Title: Theory and Experimental Platform for Bosonic Symmetry Protected Topological Phases

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Abstract: <p>Bosonic symmetric protected topological (BSPT) phases are bosonic anagolue of electron topological insulators and superconductors. Despite the theoretical progresses of classifying these states, little attention has been paid to experimental realization of BSPT states in dimensions higher than 1. We propose bilayer graphene system in a out-of-plane magnetic field with Coulomb interaction is a natural platform for BSPT states with $U(1)\times U(1)$ symmetry. We also propose that the quantum phase transition between the BSPT state and the trivial state, which may be tuned by an out-of-plane electric field, could be a novel transition with only gapless bosonic degrees of freedom. In the second part of the talk we will discuss the out-of-time-order correlation (OTOC) and its application in many-body localized and marginal MBL systems. We demonstrate, in marginal MBL systems, the scrambling time follows a stretched exponential scaling with the distance between the operators, which demonstrates Sinai diffusion of quantum information and the enhanced scrambling by the quantum criticality in non-chaotic systems.</p>

Theory And Experimental Platform For Bosonic Symmetry Protected Topological Phases

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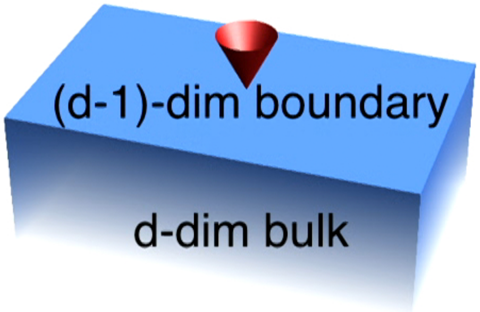
Outline

- Intro to Symmetry Protected Topological (SPT) Phases
- Bilayer Graphene as a Platform for Bosonic SPT states
- Information Scrambling in Localized Systems

"Oversimplified" Introduction to SPT States

- Topological Insulator/Superconductor
  - 1d Majorana wire, 2d QSH, 3d Bi_2Se_3, ^3He B-phase …
  - d-dimensional bulk: insulating/gapped and non-degenerate.
  - (d-1)-dimensional boundary: gapless fermions protected by symmetry → Symmetry Protected Topological (**SPT**) States.



  - Can not realize the boundary in pure (d-1)-dim → Quantum anomaly (gauge or gravitational).

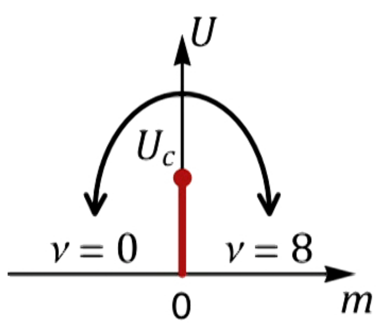
"Oversimplified" Introduction to SPT States

- Free fermion SPT: full classification   Ryu et.al. 2009, Kitev 2009
- Phase transitions between free fermion SPT phases:
  - Gapless fermions in the bulk
  - Example: Chern Insulator to Trivial Insulator

→ single Dirac fermion mass change sign

$$\mathcal{L} = \overline{\psi}(i\,\gamma^{\mu}\,\partial_{\mu} + m)\,\psi \qquad (m \to 0)$$

- Interacting fermion SPT:
  - Interaction can reduce the free fermion classification
    1d (BDI): $Z \to Z_8$ Fidkowski, Kitaev 2009
    2d (p±ip): $Z \to Z_8$ Qi 2012, Ryu 2012 …
    3d (DIII): $Z \to Z_{16}$ Fidkowski 2013, Wang 2014…
  - Interaction can make a fermionic system look like a spin (bosonic) system



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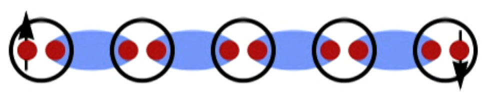
"Oversimplified" Introduction to SPT States

- Bosonic Symmetry Protected Topological (SPT) States
  - Generalization of TI/TSC to spin/boson systems
  - Bulk: gapped and non-degenerate; Boundary: gapless
  - **Always require (strong) interactions**
- Example: 1d Haldane phase of spin-1 chain   Haldane 1983



Affleck, Kennedy, Lieb, Tasaki 1987

$$H = \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \frac{1}{3} \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j \right)^2$$

  - Field theory: O(3) NLSM + Θ term ($\pi_2[S^2] = \mathbb{Z}$)

$$S = \int dx\, d\tau \, \frac{1}{g} (\partial_\mu \boldsymbol{n})^2 + \frac{i\,\Theta}{4\,\pi} \boldsymbol{n} \cdot \partial_t \boldsymbol{n} \times \partial_x \boldsymbol{n} \qquad \Theta = 2\,\pi$$

build with Neél order parameter $\boldsymbol{n} \sim (-)^i \, \boldsymbol{S}_i$

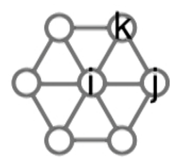
Haldane 1988, Ng 1994, Coleman 1976

"Oversimplified" Introduction to SPT States

- Higher dimensional bosonic SPT states are much more complicated, they can be classified mathematically.
  Chen, Gu, Liu, Wen 2011; Kapustin 2014; Wen 2014; Kitaev …

- What about lattice model/Hamiltonian?
  - Levin-Gu model

$$H_{\mathrm{LG}} = -\sum_i \tilde{X}_i,\ \ \tilde{X}_i = -i\, X_i \prod_{\langle jk\rangle \in \hexagon} \exp\left(\frac{i\,\pi}{4} Z_j\, Z_k\right)$$

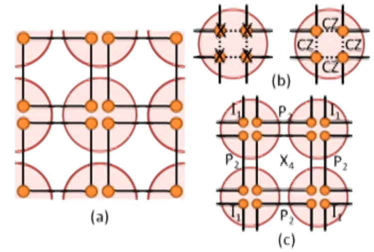


  - CZX model

$$H_{p_i} = -X_4 \otimes P_2^u \otimes P_2^d \otimes P_2^l \otimes P_2^r$$

$$X_4 = |0000\rangle\langle 1111| + |1111\rangle\langle 0000|$$

$$P_2 = |00\rangle\langle 00| + |11\rangle\langle 11|$$



  - The boundary is gapless assuming the Z2 symmetry unbroken.

Levin, Gu 2012
Chen, Liu, Wen 2012

"Oversimplified" Introduction to SPT States

- **More generic properties:**
  - The boundary of many 2d bosonic SPT states can be thought of as 1+1d O(4) WZW CFT with anisotropies:

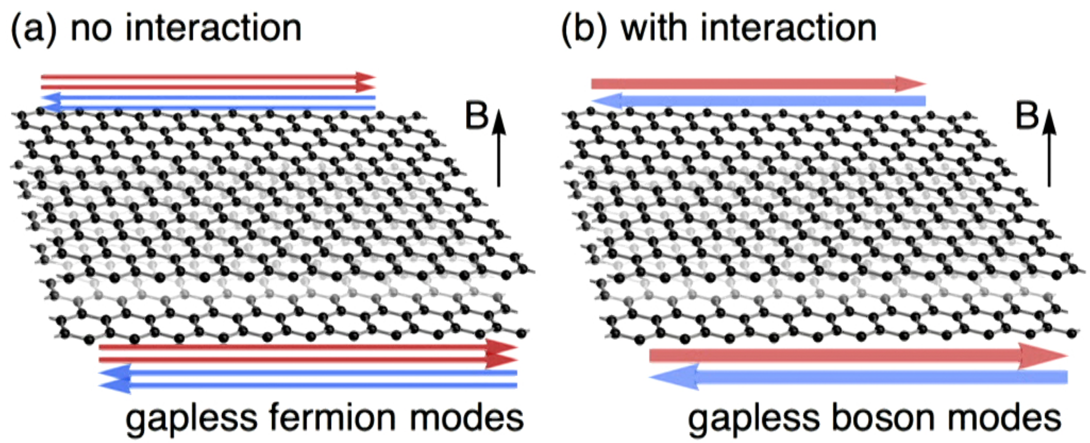
$$\mathcal{L} = \int dx d\tau \frac{1}{g}(\partial_\mu \vec{n})^2 + \int_0^1 du \frac{i2\pi}{\Omega_3} \epsilon_{abcd} n^a \partial_x n^b \partial_\tau n^c \partial_u n^d$$

  - *$SO(4) \sim SU(2)_L \times SU(2)_R$.*
  - Example: The boundary of bosonic integer quantum Hall state, corresponds to breaking the *$SU(2)_L$* symmetry completely, but break the *$SU(2)_R$* symmetry to *U(1)* charge conservation symmetry. Senthil, Levin 2012

- **Goal**: To find a realistic condensed matter system to realize/ mimic bosonic SPT state in 2d.

Realize 2d Bosonic SPT States in Bilayer Graphene

- **Proposal**:
  Bilayer graphene under (strong) magnetic field can be driven into a "bosonic" SPT state with U(1)×U(1) symmetry by Coulomb interaction. Bi, Zhang, You, Young, Balents, Liu, Xu (2016)



- **Meaning**:
  - Boundary: gapless boson modes with U(1)×U(1) symmetry, fermion modes gapped out by interaction.

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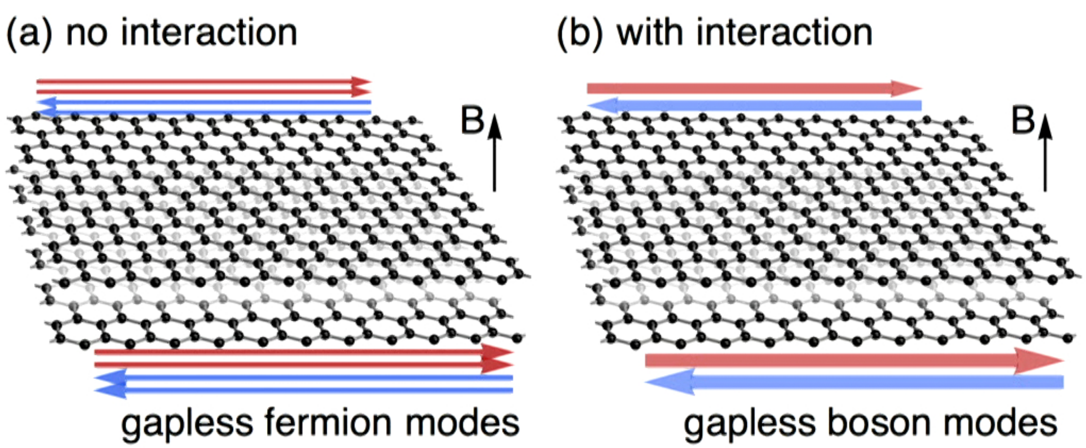
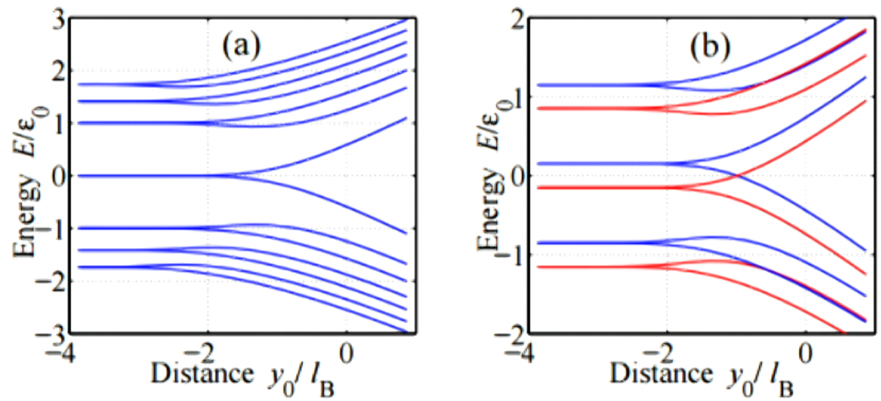


- **Meaning**:
  - Bulk: quantum phase transition between BSPT and trivial state only closes boson gap, fermions remain gapped.

Boundary Analysis

- Boundary: fermion modes gapped out under interaction, remaining gapless boson modes with U(1)×U(1) symmetry.
  - Single layer graphene under perpendicular magnetic field without interactions.



Abanin, Lee, Levitov 2006, Fertig, Brey 2006 Young et.al. 2014

  - Helical edge mode: a pair of counter-propagating fermion modes (c=1 CFT)

Boundary Analysis

- Bilayer graphene
  - Noninteracting: QSH×2, two helical edge modes (c=2)
  - Bosonization

$$H_0 = \int dx \sum_{l=1}^{2} \bar{\psi}_{l,L} iv\partial_x \psi_{l,L} - \bar{\psi}_{l,R} iv\partial_x \psi_{l,R}$$

$$\Rightarrow \quad H_0 = \int dx \frac{v}{2\pi} \sum_{l=1}^{2} \frac{1}{K}(\partial_x \theta_l)^2 + K(\partial_x \phi_l)^2 \qquad \psi_{l,L/R} \sim e^{i(\theta_l \pm \phi_l)}$$

  - Symmetry action

$$U(1)_c : \theta_l \to \theta_l + \alpha \qquad \theta_\pm = \theta_1 \pm \theta_2$$
$$U(1)_s : \phi_l \to \phi_l + \beta \quad \Rightarrow \quad \phi_\pm = (\phi_1 \pm \phi_2)/2$$

  - "-" sector is invariant under symmetry transformation, however, "+" sector is not.

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Boundary Analysis

- Bilayer graphene
  - Noninteracting: QSH×2, two helical edge modes (c=2)
  - Coulomb interaction respect the two U(1) symmetries. Hence it can only involve $\theta_- \ \& \ \phi_-$
  - Coulomb interaction is relevant → gaps out all the fermion modes → only a pair of gapless counter-propagating boson modes (c=1 CFT)

Coulomb $H_v \sim \cos(2(\phi_1 - \phi_2)) \sim \psi^\dagger_{1,L}\psi_{1,R}\psi^\dagger_{2,R}\psi_{2,L}$

⇒ $$\tilde{H} = \int dx \frac{v}{2\pi}\left(\frac{1}{\tilde{K}}(\partial_x\theta_+)^2 + \tilde{K}(\partial_x\phi_+)^2\right)$$

Boundary Analysis

- Boundary theory

$$\tilde{H} = \int dx \frac{v}{2\pi} \left(\frac{1}{\tilde{K}} (\partial_x \theta_+)^2 + \tilde{K} (\partial_x \phi_+)^2 \right)$$

- Boundary collective modes:

  - SC $\quad n_1 + i n_2 \sim e^{i\theta_+} \sim \epsilon_{\alpha\beta} \psi_{1,\alpha} \psi_{2,\beta}$
  - XY SDW $\quad n_3 + i n_4 \sim e^{i2\phi_+} \sim \sum_l (-1)^l \psi_l^\dagger \sigma^+ \psi_l$

- We can derive the boundary effective theory. It's an O(4) WZW model at level-1 (with anisotropy)

$$\mathcal{L} = \int dx d\tau \frac{1}{g} (\partial_\mu \vec{n})^2 + \int_0^1 du \frac{i2\pi}{\Omega_3} \epsilon_{abcd} n^a \partial_x n^b \partial_\tau n^c \partial_u n^d$$

Boundary Analysis

- Boundary: fermion modes gapped out under interaction, remaining gapless boson modes with U(1)×U(1) symmetry.
  - Naïve picture for why the boundary must be gapless: spin defect carries charge, charge defect carries spin

  - Edge current is transported by the bosonic edge modes (charge 2e Cooper pairs) → shot noise measurement
  - Tunneling from a normal metal → single particle gap
  - Such purely bosonic gapless boundary cannot occur with only one layer of QSH insulator

Wu, Bernevig, Zhang (2005)
Xu, Moore (2005)

Boundary Analysis

- Bulk wave function can be derived from boundary CFT correlation according to the bulk-boundary correspondence. Moore, Read (1991)

$$\langle e^{i\theta_+(z,\bar{z})} e^{-i\theta_+(0)} \rangle = |z|^{-\tilde{K}/2}$$

$$\langle e^{i\theta_+(z,\bar{z})} e^{-i2\phi_+(0)} \rangle = \bar{z}/|z|$$

$$\langle e^{i2\phi_+(z,\bar{z})} e^{-i2\phi_+(0)} \rangle = |z|^{-2/\tilde{K}}$$

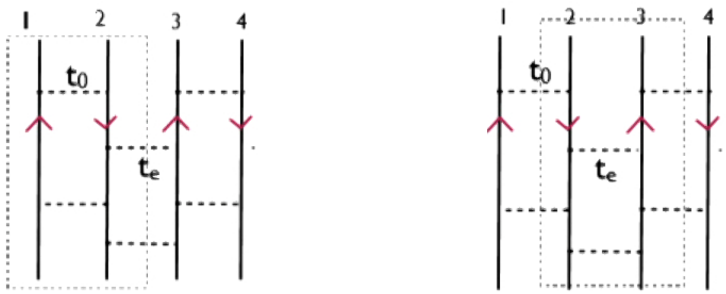
$$\Psi[w_i, z_j] = \langle \prod_i e^{i\theta_+(w_i)} \prod_j e^{i2\phi_+(z_j)} \mathcal{O}_{bg} \rangle$$

$$= F(|z_i - z_j|, |w_i - w_j|, |z_i - w_j|, \tilde{K}) \prod_{i,j} (z_i - w_j) e^{-\frac{1}{4}\sum_i (|w_i|^2 + |z_i|^2)}$$

- The last factor encodes the essential physics that the spin and charge view each other as flux. Consistent with the flux attachment picture of Senthil & Levin 2012. Senthil, Levin (2012)

Bulk Analysis

- Bulk: quantum phase transition between BSPT and trivial state only closes boson gap, fermions remain gapped.
  - Bulk theory can be build from boundary with a Chalker-Coddington / coupled-wire type of model



$t_o > t_e$, trivial $t_o < t_e$, Chern insulator

  - Example: Chern insulator & trivial insulator. We can build the bulk with coupled chiral fermions. The quantum critical point between Chern insulator and trivial insulator is precisely a 2+1d Dirac fermion.

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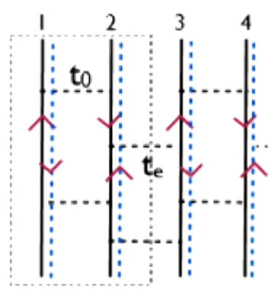
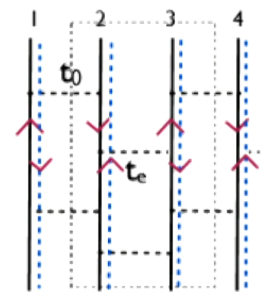


$t_o > t_e$, trivial



$t_o < t_e$, BSPT

Vishwanath, Senthil 2013

  - Boundary theory only has gapless bosons (at low energy) → expect (and supported by numerics) that bulk transition is also "bosonic" → mimic a bosonic SPT-trivial transition.

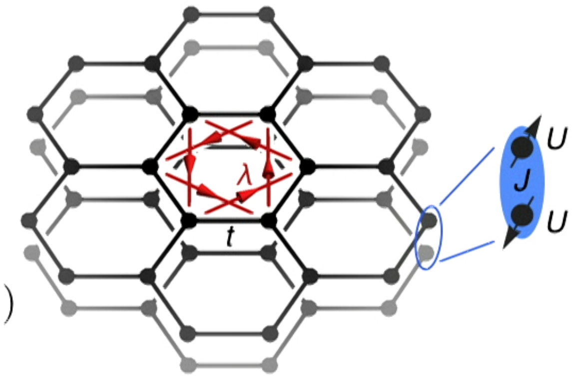
Sign Problem Free Lattice Model

- We designed a lattice model with all the key physics and with no sign problem

$$H = H_{\text{band}} + H_{\text{int}}$$

$$H_{\text{band}} = -t \sum_{\langle ij \rangle, \ell} c^\dagger_{i\ell}\, c_{j\ell} + \sum_{\langle\langle ij \rangle\rangle, \ell} i\, \lambda_{ij}\, c^\dagger_{i\ell}\, \sigma^z\, c_{j\ell}$$

$$H_{int} = J \sum_i (\mathbf{S}_{i1} \cdot \mathbf{S}_{i2} + \frac{1}{4}(n_{i1} - 1)(n_{i2} - 1))$$



- Simple limits of this model:
  - Free limit: bilayer QSH, $\sigma_{sH} = \pm 2$ (depending on λ)
  - Strong *J*-interacting limit: trivial Mott, $\sigma_{sH} = 0$

$$|\Psi\rangle = \prod_i \left(c^\dagger_{1\uparrow}\, c^\dagger_{2\downarrow} - c^\dagger_{1\downarrow}\, c^\dagger_{2\uparrow}\right) |0\rangle$$ rung singlet product state

Slagle, You, Xu (2014)

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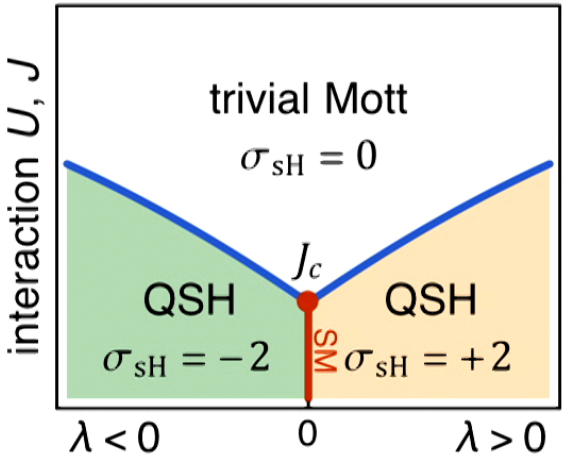

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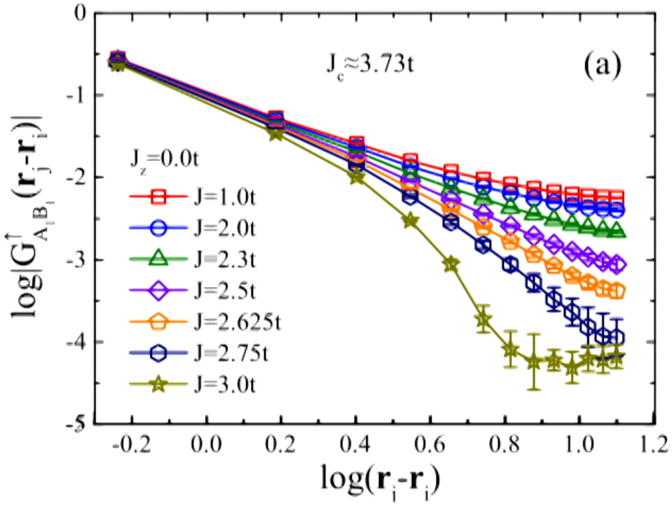
Slagle, You, Xu (2014)

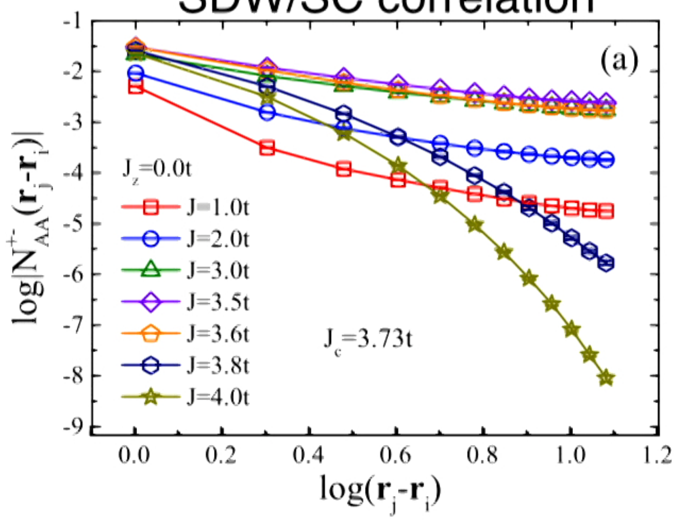
Sign Problem Free Lattice Model

- Determinant QMC (Edge)

fermion correlation



SDW/SC correlation

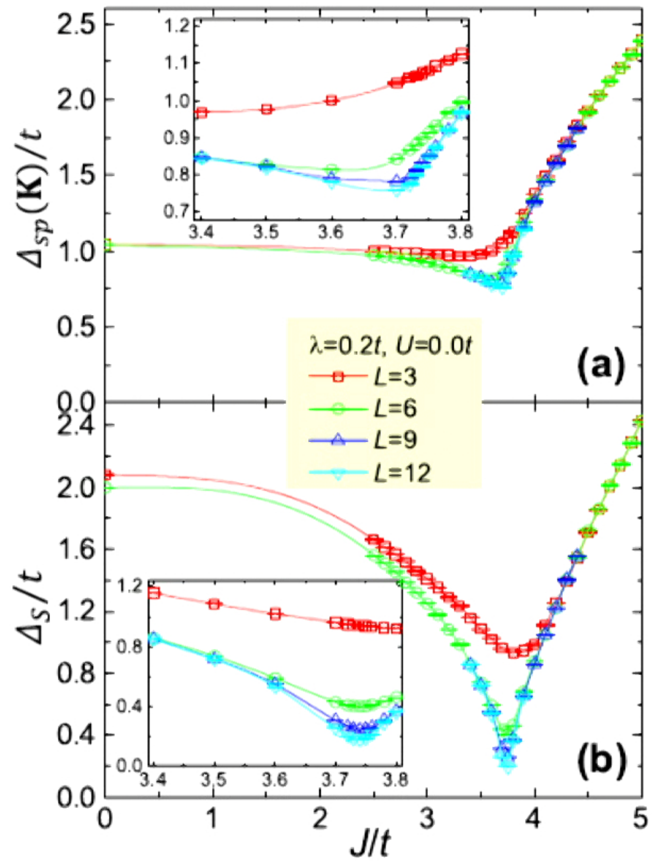


- When the fermion Green's function already decays exponentially at the boundary, bosonic modes still have power law correlation, until the system hits the bulk transition into the trivial Mott phase.

Wu et al. 2016

Sign Problem Free Lattice Model

- Determinant QMC (Bulk)
  - Fermion gap always finite.
  - Bosonic modes become gapless at the SPT-trivial critical point.
  - Fundamentally different from free fermion QSH transition.
- Because the fermionic degrees of freedom never show up at either the boundary or the bulk quantum transition, the whole system can be viewed as a bosonic SPT state.



He et al. 2015

A Theory for the Bulk Transition

- A conjectured field theory for the bulk transition:
  - O(4) NLSM with Topological Θ term.

$$\mathcal{S} = \int d^2x d\tau (\partial_\mu \vec{n})^2 + \frac{i\Theta}{2\pi^2} \epsilon_{abcd} n^a \partial_x n^b \partial_y n^c \partial_\tau n^d$$

  - Θ is the tuning parameter of the transition.
    $\Theta = 2\pi$ describes the BSPT states with SO(4) symmetry.
    $\Theta = 0$ described the trivial phase.
    $\Theta = \pi$ is the transition point.

Bi, Rasmussen, Slagle, Xu. 2013
Xu, Ludwig. 2011

  - It is conjectured that this theory is dual to Nf=2 QED3.

Xu, You. 2015; Karch, Tong. 2016; Hsin, Seiberg. 2016

NLSM with WZW term: Motivation

- Many faces of 2+1d O(5) NLSM with WZW term

$$\mathcal{S} = \int d^2x d\tau \frac{1}{g}(\partial_\mu \vec{n})^2 + \int du \frac{i2\pi k}{\Omega_4} \epsilon_{abcde} n^a \partial_x n^b \partial_y n^c \partial_\tau n^d \partial_u n^e$$

Symmetry Breaking to O(4)

O(4) NLSM with Topological $\Theta = k\pi$

Deconfined Quantum Criticality

Boundary of 3+1d BSPT with O(5) symmetry

- Lower dimensional examples show that WZW term will drastically change the dynamics of the disordered phase of NLSM.
  - 0+1d O(3) NLSM+WZW → degenerate ground states
  - 1+1d O(4) NLSM+WZW → gapless CFT
  - 2+1d O(5) NLSM+WZW → ???

NLSM with WZW term: a slightly different problem

- **Goal**: a *controlled* way to calculate the RG equation of NLSM taking account the effect of the WZW term.
  - **Technical difficulties**
  - $\pi_4(S^N) = 0 \quad \text{for} \quad N > 4$
  - 2+e expansion is not good because of the WZW term.

- **A similar problem** which has a large-N generalization.

$$\mathcal{P} \in \frac{U(N)}{U(n) \times U(N-n)} \qquad \pi_4\left(\frac{U(N)}{U(n) \times U(N-n)}\right) = \mathbb{Z}$$

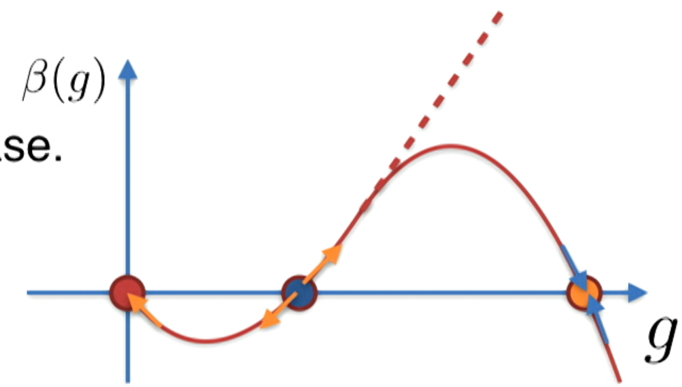
  - we are interested in fix n and take large-N. N=4, n=2 case relates to S4 by symmetry breaking.

$$S^4 \sim \frac{Sp(4)}{Sp(2) \times Sp(2)} \subset \frac{U(4)}{U(2) \times U(2)}$$

NLSM with WZW term: a slightly different problem

$$\mathcal{S} = \int d^2x d\tau \frac{1}{g}\mathrm{Tr}(\partial_\mu \mathcal{P})^2 + \int du \frac{i2\pi k}{256\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(\mathcal{P}\partial_\mu\mathcal{P}\partial_\nu\mathcal{P}\partial_\rho\mathcal{P}\partial_\sigma\mathcal{P})$$

- **Result**:
  - We need large-N and large-k and small-e generalization of the theory to have a controlled RG calculation.
  - We found a new stable fixed point in the disordered phase.
  - We have a transition between Goldstone modes and gapless disordered phase.



Bi, Rasmussen, BenTov, Xu. 2016

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Bridging Fermionic SPT and Bosonic SPT in general

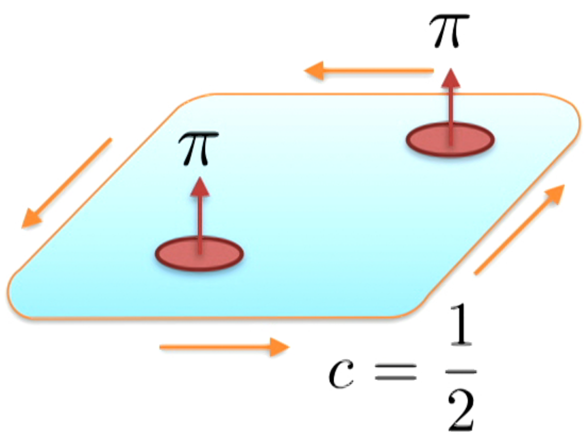
- Bosonic SPT can be built by stacking fermionic TIs/SPTs
  - Q: How many copies of FSPTs we need to form a BSPT.
- Q: How do we get rid of fermions in the spectrum? → Gauge Confinement. $\mathbb{Z}_2^f$
  - Vison has to be a *boson*!
  - Vison must carry *trivial symmetry rep.* and *gapped and non-degenerate spectrum* in order to get a gapped non-degenerate symmetric state for the confined phase
- The above criteria tells us whether in principle we can do the fermion → boson construction.
- For fermion SPTs, topological effect will dress the vison, and it's usually not a trivial boson!!

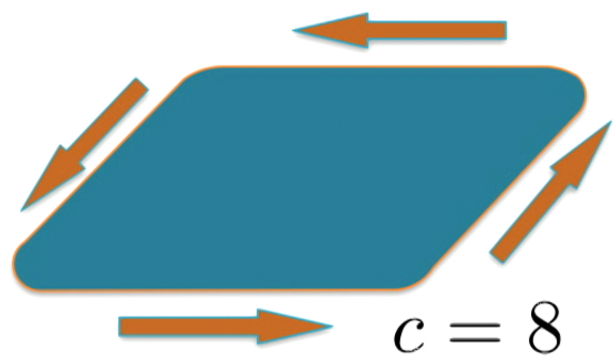
Bridging Fermionic SPT and Bosonic SPT: 2d

- For fermion SPT, topological effect will dress the vison, and it's usually not a trivial boson!!

- e.g. build p+ip SC into E8 state

Kitaev (2005)



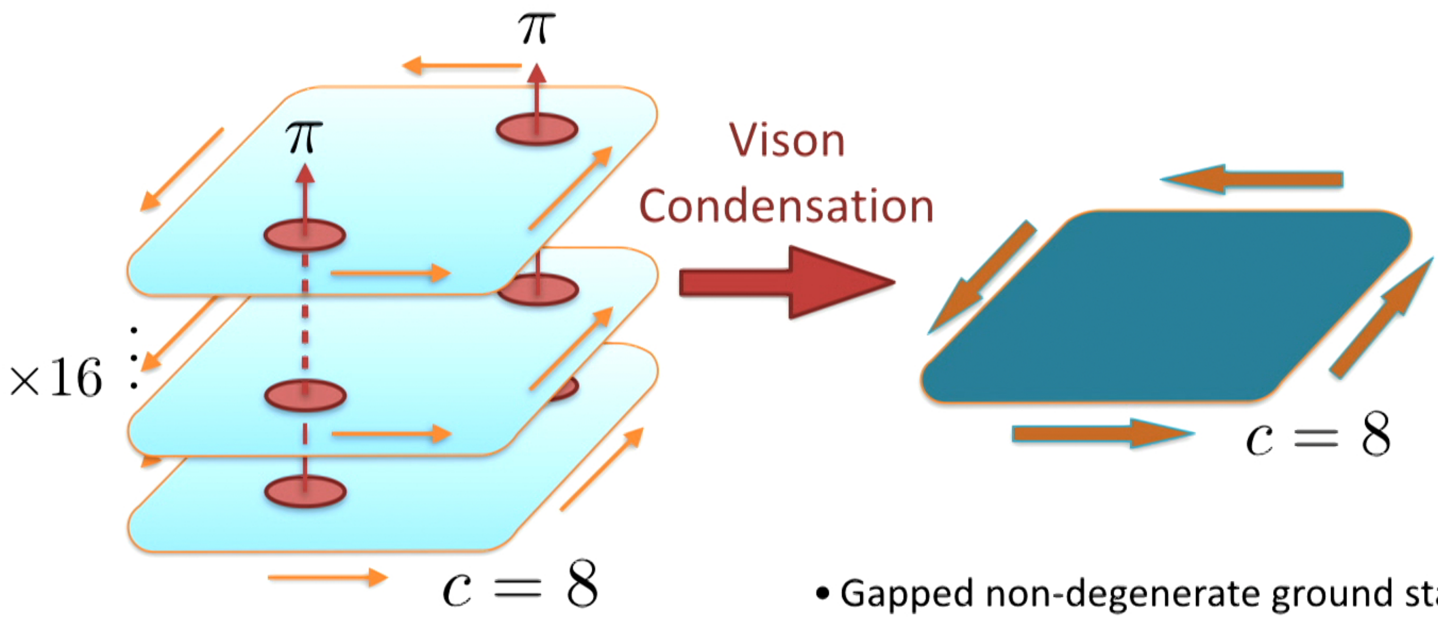


- visons trap Majorana zero modes
  non-abelian anyon

- Gapped non-degenerate ground state
- only bosonic excitations in the bulk and on the edge

Bridging Fermionic SPT and Bosonic SPT: 2d

- e.g. 16 copies p+ip SC → bosonic E8 state
- Mathematically, the gravitational anomaly on the boundaries of the two systems must match each other.



- visons are bosons

- Gapped non-degenerate ground state
- only bosonic excitations in the bulk and on the edge

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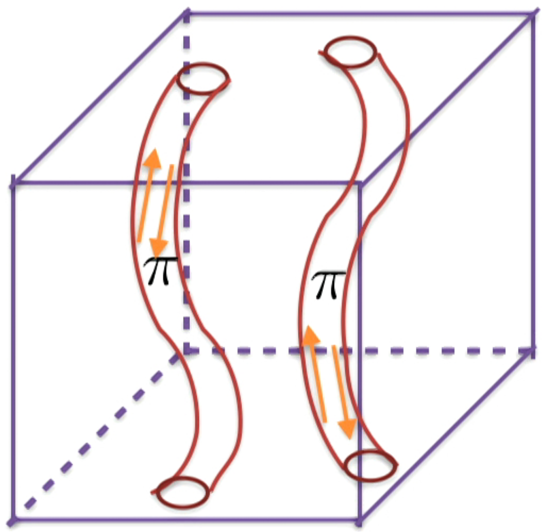
Bridging Fermionic SPT and Bosonic SPT: 3d

- e.g. 3d ^{3}He B-phase → 3d bosonic SPT with Time Reversal.



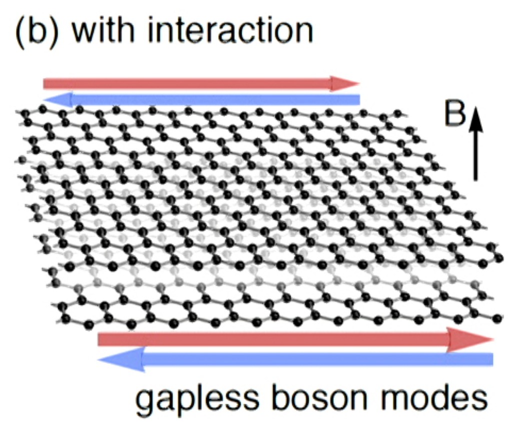
- n=1, single vison line hosts one gapless counter-propagting Majorana mode which is protected by Time Reversal Symmetry.

- For n copies of the system, no uniform time reversal invariant fermion bilinear term can totally gap out the Majorana modes along all directions.

- We can only use interaction to totally gap out the Majorana modes in the vison line. And this will require n=8. The interaction is similar to the interaction that can trivialize 8 copies of Kitaev chains.

Summary I

- **Experimental Proposal**:
  - The boundary is a conductor with single particle gap
  - Contrast between transport and tunneling



  - The competition between magnetic and electric field in the bulk may lead to a purely bosonic quantum phase transition, with gapped electron but gapless bosonic collective modes;

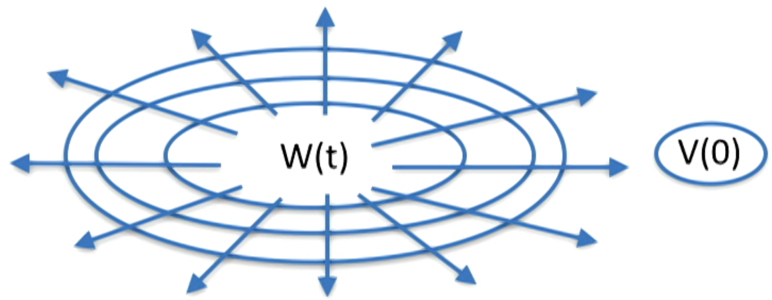
Bi, Zhang, You, Young, Balents, Liu, Xu (2016)

- **Generalities:**
  - BSPT states can be built from FSPTs in general dimensions.

You, Bi, Rasmussen, Cheng, Xu (2014)

Out of Time Order Correlation (OTOC): Introduction

- **Motivation**:
  - quantify the information scrambling and the butterfly effect in quantum many-body dynamics → Quantum Chaos

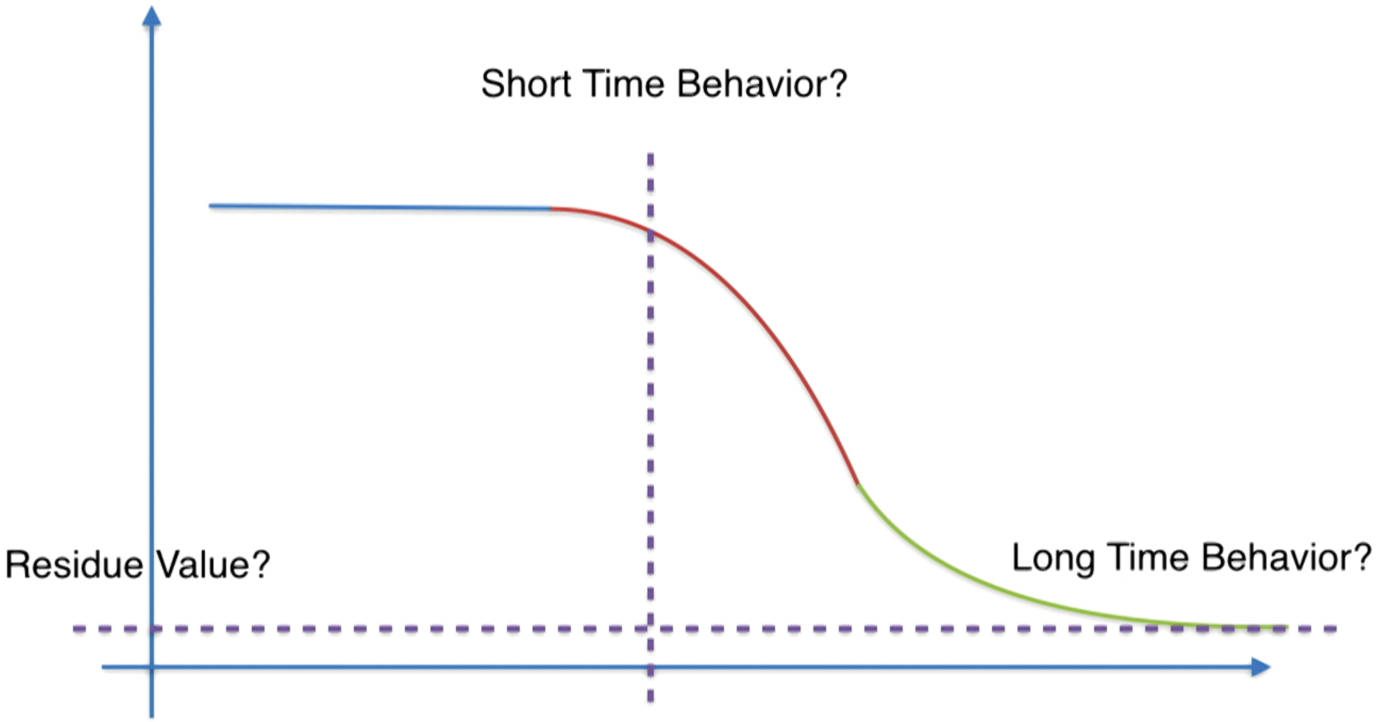


$$C(t) = \langle |[W(t), V(0)]|^2 \rangle = 2(1 - \mathrm{Re} F(t))$$

$$F(t) = \langle W(t)^\dagger V(0)^\dagger W(t) V(0) \rangle \qquad W(t) = e^{iHt} W(0) e^{-iHt}$$

OTOC: generic behaviors

$$F(t) = \langle W(t,l)^\dagger V(0,0)^\dagger W(t,l) V(0,0) \rangle$$



(Scrambling Time for Ergodic system) $t^*_{scr} = l/v_{scr}$

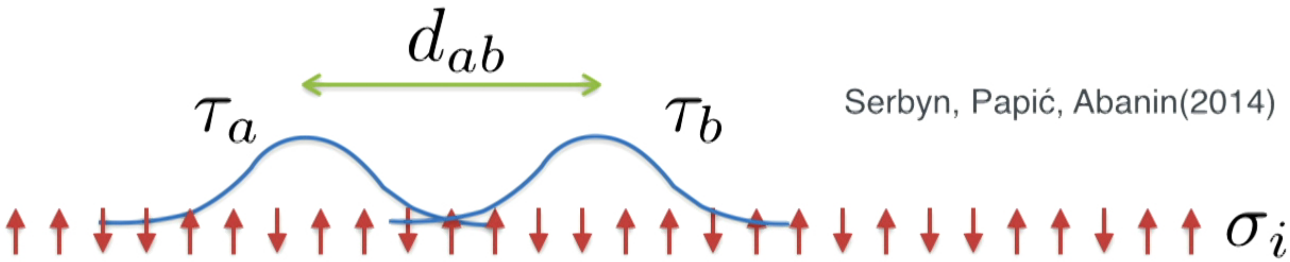
Kitaev talk at KITP(2015)

Many-body Localization: l-bit effective hamiltonian

- **Question**:
  - what's the behavior of OTOC in non-chaotic system?
- Many-body Localized system is an integrable system.

$$H_{MBL} = \sum_a \epsilon_a \tau_a + \sum_{a,b} \epsilon_{ab} \tau_a \tau_b + \sum_{a,b,c} \tau_a \tau_b \tau_c + \ldots$$

  - local integral of motions. $[\tau_a, H] = [\tau_a, \tau_b] = 0$
  - Anderson insulator: no interaction terms.
  - MBL: $\epsilon_{ab} \sim e^{-d_{ab}/\xi}$



Serbyn, Papić, Abanin(2014)

Many-body Localization protected quantum order

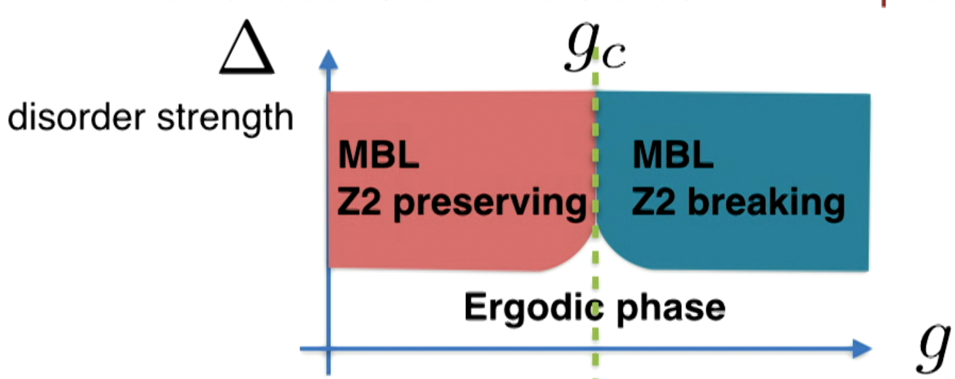
- Many-body Localized effective hamiltonian

$$H_{MBL} = \sum_a \epsilon_a \tau_a + \sum_{a,b} \epsilon_{ab} \tau_a \tau_b + \sum_{a,b,c} \tau_a \tau_b \tau_c + \dots$$

  - every excited state behaves like ground state of local hamiltonian, i.e. Area Law entanglement entropy.

$$|\Psi\rangle \sim \prod_a |\tau_a = \pm 1\rangle \qquad S_{EE} \sim \alpha L^{d-1} + \dots$$

  - Ground state order, e.g. symmetry breaking order, can be extended to excited states → MBL protected quantum order



e.g. disordered transverse field Ising mode or disordered Majorana chain

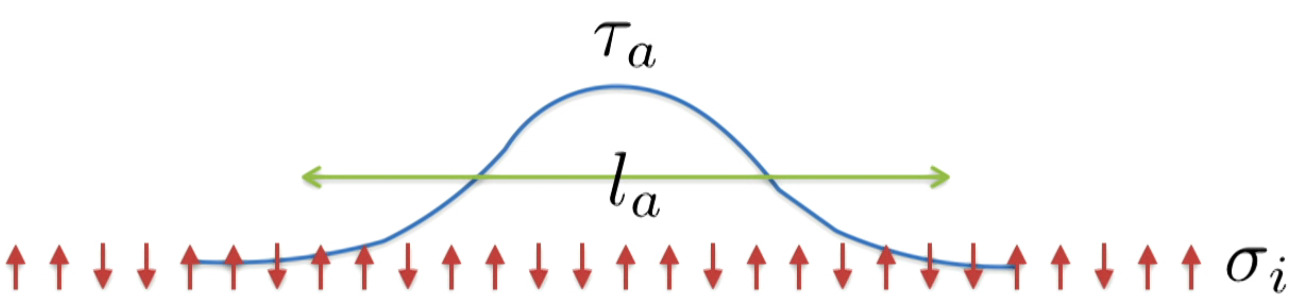
Fisher (1992)

Marginal Many-body Localization

- Marginal Many-body Localized system

$$H_{MBL} = \sum_a \epsilon_a \tau_a + \sum_{a,b} \epsilon_{ab} \tau_a \tau_b + \sum_{a,b,c} \tau_a \tau_b \tau_c + \ldots$$

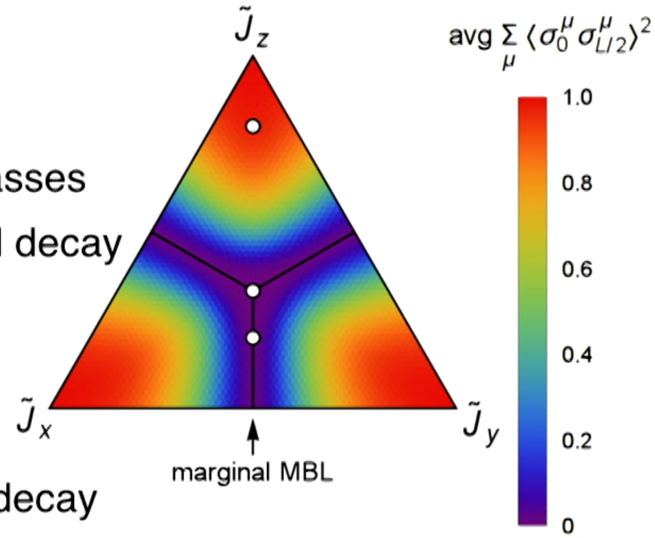
  - Quantum criticality realized in the localized states.
  - signatures: Edwars-Anderson correlations power law decay; entanglement entropy in 1d has log(L) scaling.
  - quasi-long range integral of motion (rare)
  - Single body term energy: $\epsilon_a \sim e^{-\sqrt{l_a/l_0}}$



Example: Disordered XYZ chain

- We consider a disordered spin-1/2 chain with $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry, which exhibits a rich phase diagram of localized states.

$$H = \sum_{i=1}^{L} (J_{i,x}\sigma_i^x\sigma_{i+1}^x + J_{i,y}\sigma_i^y\sigma_{i+1}^y + J_{i,z}\sigma_i^z\sigma_{i+1}^z)$$



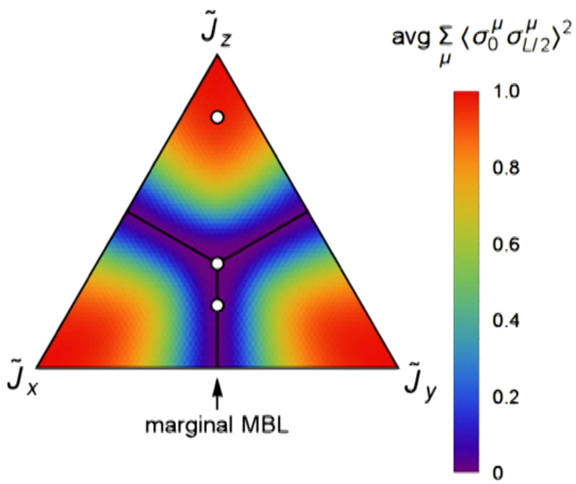
- Three different MBL $\mathbb{Z}_2$ breaking spin glasses
- Edwars-Anderson correlation: exponential decay
- $S_{EE} \sim const$

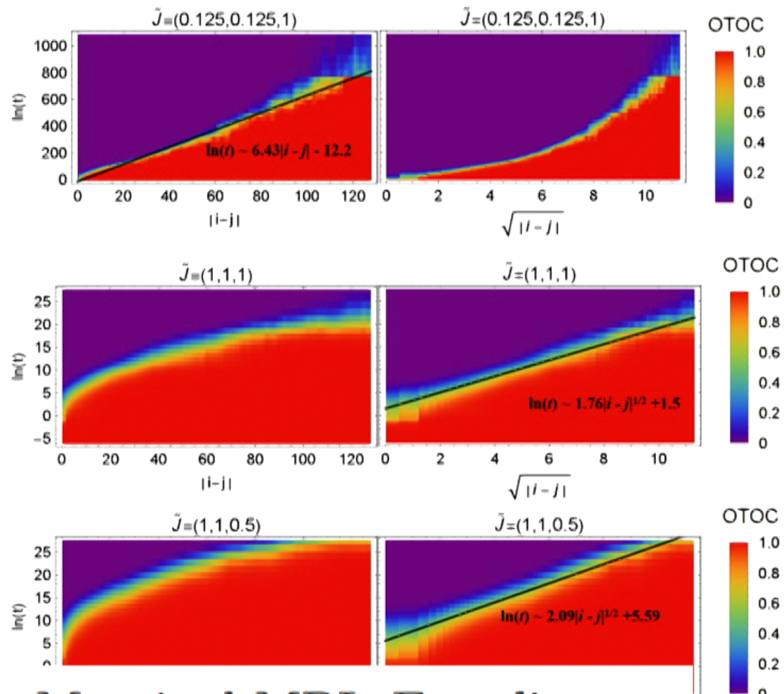
- Phase boundaries: marginal MBL
- Edwars-Anderson correlation: power law decay
- $S_{EE} = \frac{\log(2)}{6} \log(L) + ...$

You, Qi, Xu (2015)
Slagle, You, Xu. (2016)

OTOC in the Disordered XYZ chain: Results

$$H = \sum_{i=1}^{L} (J_{i,x} \sigma_i^x \sigma_{i+1}^x +$$





	Anderson	MBL	Marginal MBL	Ergodic
$\ln t_{\mathrm{scr}}$	∞	$\sim d_{WV}$	$\sim d_{WV}^{1/2}$	$\ln d_{WV}$

Slagle, Bi, You, Xu. (2016)

OTOC in l-bit formalism: derivation

$$F(t) = \mathrm{Tr}\ WVWV \prod_{\tau_A \in A_W \cap A_V} e^{4i\epsilon_A t \tau_A}$$

- The scrambling time scale is set by the following energy scale:

$$|\epsilon_A|^2_{W,V} = \sum_{\tau_A \in A_W \cap A_V} \epsilon_A^2 \Rightarrow t^*_{scr} \sim |\epsilon_A|^{-1}_{W,V}$$

- For MBL: $$|\epsilon_A|_{W,V} \sim |\epsilon_{ab}| \sim e^{-l/\xi} \qquad t^*_{scr} \sim e^{l/\xi}$$

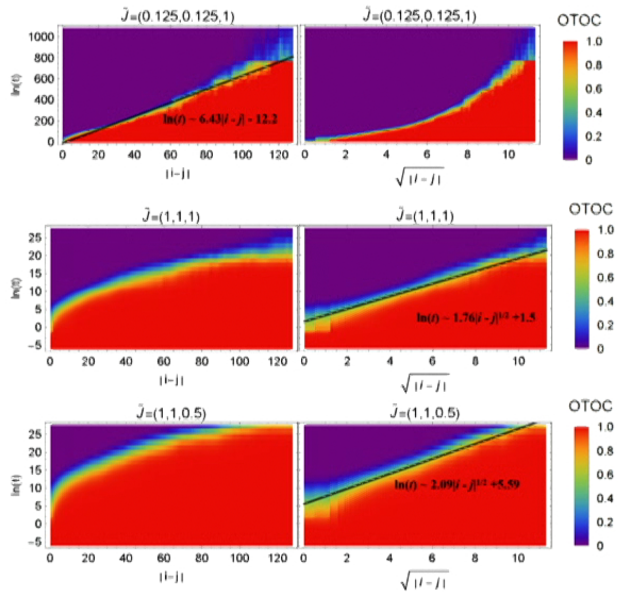
- For marginal MBL: single body term dominate

$$|\epsilon_A|_{W,V} \sim |\epsilon_a(l)| \sim e^{-\sqrt{l/l_0}} \qquad t^*_{scr} \sim e^{\sqrt{l/l_0}}$$

  - Information scrambling is an interaction effect in MBL. Also related to the log growth of entanglement entropy after quantum quench in MBL system. Serbyn, Papić, Abanin(2014)

Summary II

- OTOC has a very slow light cone behavior in localized system.
- Anderson insulator and MBL can be distinguished by OTOC measurements.
- In marginal MBL, the scrambling time exhibits a stretched exponential dependence on the distance.
- quantum criticalities will enhance information scrambling.



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Chen, Zhou, Huse, Fradkin. (2016)
Slagle, Bi, You, Xu. (2016)