Title: Capturing Topological and Symmetry Protected Physics with Entanglement and Tensor Networks

Date: Dec 12, 2016 01:00 PM

URL: http://pirsa.org/16120021

Abstract: <p>In order to create ansatz wave functions for models that realize topological or symmetry protected topological phases, it is crucial to understand the entanglement properties of the ground state and how they can be incorporated into the structure of the wave function.</p>

<p>In this first part of this talk, I will discuss entanglement properties of models of topological crystalline insulators and spin liquids and show how to incorporate topological order, symmetry fractionalization, and lattice symmetry protected topological order into tensor network wave functions.</p>

<p>In the second part of this talk, I will discuss intrinsically fermionic topological phases and an exactly solvable model we built to elucidate the structure of the ground state wave functions in these phases.</p>

<p> </p>

<p>References:</p>

<p>https://arxiv.org/abs/1507.00348</p>

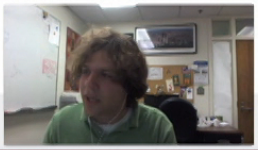
<p>https://arxiv.org/abs/1605.06125</p>

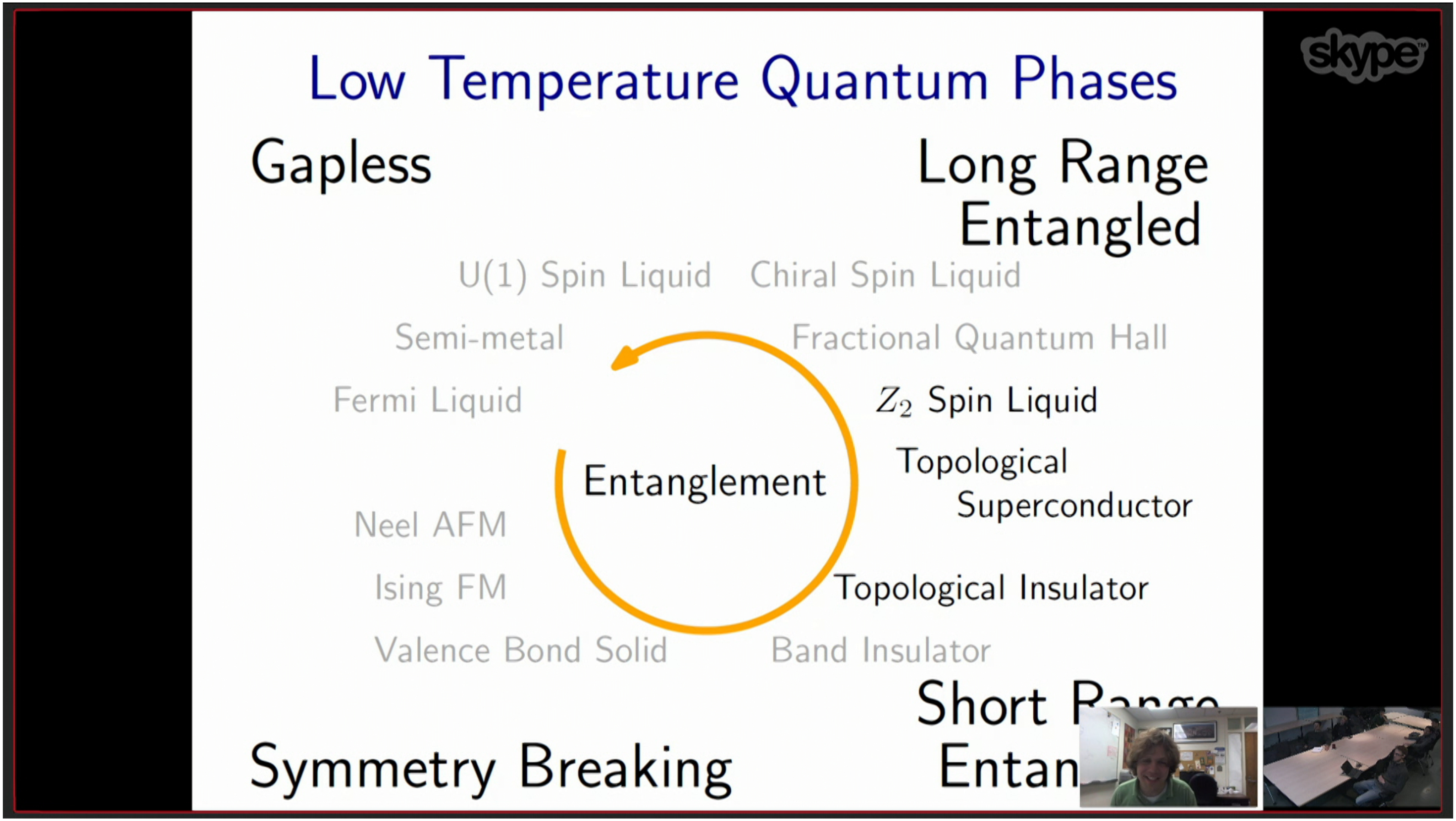
Outline

1. Featureless Phases

2. Interlude: Characterization of SETs

3. Fermionic Topological Phases: Majorana Dimer Model
   - State Consistent and Parity Conserving?
   - Local Hamiltonian?
   - Analysis of Topological Order
   - Outlook





Low Temperature Quantum Phases
Gapless
Long Range
Entangled
U(1) Spin Liquid
Chiral Spin Liquid
Semi-metal
Fractional Quantum Hall
Fermi Liquid
Z_2 Spin Liquid
Entanglement
Topological
Superconductor
Neel AFM
Ising FM
Topological Insulator
Valence Bond Solid
Band Insulator
Short Range
Symmetry Breaking
Entan

Obstructions to Featurelessness

Fundamental Result

A featureless insulator must have an integer charge per unit cell

- (Lieb, Schultz, Mattis 1961)
- (Oshikawa 1999)
- (Hastings 2003)

For certain lattices with nonsymmorphic symmetries, charge per unit cell must be multiple of 2, 4, 6, ...

- (Parameswaran, Turner, Arovas, Vishwanath 2013)
- (Watanbe, Po, Vishwanath, Zaletel 2015)

Are there additional constraints to featureless, e.g. on the honeycomb lattice?

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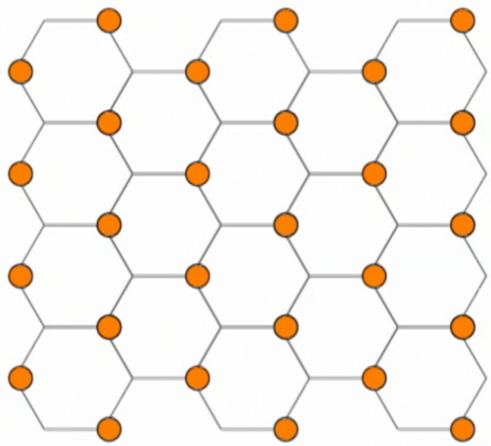
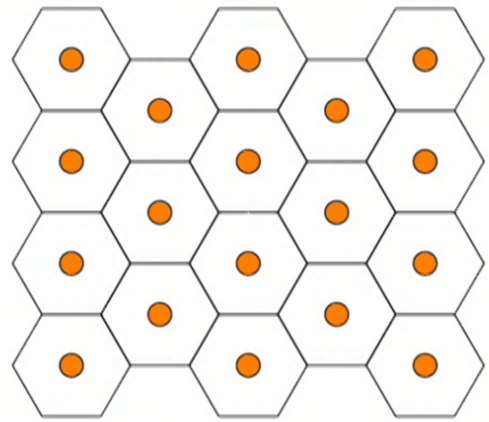
Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

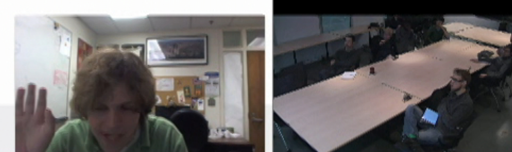


Breaks rotational symmetry



Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by (Parameswaran et al. (2013))



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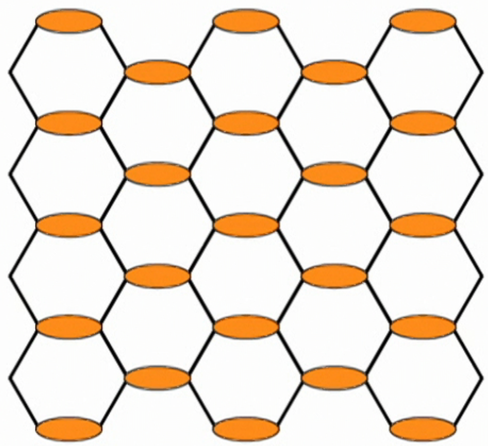

Breaks rotational symmetry

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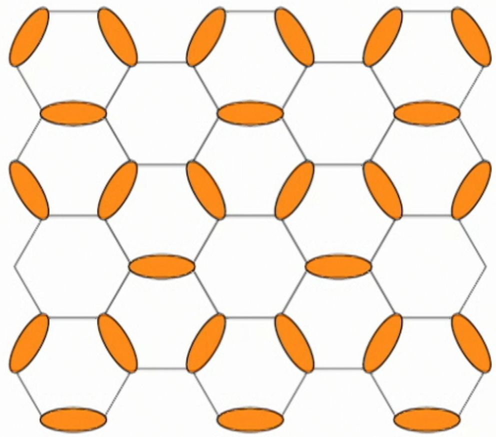
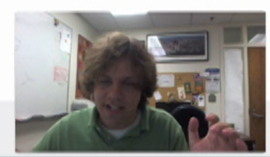


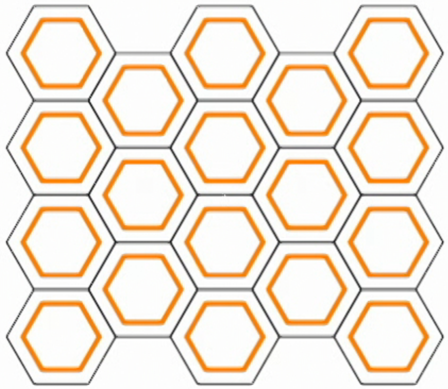
Breaks translationally symmetry, unit cell is 3 times larger

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by (Parameswaran et al. (2013))



Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



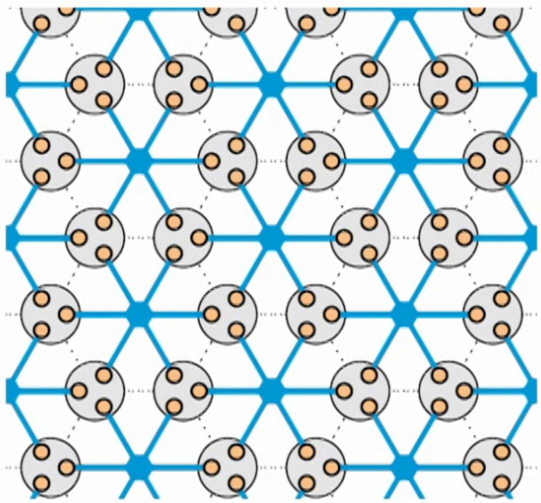
$$|\psi_+\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

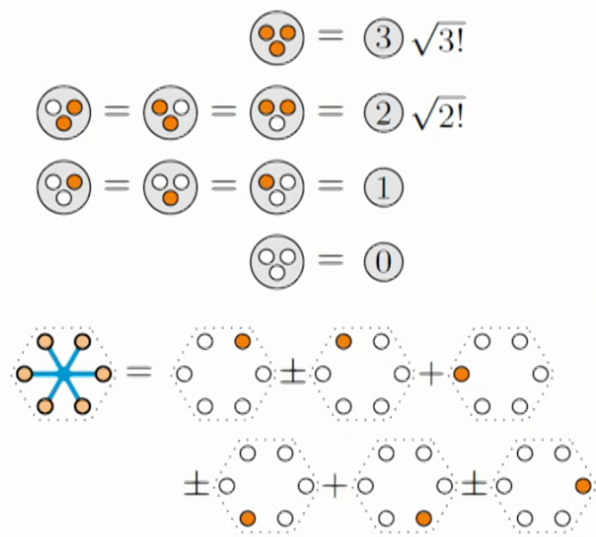
$$|\psi_-\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} (-1)^i b_i^\dagger \right) |\mathbf{0}\rangle$$

Proposed Solution by Kimchi et al. (2013)

Bosons filled into non-orthogonal, plaquette centered orbitals.

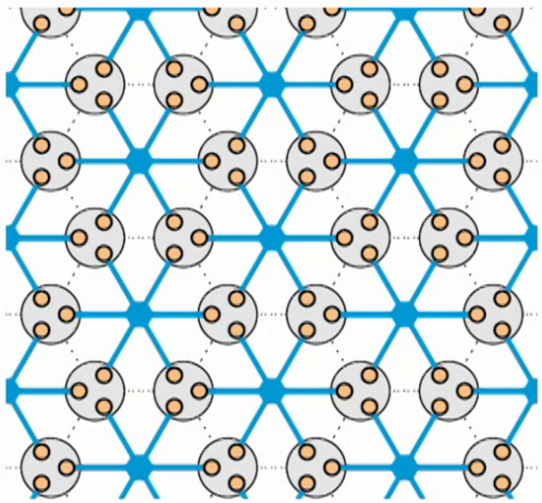
Computations on Honeycomb FBI



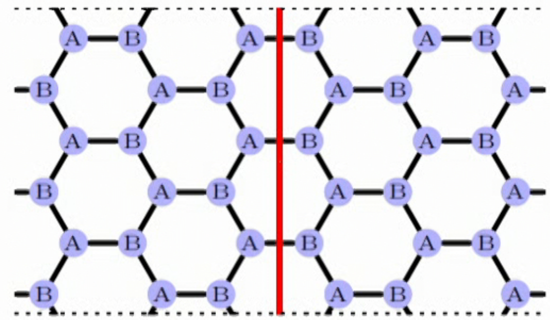


$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |0\rangle$$

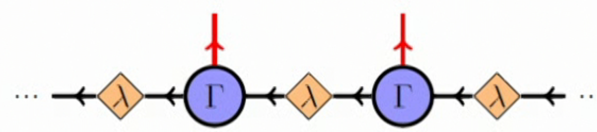
Computations on Honeycomb FBI



$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

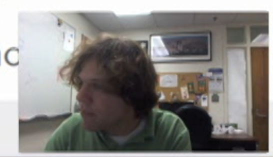


Form of a honeycomb lattice PEPS on zig-zag cylinder with width $L \leq 10$ ($L = 3$ shown)



$$\sum_i \; |\psi_L^i\rangle \quad \lambda_i \quad |\psi_R^i\rangle$$

Matrix Product state canc



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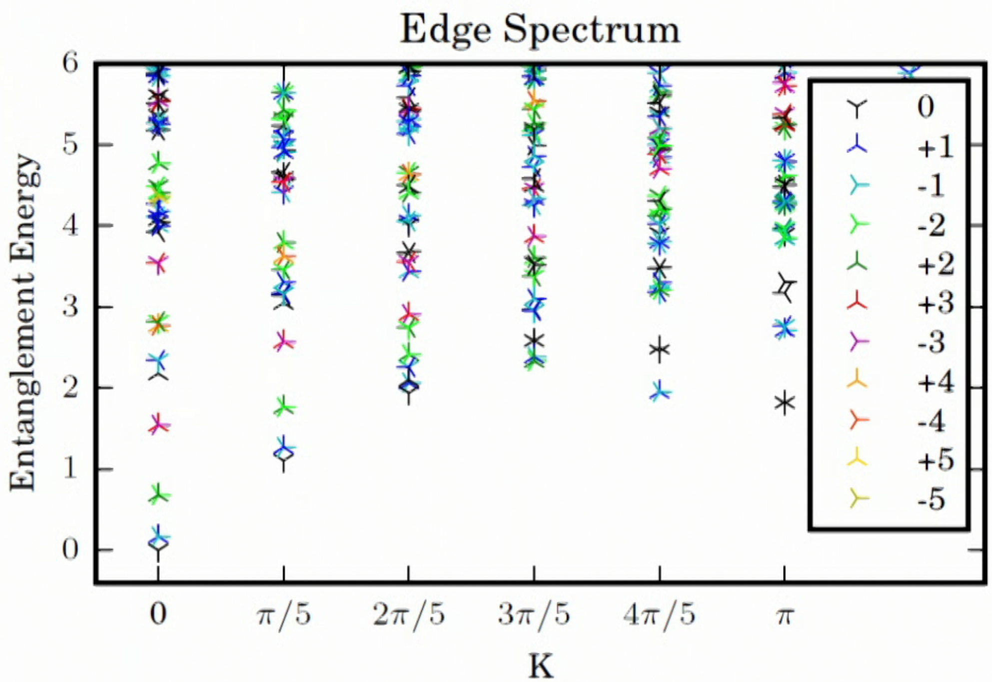
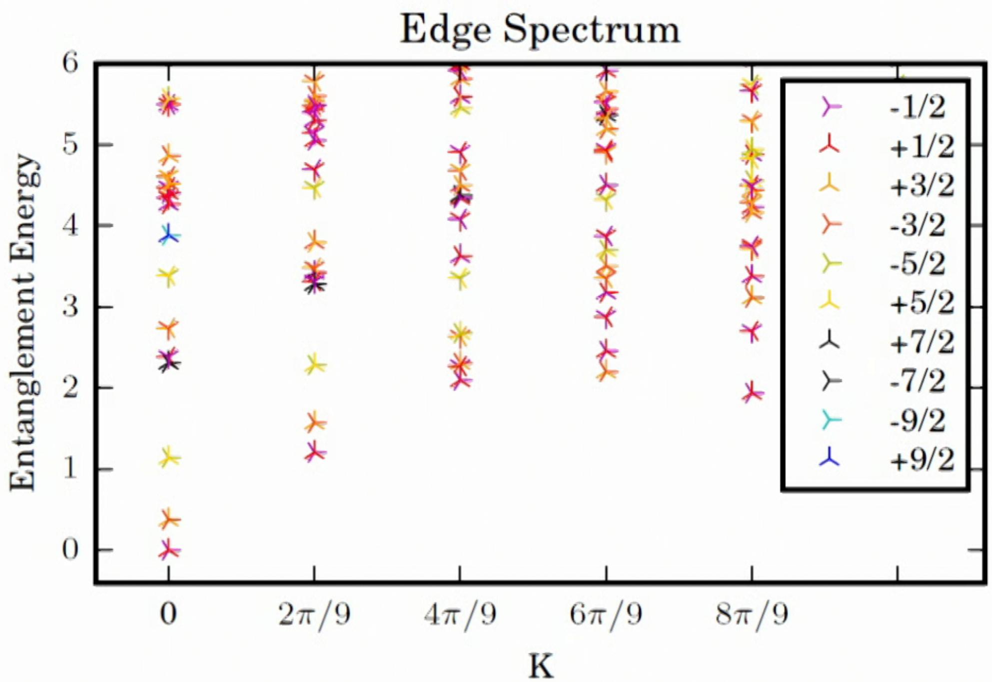
CFT Entanglement Spectrum



CFT Entanglement Spectrum



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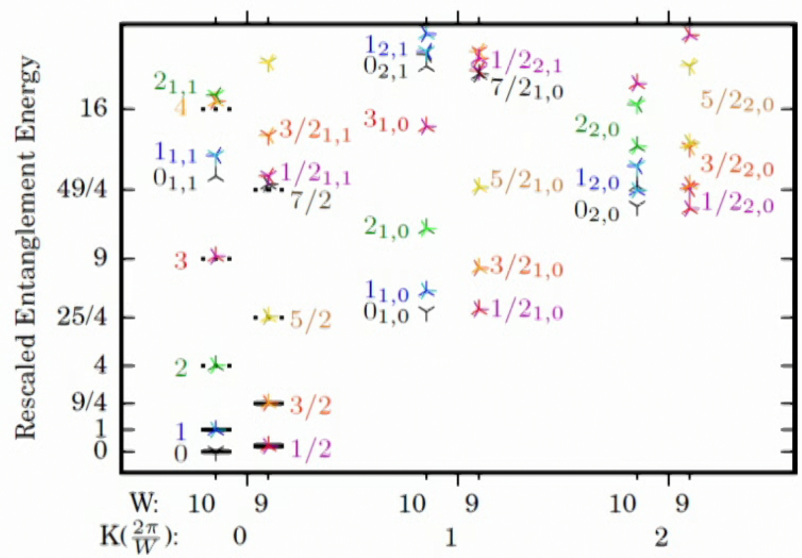
CFT Entanglement Spectrum

Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

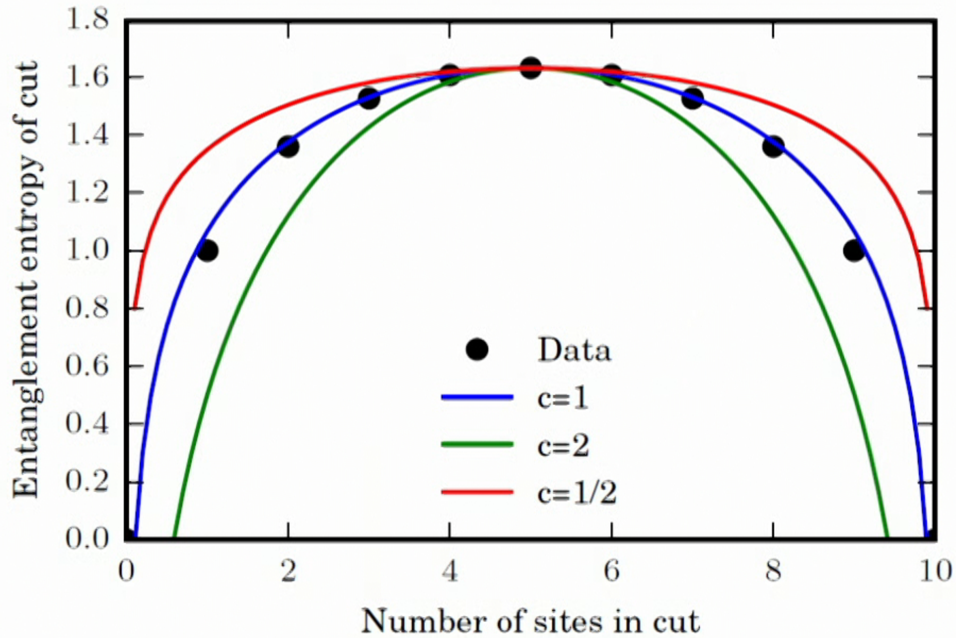
$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

Conformal primary identification in entanglement spectra



CFT Entanglement Spectrum

Conformal Charge via 'Nested Entanglement Entropy'



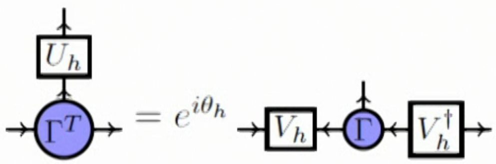
c = 1

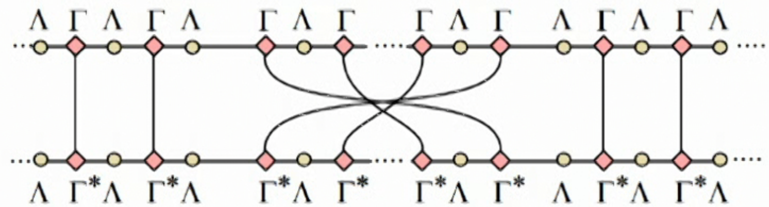
Argument for Protected Entanglement

1D SPT Order on Odd Cylinders

The cylinder geometry preserves translation, lattice inversion $\mathcal{I}$, and reflection symmetries $\mathcal{I}_x$ and $\mathcal{I}_y$ (through the cut).

If $U_h \mathcal{I}_y$ is a symmetry of the wavefunction that reverses the cylinder, then the canonical form of the MPS satisfies Pollmann et al. (2010):



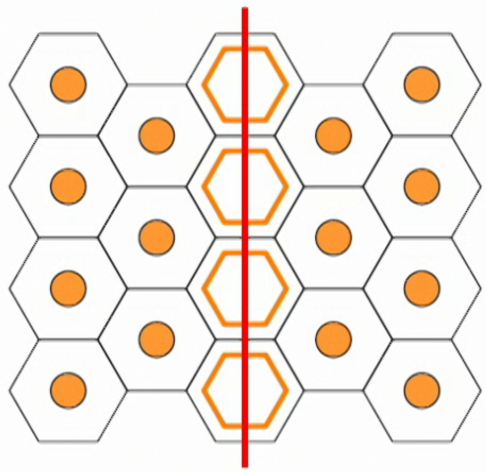


Antiunitary $V_{\mathcal{I}_y}$ satsfies $V_{\mathcal{I}_y} V^*_{\mathcal{I}_y} = -1$ for $|\psi_-\rangle$ on odd cylinders. Similarly, $V_{\pi\mathcal{I}_y} V^*_{\pi\mathcal{I}_y} = -1$ for $|\psi_+\rangle$ on odd cylinders.

$$\langle \psi_- | \mathcal{I}_y(1 \dots 2n) | \psi_- \rangle = -1$$

Argument for Protected Entanglement

Dimensional Reduction Method
(Song et al., 2016)



Trivialize state away from cut while preserving symmetry

Reflection $\mathcal{I}_y$ and translation T_y can protect the entanglement

$$H^2(\mathbb{Z}_2 \times \mathbb{Z}, U(1)) = \mathbb{Z}_2$$

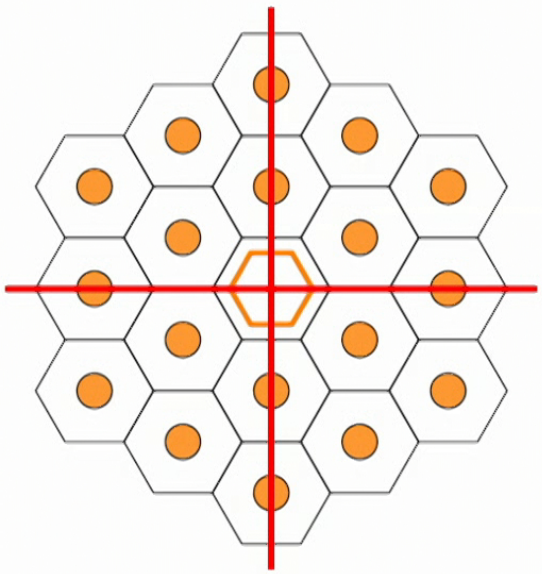
"Reflection charge per unit length" is ± 1 for $|\psi_\pm\rangle$

$H' = \sum_i h \left(b_i + b_i^\dagger \right)$ preserves entanglement degeneracy in $|\psi_-\rangle$ but not $|\psi_+\rangle$!

Similarly
$H' = \sum_i h(-1)^i \left(b_i + b_i^\dagger \right)$ for $|\psi_+\rangle$ but not $|\psi_-\rangle$.

Argument for Protected Entanglement

Dimensional Reduction Method
(Song et al., 2016)



Trivialize state away from cut while preserving symmetry

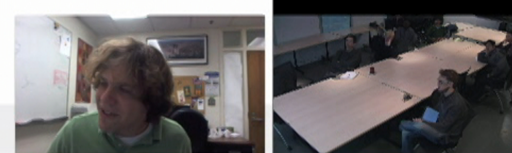
Inversion I can protect the entanglement

Nested dimensional reduction argument

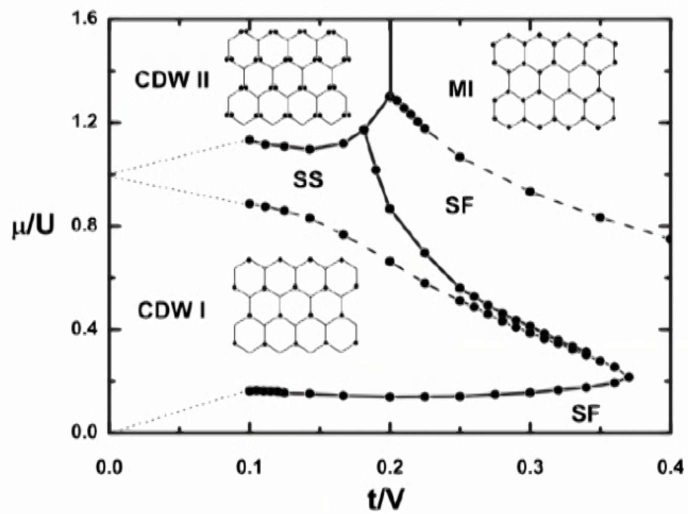
$$H^1(\mathbb{Z}_2, U(1)) = \mathbb{Z}_2$$

Inversion charge at center of rotation is -1

Detectable from wave function with partial inversion on disc centered on rotation axis

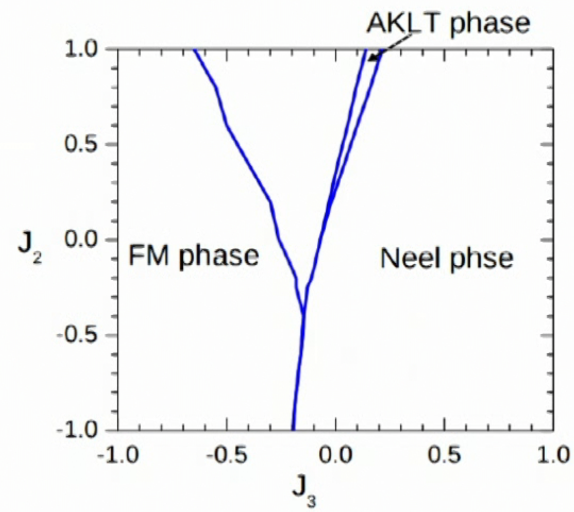


Hamiltonian??



Slice of extended soft-core Bose-Hubbard phase diagram via QMC. (Gan, et. al 2007)

Honeycomb $J_1 - J_2 - J_3$ phase diagram with spin-$\frac{3}{2}$ via tensor network renormalization (Huang, et. al. 2013)



Brute Force $SU(2)$ Tensor Classification

Featureless Quantum Insulator on the Honeycomb Lattice

Panjin Kim,[1] Hyunyong Lee,[1] Shenghan Jiang,[2] Brayden Ware,[3]
Chao-Ming Jian,[4] Michael Zaletel,[5] Jung Hoon Han,[1] and Ying Ran[2]

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(Dated: April 13, 2016)

We show how to construct fully symmetric, gapped states without topological order on a honeycomb lattice for $S = 1/2$ spins using the language of projected entangled pair states (PEPS). An explicit example is given for the virtual bond dimension $D = 4$. Four distinct classes differing by lattice quantum numbers are found by applying the systematic classification scheme introduced by two of the authors [S. Jiang and Y. Ran, Phys. Rev. B 92, 104414 (2015)]. Lack of topological degeneracy or other conventional forms of symmetry breaking, and the existence of energy gap in the proposed wave functions, are checked by numerical calculations of the entanglement entropy and various correlation functions. Our work provides the first explicit realization of a featureless quantum insulator for spin-1/2 particles on a honeycomb lattice.

Systematic construction of spin liquids on the square lattice from tensor networks with SU(2) symmetry

Matthieu Mambrini,[1] Román Orús,[2] and Didier Poilblanc[1]

[1] *Laboratoire de Physique Théorique, C.N.R.S. and Université de Toulouse, 31062 Toulouse, France*
[2] *Institute of Physics, Johannes Gutenberg University, 55099 Mainz, Germany*

(Dated: today)

We elaborate a simple classification scheme of all rank-5 SU(2)-spin rotational symmetric tensors according to i) the on-site physical spin-S, (ii) the local Hilbert space $V^{\otimes 4}$ of the four virtual (composite) spins attached to each site and (iii) the irreducible representations of the C_{4v} point

For small bond dims, find all tensors that implement lattice symmetries on-site

Use gauge redundancy to fix basis as much as possible

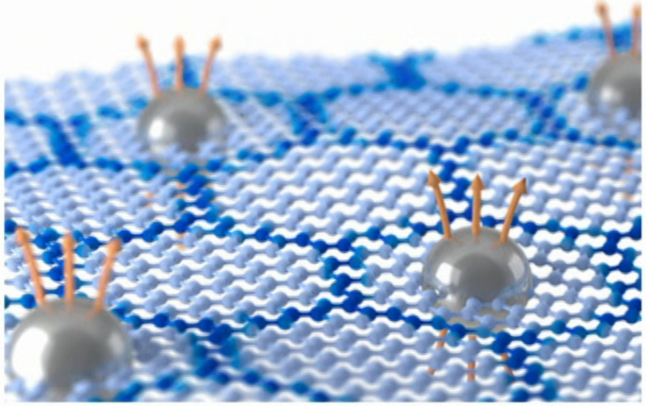
This could be a good variational class of wavefunctions

Interlude: Characterization of SETs

Symmetry and 2D Topological Phases

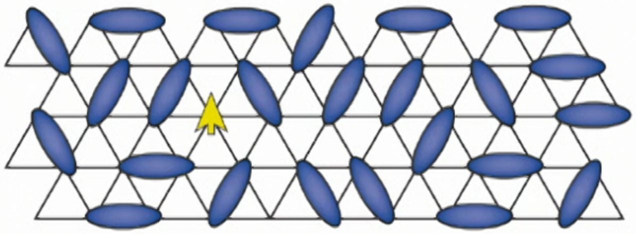
$\nu = \frac{1}{3}$ Fractional Quantum Hall

- $U(1)$ symmetry
- Quasiparticles carry fractional charge $e/3$



RVB Gapped $\mathbb{Z}_2$ Spin Liquid

- $SO(3)$ symmetry
- Spinon excitation with (fractional) spin $\frac{1}{2}$



Balents (2010)

Symmetry Enriched Topological Phases

With symmetry G, distinct *Symmetry Enriched Topological* Phases

Key feature of an SET is its symmetry fractionalization pattern:

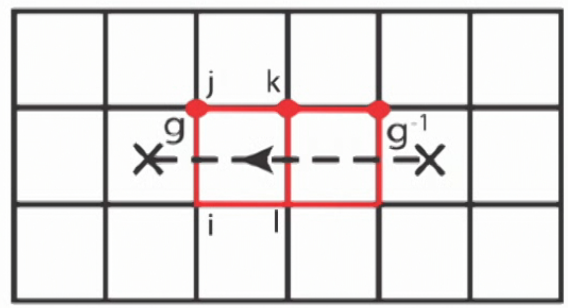
- Each anyon is assigned a projective representation of the symmetry group G
- These are constrained by compatibility under fusion

Additionally, combinations of anyons and symmetry defects have fusion, braiding, and statistics
(Barkeshli, Bonderson, Cheng, Wang (2015))

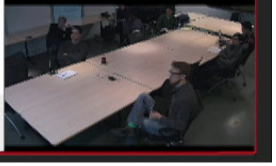
Symmetry Defects

A symmetry defect line for $g \in G$ transforms the Hamiltonian

$$H = \ldots + h_{ij} + \ldots$$
$$\Downarrow$$
$$\widetilde{H} = \ldots + g_j h_{ij} g_j^{-1} + \ldots$$

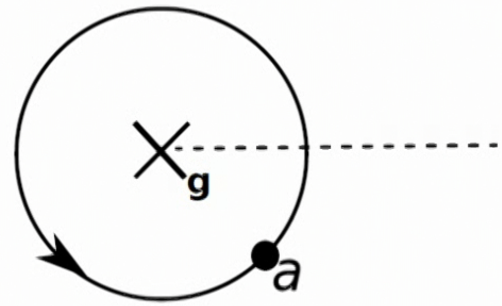


and creates a pair of symmetry defects located at the endpoints.

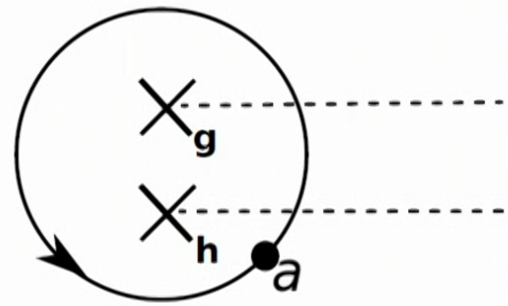


Symmetry Defects

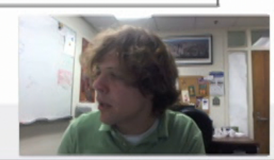
Anyons braided around defects can pick up phases or more general symmetry action.



Defects can fuse with other defects and with anyons, with fusion determined by braiding as before.



Braiding, statistics, and fusion of anyon/defect combinations can be used to distinguish SETs.



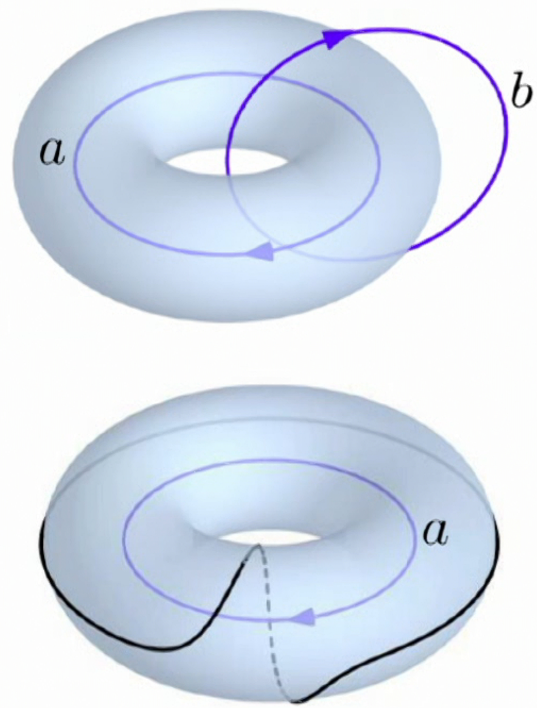
Characterizing Topological Phases with Entanglement

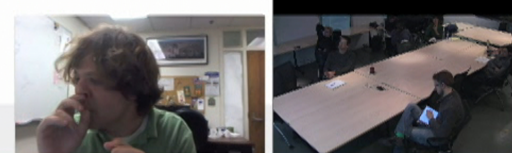
For each cycle, form basis of ground states from Minimal Entropy States (MES)

Overlap matrices between these bases related by transforming the torus under

- $\frac{\pi}{2}$ rotation gives $\mathcal{S}$
- Cut and twist gives $\mathcal{T}$
- $\frac{2\pi}{3}$ rotation gives $\mathcal{S}\mathcal{T}^{-1}$

$\mathcal{S}$ summarizes anyon braiding
$\mathcal{T}$ summarizes exchange statistics





Characterizing SPT/SET Phases

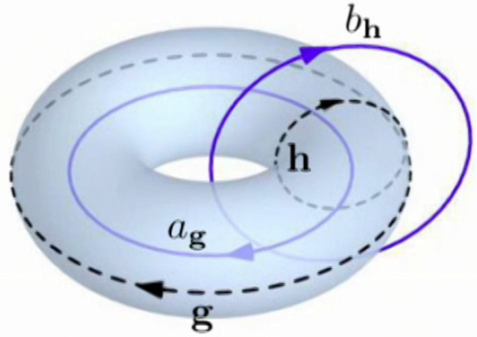
Braiding of anyons and defects can be extracted from *symmetry twisted ground states.*

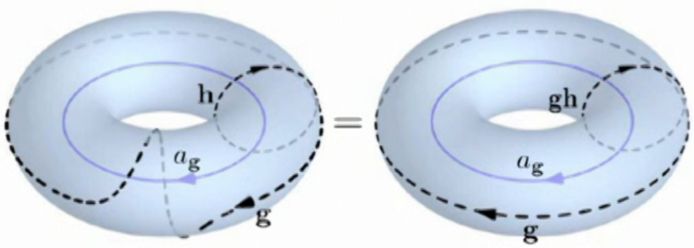
Need set of ground states for each of $|G|^2$ defect sectors labeled $\mathcal{V}_{(g,h)}$.

The number of topological ground states $\mathcal{N}_{(g,h)}$ is an SET invariant.

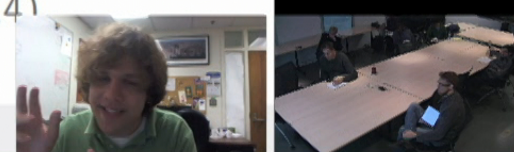
More complete info:
'extended' modular matrices

- $\frac{\pi}{2}$ rotation gives $\hat{S} : \mathcal{V}_{(g,h)} \to \mathcal{V}_{(h,g^{-1})}$
- Cut and twist gives $\hat{T} : \mathcal{V}_{(g,h)} \to \mathcal{V}_{(g,gh)}$

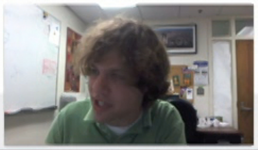


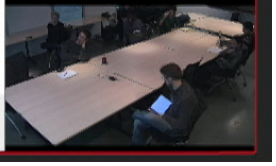


Barkeshli et al. (2014)



Fermionic Topological Phases: Majorana Dimer Model





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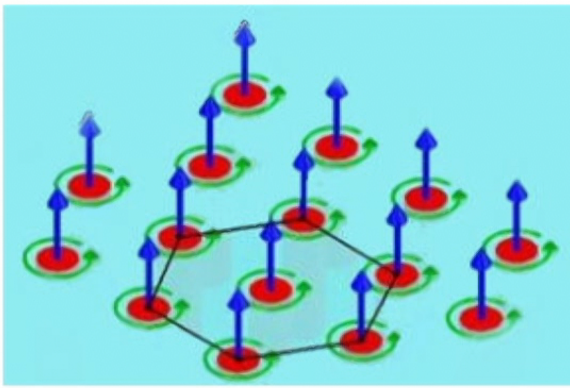
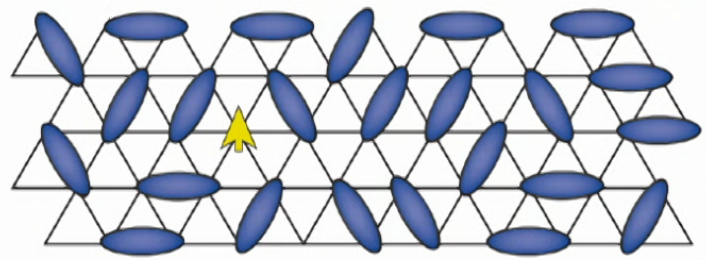
Motivation: Resonating Majorana Dimers



Triangular vortex lattice in a topological $p_x + ip_y$ superconductor

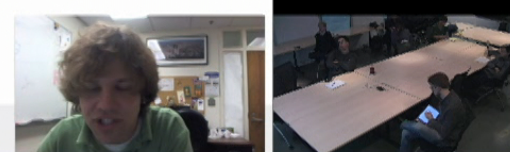
Vortices in $p_x + ip_y$ bind Majorana zero modes γ

How does $\sqrt{2}^{N_v}$ dimensional space of degenerate states split when zero modes interact?

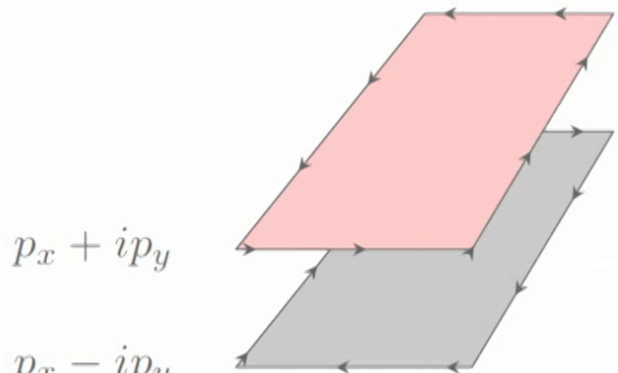


$|\mathrm{RMB}\rangle = \sum_D |\mathcal{M}(D)\rangle$

Can we deconfine the Majorana zero modes?



Motivation: Resonating Majorana Dimers



A model of deconfinement
$(p_x + ip_y) \to$ Ising?

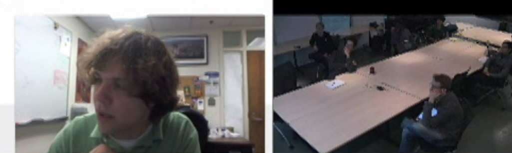
Technical Result:

Exactly solvable lattice model for

$(p_x{+}ip_y)\times(p_x{-}ip_y) \to \text{Ising}\times(p_x{-}ip_y)$

Fermionic SPT phase with $\mathbb{Z}_2^f \times \mathbb{Z}_2$ symmetry $\to$ fermionic topological phase with only $\mathbb{Z}_2^f$ symmetry

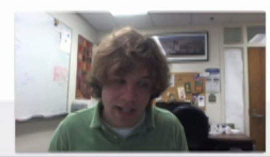
Deconfinement of Majoranas without background $p + ip$



Motivation: Resonating Majorana Dimers

Open questions

- Can tensor neworks represent topological phases beyond Levin-Wen string nets?
- Which SPT phases can be many body localized?
- Any chance this is relevant for physical $(p + ip)$ superconductors?
- Theory of phase transitions for 'fermion condensation' same as usual confinement/deconfinement transitions?

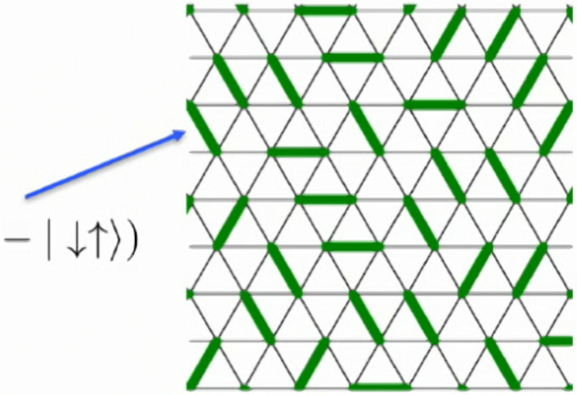



Outline

3 Fermionic Topological Phases: Majorana Dimer Model
- State Consistent and Parity Conserving?
- Local Hamiltonian?
- Analysis of Topological Order
- Outlook

Rokhsar and Kivelson's Simplification

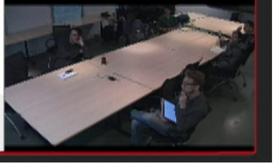
$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



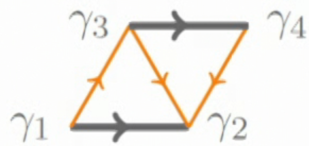
$|S(D)\rangle$

$$H^{\triangle}_{\mathrm{RK}} = \sum_p (-tB^{\triangle}_p + VC^{\triangle}_p)$$

- $|S(D)\rangle \to |D\rangle$
- $\langle S(D')|S(D)\rangle \neq 0 \to \langle D'|D\rangle = 0$
- Dimers serve as proxy for paired spin singlets
- Z_2 spin liquid on non-bipartite lattice (Moessner, Sondhi 2000)
- Can adiabatically tune back to $|RVB\rangle$ with PEPS (Schuch et. al 2012)
- Dimer models can also be viewed as models of fluctuating loops



Majorana-Dimers

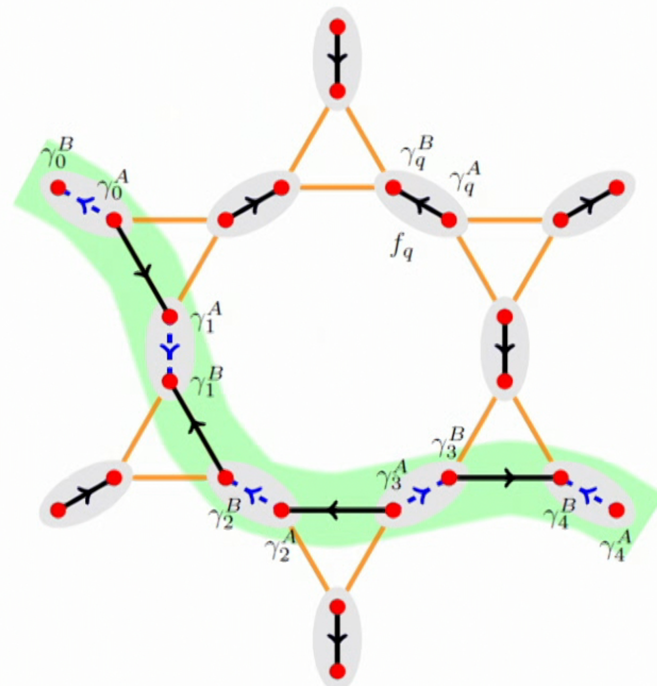


$|M(D)\rangle$ is the ground state of
$H = i\gamma_1\gamma_2 + i\gamma_3\gamma_4$

We use decorated dimers
$|\psi\rangle = \sum_D |M(D)\rangle|D\rangle$ since

$$\langle M(D')|M(D)\rangle \neq 0$$

Equivalent description: Kitaev chain decorated loops

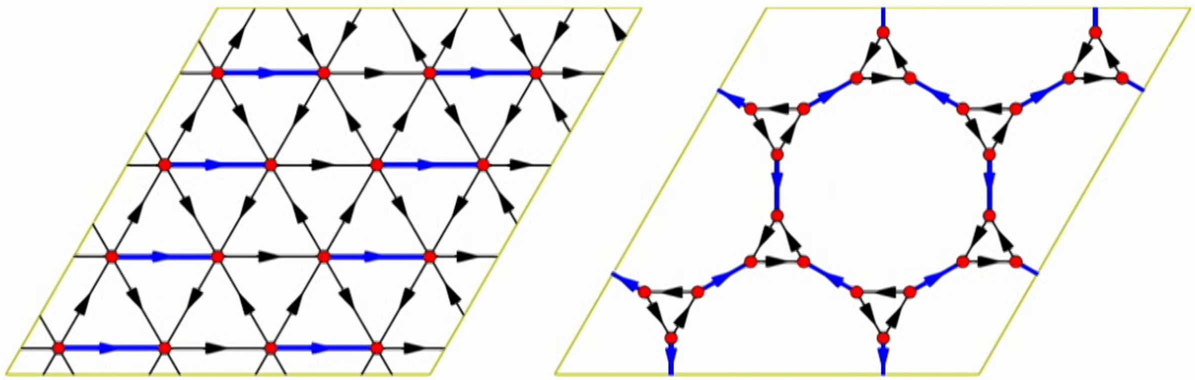


Fermion Parity Consistency

Kitaev chains have odd parity on periodic (P) boundary conditions.

Parity consistency conditon: every Kitaev chain must have antiperiodic (A) boundary conditions

Kasteleyn orientation for any planar lattice: every trivial loop has an odd number of reversed arrows



Kasteleyn orientation (arrows) and reference dimer configuration (blue bonds) on the triangular lattice and the Fisher lattice.

Fermion Parity Consistency

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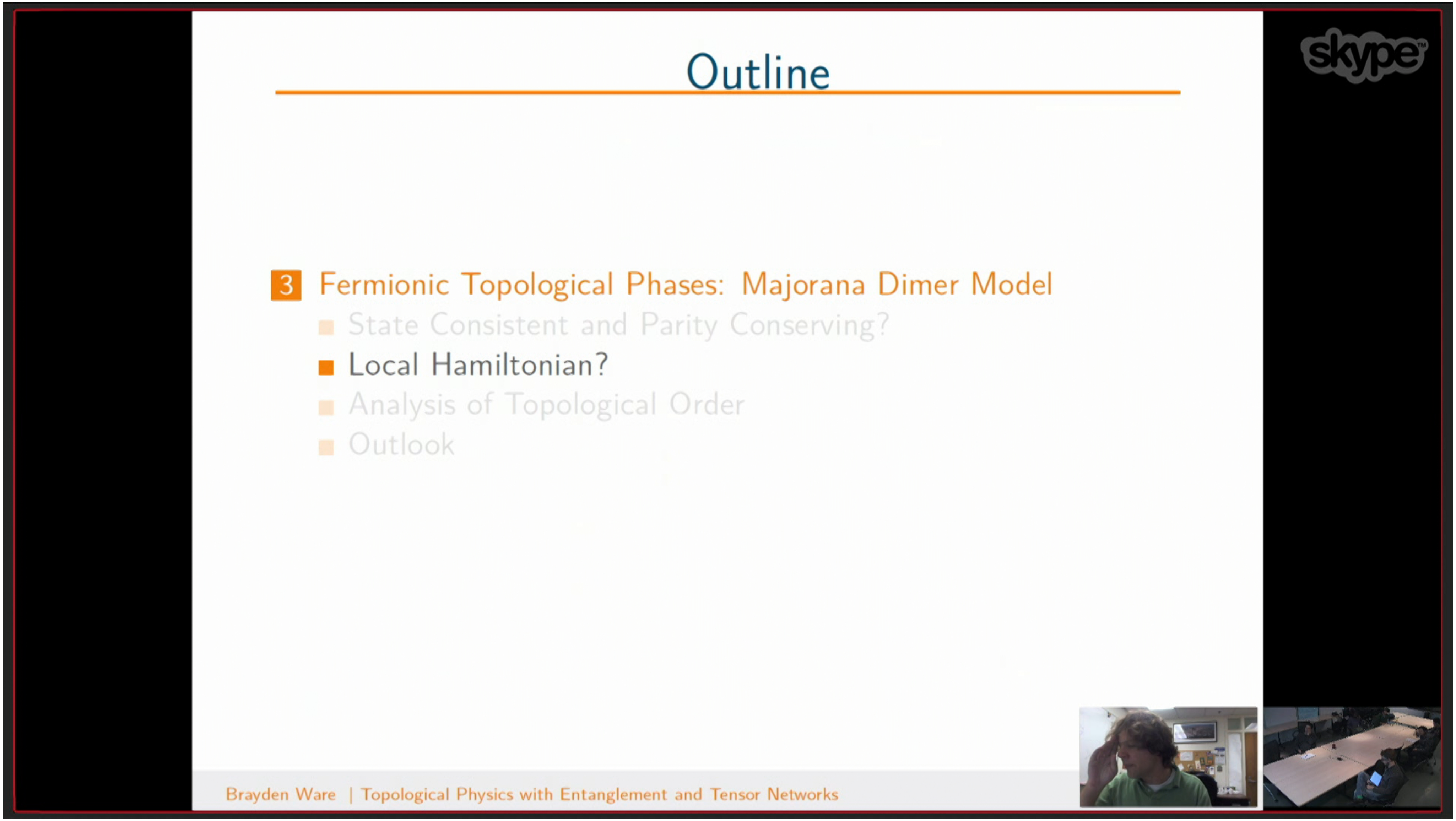
Dimer Sector	Boundary Cond. PP	PA	AP	AA
$(0,0)$	$+1$	$+1$	$+1$	$+1$
$(1,0)$	-1	$+1$	-1	$+1$
$(0,1)$	-1	-1	$+1$	$+1$
$(1,1)$	-1	$+1$	$+1$	-1

The fermion parity $P_f = \pm 1$ of a Majorana-dimer state depends on the topological sector of the bosonic dimers and the boundary conditions for the fermions.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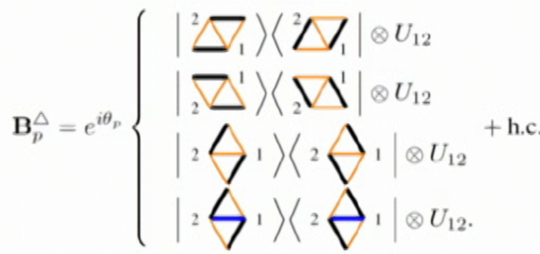
Majorana-Dimer Hamiltonian

$$H_{\rm RK}^{\triangle} = -J_e \sum_e \mathbf{A}_e^{\triangle} + \sum_p \left(-t\mathbf{B}_p^{\triangle} + VC_p^{\triangle} \right)$$

Potential energy term:

Kinetic energy (plaquette-flip) term:



Vertex term:

$$\mathbf{A}_e^{\triangle} = \frac{1-\sigma_e^z}{2}\,\frac{1+is_{ij}\gamma_i\gamma_j}{2}$$

$$U_{12} = (1 + s_{12}\gamma_1\gamma_2)/\sqrt{2}$$

Similar for Fisher Lattice with more complicated braiding operator to relocate Majorana pairings.

Numerical Results for Majorana-Dimers

3 fermion parity odd ground states

Fermion BC	Z loop sector (0, 0)	(0, 1)	(1, 0)	(1, 1)
PP	No	–	–	–
PA	+	+	No	+
AP	+	No	+	+
AA	+	+	+	NA

In each box, shown is the P symmetry charge.

More generally, fermion parity depends on boundary conditions
In the other sectors, an odd number of plaquette terms are violated, leading to volume degeneracy.
These could split into a single particle continuum. (non-universal)

Numerical Results for Majorana-Dimers

Modular matrix calculations confirm Ising $\times\ (p_x - ip_y)$ topological order.

We find that

$$S = S_{\text{Ising}^{(n/2)}} \otimes S^{PP}_{(p_x - ip_y)^n}$$

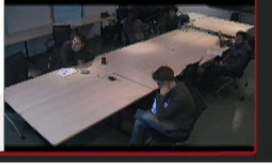
$$T = T_{\text{Ising}^{(n/2)}} \otimes T^{PP}_{(p_x - ip_y)^n}$$

where

$$S^{PP}_{(p_x - ip_y)^n} = e^{\frac{\pi i n}{4}} \qquad T^{PP}_{(p_x - ip_y)^n} = e^{-\frac{\pi i n}{12}}.$$

We did this computation with the $R_{\frac{2\pi}{3}}$ method, but modified due to the parity of the ground states in the PP sector.

$$R^3_{\frac{2\pi}{3}} = (ST^{-1})^3 = (e^{\frac{\pi i n}{3}})^3 = (-1)^n. \qquad (1)$$



Review: Topological Order in $(p_x + ip_y)$

Boundary Cond.

	PP	PA	AP	AA
P_f	-1	$+1$	$+1$	$+1$

Local excitations:

- 1
- f (Bogoliubov quasiparticle)

Defects:

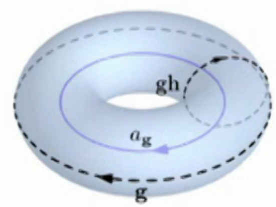
- π (Majorana zero mode)

S and T computed by explicit adiabatic transformations on the torus using free fermions (You, Cheng 2015)

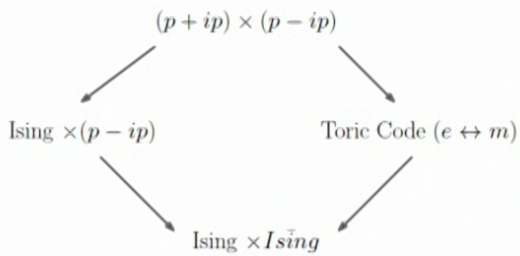
$$S = \begin{pmatrix} e^{\frac{\pi i n}{4}} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = e^{-\frac{\pi i n}{24}} \begin{pmatrix} e^{\frac{\pi i n}{8}} & 0 & 0 & 0 \\ 0 & e^{\frac{\pi i n}{8}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$c_- = \frac{1}{2}$$



Gauging and Ungauging



Ungauging

(Heinrich, Burnell, Fidkowski, Levin 2016) used to constructed toric code with $e \leftrightarrow m$.

We can construct similar tools for ungauging fermion parity to generate interesting models.

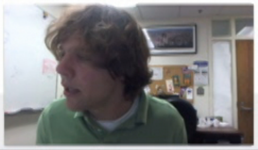
Important part of tensor network construction.

Gauging and Twisted Ground States

Therefore the $(p_x + ip_y) \times (p_x - ip_y)$ state has the following parity pattern:

	Boundary Cond. 1			
BC 2	PP	PA	AP	AA
PP	$-,-$	$+,-$	$+,-$	$+,-$
PA	$-,+$	$+,+$	$+,+$	$+,+$
AP	$-,+$	$+,+$	$+,+$	$+,+$
AA	$-,+$	$+,+$	$+,+$	$+,+$

In each box, shown is the $(-1)^{N_f^{\uparrow}}$, $(-1)^{N_f^{\downarrow}}$ symmetry charge.



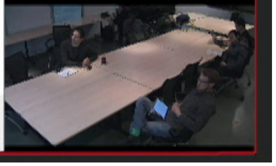
Gauging and Twisted Ground States

After gauging the Z symmetry, we 'lose' the states with odd symmetry charge. This matches the ED and is consistent with Ising $\times$ $(p_x - ip_y)$ description.

Fermion BC	Z loop sector (0, 0)	(0, 1)	(1, 0)	(1, 1)
PP	No	$-$	$-$	$-$
PA	$+$	$+$	No	$+$
AP	$+$	No	$+$	$+$
AA	$+$	$+$	$+$	NA

In each box, shown is the P symmetry charge.

In the lost sector, an odd number of plaquette terms are violated. The lowest states in the sector form a single particle continuum (flat in exactly solvable model).

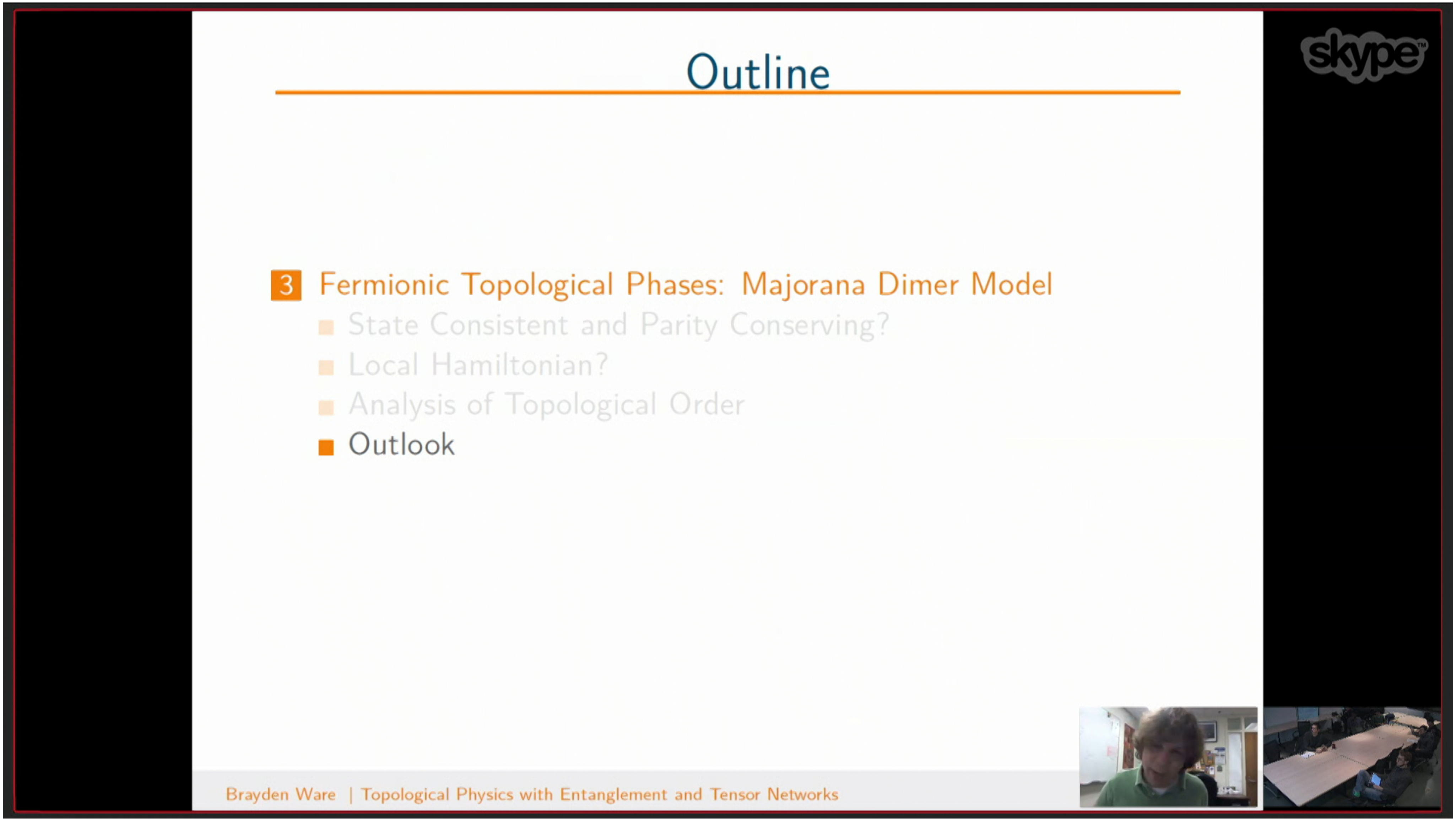


Gauging and Twisted Ground States

Why stop there? Gauge the fermion parity as well. This leaves us with the 9 ground states of the Ising string net theory. We have an explicit map to the Ising string net when we allow the orientations to be dynamical.

Fermion BC	Z loop sector (0, 0)	(0, 1)	(1, 0)	(1, 1)
PP	No	No	No	No
PA	$+$	$+$	No	$+$
AP	$+$	No	$+$	$+$
AA	$+$	$+$	$+$	NA

In each box, shown is the P symmetry charge.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Open questions

- Can tensor neworks represent topological phases beyond Levin-Wen string nets?

  Yes, but maybe not as general as any phase with $c_- = 0$
- Which SPT phases can be many body localized?

  Under most strict versions of MBL (l-bits), this still can't be.
- Do modular matrices distinguish all topological phases?

  In this case, no need for extended modular matrices.
- Any chance this is relevant for physical $(p + ip)$ superconductors?

  Not sure, but at quadratic level the pattern of coupling