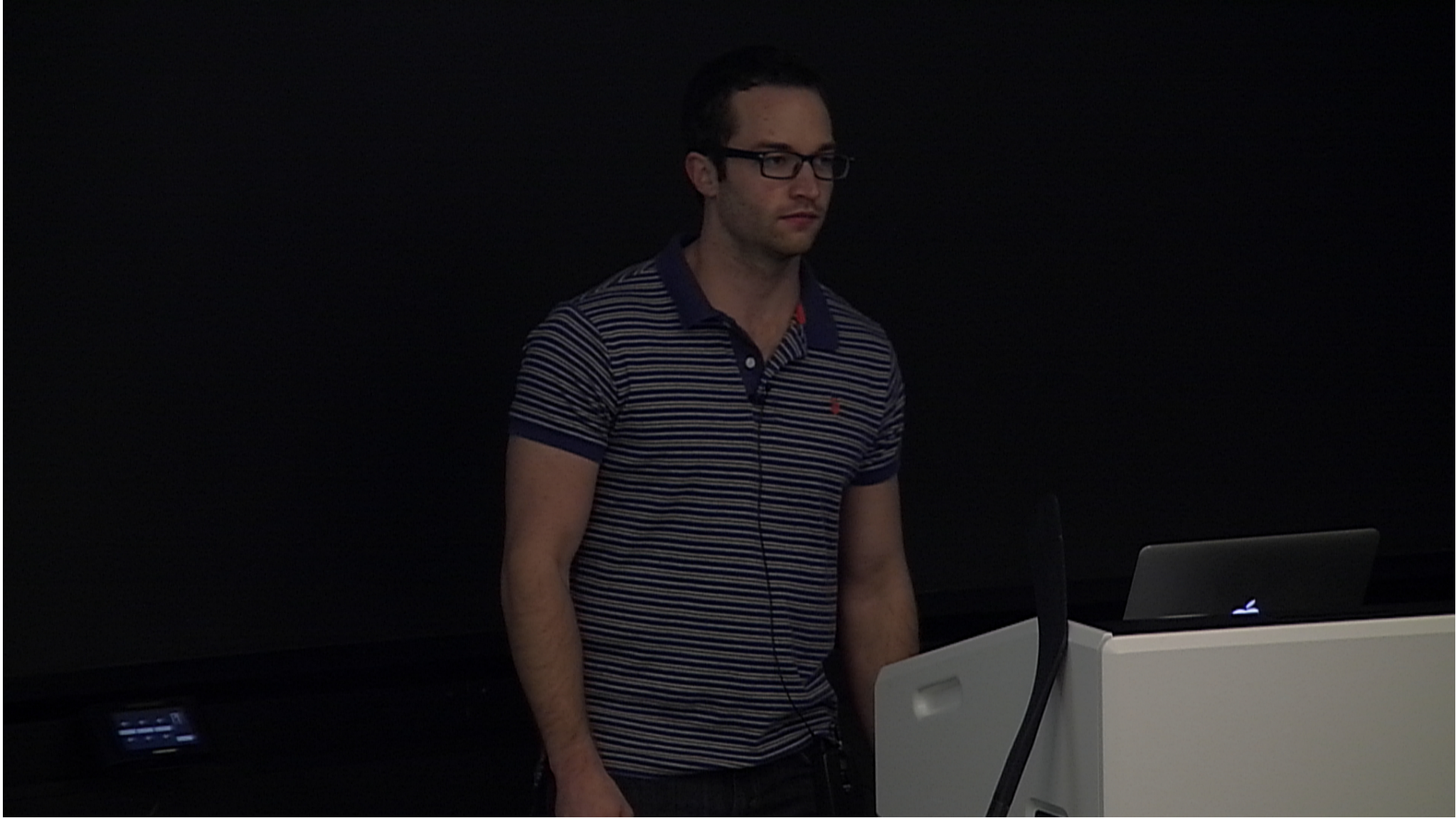


Title: 6D SCFTs and Group Theory

Date: Dec 09, 2016 02:00 PM

URL: <http://pirsa.org/16120018>

Abstract: <p>The past few years have seen a surge of interest in six-dimensional superconformal field theories (6D SCFTs). Notably, 6D SCFTs have recently been classified using F-theory, which relates these theories to elliptically-fibered Calabi-Yau manifolds. Classes of 6D SCFTs have remarkable connections to structures in group theory and therefore provide a physical link between two seemingly-unrelated mathematical objects. In this talk, we describe this link and speculate on its implications for future studies of 6D SCFTs.</p>



6D SCFTs and Group Theory

Tom Rudelius
Harvard University

Based On

- 1502.05405/hep-th
 - with Jonathan Heckman, David Morrison, and Cumrun Vafa
- 1506.06753/hep-th
 - with Jonathan Heckman
- 1601.04078/hep-th
 - with Jonathan Heckman, Alessandro Tomasiello
- 1605.08045/hep-th
 - with David Morrison
- work in progress
 - with Noppadol Mekareeya, Alessandro Tomasiello

Outline

- I. Classification of 6D SCFTs
 - i. Tensor Branches/Strings
 - ii. Gauge Algebras/Particles
- II. 6D SCFTs and Homomorphisms
 - i. $\mathfrak{su}(2) \rightarrow \mathfrak{g}_{\text{ADE}}$
 - ii. $\Gamma_{\text{ADE}} \rightarrow E_8$
- III. Implications for 6D SCFTs
 - i. C-theorems in 6D
 - ii. Classification of RG Flows

Why Study 6D SCFTs?

- Nahm: Maximal SCFT dimension is *six*
- Degrees of freedom \neq particles (but it's a QFT!)
- QFT of M5-branes is a 6D SCFT
- Compactification \Rightarrow 5D/4D/3D/2D Theories

Focus: $(1, 0)$ SCFTs

Conformal Symmetry: $\mathfrak{so}(6, 2)$

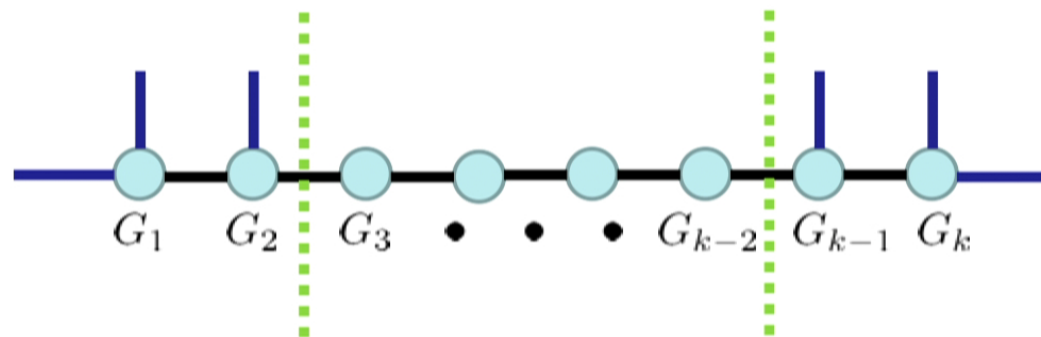
Supersymmetry: 8 Q 's and 8 S 's

R-symmetry: $\mathfrak{su}(2)_{\mathcal{R}}$

Classification of 6D SCFTs

Classification of 6D SCFTs

- 6D SCFTs can be classified via F-theory
- Nearly all F-theory conditions can be phrased in field theory terms
- 6D SCFTs = Generalized Quivers

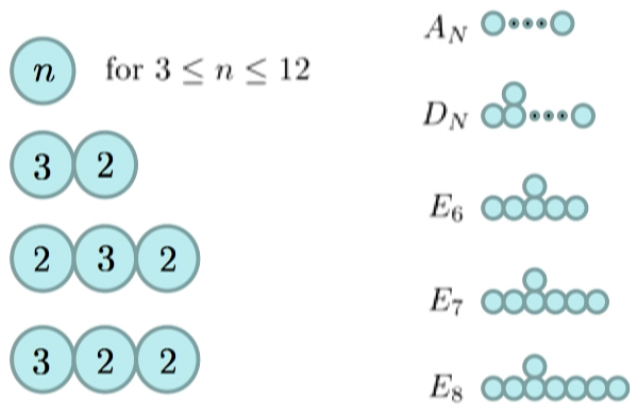


Classification of 6D SCFTs

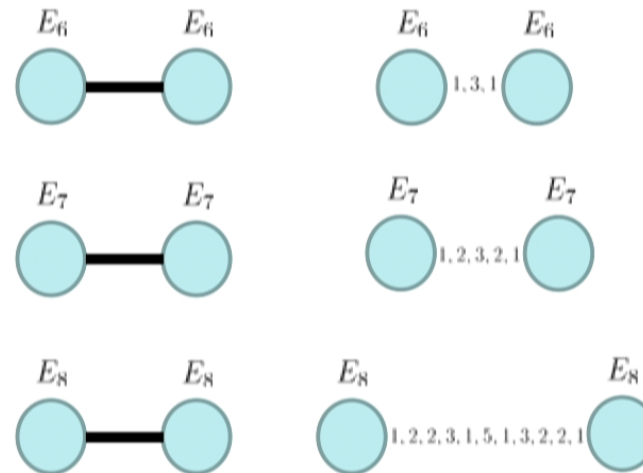
- Looks like chemistry

“Atoms”

c.f. Morrison and Taylor '12



“Radicals”



Example: All $(2, 0)$ Theories

Witten '95, Strominger '95

Type IIB on $\mathbb{C}^2/\Gamma_{ADE}$

Resolution Involves:

Bouquet of \mathbb{CP}^1 's

$$\mathbb{CP}_i^1 \cap \mathbb{CP}_j^1 = -\text{Dynkin}_{ij}$$

$$\text{Note: } \mathbb{CP}_i^1 \cap \mathbb{CP}_i^1 = -2$$



6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

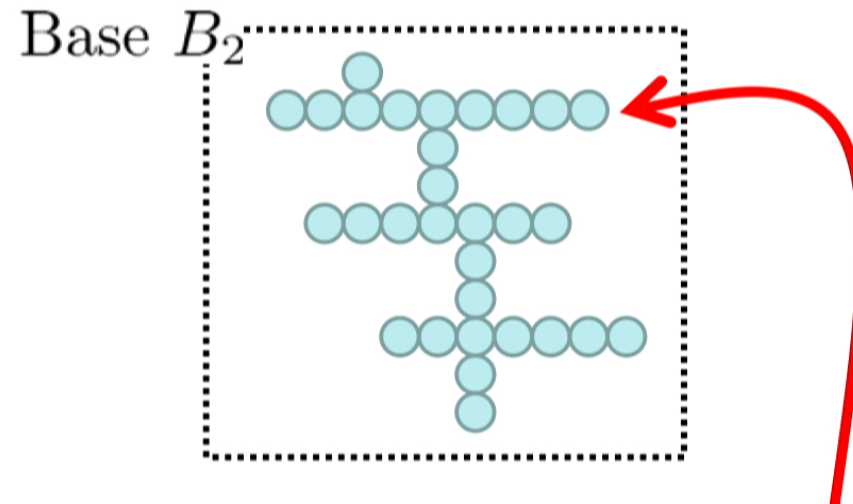
All known 6D theories have F-theory avatar*

IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

F-theory on $\mathbb{R}^{5,1} \times CY_3$

$$\begin{array}{ccc} & T^2 \rightarrow & CY_3 \\ & & \downarrow \\ & & B_2 \end{array}$$

Geometric Picture



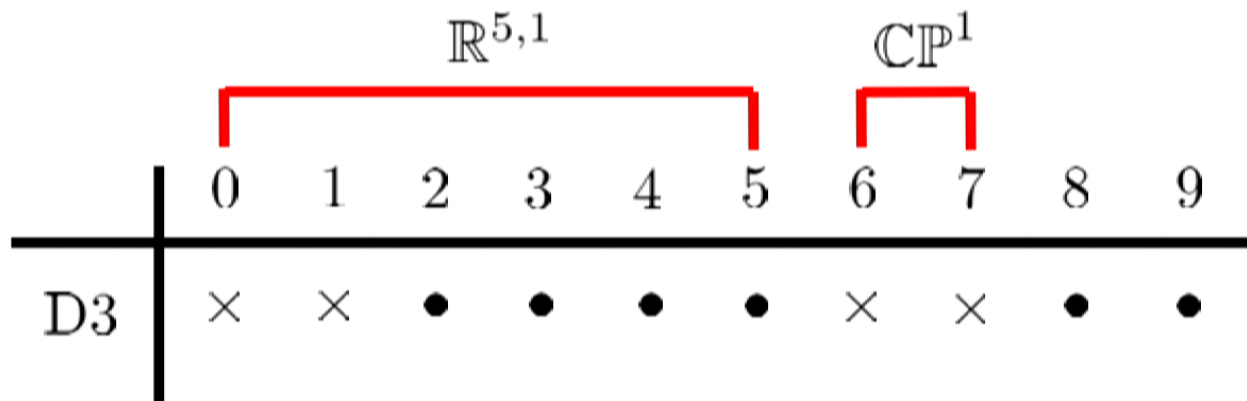
Singularities in base \Rightarrow strings (D3 / \mathbb{P}^1)

Singularities in fiber \Rightarrow particles (7-brane on \mathbb{P}^1)

Tensionless Strings in F-theory

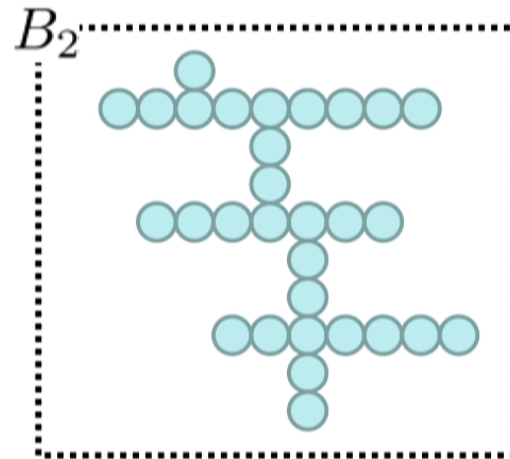
- Realized by D3-brane on collapsing \mathbb{CP}^1

$$\text{Tension} = \text{Vol}(\mathbb{CP}^1) \rightarrow 0$$



SCFT Limit

Start: A smooth base B_2

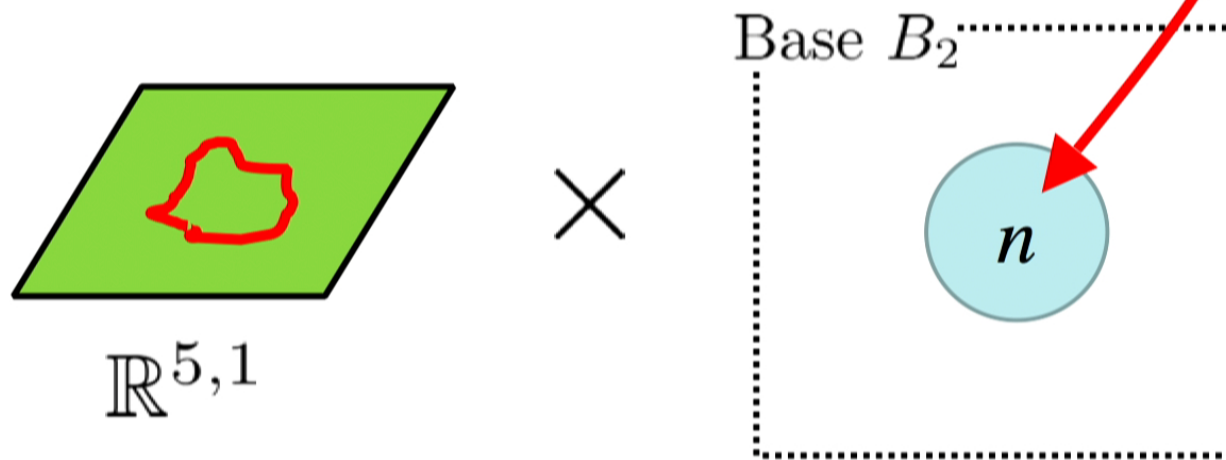


End: To get a CFT, sim. contract curves of B_2

Strings from D3 on a \mathbb{P}^1

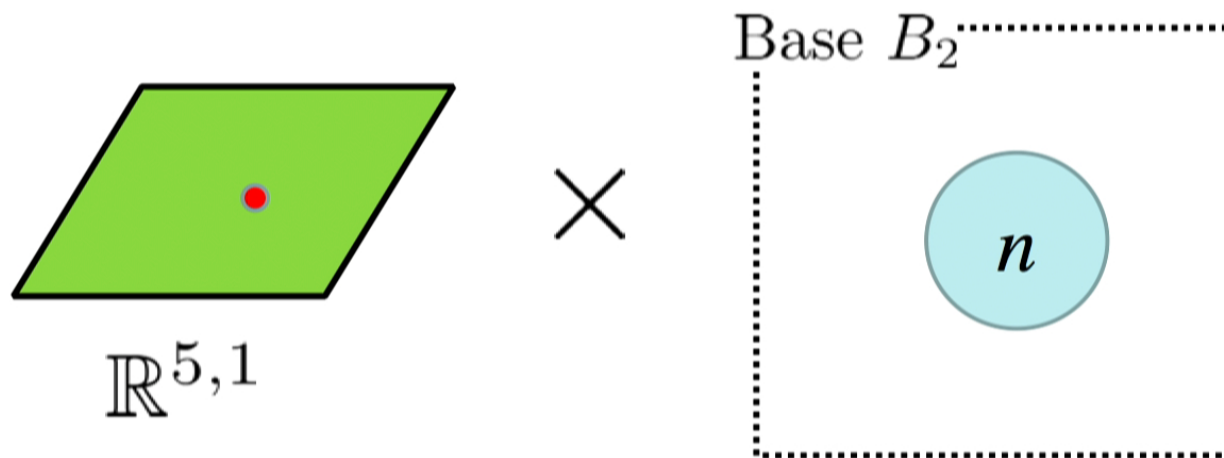
$$-\Sigma \cap \Sigma = \text{String Charge}$$

(which must be integer > 0)

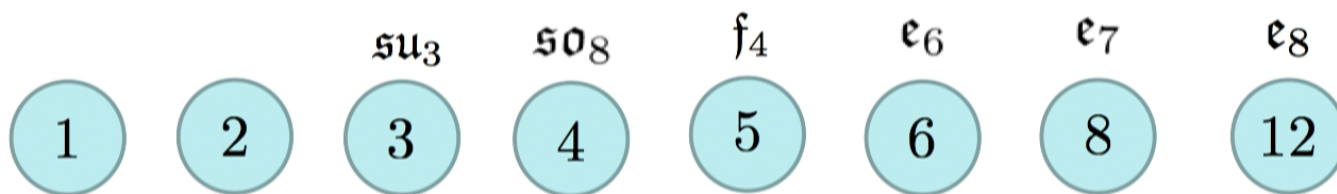


Particles from D7's on a \mathbb{P}^1

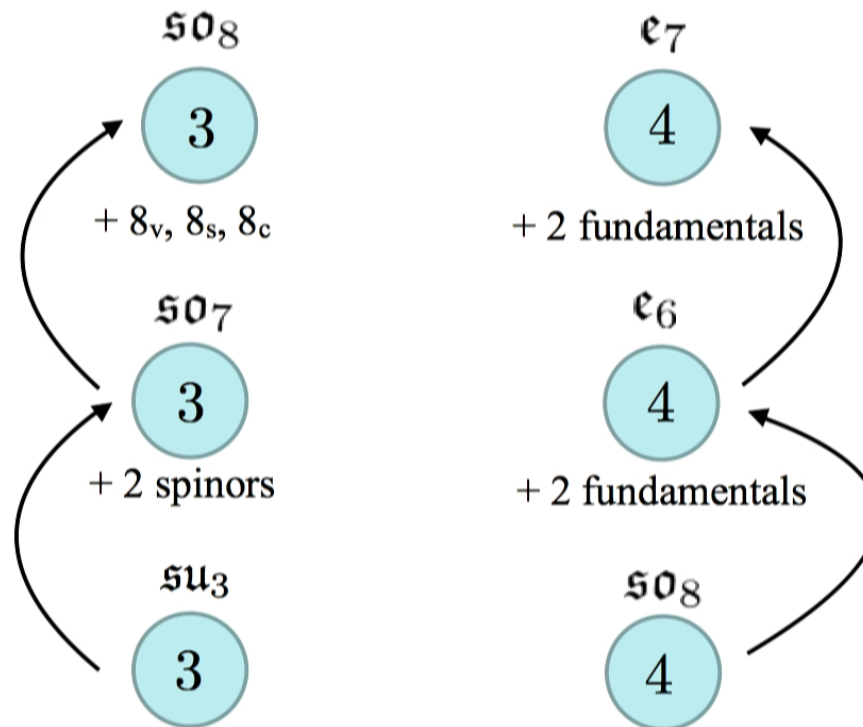
$3 \leq n \leq 12 \Rightarrow$ always have gauge fields
(elliptic fiber is singular: Morrison Taylor '12)



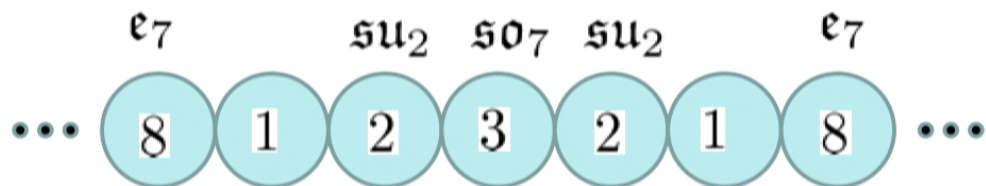
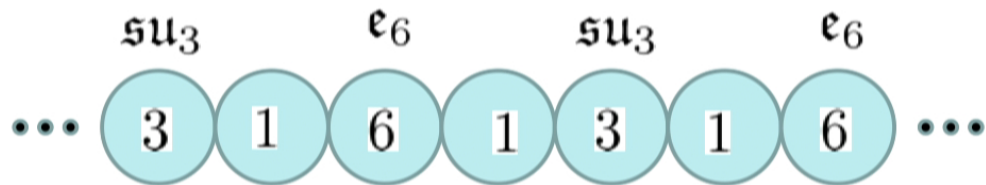
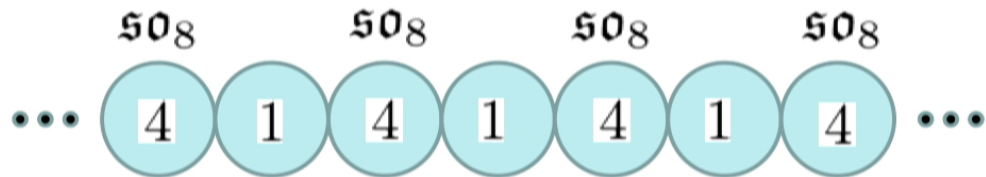
Minimal Gauge Algebras



Fiber Enhancements



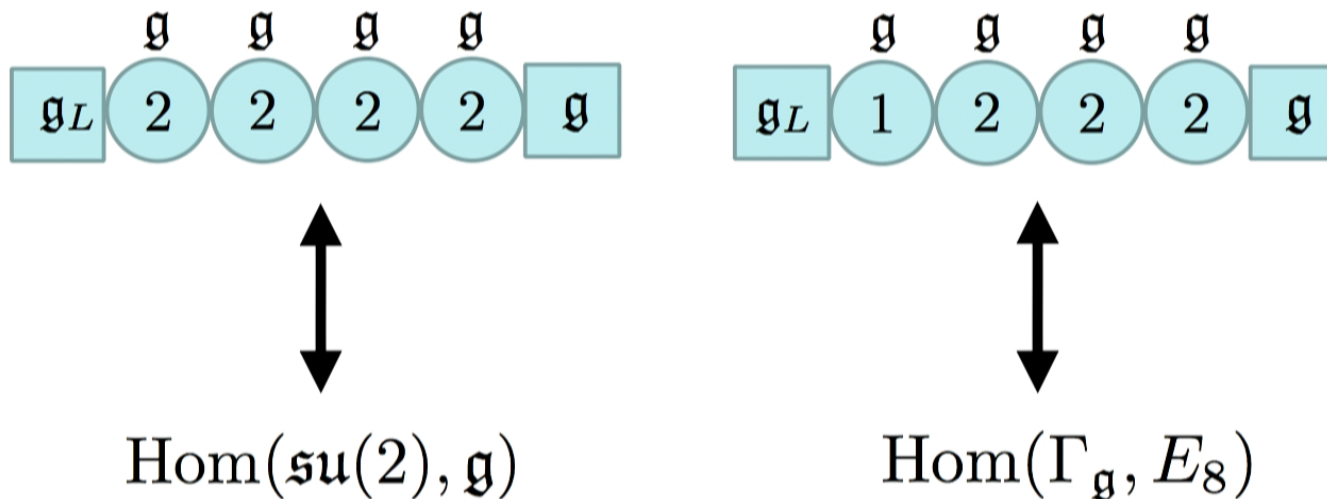
Examples



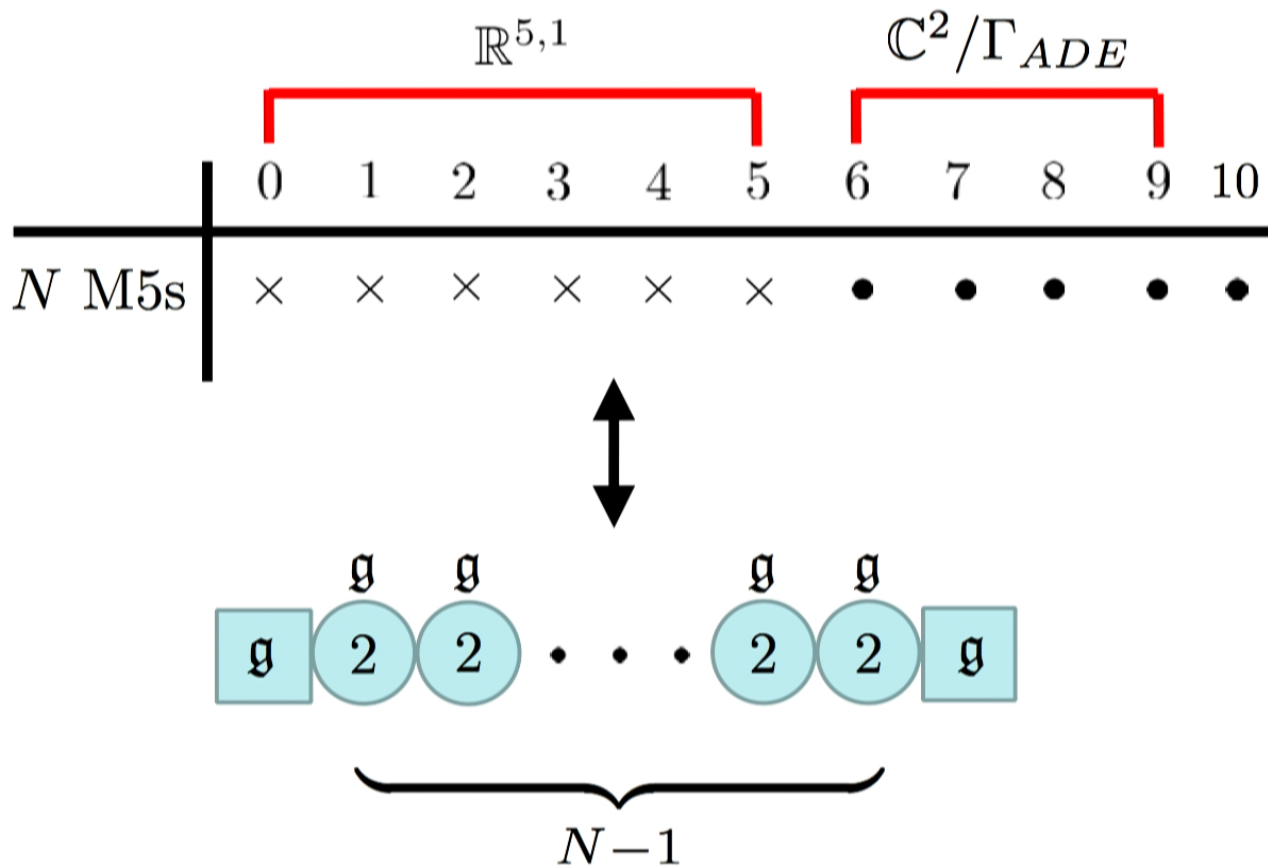
6D SCFTs and Homomorphisms

6D SCFTs and Group Theory

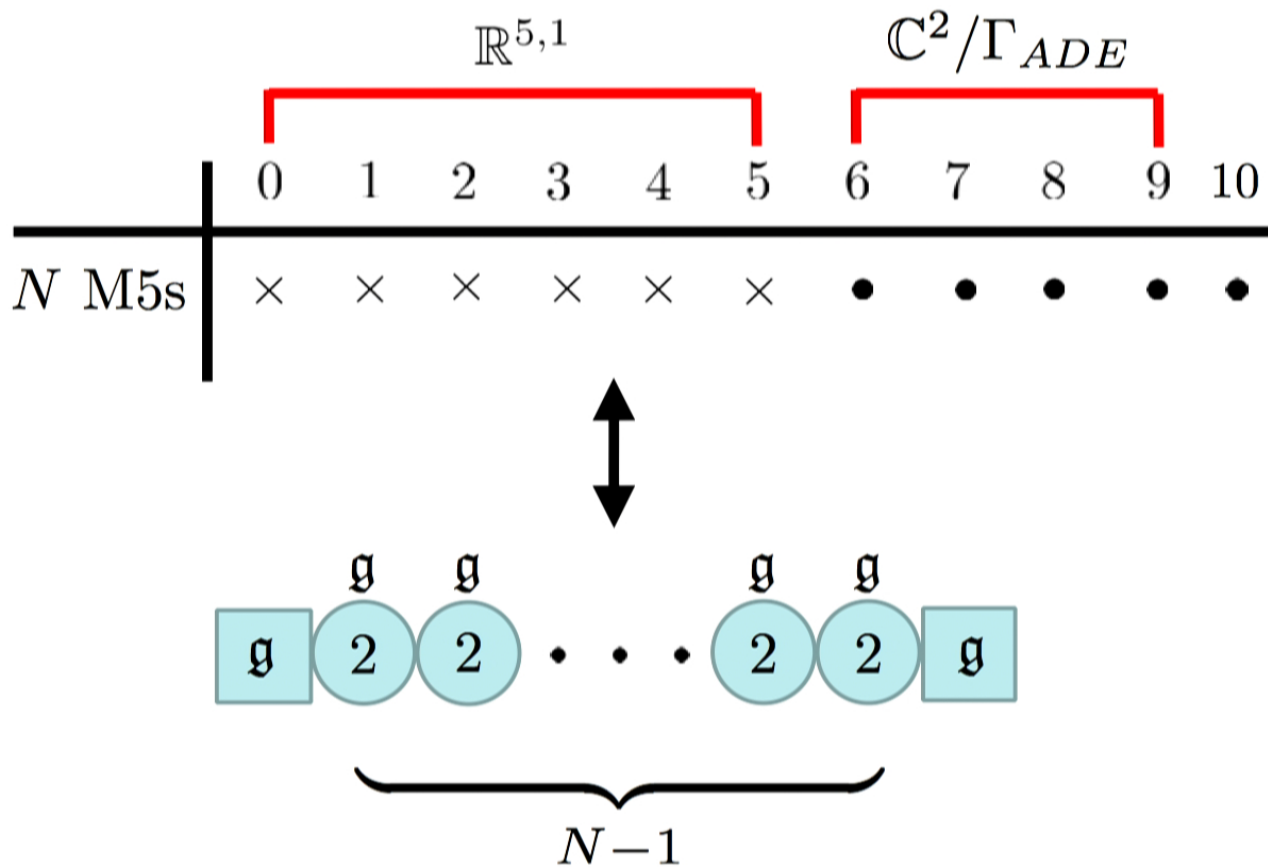
- Large classes of 6D SCFTs have connections to structures in group theory
- The correspondence has been verified explicitly



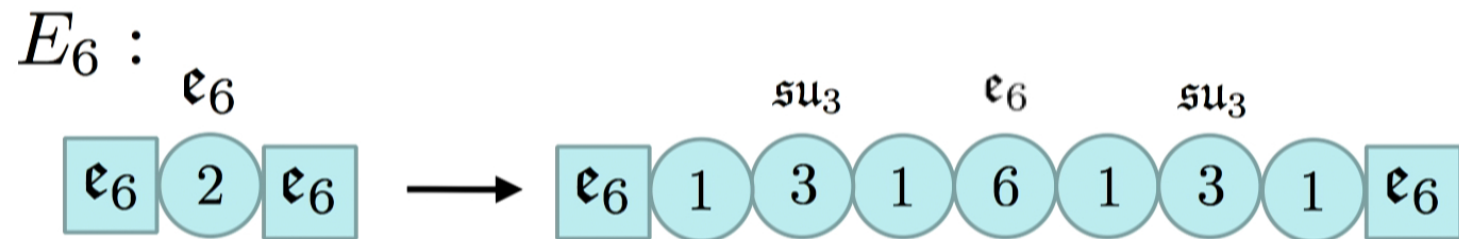
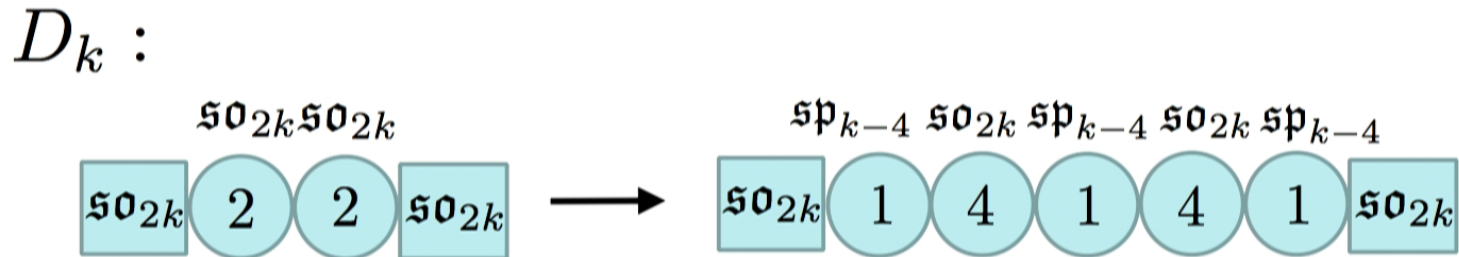
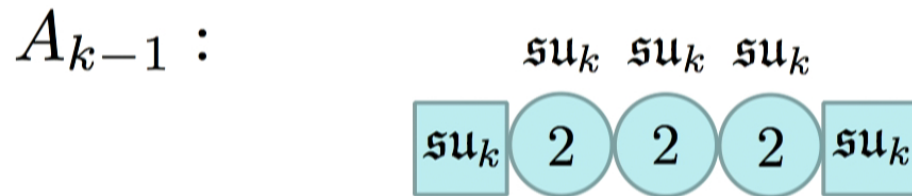
M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$



M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$



M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$



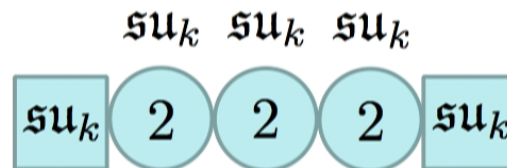
Nilpotent Deformations

- Matrix of normal deformations Φ characterizes positions of 7-branes
- View intersection points of \mathbb{CP}^1 in base as marked points
- Let adjoint field Φ have singular behavior at marked points \Rightarrow Hitchin system coupled to defects:

$$\partial_A \Phi = \sum_p \mu_{\mathbb{C}}^{(p)} \delta_{(p)} \quad F + [\Phi, \Phi^\dagger] = \sum_p \mu_{\mathbb{R}}^{(p)} \delta_{(p)}$$

M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$

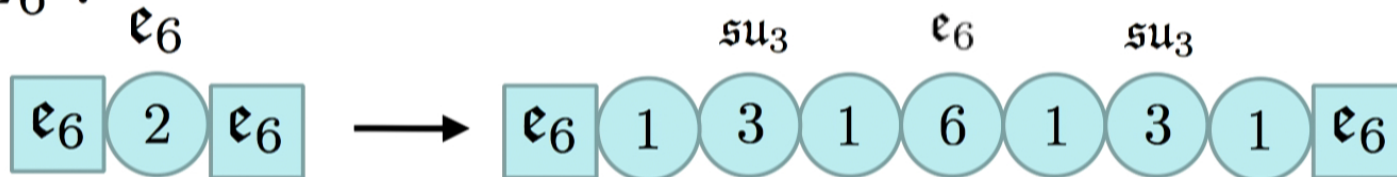
A_{k-1} :



D_k :



E_6 :



Nilpotent Deformations

- Split $\mu_{\mathbb{C}} = \mu_s + \mu_n$, consider nilpotent part μ_n , get \mathfrak{su}_2 algebra:

$$J_+ = \mu_{\mathbb{C}} \quad J_- = \mu_{\mathbb{C}}^\dagger \quad J_3 = \mu_{\mathbb{R}}$$

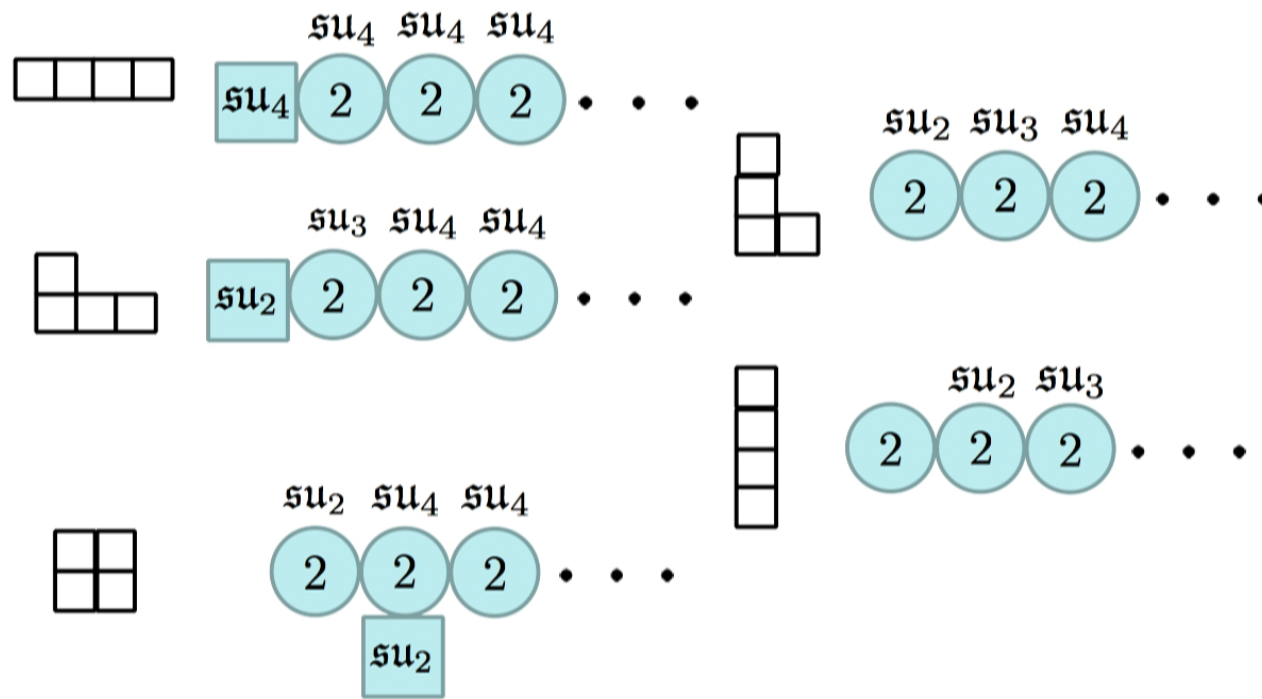
- Adjoint vevs $\Phi \sim \mu_{\mathbb{C}} \frac{dz}{z}$

\Rightarrow Classified by $\text{Hom}(\mathfrak{su}(2), \mathfrak{g})$

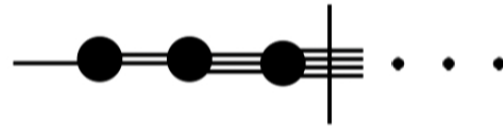
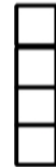
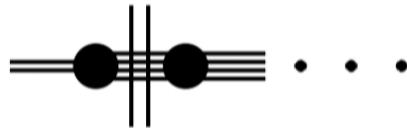
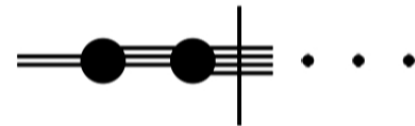
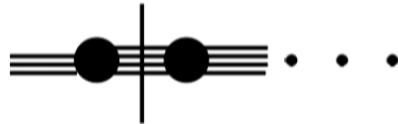
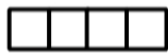
(equivalently, by nilpotent orbits J_+)

6D SCFTs and $\text{Hom}(\mathfrak{su}(2), A_{k-1})$

$\text{Hom}(\mathfrak{su}(2), A_{k-1})$ labeled by partitions of k :



Type IIA Realization



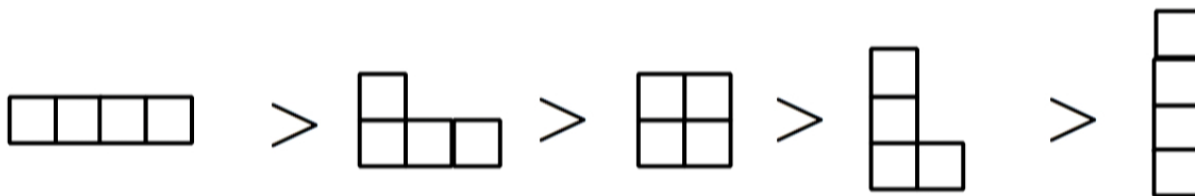
- | = D8
- = NS5
- = D6

Partial Ordering of Nilpotent Orbits

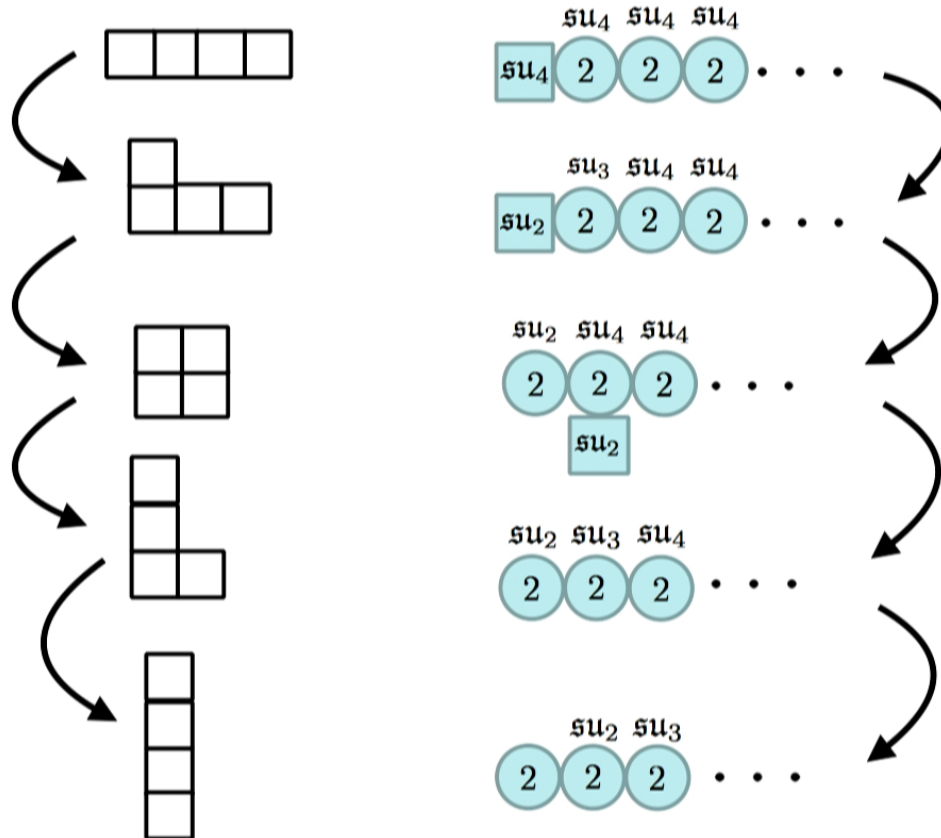
$$\mathcal{O}_\mu \geq \mathcal{O}_\nu \Leftrightarrow \bar{\mathcal{O}}_\mu \supset \mathcal{O}_\nu$$

$$\Leftrightarrow \mu \geq \nu$$

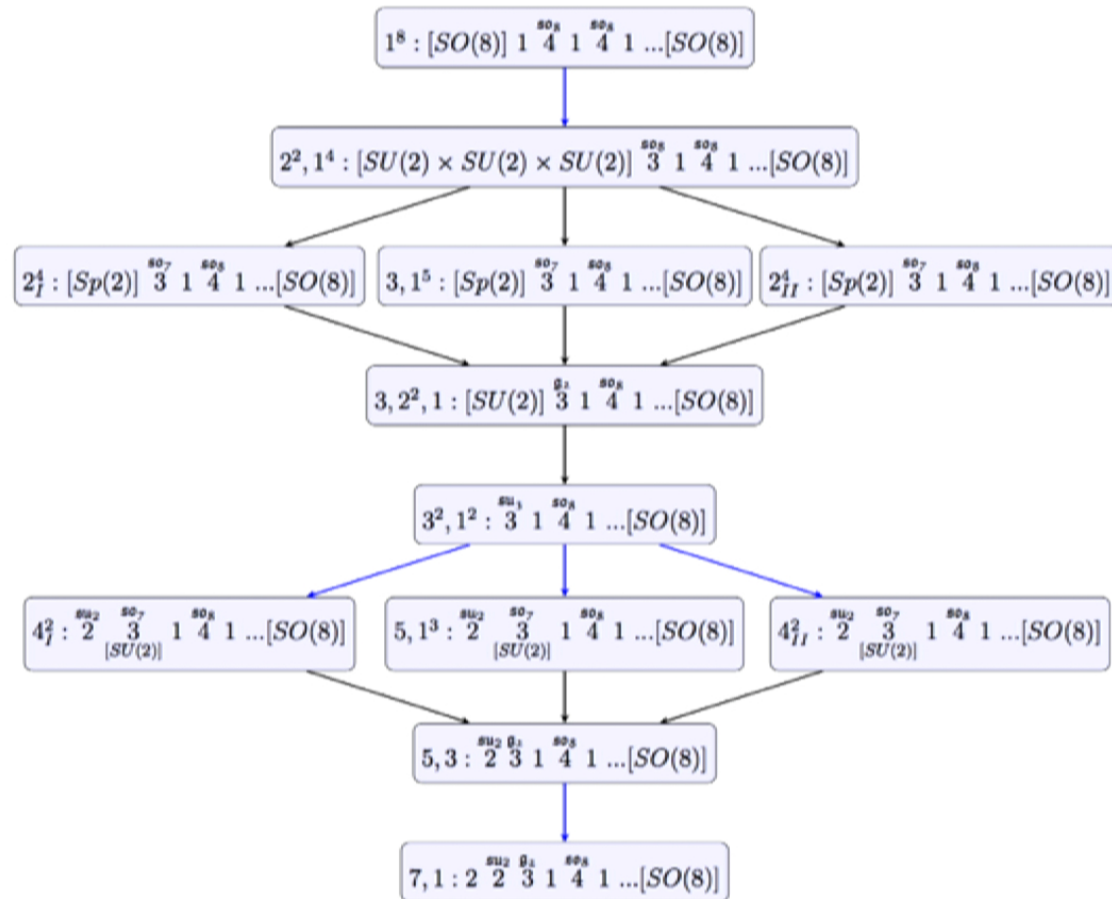
$$\Leftrightarrow \sum_{i=1}^m \mu_i^T \geq \sum_{i=1}^m \nu_i^T \quad \forall m$$



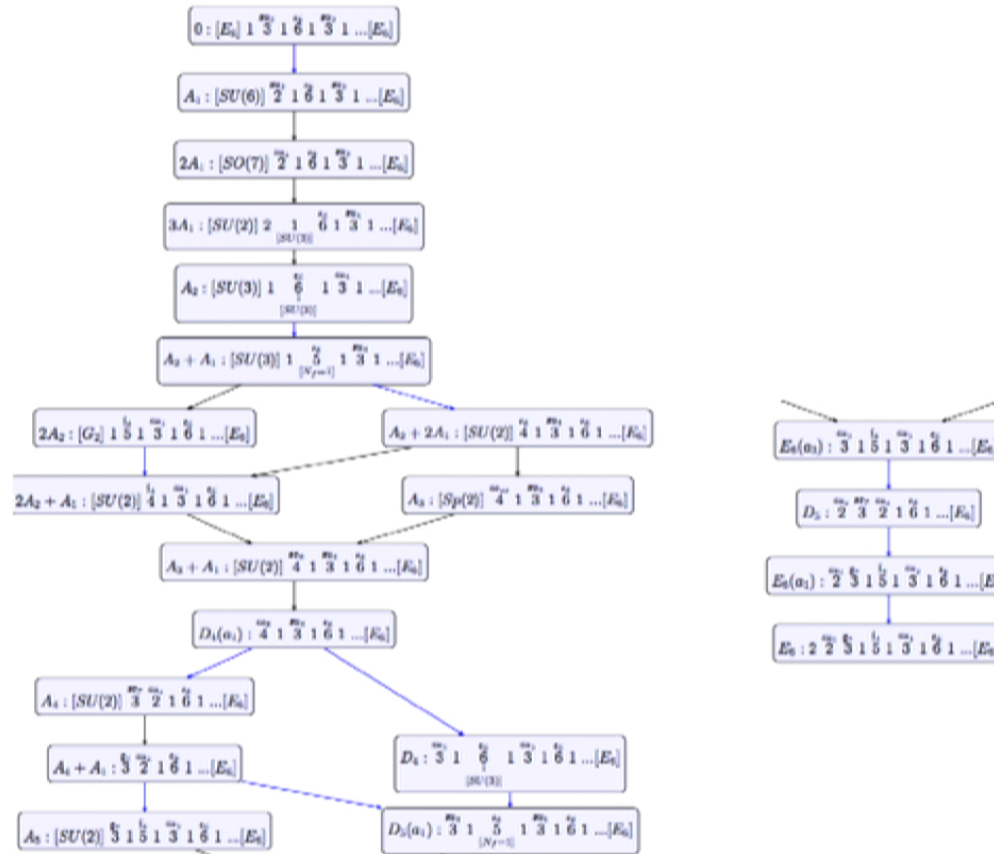
Nilpotent Hierarchy Matches RG Hierarchy!



6D SCFTs and $\text{Hom}(\mathfrak{su}(2), D_k)$

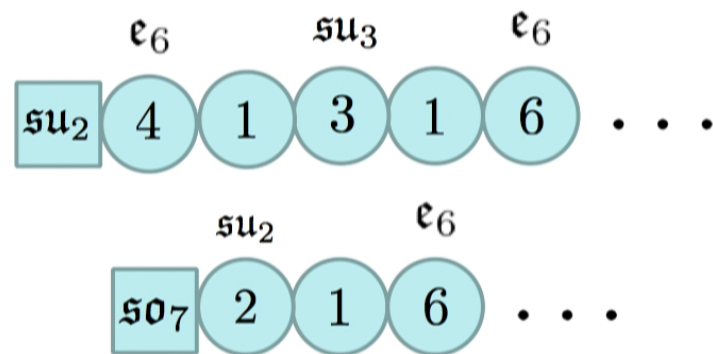


6D SCFTs and $\text{Hom}(\mathfrak{su}(2), E_6)$

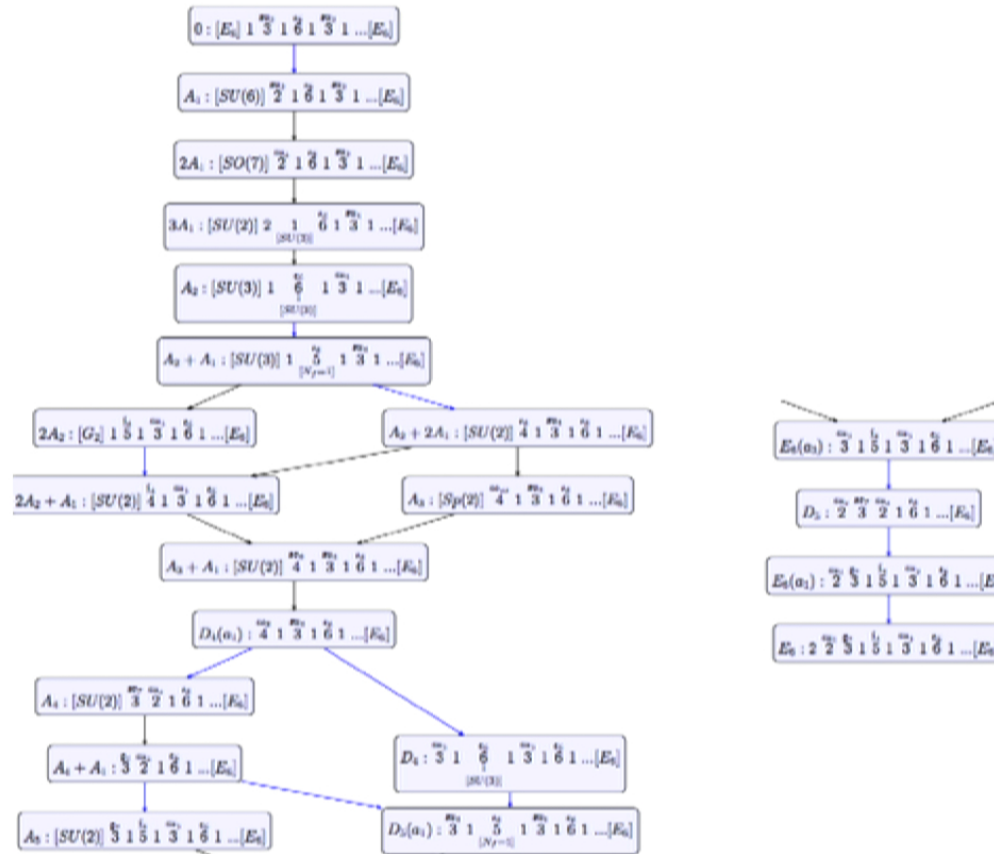


Nilpotent Orbits and Global Symmetries

- Consider nilpotent orbit $\mathcal{O}_\mu \in \mathfrak{g}$
- Let $F(\mu)$ be subgroup of G commuting with nilpotent element
- Claim: $F(\mu)$ is the global symmetry of the 6D SCFT associated with μ
- E.g.

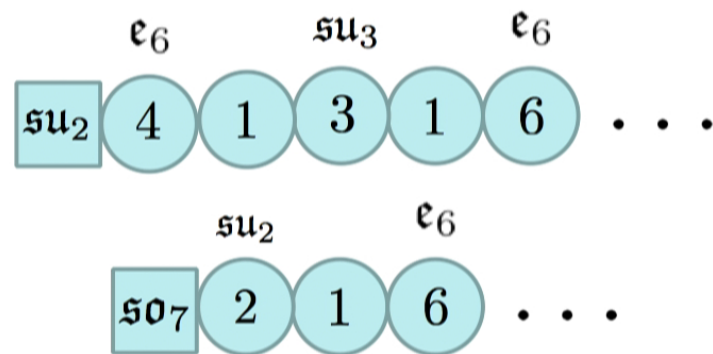


6D SCFTs and $\text{Hom}(\mathfrak{su}(2), E_6)$



Nilpotent Orbits and Global Symmetries

- Consider nilpotent orbit $\mathcal{O}_\mu \in \mathfrak{g}$
- Let $F(\mu)$ be subgroup of G commuting with nilpotent element
- Claim: $F(\mu)$ is the global symmetry of the 6D SCFT associated with μ
- E.g.



6D SCFTs and $\text{Hom}(\Gamma_{ADE}, E_8)$

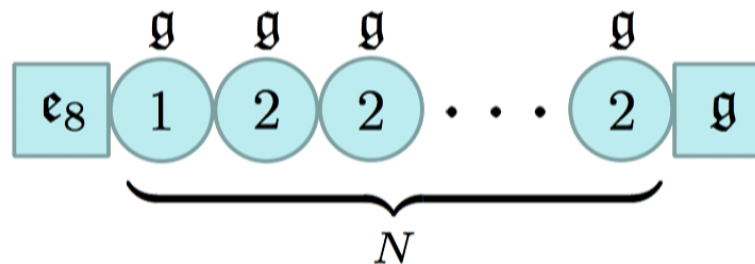
- Consider M5-branes probing Horava-Witten wall and $\mathbb{C}^2/\Gamma_{ADE}$ singularity

	$\mathbb{R}^{5,1}$						$\mathbb{C}^2/\Gamma_{ADE}$				
	0	1	2	3	4	5	6	7	8	9	10
N M5s	×	×	×	×	×	×	•	•	•	•	•
E_8 Wall	×	×	×	×	×	×	×	×	×	•	×

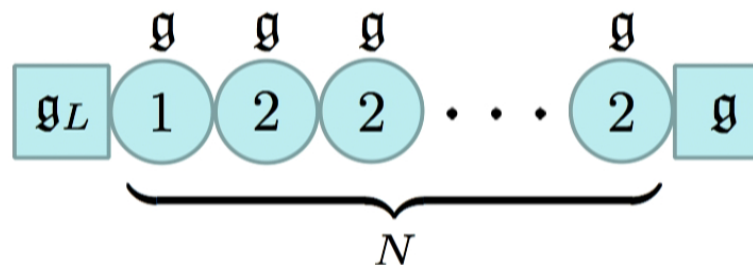
Boundary data \simeq flat E_8 connections on S^3/Γ_{ADE}

6D SCFTs and $\text{Hom}(\Gamma_{\text{ADE}}, E_8)$

- For trivial boundary data, get 6D SCFT:



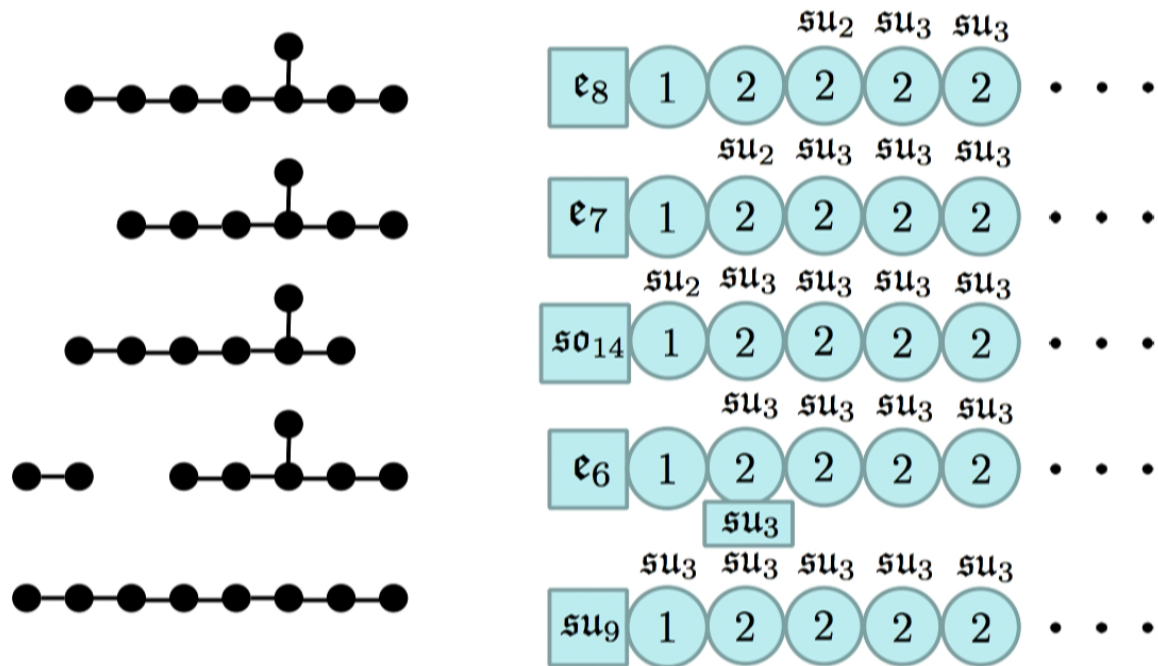
- For non-trivial boundary data, global symmetry is broken to a subgroup



6D SCFTs and $\text{Hom}(\Gamma_{\text{ADE}}, E_8)$

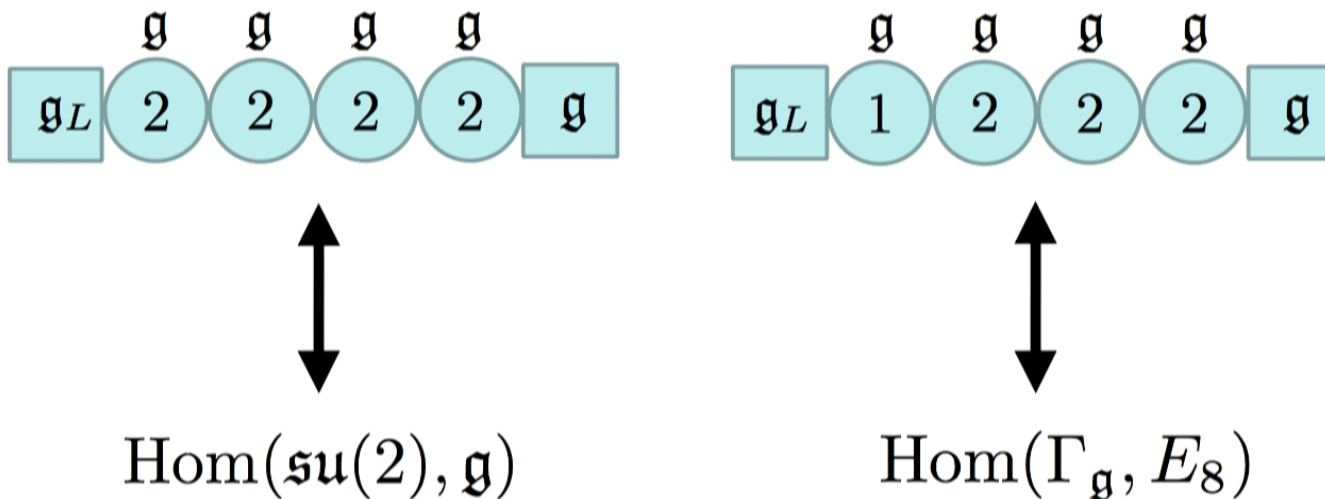
Flat E_8 connections on $S^3/\Gamma_{\text{ADE}} \Leftrightarrow \text{Hom}(\Gamma_{\text{ADE}}, E_8)$

E.g. $\Gamma_{A_2}, \text{Hom}(\mathbb{Z}_3, E_8)$:



6D SCFTs and Group Theory

- Large classes of 6D SCFTs have connections to structures in group theory
- The correspondence has been verified explicitly



Implications for 6D SCFTs

Implications for 6D SCFTs

- There is significant evidence for the a-theorem (and an infinite collection of other c-theorems) in 6D SCFTs
- Connections to group theory provide a proof in certain classes of RG flows
- We speculate that a full classification of RG flows among 6D SCFTs is possible through these connections to group theory

Implications for 6D SCFTs

- There is significant evidence for the a-theorem (and an infinite collection of other c-theorems) in 6D SCFTs
- Connections to group theory provide a proof in certain classes of RG flows
- We speculate that a full classification of RG flows among 6D SCFTs is possible through these connections to group theory

't Hooft Anomalies in 6D SCFTs

- Anomaly polynomial calculable for any 6D SCFT

Ohmori, Shimizu, Tachikawa, Yonekura '14

$$I = \alpha c_2(R)^2 + \beta c_2(R)p_1(T) + \gamma p_1(T)^2 + \delta p_2(T) + \dots$$

- Trace anomaly related to 6D Euler density

$$\langle T_{\mu}^{\mu} \rangle = \left(\frac{1}{4\pi} \right)^3 a E_6 + \dots$$

- Can be expressed in terms of anomaly polynomial:

$$a = \frac{8}{3}(\alpha - \beta + \gamma) + \delta$$

Cordova, Dumitrescu, Intriligator '15

Candidate C -Functions

Linear Combinations: $C = \vec{m} \cdot \vec{\alpha}_{\text{anomaly}}$

Given a CFT, look for numbers C such that:

$$C_{UV} > C_{IR}$$

Two Deformation Types

Complex Structure Deformation / Higgs Branch



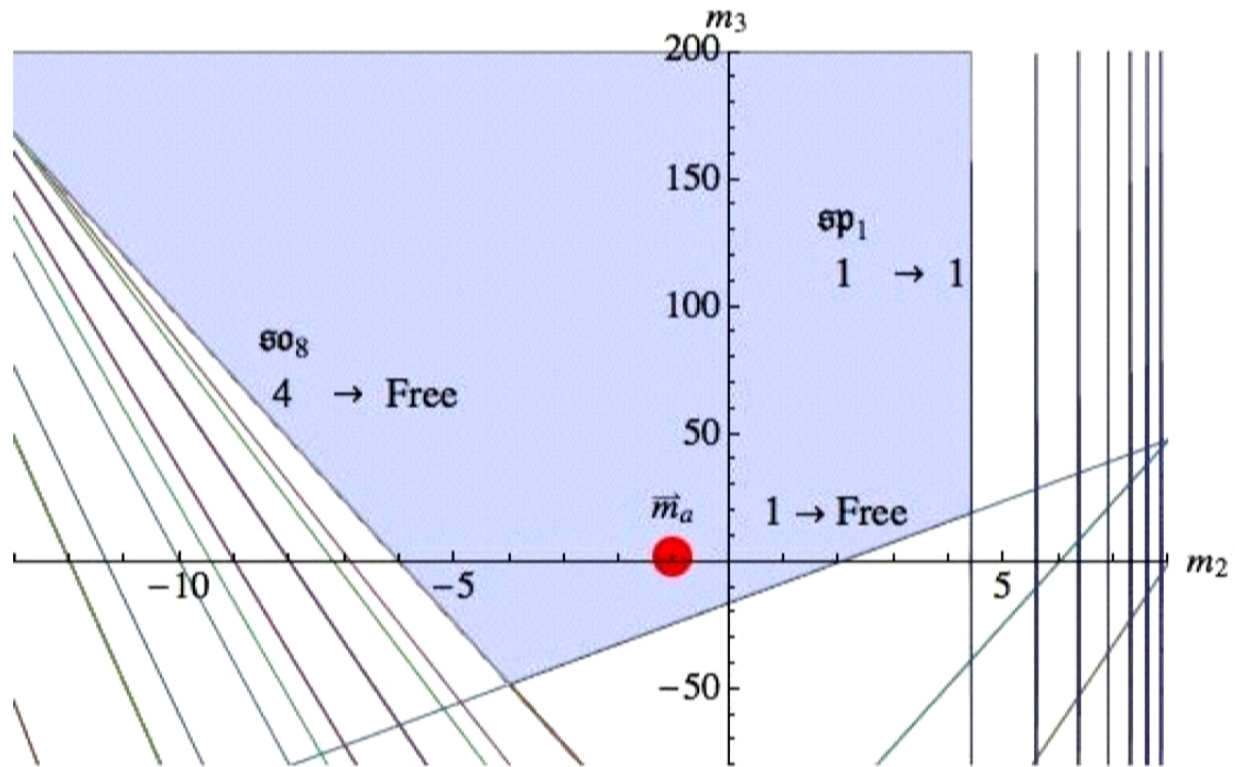
Expand a curve in base to large size / Tensor Branch



“When in doubt, use brute force.”

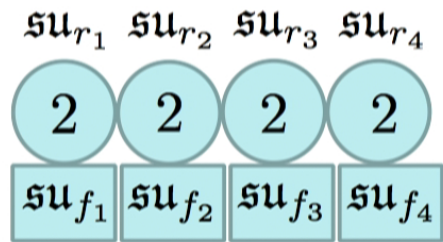
–Ken Thompson

Results of Sweep



Nilpotent Orbit SCFTs

- Can relate anomalies to data of nilpotent orbit



\Rightarrow

$$\alpha = 12 \sum_{i,j} C_{i,j}^{-1} r_i r_j + 2(N - 1) - \sum_i r_i^2$$

$$\beta = N - 1 - \frac{1}{2} \sum_i r_i^2$$

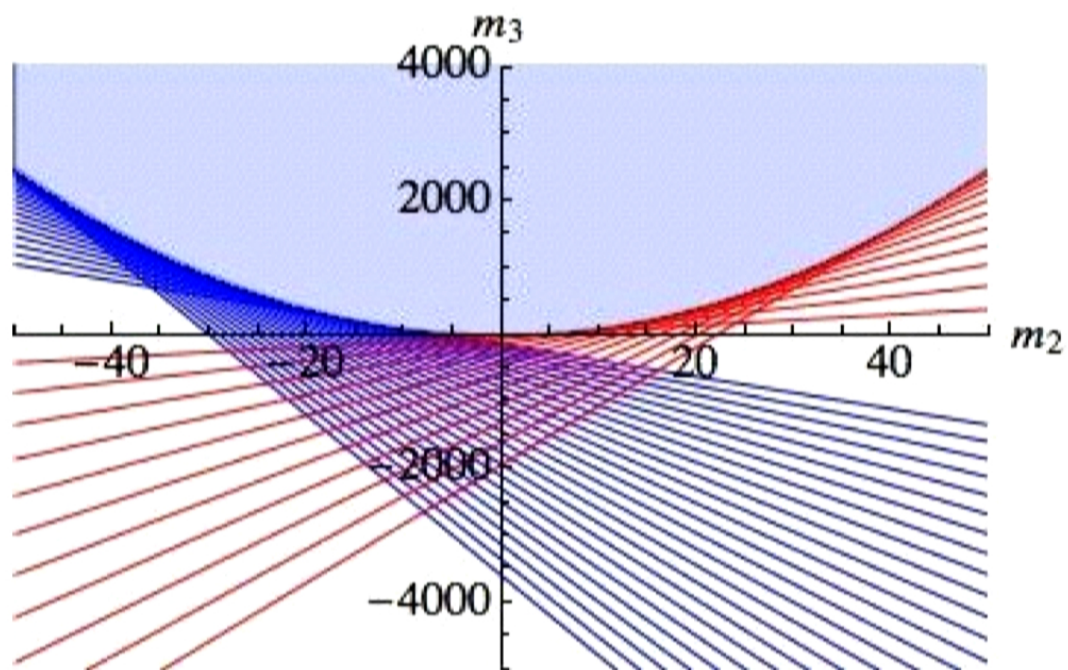
$$\gamma = \frac{1}{240} \left(\frac{7}{2} \sum_i r_i f_i + 30(N - 1) \right)$$

$$\delta = -\frac{1}{120} \left(\sum_i r_i f_i + 60(N - 1) \right)$$

Cremonesi, Tomasiello '15

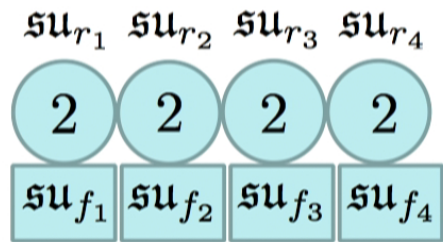
- Allows for proof of a-theorem for these flows

Tensor Branch Sweep



Nilpotent Orbit SCFTs

- Can relate anomalies to data of nilpotent orbit



$$\Rightarrow \begin{aligned} \alpha &= 12 \sum_{i,j} C_{i,j}^{-1} r_i r_j + 2(N-1) - \sum_i r_i^2 \\ \beta &= N-1 - \frac{1}{2} \sum_i r_i^2 \\ \gamma &= \frac{1}{240} \left(\frac{7}{2} \sum_i r_i f_i + 30(N-1) \right) \\ \delta &= -\frac{1}{120} \left(\sum_i r_i f_i + 60(N-1) \right) \end{aligned}$$

Cremonesi, Tomasiello '15

- Allows for proof of a-theorem for these flows

Summary and Future Research

- So far...
 - 6D SCFTs have been classified
 - There are remarkable relationships between 6D SCFTs and two classes of homomorphisms
 - There is strong evidence for c-theorems in 6D
- In the future...
 - Can we classify full set of 6D RG Flows in terms of group theory data?
 - Can we prove a-theorem for these flows?