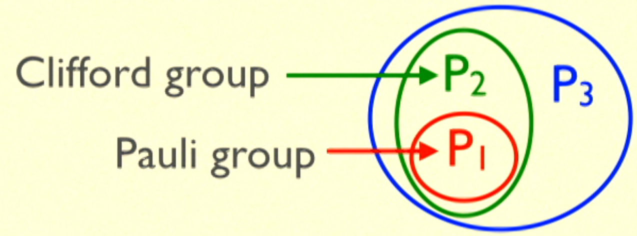

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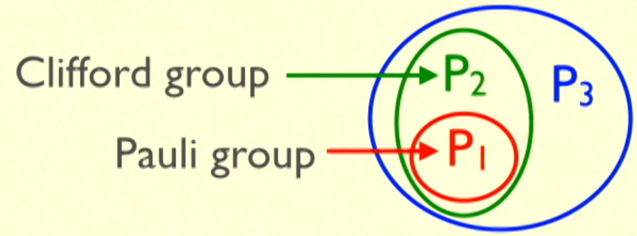

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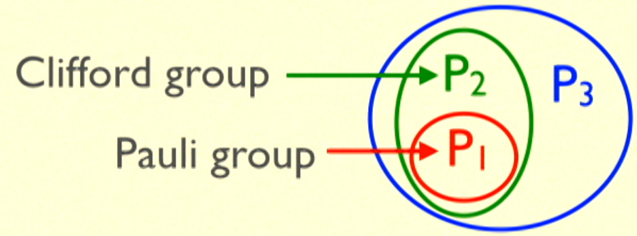

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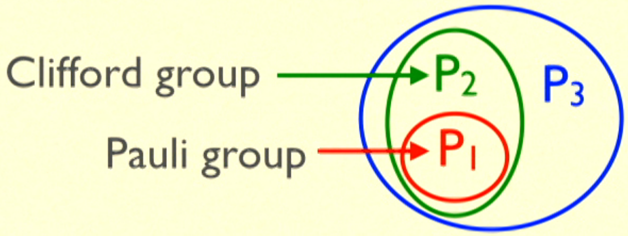

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- **Our results:** simplification of the argument by constructing explicitly a logical R_d gate.

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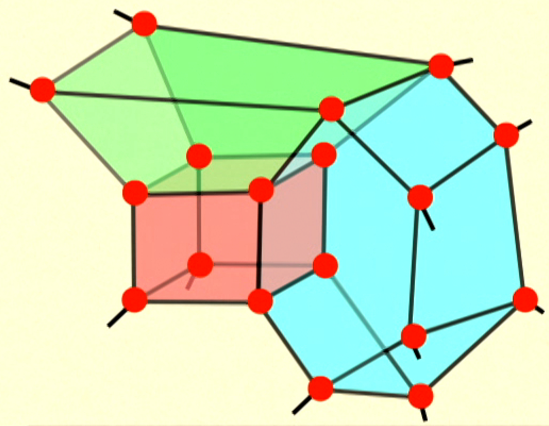

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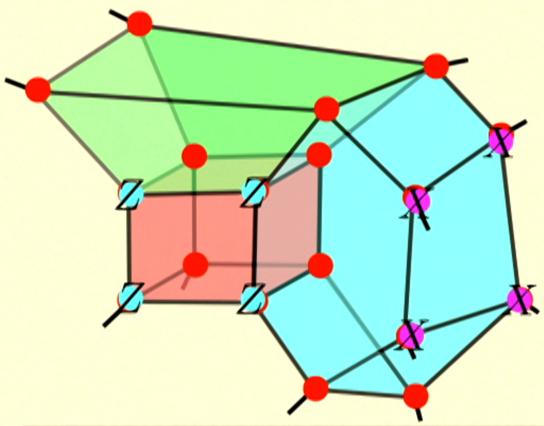

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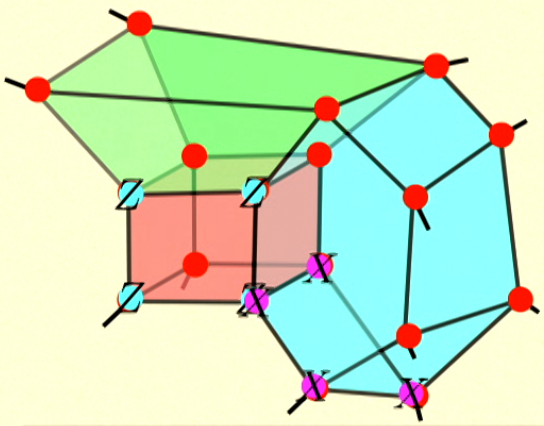

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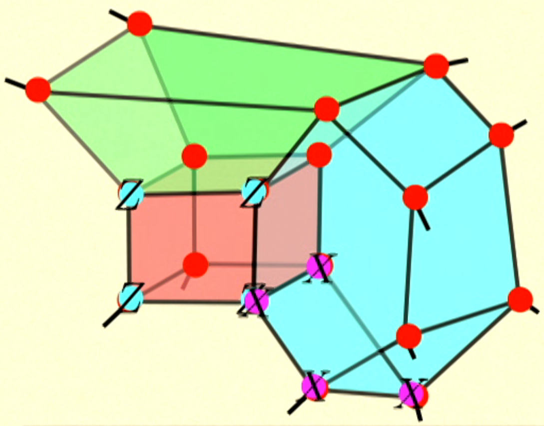

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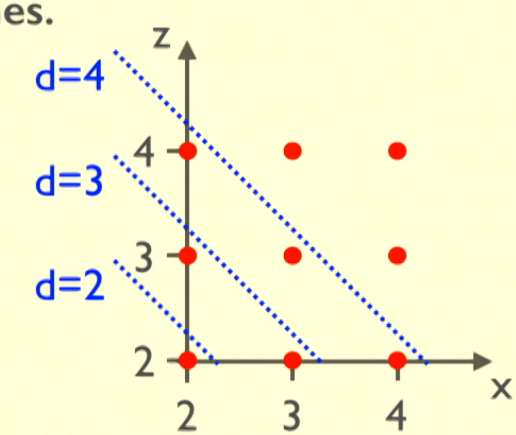

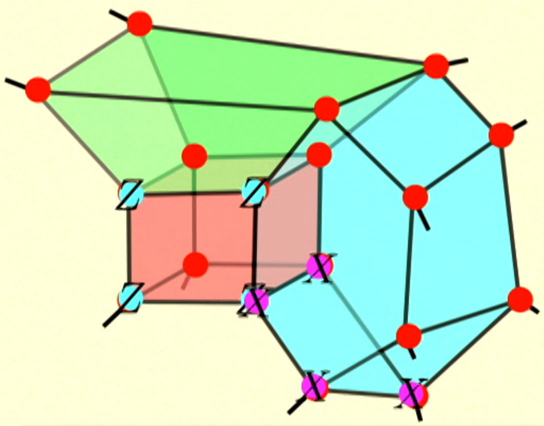

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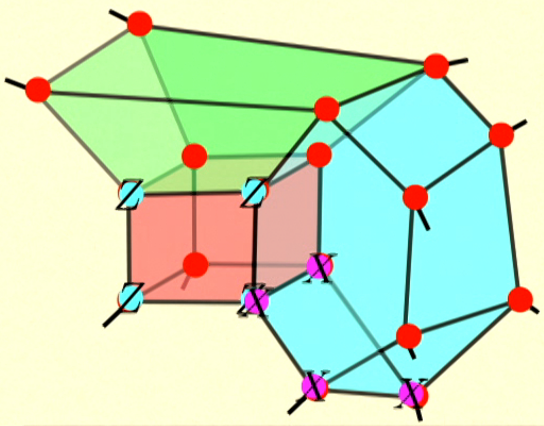

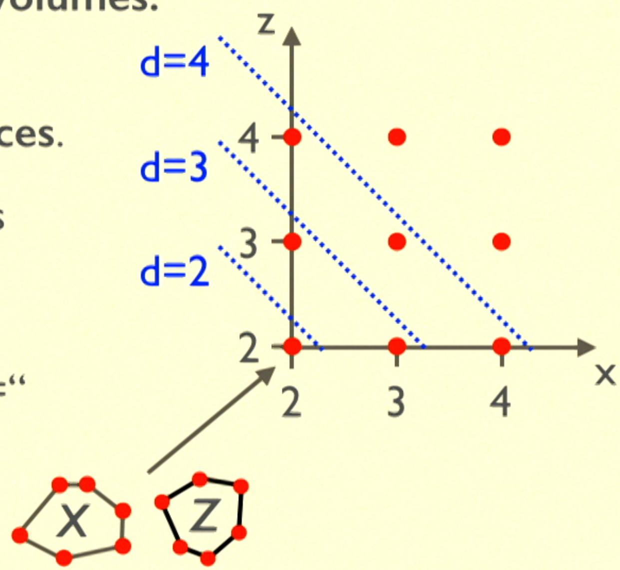

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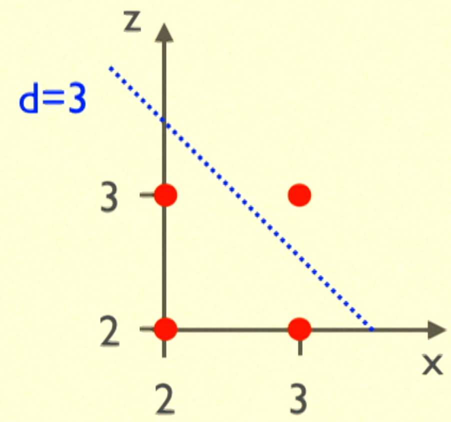
UNIVERSAL GATE SET WITH COLOR CODES IN 3D



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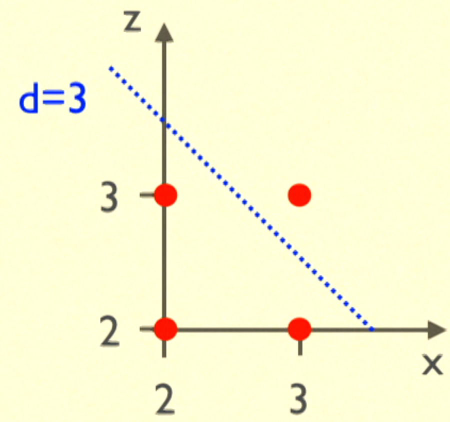

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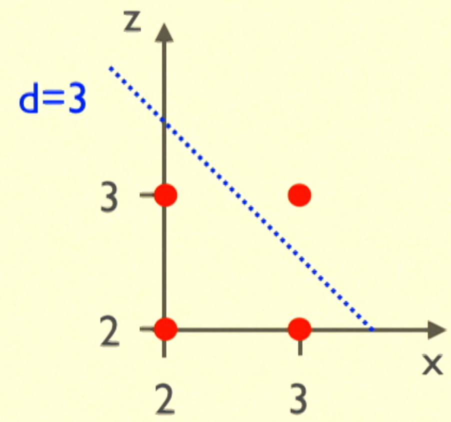

arXiv:1410.0069

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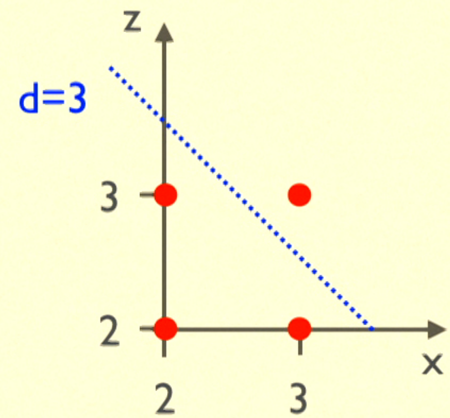

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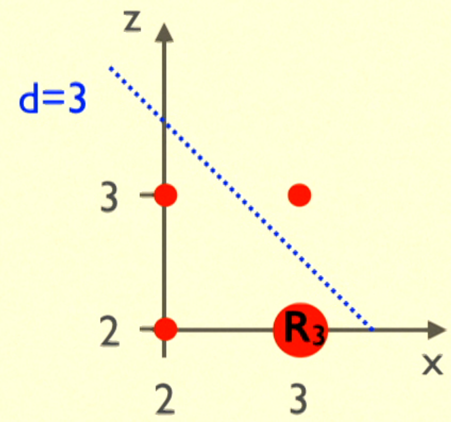

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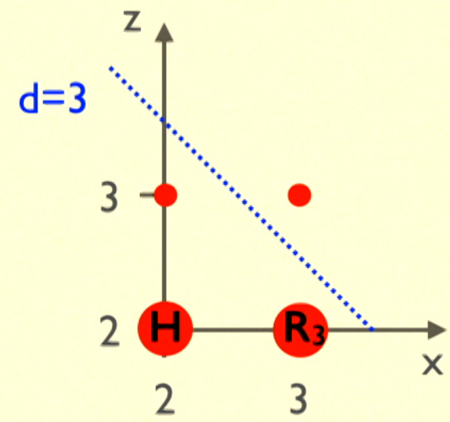

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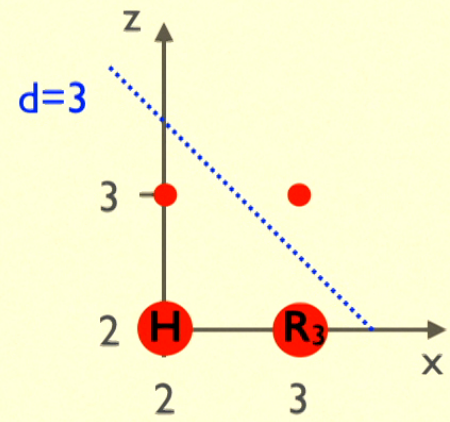
- Code switching between $CC_3(x=2, z=2)$ and $CC_3(x=3, z=2)$ possible in a fault-tolerant way. Universal gate set in 3D!



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COST OF UNIVERSALITY IN 3D COLOR CODES

- Universality with 3D color code without magic-state distillation (the main source of overhead).

work w/ **M. Beverland, F. Brandao, K. Svore**

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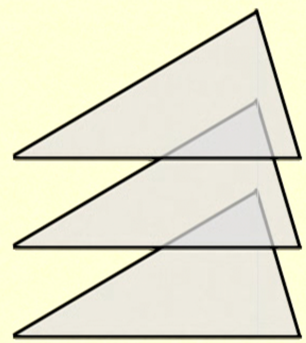

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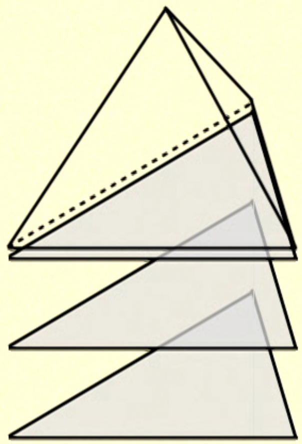

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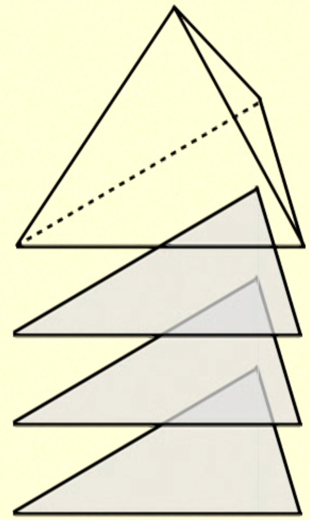

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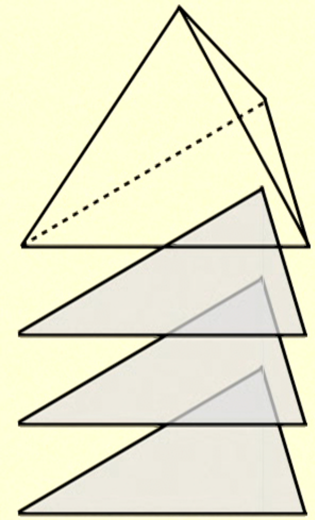

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- **Our results:** qubit overhead of code switching compares unfavourably to magic-state distillation!

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$C^{d-1}Z$ GATE IN COLOR CODE

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arXiv:**1503.02065**

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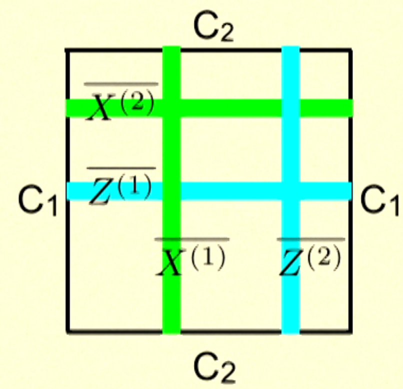

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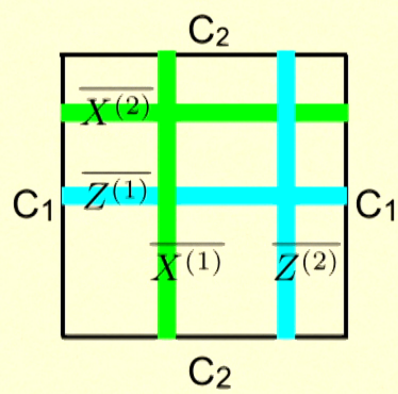

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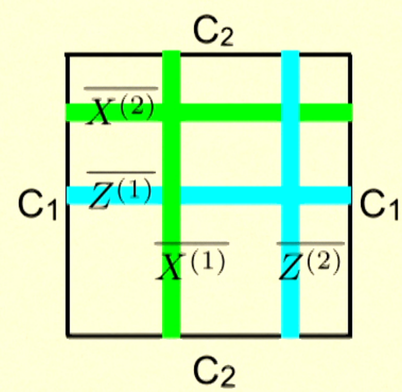

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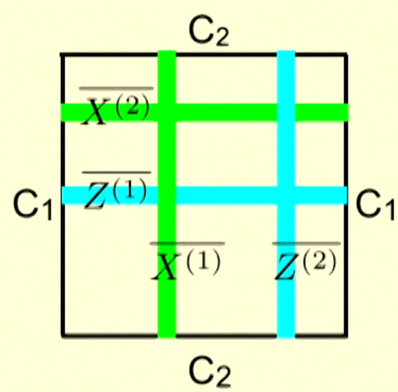

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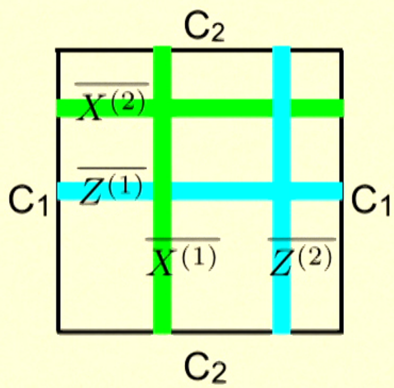

- Transversal unitary R_d implements logical $C^{d-1}Z$ gate.
- Smallest interesting instance, the [[8, 3, 2]] code, enables to distill 8 noisy T-states into less noisy CCZ magic state!

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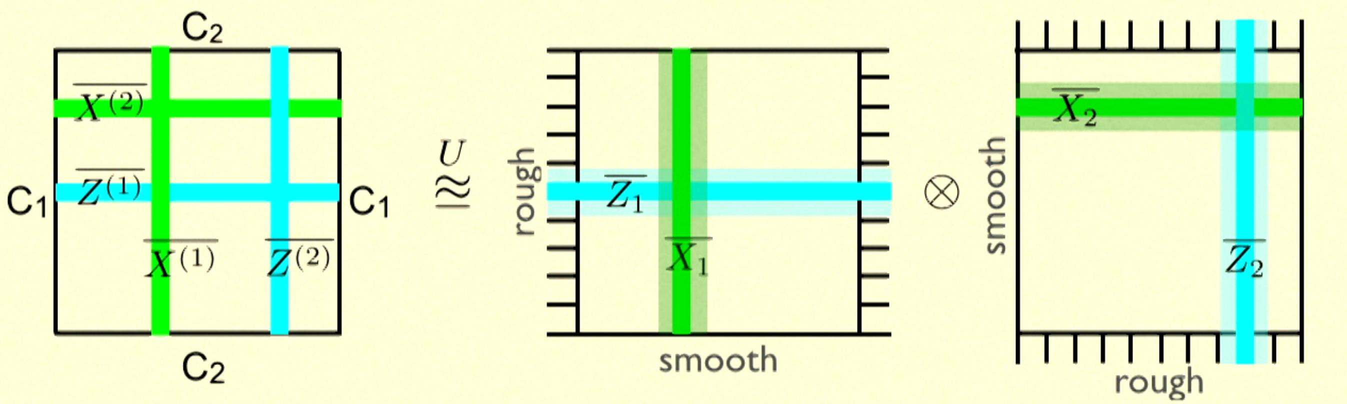
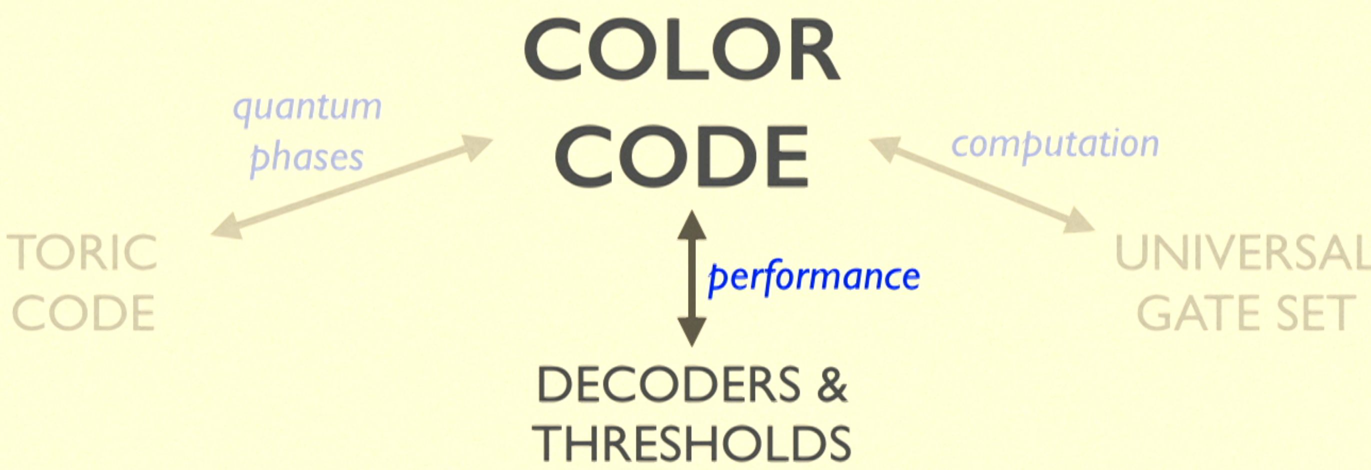
$C^{d-1}Z$ GATE IN TORIC CODE

- Recall the local unitary mapping U - the color code on a d-dim hypercube equivalent to d decoupled copies of the d-dim toric code.



- Can verify that a local unitary $UR_dU^\dagger$ implements logical $C^{d-1}Z$ gate in d copies of the toric code.

arXiv:1503.02065



- **Main results (Kubica, Delfosse, in prep.):**
  developing and benchmarking decoders of color code in $d \geq 3$ dim

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work w/ **N. Delfosse**

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- **Our results:** developing a decoder for color code in $d \geq 3$ dim and analyzing its performance.

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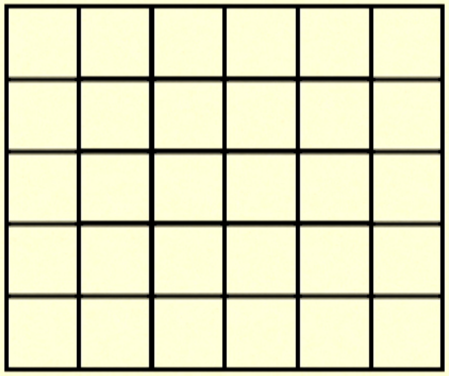
EXAMPLE: DECODING OF TORIC CODE

- CSS codes - can decode X and Z separately (might reduce performance!).



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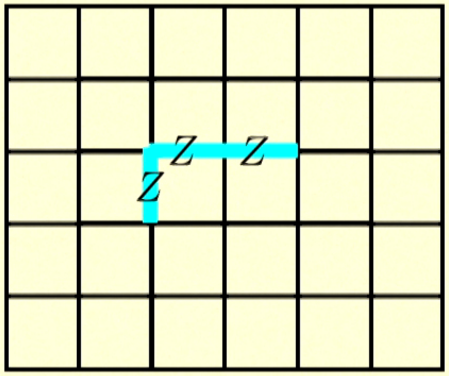

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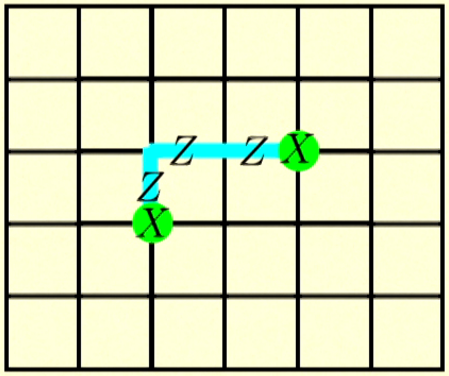

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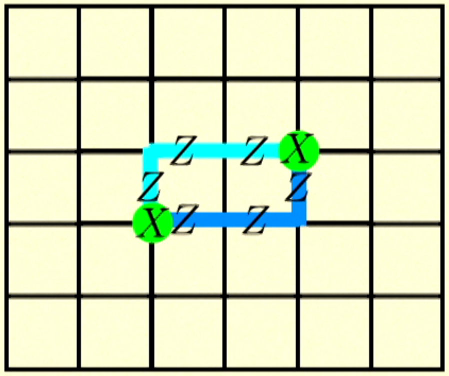

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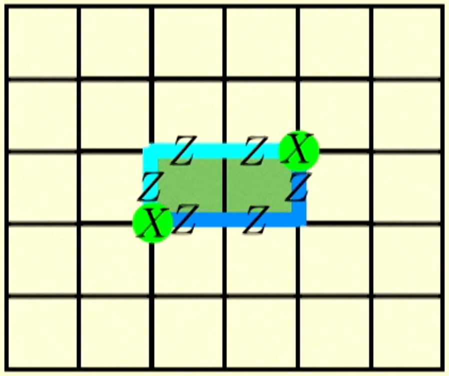

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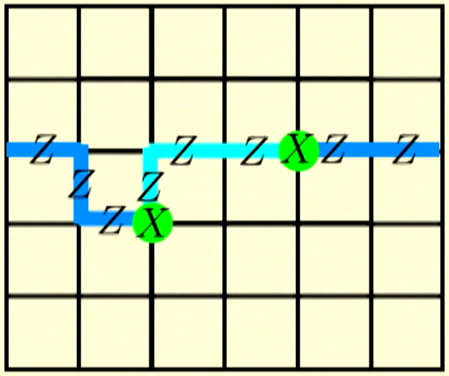

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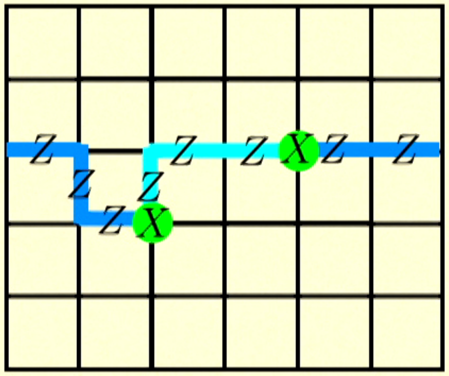

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- Minimum weight perfect matching - efficient and close to optimal.

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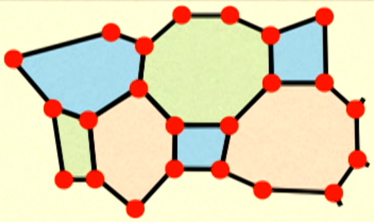

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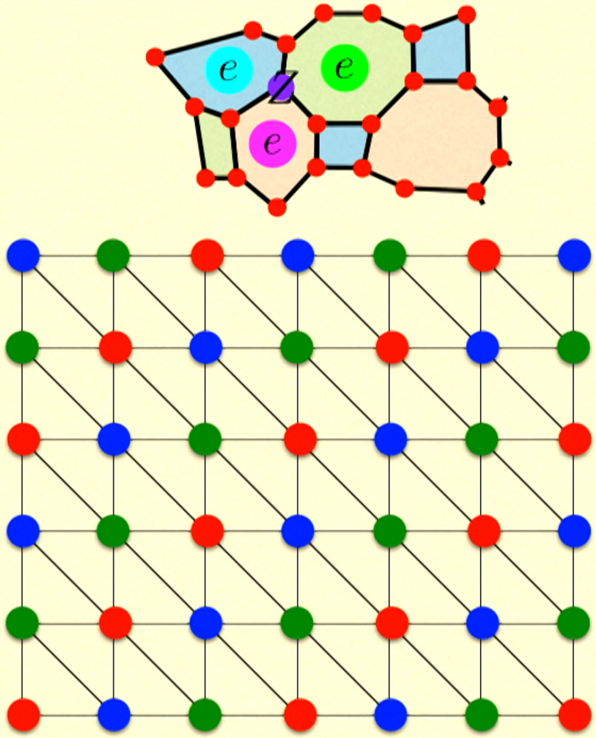

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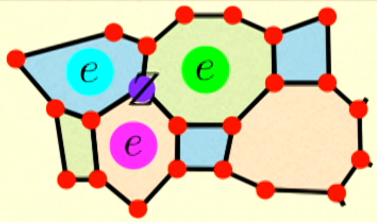

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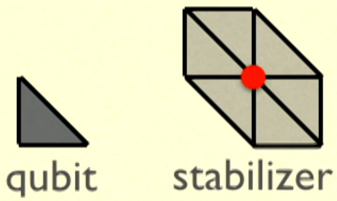

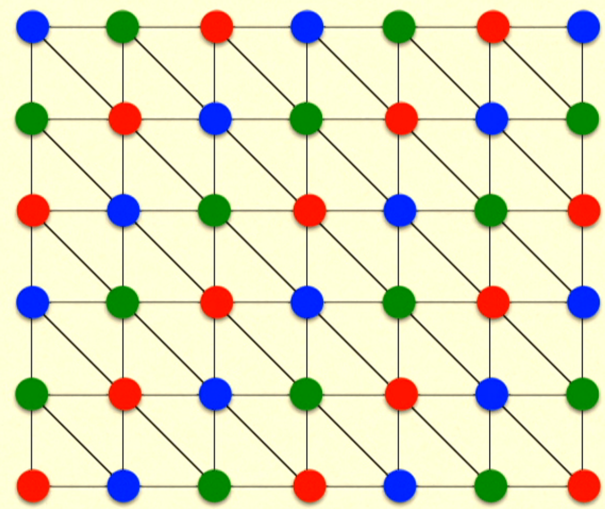

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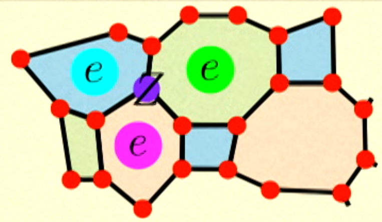

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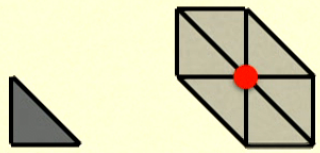
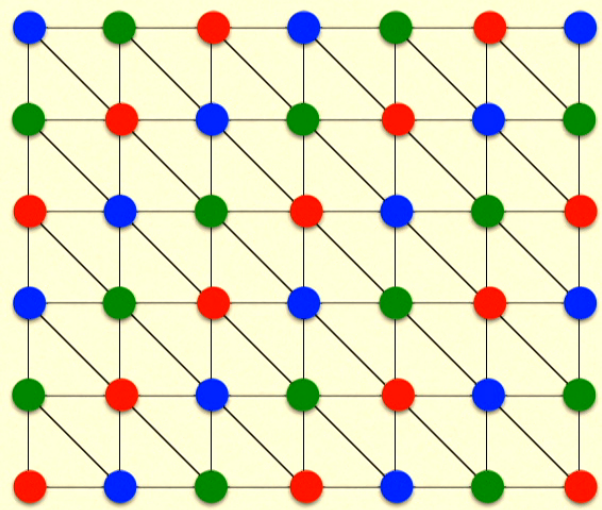


- **Delfosse'13:** project onto 3 sublattices and use toric code decoder.

error ← syndrome

2D ← 0D



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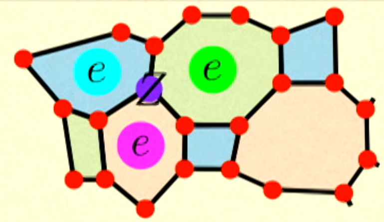

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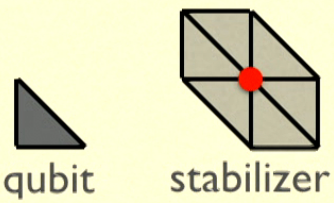

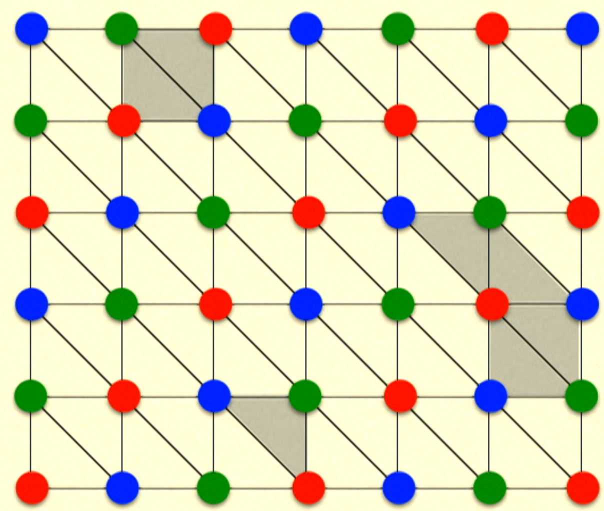

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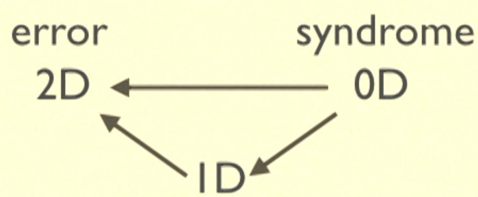

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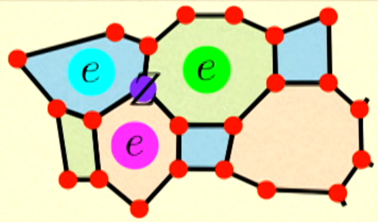

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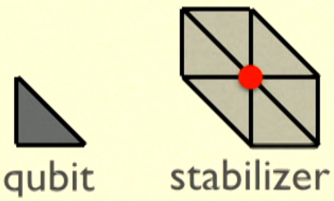

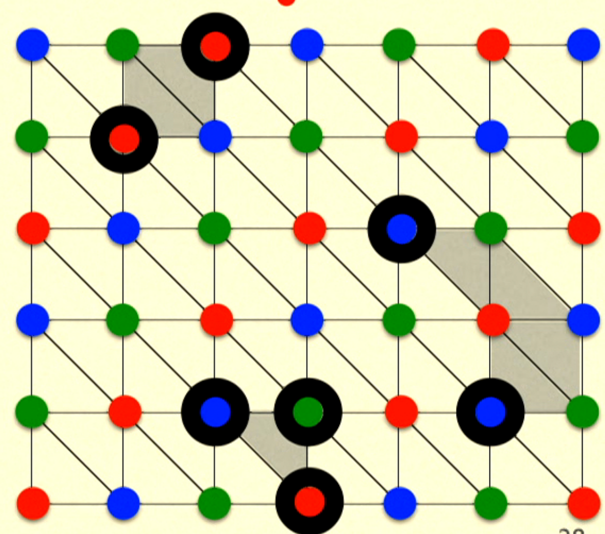

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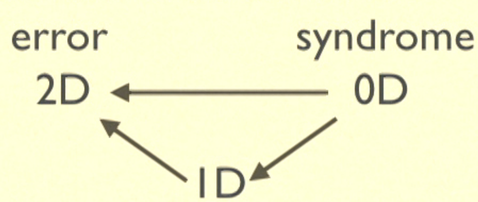

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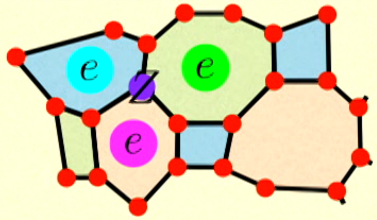

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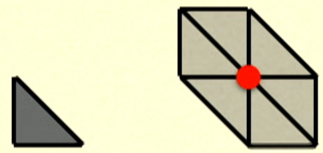
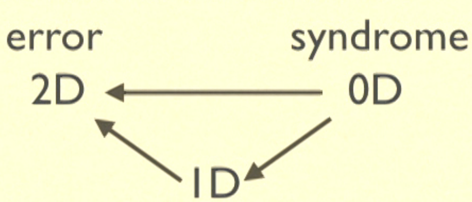
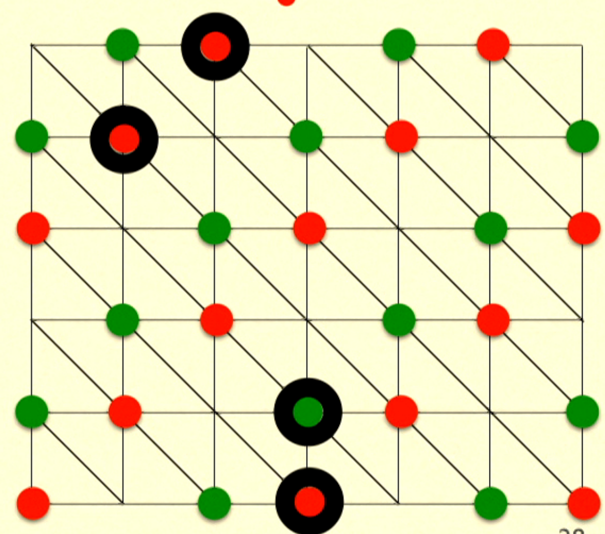


- **Delfosse'13:** project onto 3 sublattices and use toric code decoder.





work w/ **N. Delfosse**

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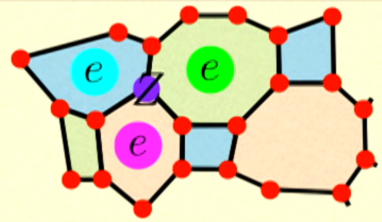

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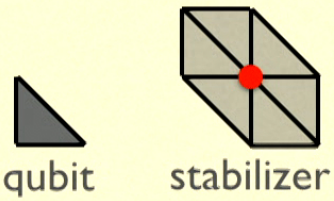

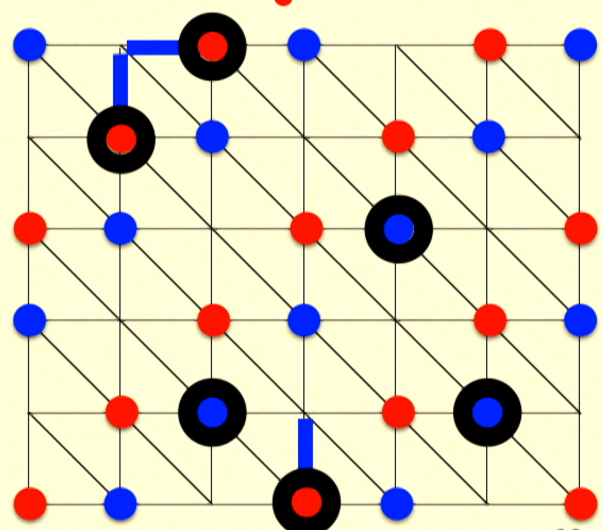

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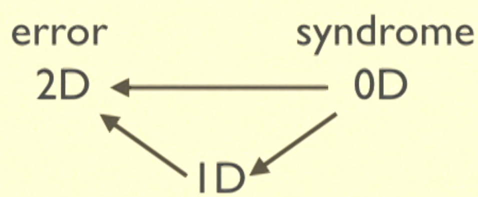

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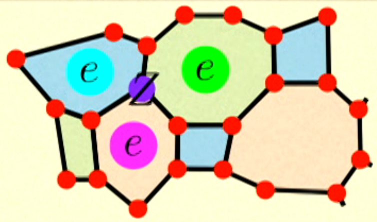

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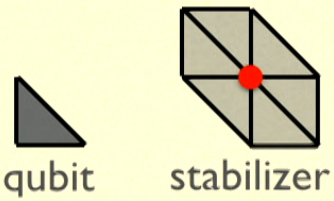

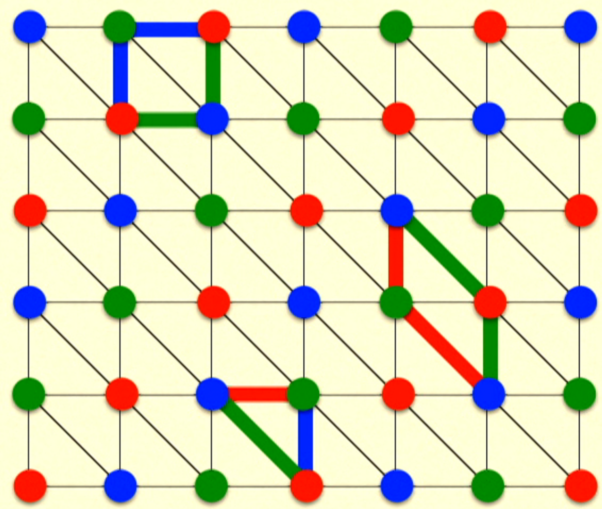

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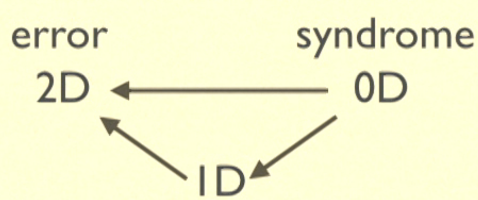

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work w/ **N. Delfosse**

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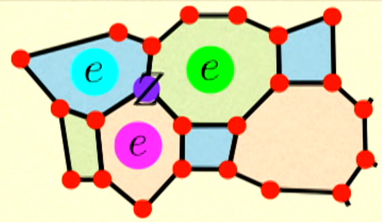

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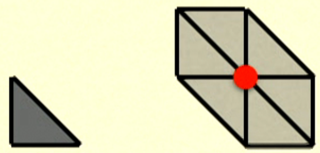

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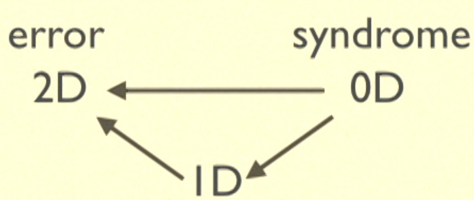

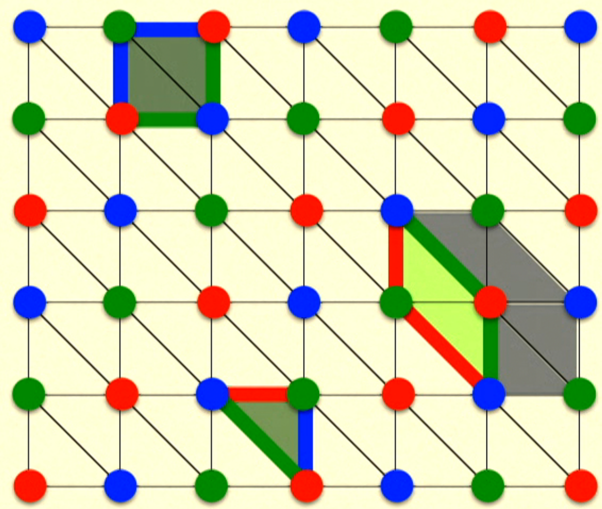

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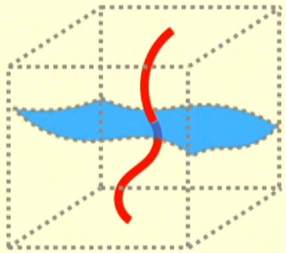
work w/ **N. Delfosse**

DECODING OF COLOR CODE IN 3D

- 3D color code has string-like (1-dim) and sheet-like (2-dim) logical operators.



29

work w/ **N. Delfosse**

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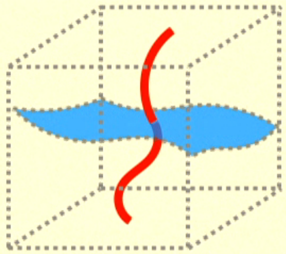

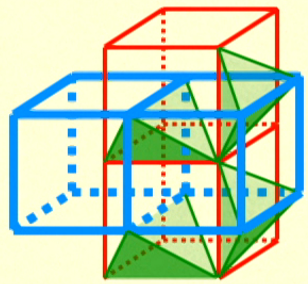

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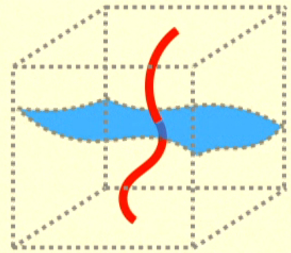
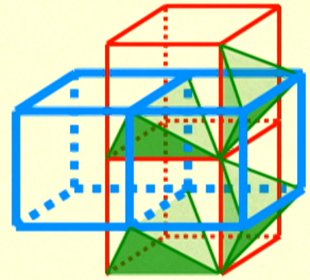
  3D   1D   0D





work w/ **N. Delfosse**

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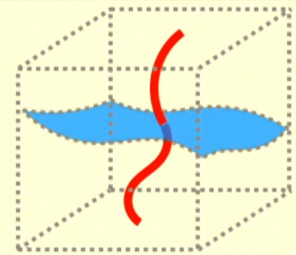

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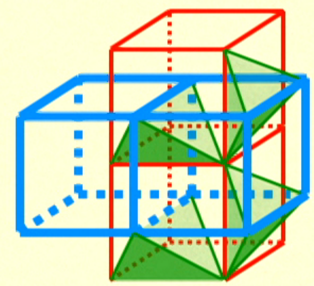

- **Decoder:** project onto 6 (pairs of colors for 0D) or 4 sublattices (triples of colors for 1D)and use toric code decoder!

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work w/ **N. Delfosse**

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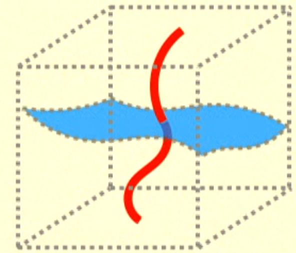

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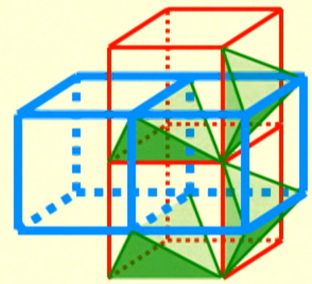
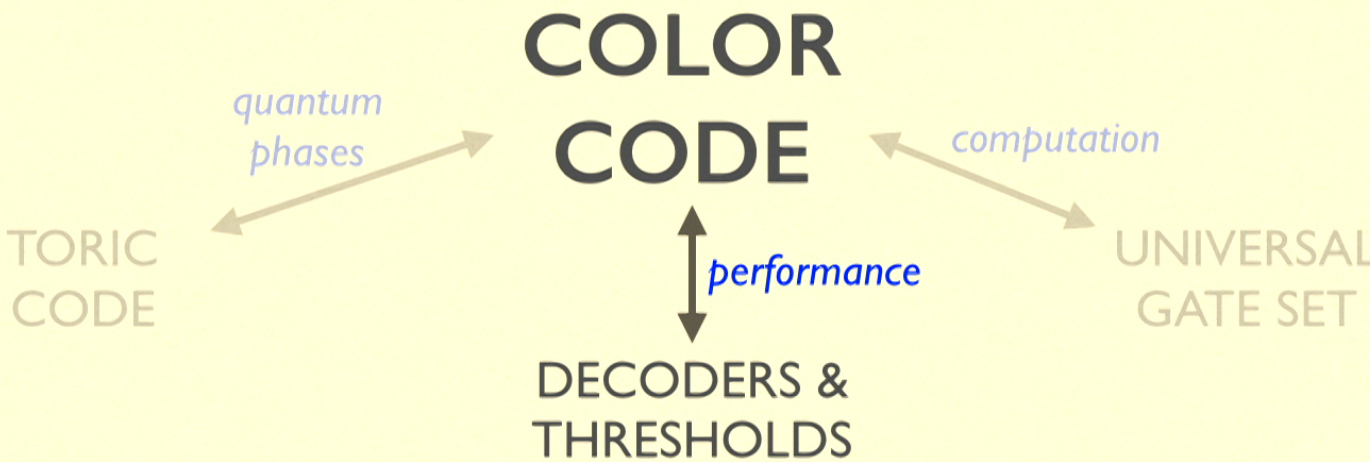


- **Decoder:** project onto 6 (pairs of colors for 0D) or 4 sublattices (triples of colors for 1D)and use toric code decoder!
- Beyond perfect matching - finding min area k-chain w/ given boundary, i.e. (k-1)-chain. Efficient for (n-1)-chains in n dim (**Sullivan PhD thesis'94**).

work w/ **N. Delfosse**



- **Main results (Kubica, Beverland, Brandao, Preskill, Svore, in prep.):**
  - investigation of phase diagrams of new statistical-mechanical models,
  - optimal error correction thresholds of 3D color code.

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THRESHOLDS FROM STATISTICAL MECHANICS

- Error correction threshold p_{th} = maximum error rate the code can tolerate. Decoding succeeds if $p < p_{th}$ (in the limit of infinite system size).

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work w/ **M. Beverland, F. Brandao, J. Preskill, K. Svore**

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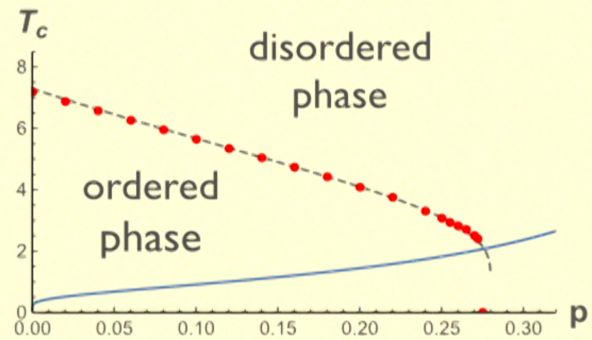

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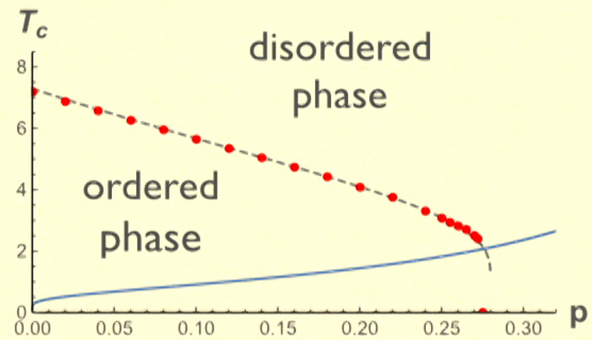

- Mapping allows to use tools and method from statistical mechanics!

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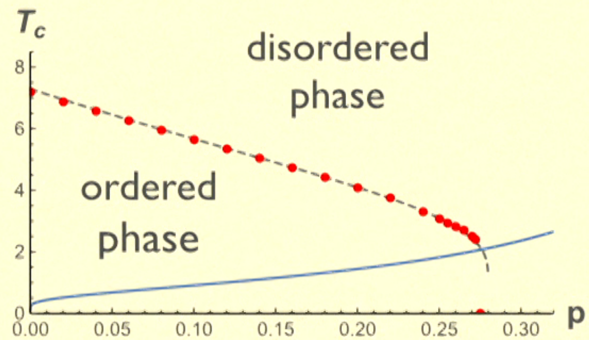

- Mapping allows to use tools and method from statistical mechanics!
- **Our results:** introducing new stat-mech models relevant for 3D color code, studying their phase diagrams, finding thresholds of 3D color code.

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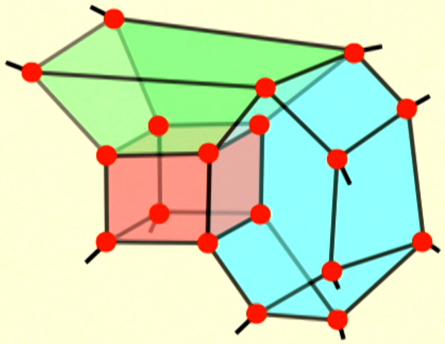
3D COLOR CODE AND RANDOM COUPLING ISING MODEL



- 3D color code - stabilizer and subsystem:
  - stabilizers on faces (A) and volumes (B),
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- Two stat-mech models to analyze:
  - 4-body random coupling Ising model (A),

$$H = -\sum \varkappa_{abcd} s_a s_b s_c s_d$$

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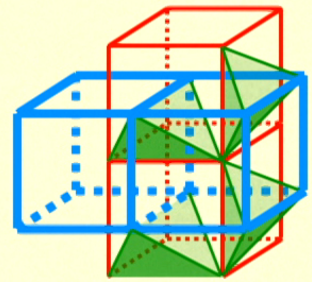

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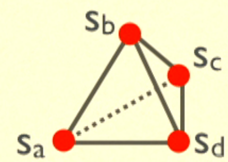

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work w/ **M. Beverland, F. Brandao, J. Preskill, K. Svore**

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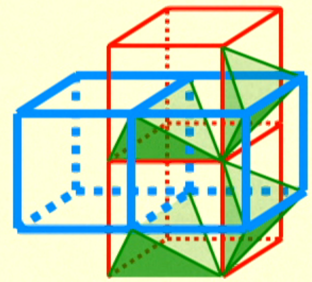

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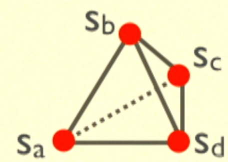

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  - local order parameter - fairly easy to analyze!

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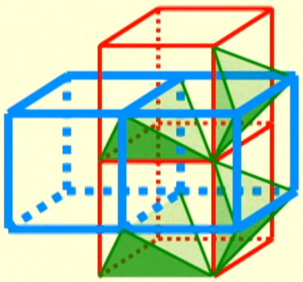

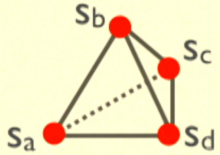

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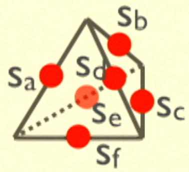

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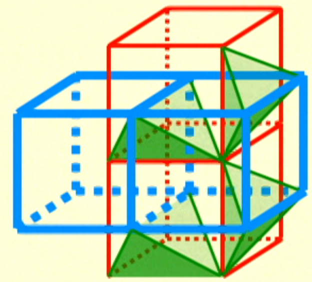

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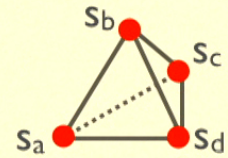

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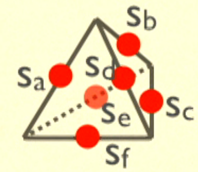

$$\varkappa_{abcdef} = \pm 1$$

  - challenging due to gauge symmetries (no local order parameter).

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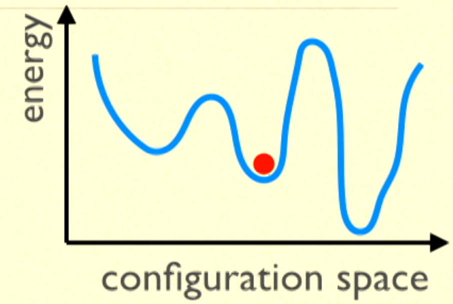
CHALLENGES



- Simulation - Monte Carlo w/ parallel tempering (replica exchange) to avoid local minima.

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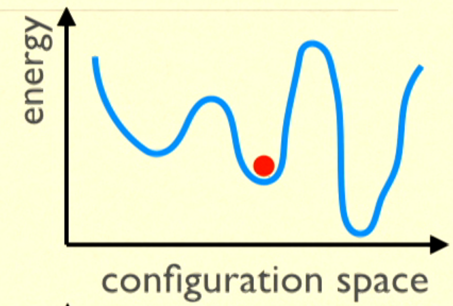

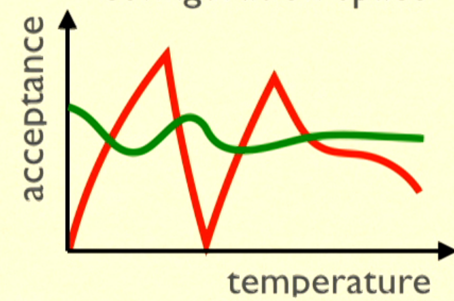

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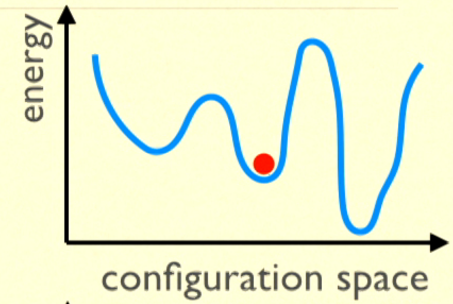

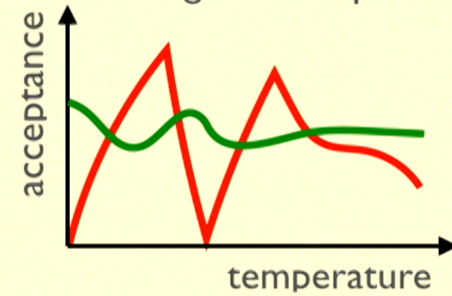

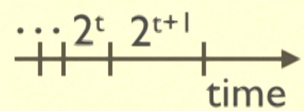

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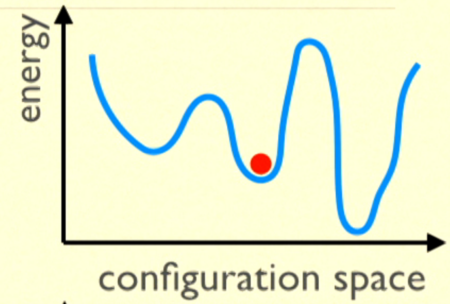
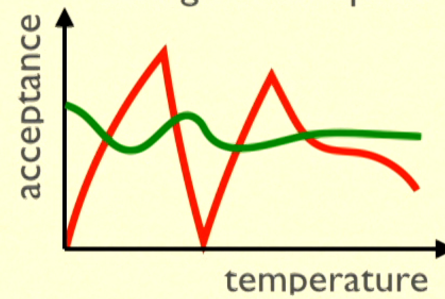
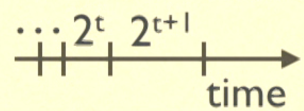
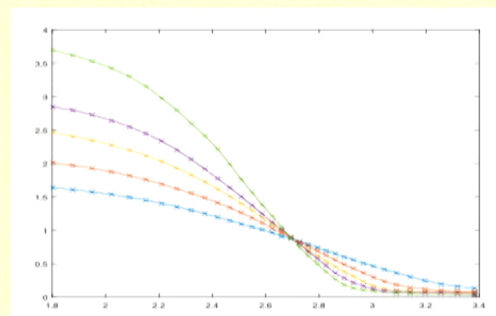
- Suitable order parameters to detect phase transition: correlation length and Wilson loops.









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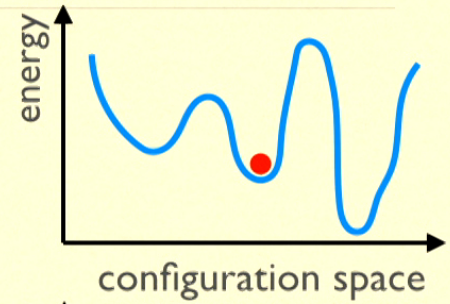
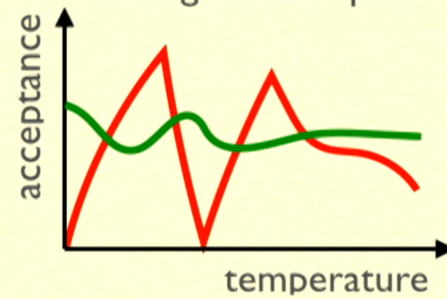
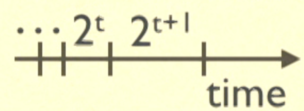
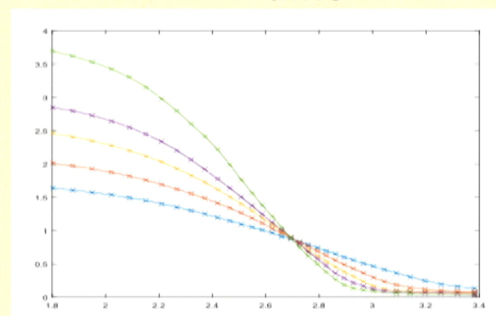
CHALLENGES

- Simulation - Monte Carlo w/ parallel tempering (replica exchange) to avoid local minima.
- How to choose temperatures for parallel tempering? Flat acceptance rate (not optimal).
- When does the system thermalize? Logarithmic binning of data. Critical slowing down!
- Suitable order parameters to detect phase transition: correlation length and Wilson loops.
- Finite-size scaling effects!
- How to analyze data? Bootstrap methods to estimate statistical errors.









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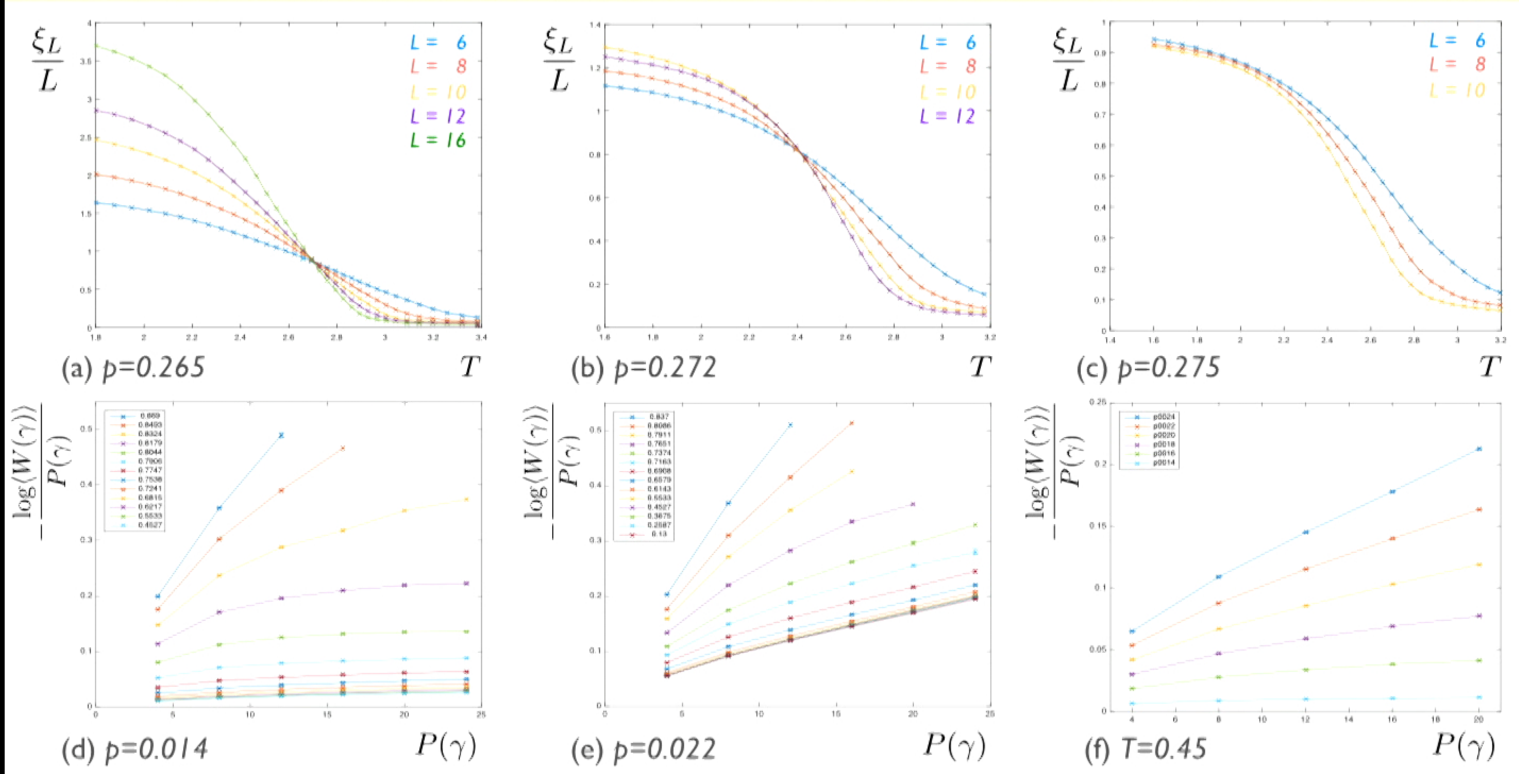
NUMERICS



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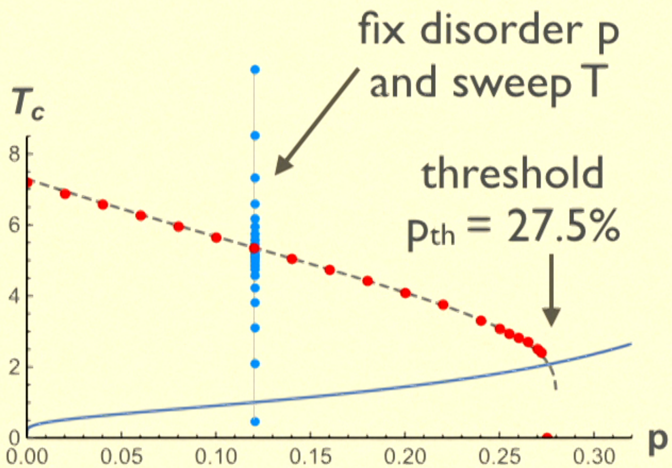
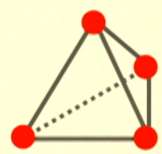
3D COLOR CODE THRESHOLDS FROM PHASE DIAGRAMS



- 4-body RCIM



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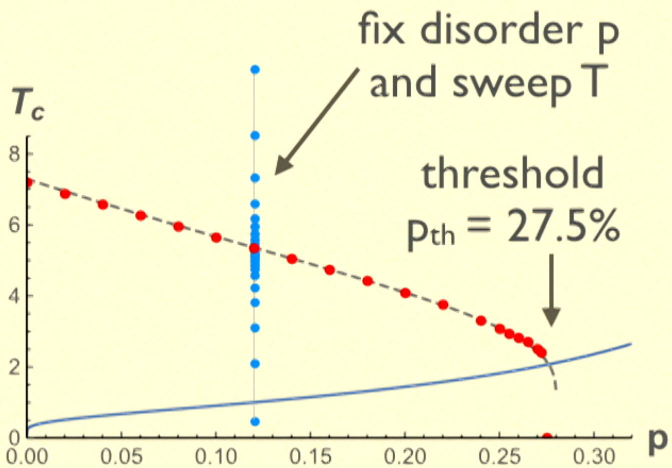
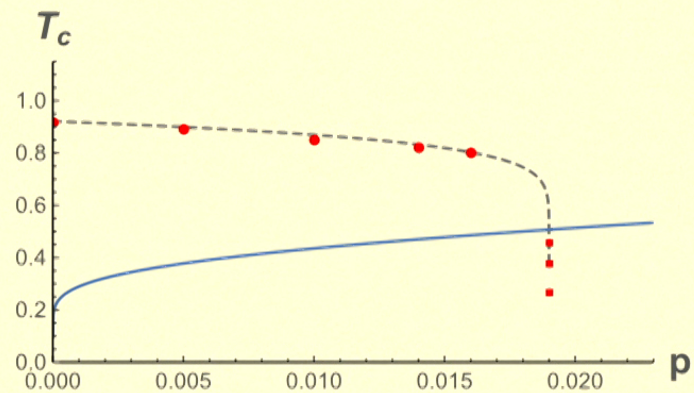
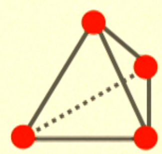
3D COLOR CODE THRESHOLDS FROM PHASE DIAGRAMS

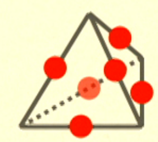




- 4-body RCIM
- threshold for sheet-like (2D) logical operators



- 6-body RCIM



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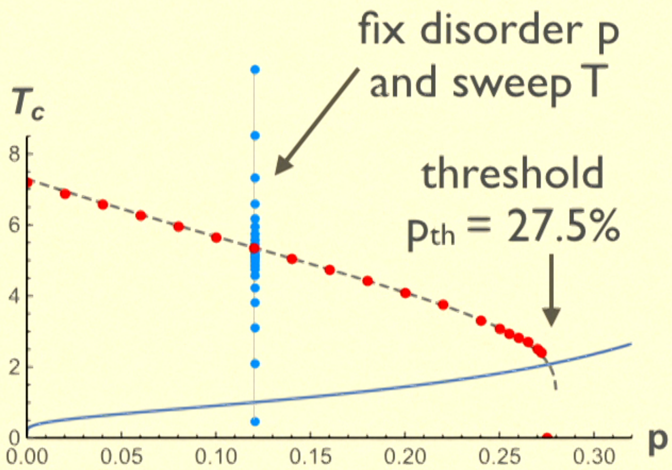
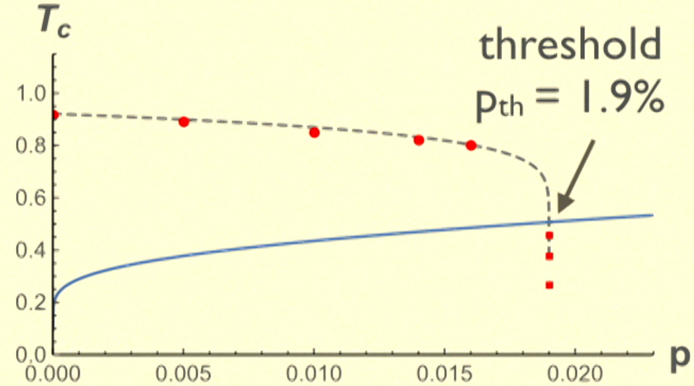
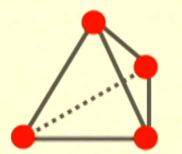
3D COLOR CODE THRESHOLDS FROM PHASE DIAGRAMS

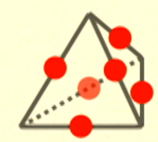




- 4-body RCIM
- threshold for sheet-like (2D) logical operators



- 6-body RCIM



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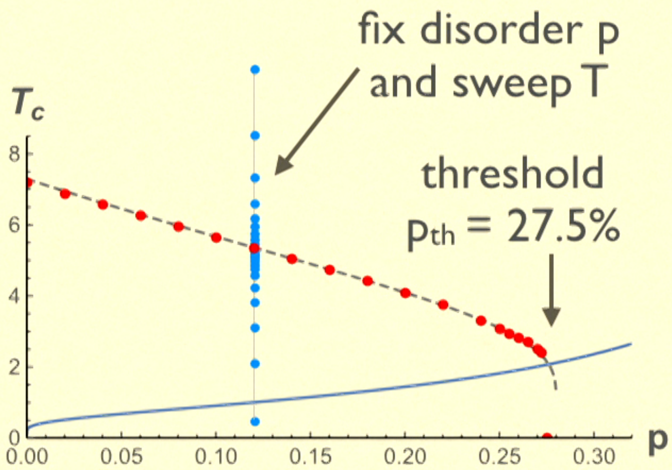
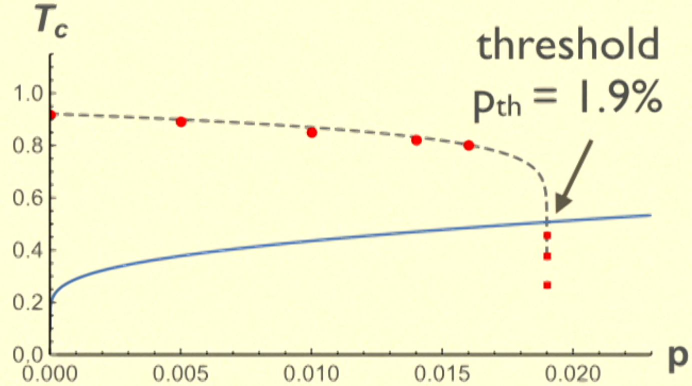
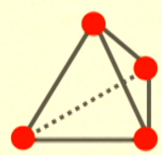
work w/ **M. Beverland, F. Brandao, J. Preskill, K. Svore**

3D COLOR CODE THRESHOLDS FROM PHASE DIAGRAMS

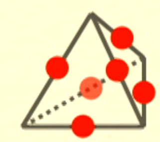




- 4-body RCIM
- threshold for sheet-like (2D) logical operators



- 6-body RCIM
- threshold for string-like (1D) logical operators



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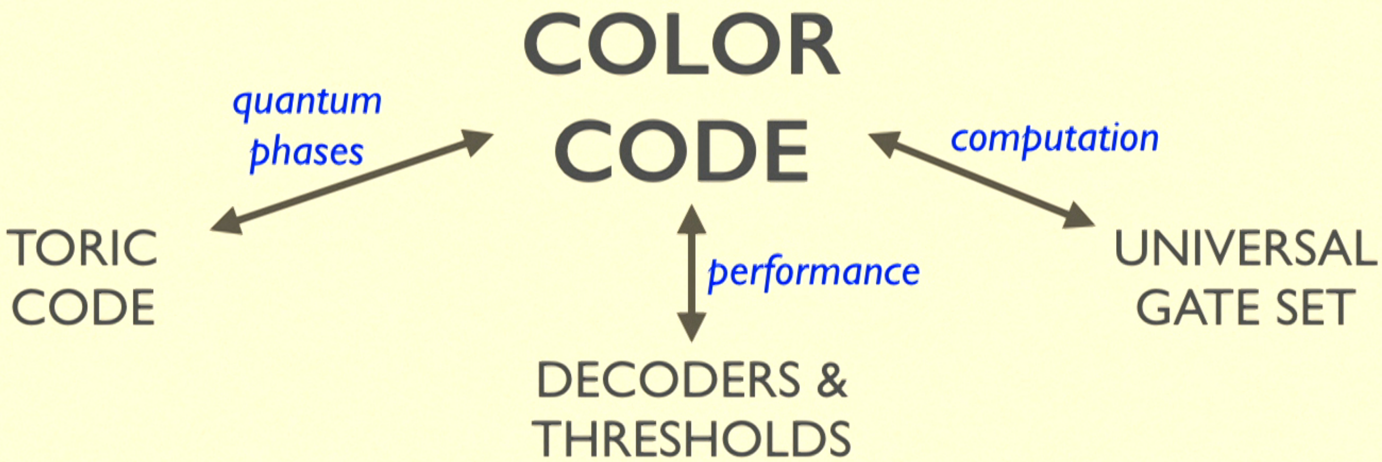
SUMMARY


COLOR CODE
quantum phases
TORIC CODE
computation
UNIVERSAL GATE SET
performance
DECODERS & THRESHOLDS

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