Title: Topological quantum mechanics and Higgs branches of 3d N=4 theories

Date: Dec 06, 2016 02:30 PM

URL: http://pirsa.org/16120005

Abstract: <p>3d N=4 theories on the sphere have interesting supersymmetric sectors described by 1d QFTs and defined as the cohomology of a certain supercharge. One can define such a 1d sector for the Higgs branch or for the Coulomb branch. We study the Higgs branch case, meaning that the 1d QFT captures exact correlation functions of the Higgs branch operators of the 3d theory. The OPE of the 1d theory gives a star-product on the Higgs branch which encodes the data of these correlation functions. When the 3d theory is superconformal, the 1d theory is topological and coincides with the known construction in flat space, where the topological 1d theory lives in the cohomology of Q+S. Our construction thus generalizes it away from the conformal point. We then focus on theories constructed from vector and hypermultiplets. Using supersymmetric localization, we explicitly describe their 1d sector as the gauged topological quantum mechanics, or equivalently a gaussian theory coupled to a matrix model. This provides a very simple technique to compute the Higgs branch correlators.</p>

Topological quantum mechanics and Higgs branches of 3d N=4 theories

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December 6, 2016, Perimeter Institute

Based on work with Silviu Pufu and Ran Yacoby, arXiv:1610.00740

Goal

Background: chiral algebras in the cohomology of 4d $\mathcal{N} = 2$ and 6d $\mathcal{N} = (2, 0)$ SCFTs (Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees); topological 1d sector in 3d $\mathcal{N} = 4$ SCFTs (Chester, Lee, Pufu, Yacoby).

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Method:

- Put theory on S^3.
- Apply supersymmetric localization.

Outline

- Cohomological reduction in SCFTs.
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- Applications.

Cohomological reduction in SCFT

- 3d $\mathcal{N} = 4$ superconformal symmetry is described by $\mathfrak{osp}(4|4)$ ($\mathfrak{osp}(4|2)$ in other notations).
- It has $\mathfrak{sp}(4)$ (alternatively $\mathfrak{sp}(2)$) $\cong \mathfrak{so}(5)$ conformal subalgebra, $\mathfrak{so}(4) \cong \mathfrak{su}(2)_H \oplus \mathfrak{su}(2)_C$ R-symmetry and Q and S supersymmetries.

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- Consider linear combinations (r has dimension of length):

$$\mathcal{Q}_1^H = Q_{11\dot{2}} + \frac{1}{2r} S^2{}_{2\dot{2}}\,, \quad \mathcal{Q}_2^H = Q_{21\dot{1}} + \frac{1}{2r} S^1{}_{2\dot{1}}\,. \qquad (1)$$

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- They are nilpotent, i.e., $(\mathcal{Q}_1^H)^2 = (\mathcal{Q}_2^H)^2 = 0$, and satisfy $\{\mathcal{Q}_1^H, \mathcal{Q}_2^H\} = \frac{4i}{r}(M_{12} - \overline{R}_{\dot{1}}{}^{\dot{1}})$.

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- Also, $\{\mathcal{Q}_1^H, \mathcal{Q}_1^{H\dagger}\} = \{\mathcal{Q}_2^H, \mathcal{Q}_2^{H\dagger}\} = 8(D - R_1{}^1)$.

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- At the origin, the cohomology is represented by the operators with $D = R_1{}^1$, the Higgs branch chiral ring operators.

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- At the origin, the cohomology is represented by the operators with $D = R_1{}^1$, the Higgs branch chiral ring operators.
- One more fact: $-\frac{1}{4}\{\mathcal{Q}_1^H, Q_{22\dot{1}}\} = \frac{1}{4}\{\mathcal{Q}_2^H, Q_{12\dot{2}}\} = P_3 + \frac{i}{2r}R_2{}^1 = \widehat{P}_3$.

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- So, local operators in the cohomology of $\mathcal{Q}_i^H$ are given by the Higgs branch chiral ring operators twisted-translated along the line $(0, 0, x_3)$.

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- Compute it!
- Example of twisted translation: $Q(x_3) = q_a u^a$, $u = (1, x_3)$. At the origin, $Q(0) = q_1$ – the chiral ring operator with the appropriate choice of Cartan in $\mathfrak{su}(2)_H$.

Operators in the cohomology

- General Higgs branch operators are constructed as gauge invariant polynomials in the hypermultiplet scalars q_a and $\widetilde{q}^a$.
- Those which are polynomials in q_1 and $\widetilde{q}_1$ are chiral if we chose $\sigma_3 \in \mathfrak{su}(2)_H$ as a Cartan element.

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- Twisted-translated operators are therefore given by gauge invariant polynomials in $Q = q_1 + q_2 x_3$ and $\widetilde{Q} = \widetilde{q}_1 + \widetilde{q}_2 x_3$.
- There is a mirror construction of supercharges $\mathcal{Q}_i^C$, whose cohomology is related to Coulomb branch.

Stereographic projection

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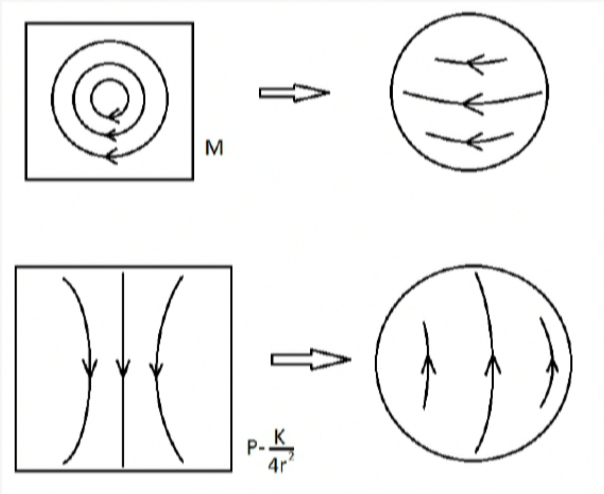
Let us put our theory on S^3 of radius r (the same r as before).

- Killing vectors and conformal Killing vectors are mixed in the process:



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- There is a subalgebra $\mathfrak{su}(2|1)_\ell \oplus \mathfrak{su}(2|1)_r \subset \mathfrak{osp}(4|4)$, parametrized by the choices of Cartan elements $h \in \mathfrak{su}(2)_H$ and $\overline{h} \in \mathfrak{su}(2)_C$.

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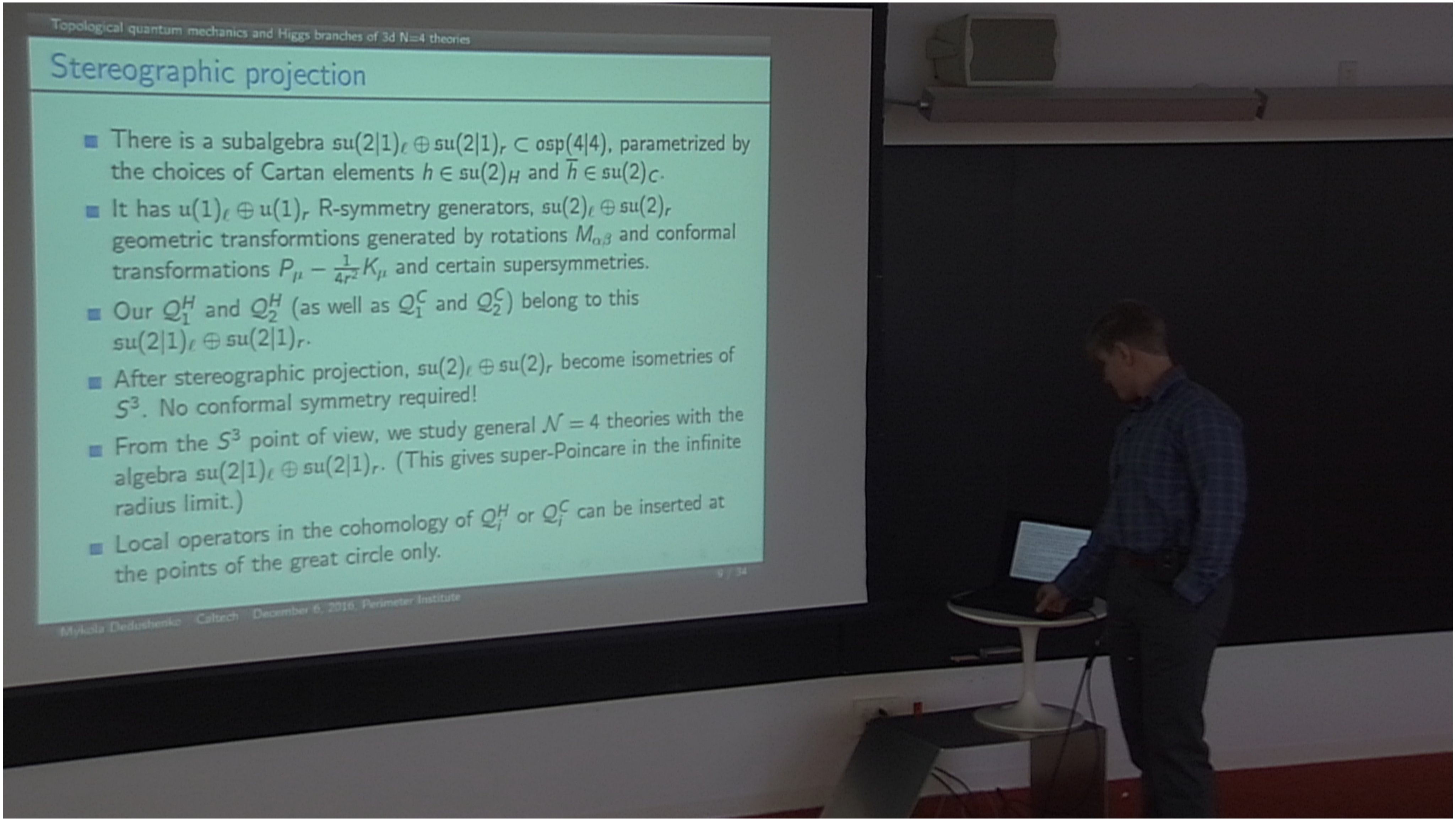
- From the S^3 point of view, we study general $\mathcal{N} = 4$ theories with the algebra $\mathfrak{su}(2|1)_\ell \oplus \mathfrak{su}(2|1)_r$. (This gives super-Poincare in the infinite radius limit.)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Local operators in the cohomology of Q_i^H or Q_i^C can be inserted at the points of the great circle only.
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Mykola Dedushenko Caltech December 6, 2016, Perimeter Institute

General setup

It is useful to consider linear combinations $\mathcal{Q}^H_\beta = \mathcal{Q}^H_1 + \beta \mathcal{Q}^H_2$ and $\mathcal{Q}^C_\beta = \mathcal{Q}^C_1 + \beta \mathcal{Q}^C_2$.
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(There is a Coulomb branch analog for everything)

Twisted rotations

A useful coordinate system on S^3 views it as a $U(1)$ fibration over the disk D^2 with the fibers shrinking at the boundary of D^2. Radial coordinate on the disk is $\theta \in [0, \pi/2]$ and angular coordinate is $\varphi \in [0, 2\pi)$. Coordinate along the fiber is $\tau \in [0, 2\pi)$. The metric is:

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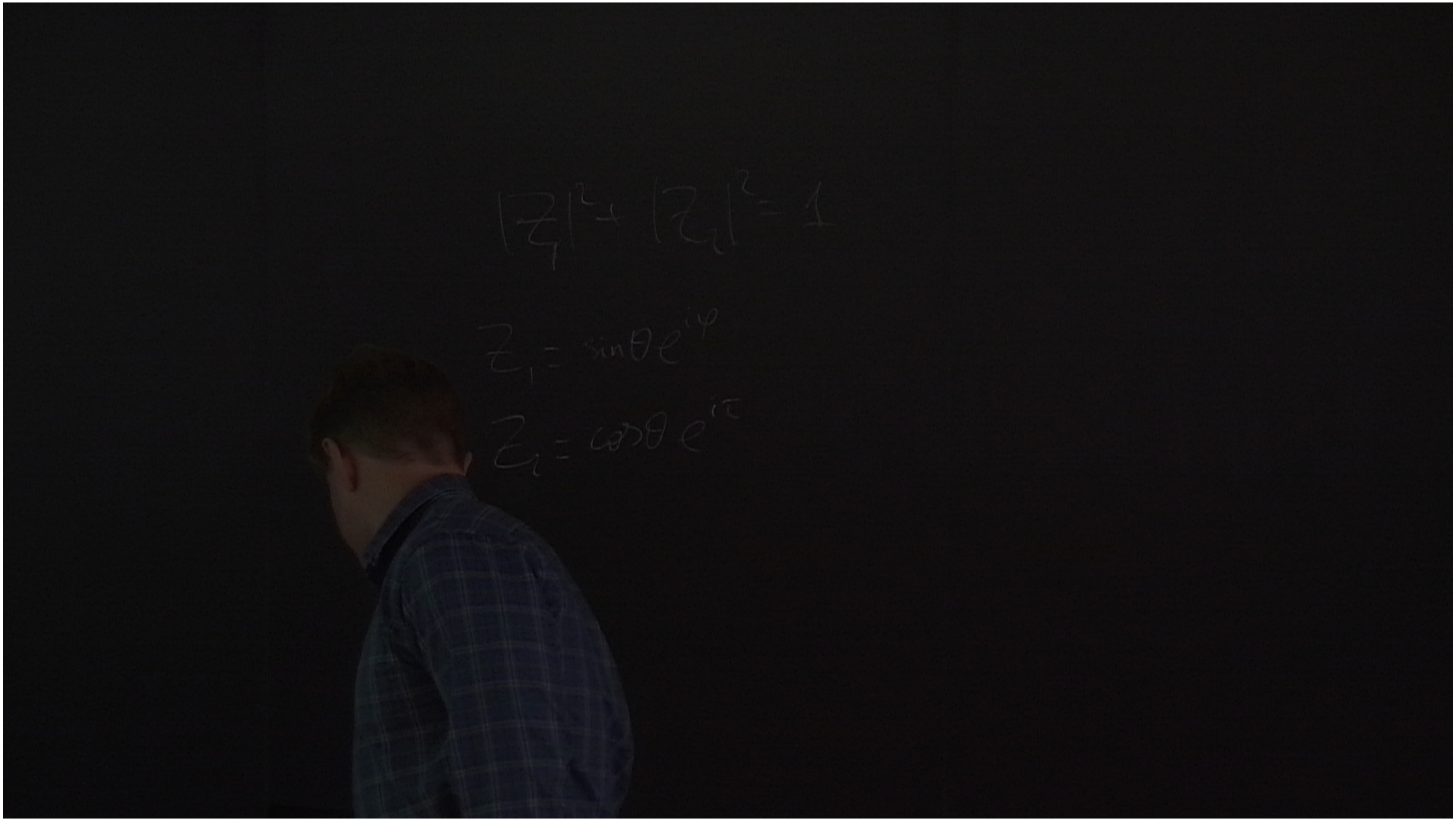
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- Both R_C and R_H are in $\mathfrak{su}(2|1)_\ell \oplus \mathfrak{su}(2|1)_r$, namely $R_\ell = R_H + R_C$ and $R_r = R_H - R_C$ generate $\mathfrak{u}(1)_\ell \oplus \mathfrak{u}(1)_r$.

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- Both R_C and R_H are in $\mathfrak{su}(2|1)_\ell \oplus \mathfrak{su}(2|1)_r$, namely $R_\ell = R_H + R_C$ and $R_r = R_H - R_C$ generate $\mathfrak{u}(1)_\ell \oplus \mathfrak{u}(1)_r$.
- Stereographic map matches twisted-translated operators on $R^1 \subset R^3$ to twisted-rotated operators on $S^1 \subset S^3$.
- $Q(\varphi) = q_1 \cos\frac{\varphi}{2} + q_2 \sin\frac{\varphi}{2}, \quad \widetilde{Q}(\varphi) = \widetilde{q}_1 \cos\frac{\varphi}{2} + \widetilde{q}_2 \sin\frac{\varphi}{2}.$

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$\mathcal{N} = 4$ theories on S^3

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- Vector multiplets in the adjoint of G and hypermultiplets in a representation $\mathcal{R}$.
- Action for hypers differs from the flat space action by curvature corrections and is invariant under the full $\mathfrak{osp}(4|4)$:

$$S_{\text{hyper}}[\mathcal{H},\mathcal{V}] = \int d^3x\sqrt{g}\,\Big[D^\mu \widetilde{q}^a D_\mu q_a - i\widetilde{\psi}^{\dot a} \not{D}\psi_{\dot a} + \frac{3}{4r^2}\widetilde{q}^a q_a + i\widetilde{q}^a D_a{}^b q_b - \frac{1}{2}\widetilde{q}^a \Phi^{\dot a\dot b}\Phi_{\dot a\dot b} q_a - i\widetilde{\psi}^{\dot a}\Phi_{\dot a}{}^{\dot b}\psi_{\dot b} + i\left(\widetilde{q}^a\lambda_a{}^{\dot b}\psi_{\dot b} + \widetilde{\psi}^{\dot a}\lambda^b{}_{\dot a} q_b\right)\Big]\,. \quad (4)$$

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$$- D^{cd}D_{cd} - i\lambda^{a\dot a}[\lambda_a{}^{\dot b},\Phi_{\dot a\dot b}] - \frac{1}{4}[\Phi^{\dot a}{}_{\dot b},\Phi^{\dot c}{}_{\dot d}][\Phi^{\dot b}{}_{\dot a},\Phi^{\dot d}{}_{\dot c}]$$
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- Can add masses by adding a background vector multiplet gauging the Cartan of flavor symmetry.
- $\Rightarrow$ Central extension of $\mathfrak{su}(2|1)_\ell \oplus \mathfrak{su}(2|1)_r$.

A remark

Any $\mathfrak{su}(2|1) \oplus \mathfrak{su}(2)$ subalgebra of $\mathfrak{su}(2|1)_\ell \oplus \mathfrak{su}(2|1)_r$ describes $\mathcal{N} = 2$ SUSY on S^3. From the point of view of any such subalgebra, $S_{\rm YM}[\mathcal{V}]$ describes an $\mathcal{N} = 2$ vector plus an adjoint $\mathcal{N} = 2$ chiral of R-charge 1. The action $S_{\rm YM}[\mathcal{V}]$ is then the standard one appearing in the $\mathcal{N} = 2$ literature.

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$$\mathcal{I} = \frac{1}{|\mathcal{W}|}\int_{\text{Cartan}} d\sigma\, \det{}'_{\text{adj}}\left[2\sinh(\pi\sigma)\right]\int D\mathcal{H} e^{-S_{\text{hyper}}[\mathcal{H},\mathcal{V}_{\text{loc}}(\sigma)]}(\dots)\,, \tag{6}$$

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- Important: $S_{\text{hyper}}[\mathcal{H}, \mathcal{V}_{\text{loc}}(\sigma)]$ is Gaussian in $\mathcal{H}$.

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- The simplest proof: check that it gives the same partition function and correlators of $\mathcal{Q}^H_\beta$-closed operators as the 3d Gaussian theory on the previous slide. (Warning: integration cycle!)

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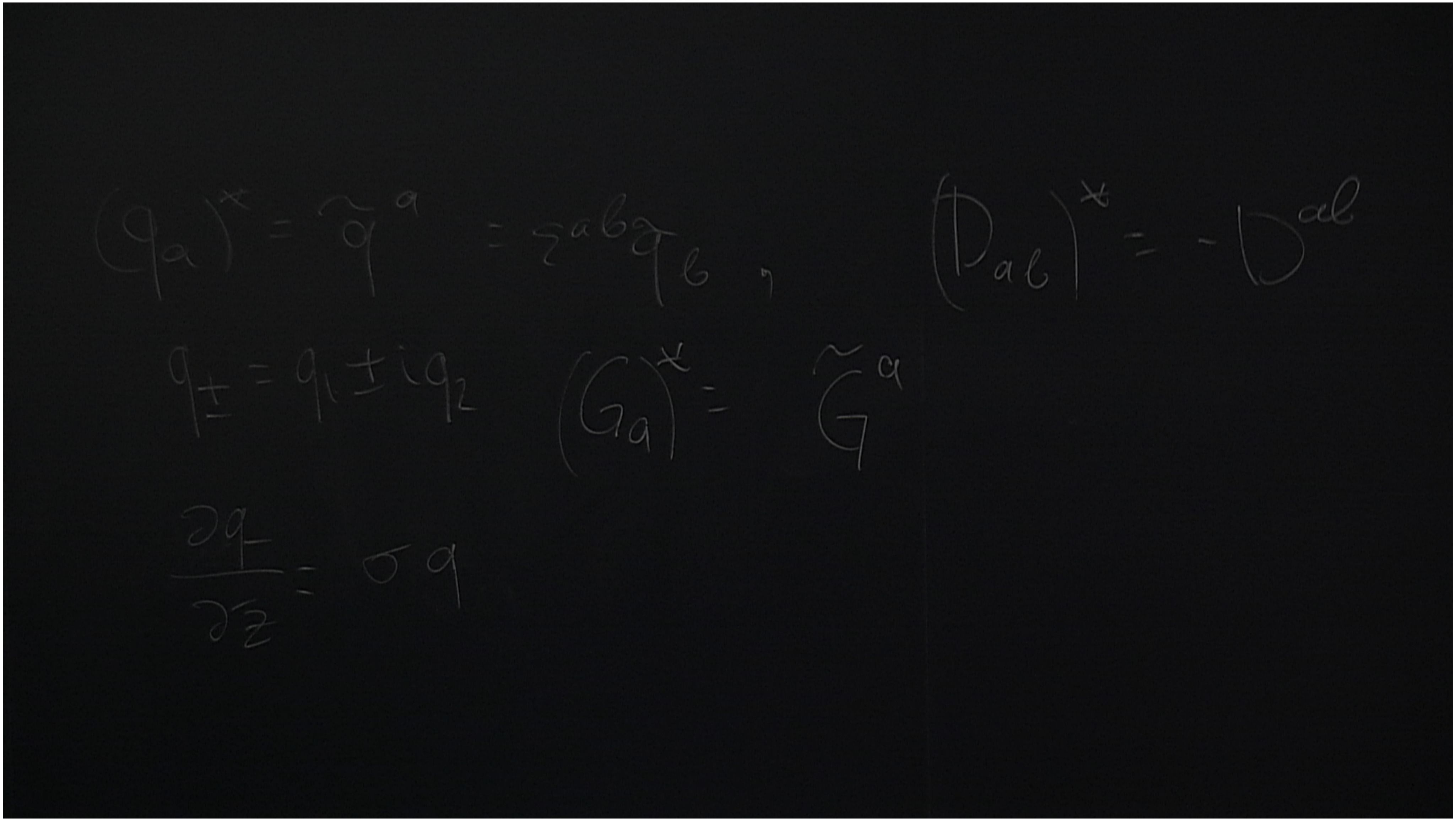
$(q_a)^* = \tilde{q}^a = \varepsilon^{ab} q_b ,$
$(D_{ab})^* = - D^{ab}$
$q_\pm = q_1 \pm i q_2$
$(G_a)^* = \tilde{G}^a$
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- $\Rightarrow \int DQD\widetilde{Q}$ goes over the non-trivial middle-dimensional integration cycle in the space of complex fields Q, $\widetilde{Q}$.
- Classical action: 3d action $\to$ action on D^2 is total derivtive $\to$ 1d action $S_\sigma[Q, \widetilde{Q}]$ on ∂D^2.

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- Fourier expansions:

$$Q(\varphi) = \sum_{n\in\mathbb{Z}+\frac{1}{2}} c_n e^{in\varphi}, \quad \widetilde{Q}(\varphi) = \sum_{n\in\mathbb{Z}+\frac{1}{2}} \widetilde{c}_n e^{-in\varphi}. \tag{9}$$

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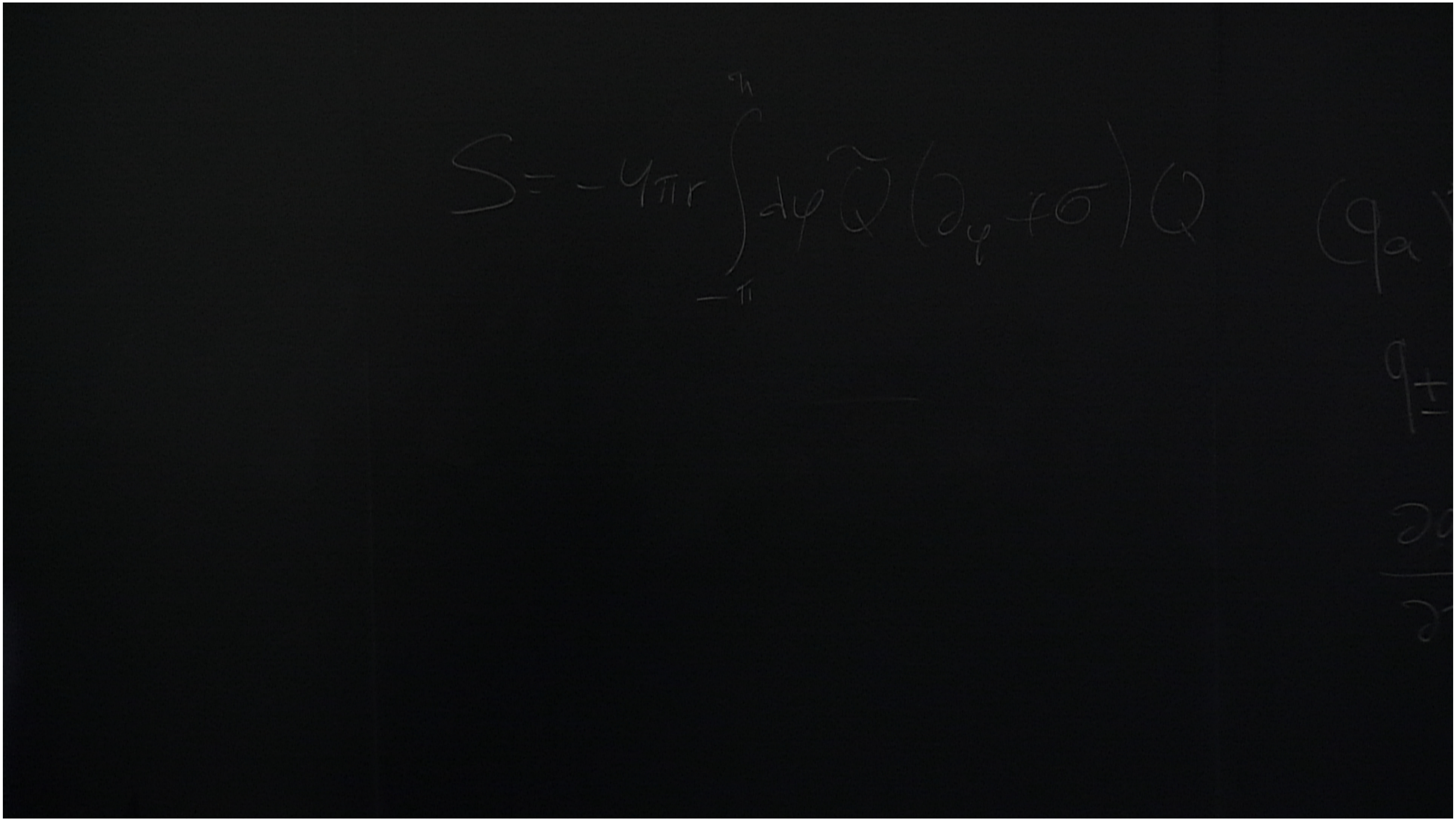
- Localization gives particular value of this phase, $e^{i\alpha_n(\sigma)} = \frac{1}{i}\frac{I_{n+\frac{1}{2}}(\sigma) - iI_{n-\frac{1}{2}}(\sigma)}{I_{n+\frac{1}{2}}(\sigma) + iI_{n-\frac{1}{2}}(\sigma)}$, but the contour of integration can be deformed to take a simpler form:

$$\widetilde{c}_n = i\,\mathrm{sgn}(n) c_n^*$$

Another remark

A cycle of integration can be found using Morse theory, as explained by Witten.
The cycle found here is very similar to his cycle, with the only difference that our fields are anti-periodic on the circle.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Gauged topological quantum mechanics

- Our 1d theory can be reinterpreted as a 1d gauge theory.
- Namely, the factor $\det'_{\rm adj}\left[2\sinh(\pi\sigma)\right]$ can be interpreted as the Faddeev-Popov determinant appearing in gauge fixing of the following action:

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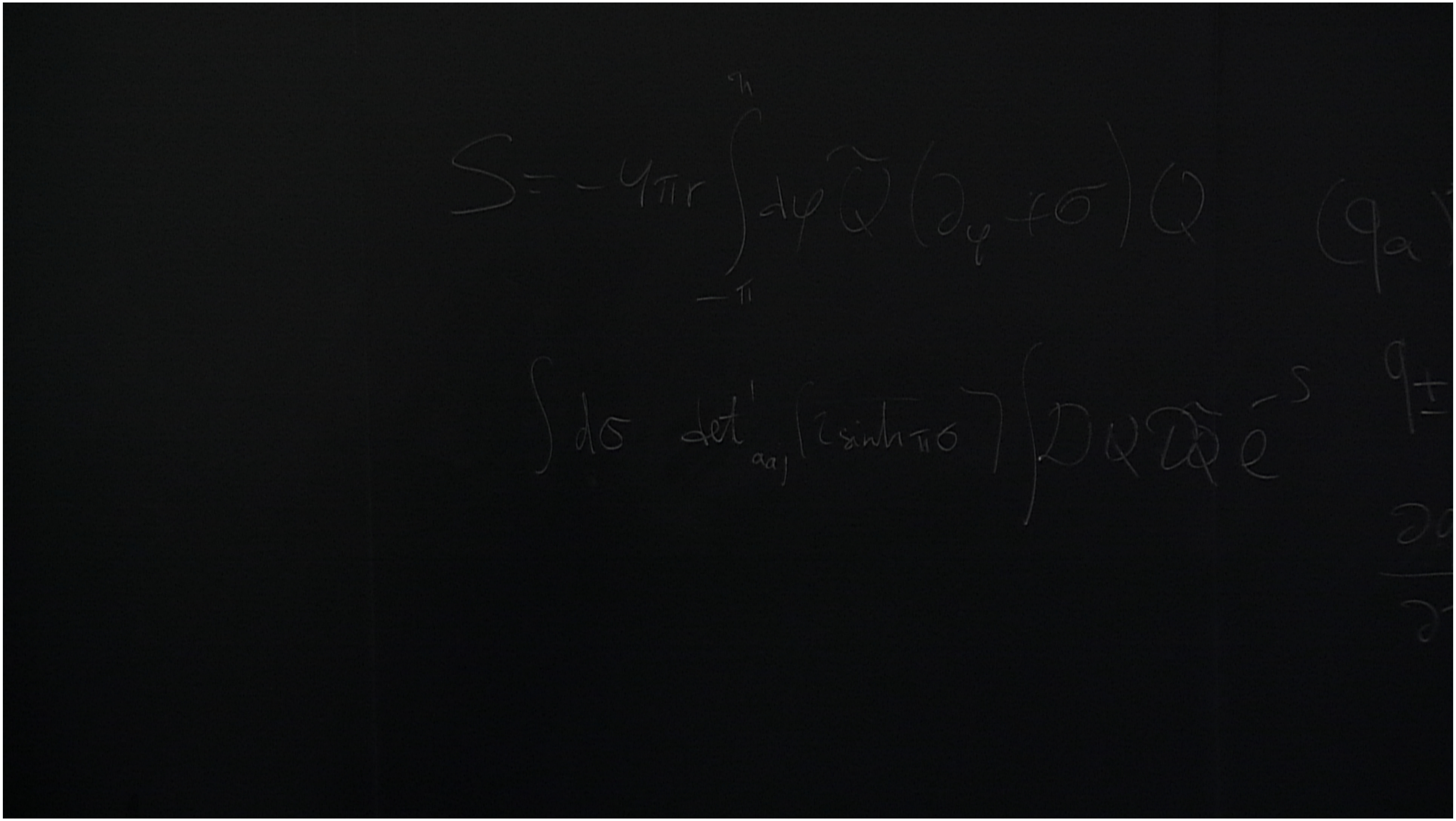

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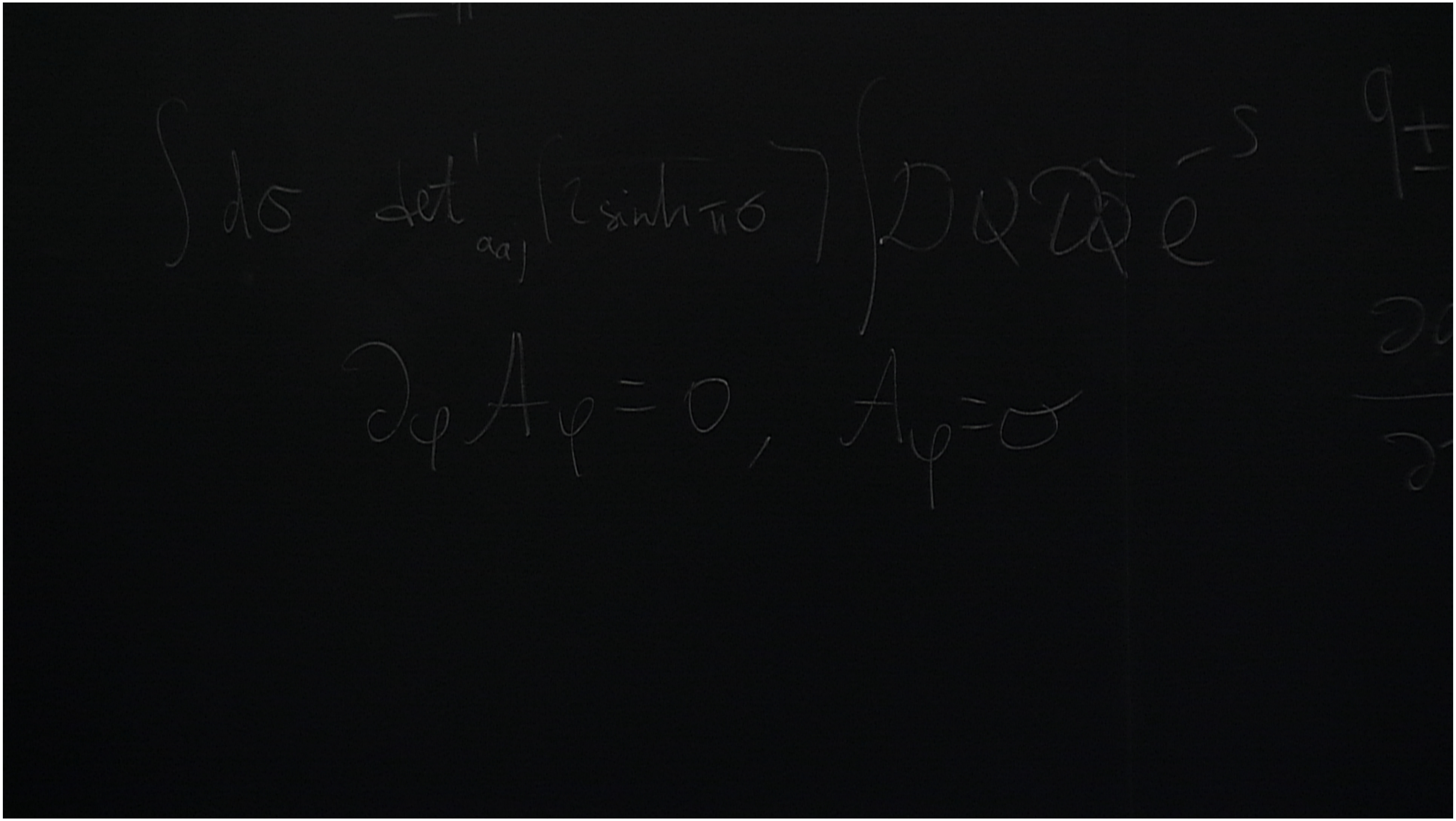
(q_a)
$\int d\sigma \, \det'_{adj} (2 \sinh \pi\sigma) \int \mathcal{D}Q \mathcal{D}\tilde{Q}\, e^{-S}$
$q_\pm$


∫dσ det'_adj [2 sinh πσ] ∫DQ DQ̃ e^-S
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- This is somewhat non-trivial and requires further studies.
- The reason is that this gauge theory is analytically continued, which is reflected in the absence of i in front of $\mathcal{A}_\varphi$ in D_φ.

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Such a picture makes certain aspects more transparent.

- The theory is topological (no metric).
- $\Rightarrow$ correlators of gauge-invariant operators are topological.
- 1d gauge field $\mathcal{A}_\varphi$ acts as a Lagrange multiplier imposing the D-term constraint $\widetilde{Q}\mathcal{R}(T)Q = 0,\ T \in \mathfrak{g}$ on the Higgs branch.

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- In particular, operators of the form $\mathcal{O}\widetilde{Q}\mathcal{R}(T)Q$ are trivial, i.e., given by linear combinations of lower dimension operators, as expected.

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- Turning on mass matrix m in the Cartan of the flavor symmetry corresponds to replacing $\sigma \to \sigma + rm$ in the gauge-fixed 1d action $S_\sigma[Q, \widetilde{Q}]$.
- In the 1d gauge theory this is $\mathcal{D}_\varphi \to \mathcal{D}_\varphi + rm$.
- F.I. parameters correspond to $4\pi r \int_{-\pi}^{\pi} d\varphi i \mathrm{tr}_\zeta \sigma$, where $\mathrm{tr}_\zeta \sigma = \sum_a \zeta_a \sigma_a$ goes over abelian factors in G.
- In the 1d gauge theory this is $4\pi r i \int \mathrm{tr}_\zeta \mathcal{A}$, the 1d analog of analytically continued Chern-Simons.

Masses and F.I. terms

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Masses and F.I. terms

- Correlation functions of twisted Higgs branch operators are still topological in the presence of F.I. terms.
- In the presence of masses, operators with non-zero flavor charges have non-topological correlation functions.
- Their position-dependence is dictated by flavor charges.
- As a 3d teory flows from the UV to the IR CFT, the 1d theory interpolates between two topological limits.

Applications

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- It computes n-point functions of twisted Higgs branch operators.
- For SCFTs, it encodes 2- and 3-point functions of arbitrary Higgs branch operators. (Simply using conformal invariance and R-symmetry)
- Gives star-product on the Higgs branch encoding the 1d operator algebra.
- F.I. and mass terms induce non-trivial deformations of the star-product.

Example: SQED with N flavors

- Fields of the 1d theory are Q^I and $\widetilde{Q}_I$ with charges $+1$ and -1.
- The path integral (with $\ell = -4\pi r$):

$$\int_{-\infty}^{\infty} d\sigma \int D\widetilde{Q}_I DQ^I \exp\left[-\ell \int d\varphi \left(\widetilde{Q}_I \partial_\varphi Q^I + \sigma \widetilde{Q}_I Q^I\right)\right]$$

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- The result is an algebra generated by the traceless bilinears

$$\mathcal{J}_I{}^J = \widetilde{Q}_I Q^J - \frac{1}{N}\delta_I^J \widetilde{Q}_K Q^K,$$

obeying $\mathcal{J}_I{}^J \mathcal{J}_J{}^K = 0$, which should be understood as relation in the chiral ring, which means that whatever definition we take for $\mathcal{J}_I{}^J \mathcal{J}_J{}^K$, this operator becomes redundant.

SQED with N flavors

- The other linearly independent operators are

$$\mathcal{J}_{I_1 I_2 \dots I_p}{}^{J_1 J_2 \dots J_p} \equiv \mathcal{J}_{(I_1}^{(J_1} \mathcal{J}_{I_2}^{J_2} \cdots \mathcal{J}_{I_p)}^{J_p)} - \text{traces}.$$

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- Introduce polarization vectors $(y^I, \overline{y}_I)$ obeying $\overline{y} \cdot y = 0$. It is convenient to introduce

$$\mathcal{J}^{(p)}(\varphi, y, \overline{y}) = \mathcal{J}_{I_1 I_2 \dots I_p}{}^{J_1 J_2 \dots J_p} y^{I_1} \dots y^{I_p} \overline{y}_{J_1} \cdots \overline{y}_{J_p},$$

and

$$\mathcal{J}(\varphi, y, \overline{y}) = \sum_{p=0}^{\infty} \ell^p \mathcal{J}^{(p)}(\varphi, y, \overline{y}).$$

SQED with N flavors

- A computation shows:

$$\langle \mathcal{J}(\varphi_1, y_1, \overline{y}_1)\mathcal{J}(\varphi_2, y_2, \overline{y}_2)\rangle$$
$$= {}_3F_2\left(\frac{N}{2}, \frac{N}{2}, 1; \frac{N}{2}\frac{N+1}{2}; -\frac{(\overline{y}_1 \cdot y_2)(y_1 \cdot \overline{y}_2)}{4}\right)$$

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$$= -\frac{N}{8\ell^3(N+1)}\left[(\overline{y}_1 \cdot y_3)(\overline{y}_3 \cdot y_2)(\overline{y}_2 \cdot y_1) - (\overline{y}_1 \cdot y_2)(\overline{y}_2 \cdot y_3)(\overline{y}_3 \cdot y_1)\right]$$

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- $\mathcal{J}_I{}^J \star \mathcal{J}_K{}^L = \mathcal{J}_{IK}{}^{JL} - \frac{1}{2\ell}\left(\delta_K^J\mathcal{J}_I{}^L - \delta_I^L\mathcal{J}_K{}^J\right) - \frac{N}{4\ell^2(N+1)}\left(\delta_I^L\delta_K^J - \frac{1}{N}\delta_I^J\delta_K^L\right)$

Turning on FI parameter

As a further illustration, consider SQED with N flavors in the presence of non-zero ζ.

Turning on FI parameter

- Star product:

$$\mathcal{J}_I{}^J \star \mathcal{J}_K{}^L = \mathcal{J}_{IK}{}^{JL} + \frac{i\zeta}{N+2}\Big(\delta_K^J \mathcal{J}_I{}^L + \delta_I^L \mathcal{J}_K{}^J - \frac{2}{N}(\delta_I^J \mathcal{J}_K{}^L + \delta_K^L \mathcal{J}_I{}^J)\Big)$$
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- All these relations identify the algebra as $\mathfrak{hs}_\lambda(\mathfrak{sl}(N))$ with $\lambda = -2i\zeta\ell/N$.

Further examples

More application can be found in the paper.

- N-node quiver with gauge group $U(1)^N/U(1)$ and N hypers with charges $(1, -1, 0, 0, \dots)$, $(0, 1, -1, 0, \dots)$, and so on. This is a mirror dual of the previous example.

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- For the $U(2)$ with adjoint and fundamental hypers flowing to $\mathcal{N}=8$ SCFT, and for the mass-deformed SQED – see paper.

Discussion

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- Doing this for the SQED, we can compare results with the N-node quiver and test mirror symmetry.
- One can identify Z of the N-node quiver with $\pm i\Phi/(8\pi)$ of the SQED.

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- Work in progress: include monopole operators.
- One has to be careful about boundary terms and understand how to compute determinants in the monopole background.
- More future directions: generalize to actions with twisted vectors and twisted hypers, in order to study more general gauge theories; include line and surface operators in the construction; study 4d $\mathcal{N} = 2$.

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The End

Thank you for your attention!