

Title: Approaching Lattice Gauge Theories with Tensor Networks – From real-time dynamics to overcoming the sign problem - Stefan Kähn

Date: Dec 15, 2016 10:00 AM

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Abstract: <p>In recent years there has been quite some effort to apply Matrix Product States (MPS) and more general Tensor Networks (TN) to lattice gauge theories. Contrary to the standard Euclidean-time Monte Carlo approach, which faces a major obstacle in the sign problem, numerical methods based on TN are free from the sign problem and allow to some extent simulating time evolution. Moreover, TN are also a suitable tool to explore proposals for potential future quantum simulators for lattice gauge theories.</p>

<p>&nbsp;</p>

<p>In this talk I am going to present some examples where these possibilities allow novel insight into lattice gauge theories. After briefly introducing MPS, I will mainly focus on two models: The first part of the talk is going to be about the Schwinger model. I will show how MPS can help to explore proposals for potential future quantum simulators for this model by studying their spectral properties and simulating adiabatic preparation protocols for the interacting vacuum.</p>

<p>Furthermore, I will show an explicit example where TN allow to overcome the Monte Carlo sign problem in a lattice calculation by studying the zero-temperature phase structure for the two-flavor case at non-zero chemical potential with MPS.</p>

<p>&nbsp;</p>

<p>In the second part, I am focusing on a non-Abelian gauge model, namely a 1+1 dimensional SU(2) lattice gauge theory. Using MPS, the phenomenon of string breaking in this theory can be studied in real time, thus allowing to gain new insight into this process. Moreover, I will show how the gauge field can be integrated out for systems with open boundary conditions and how to obtain a formulation which allows to address the model more efficiently with MPS.</p>

# Approaching Lattice Gauge Theories with Tensor Networks

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From real-time dynamics to overcoming the sign problem

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in collaboration with

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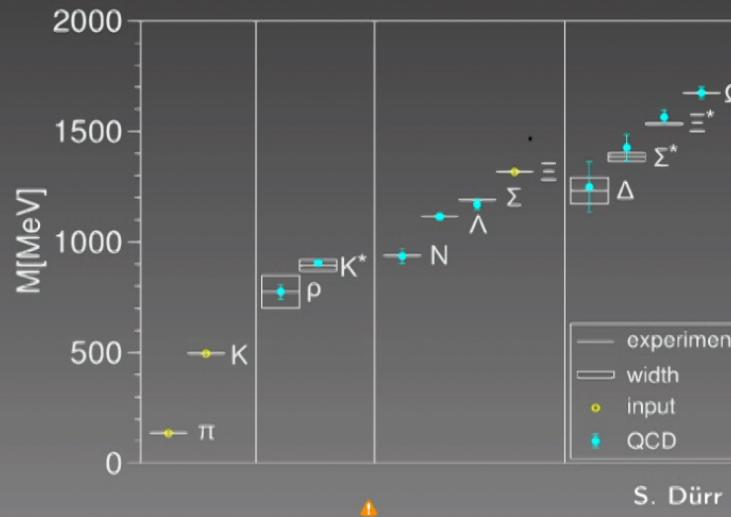
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Goethe-University, Frankfurt am Main  
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NIC, DESY Zeuthen  
CCS Tsukuba  
MPQ



## Motivation

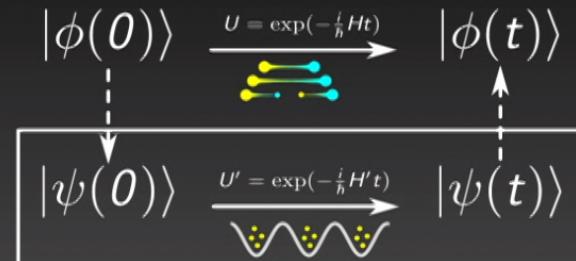
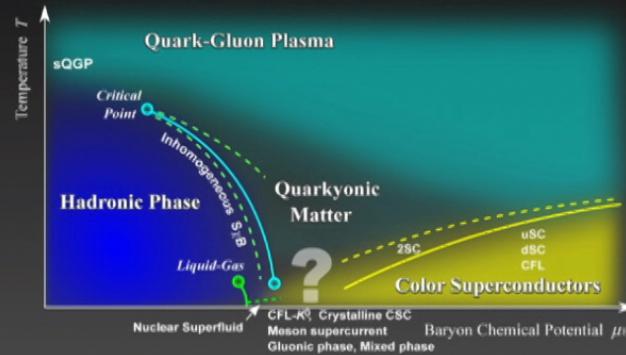
### Lattice gauge theory

- Formulate the theory on a discretized space-time lattice  
⇒ Momentum cutoff due to the lattice spacing
- Allows non-perturbative study of gauge theories
- Monte Carlo simulations in Euclidean time for mass spectra, phase diagrams, ...



S. Dürr et. al. Science 322 1224 (2008)

# Motivation



## Monte Carlo simulations

- Sign problem
- No real-time dynamics
- No access to wave function

## Quantum simulation

- Sign problem free
- Real-time dynamics
- Finite number of DOF
- Noise

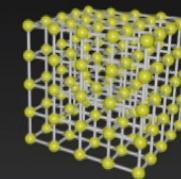
K. Fukushima and T. Hatsuda, Rep. Prog. Phys. 74, 014001 (2011)

E. A. Martinez et al. Nature 534, 516 (2016)

# Matrix Product States (MPS)

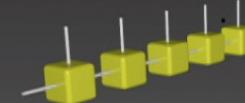
## Monte Carlo

- ✓ Expectation values
- ✗ Access to the ground state wave function
- ✗ Sign problem free



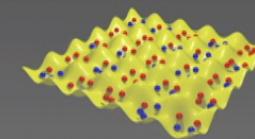
## Matrix Product States

- ✓ Ground states / Low lying excitations
- ✓ Time evolution
- ✓ Sign problem free



## Quantum simulation

- ✓ Ground states / Low lying excitations
- ✓ Time evolution
- ✓ Sign problem free
- ✓ Gauge invariance





# Outline

- 1 Motivation
- 2 Matrix Product States
- 3 The Schwinger model
  - Quantum simulation
  - Phase structure for two-flavors
- 4 SU(2) Lattice Gauge Theory
  - Dynamics of string breaking
  - Integrating out the gauge field
- 5 Conclusion & Outlook





# Matrix Product States (MPS)

## MPS ansatz

- Coming from quantum information theory
- Fulfill the area law:  $S(L) \propto L^{d'-1} \stackrel{d'=1}{=} \text{const}$
- MPS ansatz with open boundary conditions (OBC) for system with  $N$  sites

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_N} A^{i_1} A^{i_2} \dots A^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$$



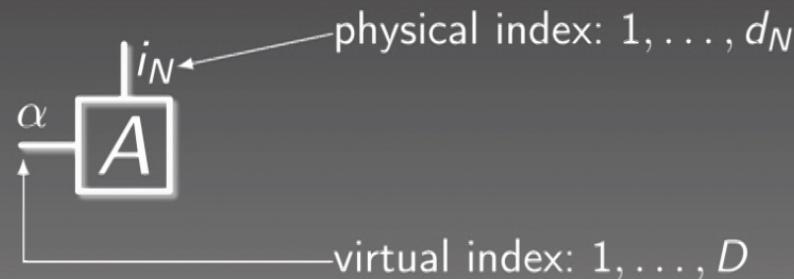
# Matrix Product States (MPS)

## MPS ansatz

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- MPS ansatz with open boundary conditions (OBC) for system with  $N$  sites

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_N} A^{i_1} A^{i_2} \dots A^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$$

- Tensor  $A^{i_N} \in \mathbb{C}^{D \times 1}$





# Matrix Product States (MPS)

## Scaling

- Number of parameters in the MPS with OBC

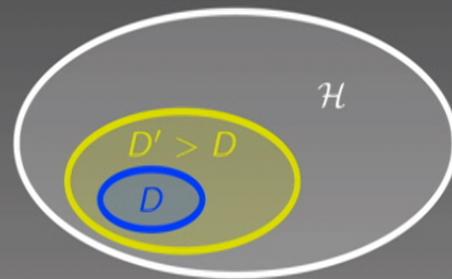
$$|\psi\rangle = \square \text{---} \square \text{---} \dots \text{---} \square \text{---} \square$$

$$(N - 2)D^2d + 2Dd = \mathcal{O}(ND^2d)$$

- Dimension of the full Hilbert space

$$d^N$$

- Quantum information: Physically relevant states  $D \ll d^{\frac{N}{2}}$



M. B. Hastings, Journal of Statistical Mechanics 2007 (2007)  
G. Vidal Phys. Rev. Lett. 93, 040502 (2004)  
F. Verstraete, D. Porras, J.I. Cirac Phys. Rev. Lett. 93, 227205 (2004)



## 1 Motivation

## 2 Matrix Product States

## 3 The Schwinger model

- Quantum simulation
- Phase structure for two-flavors

## 4 SU(2) Lattice Gauge Theory

- Dynamics of string breaking
- Integrating out the gauge field

## 5 Conclusion & Outlook





# Schwinger model

## Lattice Hamiltonian in temporal gauge

- Lattice formulation with Kogut-Susskind staggered fermions

$$H = \left[ -\frac{i}{2a} \sum_{n=1}^{N-1} \sum_{f=1}^F \left( \phi_{n,f}^\dagger L_n^+ \phi_{n+1,f} - \text{h.c.} \right) + \sum_{n=1}^N \sum_{f=1}^F \left( (-1)^n m_f + \kappa_f \right) \phi_{n,f}^\dagger \phi_{n,f} + \frac{ag^2}{2} \sum_{n=1}^{N-1} L_n^2 \right]$$

Kinetic part + Coupling to gauge field      Mass term      Chemical potential      Electric energy

- Additional constraint: Gauss Law

$$G_n |\chi\rangle = q_n |\chi\rangle, \quad [H, G_n] = 0 \quad \forall n$$

$$G_n = L_n - L_{n-1} - \sum_{f=1}^F \phi_{n,f}^\dagger \phi_{n,f} - \frac{F}{2} (1 - (-1)^n)$$

Staggered charge  $Q_n$

J. Kogut, L. Susskind, Phys. Rev. D 11 395 (1975)

# Schwinger model

## Lattice Hamiltonian in temporal gauge

- Dimensionless formulation

$$W = -i \times \sum_{n=1}^{N-1} \sum_{f=1}^F \left( \phi_{n,f}^\dagger L_n^+ \phi_{n+1,f} - \text{h.c.} \right)$$

$$+ \sum_{n=1}^N \sum_{f=1}^F \left( (-1)^n \mu_f + \nu_f \right) \phi_{n,f}^\dagger \phi_{n,f} + \sum_{n=1}^{N-1} L_n^2$$

$\frac{1}{(ag)^2}$        $\frac{2m_f/ag^2}{\uparrow}$        $\frac{2\kappa_f/ag^2}{\uparrow}$



$$\xrightarrow{a \rightarrow 0} \psi_f = \begin{pmatrix} \psi_{1,f} \\ \psi_{2,f} \end{pmatrix}$$

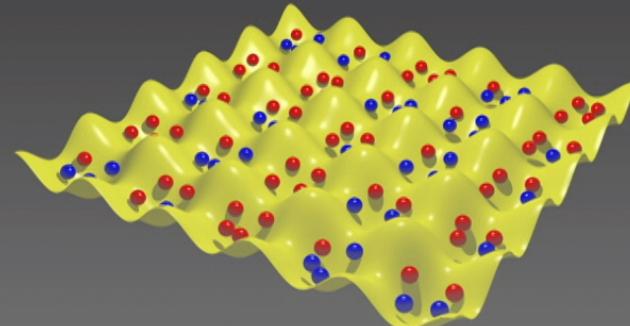
- Compact formulation:  $L_n^+ = \exp(i\theta_n)$ ,  $\theta_n \in [0, 2\pi]$

$$[\theta_n, L_m] = i\delta_{nm} \quad \Rightarrow \quad L_n^+ \text{ acts as raising operator}$$

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## Exploring a quantum simulator for the single flavor case

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## Schwinger model: Quantum simulation

### Quantum simulator

- Focus on the single-flavor case  $F = 1$
- For a single flavor the chemical potential is only a constant energy offset  
⇒ Set  $\nu = 0$
- Electric field has to be truncated to a small value to obtain finite dimensional Hilbert spaces

### Questions:

- How close is that to the full model?
- How can one prepare the interacting vacuum?
- How does gauge invariance breaking noise affect the process?

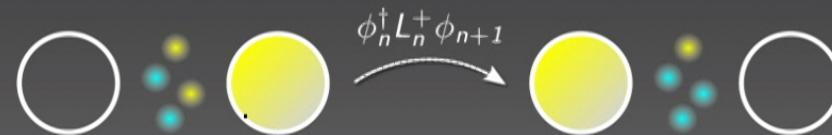


# Schwinger model: Quantum simulation

## cQED model

- Links are populated by two bosonic species
- Link operators (Schwinger representation)

$$L_n^+ = \frac{a_n^\dagger b_n}{\sqrt{\frac{N_0}{2}(\frac{N_0}{2} + 1)}}, \quad L_n = \frac{1}{2} (a_n^\dagger a_n - b_n^\dagger b_n)$$



- Number of particles per link is fixed to an even number  $N_0$
- Local Hilbert space dimension  $d = N_0 + 1$
- **Same  $U(1)$  symmetry** as the Schwinger model

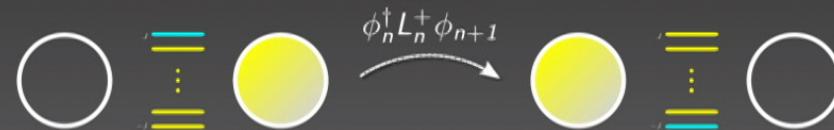
E. Zohar, J.I. Cirac, B. Reznik, Phys. Rev. A 88, 023617 (2013)

## Schwinger model: Quantum simulation

### $\mathbb{Z}_d$ model

- Use angular momentum eigenstates  $|\varphi^k\rangle$  to construct the link operators
- Identify  $|\varphi^{J+1}\rangle$  with  $|\varphi^{-J}\rangle$

$$L_n^+ = \sum_{k=-J}^J |\varphi_n^{k+1}\rangle\langle\varphi_n^k|, \quad L_n = \sum_{k=-J}^J k|\varphi_n^k\rangle\langle\varphi_n^k|$$



- $\mathbb{Z}_d$  symmetry, different from Schwinger model
- Local Hilbert space dimension  $d = 2J + 1$
- Gauss Law is fulfilled modulo  $d$ :

$$U(n)|\chi\rangle = \left[ \exp\left(i\frac{2\pi}{d}G(n)\right) - \mathbb{1} \right] |\chi\rangle = 0 \quad \forall n$$



A. De La Torre, J. Iguain, Am. J. Phys. 66, 1115 (1998)



## Schwinger model: Quantum simulation

Overview:

Model	Gauge symmetry	Link dimension $d$
Schwinger model	$U(1)$	$\infty$
cQED model	$U(1)$	$N_0 + 1$
$\mathbb{Z}_d$ model	" $\mathbb{Z}_d$ "	$2J + 1$



## Schwinger model: Quantum simulation

### Effect of the finite dimension

- Extrapolate to the continuum similar to lattice calculations
- Gather a range of values for bond dimension  $D$ , system size  $N$  and coupling strength  $x$
- Extrapolate:

Bond dimension:



$$D \in [50, 200]$$

Finite size:



$$N \in [50, 200]$$

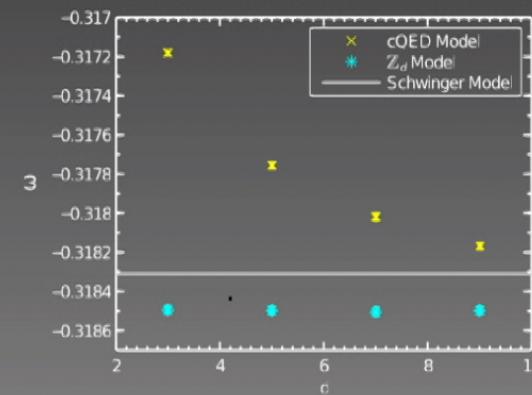
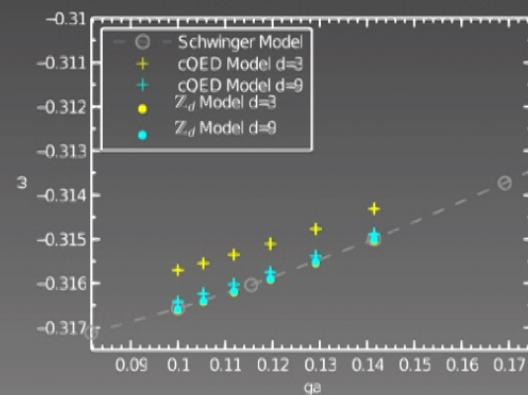
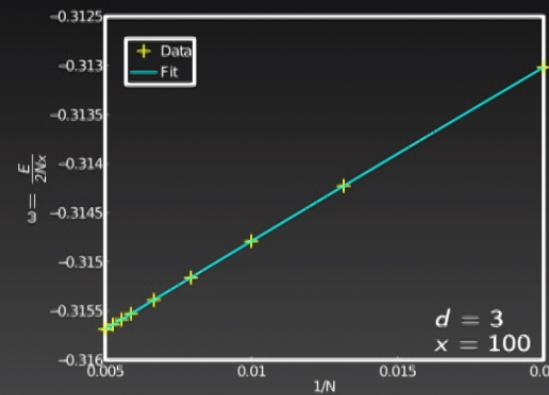
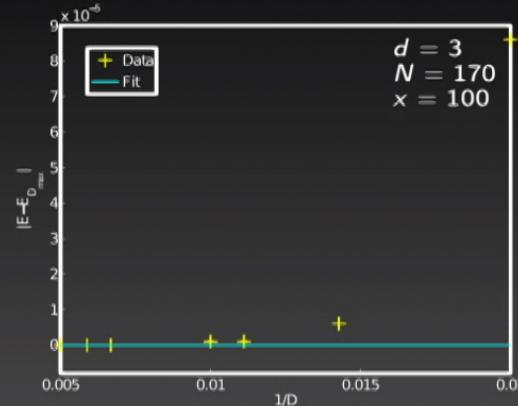


Continuum:  $x \in [50, 100]$



# Schwinger model: Quantum simulation

Spectrum



SK, J.I. Cirac, M.C. Bañuls, Phys. Rev. A 90, 042305 (2014)



## Schwinger model: Quantum simulation

### Adiabatic ground state preparation

- Start with the strong coupling ground state  $x = 0$

$$\begin{aligned} W = & -ix \sum_{n=1}^{N-1} \left( \phi_n^\dagger L_n^+ \phi_{n+1} - \text{h.c.} \right) \\ & + \sum_{n=1}^N (-1)^n \mu \phi_{n,f}^\dagger \phi_{n,f} + \sum_{n=1}^{N-1} L_n^2 \end{aligned}$$

# Schwinger model: Quantum simulation

## Adiabatic ground state preparation

- Start with the strong coupling ground state  $x = 0$

$$W = -ix \sum_{n=1}^{N-1} (\phi_n^\dagger L_n^\dagger \phi_{n+1} - \text{h.c.}) + \sum_{n=1}^N (-1)^n \mu \phi_{n,f}^\dagger \phi_{n,f} + \sum_{n=1}^{N-1} L_n^2$$

Site:	1	2	3	4	...
Fermions:					...
Charge:	0	0	0	0	...

- State fulfills Gauss Law

$$L_n - L_{n-1} = Q_n (\phi_n^\dagger \phi_n) = \phi_n^\dagger \phi_n - \frac{1}{2} [1 - (-1)^n]$$

- Adiabatically ramp  $x$  during time  $T$  to desired final value  $x_F$



## Schwinger model: Quantum simulation

### Adiabatic ground state preparation

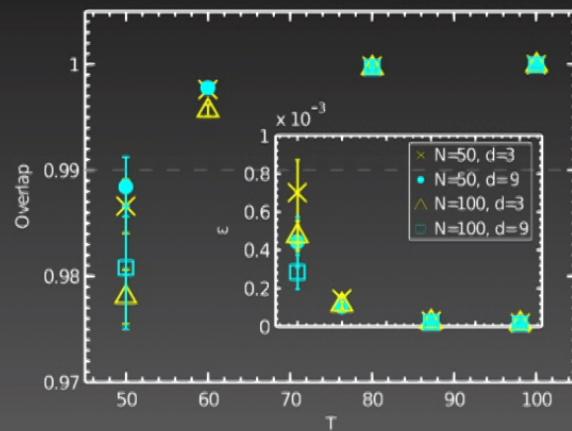
- Choice of the ramp

$$x(t) = x_F \left( \frac{t}{T} \right)^3.$$

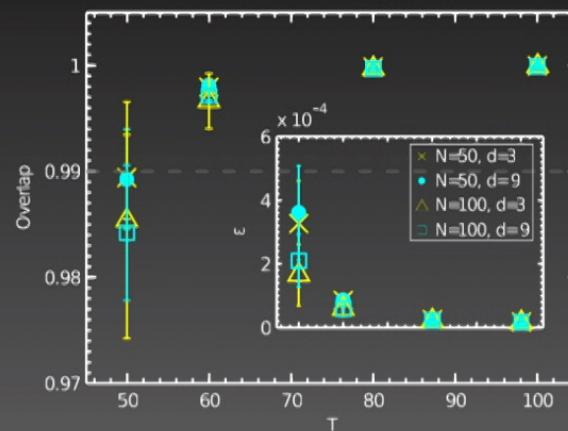
- To check the success monitor the overlap and the error in energy with respect to the variational results
- Success criterion: Overlap  $\geq 0.99$  with variational results

# Schwinger model: Quantum simulation

cQED model



$\mathbb{Z}_d$  model



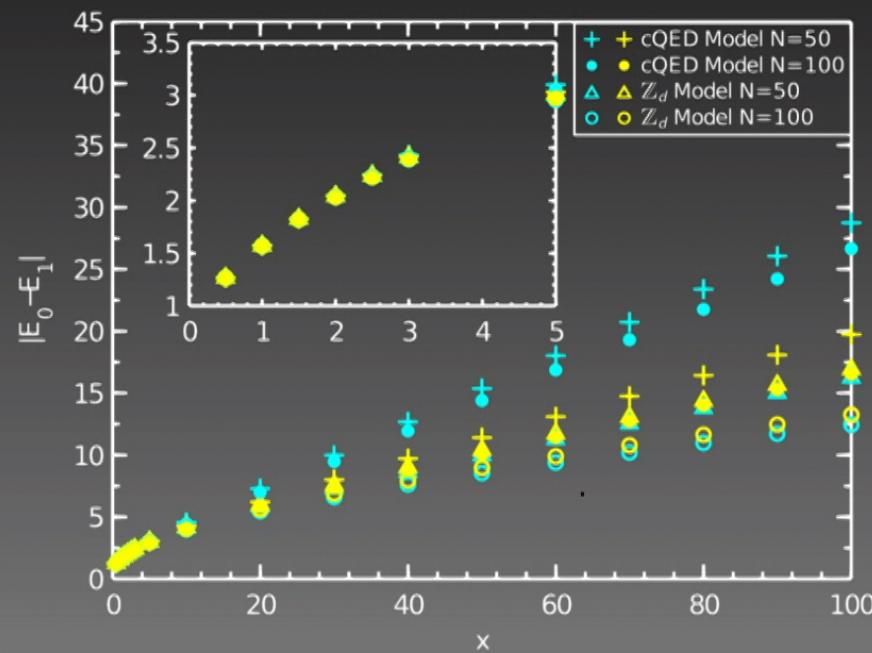
- Errors at the beginning are due to non-adiabaticity
- Almost no dependency on system size

SK, J.I. Cirac, M.C. Bañuls, Phys. Rev. A 90, 042305 (2014)

# Schwinger model: Quantum simulation

## Adiabatic ground state preparation

- Maximum ramping speed depends on the gap
- Numerical result for the gap



SK, J.I. Cirac, M.C. Bañuls, Phys. Rev. A 90, 042305 (2014)

## Schwinger model: Quantum simulation

Ground state preparation in the presence of noise

- Mimic gauge invariance breaking noise
- Assume noise strength proportional to  $x$

$$\lambda x(t) \sum_n (a_n^\dagger b_n + b_n^\dagger a_n)$$

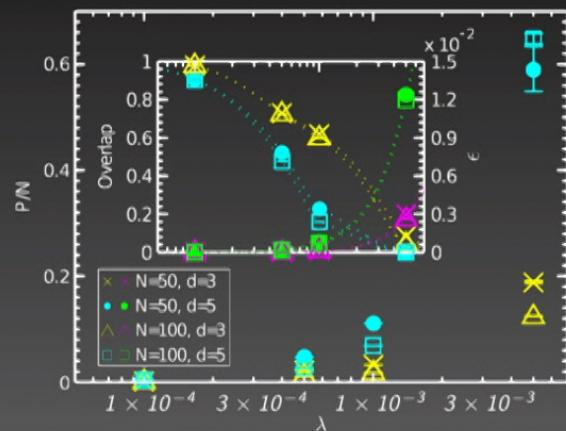
$$\lambda x(t) \sum_n (L_n^+ + L_n^-)$$



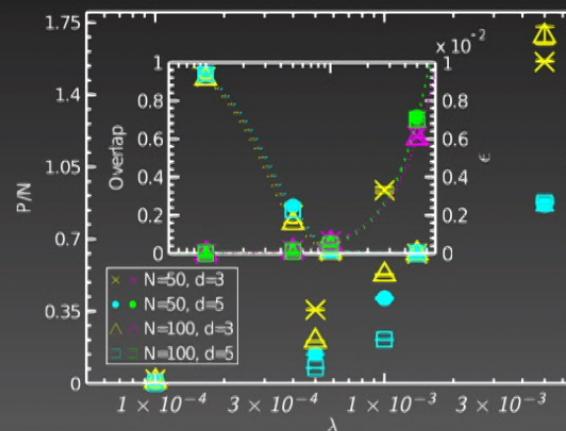
- Take  $T = 100$  which guaranteed successful evolution in the noise free case
- Monitor the violation of Gauss Law per particle  $P/N$

# Schwinger model: Quantum simulation

cQED model



$\mathbb{Z}_d$  model



- Almost no dependency on system size
- Deviation of energy  $\leq 1.5\%$

SK, J.I. Cirac, M.C. Bañuls, Phys. Rev. A 90, 042305 (2014)

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Phase structure for two flavors at  $T = 0$

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## Schwinger model: Phase structure for two-flavors

### Model system

- Two flavors  $F = 2$  at non-vanishing isospin chemical potential

$$\frac{\mu_I}{2\pi} = \frac{N}{4\pi x} (\nu_2 - \nu_1) \neq 0$$

- Cannot be accessed with Monte Carlo due to the sign problem
- For  $m/g = 0$  analytical work found an infinite number of phases characterized by the isospin number

$$\Delta N = N_1 - N_2, \quad N_i = \sum_{k=1}^N \phi_{n,i}^\dagger \phi_{n,i}$$

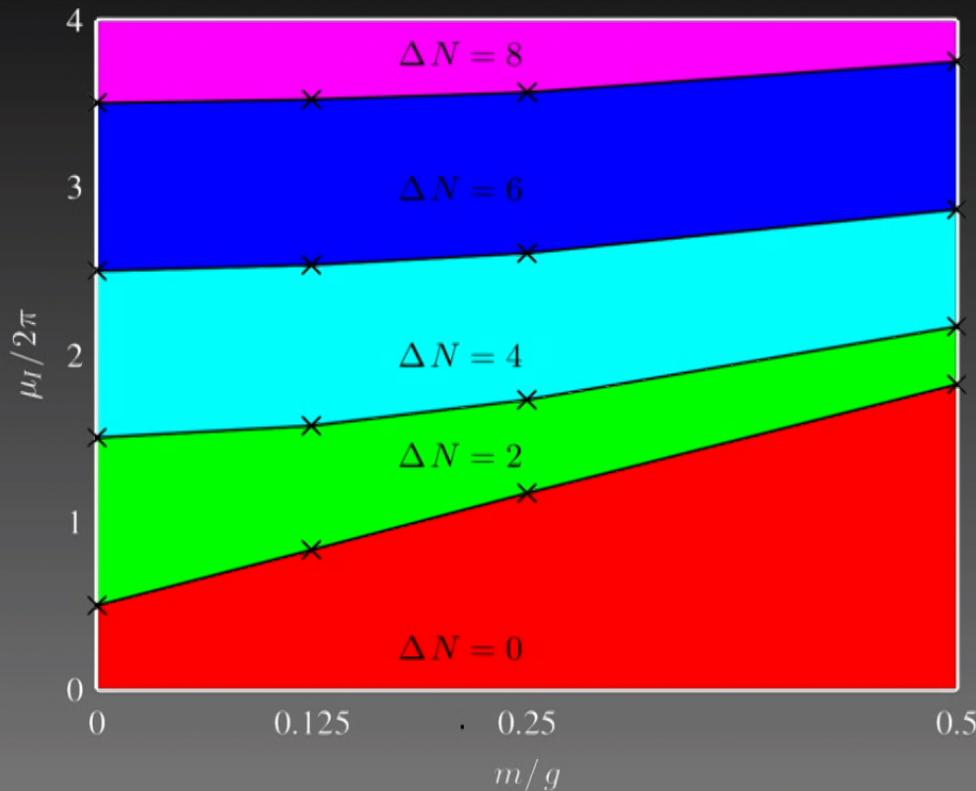
- The massive case cannot be addressed analytically

R. Narayanan, Phys. Rev. D 86, 125008 (2012)

 R. Lohmayer, R. Narayanan, Phys. Rev. D 88, 105030 (2013)

## Schwinger model: Phase structure for two-flavors

Phase structure at  $T = 0$



M.C. Bañuls, K. Cichy, J.I. Cirac, K. Jansen, SK, arXiv:1611.00705 (2016)



## 1 Motivation

## 2 Matrix Product States

## 3 The Schwinger model

- Quantum simulation
- Phase structure for two-flavors

## 4 SU(2) Lattice Gauge Theory

- Dynamics of string breaking
- Integrating out the gauge field

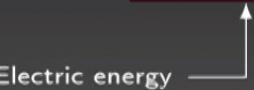
## 5 Conclusion & Outlook

# SU(2) LGT

## Lattice Hamiltonian in temporal gauge

- Lattice formulation with Kogut-Susskind staggered fermions

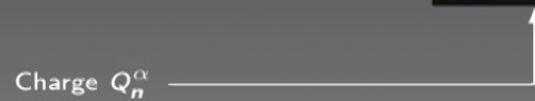
$$H = \varepsilon \sum_n \left( \phi_n^\dagger U_n \phi_{n+1} + \text{h.c.} \right) + M \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n J_n^2$$

Hopping       Staggered mass       Electric energy 

- Additional constraint: Gauss Law

$$G_n^\alpha |\chi\rangle = q_n^\alpha |\chi\rangle, \quad \alpha \in \{x, y, z\}, \quad [H, G_n^\alpha] = 0 \quad \forall \alpha, n$$

$$G_n^\alpha = L_n^\alpha - R_{n-1}^\alpha - \frac{1}{2} \phi_n^\dagger \sigma^\alpha \phi_n$$

Charge  $Q_n^\alpha$  

J. Kogut, L. Susskind, Phys. Rev. D 11 395 (1975)

# SU(2) LGT

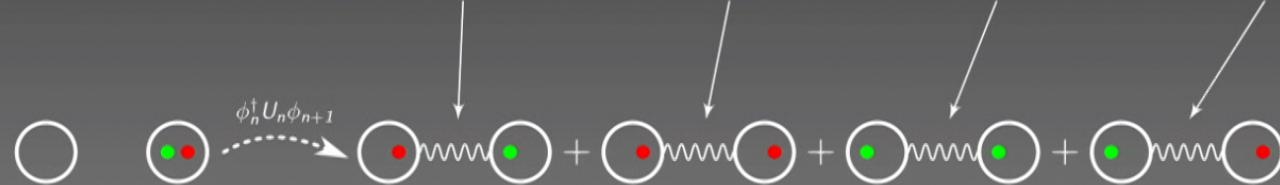
Lattice Hamiltonian in temporal gauge

- Lattice formulation with Kogut-Susskind staggered fermions

$$H = \varepsilon \sum_n \left( \phi_n^\dagger U_n \phi_{n+1} + \text{h.c.} \right) + M \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n j_n^2$$

- Hopping term

$$\begin{aligned} \phi_n^\dagger U_n \phi_{n+1} &= \begin{pmatrix} \phi_r^\dagger & \phi_g^\dagger \end{pmatrix} \begin{pmatrix} U_{n,00} & U_{n,01} \\ U_{n,10} & U_{n,11} \end{pmatrix} \begin{pmatrix} \phi_r \\ \phi_g \end{pmatrix} \\ &= \phi_r^\dagger U_{n,00} \phi_r + \phi_r^\dagger U_{n,01} \phi_g + \phi_g^\dagger U_{n,10} \phi_r + \phi_g^\dagger U_{n,11} \phi_g \end{aligned}$$



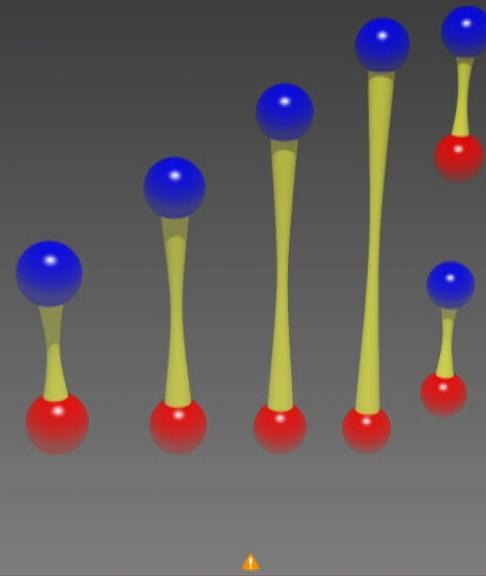
J. Kogut, L. Susskind, Phys. Rev. D 11 395 (1975)



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## Dynamics of string breaking

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# SU(2) LGT: Dynamics of string breaking

## Problem

- Link Hilbert spaces are infinite dimensional

## Truncation

- $U_n$  are nothing but Wigner matrices for spin 1/2
- Allows to define generators for left and right gauge transformations  $L_n^\alpha$  and  $R_n^\alpha$
- Suitable basis for the gauge links:  $|jmm'\rangle$
- $U_n$  can be decomposed as over the irreps for each  $j$
- Truncate at finite value  $j_{\max}$

$$d = \sum_{j=0, \frac{1}{2}, 1, \dots}^{j_{\max}} (2j+1)^2$$

- For all the following  $j_{\max} = 1/2 \Leftrightarrow d = 5$

E. Zohar, M. Burrello, Phys. Rev. D 91 054506 (2015)

# SU(2) LGT: Dynamics of string breaking

Strong coupling ground state

- Neglect the hopping

$$H = \varepsilon \sum_n \left( \phi_n^\dagger U_n \phi_{n+1} + \text{h.c.} \right) + M \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n j_n^2$$

- Strong coupling ground state

Site:	1	2	3	4	5	6	7	8
Sign:	-	+	-	+	-	+	-	+

# SU(2) LGT: Dynamics of string breaking

## Strong coupling ground state

- Neglect the hopping

$$H = \varepsilon \sum_n \left( \phi_n^\dagger U_n \phi_{n+1} + \text{h.c.} \right) + M \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n j_n^2$$

- Strong coupling ground state

Site:	1	2	3	4	5	6	7	8
Sign:	-	+	-	+	-	+	-	+
$4Q_n^\alpha Q_n^\alpha :$	0	0	0	0	0	0	0	0

- Charge square

$$4Q_n^\alpha Q_n^\alpha = (\textcolor{red}{n}_r - \textcolor{green}{n}_g)^2$$

# SU(2) LGT: Dynamics of string breaking

## Generating strings

- Manipulate fermionic content in gauge invariant way

Site:	1	2	3	4	5	6	7	8
Sign:	-	+	-	+	-	+	-	+
$4Q_n^\alpha Q_n^\alpha :$	0	0	0	0	0	0	0	0

# SU(2) LGT: Dynamics of string breaking

## Generating strings

- Manipulate fermionic content in gauge invariant way

$$S = \phi_4^\dagger U_4 \phi_5$$

Site:	1	2	3	4	5	6	7	8
+								
+								
+								
$4Q_n^\alpha Q_n^\alpha :$	0	0	0	1	1	0	0	0

⇒ Two charges connected by a flux tube

- Energy increase  $2M + \frac{ag^2}{2} J_4^2$
- Also longer strings are possible

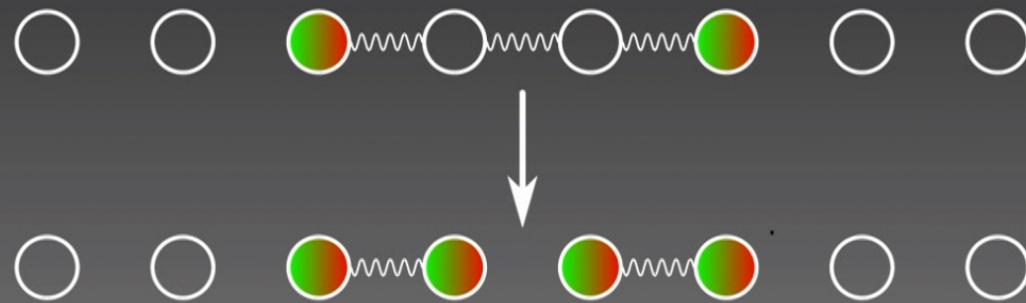
$$S = \phi_n^\dagger U_n U_{n+1} U_{n+2} \dots U_{n+l} \phi_{n+l+1}$$



# SU(2) LGT: Dynamics of string breaking

## String breaking

- Energy increase due to the string proportional to its length
- If flux energy exceeds  $2M$  system can lower its energy by creating a quark-antiquark pair and reducing the flux
- At a certain length it is favorable to break the string



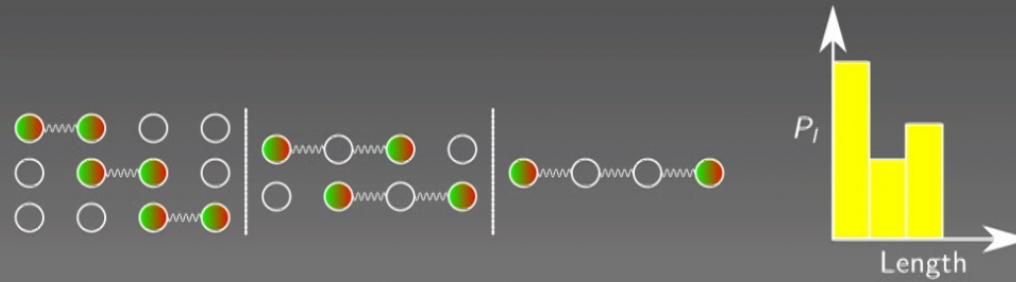


# SU(2) LGT: Dynamics of string breaking

## Detecting string breaking

- Flux configuration
- $Q_n^\alpha Q_n^\alpha$
- String: object with finite flux in some region an no flux outside  
⇒ construct projectors  $P_{nl}$
- Create histograms from the string length from these projectors

$$P_l = \frac{\sum_{n=1}^{N-l} \left( \langle \Psi | P_{nl} | \Psi \rangle - \langle \Omega | P_{nl} | \Omega \rangle \right)}{\# \text{ strings of length } l}$$



# SU(2) LGT: Dynamics of string breaking



Real-time dynamics in the presence of static external charges

## Procedure

- Compute the interacting vacuum



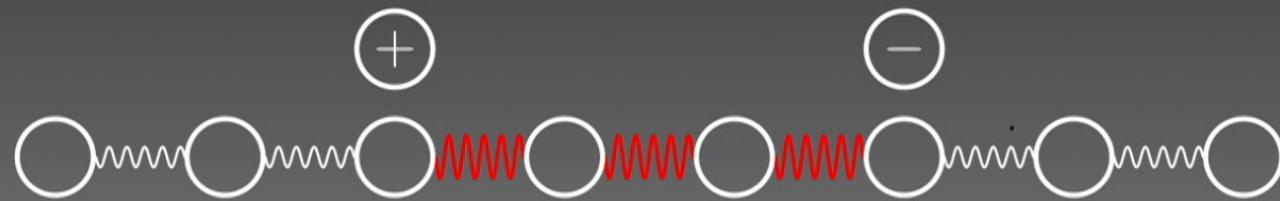


## SU(2) LGT: Dynamics of string breaking

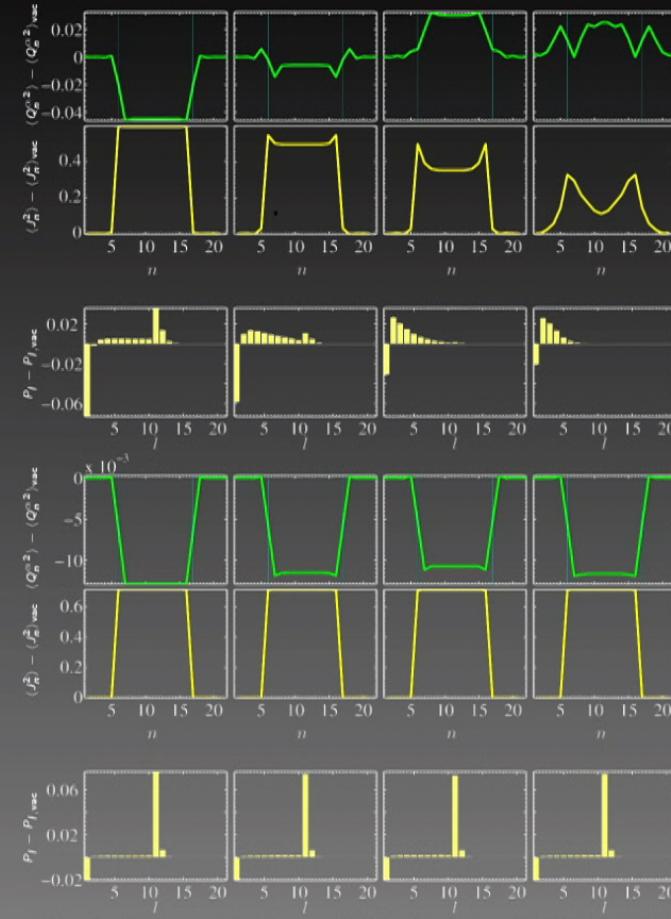
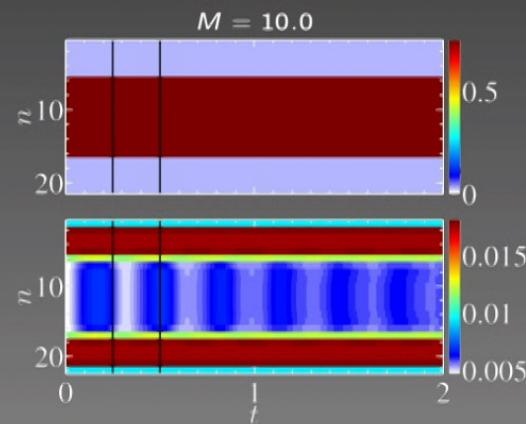
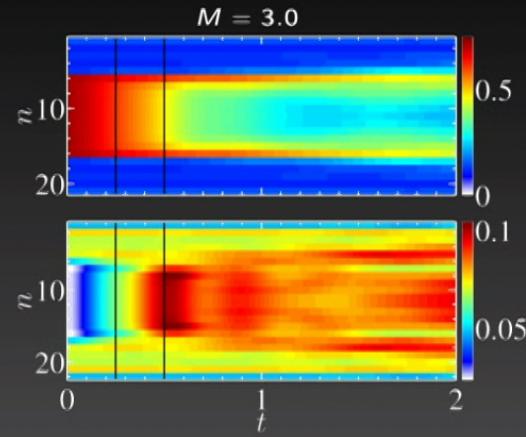
Real-time dynamics in the presence of static external charges

### Procedure

- Compute the interacting vacuum
- Generate a string on top between the two heavy external charges
- Look at real-time evolution



# SU(2) LGT: Dynamics of string breaking



SK, E. Zohar, J.I. Cirac, M.C. Bañuls, JHEP, 2015 (2015)



## SU(2) LGT: Dynamics of string breaking

Real-time dynamics in the presence of dynamical charges

### Procedure

- No external charges anymore
- Compute the interacting vacuum





## SU(2) LGT: Dynamics of string breaking

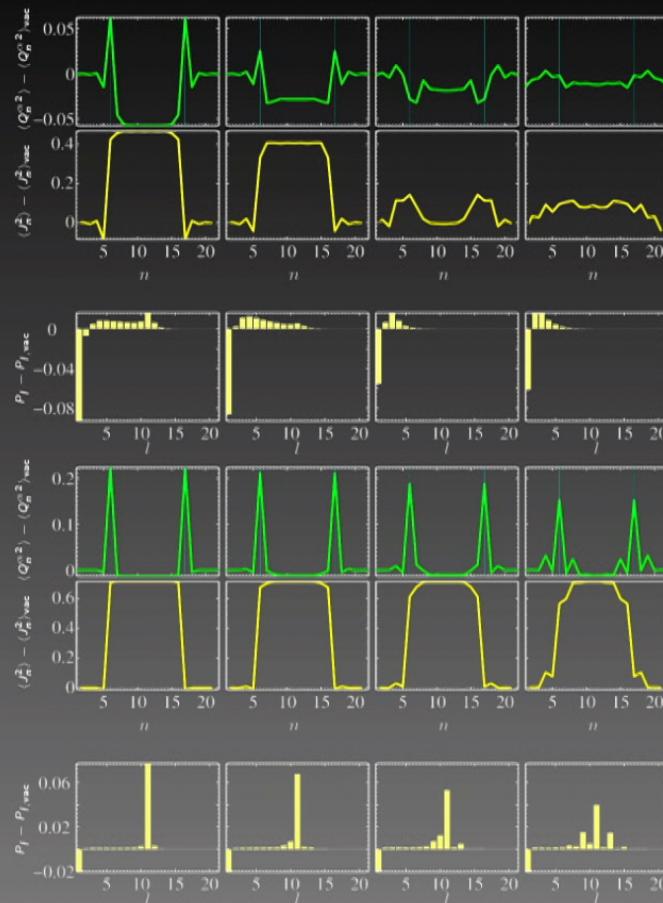
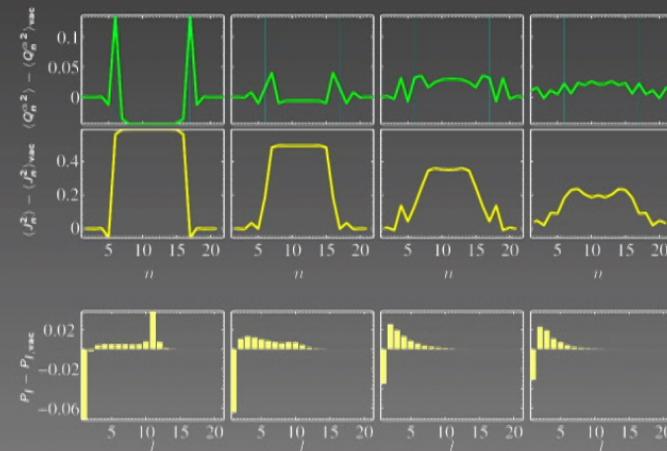
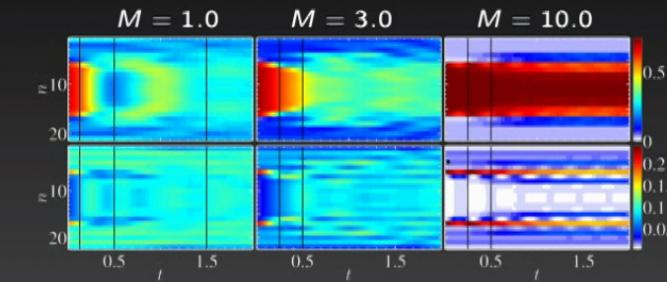
Real-time dynamics in the presence of dynamical charges

### Procedure

- No external charges anymore
- Compute the interacting vacuum
- Generate a string on top by applying a string operator
- Look at real-time evolution



# SU(2) LGT: Dynamics of string breaking



SK, E. Zohar, J.I. Cirac, M.C. Bañuls, JHEP, 2015 (2015)



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Integrating out the gauge field

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## SU(2) LGT: Integrating out the gauge field

Previous approach

- Basis for the gauge links  $|jmm'\rangle$
- Truncate at a finite value  $j_{\max}$

$$d = \sum_{j=0, \frac{1}{2}, 1, \dots}^{j_{\max}} (2j+1)^2 \xrightarrow{j_{\max} \rightarrow \infty} \mathcal{O}(j_{\max}^3)$$

⇒ Computationally expensive



# SU(2) LGT: Integrating out the gauge field

## Color neutral basis

- Hamiltonian does not contain any terms with uncontracted color indices  
⇒ Applied to a color neutral superposition the result is a color neutral superposition again



- Superposition is characterized by fermionic occupation number and by the value of  $j$

$$|0\rangle \otimes |0\rangle \otimes |2\rangle \quad \longrightarrow \quad |1\rangle \otimes |\frac{1}{2}\rangle \otimes |1\rangle$$



C.J. Hamer Nuclear Physics B 195 503 (1982)

# SU(2) LGT: Integrating out the gauge field

## Color neutral basis

- Hamiltonian does not contain any terms with uncontracted color indices  
 $\Rightarrow$  Applied to a color neutral superposition the result is a color neutral superposition again



- Superposition is characterized by fermionic occupation number and by the value of  $j$

$$|0\rangle \otimes |0\rangle \otimes |2\rangle \quad \rightarrow \quad |1\rangle \otimes |\frac{1}{2}\rangle \otimes |1\rangle$$

$$H = \varepsilon \sum_n \left( \phi_n^\dagger U_n \phi_{n+1} + \text{h.c.} \right) + M \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n J_n^2$$

- Mass term as well as electric energy are diagonal in this basis
- Matrix elements for the hopping term are available

C.J. Hamer Nuclear Physics B 195 503 (1982)

# SU(2) LGT: Integrating out the gauge field

## Color neutral basis

- Gauss Law allows to reconstruct the color electric flux for OBC

$$j_{n+1} = \begin{cases} j_n \pm \frac{1}{2} & \text{if site is singly occupied} \\ j_n & \text{else} \end{cases}$$



- To remove ambiguity introduce levels  $|1_-\rangle$ ,  $|1_+\rangle$

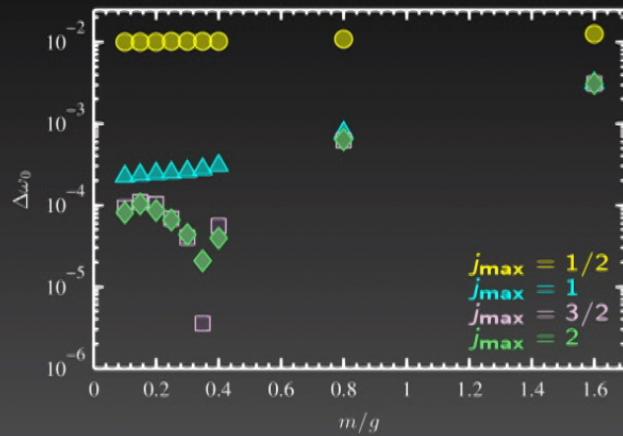


- Formulation on the gauge invariant subspace where the gauge degrees of freedom are integrated out

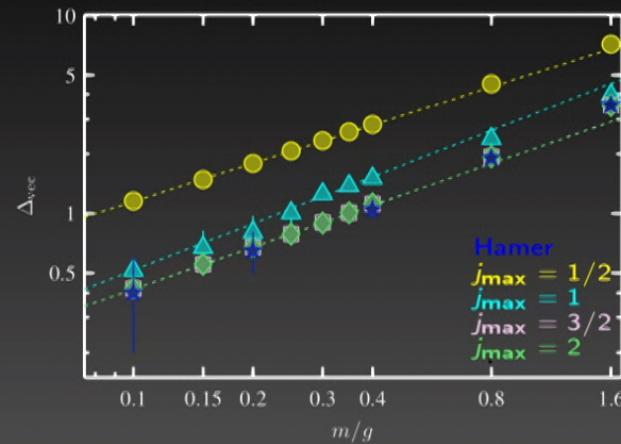


# SU(2) LGT: Integrating out the gauge field

Ground state



Vector mass gap



Critical exponent for the vector mass gap

$j_{\max}$	Exponent
$1/2$	$0.639(43)(5)$
$1$	$0.781(93)(65)$
$3/2$	$0.700(29)(11)$
$2$	$0.700(29)(12)$



C.J. Hamer Nucl. Phys. B 195, 503 (1982)



## 1 Motivation

## 2 Matrix Product States

## 3 The Schwinger model

- Quantum simulation
- Phase structure for two-flavors

## 4 SU(2) Lattice Gauge Theory

- Dynamics of string breaking
- Integrating out the gauge field

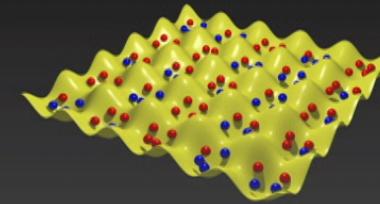
## 5 Conclusion & Outlook

## Conclusion & Outlook

### Conclusion

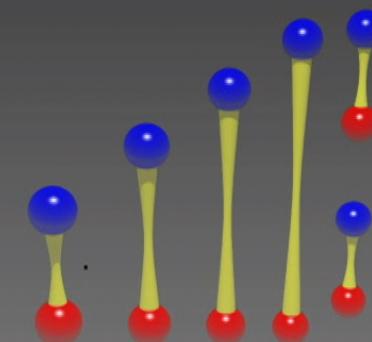
#### Schwinger model

- Powerful tool to **benchmark quantum simulators**
- **Access parameter regimes** in which Monte Carlo suffer from the **sign problem**



#### SU(2) Lattice Gauge Theory

- **Real-time dynamics** of string breaking
- **Access** to all kinds of local observables
- Spectral calculations with **efficient basis formulation**





## Conclusion & Outlook

### Outlook

- Finite temperature phase structure for the two-flavor Schwinger model
- Background field the two-flavor Schwinger model
- Finite temperature for SU(2) Lattice Gauge Theory
- **Accessing 2+1 dimensions with PEPS**



Thank you for your attention!

