Title: Low energy field theories for non-Fermi liquids

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Abstract: <p> Non-Fermi liquids are exotic metallic states which do not support well defined quasiparticles. Due to strong quantum fluctuations and the presence of extensive gapless modes near the Fermi surface, it has been difficult to understand universal low energy properties of non-Fermi liquids reliably. In this talk, we will discuss recent progress made on field theories for non-Fermi liquids. Based on a dimensional regularization scheme which tunes the number of co-dimensions of Fermi surface, critical exponents that control scaling behaviors of physical observables can be computed in controlled ways. The systematic expansion also provides important insight into strongly interacting metals. This allows us find the non-perturbative solution for the strange metal realized at the antiferromagnetic quantum critical point in 2+1 dimensions and predict the exact critical exponents.</p>

Low Energy Effective Theories for non-Fermi liquids

Sung-Sik Lee

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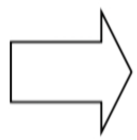
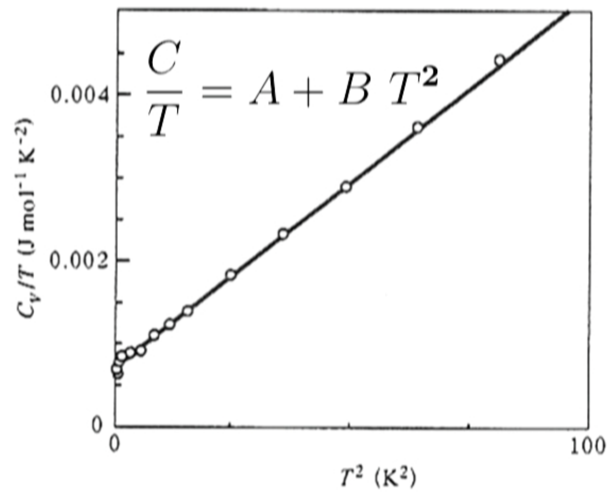
Perimeter Institute

In condensed matter physics, we aim to understand collective behaviors of many particles

Copper





The microscopic theory :

$$H = \sum_i \frac{p_i^2}{2m} + \sum_I \frac{p_I^2}{2M} + \sum_{i>j} \frac{e^2}{|r_i - r_j|} + \sum_{I>J} \frac{Z^2 e^2}{|R_I - R_J|} - \sum_{i,I} \frac{Z e^2}{|R_I - r_i|}$$

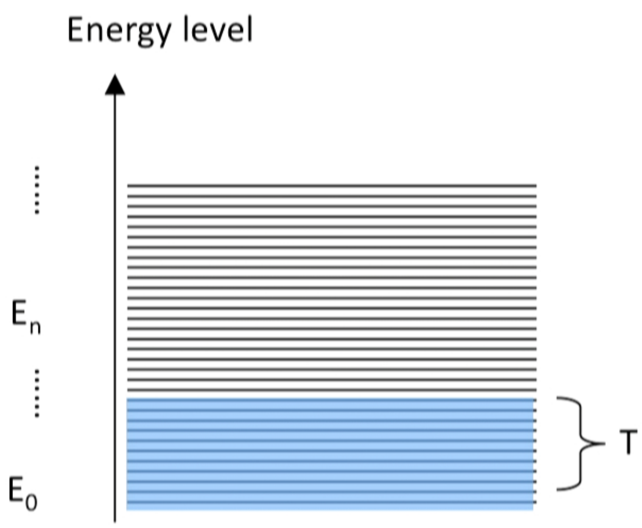
$$H|\Psi >= E|\Psi >$$

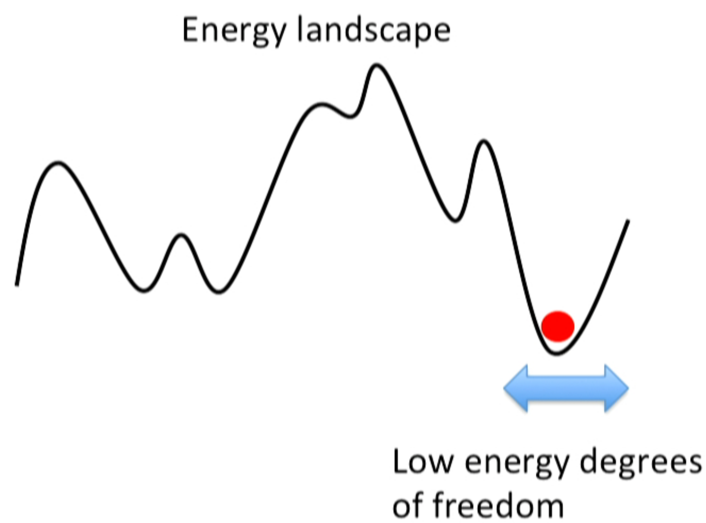
- It is in general impossible to solve the Schrodinger equation for 10^{23} interacting particles
  - Size of Hilbert space is too big
  - The full many-body wavefunction is too complicated to be useful

Useful strategies :

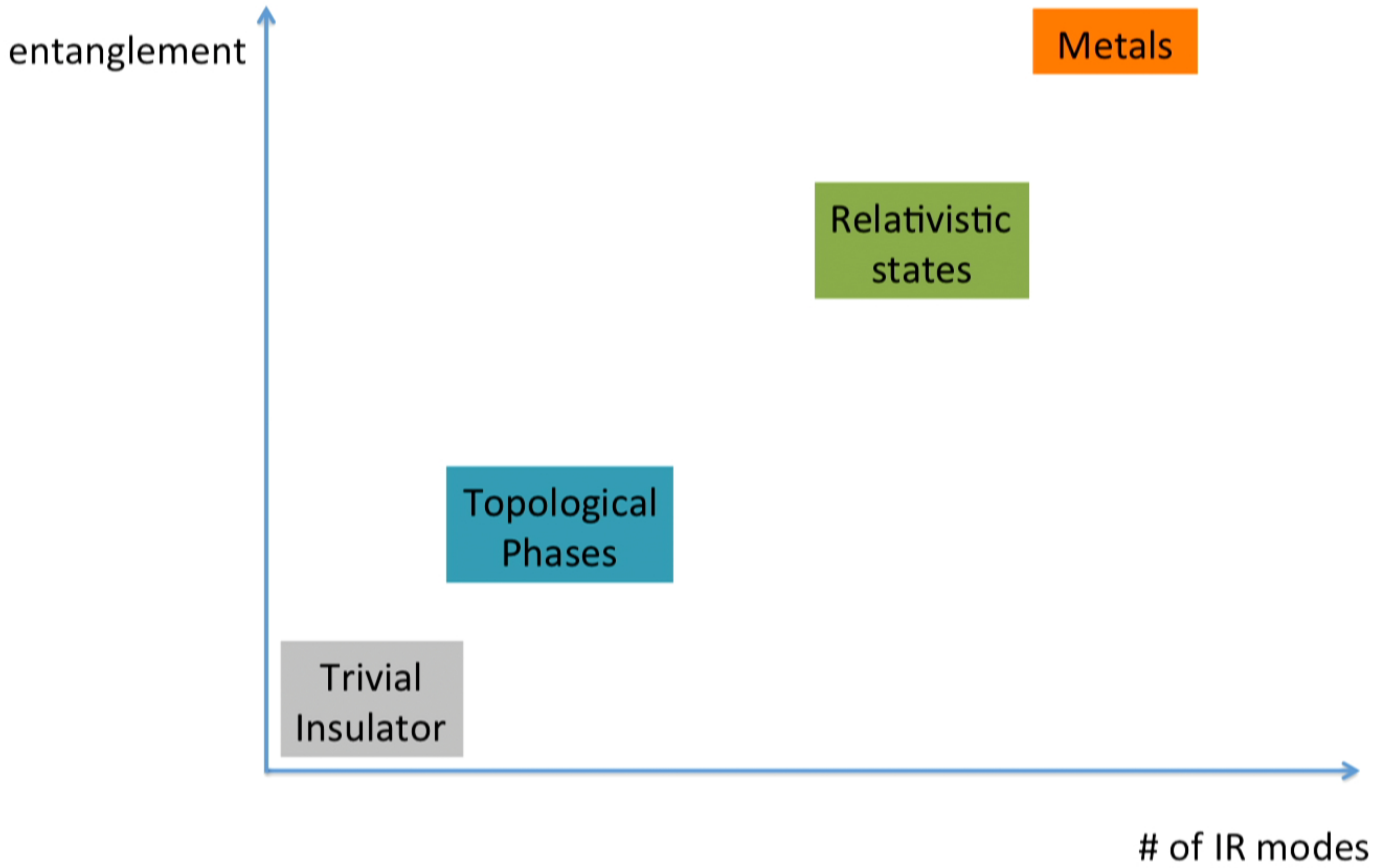
Capture universal low energy properties using effective theories

Look for simple organizing principles that emerge dynamically





Different phases of matter

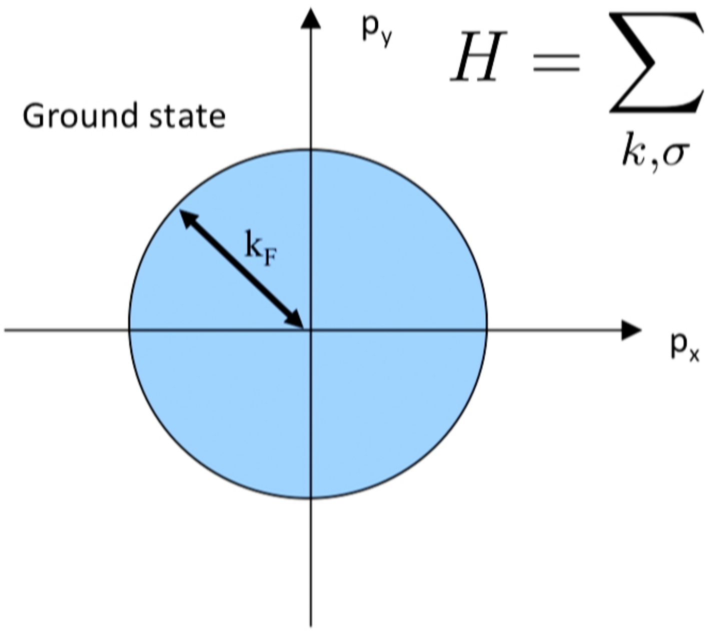


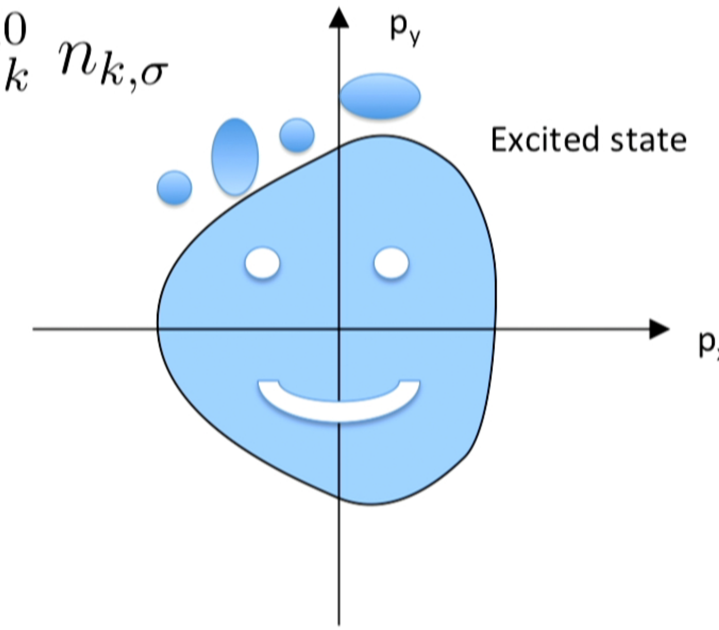
This talk :

Understanding universal properties of metals based on low energy effective field theories

Fermi Gas

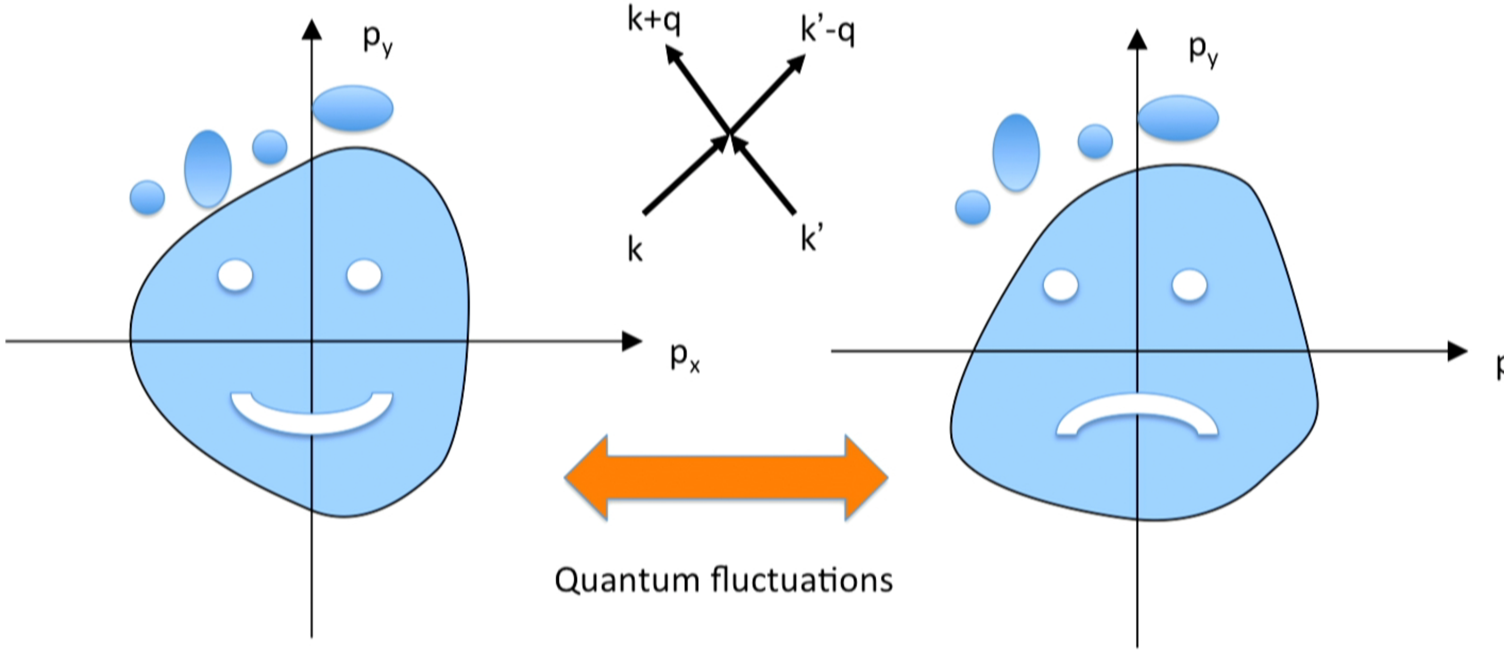
$$H = \sum_{k,\sigma} \epsilon_k^0 \; n_{k,\sigma}$$





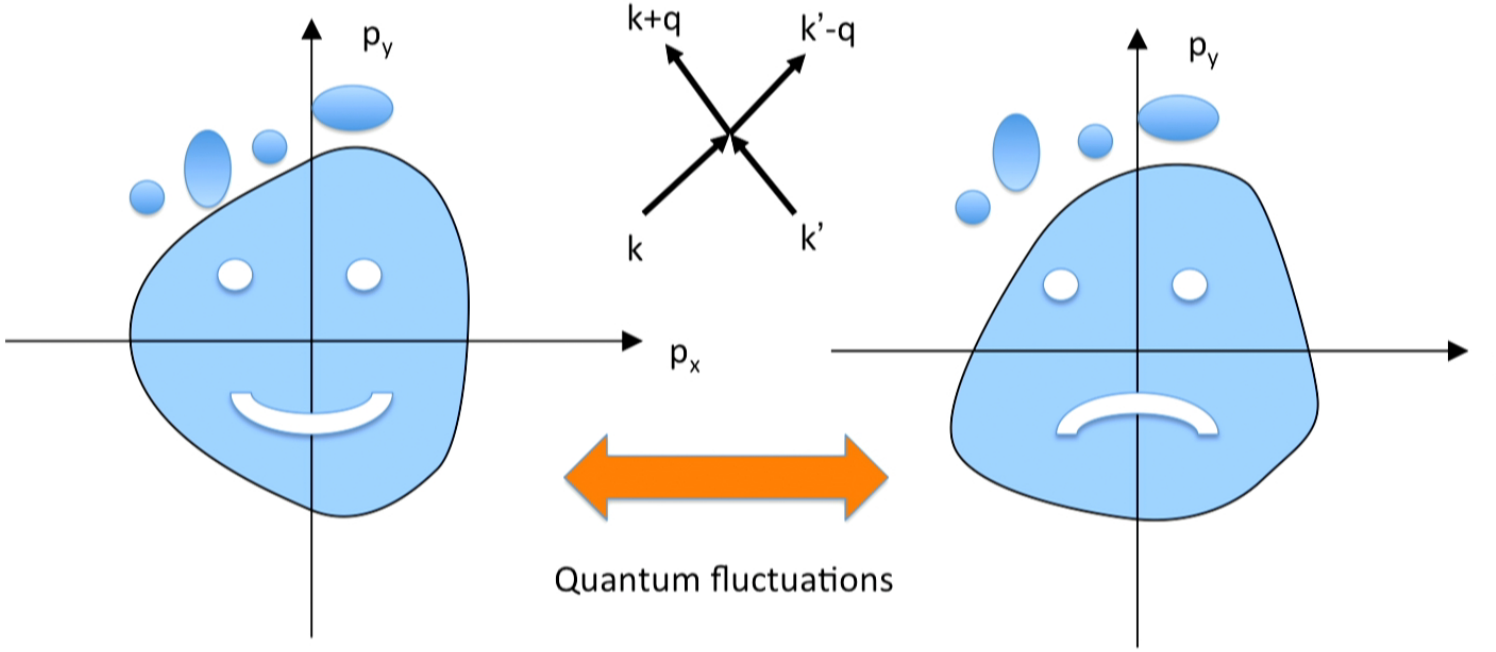
Many-body eigenstates are labeled by occupation numbers of single-particle states $|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \ldots >$

Interacting Fermions



Shape of Fermi surface is subject to quantum fluctuations

Interacting Fermions



Shape of Fermi surface is subject to quantum fluctuations

Fermi Liquids

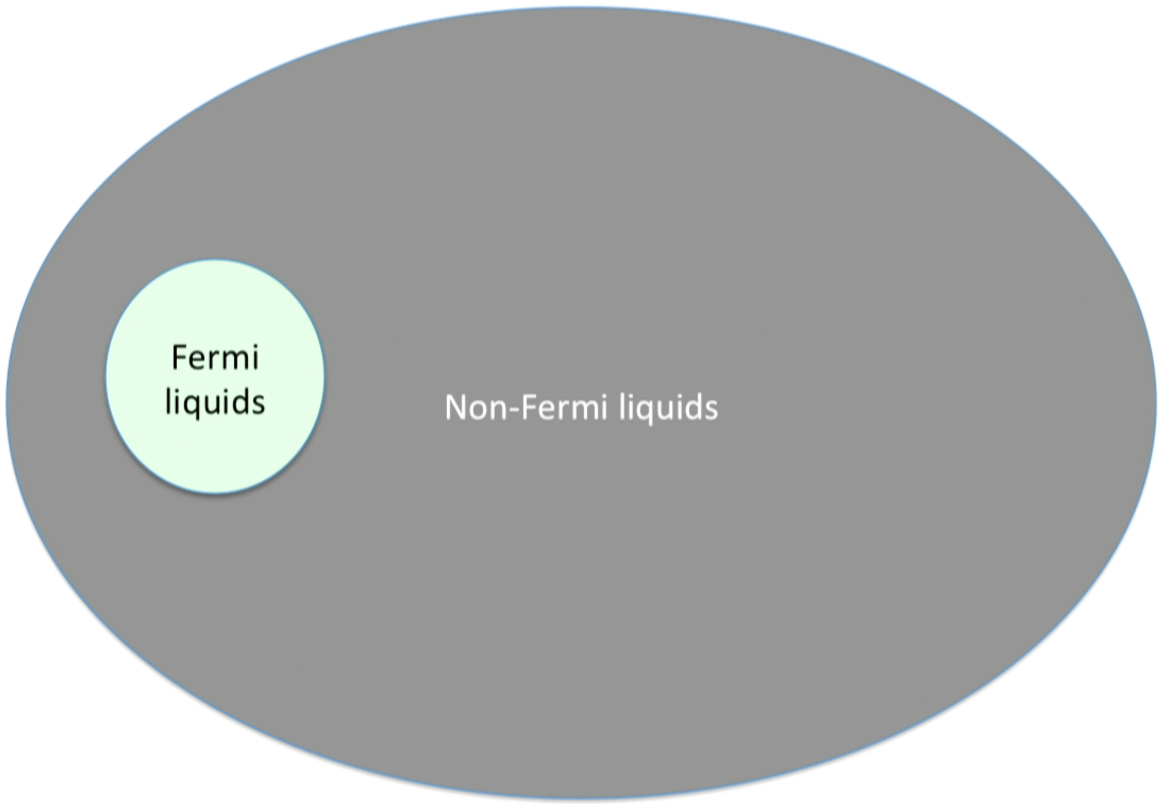
[Landau]

- In a class of metals, the low temperature properties of interacting fermions are remarkably similar to those of the non-interacting counterpart
  - Specific heat : C ~ T
  - Magnetic susceptibility : χ ~ const.
- Landau postulated that low energy eigenstates of interacting fermions are still labeled by single particle occupation numbers

$$|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \dots >'$$

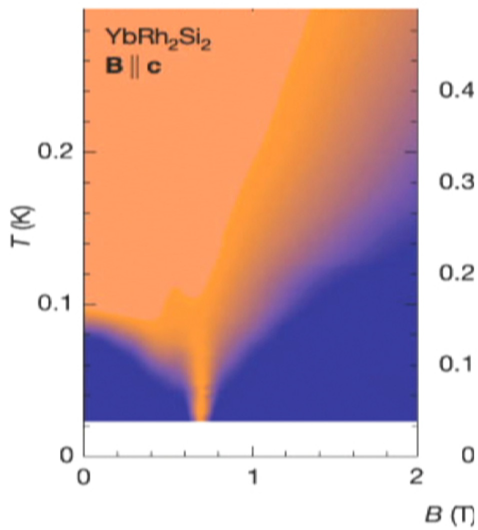
- The well-defined single-particle excitations are dressed fermions with renormalized mass but with the same charge and statistics : quasiparticles
- The quasiparticle paradigm has been very successful in explaining behaviors of simple metals

Landscape of metals

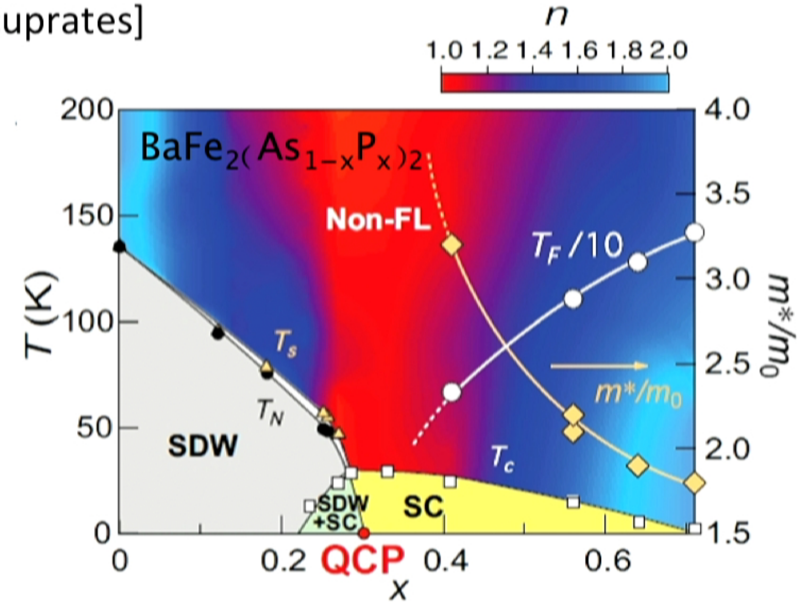


Breakdown of Fermi liquid near Quantum Critical Point

[heavy fermion; pnictides; cuprates]

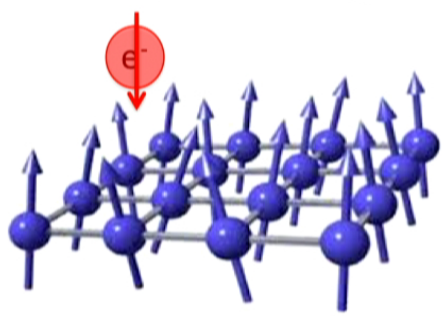


[Custers et al.(2003)]

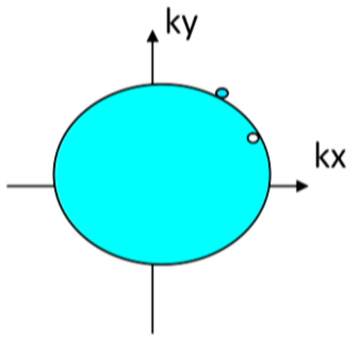


[Hashimoto et al. Science 336, 1554 (2012)]

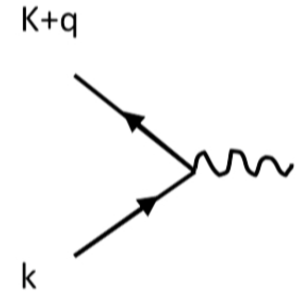
- At quantum critical point, order parameter fluctuations become gapless
- Specific heat : C ~ T log(1/T),
- Resistivity : ρ ~ T^n , n < 2



Soft collective mode cause fluctuations of FS



Fermi surface
+ gapless boson



- Non-forward scatterings are enhanced by collective modes
- Bare fermion decays into a complicated superposition of multi-particle states

Wanted : theories that replace the Fermi liquid theory

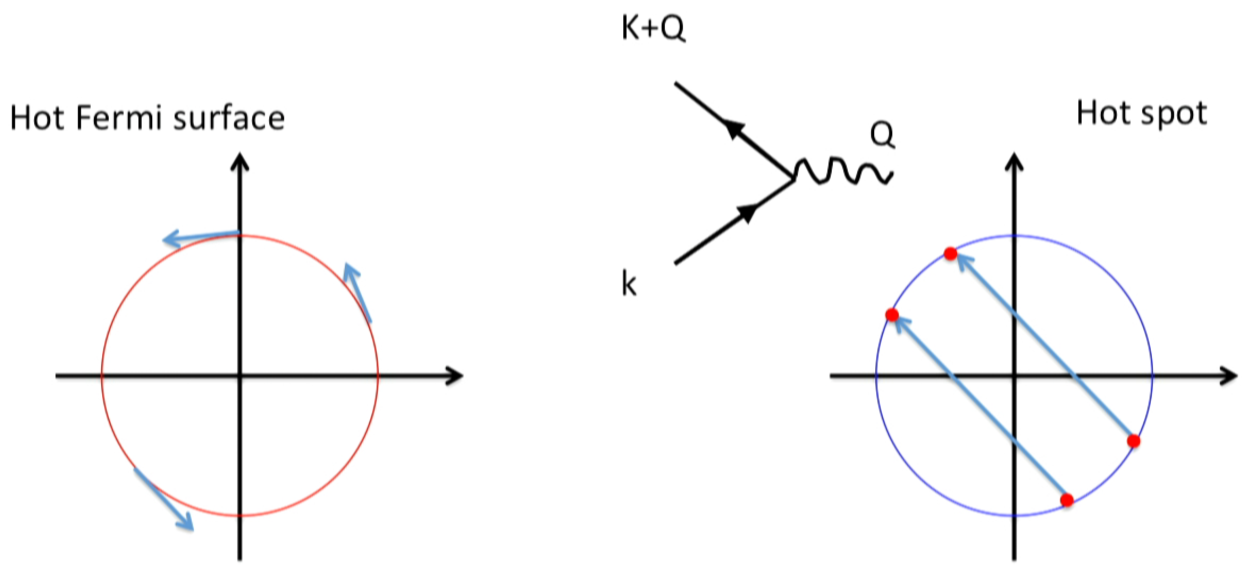
Non-Fermi liquids in 2+1D

- Most interesting dimensions :
  - Extended Fermi surface
  - Strong quantum fluctuations at low energies
- In general, a small parameter is needed to study the theory in a controlled way
- Recently, non-perturbative solutions are found for some cases

Some surprises for theories of NFL

- NFL with N vector flavors behaves as a matrix model in the large N limit
  - Angle around Fermi surface effectively plays the role of an additional internal index, promoting vector field to matrix field
- Weak coupling expansion near the upper critical dimensions ≠ Loop expansion
  - High loop graphs are enhanced by IR singularities
  - Expansion in fractional powers of coupling

Critical surface vs Hot spots



Q=0
Nematic, ferromagnetic QCP
Spin liquids with emergent gauge boson

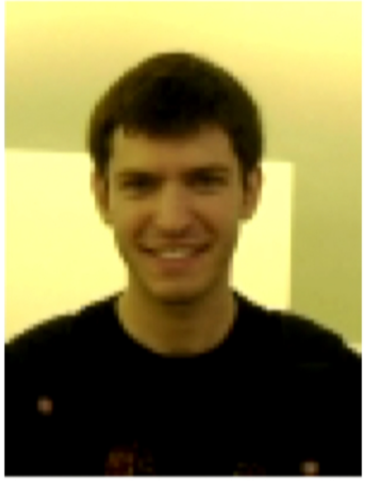
Q≠0
Spin & CDW QCP

Antiferromagnetic quantum critical metal

Shouvik Sur

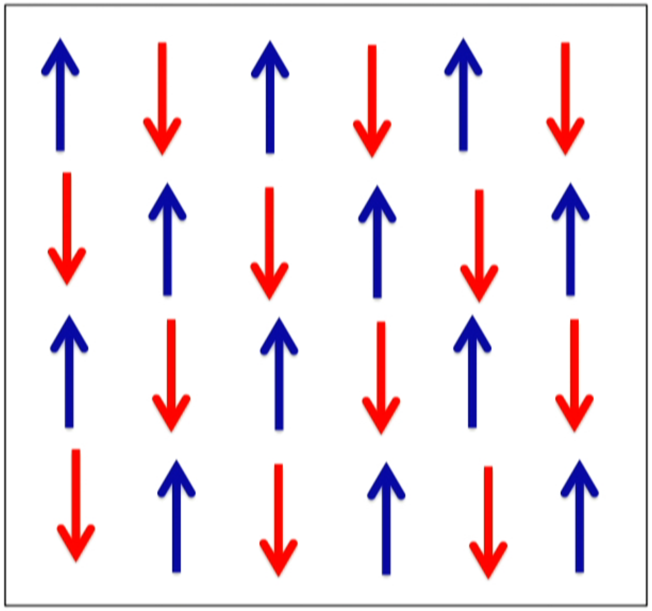
(McMaster-> Tallahassee)

Andres Schlief



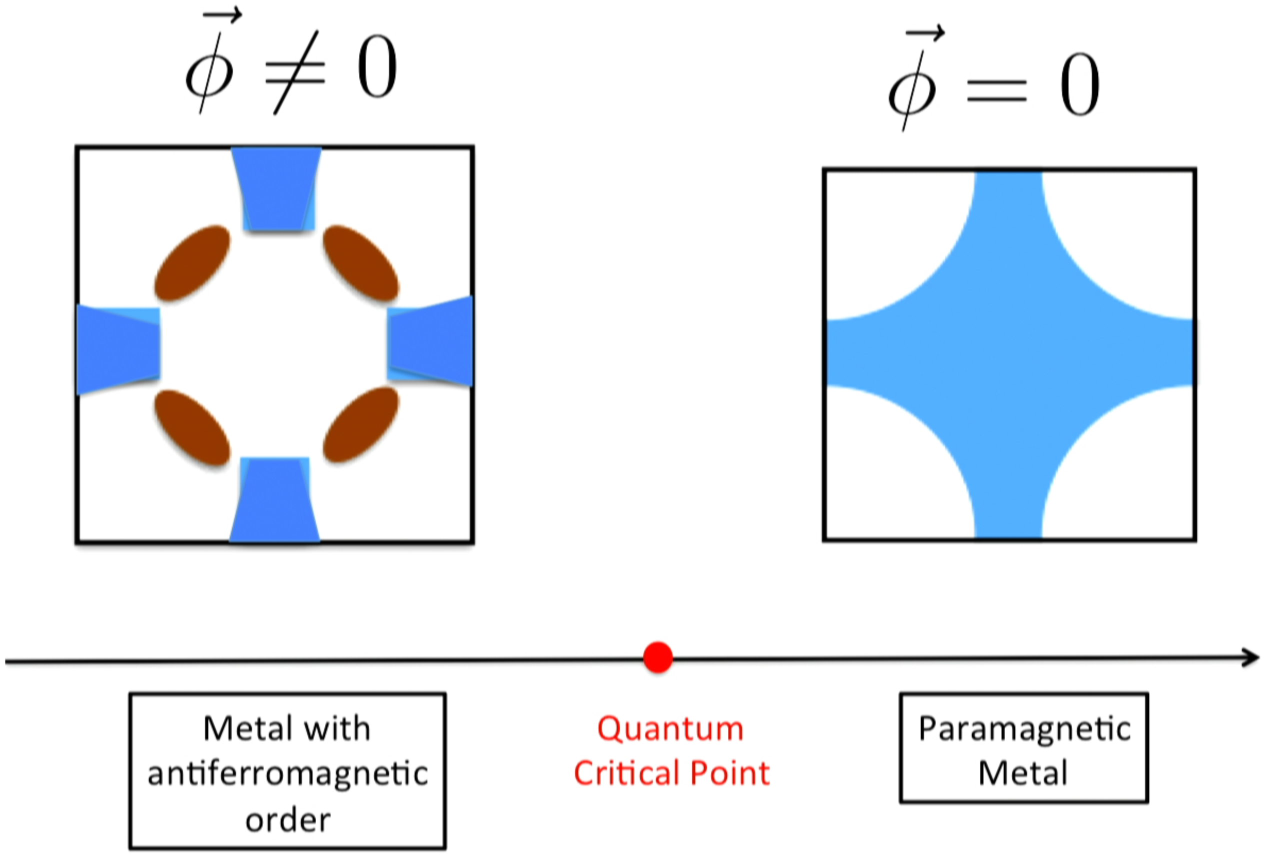
Peter Lunts

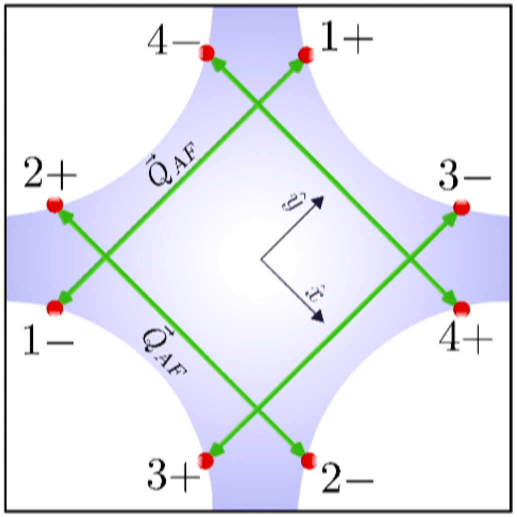
Antiferromagnetic Order



$$< \vec{S}(\vec{r}) >= \vec{\phi}\, e^{i\vec{Q}\cdot\vec{r}}$$

Antiferromagnetic phase transition in metal





Minimal Theory for AF QCP in 2+1D

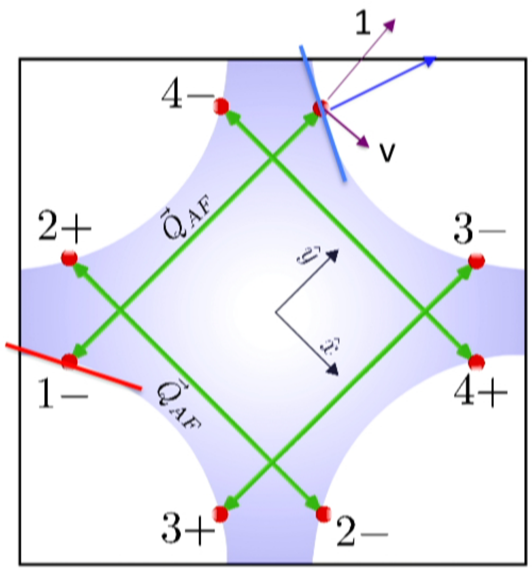
[Abanov, Chubukov]

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = v k_x \pm k_y$$

$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + v k_y$$

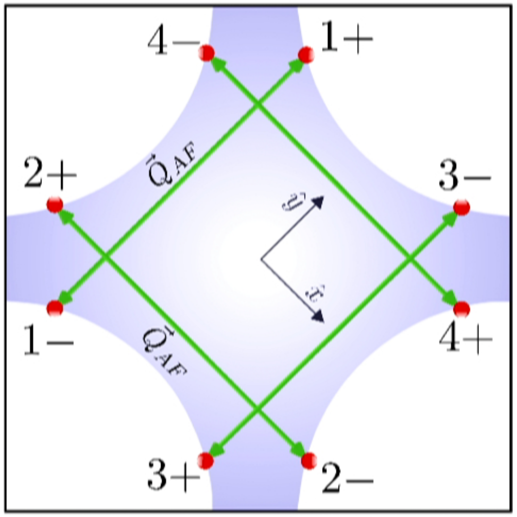
$$\begin{aligned}
\mathcal{S} = & \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3}\, \psi_{l,\sigma}^{(m)*}(k) \left[i k_0 + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k) \\
& + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q) \\
& + g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right] \\
& + \frac{u_0}{4!} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(k_1+q) \cdot \vec{\Phi}(k_2-q) \right] \left[\vec{\Phi}(-k_1) \cdot \vec{\Phi}(-k_2) \right]
\end{aligned}$$

Parameters of the theory



- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : coupling bet'n fermion and boson
- u : quartic boson coupling

- If v=0, hot spots connected by Q_{AF} are nested



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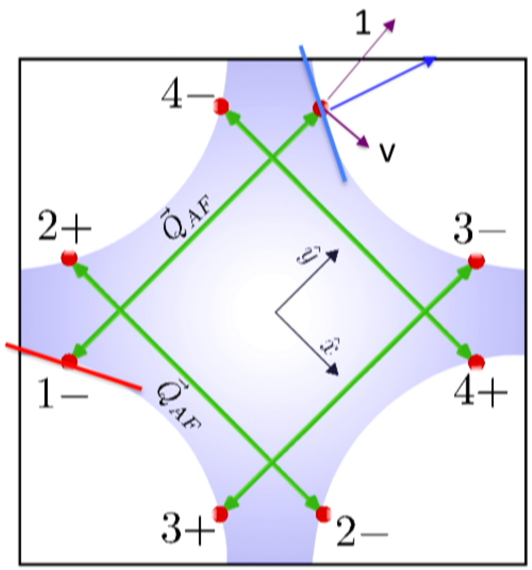

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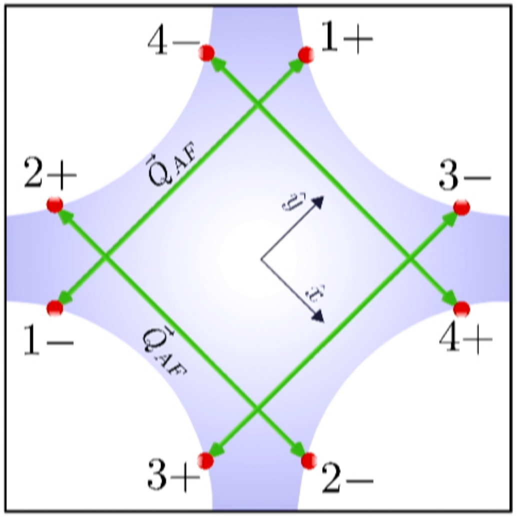

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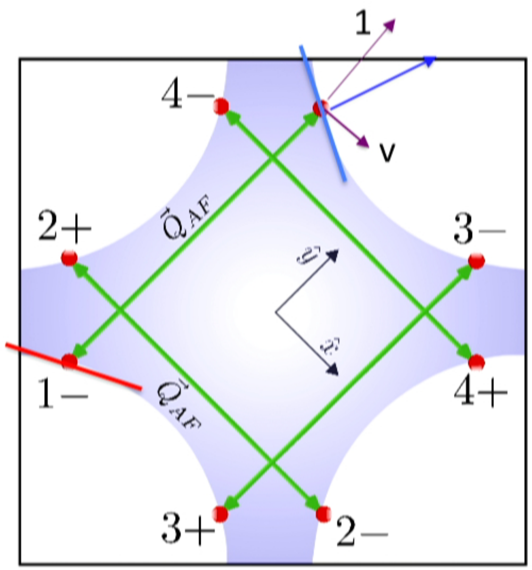

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Strong quantum fluctuations in 2+1D

$$\mathcal{S} = \sum_{l=1}^{4}\sum_{m=\pm}\sum_{\sigma=\uparrow,\downarrow}\int \frac{d^3k}{(2\pi)^3}\,\psi^{(m)*}_{l,\sigma}(k)\left[ik_0 + e^m_l(\vec{k})\right]\psi^{(m)}_{l,\sigma}(k)$$
$$+\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}\left[q_0^2 + c^2|\vec{q}|^2\right]\vec{\Phi}(-q)\cdot\vec{\Phi}(q)$$
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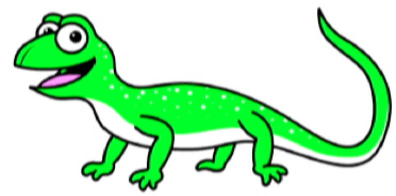
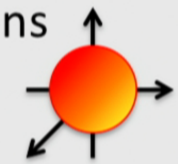
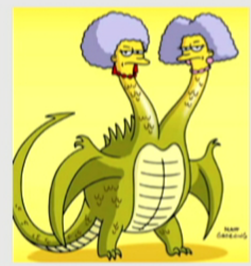
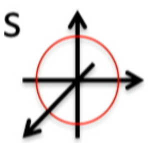
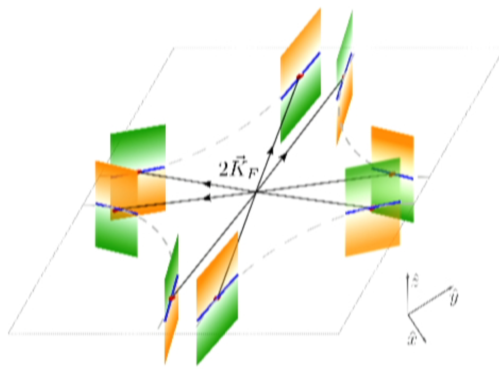
$$+\frac{u_0}{4!}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\left[\vec{\Phi}(k_1+q)\cdot\vec{\Phi}(k_2-q)\right]\left[\vec{\Phi}(-k_1)\cdot\vec{\Phi}(-k_2)\right]$$

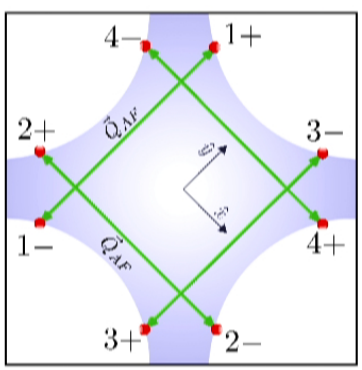
Interactions are relevant at the Gaussian fixed point

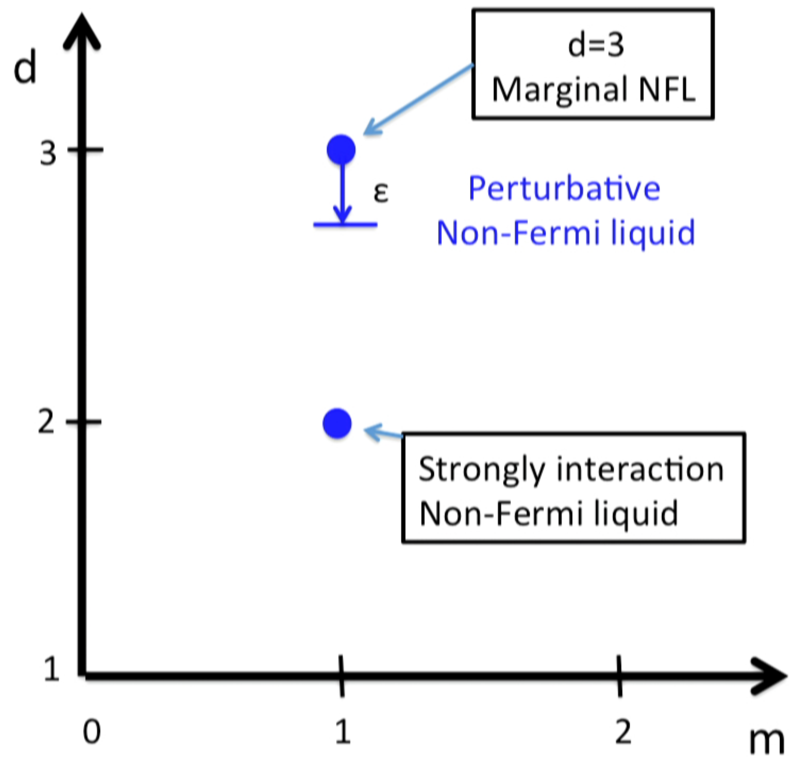
How to tame quantum fluctuations

Dynamical tuning	Modify the bare dispersion $\|q\|^2\phi^2 \rightarrow \|q\|^{1+\epsilon}\phi^2$	
Dim. reg.	Tune the number of dimensions 	
Co-dim. reg.	Tune the number of co-dimensions 	 √

A continuous interpolation between 2d Fermi surface and 3d metal with line nodes







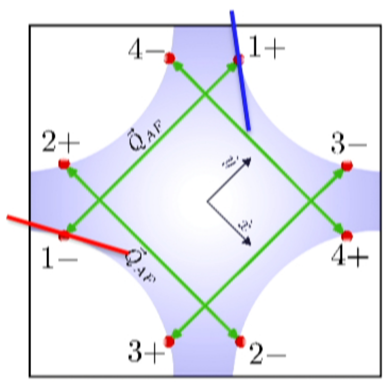
[Sur, Lee (14)]

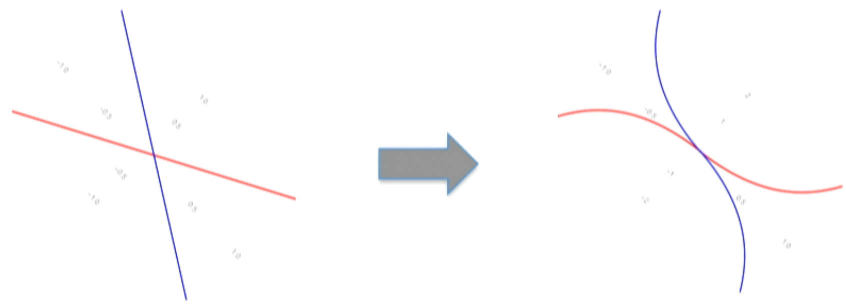
Lesson from the ε-expansion

[Sur, Lee (14); Lunts, Andres, Lee, in preparation]

- Emergent locality
  - Fermi surface is dynamically nested near the hot spots
  - the speed of the collective mode becomes zero
- Interaction is renormalized to form a balance with vanishing speed

$$v, c \to 0 \ \left(\frac{v}{c} \to 0\right), \quad g \to 0 \left(\frac{g^2}{v} \to O(\epsilon)\right)$$





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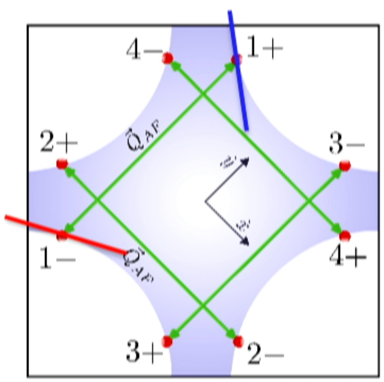

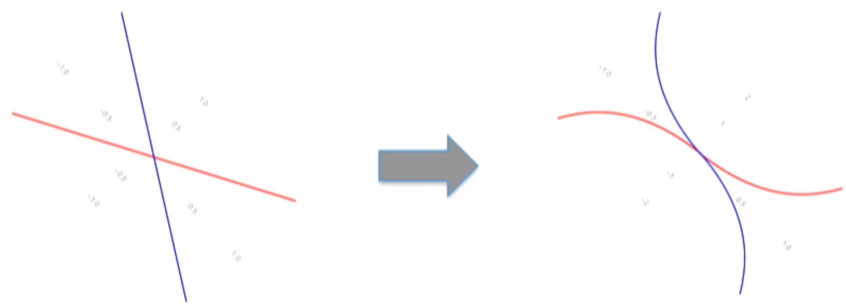

Plan to tackle 2+1D

- We will make an Ansatz by assuming that
  - Interaction plays the dominant role $g^2/v \sim 1$
  - Collective mode is damped by the particle-hole excitations
  - Emergent locality + Hierarchy in the velocities : $v << c << 1$

- Then we check that the Ansatz is self-consistent

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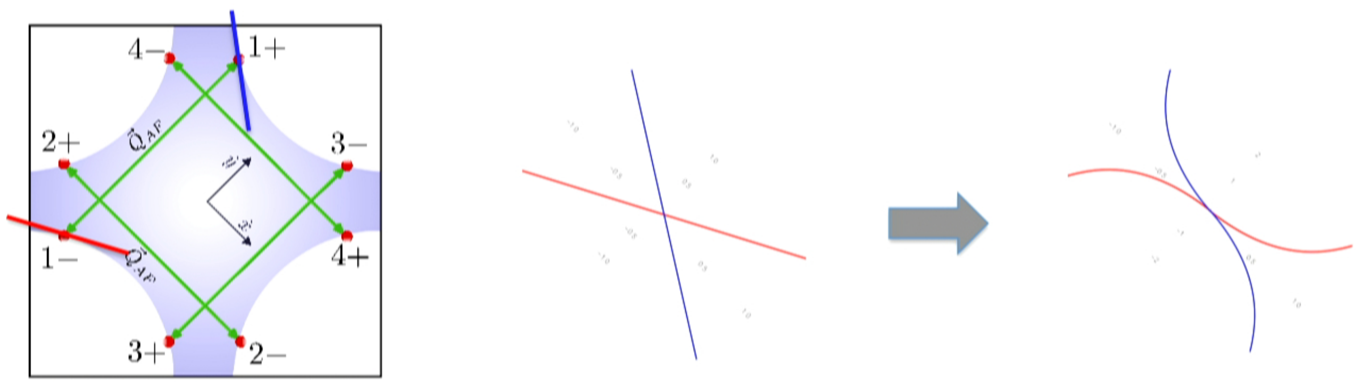

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Interaction driven scaling

[Sur, Lee (13)]

Scaling which leaves the fermion-boson coupling and fermion kinetic term marginal at the expense of dropping boson kinetic energy as irrelevant term

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+ \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

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$$+ \frac{u_0}{4!} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(k_1+q) \cdot \vec{\Phi}(k_2-q) \right] \left[\vec{\Phi}(k_1) \cdot \vec{\Phi}(k_2) \right]$$

Interaction driven scaling

$$[k_0] = [k_x] = [k_y] = 1,$$

$$[\psi(k)] = [\phi(k)] = -2.$$

- Keeping Yukawa coupling and fermion kinetic term as marginal operators uniquely fixes the scaling
- Electron keeps the classical scaling dimension
- Collective mode has anomalous dimension ½ compared to the classical scaling

Minimal action

$$\mathcal{S} \;=\; \sum_{n=1}^{4}\sum_{\sigma=\uparrow,\downarrow}\int dk\;\bar{\Psi}_{n,\sigma}(k)\Big[i\gamma_0 k_0 + i\gamma_1\varepsilon_n(\vec{k})\Big]\Psi_{n,\sigma}(k)$$

$$+i\sqrt{\frac{\pi v}{2}}\sum_{n=1}^{4}\sum_{\sigma,\sigma'}\int dkdq\;\Big[\bar{\Psi}_{\bar{n},\sigma}(k+q)\Phi_{\sigma,\sigma'}(q)\gamma_1\Psi_{n,\sigma'}(k)\Big]$$

- Entire boson action is irrelevant
- Yukawa coupling is rescaled such that $g^2/v \sim 1$
- v is the only dimensionless parameter

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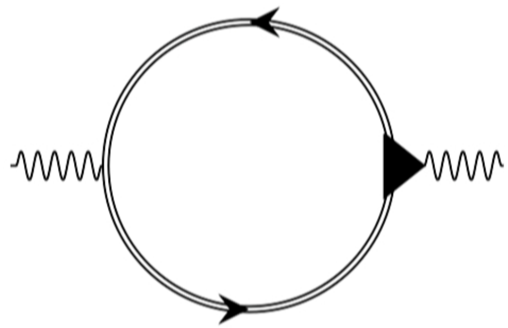
Self-consistent boson propagator



$$D(q)^{-1} = m_{CT} - \pi v \sum_n \int dk\ \mathrm{Tr}\left[\gamma_1 G_{\bar{n}}(k+q)\Gamma(k,q)G_n(k)\right]$$

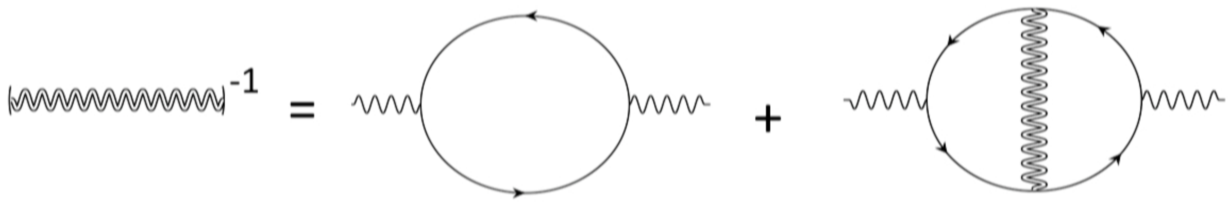
- In general, it is hard to solve the self-consistent equation because G(k), Γ(k,q) depend on D(q)

Plan to tackle 2+1D

- We will make an Ansatz by assuming that
  - Interaction plays the dominant role $g^2/v \sim 1$
  - Collective mode is damped by the particle-hole excitations
  - Emergent locality + Hierarchy in the velocities : $v << c << 1$

- Then we check that the Ansatz is self-consistent

Small v limit



$$D(q)^{-1} = |q_0| + c(v)\Big[|q_x| + |q_y|\Big],$$

$$c(v) = \frac{1}{4}\sqrt{v\log(1/v)}$$

- Boson propagator is entirely generated from particle-hole fluctuations
- v << c << 1 in the small v limit

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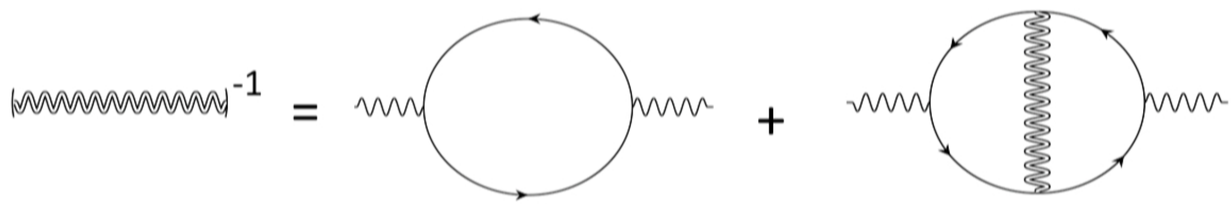

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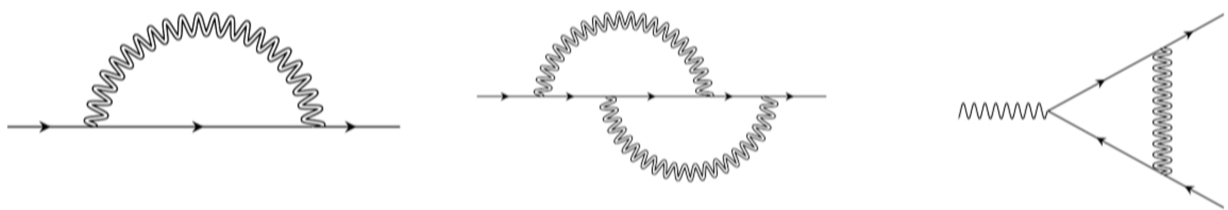
Flow of v

- In the small v limit, v indeed flows to zero in the low energy limit, which completes the cycle of self-consistency



$$\frac{dv}{d\ln\mu} = \frac{6}{\pi^2}\, v^2 \log\left(\frac{1}{c(v)}\right) \qquad v = \frac{\pi^2}{3}\left(\log\frac{1}{\mu}\ \ \log\log\frac{1}{\mu}\right)^{-1}$$

Critical exponents

$$z = 1 + \frac{3}{4\pi}\frac{v}{c(v)},$$

$$\eta_\phi = \frac{1}{4\pi}\frac{v}{c(v)}\log\left(\frac{c(v)}{v}\right),$$

$$\eta_\psi = -\frac{3}{8\pi}\frac{v}{c(v)}$$

- Deviation from the interaction driven scaling dimensions are controlled by v/c, which flows to zero
- This confirms that the interaction driven scaling is exact

[Andres, Lunts, Lee, 1608.06927]

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Critical exponents

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\begin{aligned}
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$$

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Spectral function near hot spots

$$G^R_{1+}(\omega,\vec{k}) = \frac{1}{F_\psi(\omega)\left[\omega\ F_z(\omega)\left(1+i\frac{\sqrt{3}\pi}{2}\frac{1}{\sqrt{\log\frac{1}{\omega}}\ \log\log\frac{1}{\omega}}\right)-\left(\frac{\pi^2}{3}\frac{k_x}{\log\frac{1}{\omega}\ \log\log\frac{1}{\omega}}+k_y\right)\right]}$$

$$F_\psi(\omega) = \left(\log\frac{1}{\omega}\right)^{\frac{3}{8}}, \quad F_z(\omega) = e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}$$

- The deviation from the Fermi liquid is stronger than marginal Fermi liquid due to slow decay of v/c

$$\frac{v}{c} = \frac{4\pi}{\sqrt{3}}\left(\log^{1/2}\frac{1}{\mu}\ \log\log\frac{1}{\mu}\right)^{-1}$$

Spin structure factor

$$D^R(\omega, \vec{q}) = \frac{1}{F_\phi(\omega)\left(-i\omega F_z(\omega) + \frac{\pi}{4\sqrt{3}}\frac{|q_x|+|q_y|}{\left(\log\frac{1}{\omega}\right)^{1/2}}\right)}$$

$$F_\phi(\omega) = e^{\frac{2}{\sqrt{3}}\left(\log\frac{1}{\omega}\right)^{1/2}}$$

- Spin fluctuations are strongly damped by particle-hole excitations

Specific heat

$$c \sim T F_z(T)(\log 1/T)^{1/2}$$

$$F_z(\omega) = e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}$$

- Collective mode dominates the specific heat (more important than the contribution from the cold electrons on the extended Fermi surface)
- Deviation from Fermi liquid is stronger than log T !
- More accurate experiments needed

Fate in the low T limit

- In the low T limit, superconductivity is expected to set in
- Electrons away from the hot spots support well-defined quasiparticles and are prone to BCS instability
- T_c will be parametrically enhanced near the critical point but T_c is non-universal : the temperature window for the critical scaling depends on specific materials

Summary

- Low energy effective theories for NFLs
- Co-dimensional reg. give perturbative NFLs
- Antiferromagnetic critical metal in 2+1D
  - Exact critical exponents are predicted based on a non-perturbative solution
  - Precise experiments to test the predictions are needed

Spin structure factor

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